Introducing Holography

(a short review of gauge/gravity duality)

Francesco Nitti

Laboratoire APC & Universié Paris Cité

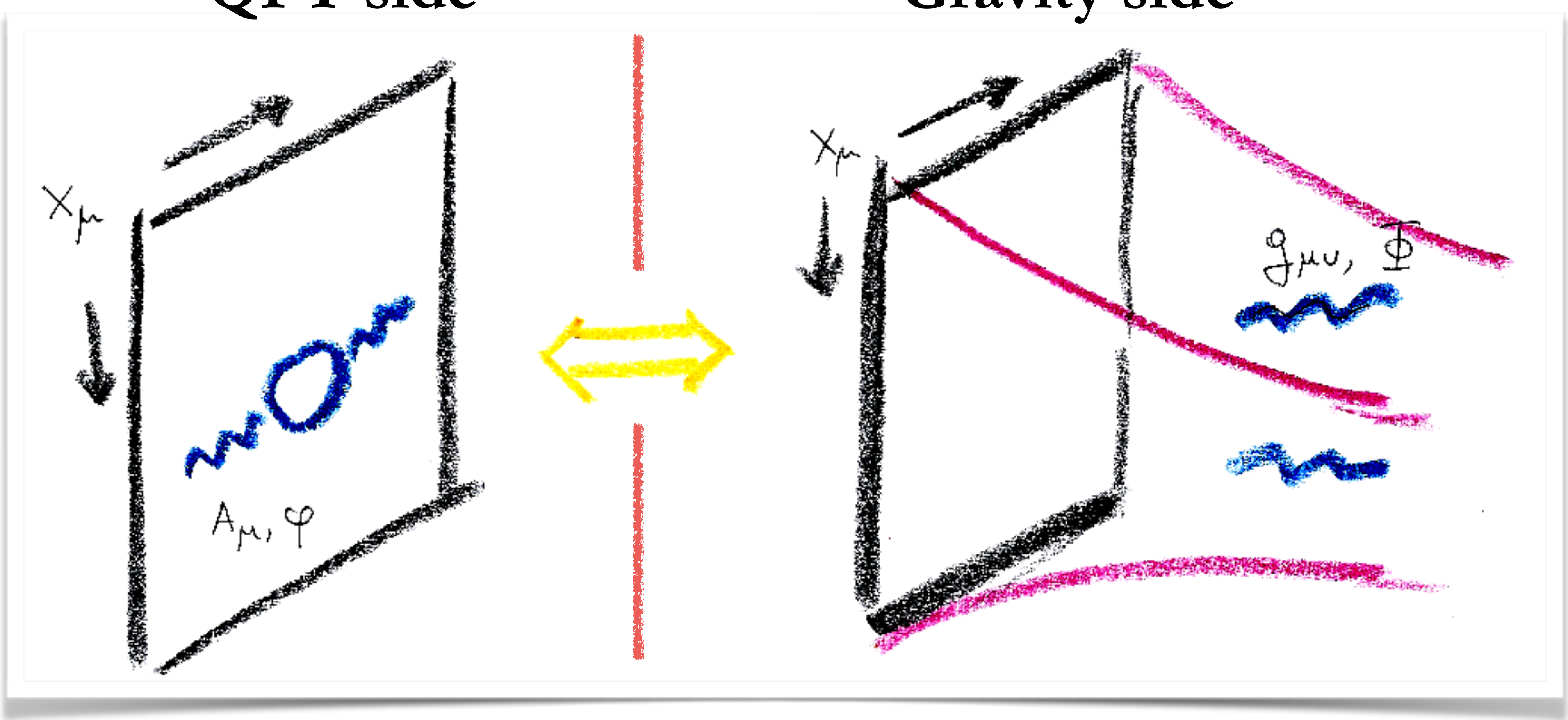
TUG, Montpellier, 4 Octobre 2022

Gauge/Gravity Duality

- In **1997** Maldacena (followed by Witten, Gubser, Klebanov, Polyakov and then many others) wrote a paper which changed the way we think about both **gravity** and **quantum field theory**
- He conjectured that a certain physical system can have two (very) different but equivalent descriptions:

1. As **quantum field theory** in **4d flat spacetime**
2. As a **gravitational theory** on a **curved spacetime 10d spacetime**

QFT side **Gravity side**



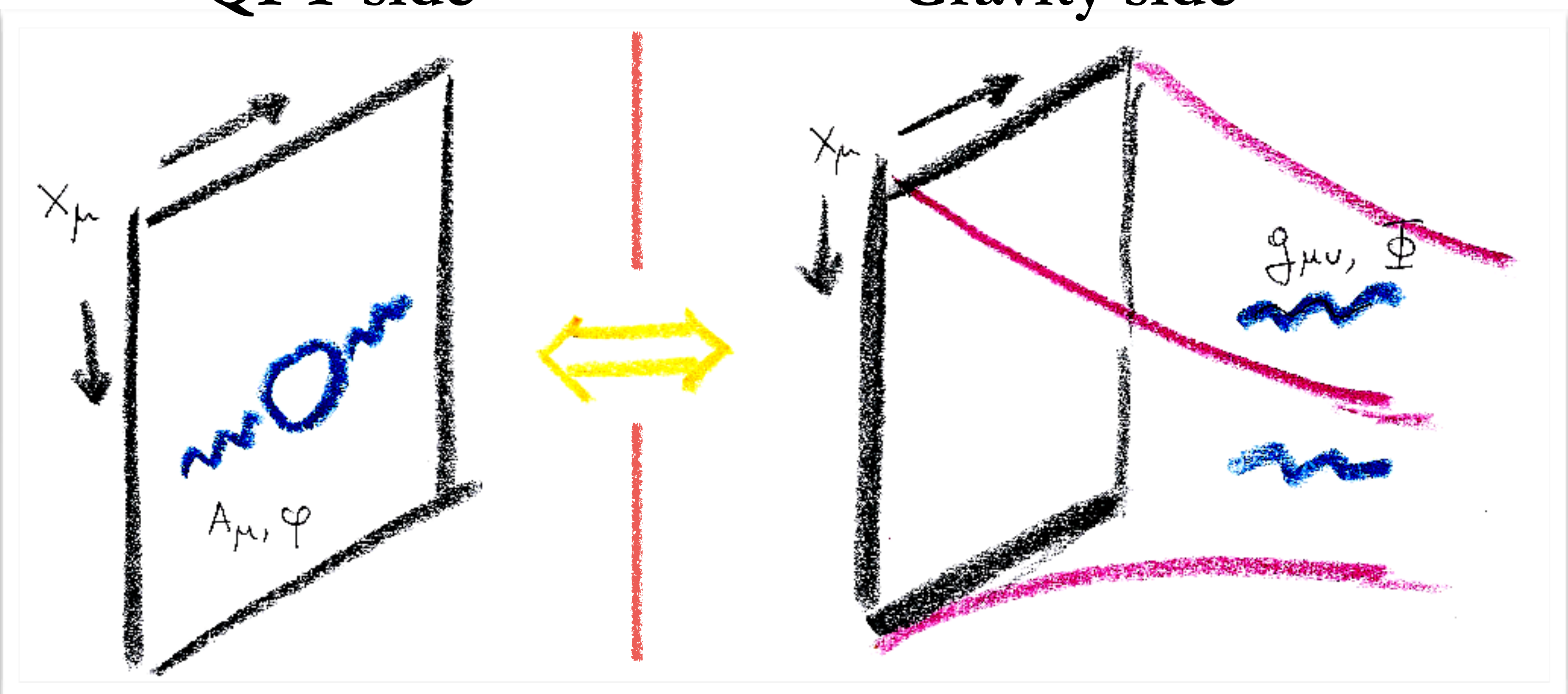
Gauge/Gravity Duality

- The system in question was, on one side, a specific **gauge theory:** maximally supersymmetric Yang-Mills theory with gauge group **SU(N).** This theory is conformally invariant (it is a **CFT**)
- On the other side, it was (type IIB) **string theory** on $AdS_5 \times S^5$

(hence the name **AdS/CFT duality**)

- These are **the same theory** written in terms of different variables
- Since then, this has been generalized to many other cases (with less or no supersymmetry, no conformal invariance…).

QFT side **Gravity side**



Outline

- Basic ingredients (*what is a CFT? how does AdS space look like?*)
- Statement of the correspondence *(fields and operators)*
- How to compute field theory observables from gravity
- Curved-spacetime holography and applications to cosmology

Conformal Field Theory

• CFT : a relativistic quantum field theory which has extra invariance:

Scale transformations $x^\mu \to \lambda x^\mu$

(also special conformal transformations) $x^\mu \to \dfrac{x^\mu - b^\mu x^2}{1 - 2b \cdot x + b^2 x^2}$

• Scale invariance ➡ absence of a mass scale

Conformal Field Theory

- CFT : a relativistic quantum field theory which has extra invariance:

Scale transformations $x^\mu \to \lambda x^\mu$

(also special conformal transformations) $x^\mu \to \frac{x^\mu - b^\mu x^2}{1 - 2b \cdot x + b^2 x^2}$

in 4d: 5 extra parameters

15 spacetime symmetries

- Scale invariance ➡ absence of a mass scale

Conformal Field Theory

• CFT : a relativistic quantum field theory which has extra invariance:

Scale transformations $x^\mu \to \lambda x^\mu$

in 4d: 5 extra parameters

15 spacetime symmetries

(also special conformal transformations) $x^\mu \to \dfrac{x^\mu - b^\mu x^2}{1 - 2b \cdot x + b^2 x^2}$

• Scale invariance ➡ absence of a mass scale

• Examples of **classical** CFTs: Massless QED, massless φ^4 theory…

Conformal Field Theory

- CFT : a relativistic quantum field theory which has extra invariance:

Scale transformations $x^\mu \to \lambda x^\mu$

in 4d: 5 extra parameters

15 spacetime symmetries

(also special conformal transformations) $x^\mu \to \frac{x^\mu - b^\mu x^2}{1 - 2b\cdot x + b^2 x^2}$

- Scale invariance → absence of a mass scale

- Examples of **classical** CFTs: Massless QED, massless φ^4 theory…

- It contains operators which transform covariantly:

  $e.g.\quad F_{\mu\nu}F^{\mu\nu}, \bar{\psi}\psi\ldots$

$$O(\lambda x^\mu) = \lambda^{-\Delta} O(x^\mu)$$

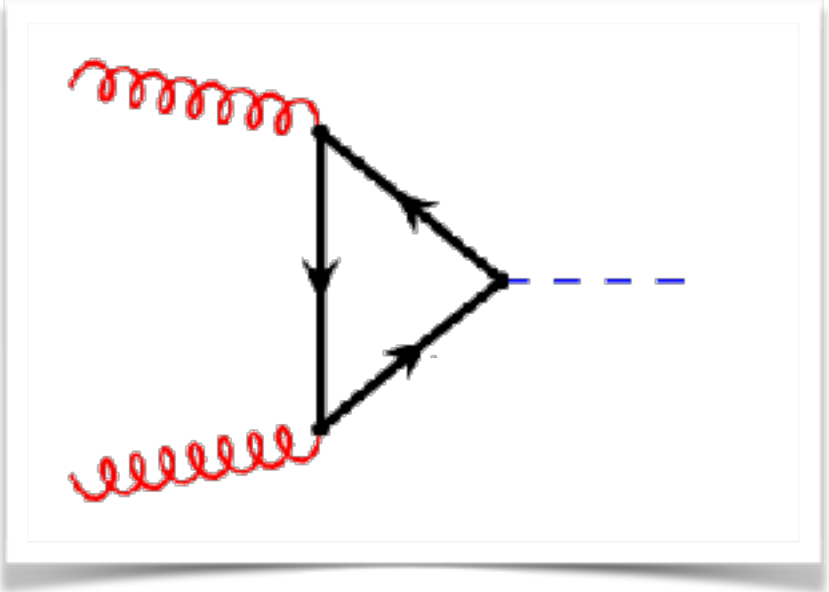
weight (Δ)

- Correlation functions are simple:

$$\langle O(x) O(y)\rangle = \frac{1}{|x-y|^{2\Delta}}$$

Quantum CFT

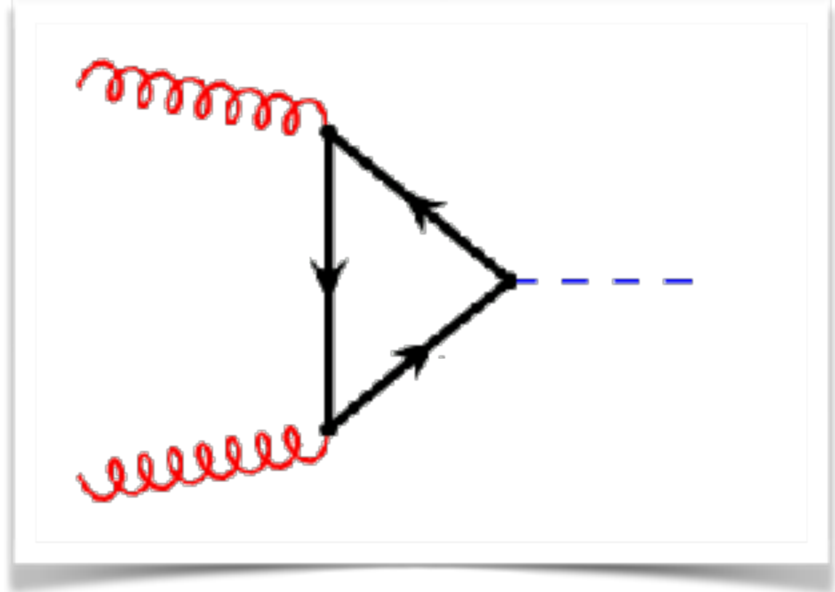
- Conformal invariance is typically broken by quantum effects



- Very hard to construct a 4d QFT which are conformal at the full quantum level (supersymmetry helps)

Quantum CFT

- Conformal invariance is typically broken by quantum effects



- Very hard to construct a 4d QFT which are conformal at the full quantum level (supersymmetry helps)

- One example is Maximally SUSY Yang-Mills in 4d: it is a gauge theory with scalars and fermions (all in adjoint representation of SU(N))

- 1 gauge field A_μ
- 6 real scalars φ^I
- 4 Weyl fermions ψ_i

- Parameters:

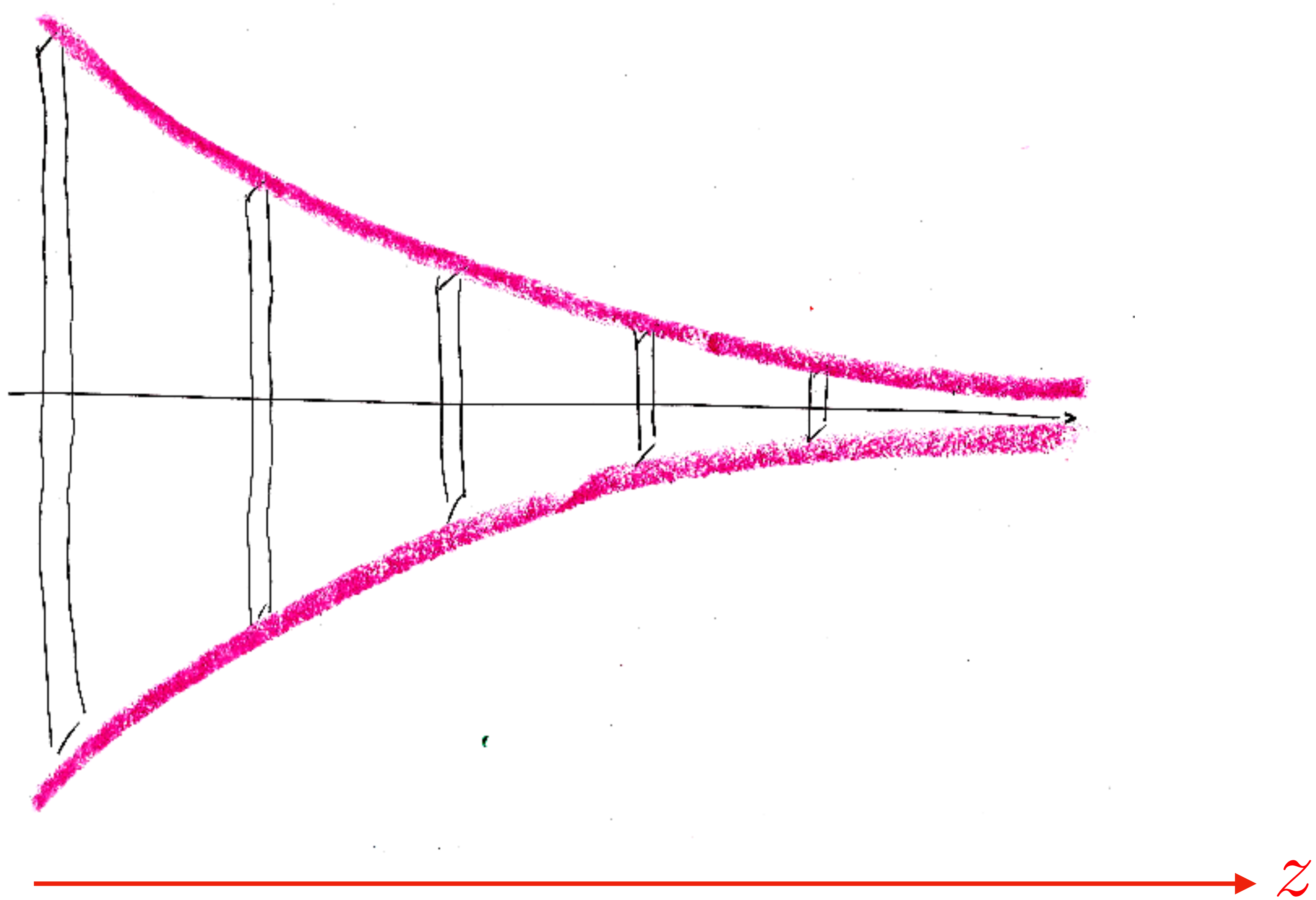
  gauge coupling: g

  size of the gauge group: N

Anti-de Sitter Spacetime

- Maximally symmetric, negative curvature spacetime
- Solution of Einstein equation with negative cosmological constant

