PATRICK PETER

GReCO


Théorie, Univers et Gravitation

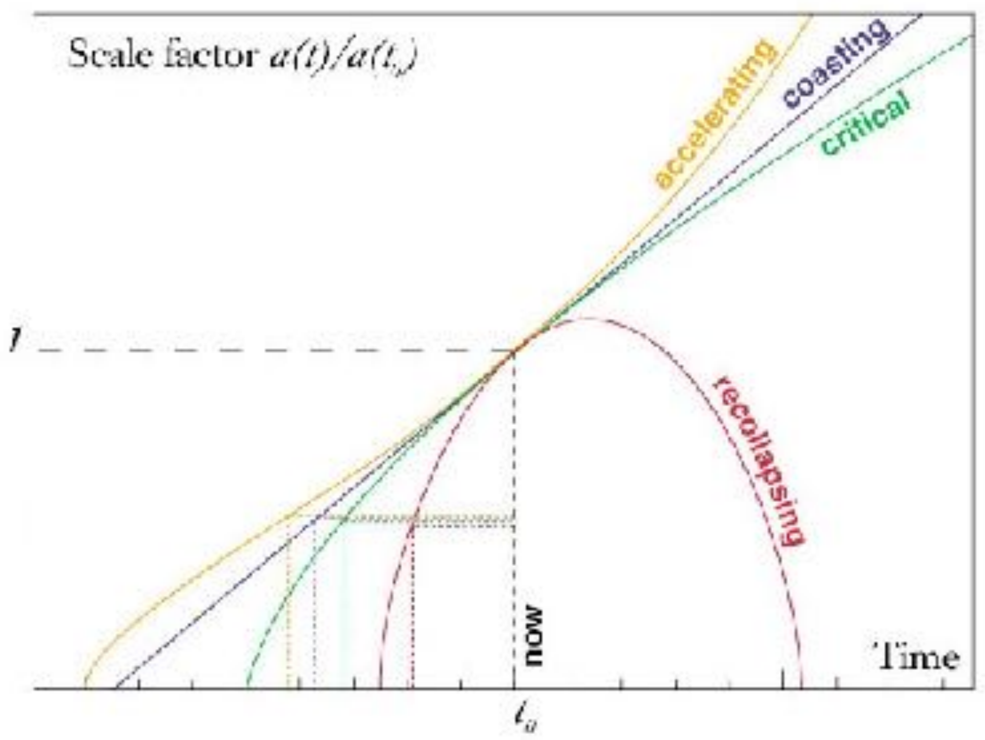
Montpellier, 4-6 octobre 2022

Organisateurs : Laura Bernard (LUTH), Philippe Brax (IPhT), Cédric Deffayet (IAP), Julien Lavalle (LUPM), Pasquale Serpico (LAPTh), Vincent Vennin (APC), Filippo Vernizzi (IPhT)


Buveurs tres illustres...

Bouncing universes

Limitations and puzzles of the standard model



- Singularity $\exists t_{(\pm\infty)}; a(t) \to 0$

- Horizon

$$\frac{d\Omega_{\mathcal{K}}}{d\ln a} = (3w+1)(1-\Omega_{\mathcal{K}})\Omega_{\mathcal{K}}$$

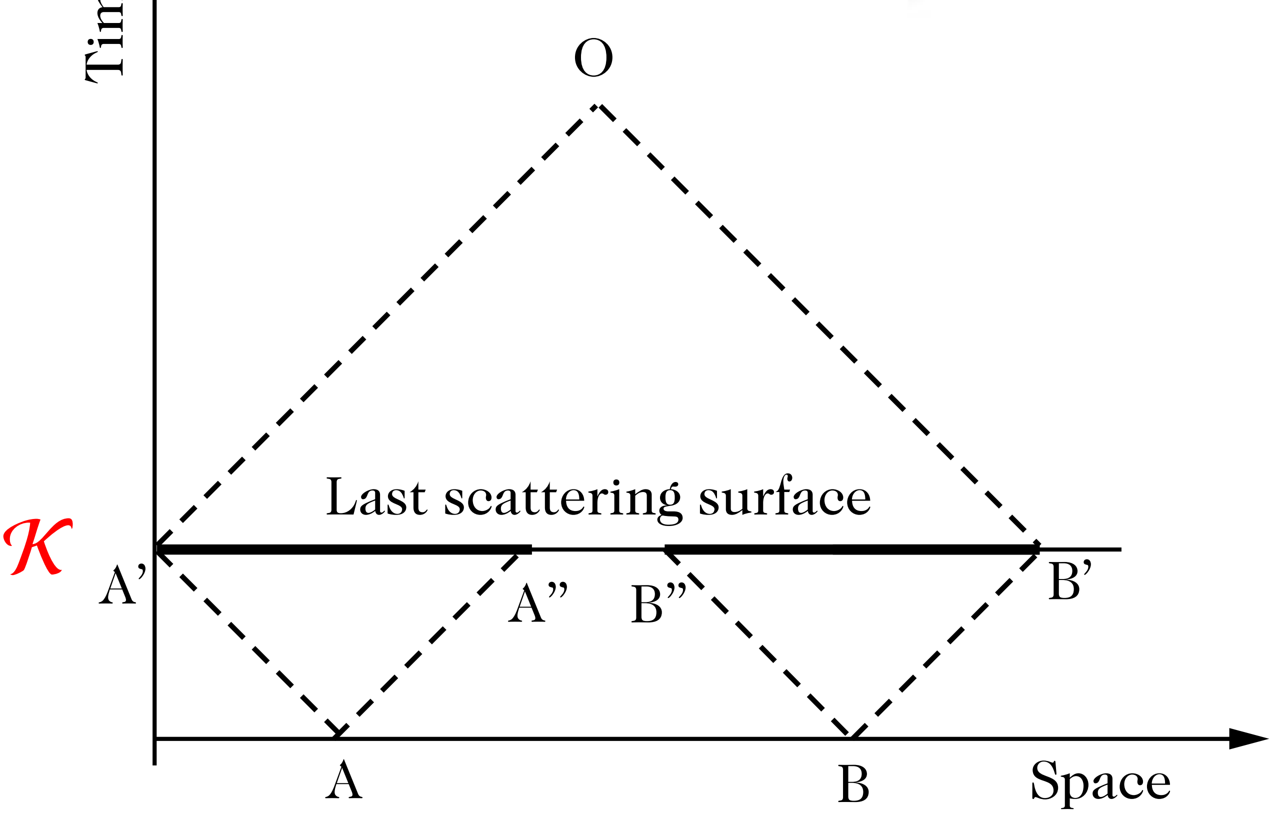
- Flatness Unstable fixed point!!

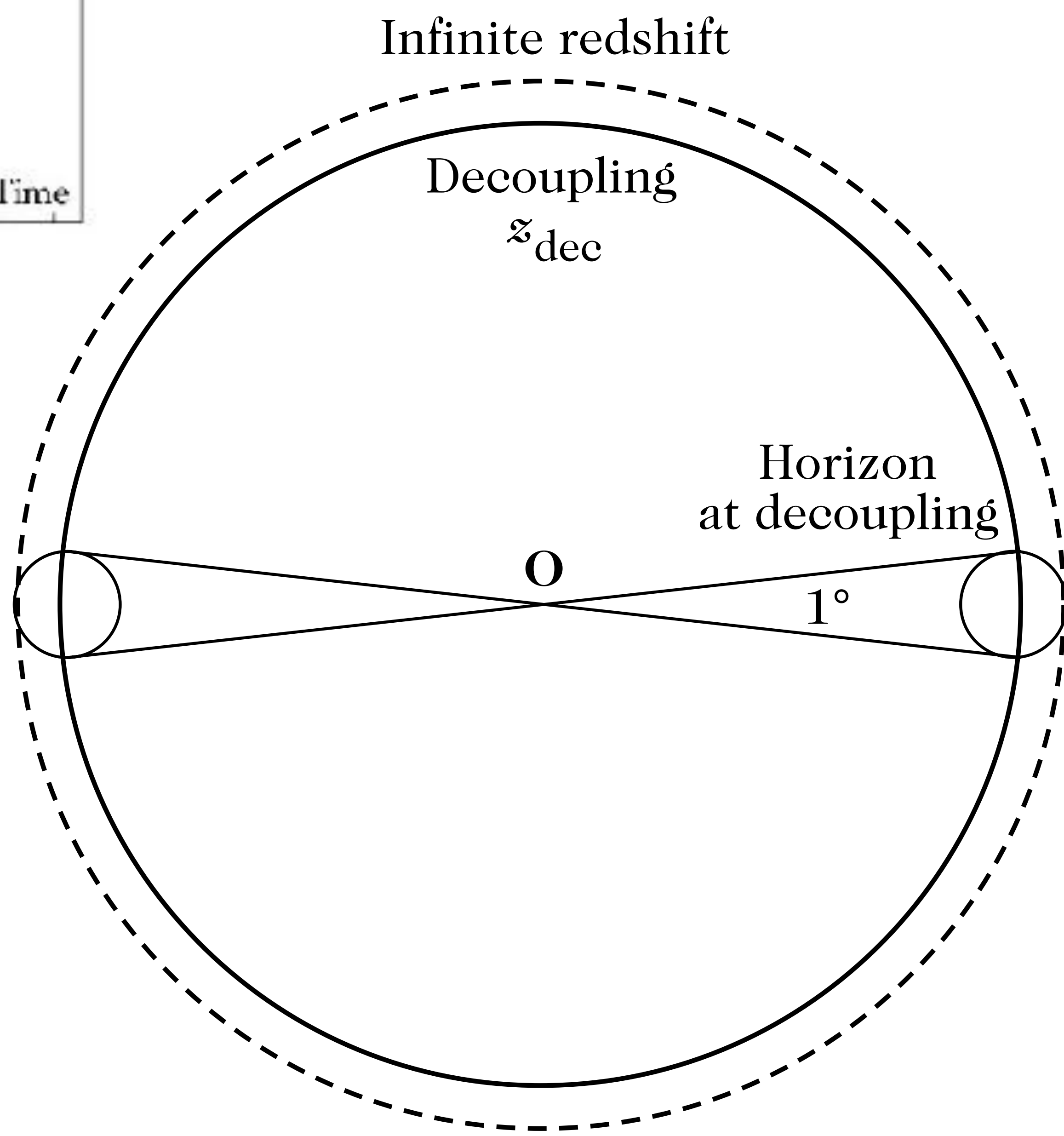
$$|\Omega(z_{\rm eq}) - 1| < 3 \times 10^{-5}$$

