Big-bang nucleosynthesis

Cyril Pitrou

TUG, Montpellier, 5/10/2022

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The Origin of Chemical Elements

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AND

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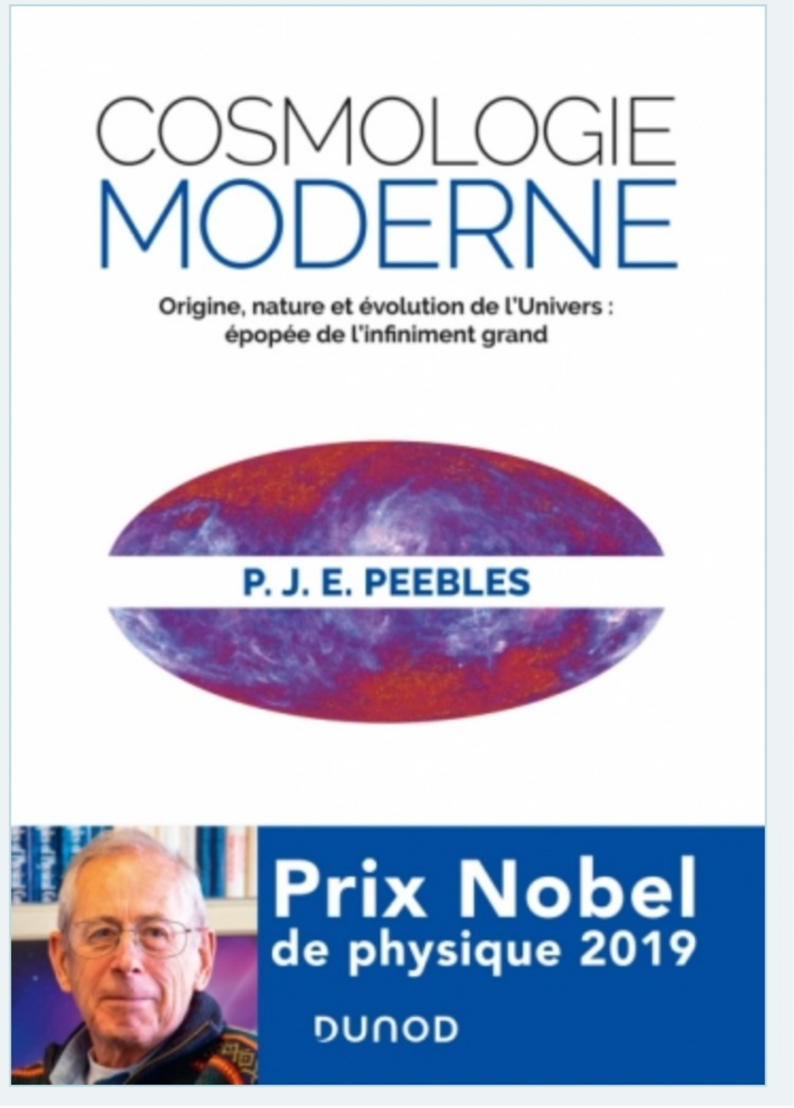
AND

G. GAMOW
The George Washington University, Washington, D. C.

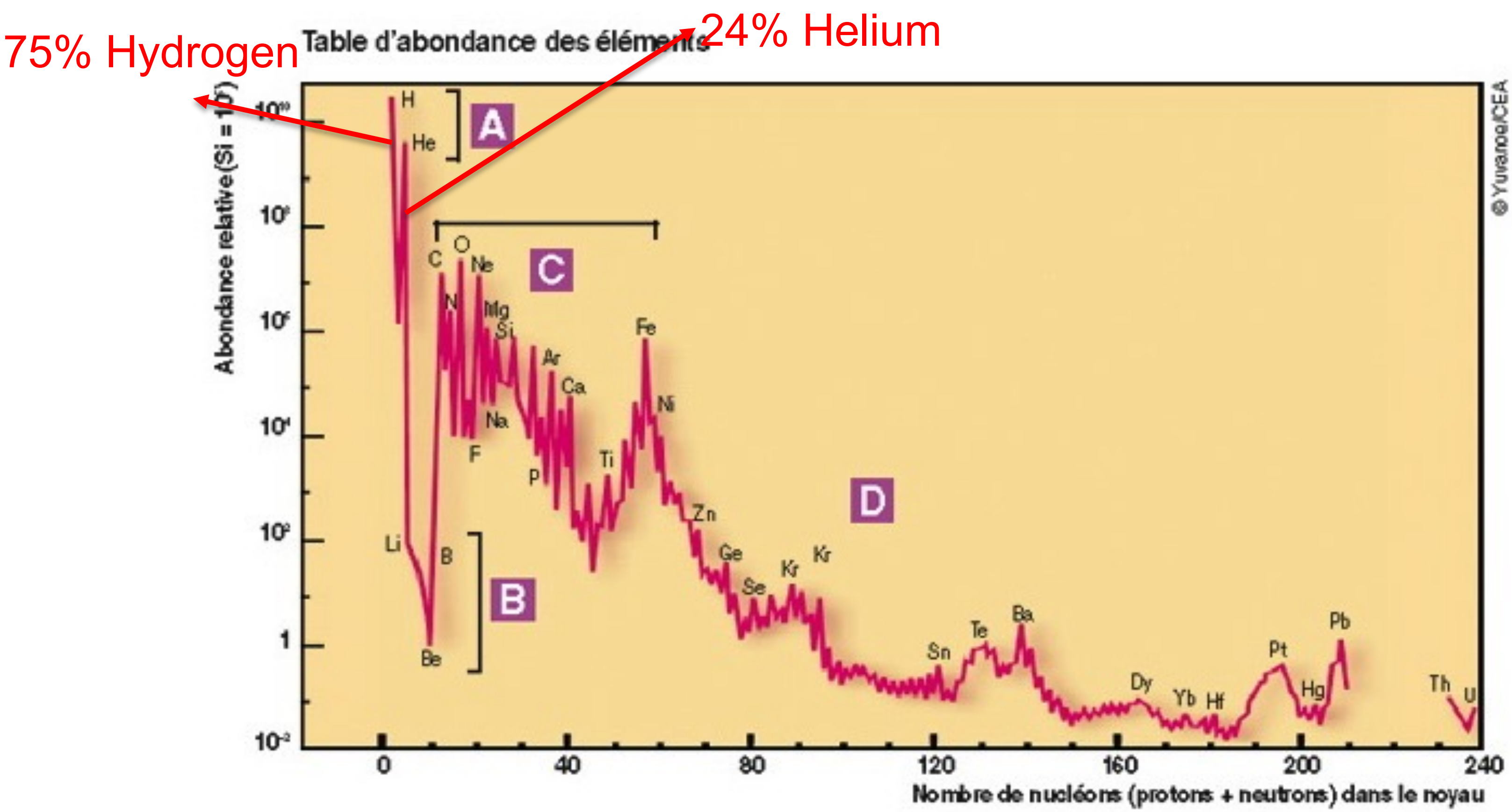
February 18, 1948

To know more about the History of BBN :



❑ Origin of elements

1. Very light elements : Primordial Universe
2. Some light elements : spallation from cosmic rays
3. Heavy elements : stars and star explosions



Outline

1. BBN in a nutshell
2. The hunt for precision
3. Cosmological parameters

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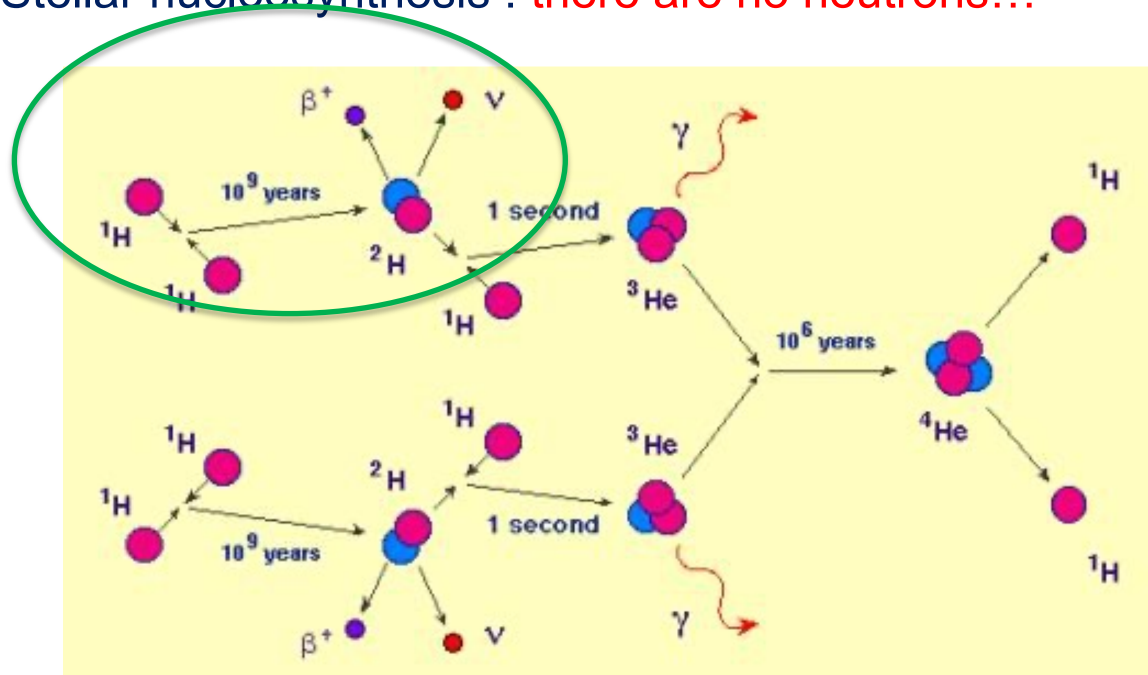
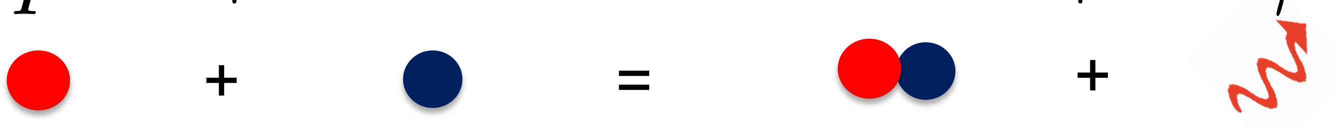
❑ Why primordial synthesis ?

1. Stellar nucleosynthesis : there are no neutrons…



2. Primordial nucleosynthesis : there was plenty of neutrons !

$$p + n \leftrightarrow D + \gamma$$



2.2 MeV

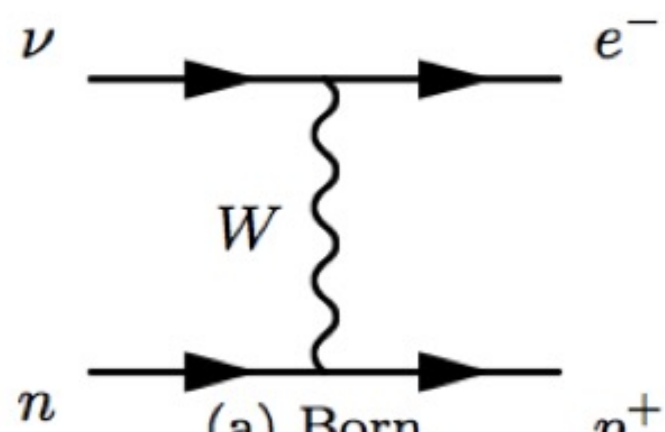
When does it happen ? How many neutrons were available ?

❑ Neutron/proton conversions

$$n + \nu_e \longleftrightarrow p + e^-$$

$$n \longleftrightarrow p + e^- + \bar{\nu}_e$$

$$n + e^+ \longleftrightarrow p + \bar{\nu}_e$$



(a) Born

- If enough interactions, then statistical equilibrium

$$n = e^{-\frac{E}{k_B T}}$$

Baryons are non-relativistic : $E \simeq m$

$$m_p = 938.2\,\mathrm{MeV}$$

$$m_n = m_p + 1.3\,\mathrm{MeV}$$

Protons $n_p = e^{-m_p/T}$

Neutrons $n_n = e^{-m_n/T} = n_p e^{-(m_n - m_p)/T}$

❑ Friedmann equation in radiation era

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$= \frac{8\pi G}{3}\left(\frac{\rho^{\text{today}}_{\text{matter}}}{a^3} + \frac{\rho^{\text{today}}_{\text{rad}}}{a^4}\right)$$

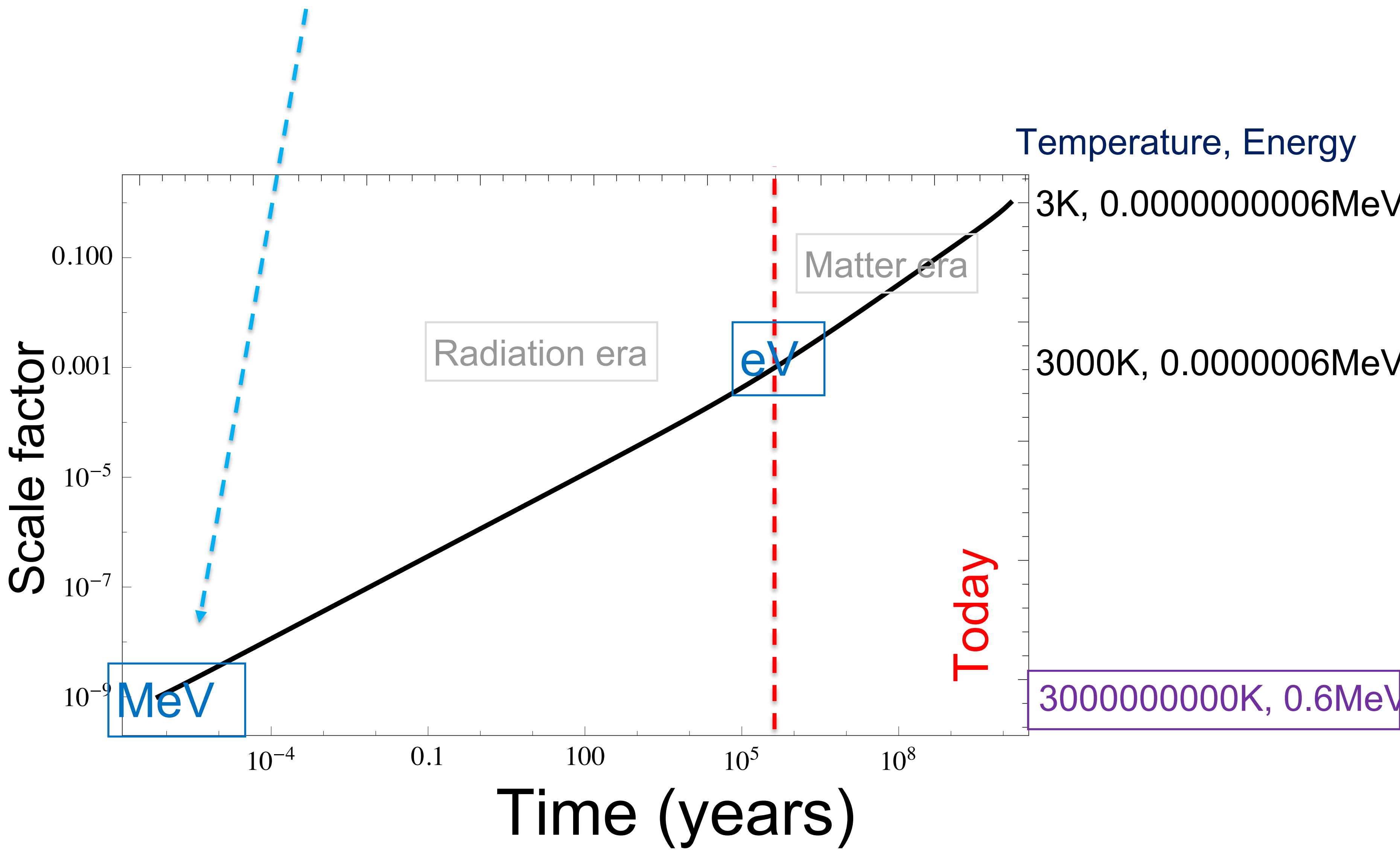
(The matter term $\frac{\rho^{\text{today}}_{\text{matter}}}{a^3}$ is crossed out in red.)