- Metric of AdS_5 in local coordinates: $$ds^2 = \ell^2 \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$



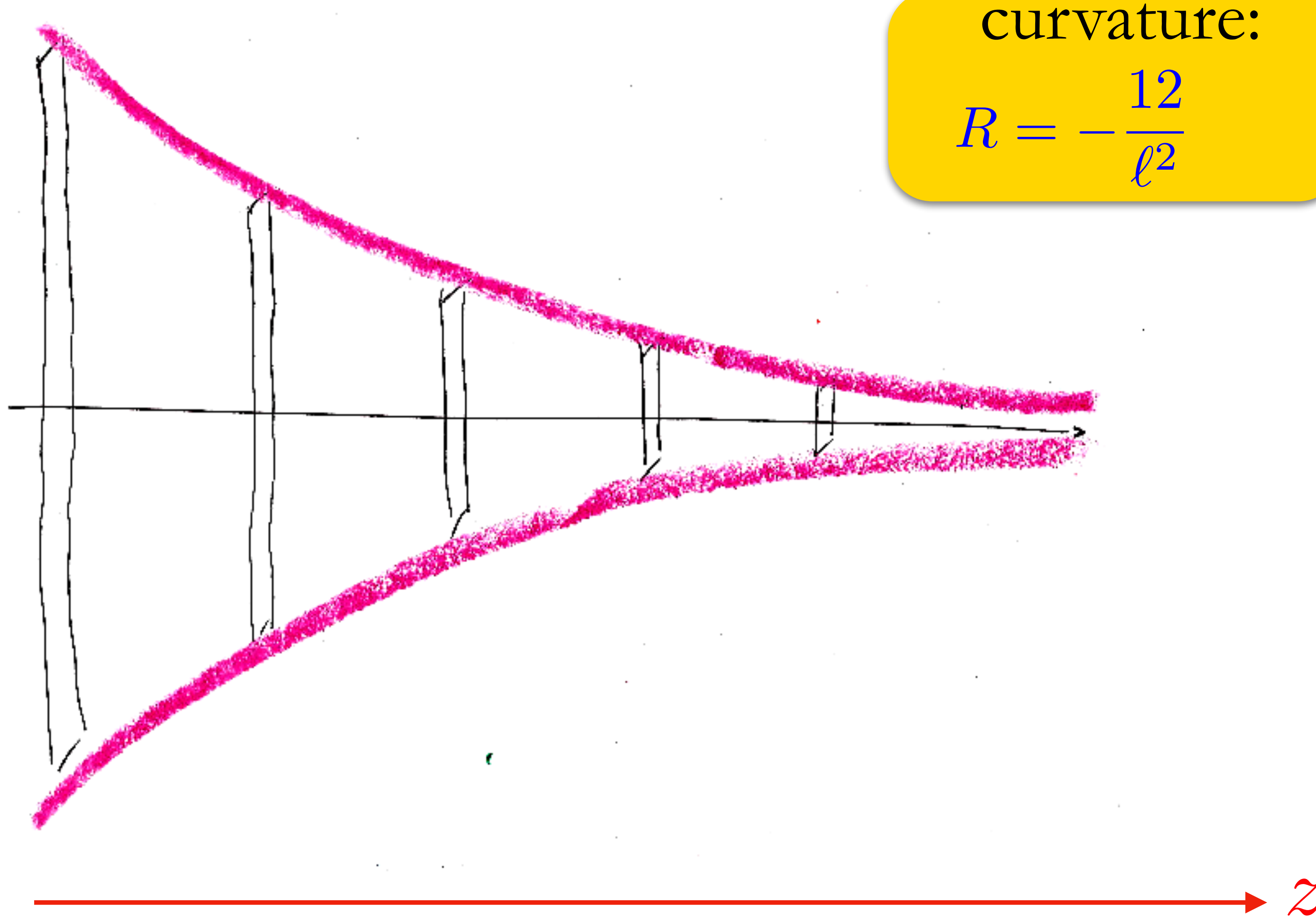
Anti-de Sitter Spacetime

- Maximally symmetric, negative curvature spacetime
- Solution of Einstein equation with negative cosmological constant

- Metric of AdS_5 in local coordinates: $$ds^2 = \ell^2 \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$

curvature:

$$R = -\frac{12}{\ell^2}$$

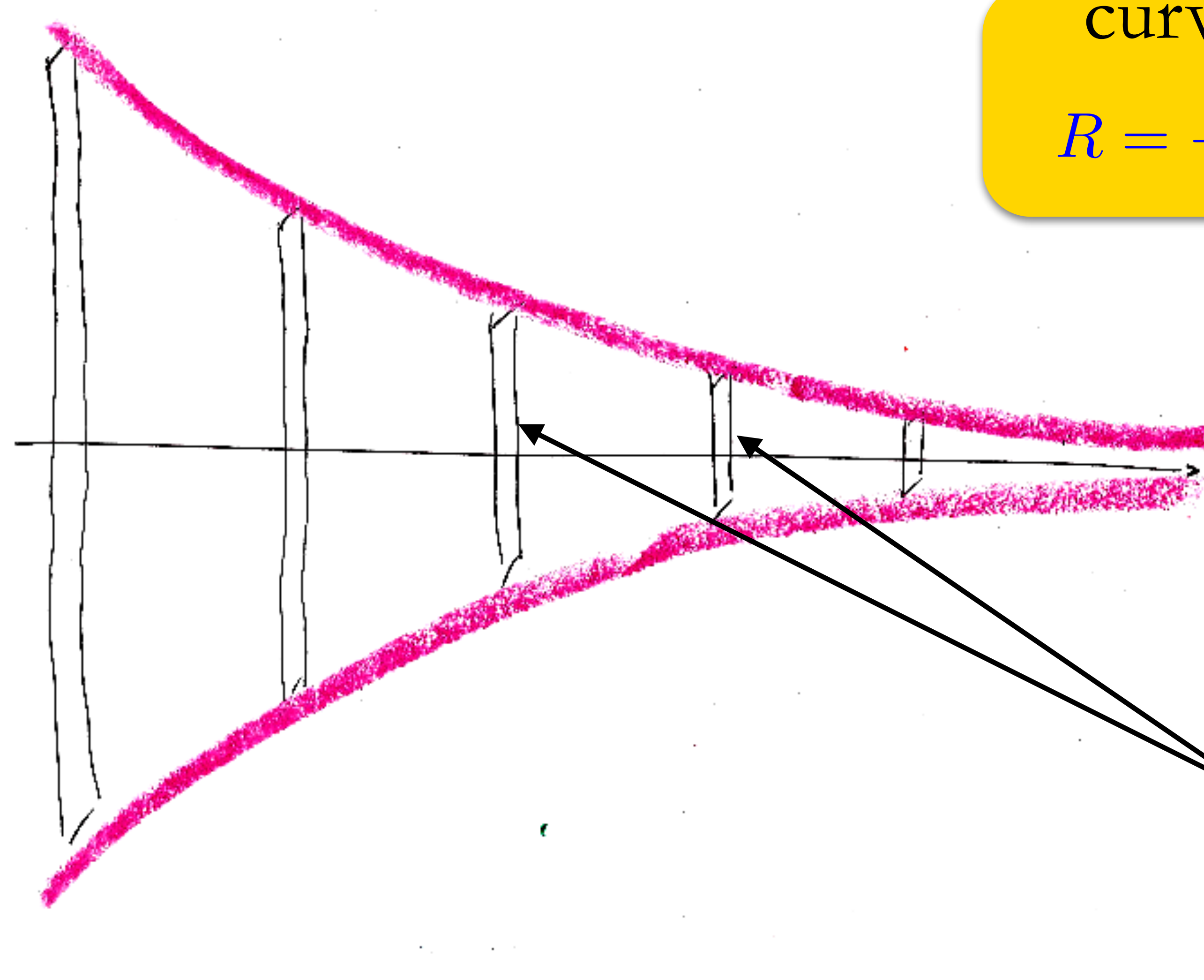


Anti-de Sitter Spacetime

- Maximally symmetric, negative curvature spacetime
- Solution of Einstein equation with negative cosmological constant

- Metric of AdS_5 in local coordinates: $$ds^2 = \ell^2 \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$

curvature:
$$R = -\frac{12}{\ell^2}$$



constant-z slices
= 4d Minkowski

z

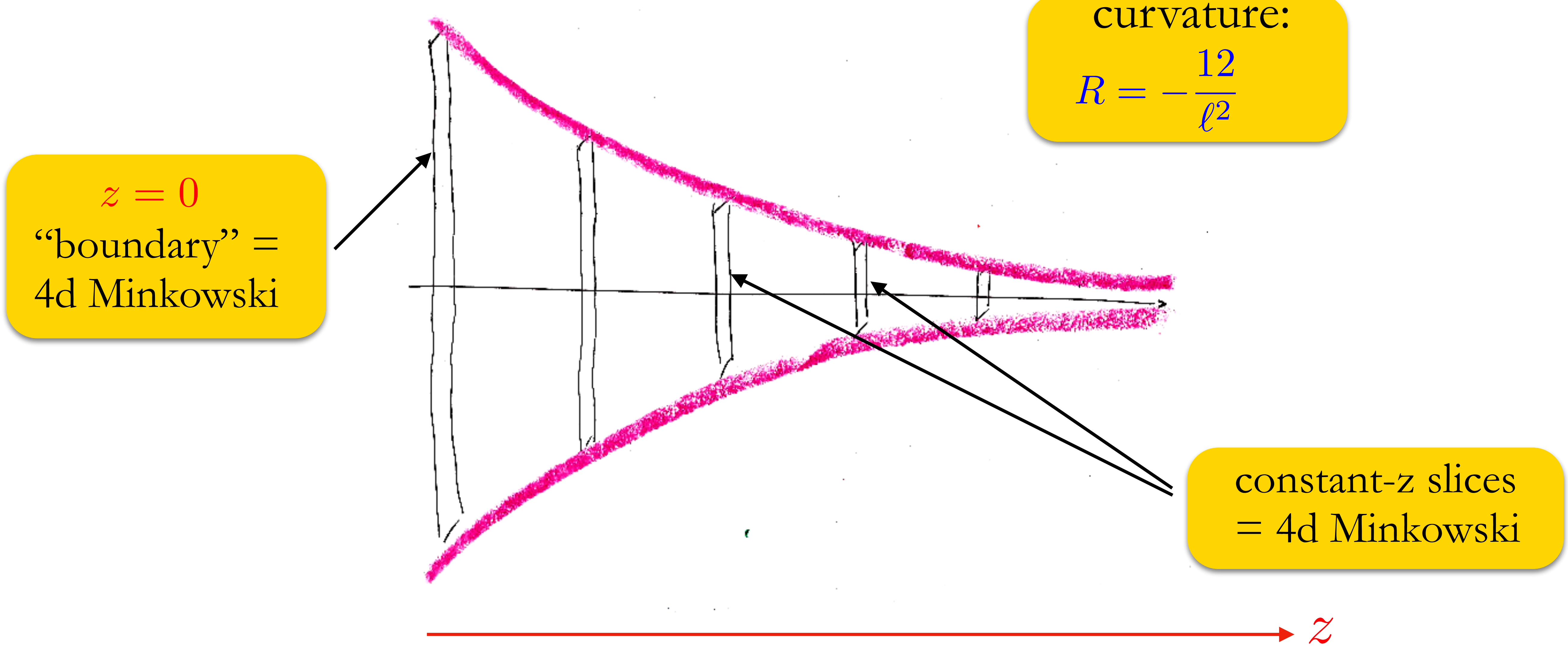
Anti-de Sitter Spacetime

- Maximally symmetric, negative curvature spacetime
- Solution of Einstein equation with negative cosmological constant

- Metric of AdS_5 in local coordinates: $$ds^2 = \ell^2 \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$

curvature:

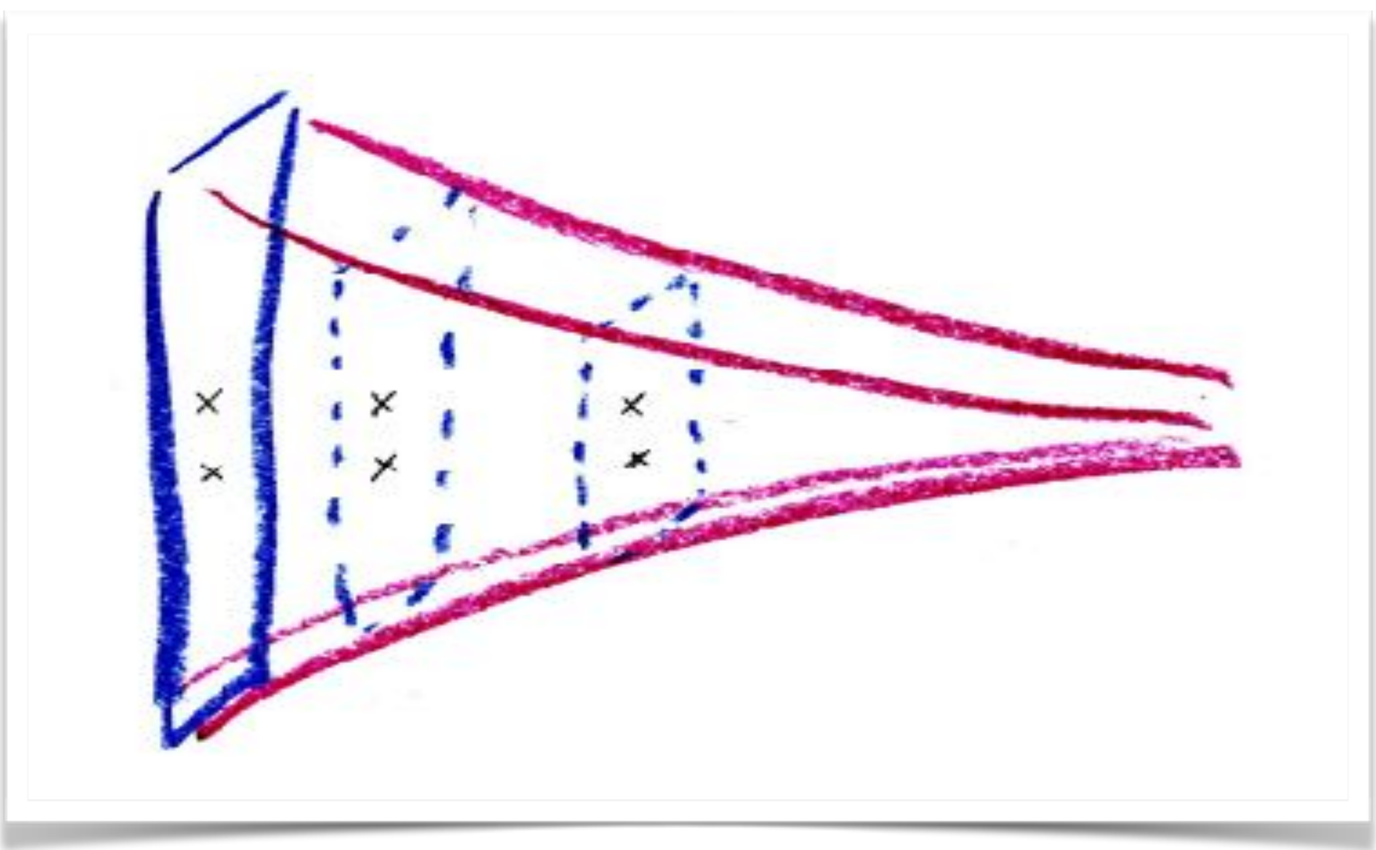
$$R = -\frac{12}{\ell^2}$$



Anti-de Sitter Spacetime

- Maximally symmetric, negative curvature spacetime
- Solution of Einstein equation with negative cosmological constant

