

Quantum of action in entangled relativity

Olivier Minazzoli

ominazzoli@gmail.com

ARTEMIS, Observatoire de la Côte d'Azur, Nice, France

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Action and field equations

Action

$$S = -\frac{\xi}{2c} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R}$$

S: Action [ML^2T^{-1}] ξ dimensionfull constant ($[\xi] = [\kappa]$)

\mathcal{L}_m : matter Lagrangian density

g : spacetime metric determinant

$$\text{GR: } S = \frac{1}{c} \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)$$

Teasing the conclusion for the classical part

$$\boxed{\text{GR:}} \quad \mathcal{L} = \frac{R}{2\kappa} + \mathcal{L}_m \quad \longrightarrow \quad \mathcal{L} = -\frac{\xi}{2} \frac{\mathcal{L}_m^2}{R} \quad \boxed{\text{:ER}}$$

The theory is interesting for the following reasons:

- Same “ingredients” as GR, yet totally different!
- Reduces (or converges) to GR for fairly generic situations.
- **No free** (theoretical) parameter (at the classical level).
- The theory cannot be defined without matter \Rightarrow it does not blatantly violate the *principle of relativity of inertia* that Einstein named *Mach's Principle*.

Action and field equations

Action

$$S = -\frac{\xi}{2c} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad (1)$$

S: Action [ML^2T^{-1}] ξ dimensionfull constant ($[\xi] = [\kappa]$)

\mathcal{L}_m : matter Lagrangian density

g : spacetime metric determinant

$$\text{GR:} \quad S = \frac{1}{c} \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (2)$$

Metric field equation (ξ not in there!)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{R}{\mathcal{L}_m}T_{\mu\nu} + \frac{R^2}{\mathcal{L}_m^2}(\nabla_\mu\nabla_\nu - g_{\mu\nu}\square)\frac{\mathcal{L}_m^2}{R^2} \quad (3)$$

Action and field equations

Matter field equation (\forall tensorial material field χ)

$$\frac{\partial \mathcal{L}_m}{\partial \chi} - \frac{1}{\sqrt{-g}} \partial_\sigma \left(\frac{\partial \sqrt{-g} \mathcal{L}_m}{\partial (\partial_\sigma \chi)} \right) = \frac{\partial \mathcal{L}_m}{\partial (\partial_\sigma \chi)} \frac{R}{\mathcal{L}_m} \partial_\sigma \left(\frac{\mathcal{L}_m}{R} \right) \quad (4)$$

Conservation equation

$$\nabla_\sigma \left(\frac{\mathcal{L}_m}{R} T^{\alpha\sigma} \right) = \mathcal{L}_m \nabla^\alpha \left(\frac{\mathcal{L}_m}{R} \right) \quad (5)$$

$$T_{\mu\nu} \equiv - \frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}} \quad (6)$$

Equivalent actions

Original action (exists provided that $\mathcal{L}_m \neq \emptyset$ in the action)

$$S = -\frac{\xi}{2c} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad (7)$$

Equivalent action (provided that $\mathcal{L}_m \neq \emptyset$ in the action)

$$S = \frac{1}{c} \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (8)$$

κ : dimensionfull scalar field \uparrow Cauchy well-posed \uparrow

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} + \kappa^2 [\nabla_\mu \nabla_\nu - g_{\mu\nu} \square] \frac{1}{\kappa^2} \quad (9)$$

$$\kappa = -\frac{R}{\mathcal{L}_m} \quad \left(\kappa = -\frac{R}{T} \text{ in GR} \right) \quad (10)$$

Equivalent actions

Original action (exists provided that $\mathcal{L}_m \neq \emptyset$ in the action)

$$S = -\frac{\xi}{2c} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad (11)$$