$$|\Omega(z_{\rm Planck}) - 1| < 10^{-60}$$

- Monopoles

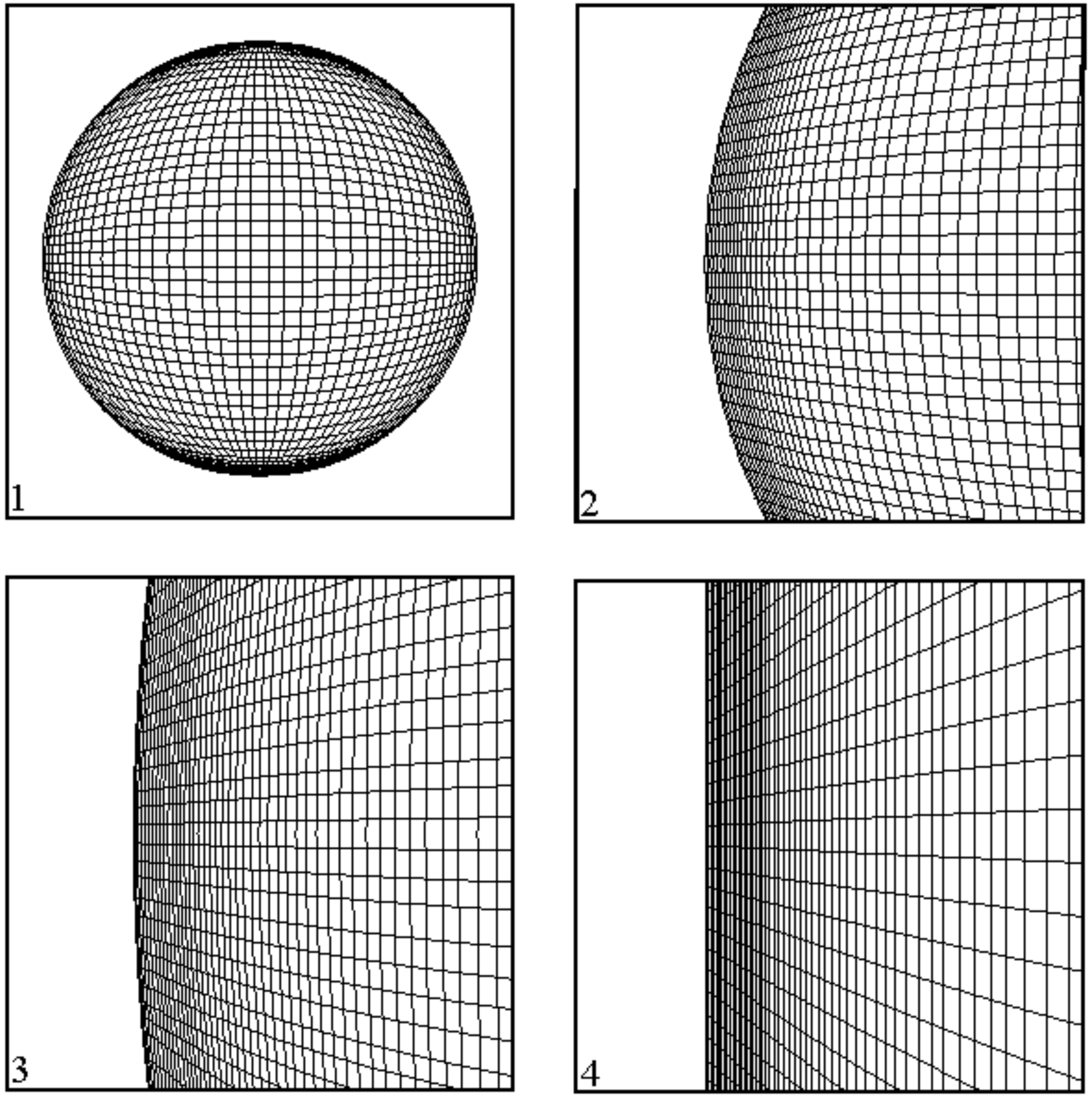
- **Validity of classical GR?**

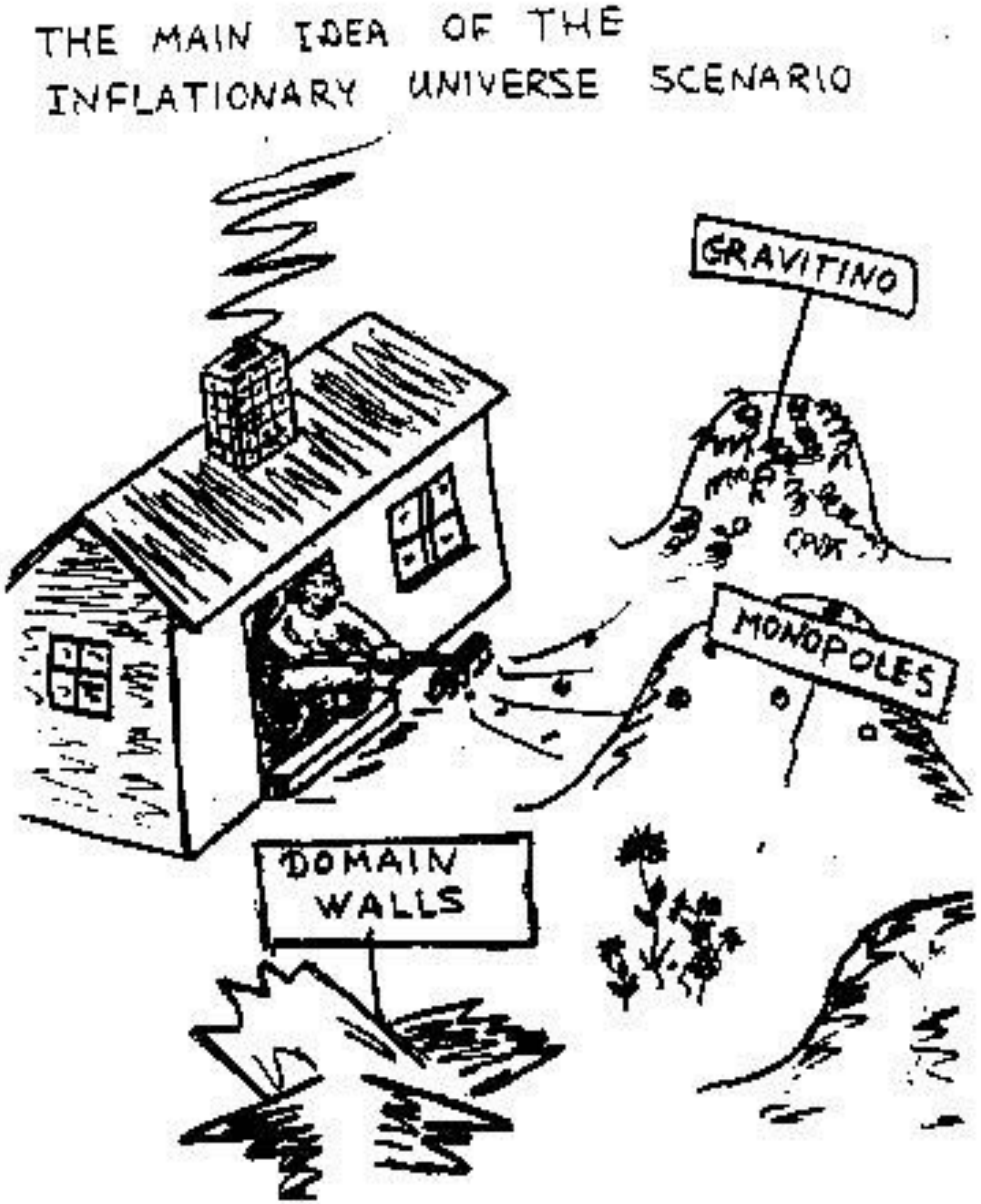




Standard paradigm: inflation!