- Metric of AdS_5 in local coordinates: $$ds^2 = \ell^2 \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$



Moving towards the boundary

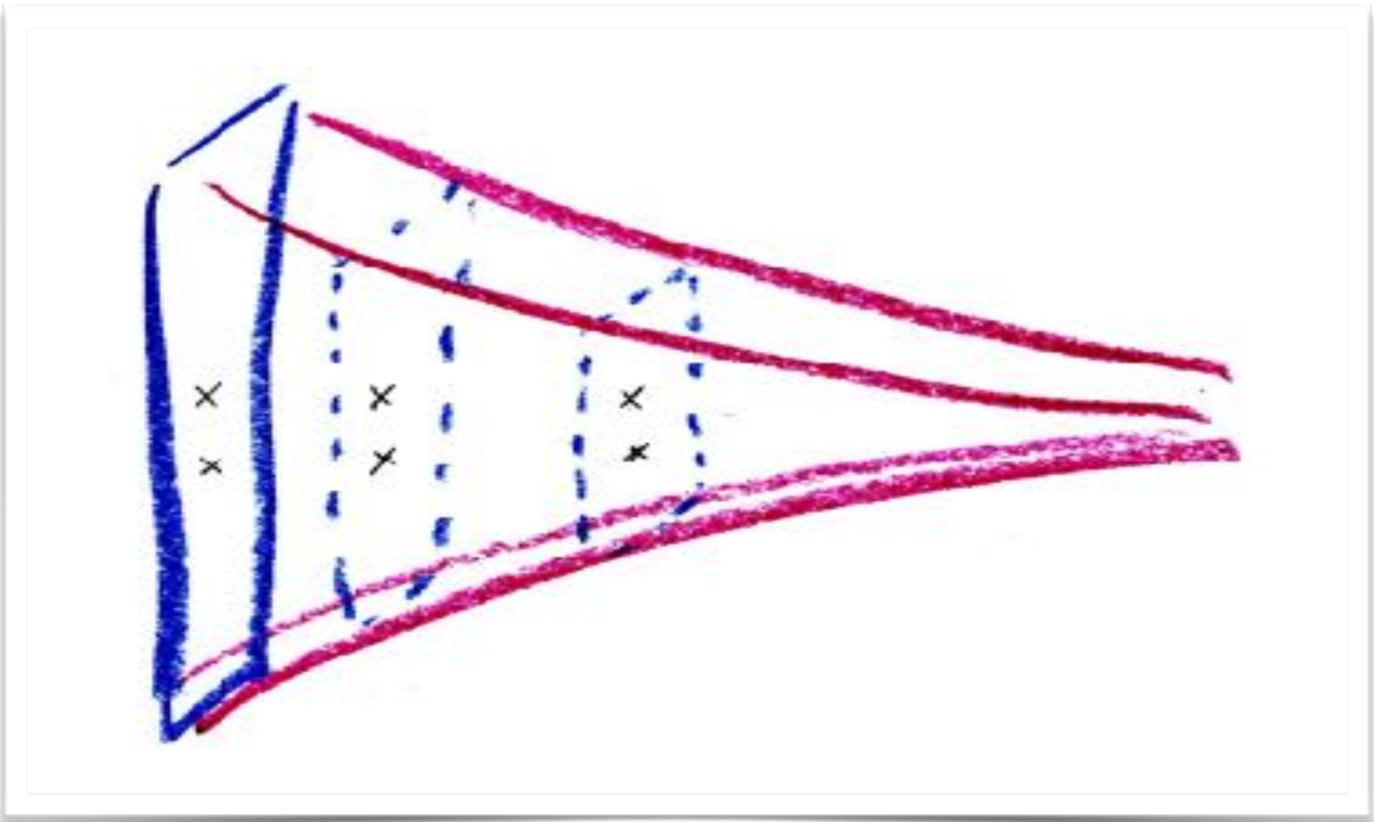
=

moving to shorter distances in 4d spacetime

Anti-de Sitter Spacetime

- Maximally symmetric, negative curvature spacetime
- Solution of Einstein equation with negative cosmological constant

- Metric of AdS_5 in local coordinates: $$ds^2 = \ell^2 \frac{dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$



Moving towards the boundary

=

moving to shorter distances in 4d spacetime

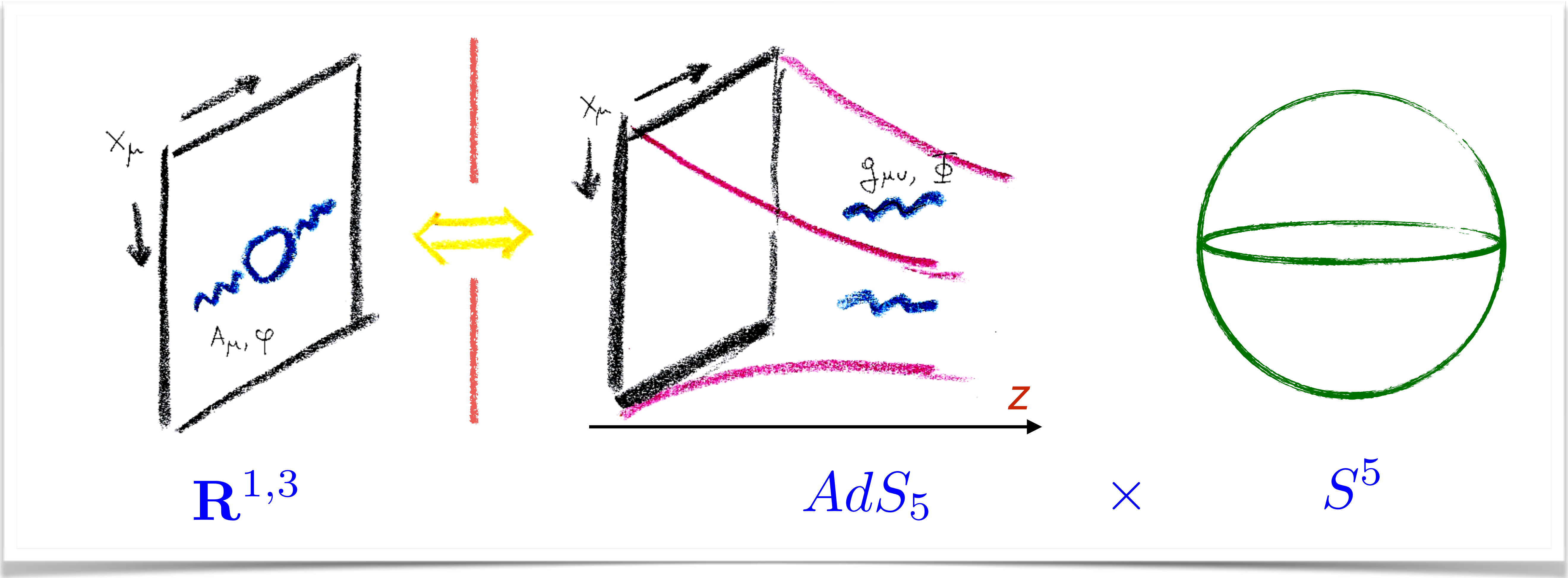
- **Isometry group = 4d conformal group**

scaling isometry: $x^\mu \to \lambda x^\mu, \; z \to \lambda z$

The Maldacena Correspondence

Gauge theory side:
N=4 SuperYM

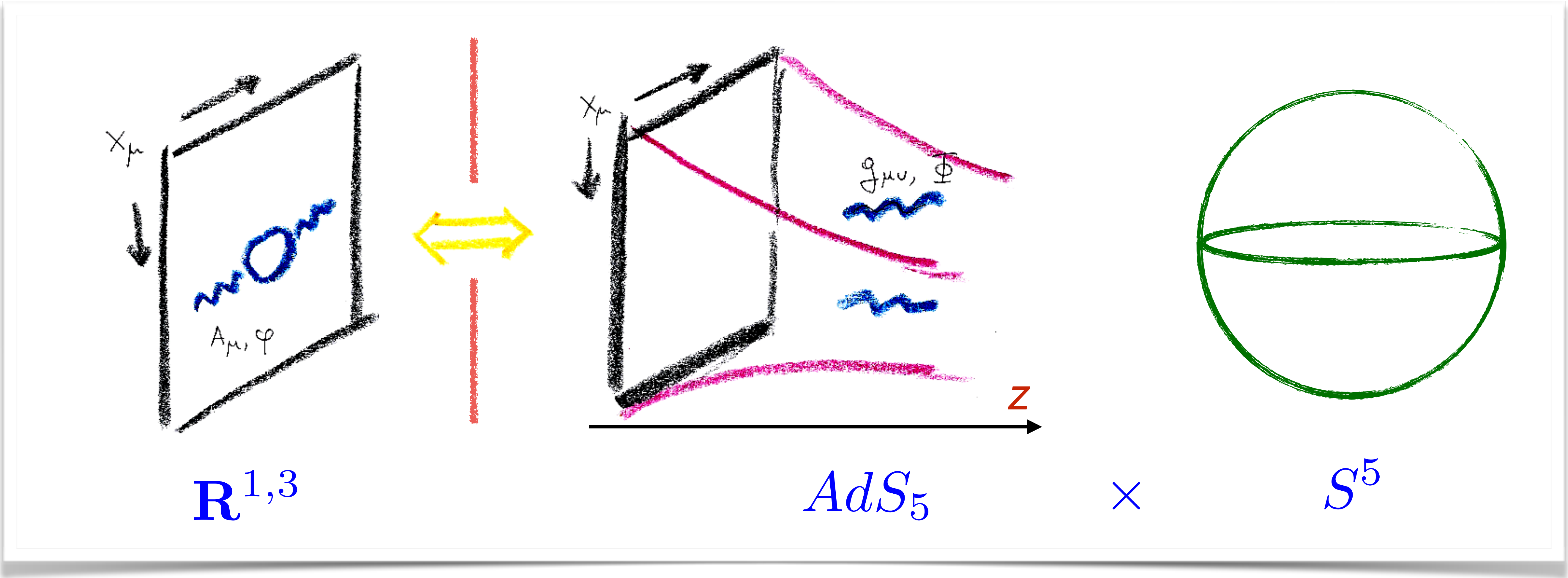
Gravity side:
Type IIB string theory



The Maldacena Correspondence

Gauge theory side:
N=4 SuperYM

Gravity side:
Type IIB string theory

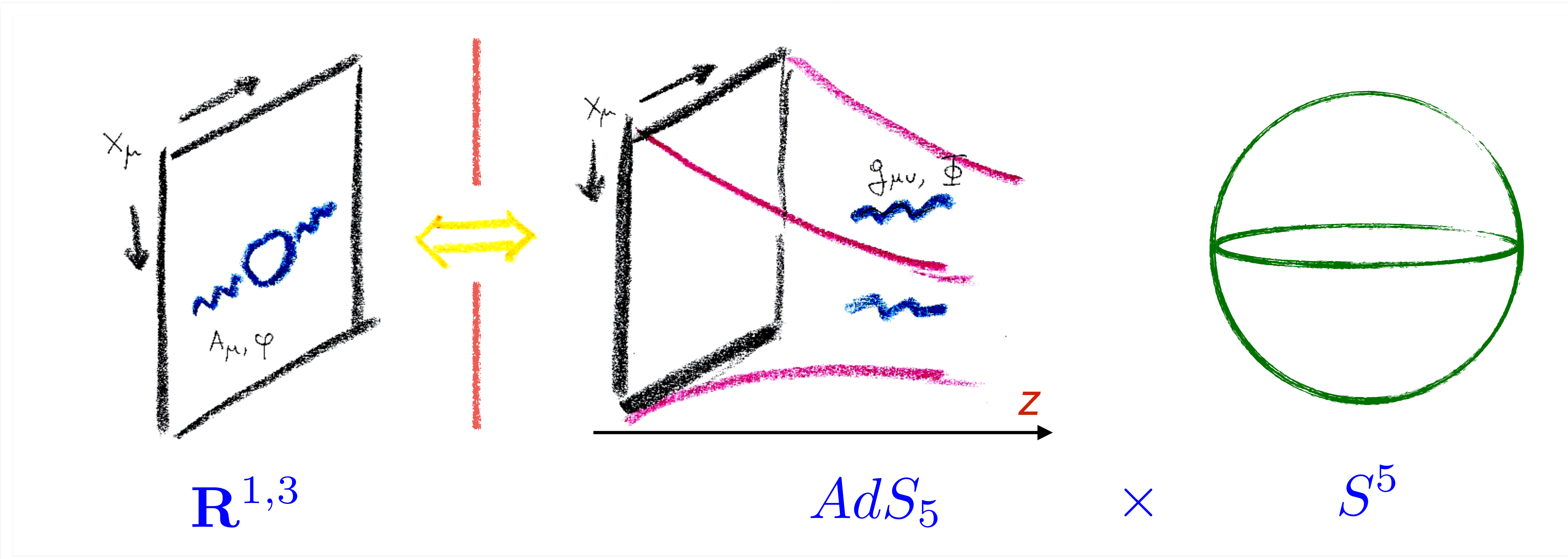


- The **boundary of AdS** is identified with **the space the CFT lives on**

The Maldacena Correspondence

Gauge theory side:
N=4 SuperYM

Gravity side:
Type IIB string theory



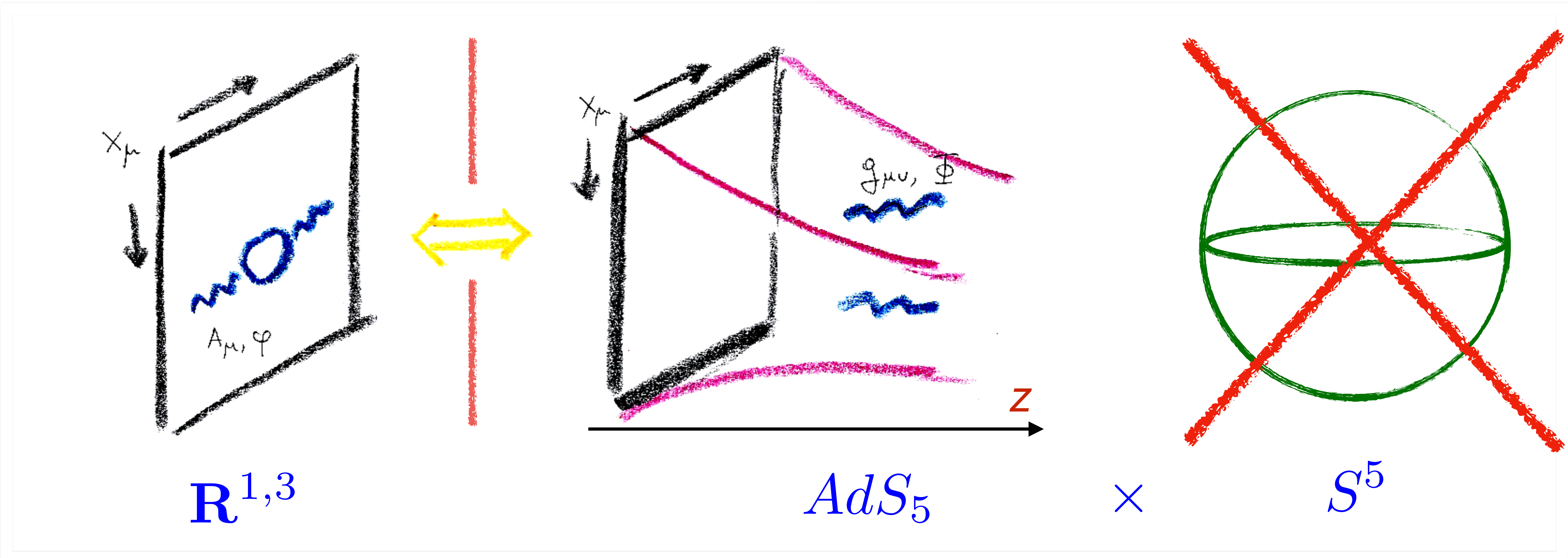
- The **boundary of AdS** is identified with **the space the CFT lives on**