Equivalent action (provided that $\mathcal{L}_m \neq \emptyset$ in the action)

$$S = \frac{1}{c} \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (12)$$

κ : dimensionfull scalar field \uparrow Cauchy well-posed \uparrow

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} + \kappa^2 [\nabla_\mu \nabla_\nu - g_{\mu\nu} \square] \frac{1}{\kappa^2} \quad (13)$$

$$\kappa = -\frac{R}{\mathcal{L}_m} \propto G \quad \left(\kappa = -\frac{R}{T} \propto G \text{ in GR} \right) \quad (14)$$

Conformal transformation

$$\tilde{g}_{\alpha\beta} = e^{-2\varphi/\sqrt{3}} g_{\alpha\beta}$$

$$\kappa = \bar{\kappa} e^{-\varphi/\sqrt{3}}$$

$\bar{\kappa}$: normalisation dimension-full constant ($[\bar{\kappa}] = [G/c^4]$).

With electromagnetic field

$$S = \frac{1}{c} \frac{\xi}{\bar{\kappa}} \int d^4x \sqrt{-\tilde{g}} \times \left[\frac{1}{2\bar{\kappa}} \left(\tilde{R} - 2\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right) - e^{-2\alpha\varphi} \frac{\tilde{F}^2}{2\mu_0} \right] \quad (15)$$

$$\tilde{F}^2 = \tilde{g}^{\alpha\sigma} \tilde{g}^{\beta\epsilon} \tilde{F}_{\sigma\epsilon} \tilde{F}_{\alpha\beta}$$

$$\tilde{F}_{\alpha\beta} := F_{\alpha\beta}$$

Equivalent to usual **Einstein-Maxwell-dilaton theory** with $\alpha = 1/(2\sqrt{3})$. (e.g $\alpha = 1$ for bosonic string & $\alpha = \sqrt{3}$ for 5D KK).

Intrinsic (built-in) decoupling

$$\nabla_\sigma (\kappa^{-1} T^{\alpha\sigma}) = \mathcal{L}_m \nabla^\alpha \kappa^{-1} \quad (16)$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} + \kappa^2 [\nabla_\mu \nabla_\nu - g_{\mu\nu} \square] \frac{1}{\kappa^2} \quad (17)$$

The trace of the metric field equation gives an equation on the scalar dof κ

$$3\kappa^2 \square \frac{1}{\kappa^2} = \kappa (T - \mathcal{L}_m) \quad (18)$$

$\mathcal{L}_m = T \Rightarrow \kappa = \text{cste}$ **can be solution**
 \Rightarrow equations of general relativity

E.g. **Dust with null radiation**

($\mathcal{L}_m = -\rho + F^2/4 = -\rho = T$, since $F^2 \propto E^2 - B^2 = 0$ for null radiation)

Consequences of the intrinsic decoupling

The phenomenology of the theory reduces (or converges) toward the one of general relativity whenever $\mathcal{L}_m = T$ on-shell.

- For a universe made of dust and EM radiation, the scalar degree of freedom freezes and one gets GR back at the cosmological level. Minazzoli [2021]
- Neutron stars are at max a few percent different from the ones of GR. Arruga and Minazzoli [2021], Arruga et al. [2021]
- Exterior of (spherical) black holes cannot be distinguished from the ones of GR in astrophysical conditions. Minazzoli and Santos [2021]
- Gravitational waves emitted from the fusion of black-holes are indistinguishable from the ones of GR. Hirschmann et al. [2018], Khalil et al. [2018]

Conclusion for the classical part

$$\boxed{\text{GR:}} \quad \mathcal{L} = \frac{R}{2\kappa} + \mathcal{L}_m \quad \longrightarrow \quad \mathcal{L} = -\frac{\xi}{2} \frac{\mathcal{L}_m^2}{R} \quad \boxed{\text{:ER}}$$

The theory is interesting for the following reasons:

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Path integral formulation

Action of entangled relativity

$$S = -\frac{\xi}{2c} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad (19)$$

Path integral

$$\mathcal{Z} = \int [\mathcal{D}F_1 \dots \mathcal{D}F_N] e^{\pm i\theta} \quad (20)$$

where the quantum phase is $\theta = S/\alpha$ (21)

α : quantum of action $\neq \hbar$ a priori.