Radiation dominated universe $a \propto t^{1/2}$

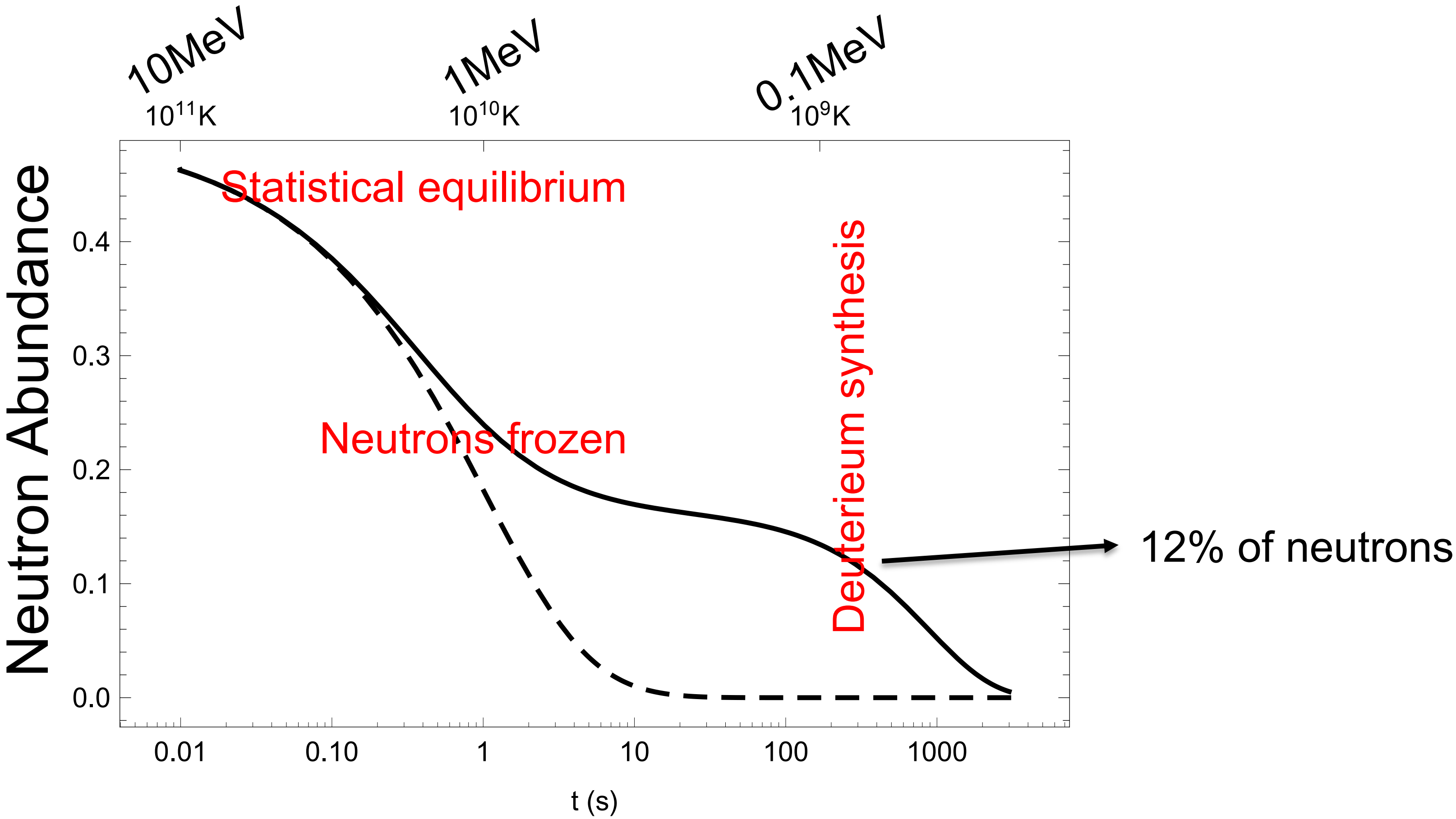
$$\rho_{\text{rad}} \propto T^4 \propto a^{-4}$$

$$\boxed{T^2 \propto 1/t}$$

❑ The MeV scale

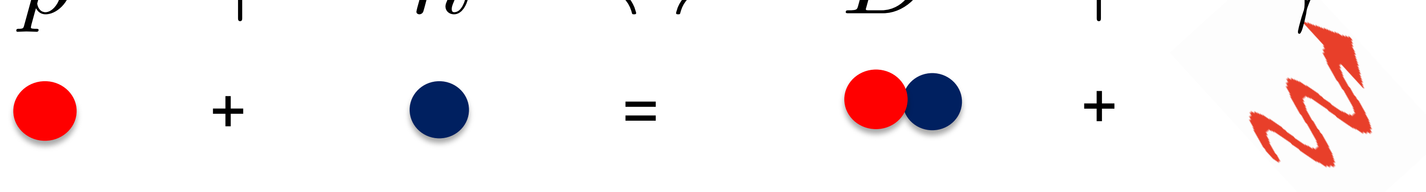


Evolution of neutrons

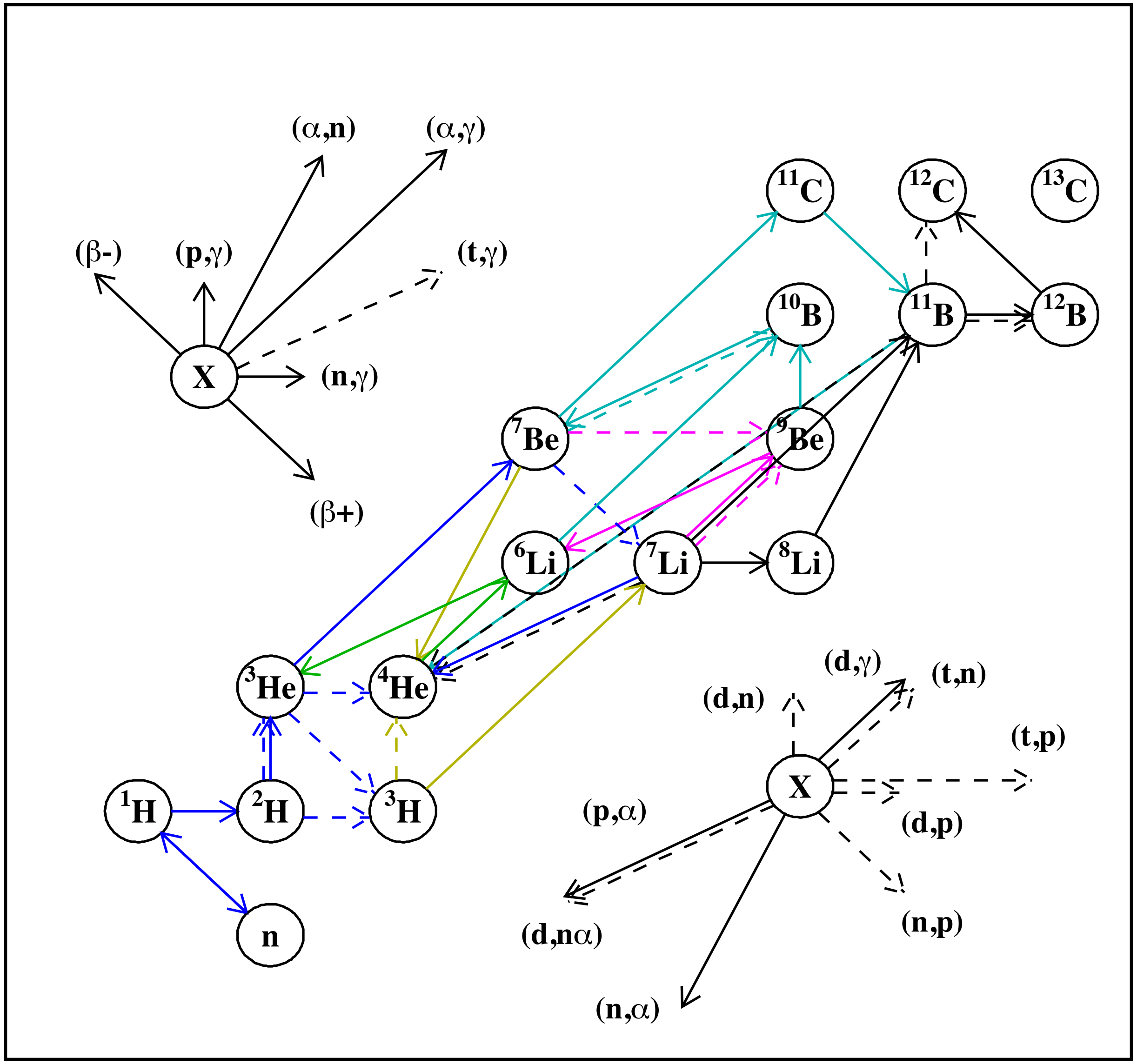


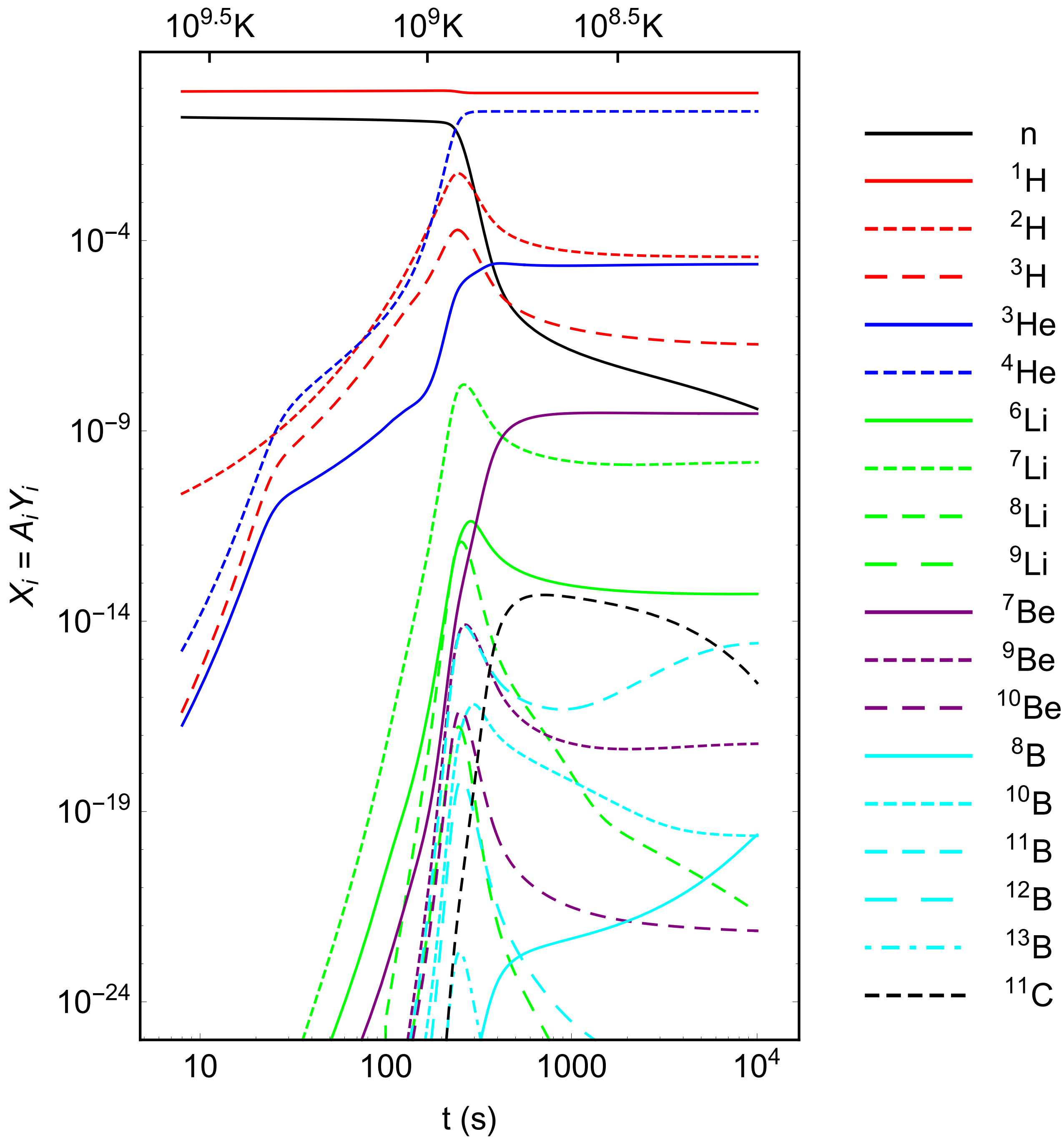
Deuterium

$$p + n \leftrightarrow D + \gamma$$



Nuclear network



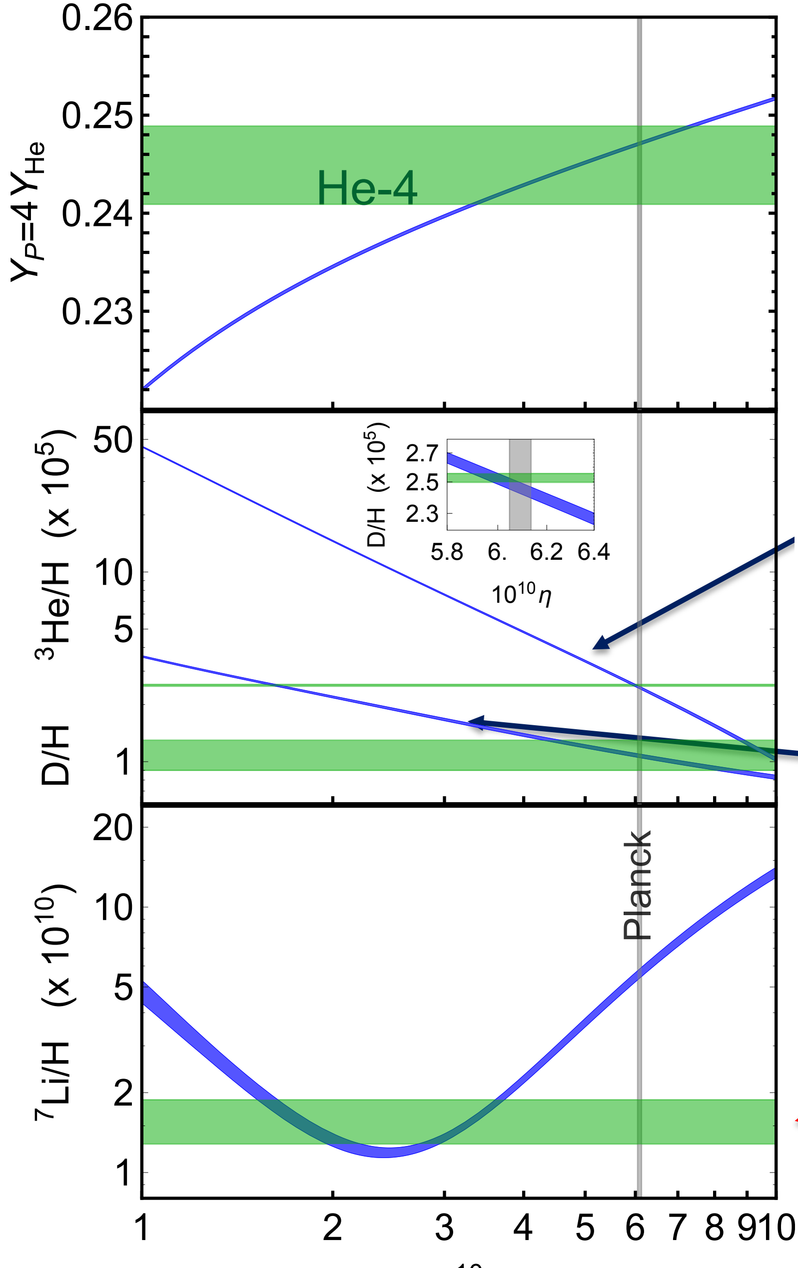


Public BBN codes:

PRIMAT

AlterBBN

PArthENoPE



Baryons abundance

Aver et al. 2020

$$Y_p = 0.2453 \pm 0.0034$$ 1.4 %

Deuterium

$$\mathrm{D/H} = (2.527 \pm 0.030) \times 10^{-5}$$ 1.2 %

Cooke et al. 2016

He-3

Li-7

Lithium problem

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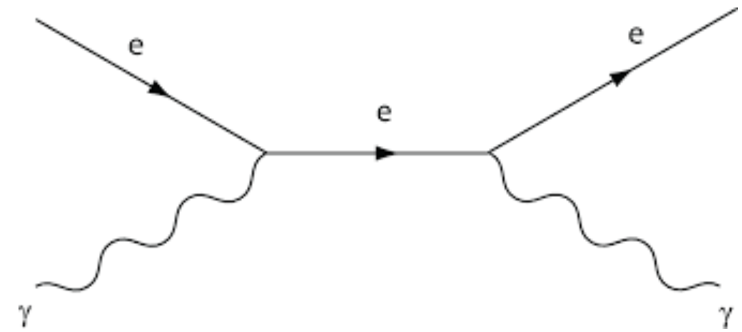
Hunt for precision