- Both sides obtained as a low energy limit of string theory in 10d
- The gravity side looks much more complicated than the field theory side: more dimensions, curved space, quantum gravity, many more fields

The Maldacena Correspondence

Gauge theory side:
N=4 SuperYM

Gravity side:
Type IIB string theory

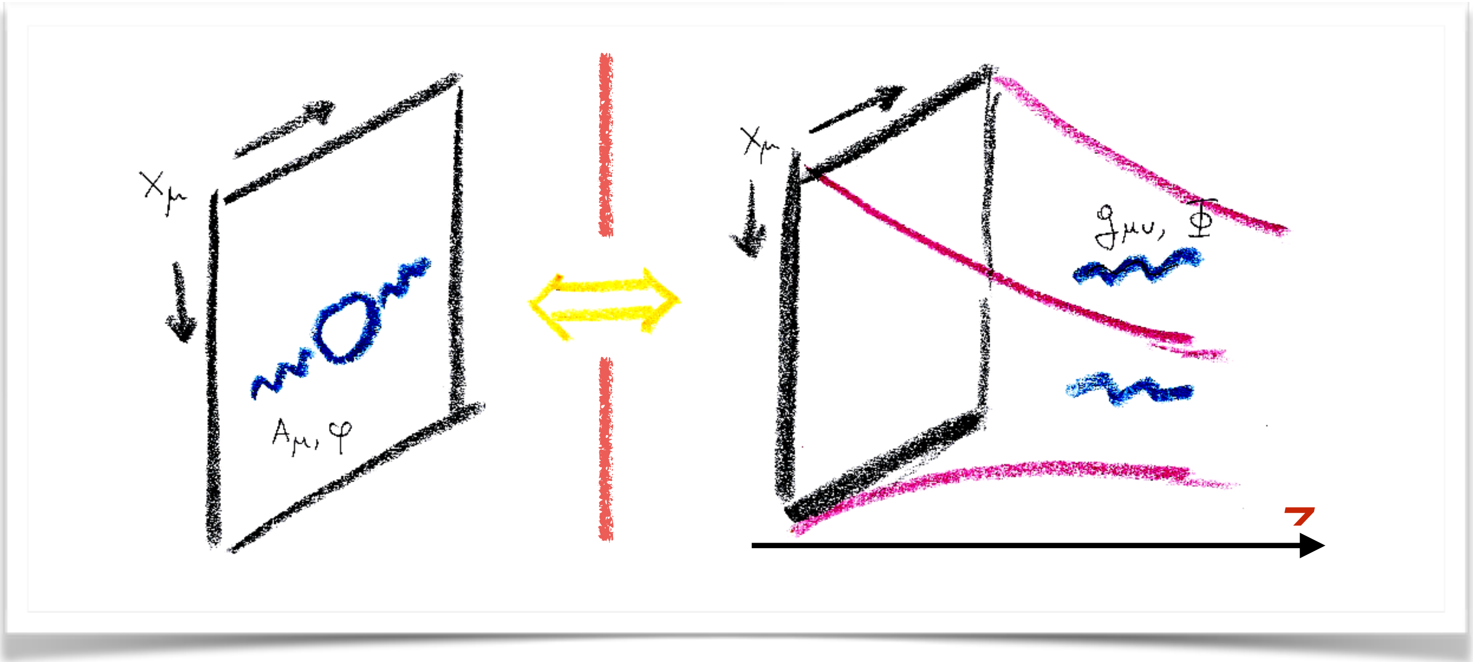


- The **boundary of AdS** is identified with **the space the CFT lives on**

- Both sides obtained as a low energy limit of string theory in 10d
- The gravity side looks much more complicated than the field theory side: more dimensions, curved space, quantum gravity, many more fields

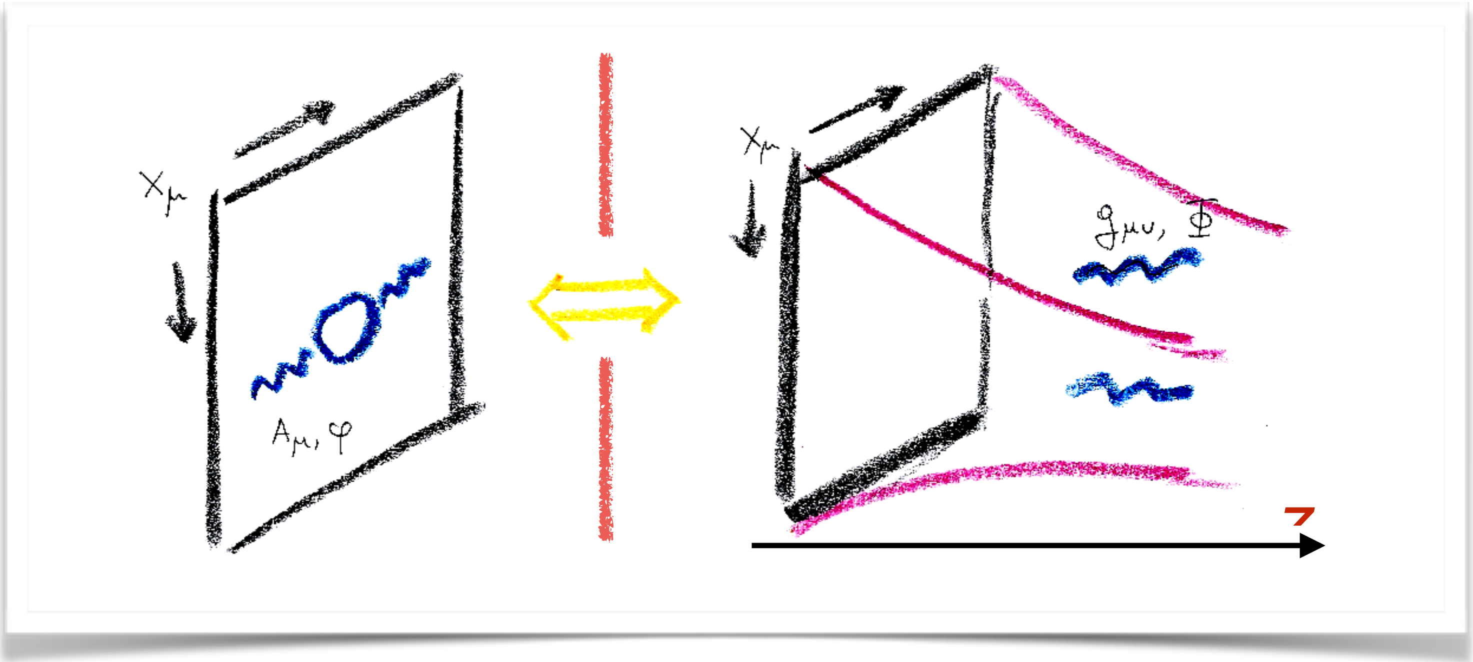
Large-N limit

Super-YM **Strings on AdS**



Large-N limit

Super-YM **Strings on AdS**

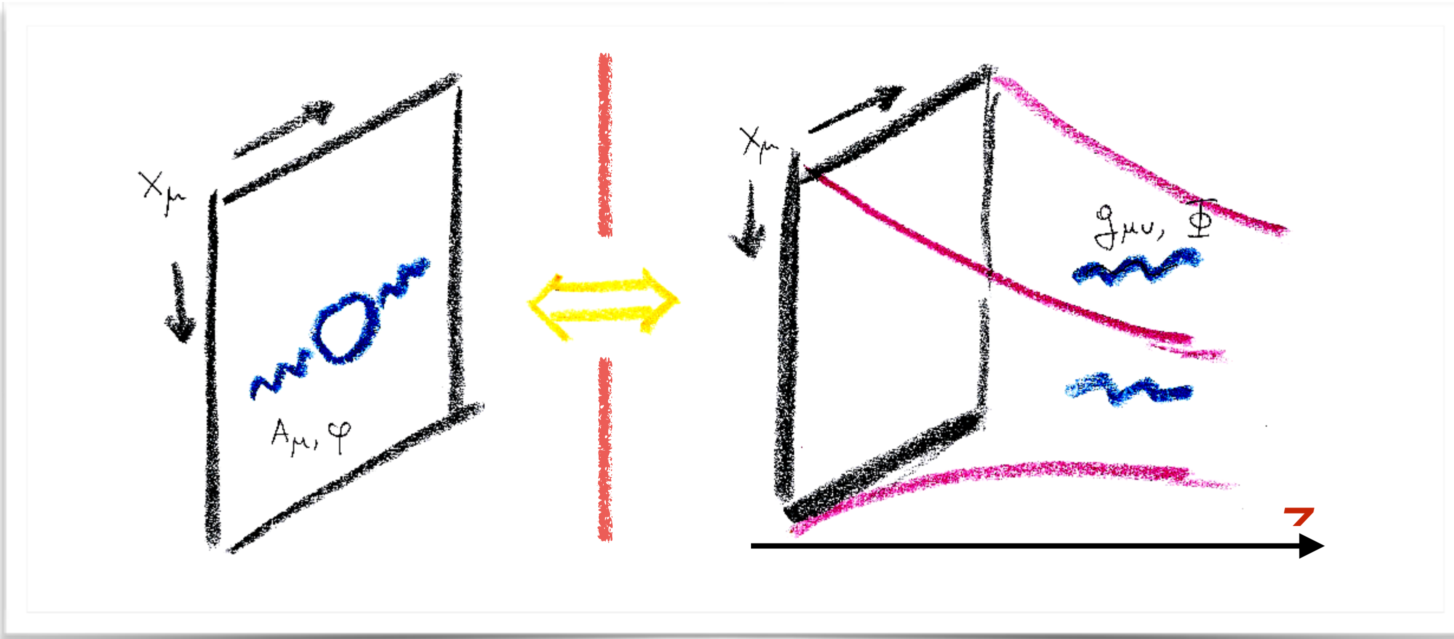


Two parameters: N

g^2_{YM}

Large-N limit

Super-YM **Strings on AdS**



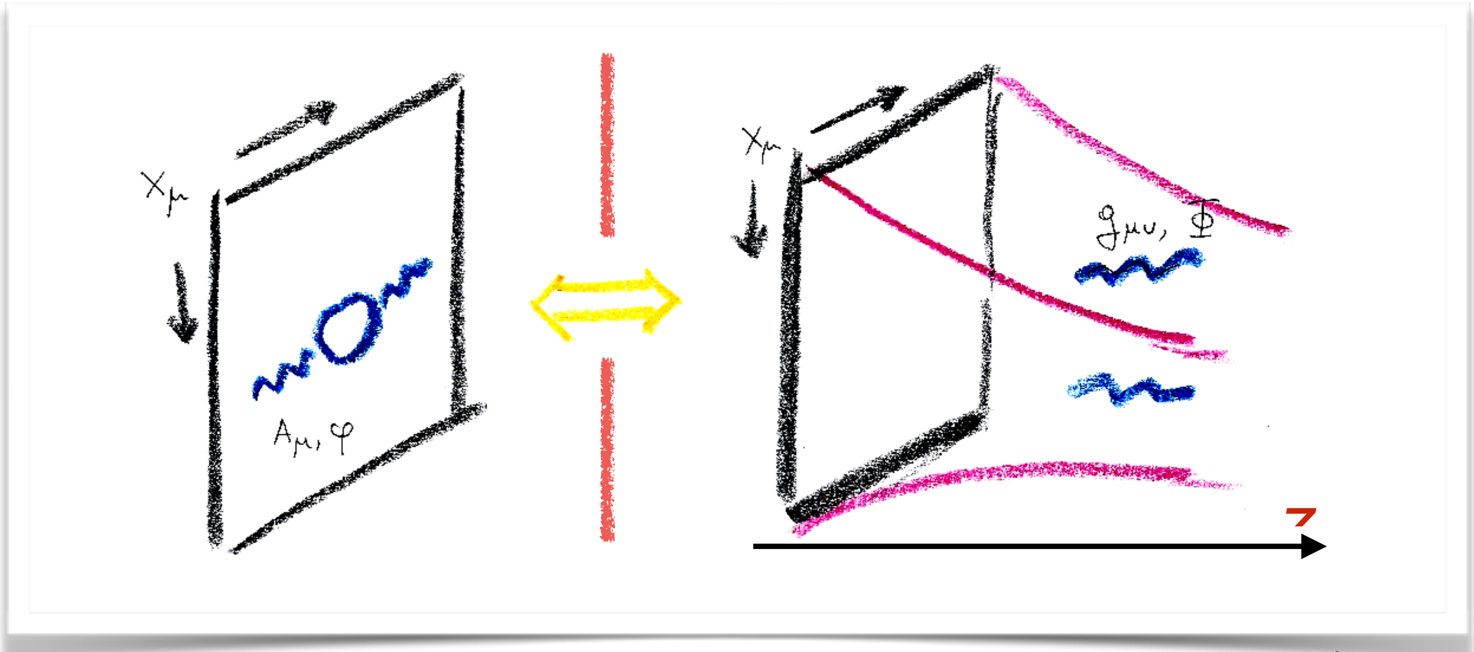
Two parameters:

$N \longleftrightarrow (M_p \ell)^4$

$g_{YM}^2 \longleftrightarrow g_s$

Large-N limit

Super-YM **Strings on AdS**



Two parameters:

N ⟷ $(M_p \ell)^4$

g^2_{YM} ⟷ g_s

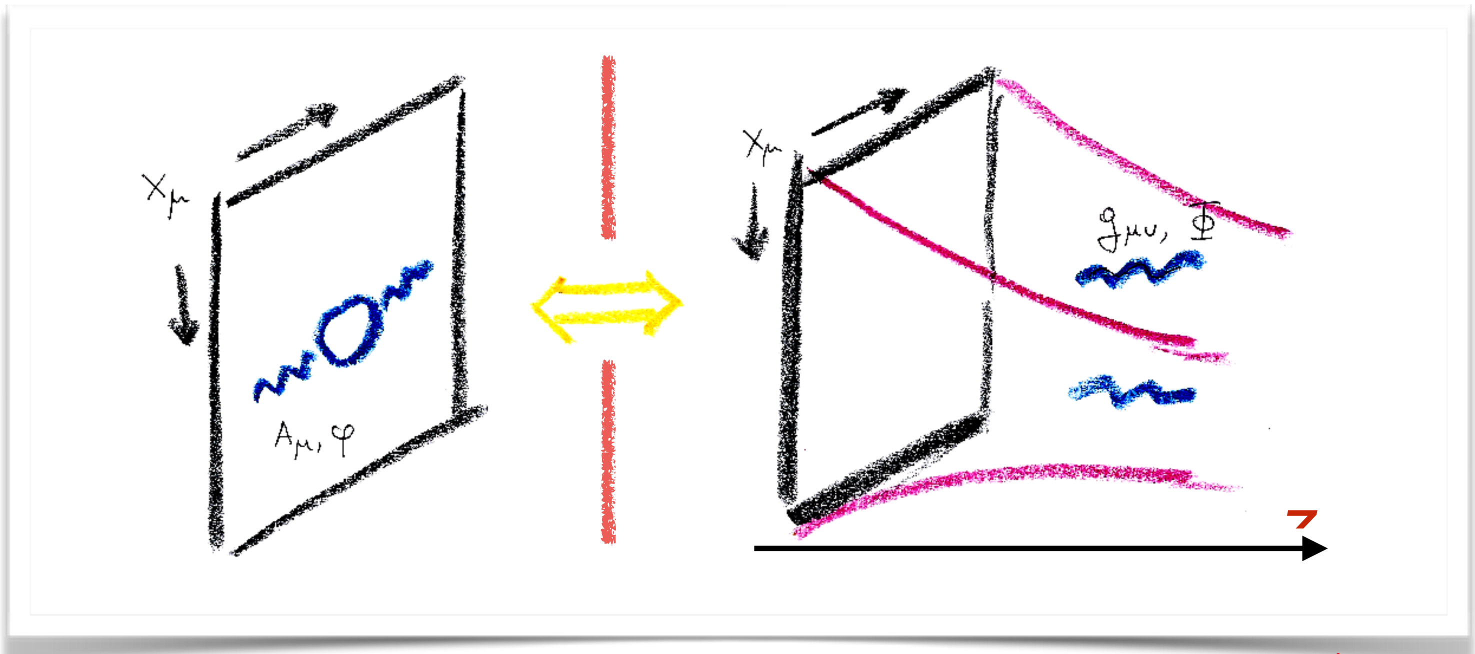
- **Large N : gravity side can be treated classically**