Semi-classical limits where the background of $\kappa \sim \text{constant}$

$$\theta = \frac{1}{\alpha c} \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)$$

Semi-classical limits where the background of $\kappa \sim \text{constant}$

$$\theta = \frac{1}{\alpha c} \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (22)$$

$$\sim \boxed{\frac{\xi}{\kappa \alpha c} \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)} \quad (23)$$

$$(24)$$

Semi-classical limits where the background of $\kappa \sim \text{constant}$

$$\theta = \frac{1}{\alpha c} \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (25)$$

$$\sim \boxed{\frac{\xi}{\kappa \alpha c} \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)} \quad (26)$$

$$=: \frac{1}{c \hbar} \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (27)$$

Semi-classical limits where the background of $\kappa \sim \text{constant}$

$$\theta = \frac{1}{\kappa \alpha c} \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (28)$$

$$\sim \boxed{\frac{\xi}{\kappa \alpha c} \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right)} \quad (29)$$

$$=: \frac{1}{c \hbar} \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (30)$$

$$\Rightarrow \boxed{\hbar := \frac{\kappa \alpha}{\xi}} \quad (31)$$

$$\boxed{\text{Which means: } \hbar \propto G \text{ !!!}} \quad (32)$$

Final conclusion

Check out the paper (2206.03824)

$$\boxed{\text{CT:}} \quad \theta = \frac{1}{c\hbar} \int d_g^4 x \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \rightarrow \theta = -\frac{1}{2\varepsilon^2} \int d_g^4 x \frac{\mathcal{L}_m^2}{R} \quad \boxed{\text{:ER}}$$

The theory is interesting for the following reasons:

- Same “ingredients” as GR, yet totally different!
- Reduces (or converges) to GR for fairly generic situations.
- The theory cannot be defined without matter \Rightarrow it does not blatantly violate the *principle of relativity of inertia* that Einstein named *Mach's Principle*.
- Reduces the number of universal constants by one !
- Links G and \hbar ! (and this is testable in principle !!!)
- Opens up an entire world of possibilities/questions.

We were looking for a PhD student... and still are...
Contact me: ominazzoli@gmail.com

Date limite de candidature (à 23h59) 1 juin 2022

 Version Française

 English Version

Gravitational and quantum phenomenology of entangled relativity

Keywords

Relativistic gravitation, Fundamental physics, Quantum field theory, Neutron stars

Profile and skills required

Background in theoretical or astrophysical physics. Comfortable with general relativity. Ideally with basis in data analysis.

Project description

Despite its incredible successes, there exist several indications already that general relativity may actually be a limit of an even more general theory. As for Newtonian gravity in the early 19's – which was fundamentally not compatible with the laws of electromagnetism and special relativity – one has to dig into the inconsistencies of general relativity to come to that conclusion. The first indication that comes to mind – and which was notably celebrated by the Nobel committee in 2020 by giving the Nobel prize in

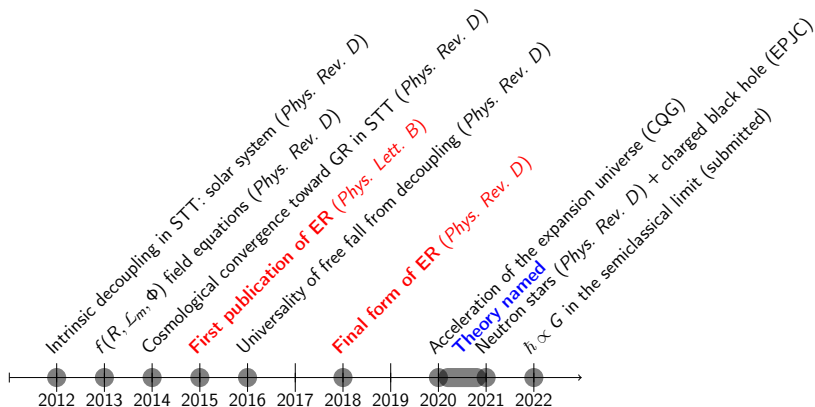
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Chronology of entangled relativity