- Plasma physics
- Neutrino physics
- Weak rates
- Nuclear rates

❑ QED reactions :



- Momentum exchange :

$$\gamma + e^{\pm} \leftrightarrow \gamma + e^{\pm}$$

- Annihilations

$$\gamma + \gamma \leftrightarrow e^{\pm} + e^{\mp}$$

Collisions

$$\begin{cases} \dot{\rho}_{e^\pm} + 3H(\rho_{e^\pm} + P_{e^\pm}) = \dot{q}_{e^\pm} \\ \dot{\rho}_\gamma + 4H(\rho_\gamma) = \dot{q}_\gamma \end{cases}$$

❑ Temperature evolution, *method 1*

$$\dot{q}_\gamma + \dot{q}_{e^\pm} = 0$$

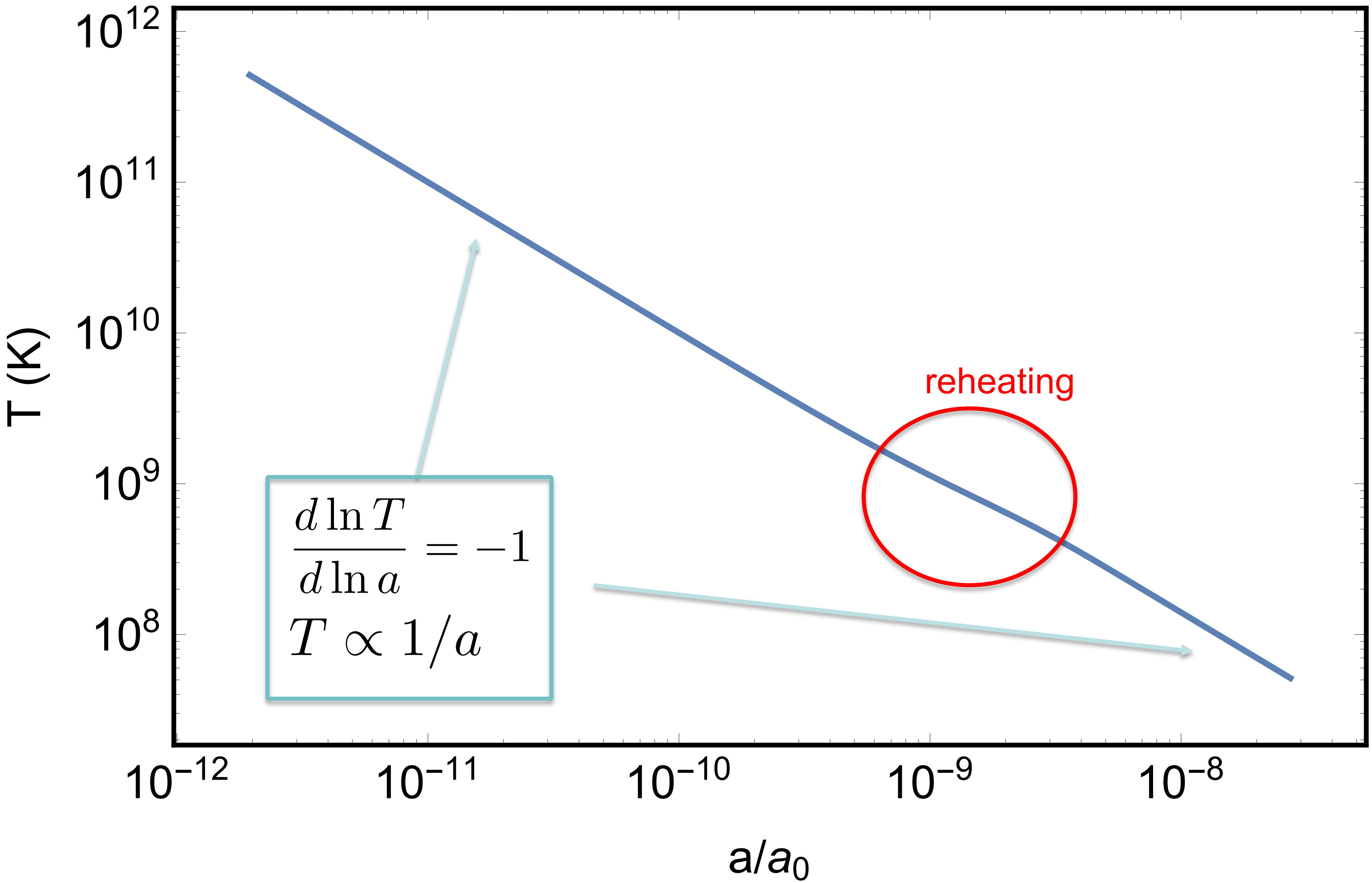
$$\dot{\rho}_{e^\pm} + \dot{\rho}_\gamma + 3H(\rho_{e^\pm} + P_{e^\pm}) + 4H\rho_\gamma = 0$$

1. Using that thermodynamic quantites are functions of temperature only

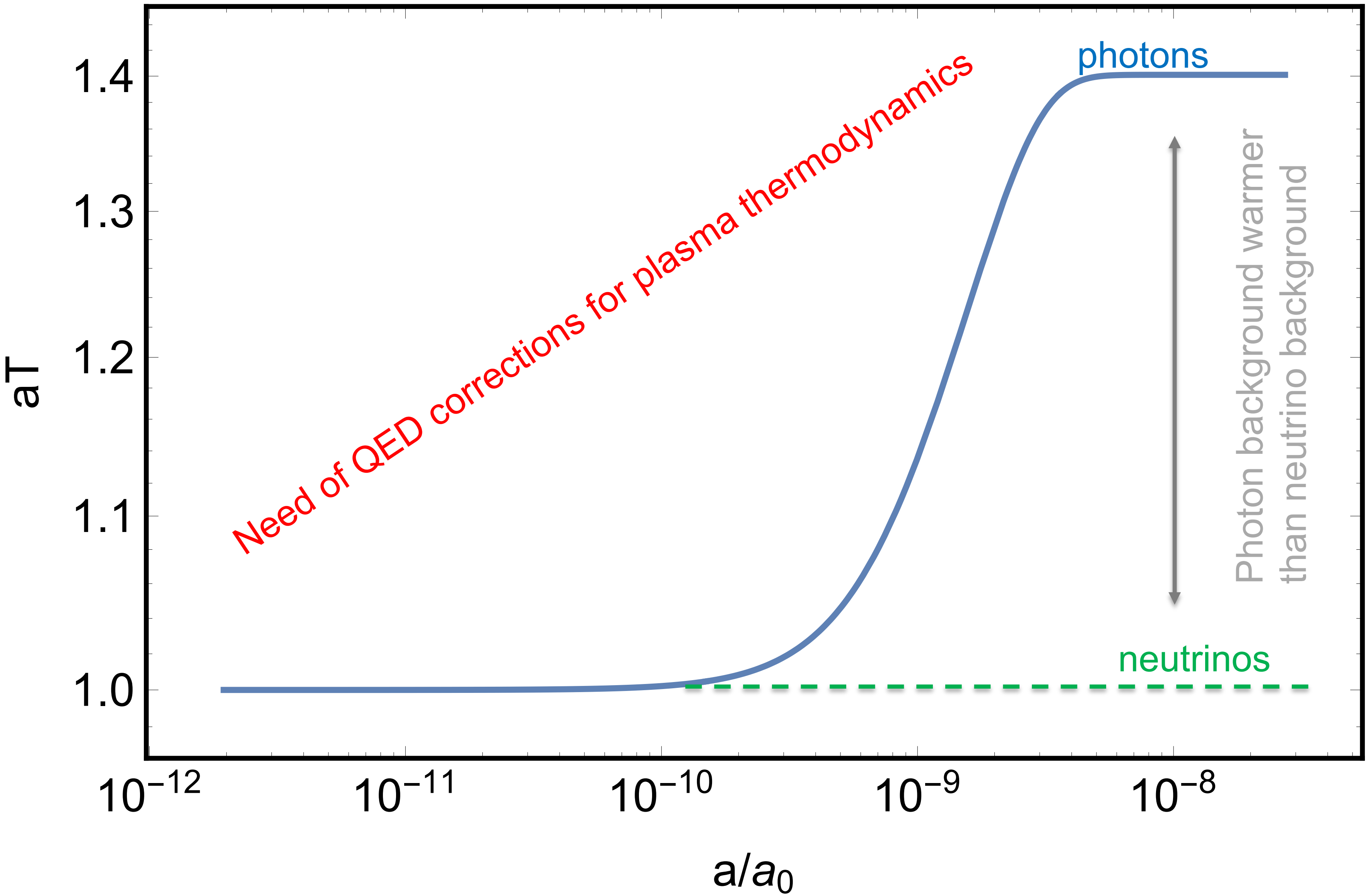
$$\dot{\rho} = \dot{T} d\rho(T)/dT$$

2. Trading time for scale factor with : $H = \frac{d \ln a}{dt}$

$$\frac{d \ln T}{d \ln a} = -\frac{3[\rho_{e^\pm}(T) + P_{e^\pm}(T)] + 4\rho_\gamma(T)}{T[\rho'_{e^\pm}(T) + \rho'_\gamma(T)]}$$



T (K)
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
a/a_0
reheating
$\frac{d \ln T}{d \ln a} = -1$
$T \propto 1/a$



photons
Need of QED corrections for plasma thermodynamics
Photon background warmer than neutrino background
neutrinos
aT
1.4
1.3
1.2
1.1
1.0
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
a/a_0

Hunt for precision

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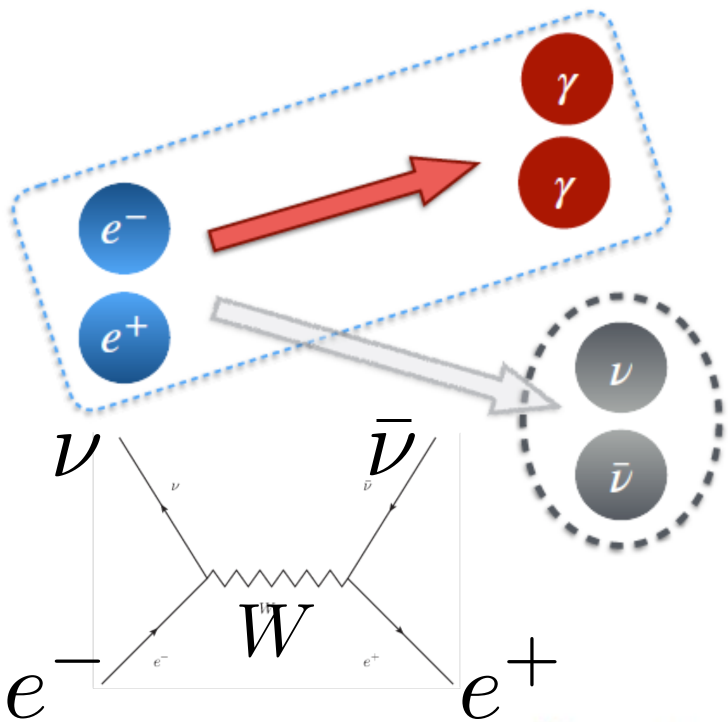
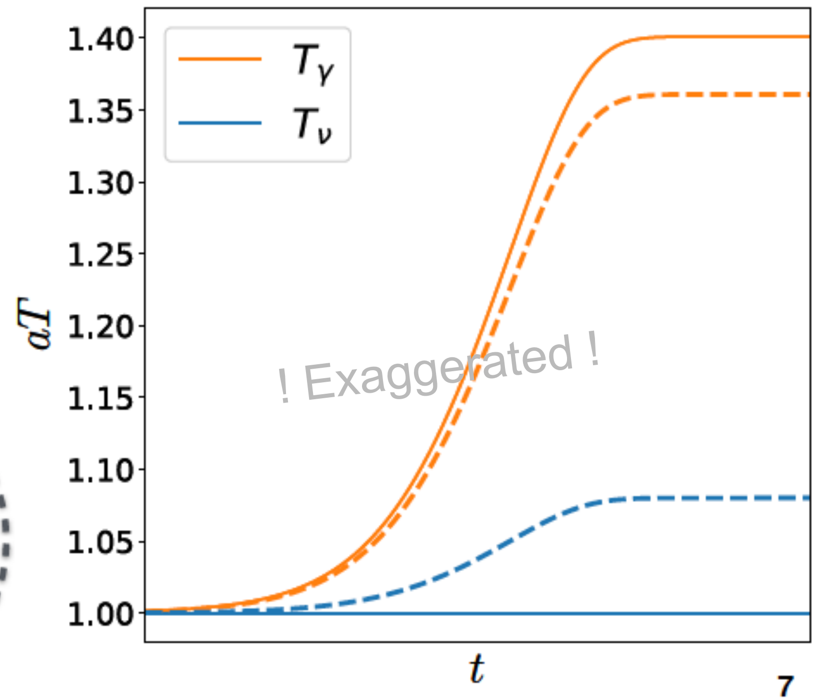
Beyond the instantaneous decoupling approximation

- Overlap between decoupling and $e^{\pm}$ annihilations

  $\Longrightarrow$ smaller T_γ and increased T_ν





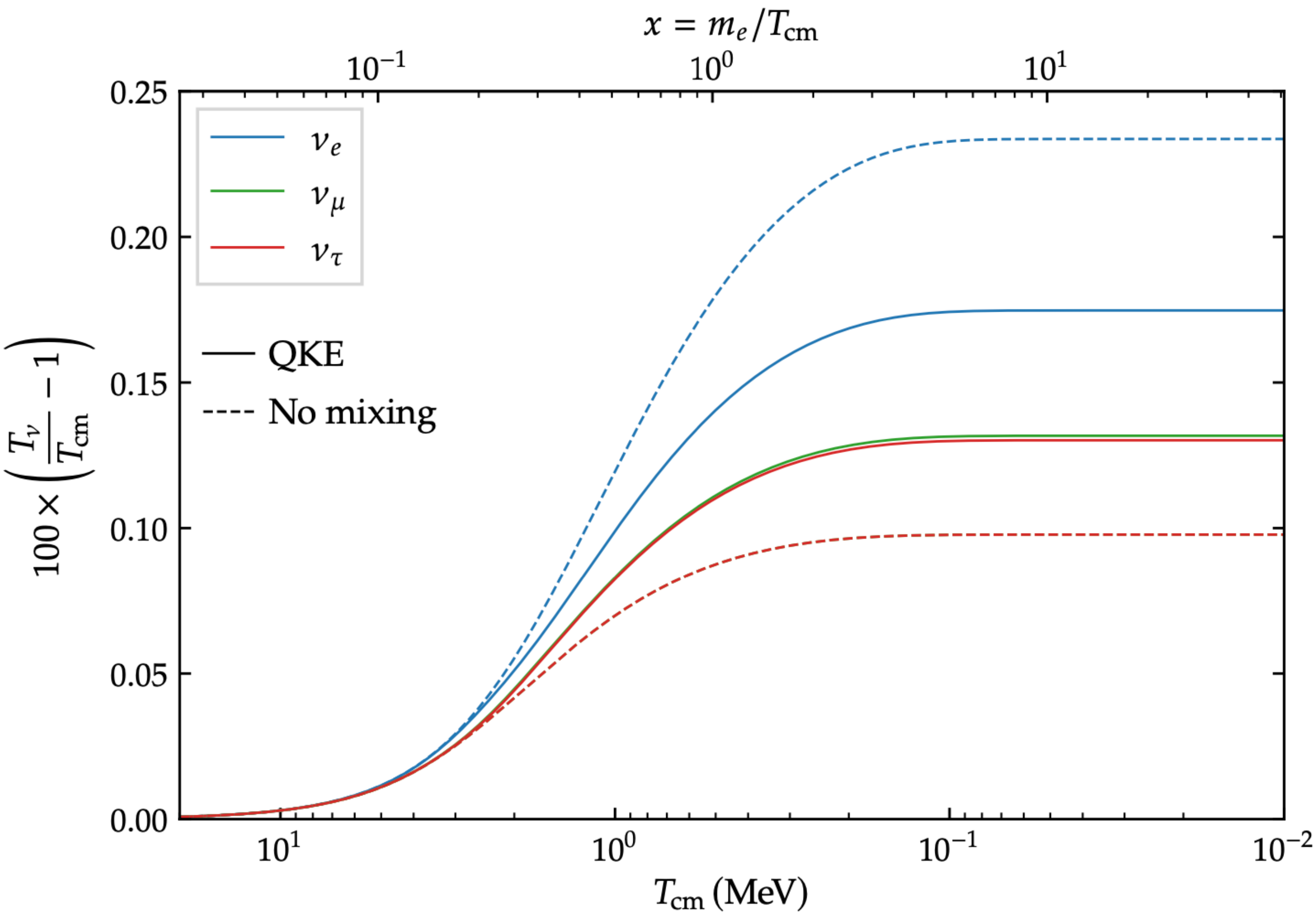
definition

$$\rho_R = \rho_\nu + \rho_\gamma = \rho_\gamma \left(1 + \frac{7}{8} N_{\rm eff} \left(\frac{4}{11}\right)^{4/3}\right)$$