- **Large $g^2_{YM} N$: neglect massive string spectrum**

$g^2_{YM} N$ is the *true* coupling of large-N gauge theories ('t Hooft)

Large-N limit

Super-YM | **Strings on AdS**



Two parameters:

$N \longleftrightarrow (M_p \ell)^4$

$g^2_{YM} \longleftrightarrow g_s$

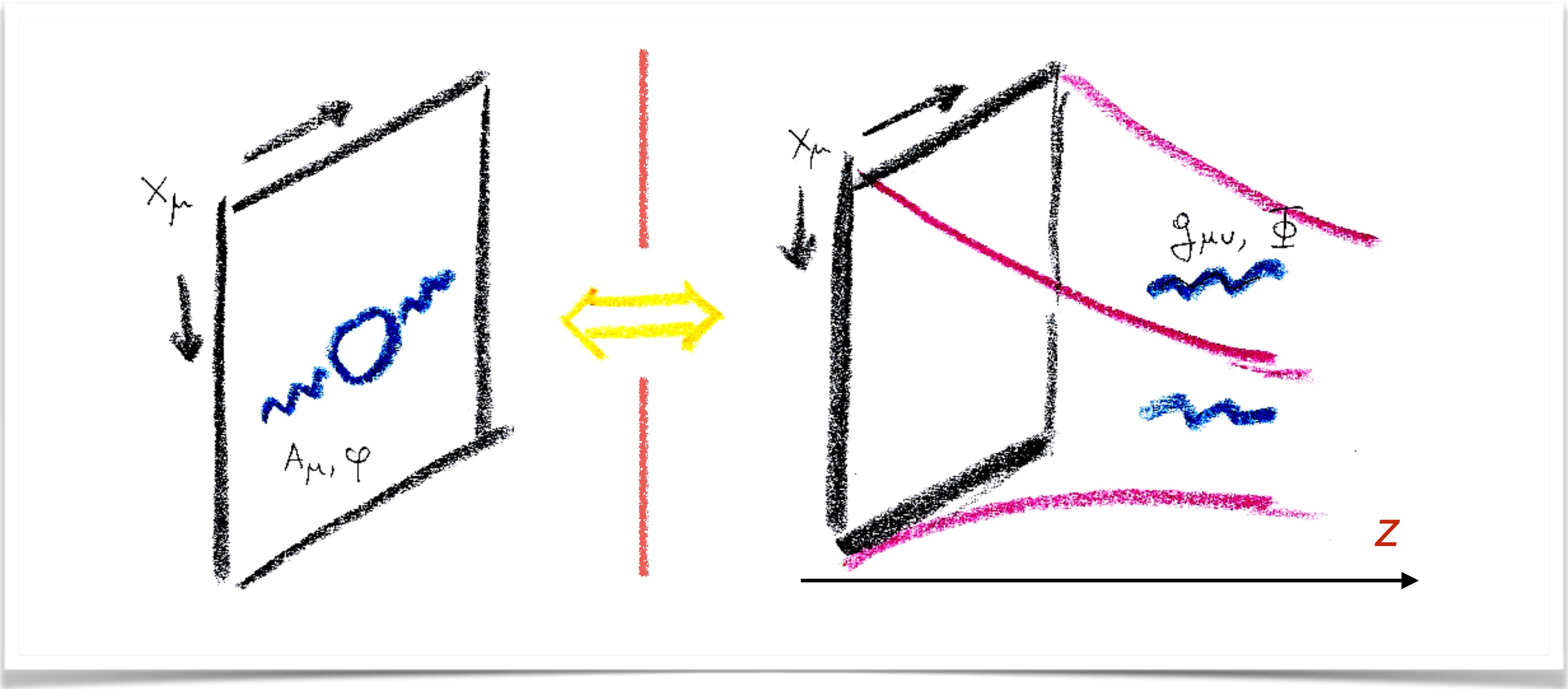
- **Large N : gravity side can be treated classically**

- **Large $g^2_{YM} N$: neglect massive string spectrum**

$g^2_{YM} N$ is the *true* coupling of large-N gauge theories ('t Hooft)

Large N, strongly coupled SYM theory is described by classical GR

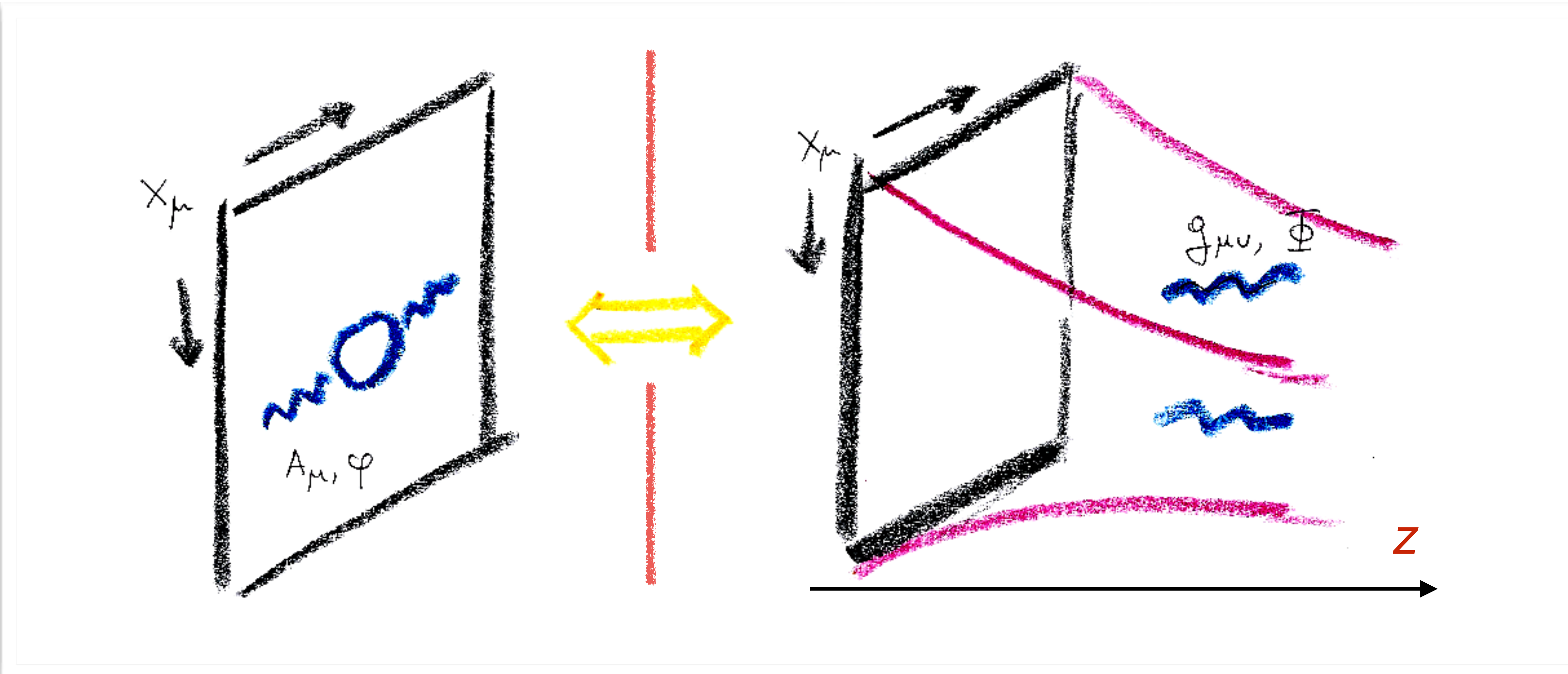
What is it good for ?



Strongly coupled large N Super-YM ← Classical GR on AdS

Perturbative Super-YM → Quantum gravity/ strings on AdS

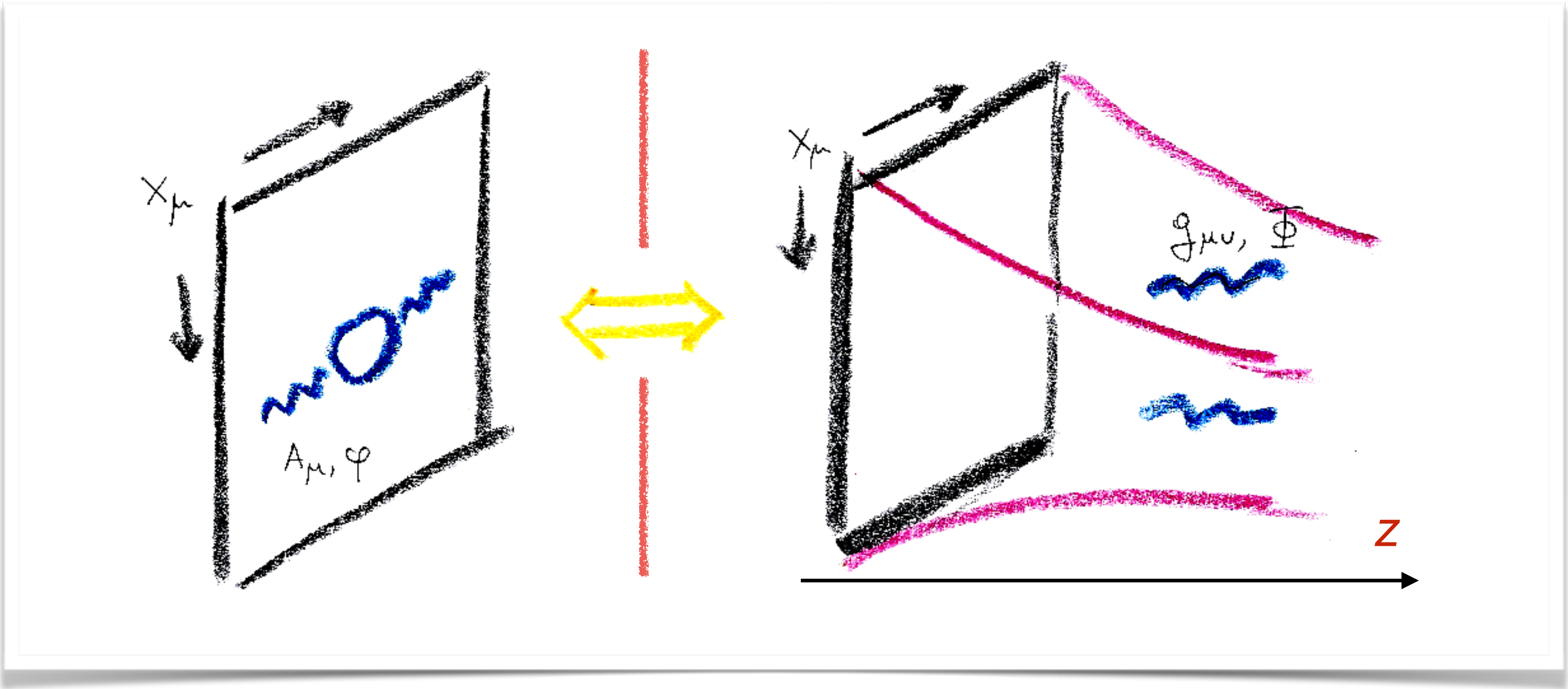
What is it good for ?



Strongly coupled large N Super-YM ← Classical GR on AdS

Perturbative Super-YM → Quantum gravity/ strings on AdS

What is it good for ?



Strongly coupled large N Super-YM ⟵ Classical GR on AdS

Perturbative Super-YM ⟶ Quantum gravity/ strings on AdS

Field/operator correspondence

- The **boundary of AdS** is identified with **the space the CFT lives on**

- **QFT side observables**: correlation functions of local operators
- **Gravity side: no local observables** due to diff invariance.

  Only boundary observables (diffs that act on the boundary are not gauge transformations but change the state).

Field/operator correspondence

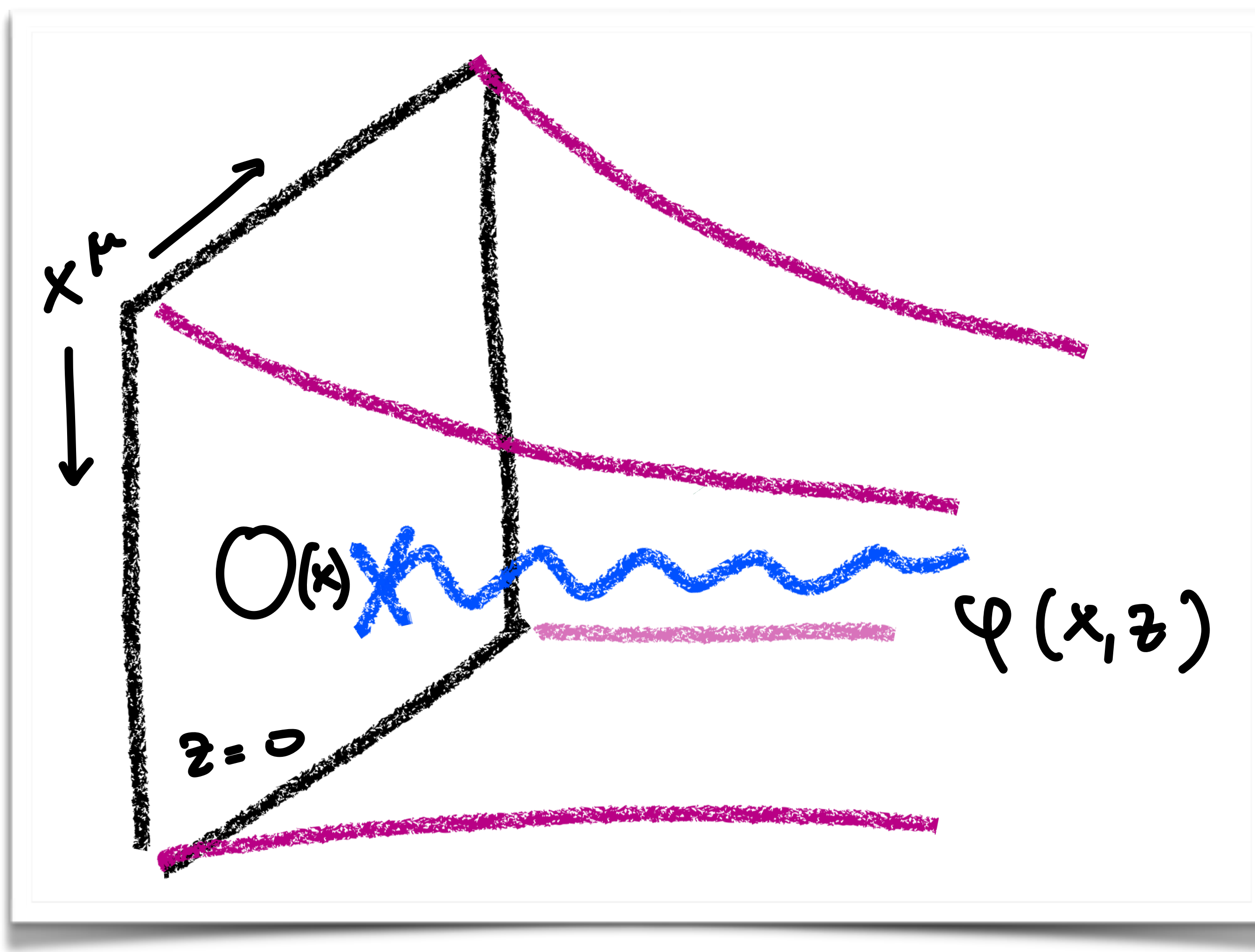
• The **boundary of AdS** is identified with **the space the CFT lives on**

• **QFT side observables**: correlation functions of local operators

• **Gravity side: no local observables** due to diff invariance.

Only boundary observables (diffs that act on the boundary are not gauge transformations but change the state).