Arruga and Minazzoli [2021], Arruga et al. [2021], Harko et al. [2013], Ludwig et al. [2015], Minazzoli [2014, 2018, 2021], Minazzoli and Hees [2013, 2014, 2016], Minazzoli and Santos [2021]

Semi-classical limits where the background of $\kappa \sim$ constant

$$\hbar = \frac{\kappa \mathbf{a}}{\xi} \Rightarrow \boxed{\frac{c\hbar}{\kappa} =: \varepsilon^2 := \frac{c\mathbf{a}}{\xi}} \quad (33)$$

where ε is the Plank energy.

Only one dimensional constant in the quantum phase:
Planck energy

$$\theta = -\frac{1}{2\varepsilon^2} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad (34)$$

or

$$\theta = \int d^4x \sqrt{-g} \left(\frac{R}{2\ell^2} + \frac{\mathcal{L}_m}{\ell\varepsilon} \right) \quad (35)$$

with $\ell := \kappa\varepsilon$.

Remark 1: comparison of κ

Equations of entangled relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu} + \kappa^2 [\nabla_\mu \nabla_\nu - g_{\mu\nu}\square] \frac{1}{\kappa^2} \quad (36)$$

$$3\kappa^2\square\frac{1}{\kappa^2} = \kappa(T - \mathcal{L}_m) \quad (37)$$

$$\boxed{\kappa = -\frac{R}{\mathcal{L}_m}} \quad (38)$$

Equations of general relativity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu} \quad (39)$$

$$\boxed{\kappa = -\frac{R}{T}} \quad (40)$$

Remark 2: More than Lagrangian equivalence \rightarrow algebraic equivalence

Both actions

$$S = -\frac{\xi}{2c} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad (41)$$

$$S = \frac{1}{c} \int d^4x \sqrt{-g} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) \quad (42)$$

$$\begin{aligned} \frac{\xi}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right) &= \xi \left(\frac{R}{2\kappa^2} + \frac{\mathcal{L}_m}{\kappa} \right) \\ &= \xi \left(\frac{\mathcal{L}_m^2}{2R} - \frac{\mathcal{L}_m^2}{R} \right) = -\frac{\xi}{2c} \frac{\mathcal{L}_m^2}{R} \end{aligned} \quad (43)$$

Inertia can be defined *ex nihilo* in general relativity: violation of Einstein-Mach principle

Einstein believed in the *relativity of inertia*

“c. Mach’s Principle. [Spacetime] is completely determined by [matter] [...]. With (c), according to the field equations of gravitation, **there can be no [spacetime] without matter.**” Einstein [1918a]

To the press during his first visit in the US in 1921:

“It was formerly believed that if all material things disappeared out of the universe, time and space would be left. According to relativity theory, however, time and space disappear together with the things.”
Robinson [2018]

The actual reason for the cosmological constant

The cosmological constant was meant (but failed) to satisfy Mach's Principle of Einstein

Response to the paper of de Sitter. Einstein [1918b]

"If the de Sitter solution were valid everywhere, it would show that the introduction of the λ -term does not fulfill the purpose I intended. Because, in my opinion, the general theory of relativity is a satisfying system only if it shows that the physical qualities of space are *completely* determined by matter alone. Therefore no $g_{\mu\nu}$ -field must exist (that is, no space-time continuum is possible) without matter that generates it."