Phase of accelerated expansion





Standard Model Failures and inflationary solutions

Singularity Not solved... actually not addressed!

Horizon $d_{\mathrm{H}} \equiv a(t) \int_{t_{\mathrm{i}}}^{t} \frac{\mathrm{d}\tau}{a(\tau)}$ can be made as big as one wishes

Flatness $\frac{\mathrm{d}}{\mathrm{d}t} |\Omega - 1| = -2 \frac{\ddot{a}}{\dot{a}^3}$ $\ddot{a} > 0 \ \ \& \ \ \dot{a} > 0$

accelerated expansion (inflation)

Homogeneity & Isotropy

Initial Universe = very small patch
Accelerated expansion drives the shear to zero... $\Longrightarrow$ vacuum state!
+ attractor

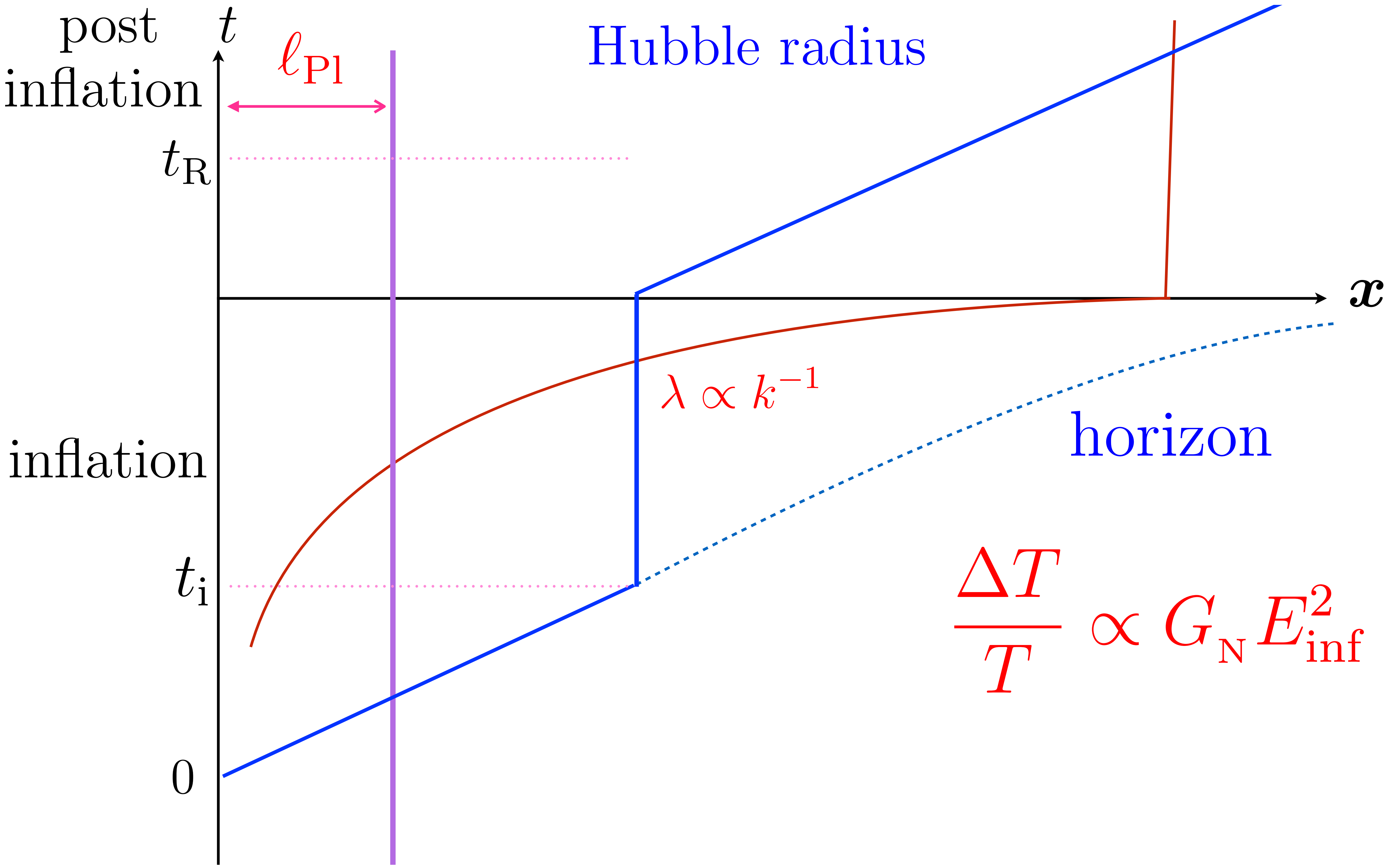
Perturbations Bonus of the theory: superb predictions!!!

Inflation:

- ☺ solves cosmological puzzles
- ☺ uses GR + scalar fields [(semi-)classical]
- ☺ can be implemented in high energy theories???
- ☺ makes falsifiable predictions ...
- ☺ ... consistent with all known observations

Alternative model???

From R. Brandenberger, *in* M. Lemoine, J. Martin & PP (Eds.), “Inflationary cosmology”, Lect. Notes Phys. **738** (Springer, Berlin, 2007).



- Singularity $\exists t_{(\pm\infty)}\,;\, a(t) \rightarrow 0$
- Trans-Planckian

$$\exists t ; \ell(t) = \ell_0 \frac{a(t)}{a_0} \leq \ell_{\rm Pl}$$

- Hierarchy (amplitude)?

$$\frac{V(\varphi)}{\Delta\varphi^4} \leq 10^{-12}$$

- Classical GR?

$$\frac{\Delta T}{T} \propto G_{\rm N} E_{\rm inf}^2 \sim \left(\frac{E_{\rm inf}}{M_{\rm Pl}}\right)^2 \longrightarrow E_{\rm inf} \simeq 10^{-3} M_{\rm Pl}$$

- η problem & Lyth bound
- Initial condition & entropy
- Eternal inflation & measure (anthropic)

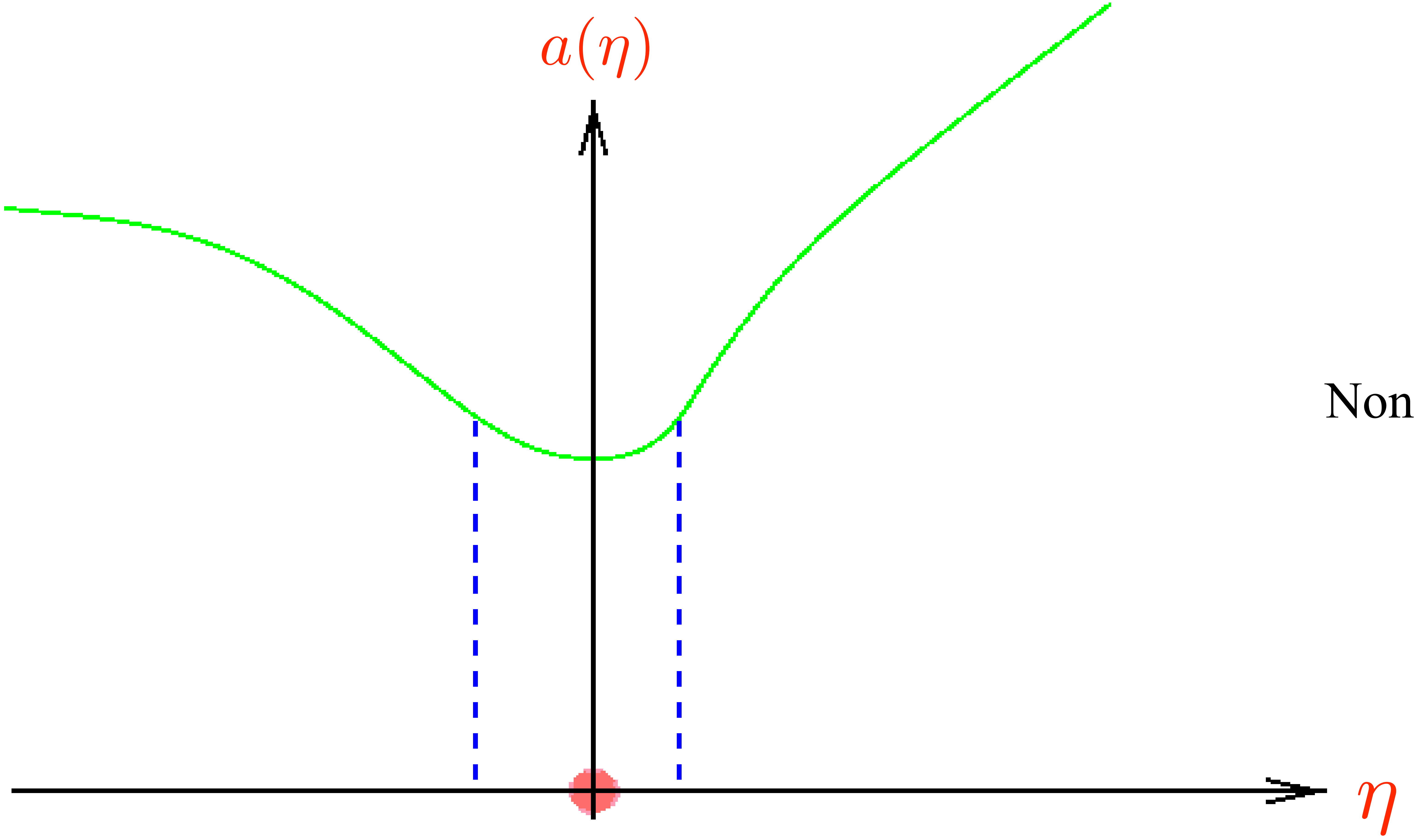
Inflation

- ☺ solves cosmological puzzles
- ☺ uses GR + scalar fields [(semi-)classical]
- ☺ can be implemented in high energy theories???
- ☺ makes falsifiable predictions ...
- ☺ ... consistent with all known observations

Alternative model???