QFT data are boundary conditions for gravity fields



Statement of the equivalence

CFT operator $O(x)$ **(gauge-invariant)** ⟷ **Dynamical field** $\varphi(x, z)$

Statement of the equivalence

CFT operator $O(x)$ **(gauge-invariant)** ⟷ **Dynamical field** $\varphi(x, z)$

• The CFT functional integral is defined by coupling operators to sources

$$Z_{CFT}[J(x)] = \int \mathcal{D}[\psi] exp \left[iS_{CFT} + i \int d^4x J(x) O(x) \right]$$

$$\langle O(x_1) O(x_2) ... \rangle = \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} \dots Z[J]$$

Statement of the equivalence

CFT operator $O(x)$ **(gauge-invariant)** ⟷ **Dynamical field** $\varphi(x,z)$

• The CFT functional integral is defined by coupling operators to sources

$$Z_{CFT}[J(x)] = \int \mathcal{D}[\psi] exp\left[iS_{CFT} + i\int d^4x J(x) O(x) \right]$$

$$\langle O(x_1) O(x_2) \ldots \rangle = \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} \ldots Z[J]$$

• On the gravity side $J(x)$ is a **boundary condition** for the field $\varphi(x,z)$

Statement of the equivalence

CFT operator $O(x)$ **(gauge-invariant)** ⟷ **Dynamical field** $\varphi(x,z)$

• The CFT functional integral is defined by coupling operators to sources

$$Z_{CFT}[J(x)] = \int \mathcal{D}[\psi] exp\left[iS_{CFT} + i\int d^4x J(x) O(x) \right]$$

$$\langle O(x_1) O(x_2) ... \rangle = \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} \dots Z[J]$$

• On the gravity side $J(x)$ is a **boundary condition** for the field $\varphi(x,z)$

near-boundary solution of field equations on AdS

Statement of the equivalence

CFT operator $O(x)$ (gauge-invariant) ⟷ **Dynamical field** $\varphi(x,z)$

- The CFT functional integral is defined by coupling operators to sources

$$Z_{CFT}[J(x)] = \int \mathcal{D}[\psi] exp\left[iS_{CFT} + i\int d^4x J(x) O(x) \right]$$

$$\langle O(x_1) O(x_2) ... \rangle = \frac{\delta}{\delta J(x_1)} \frac{\delta}{\delta J(x_2)} \dots Z[J]$$

- On the gravity side $J(x)$ is a **boundary condition** for the field $\varphi(x,z)$

$$\varphi(x,z) \simeq z^{(4-\Delta)} J(x) + \dots \qquad z \to 0$$

near-boundary solution
of field equations on AdS

Statement of the equivalence

CFT operator $O(x)$ (gauge-invariant) ⟷ **Dynamical field** $\varphi(x,z)$

• The CFT functional integral is defined by coupling operators to sources

$$Z_{CFT}[J(x)] = \int \mathcal{D}[\psi] exp\left[iS_{CFT} + i\int d^4x J(x) O(x) \right]$$

$$\langle O(x_1)O(x_2)...\rangle = \frac{\delta}{\delta J(x_1)}\frac{\delta}{\delta J(x_2)}\ldots Z[J]$$

• On the gravity side $J(x)$ is a **boundary condition** for the field $\varphi(x,z)$

$$\varphi(x,z) \simeq z^{(4-\Delta)} J(x) + \ldots \qquad z \to 0$$

near-boundary solution of field equations on AdS

• The weight Δ of $O(x)$ is related to the mass of the field $\varphi(x,z)$:

$$\Delta(\Delta - 4) = m^2\ell^2$$

Statement of the equivalence

CFT operator $O(x)$ **(gauge-invariant)** ⟷ **Dynamical field** $\varphi(x,z)$

• The CFT functional integral is defined by coupling operators to sources

$$Z_{CFT}[J(x)] = \int \mathcal{D}[\psi] exp \left[iS_{CFT} + i \int d^4x J(x) O(x) \right]$$

• Gravity side path integral defined with fixed boundary conditions:

$$Z_{grav}[J(x)] = \int_{\varphi \to z^{(4-\Delta)} J(x)} \mathcal{D}[\varphi] exp\, iS_{grav}[\varphi(x,z)]$$

Statement of the equivalence

CFT operator $O(x)$ (gauge-invariant) ⟷ **Dynamical field** $\varphi(x,z)$

• The CFT functional integral is defined by coupling operators to sources

$$Z_{CFT}[J(x)] = \int \mathcal{D}[\psi] exp \left[iS_{CFT} + i \int d^4x J(x) O(x) \right]$$

• Gravity side path integral defined with fixed boundary conditions:

$$Z_{grav}[J(x)] = \int_{\varphi \to z^{(4-\Delta)} J(x)} \mathcal{D}[\varphi] exp\, iS_{grav}[\varphi(x,z)]$$

AdS/CFT Conjecture: $Z_{CFT}[J(x)] = Z_{grav}[J(x)]$

Statement of the equivalence

CFT operator $O(x)$ (gauge-invariant) ⟷ **Dynamical field** $\varphi(x,z)$

• The CFT functional integral is defined by coupling operators to sources

$$Z_{CFT}[J(x)] = \int \mathcal{D}[\psi] exp \left[iS_{CFT} + i \int d^4x J(x) O(x) \right]$$

• Gravity side path integral defined with fixed boundary conditions:

$$Z_{grav}[J(x)] = \int_{\varphi \to z^{(4-\Delta)} J(x)} \mathcal{D}[\varphi] exp\, iS_{grav}[\varphi(x,z)]$$

AdS/CFT Conjecture:

$$Z_{CFT}[J(x)] = Z_{grav}[J(x)]$$

Large N limit:
saddle point approximation

$$Z_{grav}[J(x)] \simeq exp\, iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi \to z^{(4-\Delta)} J}$$

Computing CFT observables from Gravity side

$$Z_{grav}[J(x)] \simeq exp\, iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi \to z^{(4-\Delta)} J}$$

- Establish dictionary
- Write action for $\varphi(x,z)$
- Compute field equations
- Impose boundary condition at $z \to 0$
- Impose some kind of regularity condition at $z \to +\infty$
- Evaluate the action on the solution
- Differentiate wrt boundary data

$$O(x) \leftrightarrow \varphi(x.z)$$

$$S_{grav} = -M_p^3 \int d^4x dz \sqrt{g}\left[g^{ab}\partial_a\varphi\partial_b\varphi + m^2\varphi^2\right]$$

$$\Box_g \varphi = m^2 \varphi$$

$$\varphi(x,z) \to z^{(4-\Delta)} J(x)$$

Determine the solution $\varphi_{class}^{(J(x))}(x,z)$

$$S_{on-shell}[J(x)] = S_{grav}[\varphi_{class}^{J(x)}]$$

$$\langle O(x_1)\dots O(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} S_{on-shell}[J(x)]\right]_{J=0}$$

Computing CFT observables from Gravity side

$$Z_{grav}[J(x)] \simeq exp\, iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi\to z^{(4-\Delta)}J}$$

- Establish dictionary $O(x) \leftrightarrow \varphi(x.z)$
- Write action for $\varphi(x,z)$ $$S_{grav} = -M_p^3 \int d^4x dz \sqrt{g}\,[g^{ab}\partial_a\varphi\partial_b\varphi + m^2\varphi^2]$$
- Compute field equations $$\Box_g\varphi = m^2\varphi$$
- Impose boundary condition at $z \to 0$ $$\varphi(x,z) \to z^{(4-\Delta)}J(x)$$
- Impose some kind of regularity condition at $z \to +\infty$

Determine the solution $\varphi_{class}^{(J(x))}(x,z)$

- Evaluate the action on the solution $$S_{on-shell}[J(x)] = S_{grav}[\varphi_{class}^{J(x)}]$$
- Differentiate wrt boundary data $$\langle O(x_1)\dots O(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta J(x_1)}\dots\frac{\delta}{\delta J(x_n)}S_{on-shell}[J(x)]\right]_{J=0}$$

Computing CFT observables from Gravity side

$$Z_{grav}[J(x)] \simeq exp\ iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi\to z^{(4-\Delta)}J}$$

- Establish dictionary

$$O(x) \leftrightarrow \varphi(x.z)$$

- Write action for $\varphi(x,z)$

$$S_{grav} = -M_p^3 \int d^4x dz \sqrt{g}\left[g^{ab}\partial_a\varphi\partial_b\varphi + m^2\varphi^2\right]$$

- Compute field equations

$$\Box_g \varphi = m^2 \varphi$$

- Impose boundary condition at $z \to 0$

$$\varphi(x,z) \to z^{(4-\Delta)} J(x)$$

- Impose some kind of regularity condition at $z \to +\infty$

Determine the solution $\varphi_{class}^{(J(x))}(x,z)$

- Evaluate the action on the solution

$$S_{on-shell}[J(x)] = S_{grav}[\varphi_{class}^{J(x)}]$$

- Differentiate wrt boundary data

$$\langle O(x_1)\dots O(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)} S_{on-shell}[J(x)]\right]_{J=0}$$

Computing CFT observables from Gravity side

$$Z_{grav}[J(x)] \simeq exp\; iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi\to z^{(4-\Delta)}J}$$

- Establish dictionary

$$O(x) \leftrightarrow \varphi(x.z)$$

- Write action for $\varphi(x,z)$

$$S_{grav} = -M_p^3 \int d^4x dz\,\sqrt{g}\,\left[g^{ab}\partial_a\varphi\partial_b\varphi + m^2\varphi^2\right]$$

- Compute field equations

$$\Box_g\varphi = m^2\varphi$$

- Impose boundary condition at $z \to 0$

$$\varphi(x,z) \to z^{(4-\Delta)}J(x)$$

- Impose some kind of regularity condition at $z \to +\infty$

Determine the solution $\varphi_{class}^{(J(x))}(x,z)$

- Evaluate the action on the solution

$$S_{on-shell}[J(x)] = S_{grav}[\varphi_{class}^{J(x)}]$$

- Differentiate wrt boundary data

$$\langle O(x_1)\ldots O(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)}S_{on-shell}[J(x)]\right]_{J=0}$$

Computing CFT observables from Gravity side

$$Z_{grav}[J(x)] \simeq exp\, iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi \to z^{(4-\Delta)} J}$$

- Establish dictionary

$$O(x) \leftrightarrow \varphi(x.z)$$

- Write action for $\varphi(x,z)$

$$S_{grav} = -M_p^3 \int d^4x dz\, \sqrt{g}\, [g^{ab}\partial_a\varphi\partial_b\varphi + m^2\varphi^2]$$

- Compute field equations

$$\Box_g \varphi = m^2 \varphi$$

- Impose boundary condition at $z \to 0$

$$\varphi(x,z) \to z^{(4-\Delta)} J(x)$$

- Impose some kind of regularity condition at $z \to +\infty$

Determine the solution $\varphi_{class}^{(J(x))}(x,z)$

- Evaluate the action on the solution

$$S_{on-shell}[J(x)] = S_{grav}[\varphi_{class}^{J(x)}]$$

- Differentiate wrt boundary data

$$\langle O(x_1)\dots O(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta J(x_1)}\dots\frac{\delta}{\delta J(x_n)} S_{on-shell}[J(x)]\right]_{J=0}$$

Computing CFT observables from Gravity side

$$Z_{grav}[J(x)] \simeq exp\; iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi \to z^{(4-\Delta)} J}$$

- Establish dictionary

$$O(x) \leftrightarrow \varphi(x.z)$$

- Write action for $\varphi(x,z)$

$$S_{grav} = -M_p^3 \int d^4x dz\, \sqrt{g}\, [g^{ab}\partial_a\varphi\partial_b\varphi + m^2\varphi^2]$$

- Compute field equations

$$\Box_g \varphi = m^2 \varphi$$

- Impose boundary condition at $z \to 0$

$$\varphi(x,z) \to z^{(4-\Delta)} J(x)$$

- Impose some kind of regularity condition at $z \to +\infty$

Determine the solution $\varphi_{class}^{(J(x))}(x,z)$

- Evaluate the action on the solution

$$S_{on-shell}[J(x)] = S_{grav}[\varphi_{class}^{J(x)}]$$

- Differentiate wrt boundary data

$$\langle O(x_1)\dots O(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} S_{on-shell}[J(x)]\right]_{J=0}$$

Computing CFT observables from Gravity side

$$Z_{grav}[J(x)] \simeq exp\; iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi\to z^{(4-\Delta)}J}$$

- Establish dictionary

$$O(x) \leftrightarrow \varphi(x.z)$$

- Write action for $\varphi(x,z)$

$$S_{grav} = -M_p^3 \int d^4x dz \sqrt{g}\,[g^{ab}\partial_a\varphi\partial_b\varphi + m^2\varphi^2]$$

- Compute field equations

$$\Box_g \varphi = m^2\varphi$$

- Impose boundary condition at $z\to 0$

$$\varphi(x,z) \to z^{(4-\Delta)}J(x)$$

- Impose some kind of regularity condition at $z\to +\infty$

Determine the solution $\varphi_{class}^{(J(x))}(x,z)$

- Evaluate the action on the solution

$$S_{on-shell}[J(x)] = S_{grav}[\varphi_{class}^{J(x)}]$$

- Differentiate wrt boundary data

$$\langle O(x_1)\ldots O(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)} S_{on-shell}[J(x)]\right]_{J=0}$$

Computing CFT observables from Gravity side

$$Z_{grav}[J(x)] \simeq exp\, iS_{grav}[\varphi_{class}(x,z)]\Big|_{\varphi \to z^{(4-\Delta)}J}$$

- Establish dictionary

$$O(x) \leftrightarrow \varphi(x.z)$$

- Write action for $\varphi(x,z)$

$$S_{grav} = -M_p^3 \int d^4x dz \sqrt{g}\, [g^{ab}\partial_a\varphi\partial_b\varphi + m^2\varphi^2]$$

- Compute field equations

$$\Box_g \varphi = m^2 \varphi$$

- Impose boundary condition at $z \to 0$

$$\varphi(x,z) \to z^{(4-\Delta)} J(x)$$

- Impose some kind of regularity condition at $z \to +\infty$

Determine the solution $\varphi^{(J(x))}_{class}(x,z)$

- Evaluate the action on the solution

$$S_{on-shell}[J(x)] = S_{grav}[\varphi^{J(x)}_{class}]$$

- Differentiate wrt boundary data

$$\langle O(x_1)\ldots O(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta J(x_1)} \cdots \frac{\delta}{\delta J(x_n)} S_{on-shell}[J(x)]\right]_{J=0}$$

Doing away with conformal invariance

Breaking conformal invariance ⟷ Deformations away from AdS

Doing away with conformal invariance

Breaking conformal invariance ⟷ **Deformations away from AdS**

QFT side

Turn on a relevant coupling

$$S = S_{CFT} + g \int d^4x \, O(x)$$

Doing away with conformal invariance

Breaking conformal invariance ⟷ **Deformations away from AdS**

QFT side

Turn on a relevant coupling

$$S = S_{CFT} + g \int d^4x \, O(x)$$

Gravity side

Turn on scalar field $\varphi(z)$ dual to O and require

$$\varphi(z) \to g z^{(4-\Delta)} \qquad z \to 0$$

Doing away with conformal invariance

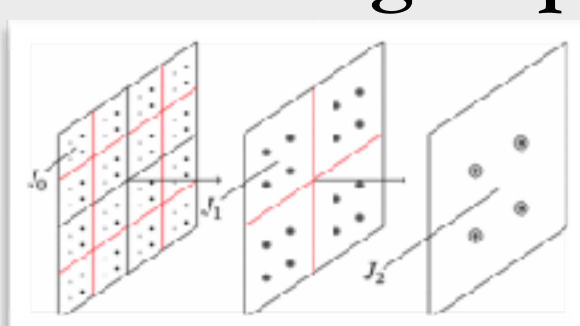
Breaking conformal invariance ⟷ **Deformations away from AdS**

QFT side

Turn on a relevant coupling

$$S = S_{CFT} + g \int d^4x\, O(x)$$

The theory flows to the IR along a **renormalization group flow**



Gravity side

Turn on scalar field $\varphi(z)$ dual to O and require

$$\varphi(z) \to g z^{(4-\Delta)} \qquad z \to 0$$

The solution to the classical equations becomes

$$ds^2 = a^2(z) \left[dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right]$$

holographic RG flow

Doing away with conformal invariance

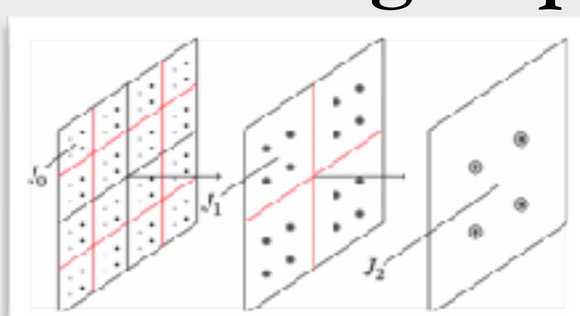
Breaking conformal invariance ⟷ **Deformations away from AdS**