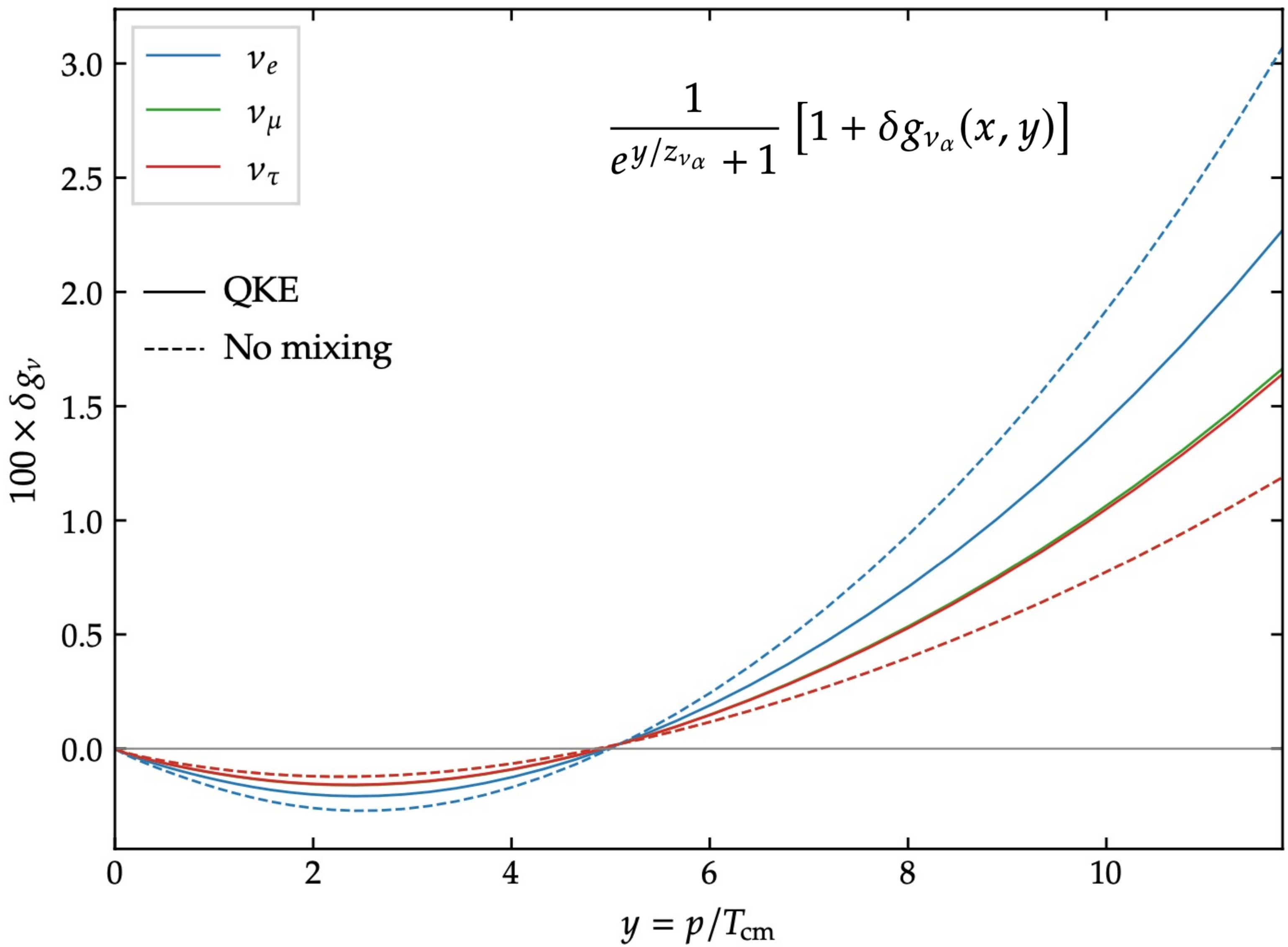
Froustey et al. 2020

$$N_{\rm eff} = 3.0440$$

Neutrino mixing (aka oscillations) effects



Neutrino Fermi-Dirac spectrum distortions



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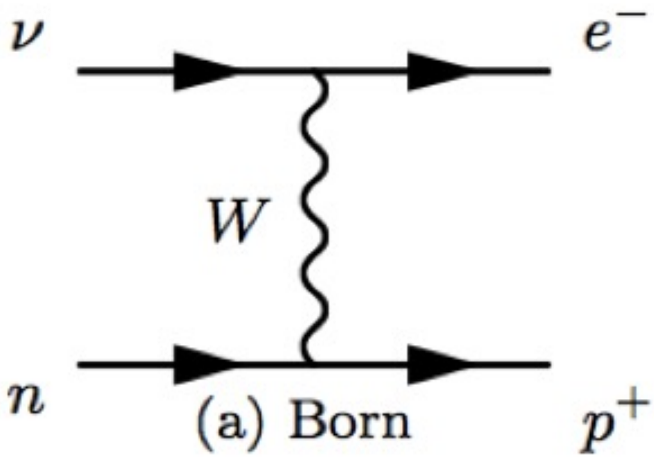


❑ General expression of weak rates

$$\dot{n}_n + 3Hn_n = -n_n\Gamma_{n\to p} + n_p\Gamma_{p\to n}$$
$$\dot{n}_p + 3Hn_p = -n_p\Gamma_{p\to n} + n_n\Gamma_{n\to p}$$

Weak interaction Matrix element

$$n_n\Gamma = \int \Pi_i[\mathrm{d}^3\mathbf{p}_i](2\pi)^4\delta^4\left(\underline{p}_n - \underline{p}_p + \alpha_\nu \underline{p}_\nu + \alpha_e \underline{p}_e\right)|M|^2 f_n(E_n)[1-f_p(E_p)]f_\nu(\alpha_\nu E_\nu)f_e(\alpha_e E_e)$$

momentum conservation

$$\begin{cases} \alpha_i = 1 & \text{if initial particle} \\ \alpha_i = -1 & \text{if final particle} \end{cases}$$

isotropy

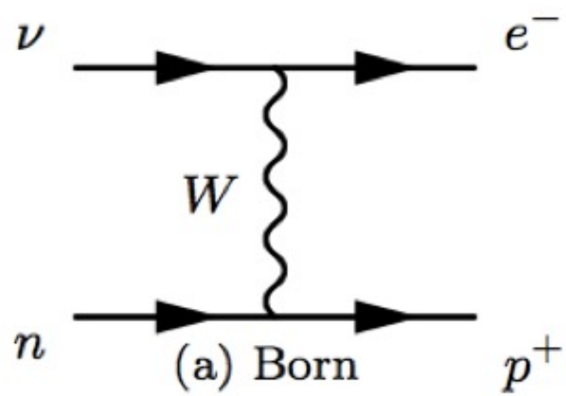
$$[\mathrm{d}^3\mathbf{p}] \equiv \frac{\mathrm{d}^3\mathbf{p}}{2E(2\pi)^3} = \frac{4\pi p^2 \mathrm{d}p}{2E(2\pi^3)}$$

$$g(-E) = 1 - g(E)$$

Fermi-Dirac Property

❑ Interaction Hamiltonian

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} J^{\mu}_{e\nu} J_{pn,\,\mu}$$



$$J^{\mu}_{e\nu} = \bar{\boldsymbol{\nu}}\gamma^{\mu}(1-\gamma^{5})\mathbf{e}$$

$$J^{\mu}_{pn} = \quad V_{ud}\,\bar{\mathbf{p}}\left(\gamma^{\mu}(1-g_A\gamma^{5}) + \mathrm{i}\,\frac{f_{\rm wm}}{m_N}2\Sigma^{\mu\nu}q_{\nu}\right)\mathbf{n}$$

CKM angle

Axial current coupling

Weak-Magnetism

❑ BORN approximation

Simple integral on electron momentum :

$$\overline{\Gamma}_{n\to p} = \overline{\Gamma}_{n\to p+e} + \overline{\Gamma}_{n+e\to p}$$

$$= K \int_0^\infty p^2 \mathrm{d}p [\chi_+(E) + \chi_+(-E)] ,$$

Fermi-Dirac distributions

$$\chi_\pm(E) \equiv (E_\nu^\mp)^2 g_\nu(E_\nu^\mp) g(-E) ,$$

$$E_\nu^\mp \equiv E \mp \Delta ,$$

$$K \equiv \frac{4G_W^2(1+3g_A^2)}{(2\pi)^3} .$$

Axial coupling constant

$$G_W = G_F V_{ud}$$

→ CKM angle

Neutron lifetime as a proxy

$$K \equiv \frac{4G_W^2(1+3g_A^2)}{(2\pi)^3}$$

$$K = 1/(\tau_n \lambda_0 m_e^5)$$

$$\lambda_0 \simeq 1.75474$$

Best values are :

$$\tau_n \simeq 879.4 \pm 0.6\,\mathrm{s}$$

$$V_{ud} = 0.97420(20)$$

$$g_A = 1.2723(23)$$

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PDG 2022

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$$\boxed{\tau_n = 878.4 \pm 0.5\,\mathrm{s}}$$

$$V_{ud} = 0.97420(20)$$

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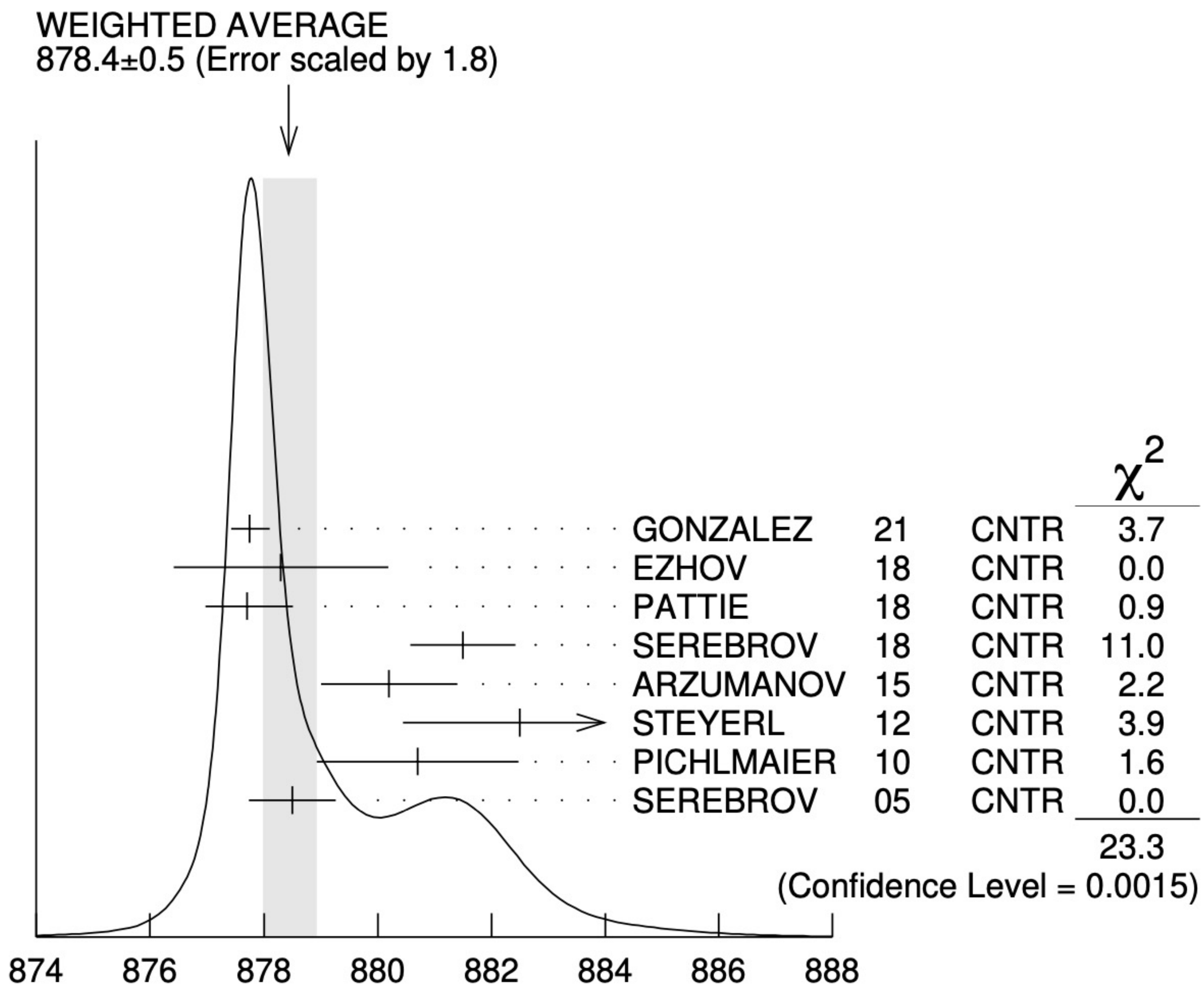
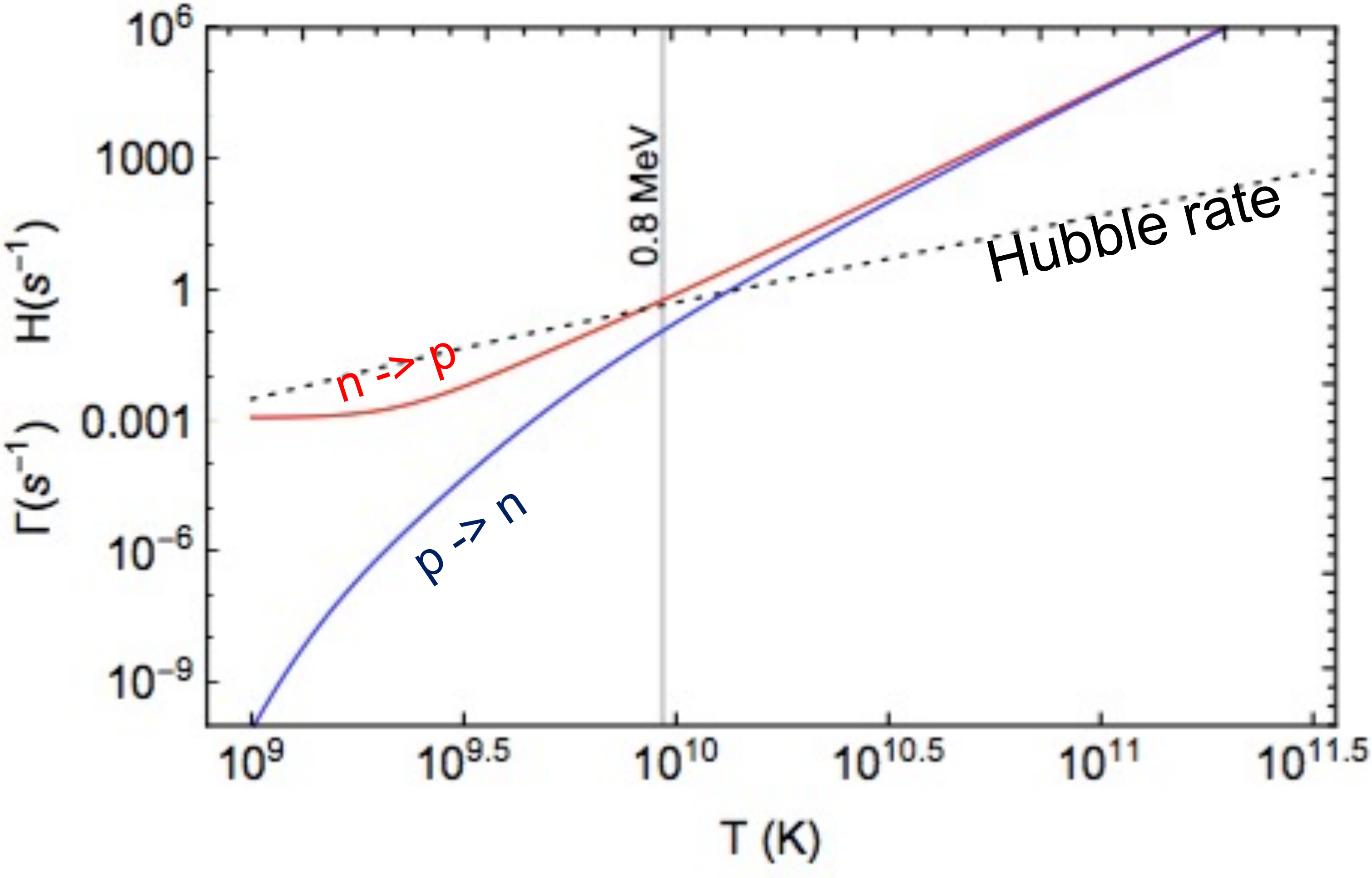
Particle Data Group on neutron lifetime