- ➡ singularity, initial conditions & homogeneity
- ➡ string based ideas (PBB, other brane models, string gas, …)
- ➡ Quantum gravity / cosmology
- ➡ provide challengers / new ingredients!

➡ bouncing cosmology



Non singular bounce

A brief history of bouncing cosmology

- R. C. Tolman, "*On the Theoretical Requirements for a Periodic Behaviour of the Universe*", PRD 38, 1758 (1931)
- G. Lemaître, "*L'Univers en expansion*", Ann. Soc. Sci. Bruxelles (1933)

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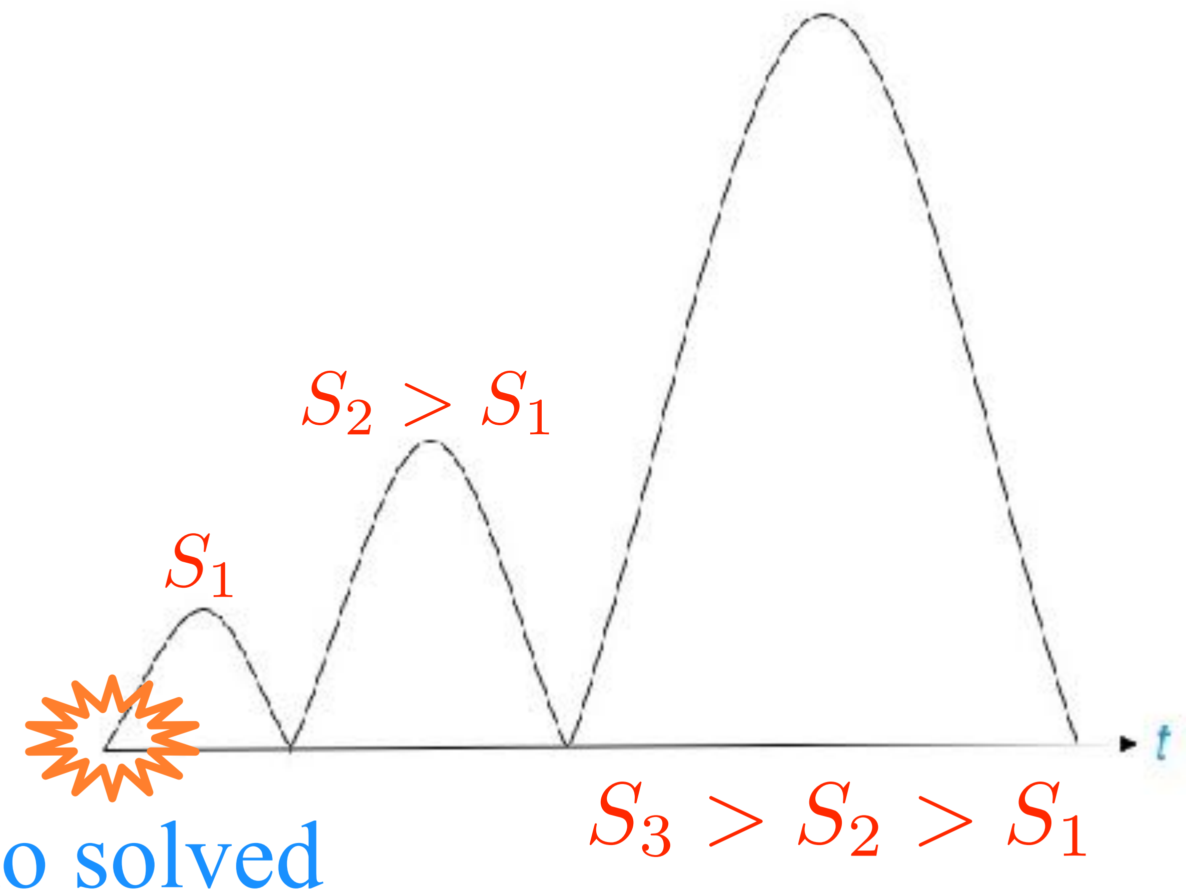


…

Singularity pb no solved

- A. A. Starobinsky, "*On one non-singular isotropic cosmological model*", Sov. Astron. Lett. 4, 82 (1978)
- V. N. Melnikov, S.V. Orlov, Phys. Lett. A 70, 263 (1979).
- M. Novello & J. M. Salim, Phys. Rev. D20, 377 (1979).
- R. Durrer & J. Laukerman, "*The oscillating Universe: an alternative to inflation*", Class. Quantum Grav. 13, 1069 (1996)

- Many new ideas, models…

- M. Novello & S.E. Perez Bergliaffa, "*Bouncing cosmologies*", Phys. Rep. **463**, 127 (2008)
- D. Battefeld & PP, "*A Critical Review of Classical Bouncing Cosmologies*", Phys. Rep. **571**, 1 (2015)
- R. Brandenberger & PP, "*Bouncing cosmologies: Progress and problems*", Found. Phys. (2017)

PP & N. Pinto-Neto, *Phys. Rev.* **D78**, 063506 (2008)

Standard Model Failures and bouncing solutions

Singularity Merely a non issue in the bounce case!

Horizon $d_{\mathrm{H}} \equiv a(t)\int_{t_{\mathrm{i}}}^{t}\frac{\mathrm{d}\tau}{a(\tau)}$ can be made divergent easily if $t_{\mathrm{i}} \rightarrow -\infty$

Flatness $\frac{\mathrm{d}}{\mathrm{d}t}|\Omega-1| = -2\frac{\ddot{a}}{\dot{a}^3}$ $\ddot{a} < 0 \;\&\; \dot{a} < 0$

accelerated expansion (inflation) or decelerated contraction (bounce)

Homogeneity Large & flat Universe + low initial density + diffusion

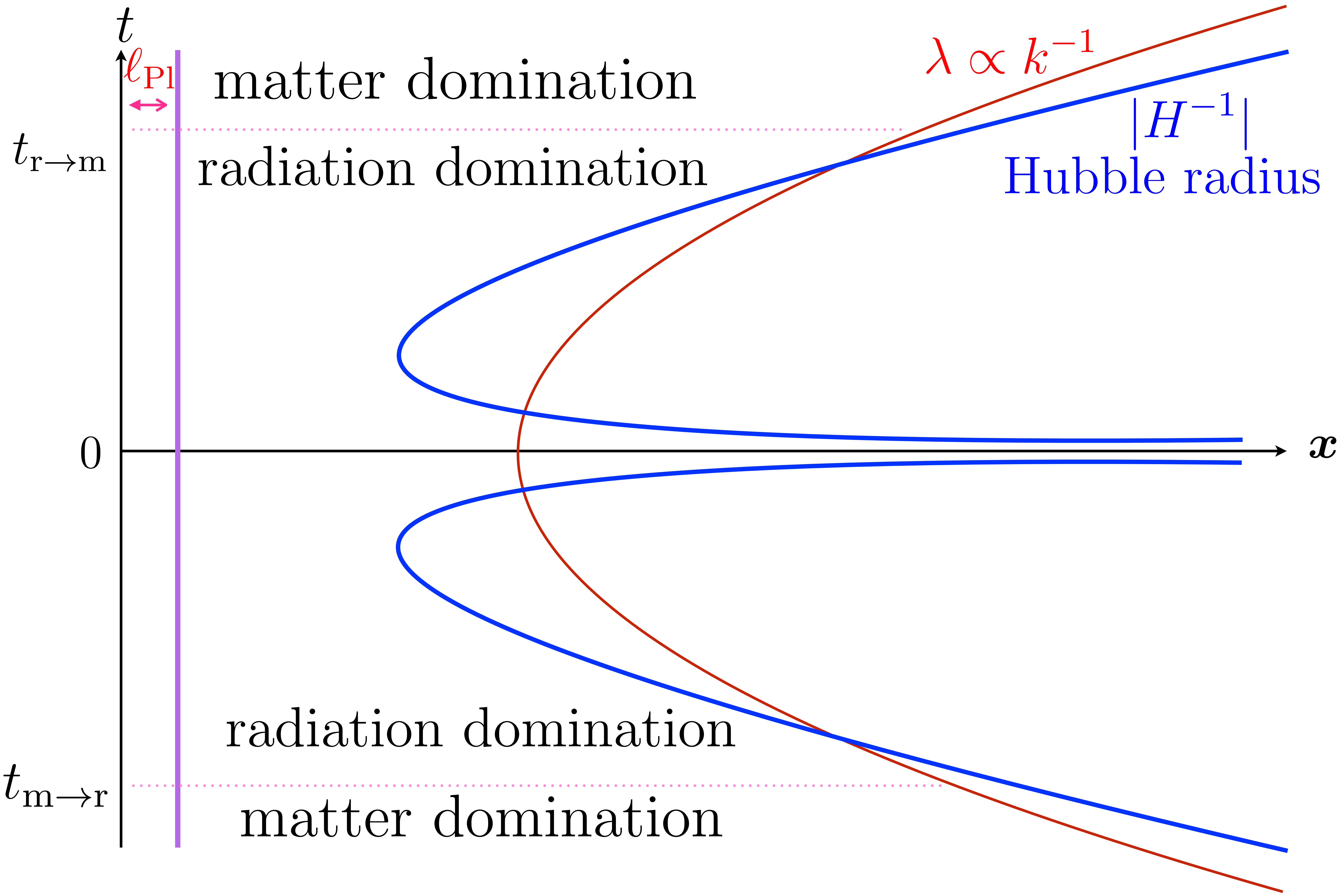
$$\frac{t_{\text{dissipation}}}{t_{\text{Hubble}}} \propto \frac{\lambda}{R_{\mathrm{H}}^{1/3}}\left(1+\frac{\lambda}{AR_{\mathrm{H}}^{2}}\right)$$