QFT side

Turn on a relevant coupling

$$S = S_{CFT} + g \int d^4x\, O(x)$$

The theory flows to the IR along a **renormalization group flow**



Gravity side

Turn on scalar field $\varphi(z)$ dual to O and require

$$\varphi(z) \to g z^{(4-\Delta)} \qquad z \to 0$$

The solution to the classical equations becomes

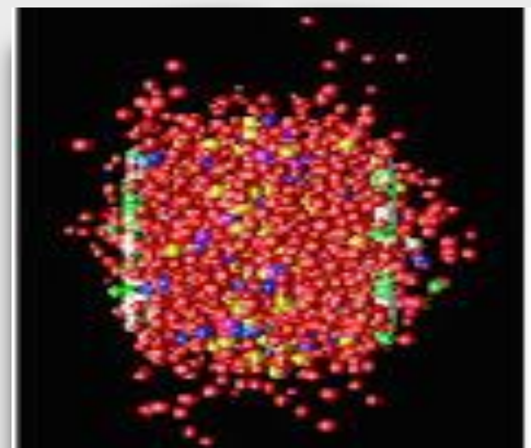
$$ds^2 = a^2(z)\left[dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right]$$

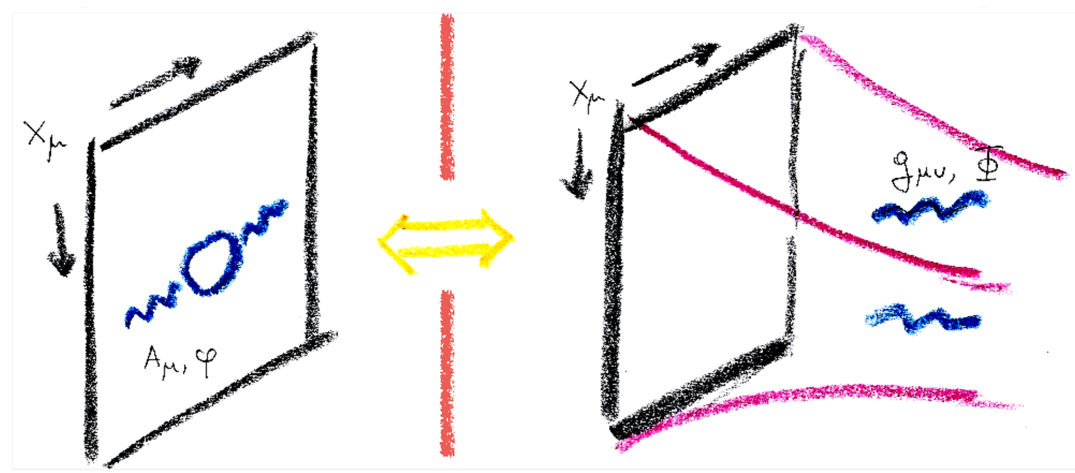
holographic RG flow

Examples of mass gap, confinement, spontaneous symmetry breaking…

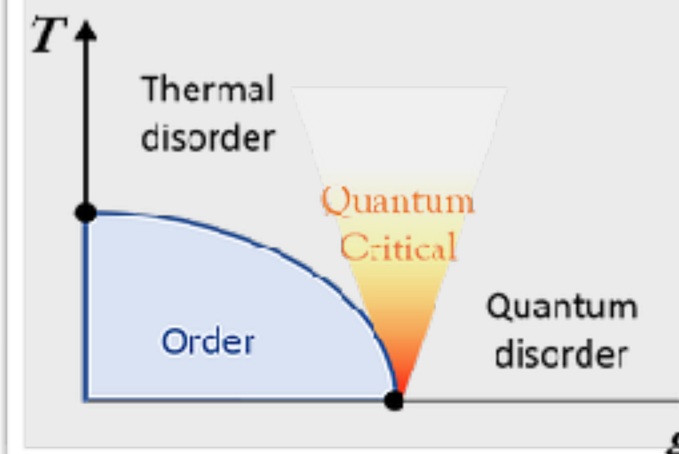
Applications

Theory of strong interactions (QCD)

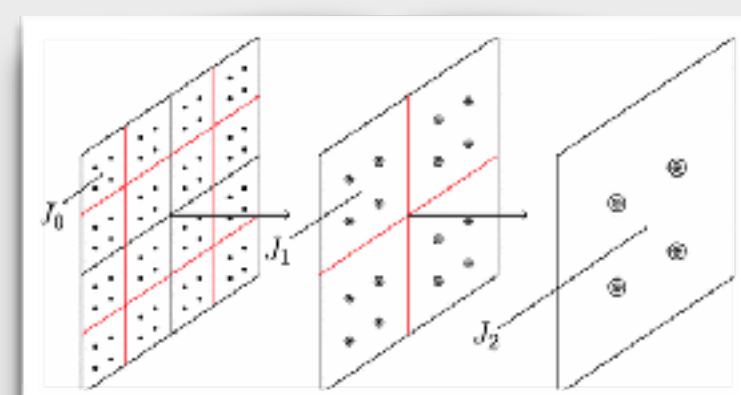




Strongly coupled systems in condensed matter



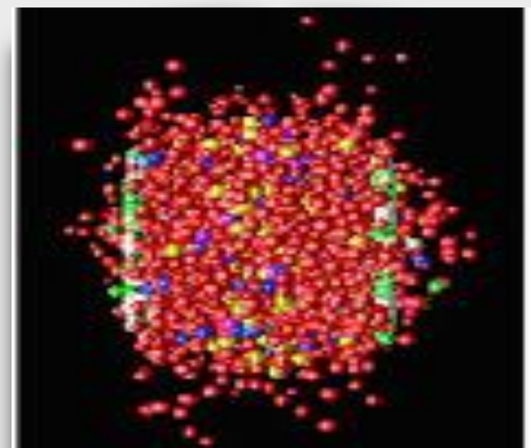
Cosmology and modified gravity

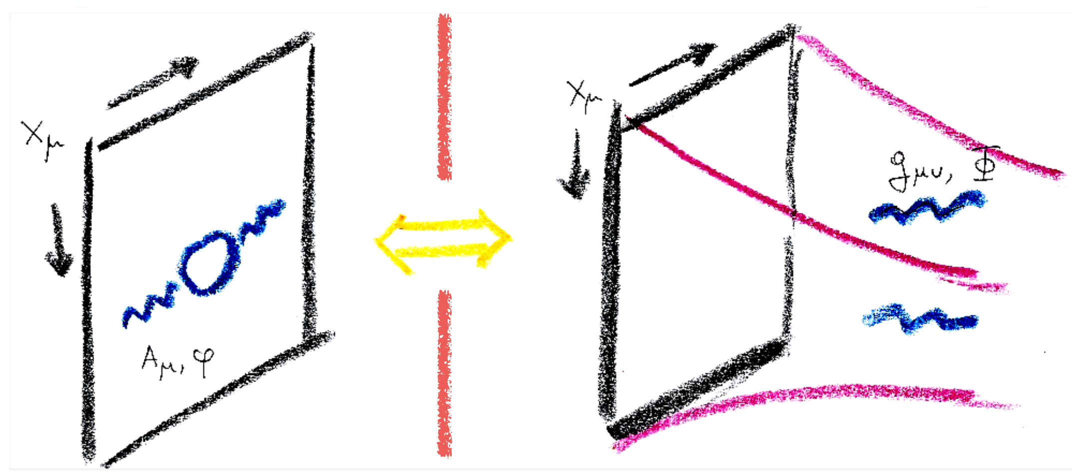


Renormalization group

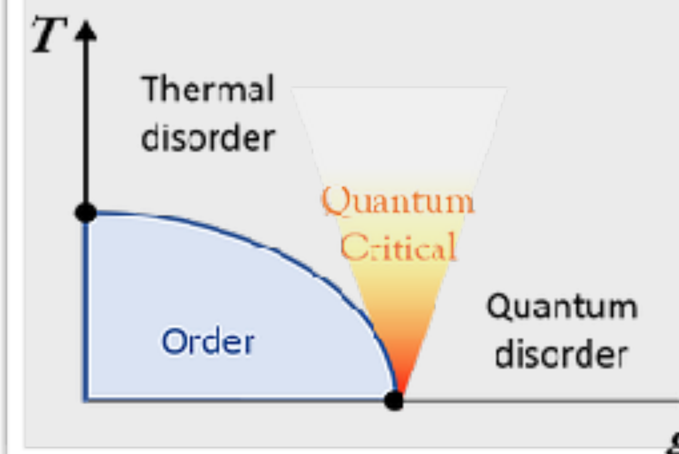
Applications

Theory of strong interactions (QCD)

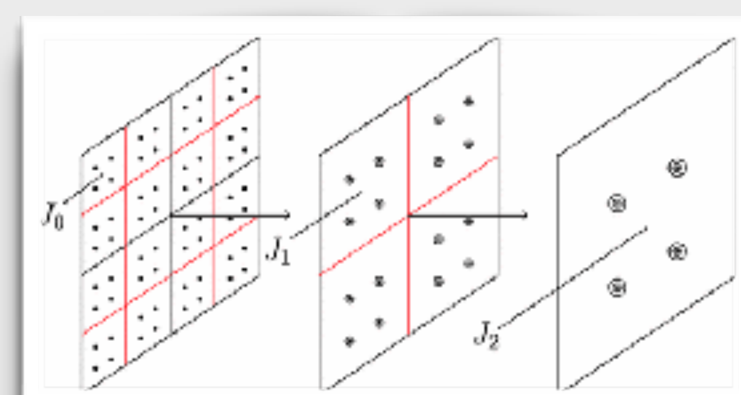




Strongly coupled systems in condensed matter



Cosmology and modified gravity



Renormalization group

The metric and the stress tensor

- The CFT has a **stress tensor** operator: its source is the **boundary metric**

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \delta g^{(0)}_{\mu\nu} \;\Rightarrow\; S_{CFT} \to S_{CFT} + \int d^4x \, T^{\mu\nu} \delta g^{(0)}_{\mu\nu}$$

$\delta g^{(0)}_{\mu\nu}(x)$: boundary value of the gravity side metric $g_{ab}(x, z)$

The metric and the stress tensor

- The CFT has a **stress tensor** operator: its source is the **boundary metric**

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \delta g^{(0)}_{\mu\nu} \;\Rightarrow\; S_{CFT} \to S_{CFT} + \int d^4x \, T^{\mu\nu} \delta g^{(0)}_{\mu\nu}$$

$\delta g^{(0)}_{\mu\nu}(x)$: boundary value of the gravity side metric $g_{ab}(x, z)$

5d metric must be among the dynamical fields

$$S_{grav} \supset \int d^5x \sqrt{g} \, (R - 2\Lambda + \ldots)$$

The metric and the stress tensor

- The CFT has a **stress tensor** operator: its source is the **boundary metric**

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \delta g^{(0)}_{\mu\nu} \;\Rightarrow\; S_{CFT} \to S_{CFT} + \int d^4x \, T^{\mu\nu} \delta g^{(0)}_{\mu\nu}$$

$\delta g^{(0)}_{\mu\nu}(x)$: boundary value of the gravity side metric $g_{ab}(x,z)$

5d metric must be among the dynamical fields

$$S_{grav} \supset \int d^5x \sqrt{g}\,(R - 2\Lambda + \ldots)$$

must be diff-invariant for stress-tensor conservation

The metric and the stress tensor

- The CFT has a **stress tensor** operator: its source is the **boundary metric**

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \delta g^{(0)}_{\mu\nu} \;\Rightarrow\; S_{CFT} \to S_{CFT} + \int d^4x\, T^{\mu\nu} \delta g^{(0)}_{\mu\nu}$$

$\delta g^{(0)}_{\mu\nu}(x)$: boundary value of the gravity side metric $g_{ab}(x,z)$

5d metric must be among the dynamical fields

$$S_{grav} \supset \int d^5x \sqrt{g}\,(R - 2\Lambda + \dots)$$

must be diff-invariant for stress-tensor conservation

Near boundary $z \to 0$

$$ds^2 = \frac{\ell^2}{z^2}\left[dz^2 + g_{\mu\nu}(x,z)dx^\mu dx^\nu\right]$$

$$g_{\mu\nu}(x,z) = g^{(0)}_{\mu\nu}(x) + z^2 g^{(2)}_{\mu\nu}(x) + z^4 \log z\, g^{(4)}_{\mu\nu} + z^4 h^{(4)}_{\mu\nu}(x) + \dots$$

The metric and the stress tensor

- The CFT has a **stress tensor** operator: its source is the **boundary metric**

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \delta g^{(0)}_{\mu\nu} \;\Rightarrow\; S_{CFT} \to S_{CFT} + \int d^4x \, T^{\mu\nu} \delta g^{(0)}_{\mu\nu}$$

$\delta g^{(0)}_{\mu\nu}(x)$: boundary value of the gravity side metric $g_{ab}(x,z)$

5d metric must be among the dynamical fields

$$S_{grav} \supset \int d^5x \sqrt{g} \, (R - 2\Lambda + \ldots)$$

must be diff-invariant for stress-tensor conservation

Near boundary $z \to 0$

$$ds^2 = \frac{\ell^2}{z^2}\left[dz^2 + g_{\mu\nu}(x,z) dx^\mu dx^\nu\right]$$

$$g_{\mu\nu}(x,z) = g^{(0)}_{\mu\nu}(x) + z^2 g^{(2)}_{\mu\nu}(x) + z^4 \log z \, g^{(4)}_{\mu\nu} + z^4 h^{(4)}_{\mu\nu}(x) + \ldots$$

metric on which the QFT lives (arbitrary!)