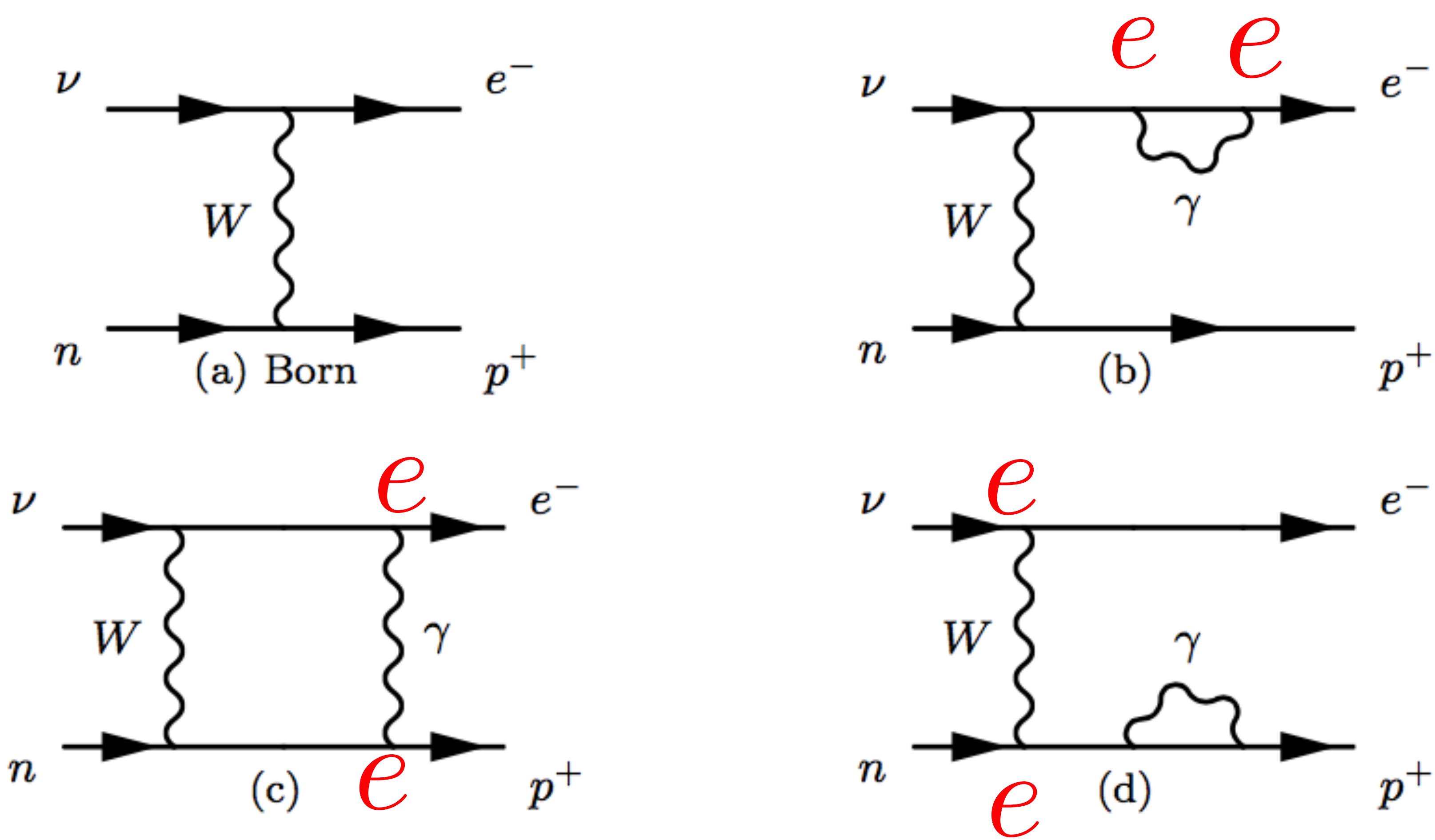


❑ BORN approximation rates vs Hubble rate

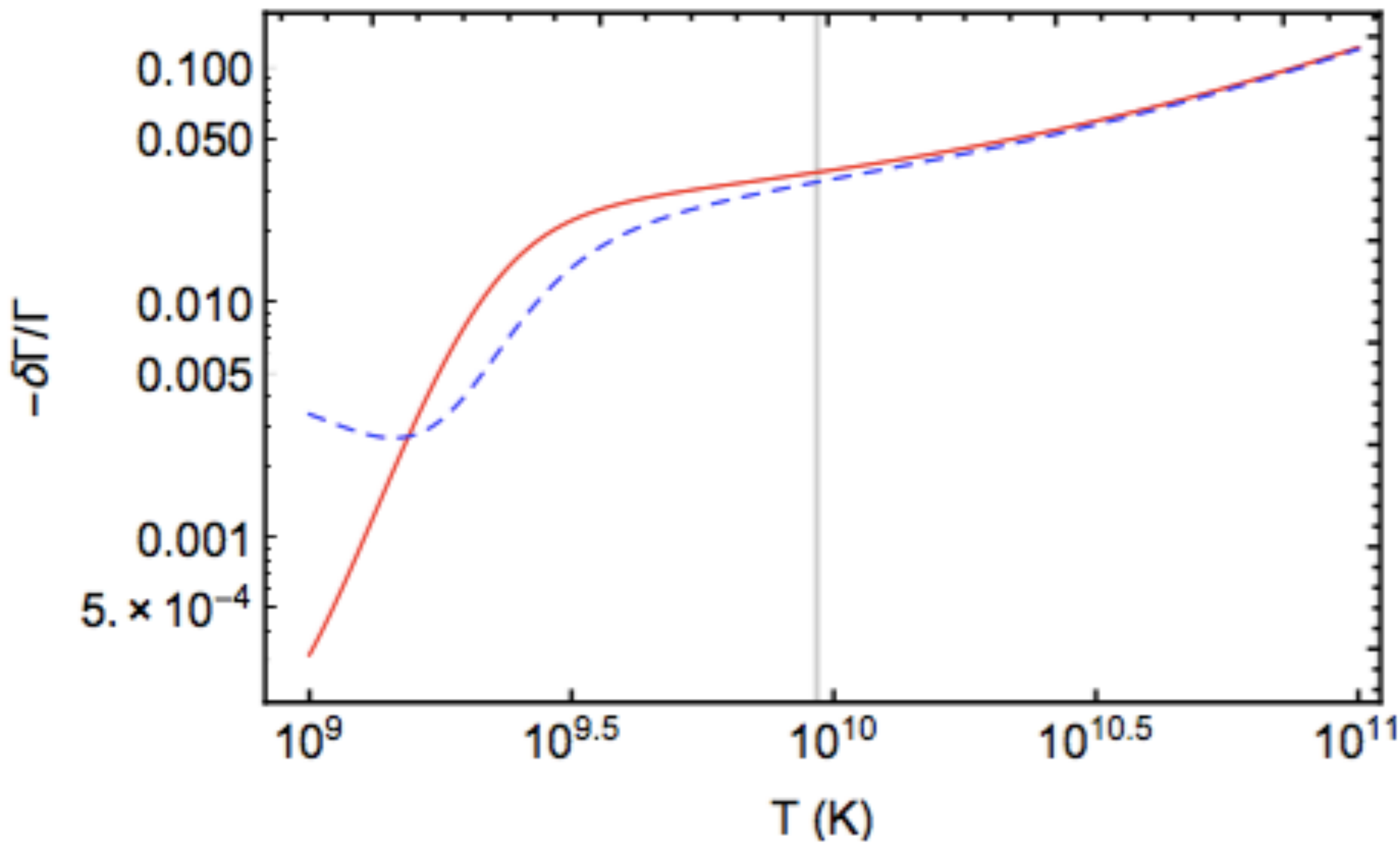


❑ Radiative corrections

$$\frac{e^2}{4\pi} = \alpha_{\rm FS} \simeq \frac{1}{137}$$



❑ Total corrections



Hunt for precision

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❑ Evolution of abundances

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} + 3Hn_i = \mathcal{J}_i$$ → Source from nuclear reactions

$$\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t} + 3Hn_\mathrm{b} = 0,$$ Baryons are only diluted

$$Y_i \equiv n_i/n_b$$ → Removes dilution

- Two-body reactions of the type $i + j \leftrightarrow k + l$

$$\mathcal{J}_i \supset n_k n_l \gamma_{kl\to ij} - n_i n_j \gamma_{ij\to kl}$$

$$\gamma_{ij\to kl} \equiv \langle \sigma v \rangle_{ij\to kl}\,.$$

Average of cross-section over Maxwell-Boltzmann distribution

- General form

$$\dot{Y}_{i_1} = \sum_{i_2\ldots i_p, j_1\ldots j_q} N_{i_1} \left(\Gamma_{j_1\ldots j_q \to i_1\ldots i_p} \frac{Y_{j_1}^{N_{j_1}} \ldots Y_{j_q}^{N_{j_q}}}{N_{j_1}! \ldots N_{j_q}!} - \Gamma_{i_1\ldots i_p \to j_1\ldots j_q} \frac{Y_{i_1}^{N_{i_1}} \ldots Y_{i_p}^{N_{i_p}}}{N_{i_1}! \ldots N_{i_p}!} \right)$$

At nuclear statistical equilibrium (NSE), the densities of nuclei satisfy

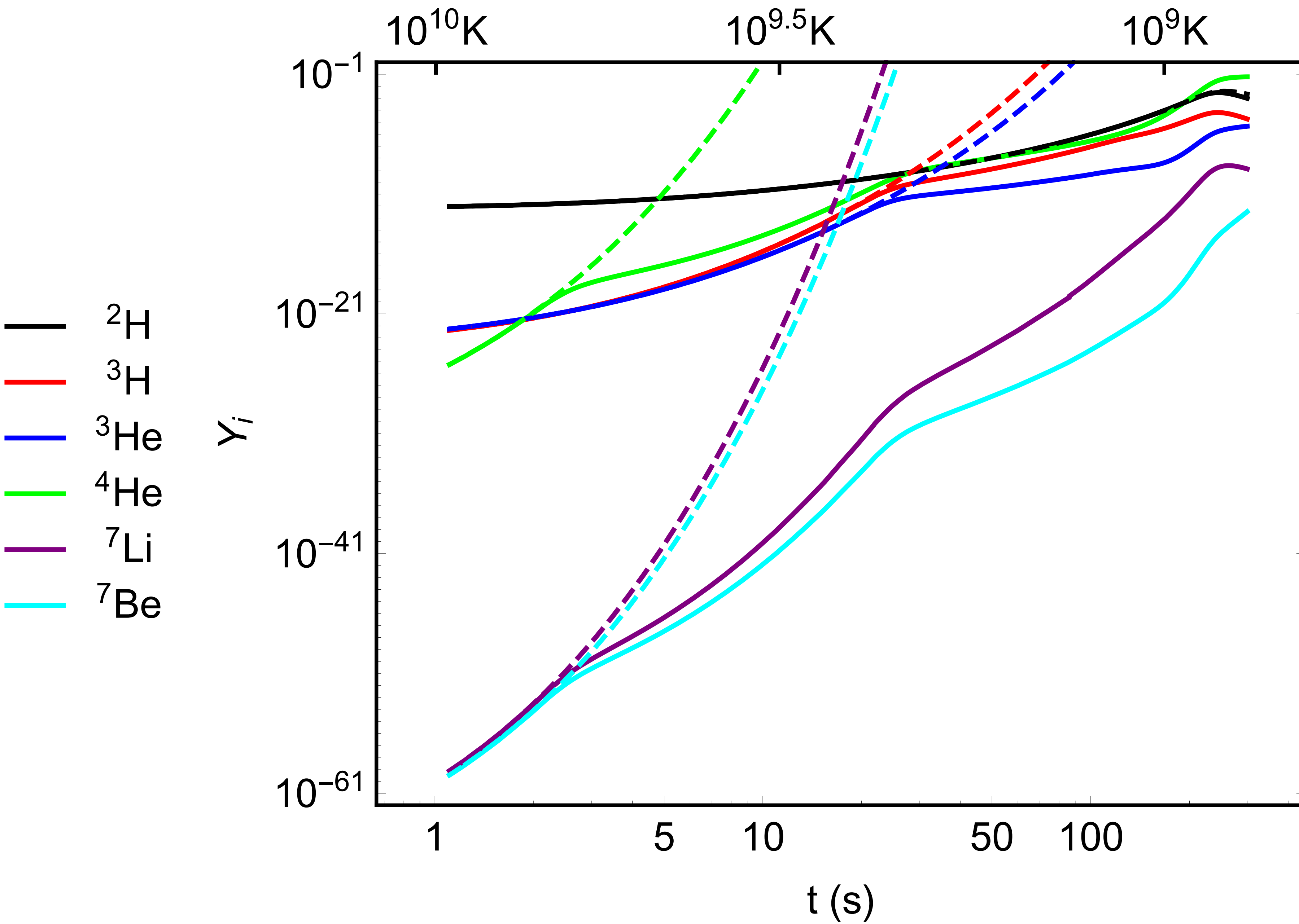
$$n_i^{\rm NSE} = \frac{g_i m_i^{3/2}}{2^{A_i}} \left(\frac{n_p}{m_p^{3/2}}\right)^{Z_i} \left(\frac{n_n}{m_n^{3/2}}\right)^{A_i - Z_i} \left(\frac{2\pi}{T}\right)^{\frac{3(A_i-1)}{2}} \mathrm{e}^{B_i/T}$$

Number protons — Number neutrons — Binding energy

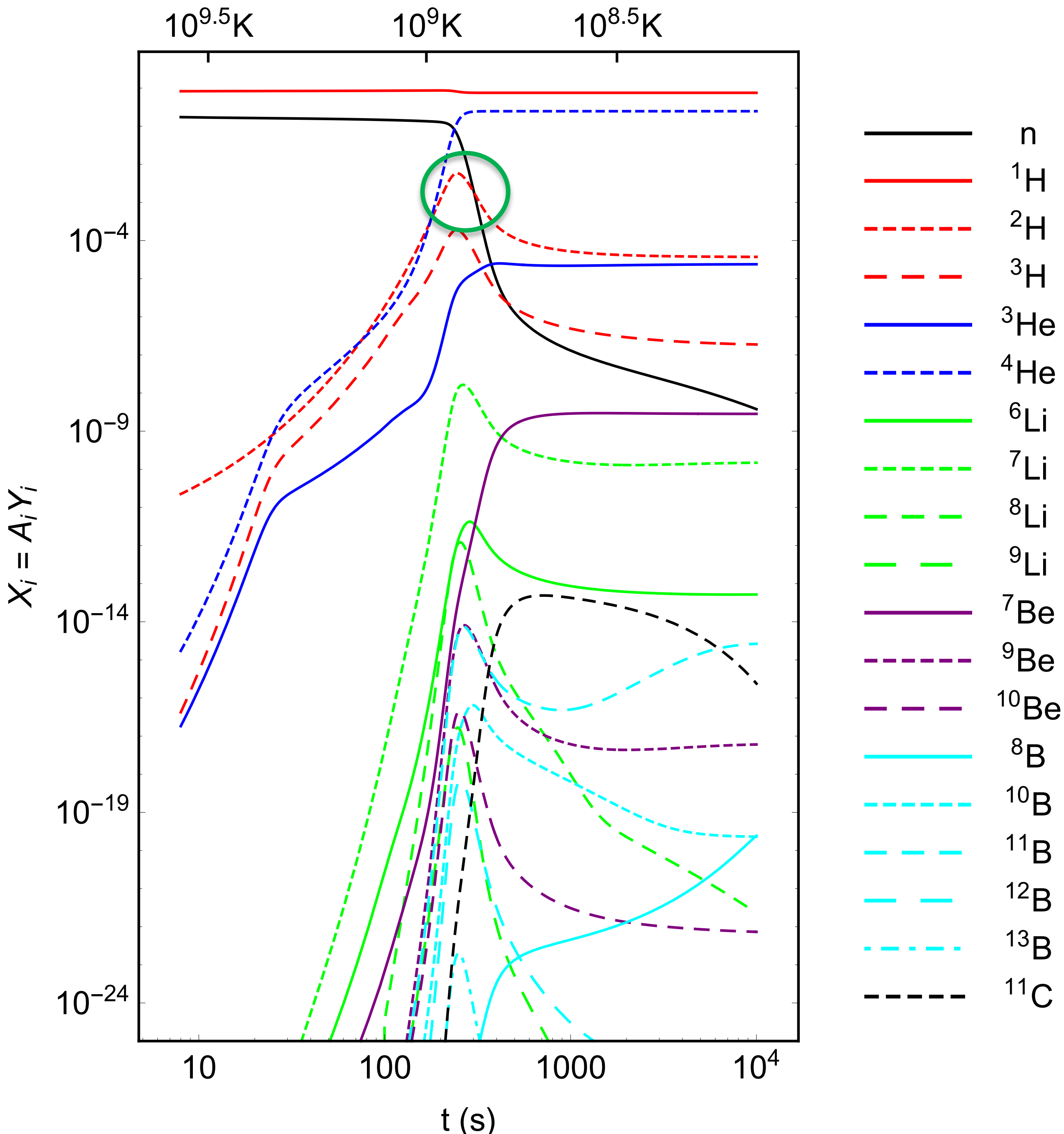
$$B_i \equiv Z_i m_p + (A_i - Z_i) m_n - m_i$$