enough time to dissipate any wavelength

$\Longrightarrow$ quantum vacuum fluctuations...

Isotropy Potentially problematic: model dependent

Others dark matter/energy, baryogenesis, ...


t
$\ell_{\rm Pl}$
matter domination
$t_{\rm r\to m}$
radiation domination
$\lambda \propto k^{-1}$
$|H^{-1}|$
Hubble radius
0
$\boldsymbol{x}$
radiation domination
$t_{\rm m\to r}$
matter domination

PP & N. Pinto-Neto, *Phys. Rev.* **D78**, 063506 (2008)

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$$d_{\mathrm{H}}^{\mathrm{cont}} = \frac{3(1+w)}{1+3w} t_{\mathrm{end}} \left[1 - \left(\frac{t_{\mathrm{ini}}}{t_{\mathrm{end}}} \right)^{(1+3w)/[3(1+w)]} \right]$$

$$t_{\mathrm{ini}} \to -\infty$$

PP & N. Pinto-Neto, *Phys. Rev.* **D78**, 063506 (2008)

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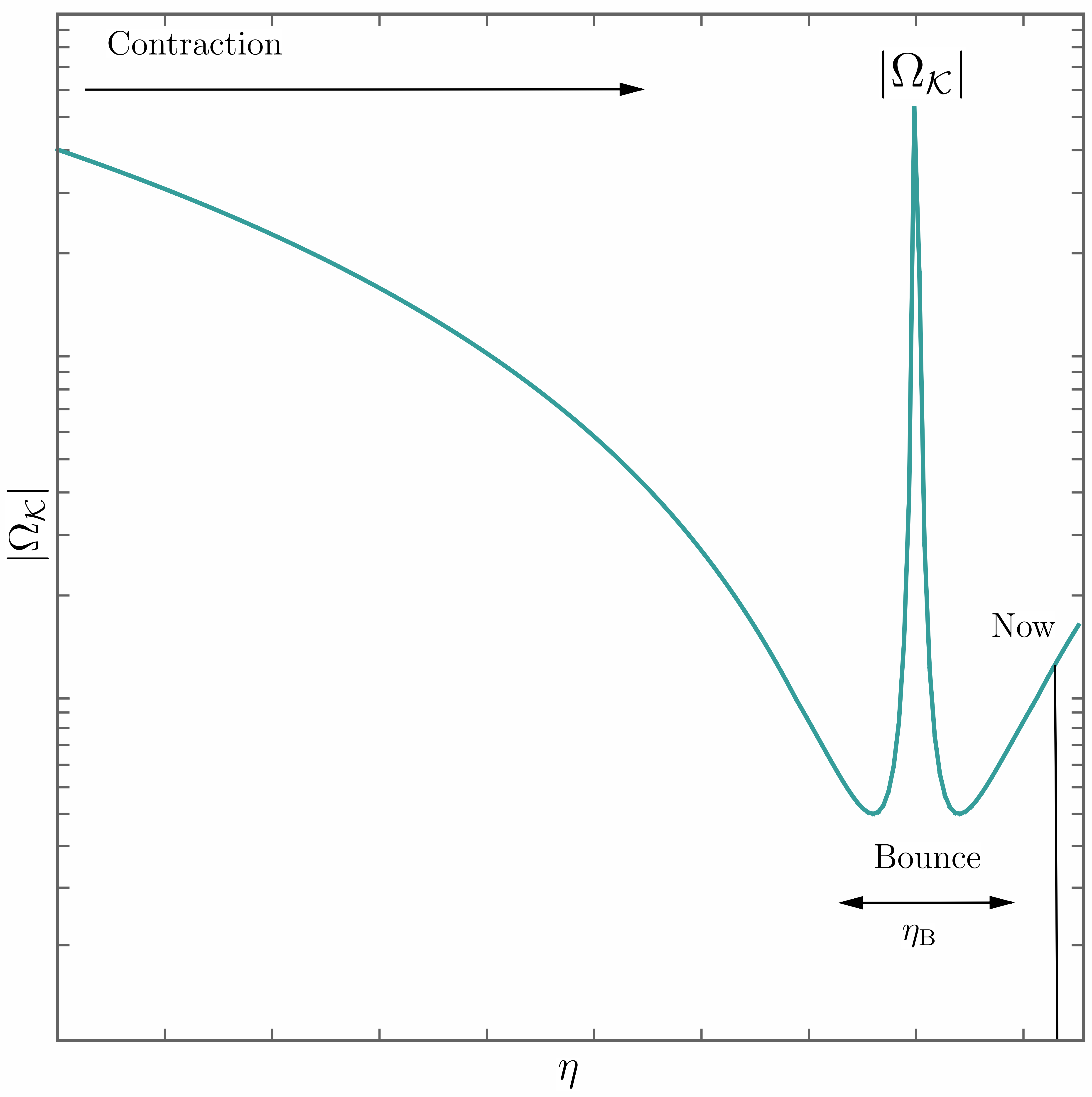
$$H^2 = \frac{1}{3}\left[-\frac{3\mathcal{K}}{a^2} + \frac{\rho_{\mathrm{m}0}}{a^3} + \frac{\rho_{\mathrm{r}0}}{a^4} + \frac{\rho_{\theta 0}}{a^6} + \ldots + \frac{\rho_{\phi 0}}{a^{3(1+w_\phi)}}\right]$$

Critical density

$$\rho_{\mathrm{c}} \equiv \frac{3H^2}{8\pi G_{\mathrm{N}}} \quad \Longrightarrow \quad \Omega \equiv \frac{\rho}{\rho_{\mathrm{c}}}$$

Density parameter



PP & N. Pinto-Neto, *Phys. Rev.* **D78**, 063506 (2008)

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Shear Potentially problematic: model dependent

PP & N. Pinto-Neto, *Phys. Rev.* **D78**, 063506 (2008)

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Implementing a bounce

Quantized scalar field effect model:

Parker & Fulling '73: massive scalar field, if $\langle a^\dagger a \rangle \gg 1$, then solution ($\kappa > 0$)

$$a(t) = \left(\frac{|B_2|^2 - |B_1|^2}{m^2 |B_2|^2} + \frac{8\pi G m^2 |B_2|^2 t^2}{3} \right)^{1/2};$$