The metric and the stress tensor

- The CFT has a **stress tensor** operator: its source is the **boundary metric**

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \delta g^{(0)}_{\mu\nu} \;\Rightarrow\; S_{CFT} \to S_{CFT} + \int d^4x\, T^{\mu\nu} \delta g^{(0)}_{\mu\nu}$$

$\delta g^{(0)}_{\mu\nu}(x)$: boundary value of the gravity side metric $g_{ab}(x,z)$

5d metric must be among the dynamical fields

$$S_{grav} \supset \int d^5x \sqrt{g}\,(R - 2\Lambda + \ldots)$$

must be diff-invariant for stress-tensor conservation

Near boundary $z \to 0$

$$ds^2 = \frac{\ell^2}{z^2}\left[dz^2 + g_{\mu\nu}(x,z)dx^\mu dx^\nu\right]$$

$$g_{\mu\nu}(x,z) = g^{(0)}_{\mu\nu}(x) + z^2 g^{(2)}_{\mu\nu}(x) + z^4 \log z\, g^{(4)}_{\mu\nu} + z^4 h^{(4)}_{\mu\nu}(x) + \ldots$$

Determined by $g^{(0)}_{\mu\nu}$ solving Einstein eq order by order

The metric and the stress tensor

- The CFT has a **stress tensor** operator: its source is the **boundary metric**

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \delta g^{(0)}_{\mu\nu} \;\Rightarrow\; S_{CFT} \to S_{CFT} + \int d^4x\, T^{\mu\nu} \delta g^{(0)}_{\mu\nu}$$

$\delta g^{(0)}_{\mu\nu}(x)$: boundary value of the gravity side metric $g_{ab}(x,z)$

5d metric must be among the dynamical fields

$$S_{grav} \supset \int d^5x \sqrt{g}\,(R - 2\Lambda + \ldots)$$

must be diff-invariant for stress-tensor conservation

Near boundary $z \to 0$

$$ds^2 = \frac{\ell^2}{z^2}\left[dz^2 + g_{\mu\nu}(x,z)dx^\mu dx^\nu\right]$$

$$g_{\mu\nu}(x,z) = g^{(0)}_{\mu\nu}(x) + z^2 g^{(2)}_{\mu\nu}(x) + z^4 \log z\, g^{(4)}_{\mu\nu} + z^4 h^{(4)}_{\mu\nu}(x) + \ldots$$

Determined by what happens in the interior (the **state** of the QFT)

The metric and the stress tensor

- The CFT has a **stress tensor** operator: its source is the **boundary metric**

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \delta g^{(0)}_{\mu\nu} \;\Rightarrow\; S_{CFT} \to S_{CFT} + \int d^4x \, T^{\mu\nu} \delta g^{(0)}_{\mu\nu}$$

$\delta g^{(0)}_{\mu\nu}(x)$: boundary value of the gravity side metric $g_{ab}(x, z)$

5d metric must be among the dynamical fields

$$S_{grav} \supset \int d^5x \sqrt{g} \, (R - 2\Lambda + \ldots)$$

must be diff-invariant for stress-tensor conservation

Near boundary $z \to 0$

$$ds^2 = \frac{\ell^2}{z^2}\left[dz^2 + g_{\mu\nu}(x, z) dx^\mu dx^\nu\right]$$

$$g_{\mu\nu}(x, z) = g^{(0)}_{\mu\nu}(x) + z^2 g^{(2)}_{\mu\nu}(x) + z^4 \log z \, g^{(4)}_{\mu\nu} + z^4 h^{(4)}_{\mu\nu}(x) + \ldots$$

Determined by what happens in the interior (the **state** of the QFT)

$$\langle T_{\mu\nu} \rangle = h^{(4)}_{\mu\nu} + \text{stuff depending on } g^{(0)}_{\mu\nu}$$

Holography for curved space QFT

- The **boundary of AdS** is identified with **the space the CFT lives on**
- We can endow it with **any metric**
- We can study **strongly coupled QFTs on curved spacetime** by doing a classical GR calculation

Holography for curved space QFT

- The **boundary of AdS** is identified with **the space the CFT lives on**
- We can endow it with **any metric**
- We can study **strongly coupled QFTs on curved spacetime** by doing a classical GR calculation

- For example: **put QFT on 4d de Sitter**

Holography for curved space QFT

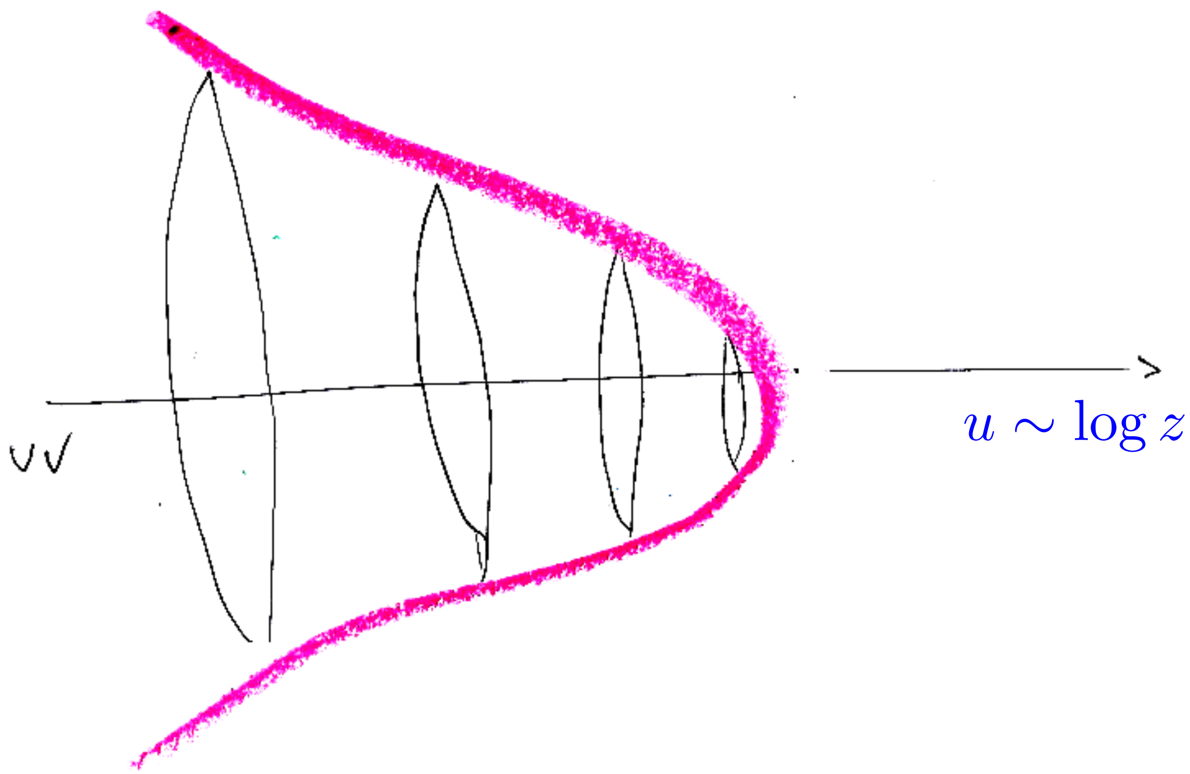
- The **boundary of AdS** is identified with **the space the CFT lives on**
- We can endow it with **any metric**
- We can study **strongly coupled QFTs on curved spacetime** by doing a classical GR calculation

- For example: **put QFT on 4d de Sitter**

AdS_5 can be written in dS_4 slices:

$$ds^2 = du^2 + (\sinh u)^2\, ds_4^2$$

$$ds_4^2 = -d\tau^2 + e^{H\tau} d\vec{x}^2$$



Holography for curved space QFT

- The **boundary of AdS** is identified with **the space the CFT lives on**
- We can endow it with **any metric**
- We can study **strongly coupled QFTs on curved spacetime** by doing a classical GR calculation

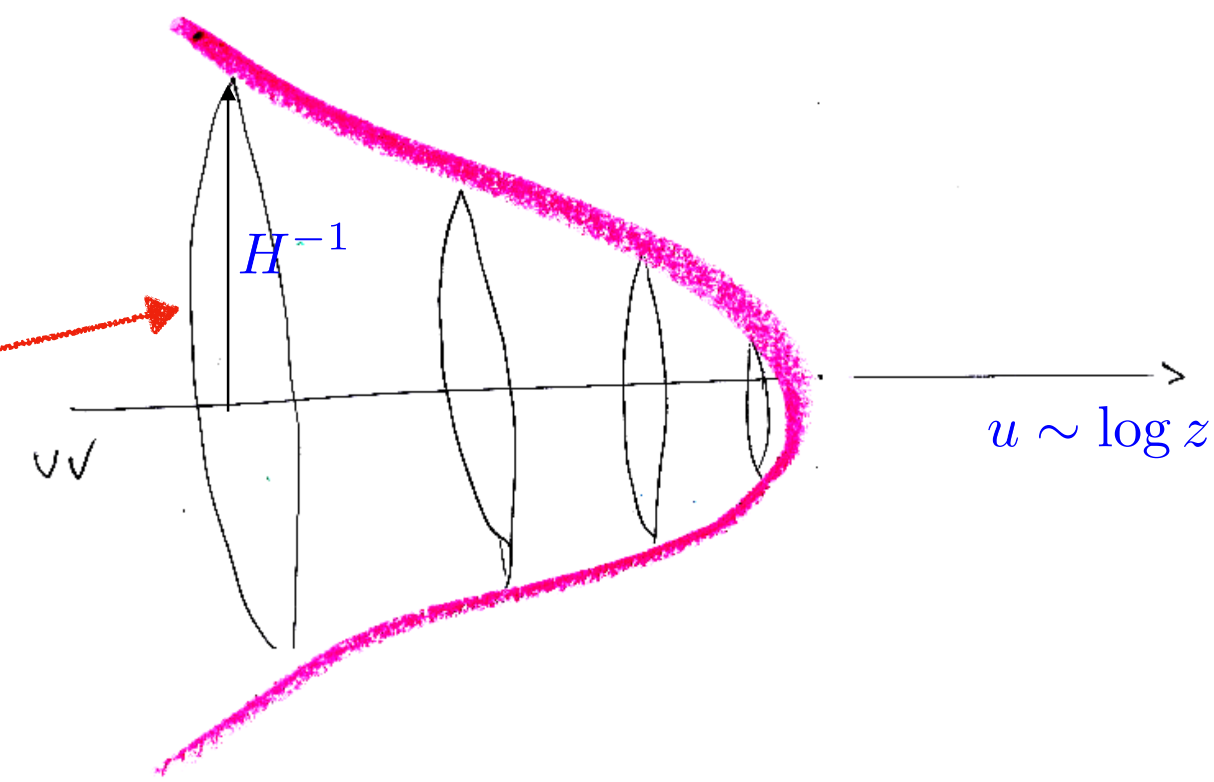
- For example: **put QFT on 4d de Sitter**

AdS_5 can be written in dS_4 slices:

$$ds^2 = du^2 + (\sinh u)^2\, ds_4^2$$

$$ds_4^2 = -d\tau^2 + e^{H\tau} d\vec{x}^2$$

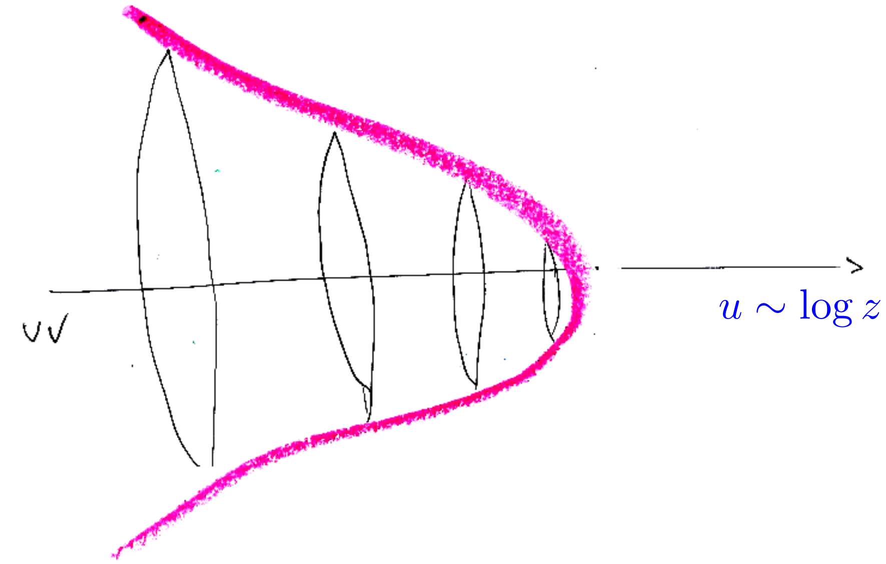
$g_{\mu\nu}^{(0)}$



Holography for curved space QFT

$$ds^2 = du^2 + (\sinh u)^2\, ds_4^2$$

$$ds_4^2 = -d\tau^2 + e^{H\tau} d\vec{x}^2$$



- Now the on-shell action will be a function of the boundary curvature
- One can show that it takes the form:

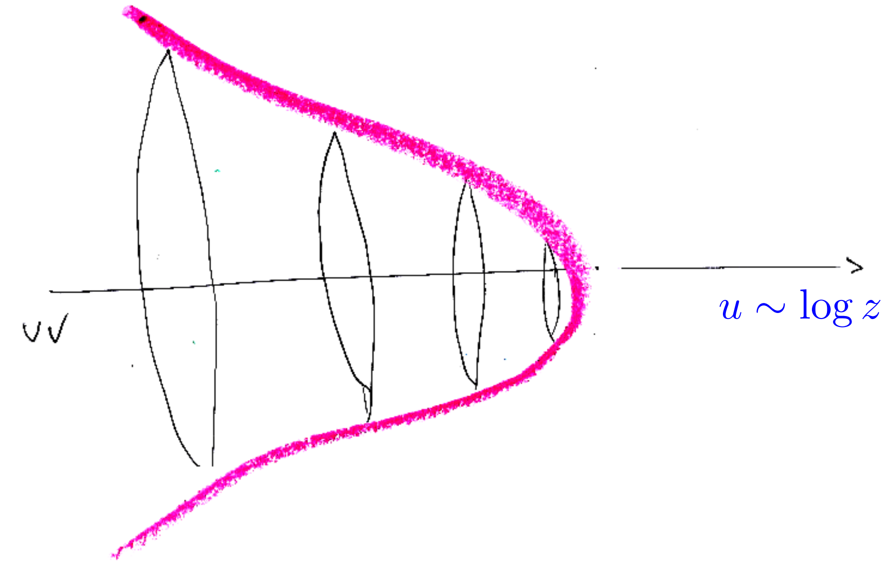
$$S_{on-shell}[g^{(0)}] = \int d^4x \sqrt{g^{(0)}} F(R^{(0)})$$

$$R^{(0)} = 12H^2$$

Holography for curved space QFT

$$ds^2 = du^2 + (\sinh u)^2\, ds_4^2$$

$$ds_4^2 = -d\tau^2 + e^{H\tau} d\vec{x}^2$$



- Now the on-shell action will be a function of the boundary curvature
- One can show that it takes the form:

$$S_{on-shell}[g^{(0)}] = \int d^4x \sqrt{g^{(0)}}\, F(R^{(0)}) \qquad R^{(0)} = 12H^2$$

According to holography, this gives the **full answer** of doing the QFT path integral on a constant curvature spacetime

$$= \log Z_{CFT}[g^{(0)}]$$

Application: QFT backreaction on de Sitter

Take **dynamical 4d gravity** coupled to a 4d QFT which has a holographic description.

$$S_{4d} = M_4^2 \int \sqrt{g^0}(R^{(0)} - 2\lambda_4)$$

Application: QFT backreaction on de Sitter

Take **dynamical 4d gravity** coupled to a 4d QFT which has a holographic description.

$$S_{4d} = M_4^2 \int \sqrt{g^0}(R^{(0)} - 2\lambda_4)$$

Integrating out the QFT gives an effective gravity action of the type $f(R)$

$$S_{eff}[g^{(0)}] = S_{4d}[g^{(0)}] + \int d^4x \sqrt{g^{(0)}} F(R^{(0)})$$

Now solve the classical EOM for constant-curvature metrics

Application: QFT backreaction on de Sitter

Take **dynamical 4d gravity** coupled to a 4d QFT which has a holographic description.

$$S_{4d} = M_4^2 \int \sqrt{g^0}(R^{(0)} - 2\lambda_4)$$

Integrating out the QFT gives an effective gravity action of the type $f(R)$

$$S_{eff}[g^{(0)}] = S_{4d}[g^{(0)}] + \int d^4x \sqrt{g^{(0)}} F(R^{(0)})$$

Now solve the classical EOM for constant-curvature metrics

Equivalent to solving semiclassical 4d Einstein eq: $G^{(0)}_{\mu\nu} = \frac{1}{2M_4^2}\langle T_{\mu\nu}\rangle_{CFT}$

Application: QFT backreaction on de Sitter

Take **dynamical 4d gravity** coupled to a 4d QFT which has a holographic description.

$$S_{4d} = M_4^2 \int \sqrt{g^0}(R^{(0)} - 2\lambda_4)$$

Integrating out the QFT gives an effective gravity action of the type $f(R)$

$$S_{eff}[g^{(0)}] = S_{4d}[g^{(0)}] + \int d^4x \sqrt{g^{(0)}} F(R^{(0)})$$

Now solve the classical EOM for constant-curvature metrics

Equivalent to solving semiclassical 4d Einstein eq: $G^{(0)}_{\mu\nu} = \frac{1}{2M_4^2} \langle T_{\mu\nu} \rangle_{CFT}$

One can start with $g^{(0)}_{\mu\nu}$ having a dS solution and ask whether, after the field theory backreats, dS is **still** a solution (spoiler: generically yes, with smaller H)

J.K.Ghosh, E.Kiritsis, FN, L.Witkowski, ArXiv:2003.09435

Full dS invariance preserved at every step of the calculation

Conclusion

- Holography: a connection between QFT and (quantum) gravity
- Allows one to model strongly coupled QFTs by classical GR
- (In principle) one can learn about QG from perturbative QFT
- Applications to QCD, BSM, modified gravity, cosmology, condensed matter, quantum information theory…

Conclusion

- Holography: a connection between QFT and (quantum) gravity
- Allows one to model strongly coupled QFTs by classical GR
- (In principle) one can learn about QG from perturbative QFT
- Applications to QCD, BSM, modified gravity, cosmology, condensed matter, quantum information theory…

Bibliography

- Aharoni *et al* , ArXiv: hep-th/9905111
- Penedones, ArXiv: 1608.04948
- Nastase, *Introduction to the AdS/CFT correspondence,* 2015 Cambridge University Press