Actual metric field equation

From the action ($\forall \mathcal{L}_m$ on-shell)

$$\frac{\mathcal{L}_m^2}{R^2} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = -\frac{\mathcal{L}_m}{R} T_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \frac{\mathcal{L}_m^2}{R^2} \quad (44)$$

$$\times \frac{R^2}{\mathcal{L}_m^2} \quad (\forall \mathcal{L}_m \neq 0 \text{ on-shell}) \quad (45)$$

Only $\forall \mathcal{L}_m \neq 0$ on-shell

$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{R}{\mathcal{L}_m} T_{\mu\nu} + \frac{R^2}{\mathcal{L}_m^2} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \frac{\mathcal{L}_m^2}{R^2} \quad (46)$$

Equivalent actions

Original action

$$S = -\frac{\xi}{2} \int d^4x \sqrt{-g} \frac{\mathcal{L}_m^2}{R} \quad (47)$$

Equivalent action (provided that $\mathcal{L}_m \neq \emptyset$ in the action)

$$S = \int d^4x \sqrt{-g} \frac{\xi}{\bar{\kappa}} \left(\theta^2 \frac{R}{2\bar{\kappa}} + \theta \mathcal{L}_m \right) \quad (48)$$

κ : dimensionfull scalar field

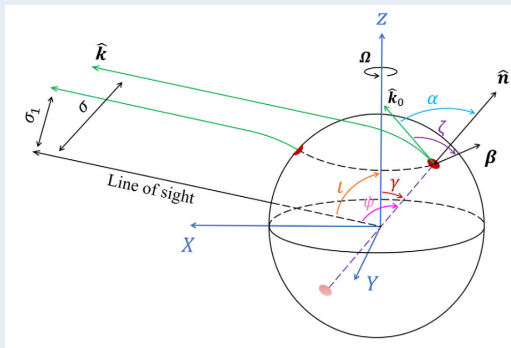
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \bar{\kappa} \frac{T_{\mu\nu}}{\theta} + \frac{1}{\theta^2} [\nabla_\mu \nabla_\nu - g_{\mu\nu} \square] \theta^2 \quad (49)$$

$$\theta = -\bar{\kappa} \frac{\mathcal{L}_m}{R} \quad (50)$$

Neutron stars

Toward experimental tests

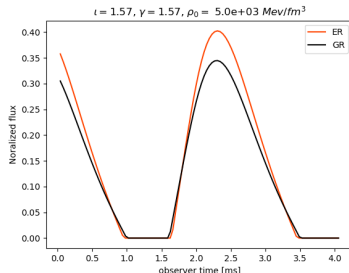
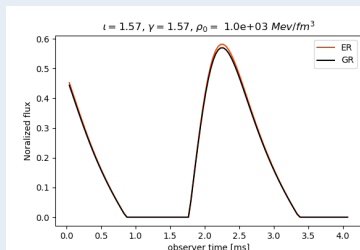
Schematic illustration for the x-rays emitted from a hot spot on a rotating NS and reaching the observer at infinity



Xu et al. [2020]

Next step: simulate X-ray pulse profiles (to be eventually tested with the NICER instrument)

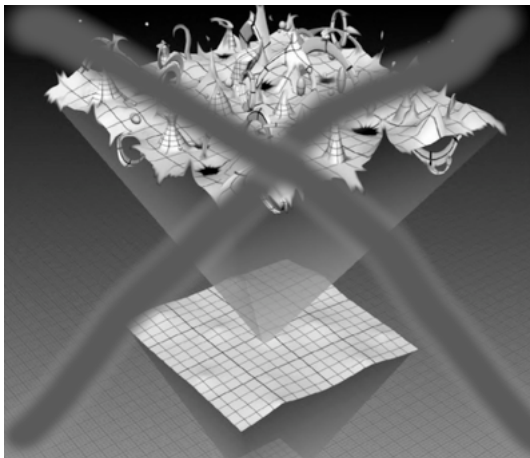
Preview: unpublished results of Denis Arruga



Takes into account the non-conservation of the photon number close to the neutron star, which follows from $\nabla_\nu (\sqrt{\Phi} F^{\mu\nu}) = 0$. Same assumptions as in Silva and Yunes [2019] otherwise.

Planck scale

No reason (a priori) to expect weird spacetime topology close to Planck scale



Spacetime might still be well-behaved in the deep UV of matter