Demo tools :

- Non-relativistic number density $n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \mathrm{e}^{(\mu_i - m_i)/T}$
- Chemical equilibrium $\mu_i = Z_i \mu_p + (A_i - Z_i)\mu_n$

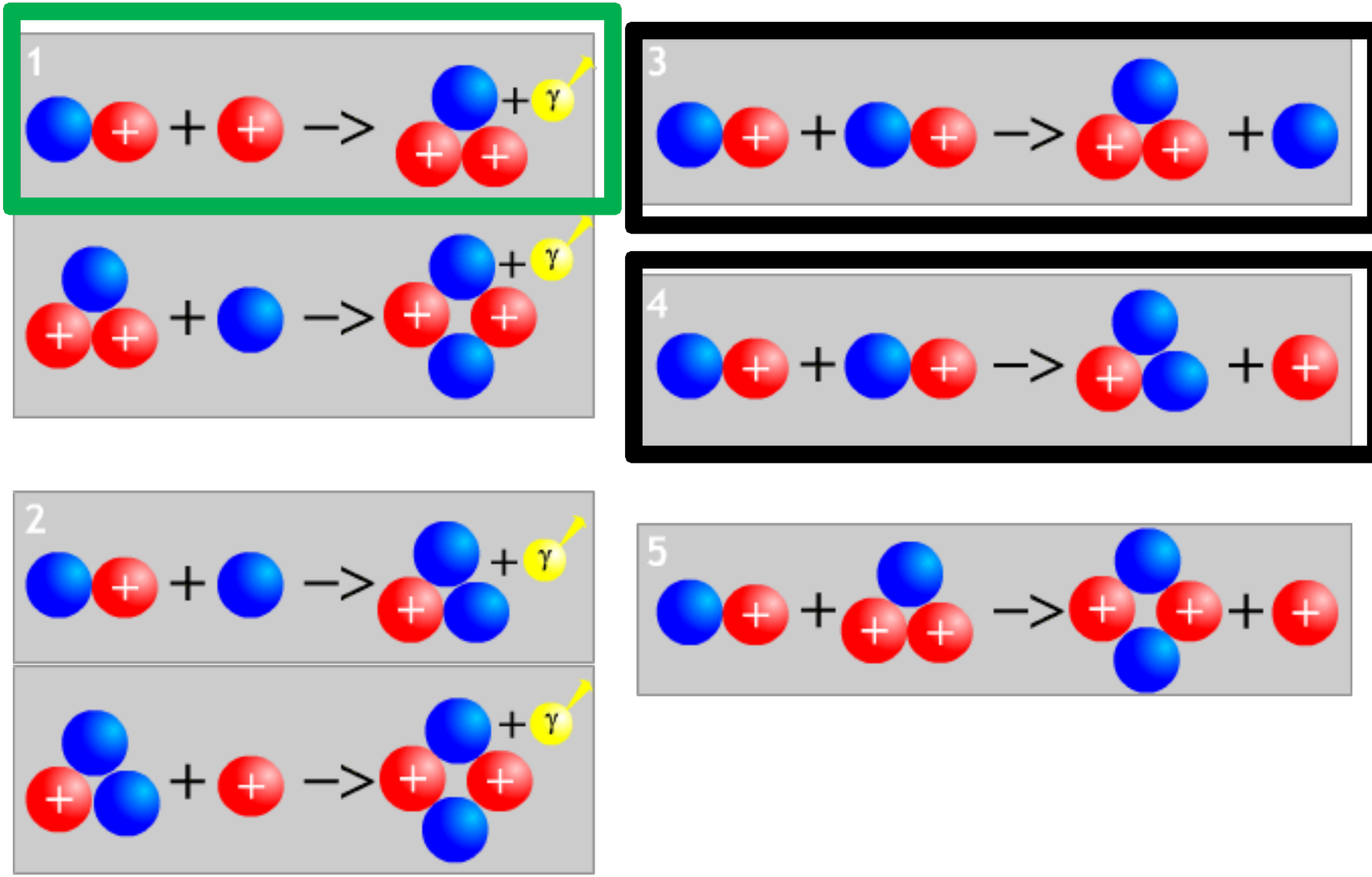


^{2}H
^{3}H
^{3}He
^{4}He
^{7}Li
^{7}Be
10^{10}K
$10^{9.5}$K
10^{9}K
10^{-1}
10^{-21}
10^{-41}
10^{-61}
Y_i
1
5
10
50
100
t (s)



10^9.5K
10^9K
10^8.5K
10^−4
10^−9
10^−14
10^−19
10^−24
X_i = A_i Y_i
10
100
1000
10^4
t (s)
n
1H
2H
3H
3He
4He
6Li
7Li
8Li
9Li
7Be
9Be
10Be
8B
10B
11B
12B
13B
11C

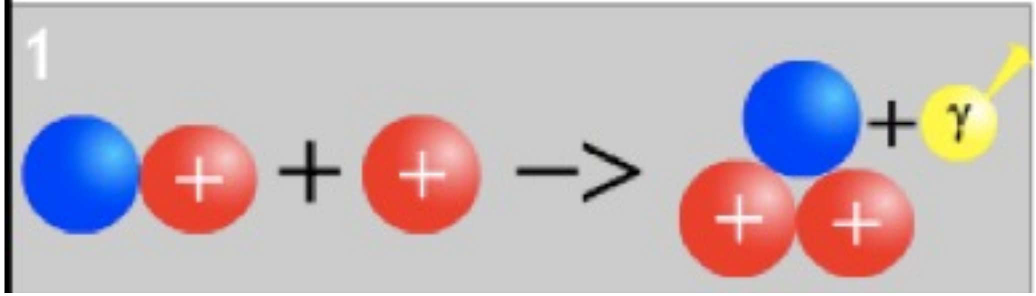
Main reactions for D



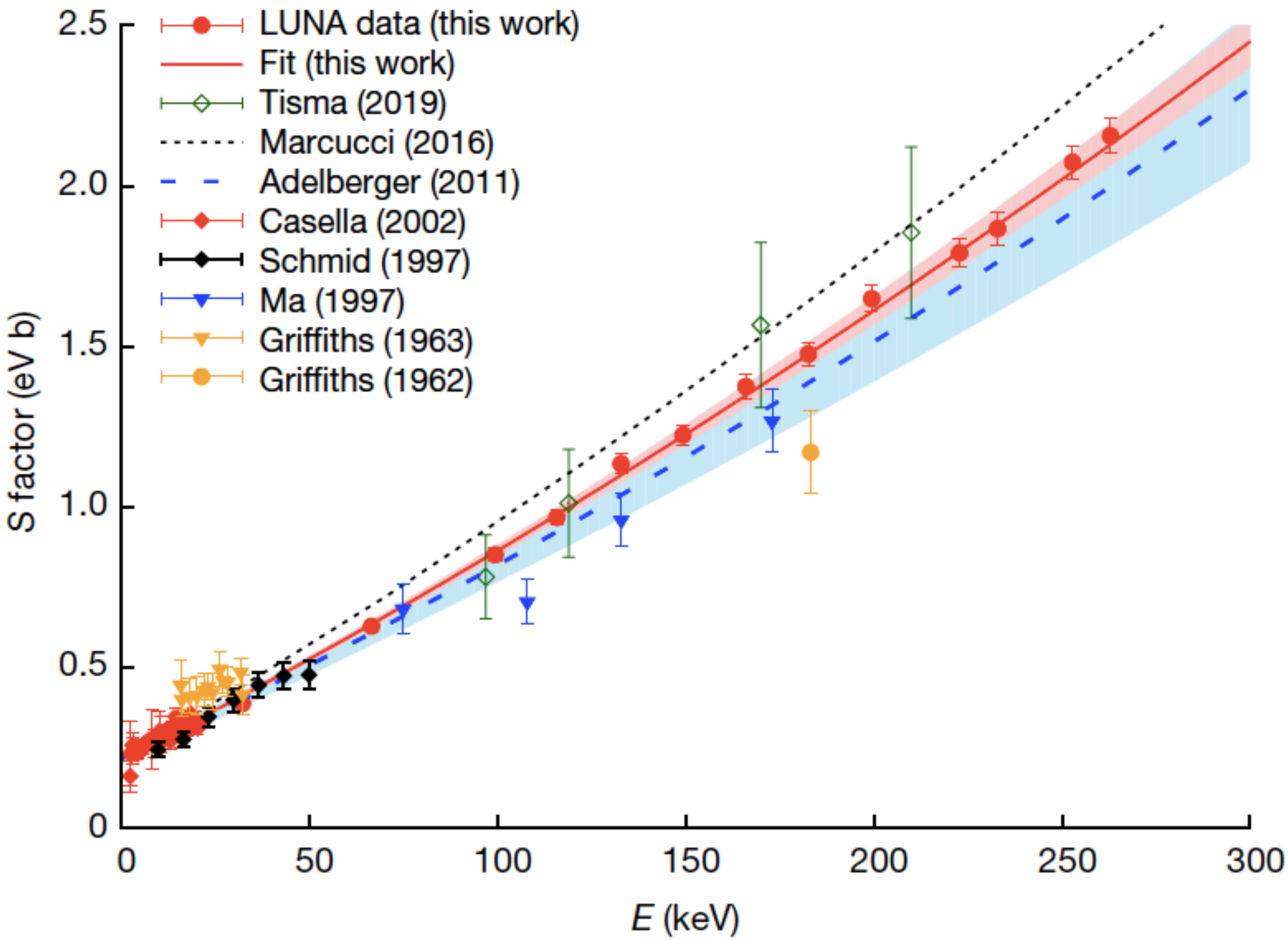
Final deuterium sensitivity

$$\frac{\Delta(\mathrm{D/H})}{\mathrm{D/H}} = -0.32\frac{\Delta\langle\sigma v\rangle_{\mathrm{D(p,\gamma)^3He}}}{\langle\sigma v\rangle_{\mathrm{D(p,\gamma)^3He}}}$$

$$\frac{\Delta(\mathrm{D/H})}{\mathrm{D/H}} = -0.54\frac{\Delta\langle\sigma v\rangle_{\mathrm{D(d,n)^3He}}}{\langle\sigma v\rangle_{\mathrm{D(d,n)^3He}}} - 0.46\frac{\Delta\langle\sigma v\rangle_{\mathrm{D(d,p}}}{\langle\sigma v\rangle_{\mathrm{D(d,p)^3H}}}$$


1
+
->
+
γ

2020


LUNA data (this work)
Fit (this work)
Tisma (2019)
Marcucci (2016)
Adelberger (2011)
Casella (2002)
Schmid (1997)
Ma (1997)
Griffiths (1963)
Griffiths (1962)
S factor (eV b)
2.5
2.0
1.5
1.0
0.5
0
0
50
100
150
200
250
300
E (keV)

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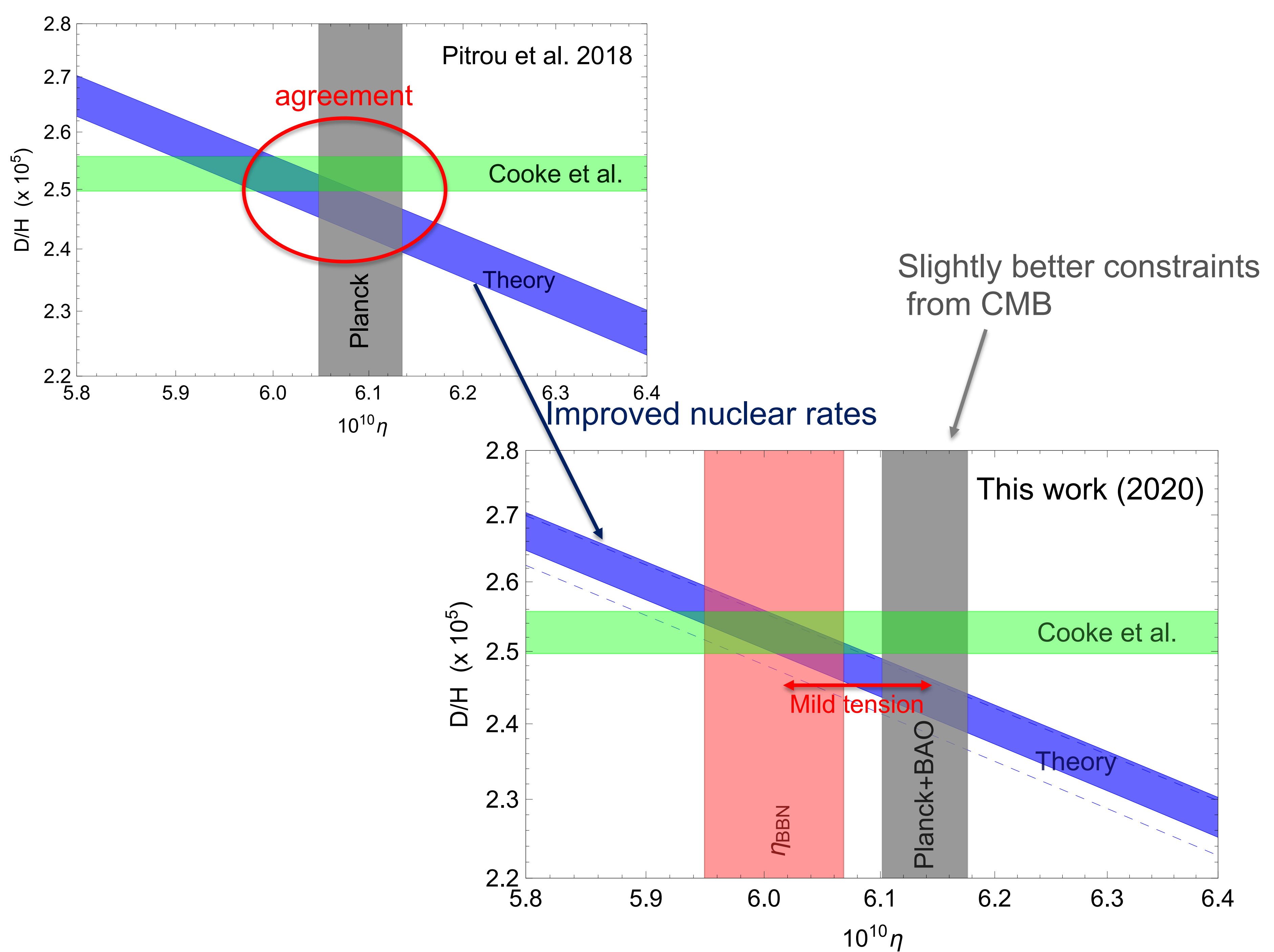

Pitrou et al. 2018
agreement
Cooke et al.
Planck
Theory
D/H (x 10^5)
10^10 η
Improved nuclear rates
Slightly better constraints from CMB
This work (2020)
Cooke et al.
Mild tension
η_BBN
Planck+BAO
Theory
D/H (x 10^5)
10^10 η

A new tension in the cosmological model from primordial deuterium?