Coherence of the quantum state crucial

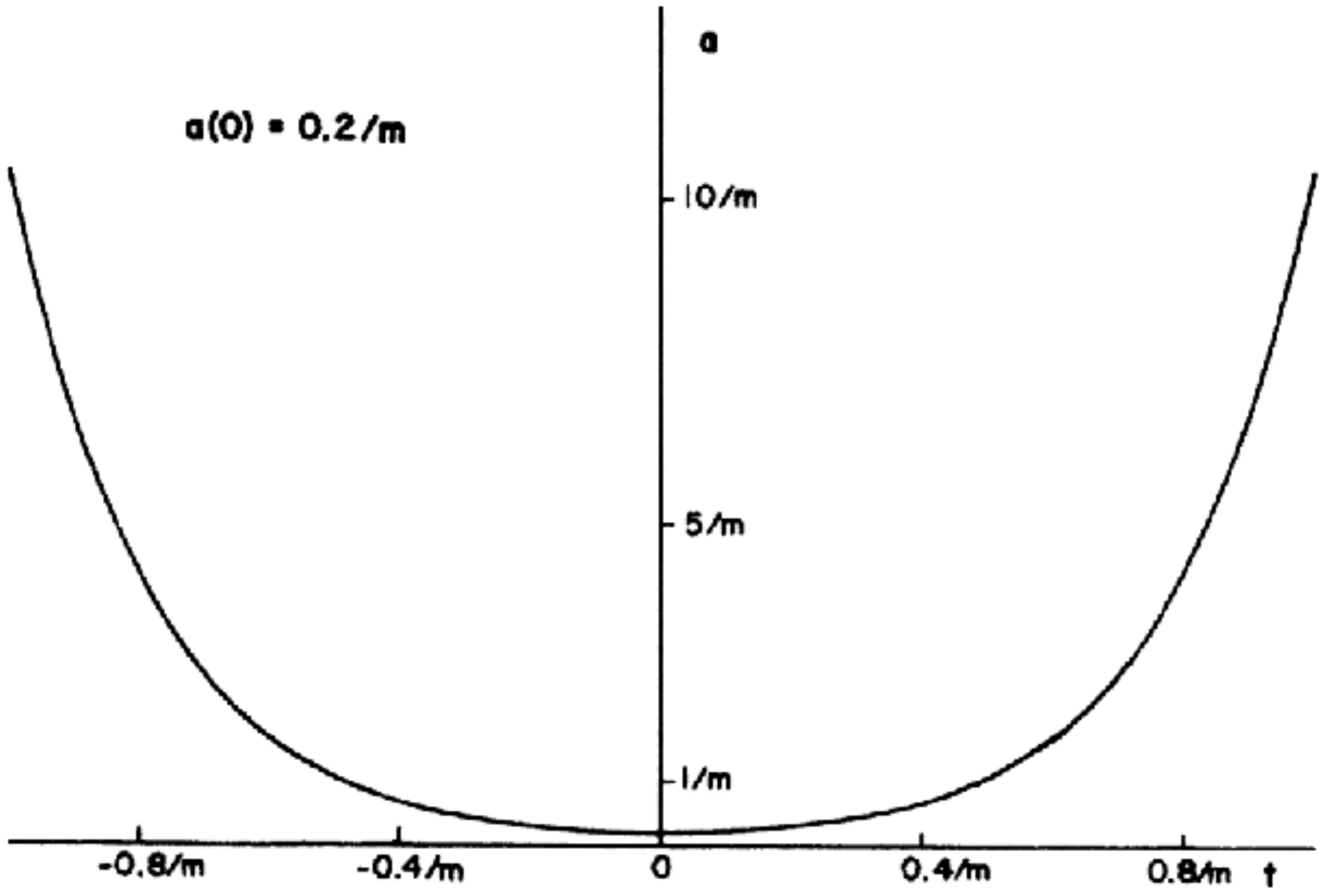

FIG. 1. Solution with $a(0) = 0.2m^{-1}$: time-symmetric expansion from the minimum radius.

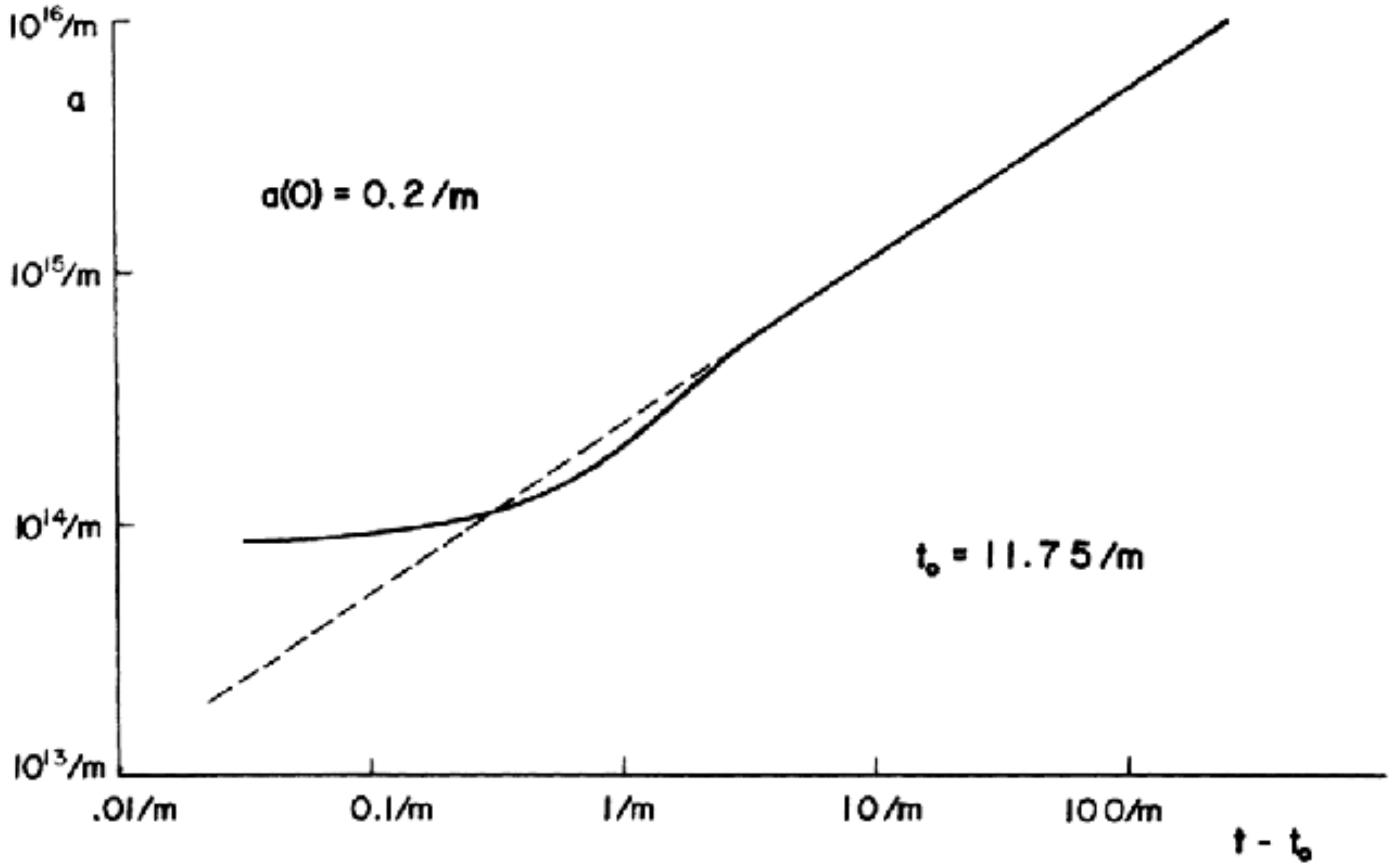

FIG. 2. Solution with $a(0) = 0.2m^{-1}$ (solid curve): approach to a Friedmann solution (dashed curve). The horizontal and vertical scales are logarithmic, and the time origin has been shifted to the initial singularity of the Friedmann curve, so that the latter becomes a straight line of slope $\frac{2}{3}$. (The deviation of the Friedmann solution from the $a \propto t^{2/3}$ law due to the three-space curvature of the closed universe is negligible in the range of t plotted.)

Implementing a bounce

Model listing:

Quantum gravity

LQG & LQC

Canonical quantum gravity (WdW)