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Primordial Deuterium after LUNA: concordances and error budget

Ofelia Pisanti, Gianpiero Mangano, Gennaro Miele, and Pierpaolo Mazzella
Dipartimento di Fisica E. Pancini, Università di Napoli Federico II, and INFN, Sezione di Napoli, Via Cintia, I-80126 Napoli, Italy

after the LUNA results, the value of Deuterium is quite precisely fixed, and points to a value of the baryon density in excellent agreement with the Planck result,

The Impact of New $d(p,\gamma)^3$He Rates on Big Bang Nucleosynthesis

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Brian D. Fields

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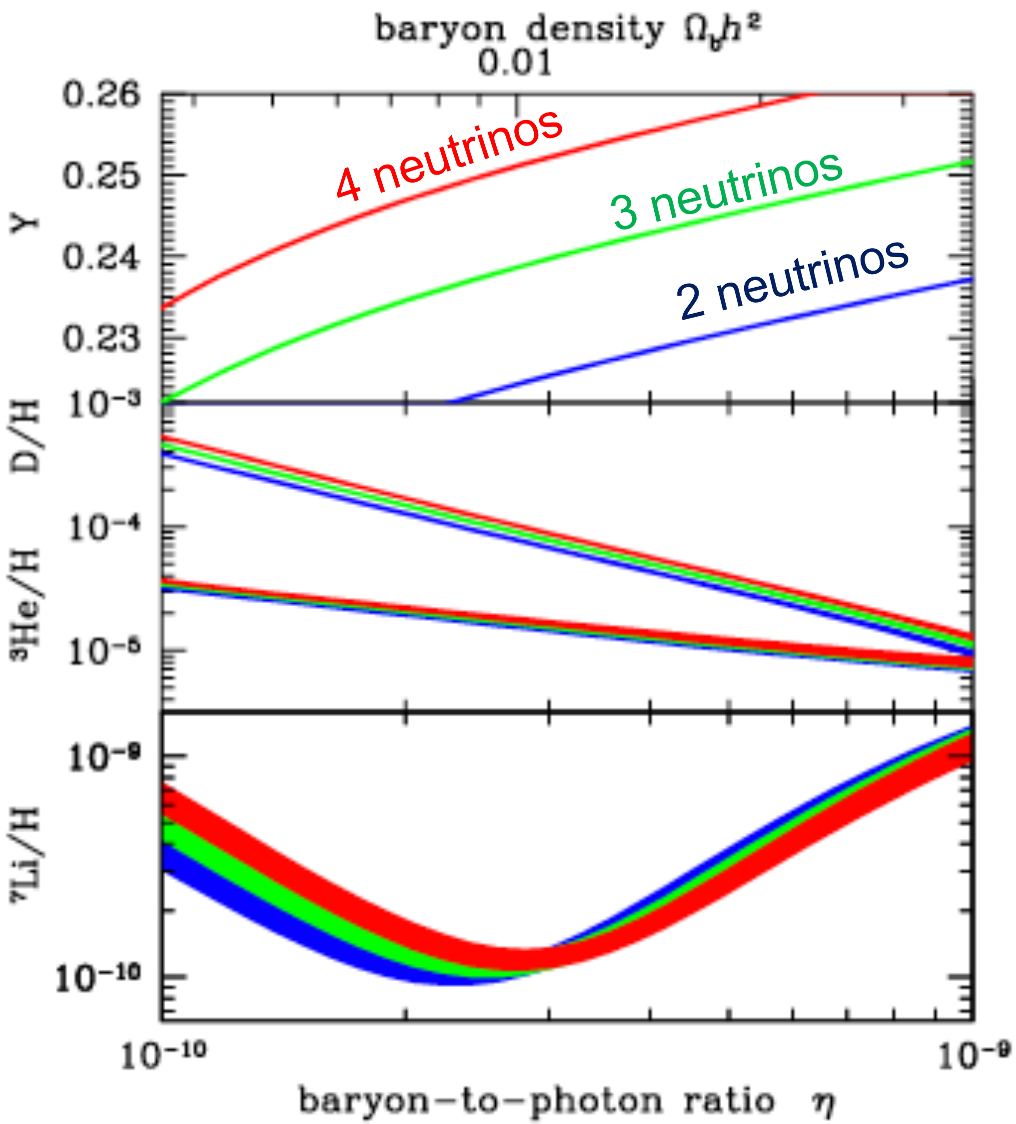
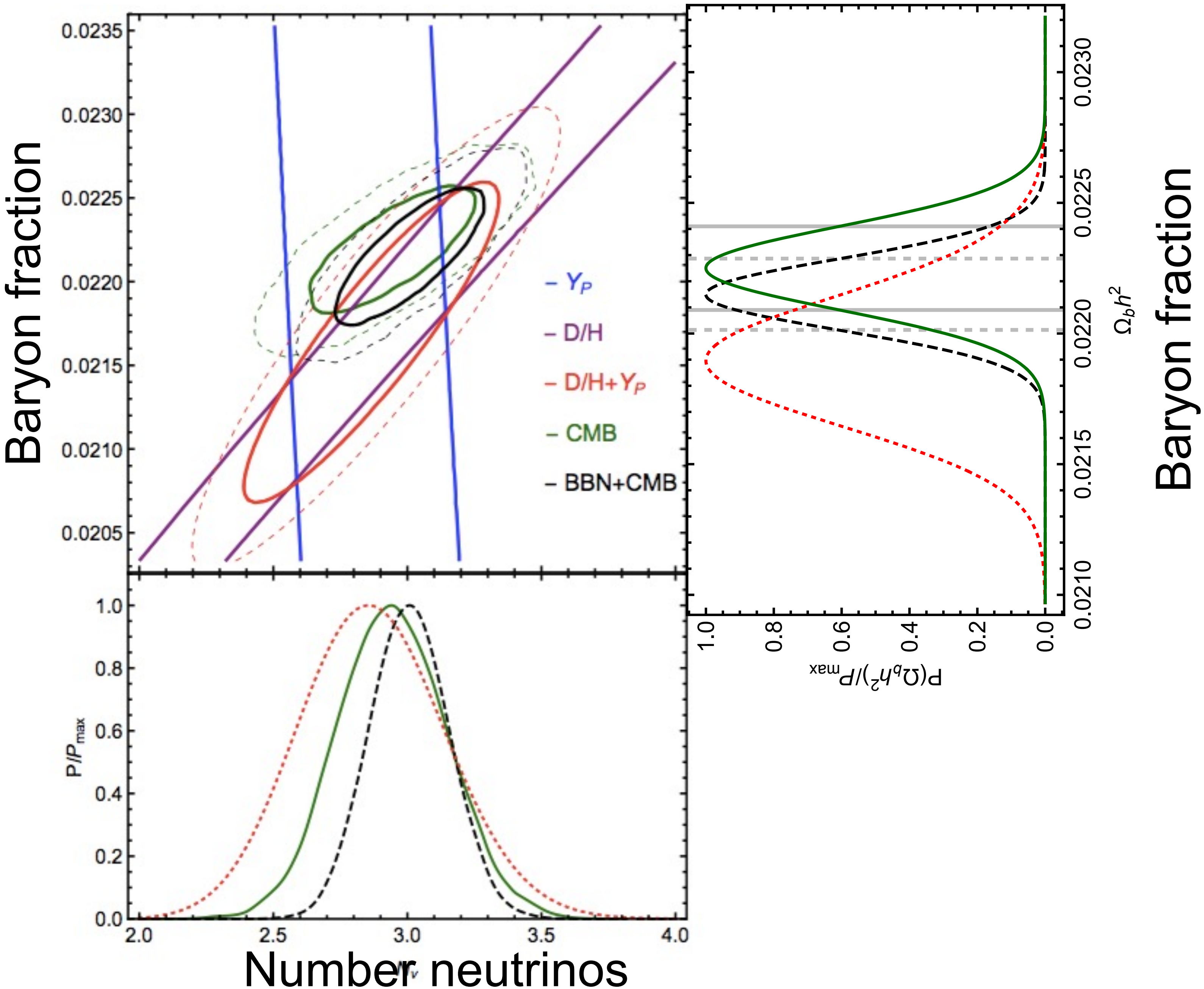
Finally, we note that the observed deuterium abundance continues to be more precise than the BBN+CMB prediction, whose error budget is now dominated by $d(d,n)^3$He and $d(d,p)^3$H.

❑ Dependance on number of neutrino species

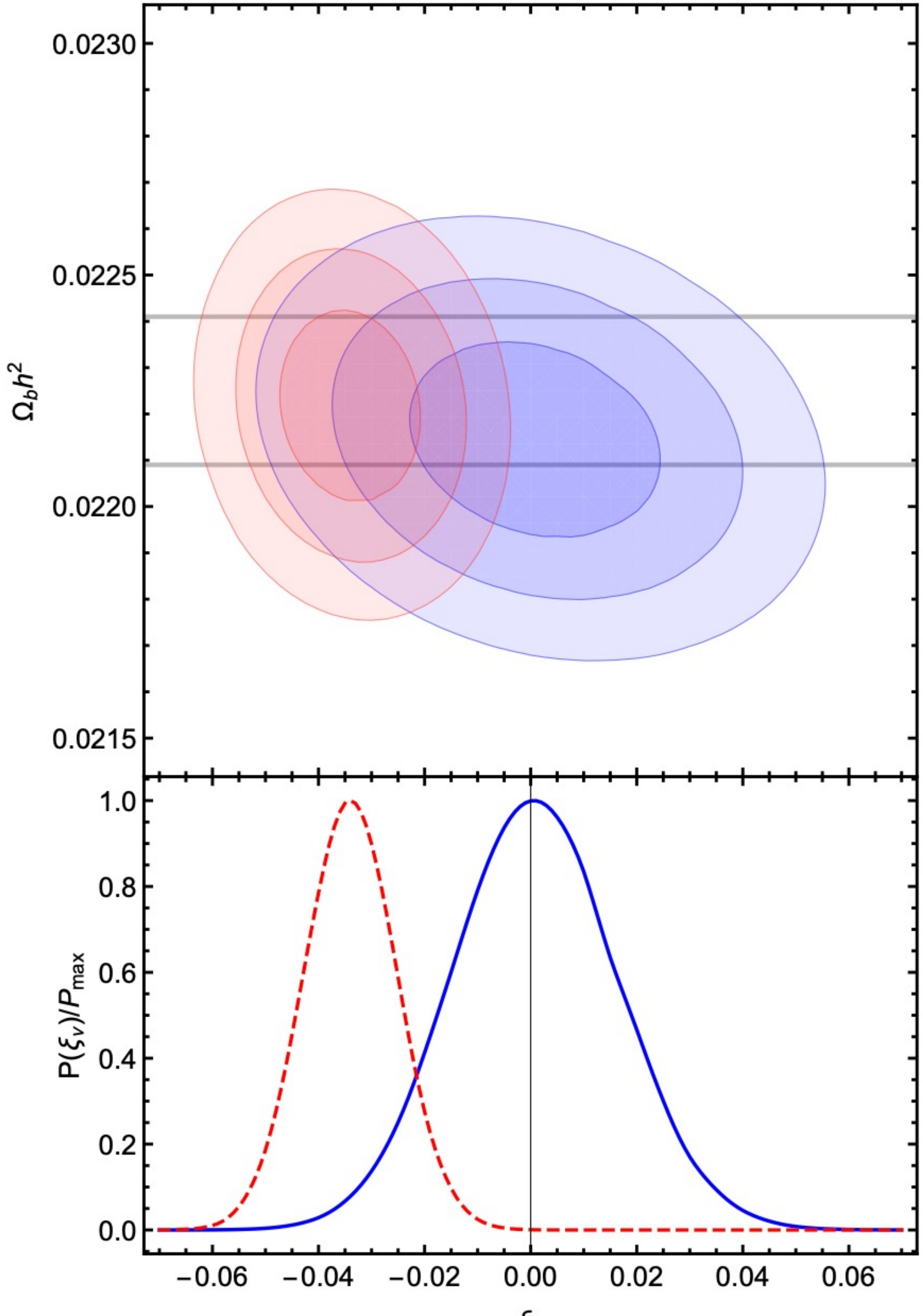


Cyburt et al. 2015



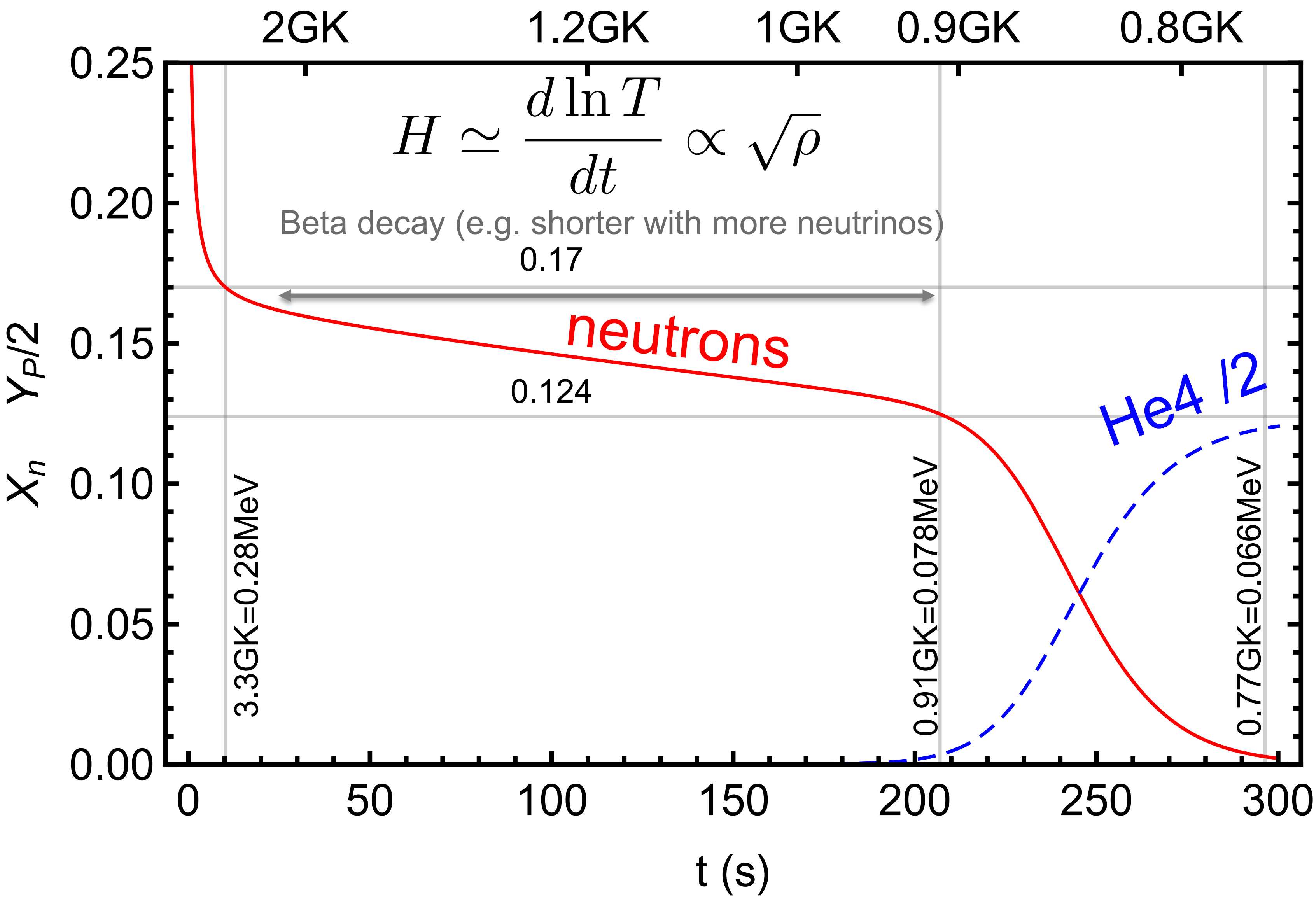
0.0235
0.0230
0.0225
0.0220
0.0215
0.0210
0.0205
Baryon fraction
– Y_P
– D/H
– D/H+Y_P
– CMB
– BBN+CMB
1.0
0.8
0.6
0.4
0.2
0.0
P/P_{max}
2.0
2.5
3.0
3.5
4.0
N_ν
Number neutrinos
$P(\Omega_b h^2)/P_{max}$
$\Omega_b h^2$
Baryon fraction