String theory

Non relativistic quantum gravity

Implementing a bounce

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Quantum gravity

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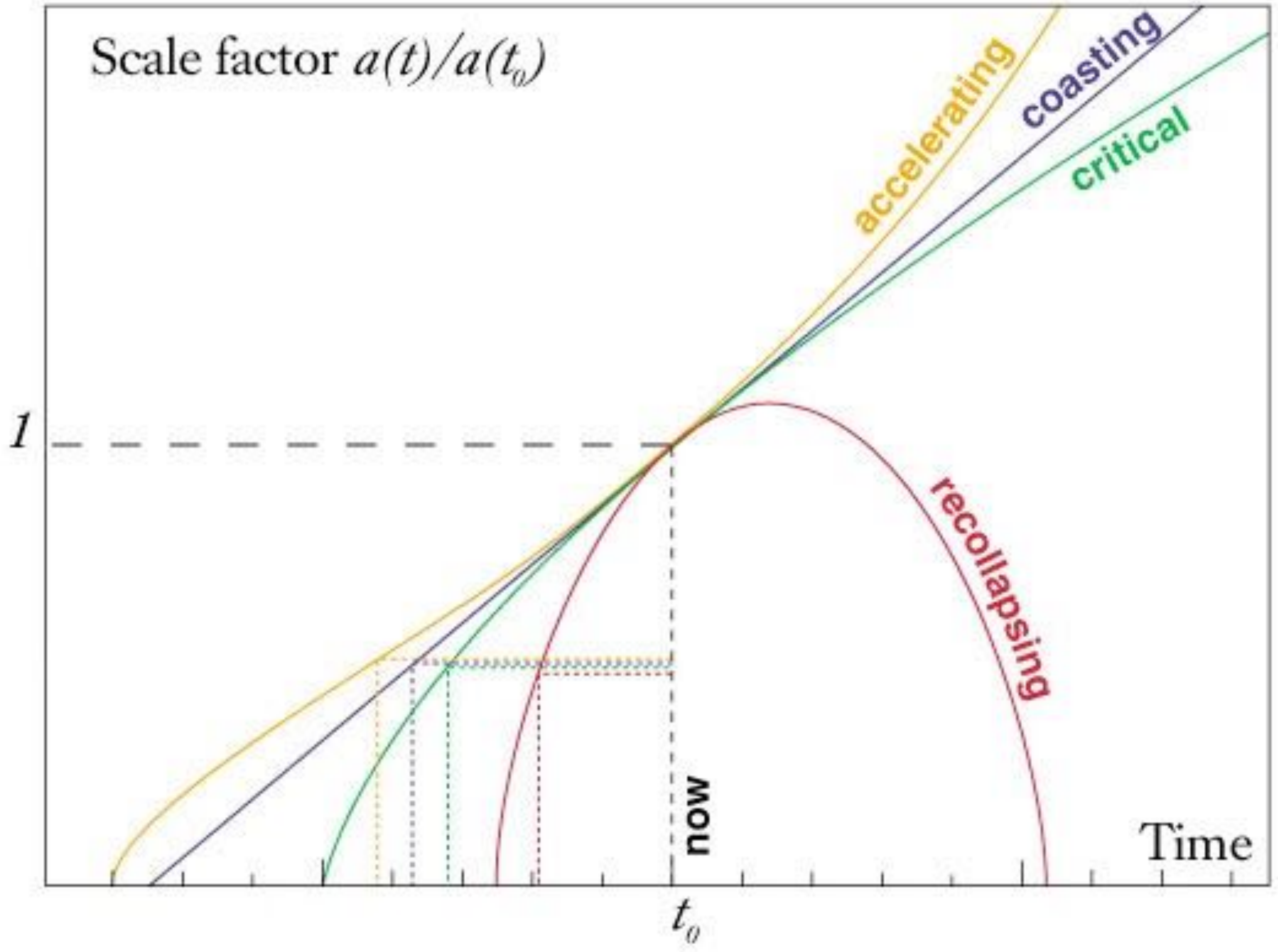
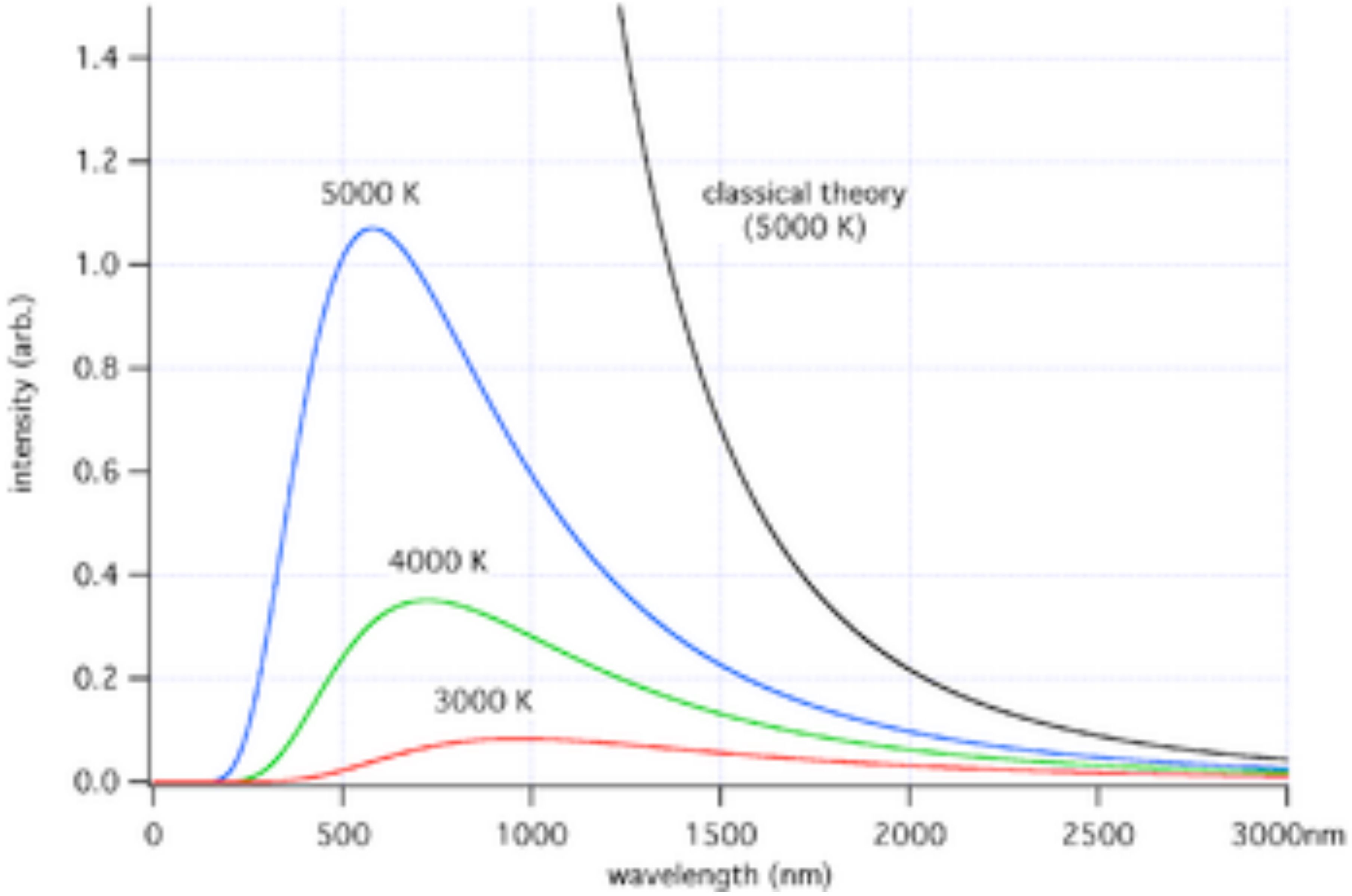
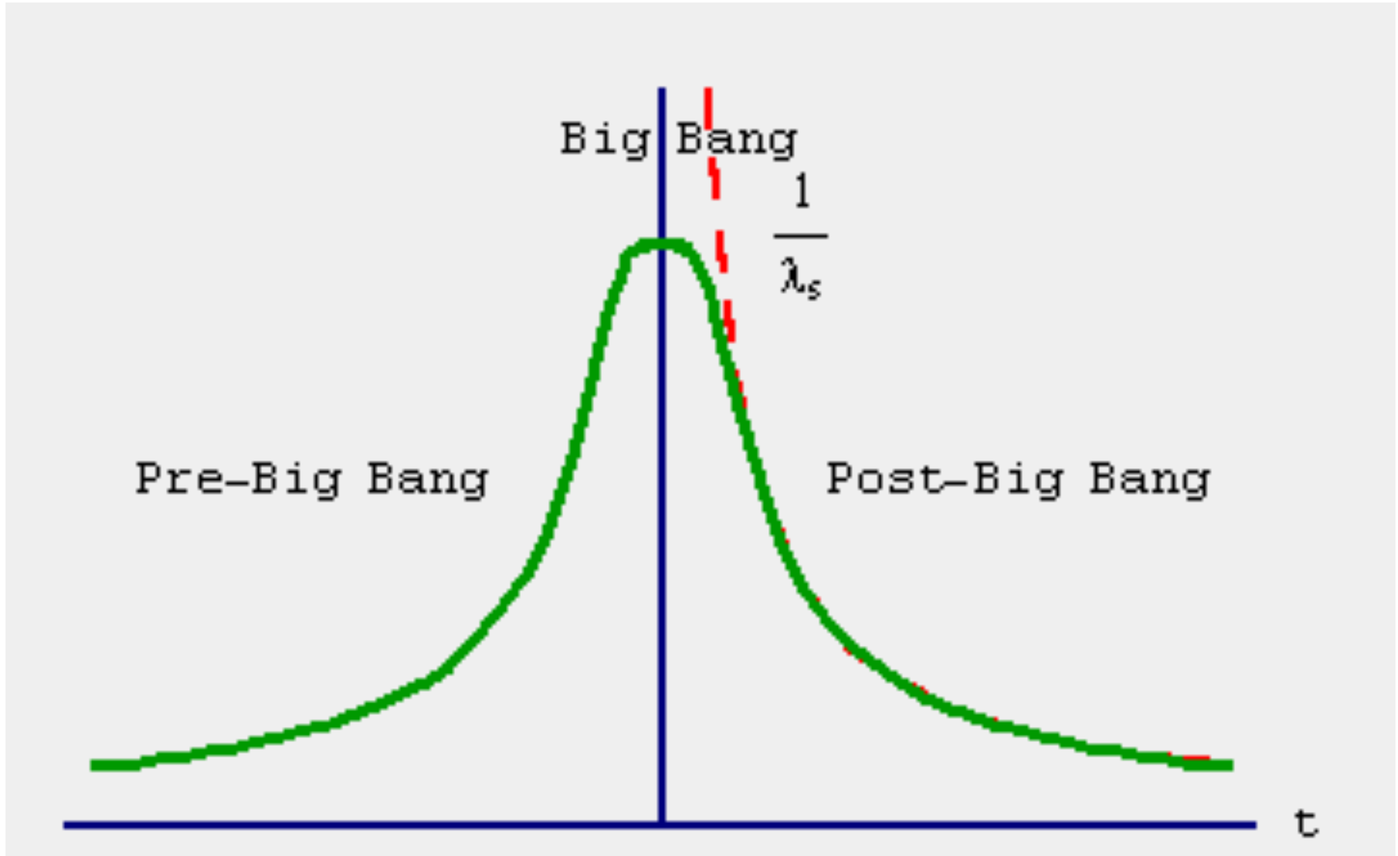
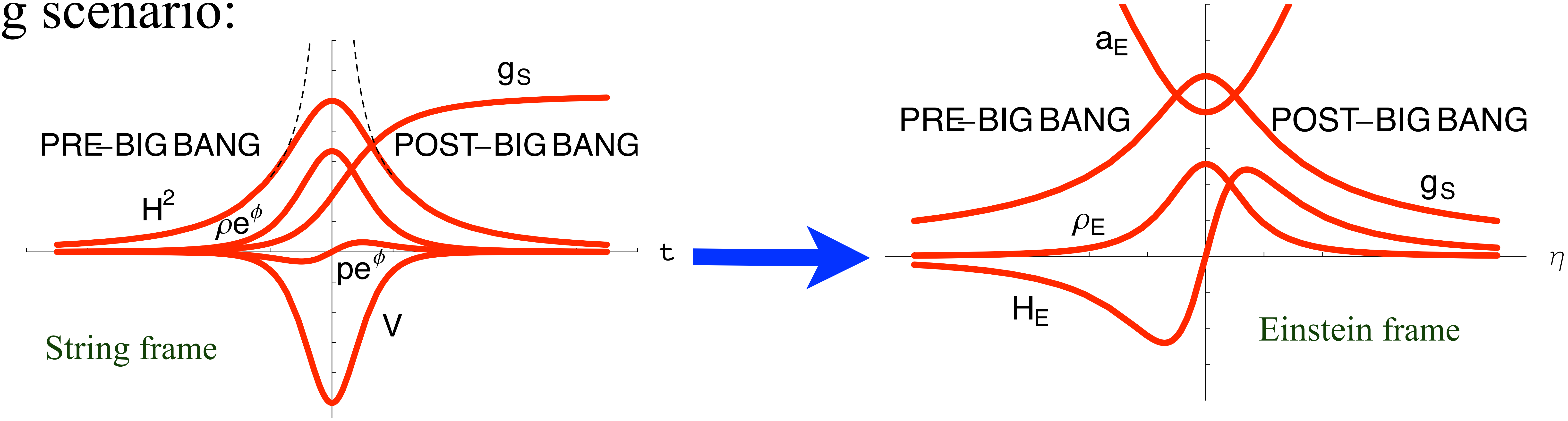