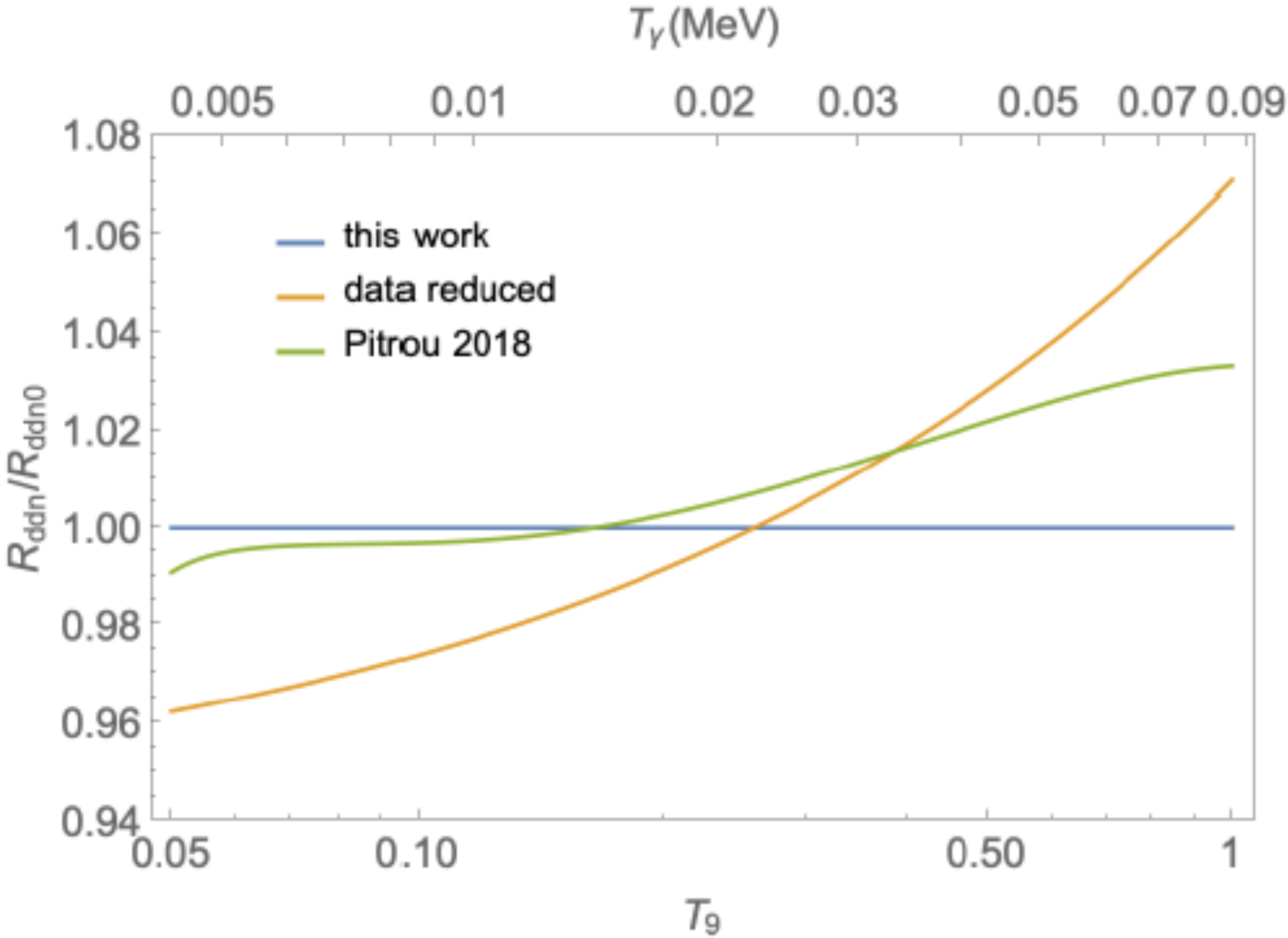
Thank you



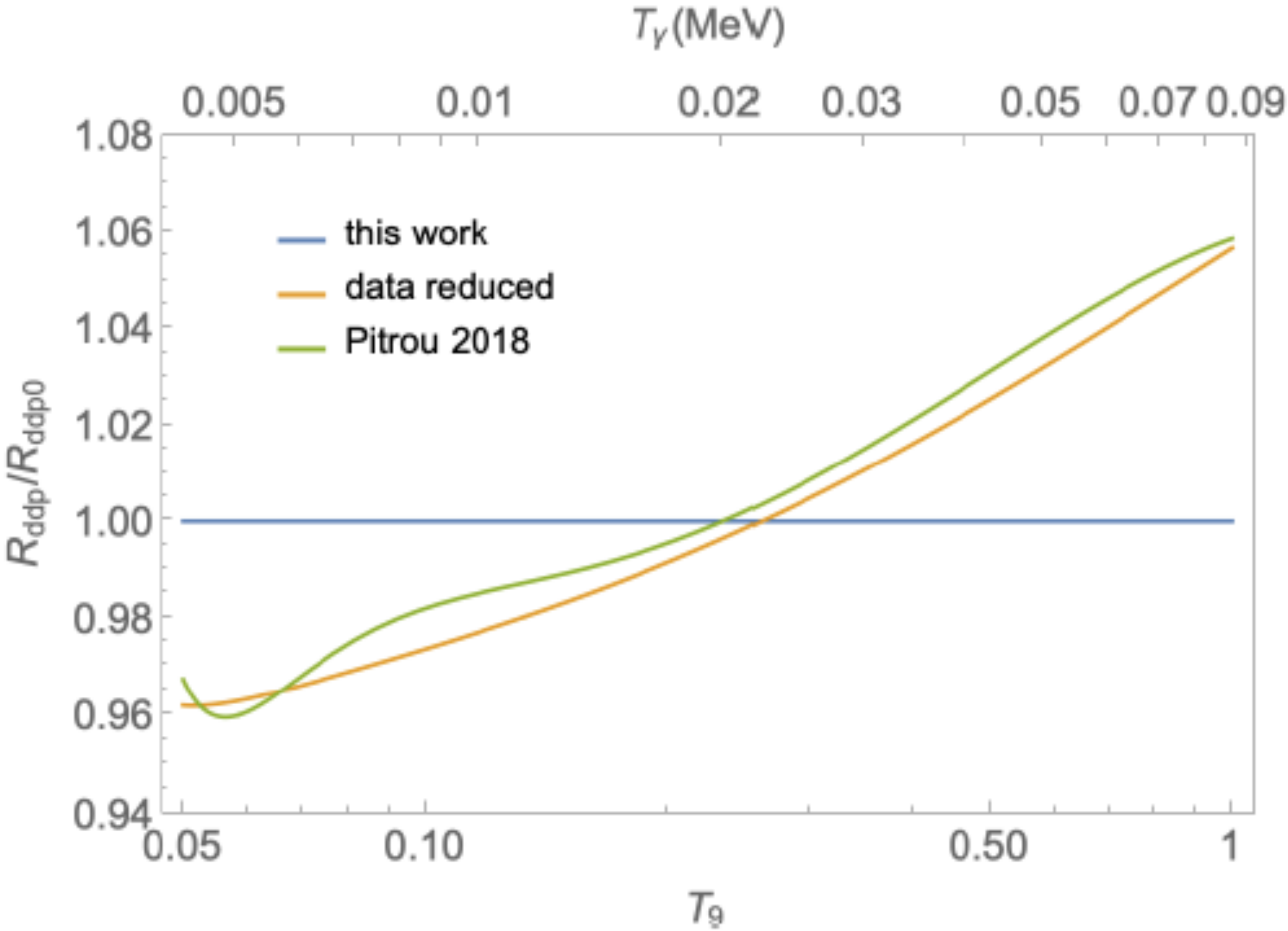
0.0230
0.0225
0.0220
0.0215
$\Omega_b h^2$
1.0
0.8
0.6
0.4
0.2
0.0
$P(\xi_\nu)/P_{max}$
−0.06
−0.04
−0.02
0.00
0.02
0.04
0.06

❑ Effect on beta decay

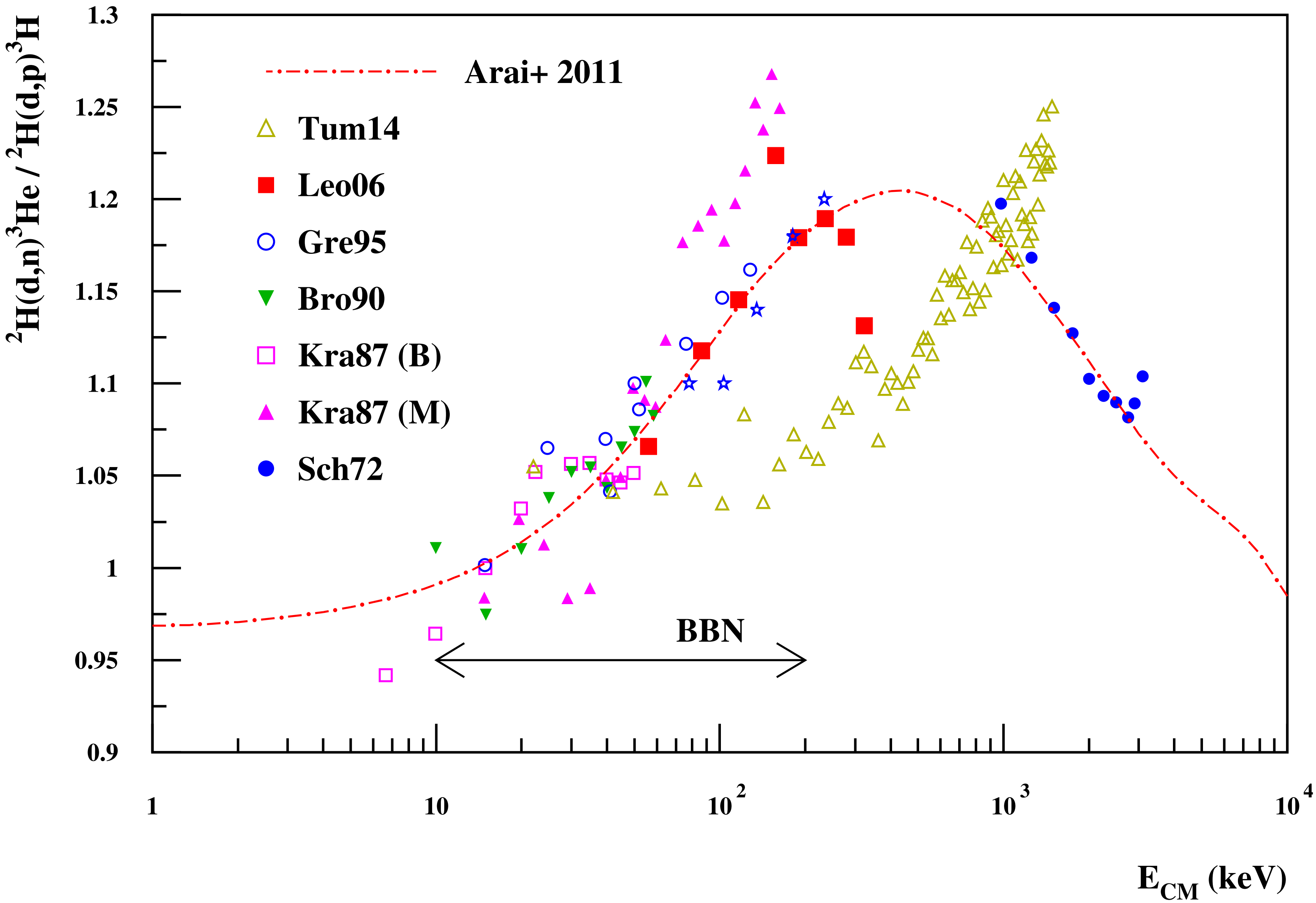




T_γ(MeV)
0.005
0.01
0.02
0.03
0.05
0.07
0.09
1.08
1.06
1.04
1.02
1.00
0.98
0.96
0.94
this work
data reduced
Pitrou 2018
R_{ddn}/R_{ddn0}
0.05
0.10
0.50
1
T_9

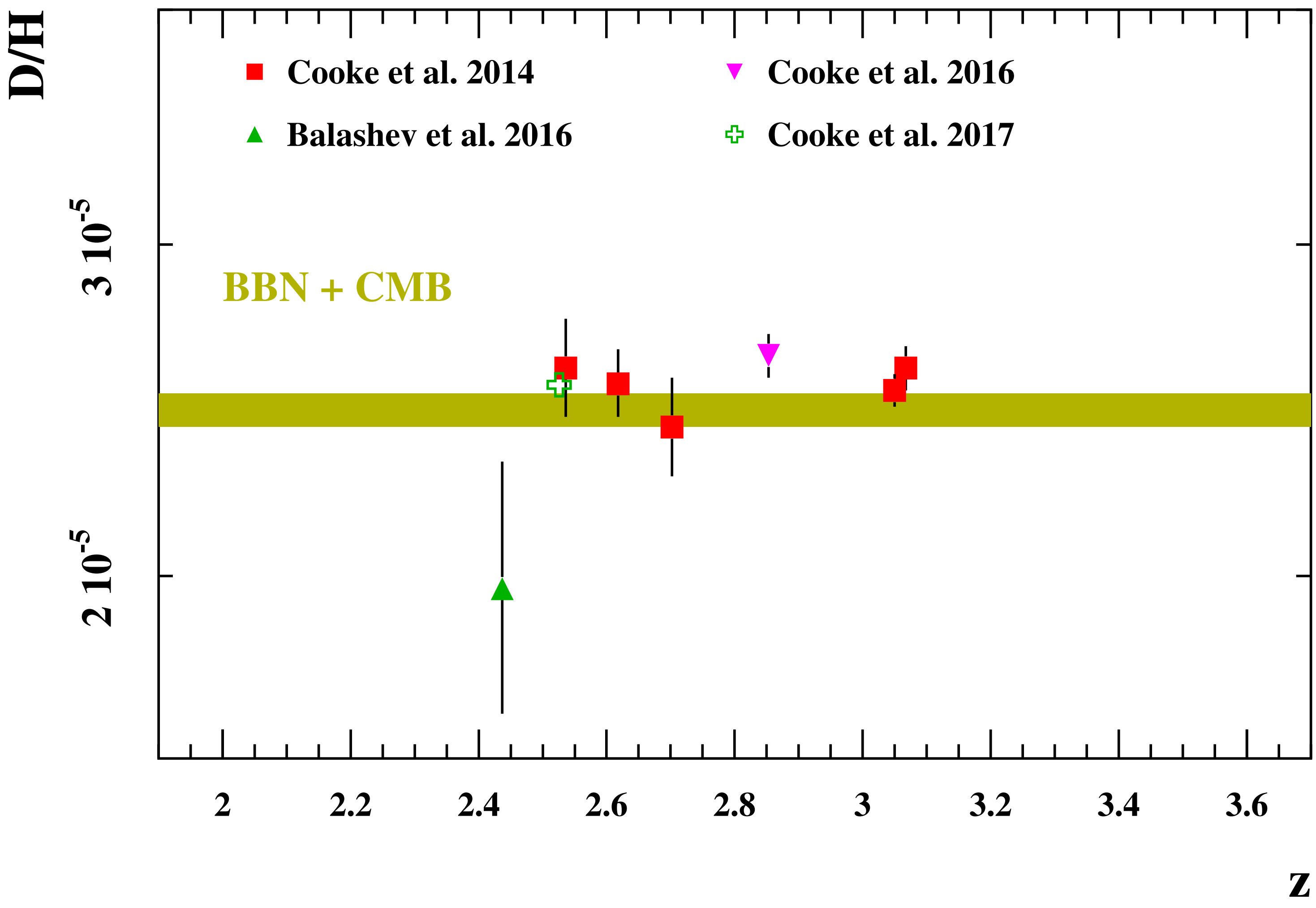


T_γ(MeV)
0.005
0.01
0.02
0.03
0.05
0.07
0.09
1.08
1.06
1.04
1.02
1.00
0.98
0.96
0.94
this work
data reduced
Pitrou 2018
R_{ddp}/R_{ddp0}
0.05
0.10
0.50
1
T_9



Arai+ 2011
Tum14
Leo06
Gre95
Bro90
Kra87 (B)
Kra87 (M)
Sch72
BBN
^{2}H(d,n)^{3}He / ^{2}H(d,p)^{3}H
E_{CM} (keV)
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
1
10
10^2
10^3
10^4

Quasar absorption lines



❑ Helium production