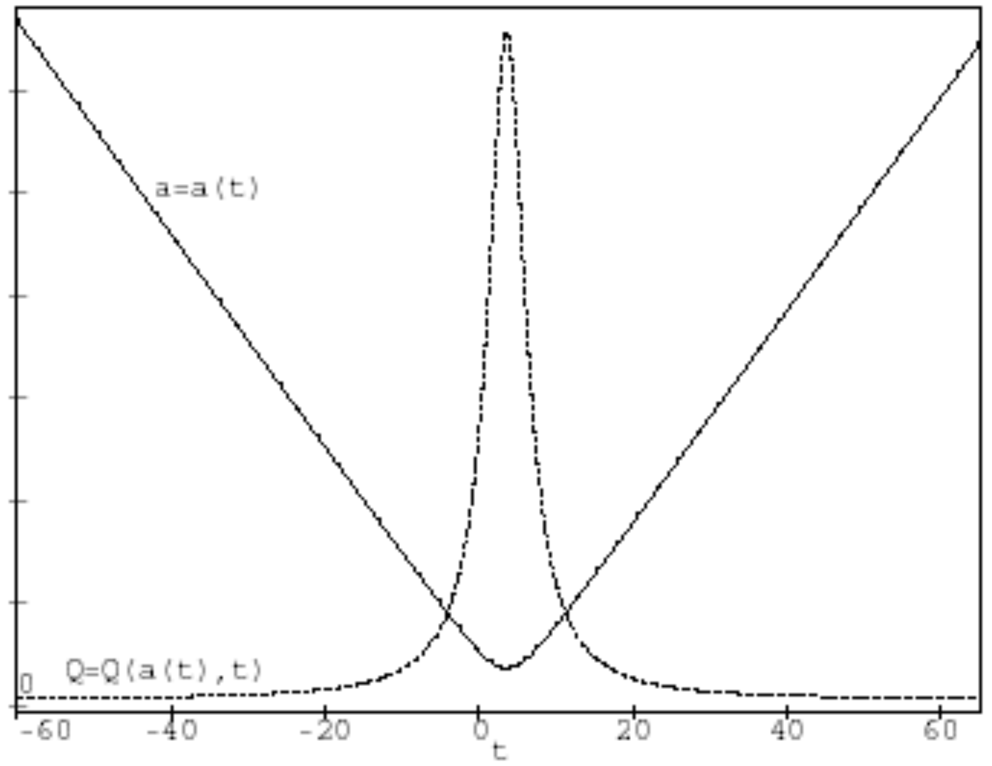


M.Gasperini & G. Veneziano, *Phys. Rep.* **373**, 1 (2003), hep-th/0207130 & hep-th/0703055

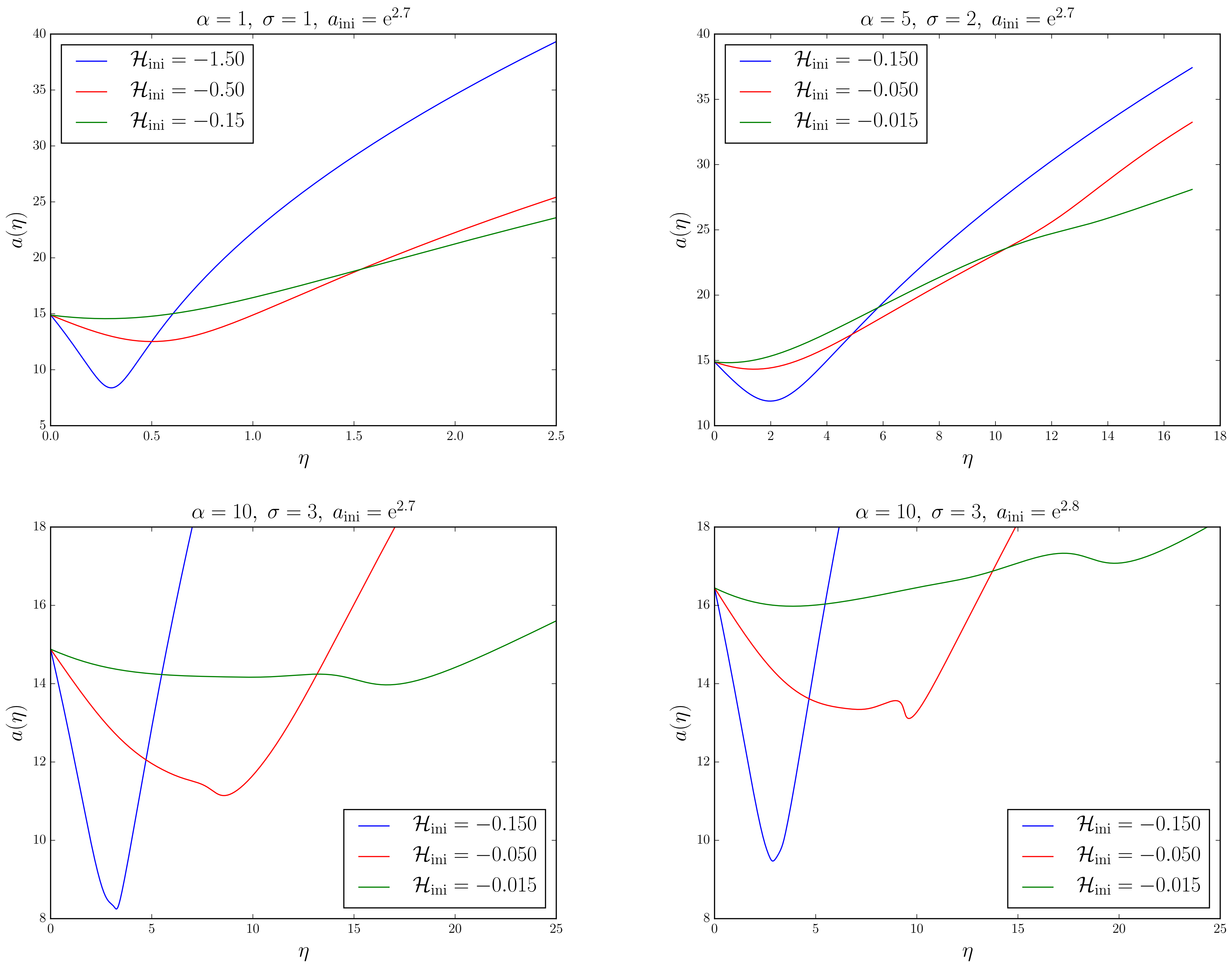
Pre Big Bang scenario:



quantum cosmology:



J. Acacio de Barros, N. Pinto-Neto & M. Sagorio-Leal
Phys. Lett. A**241**, 229 (1998)



S. Vitenti & PP
Mod.Phys.Lett. A**31**, 1640006 (2016).

Model listing:

Quantum gravity

LQG & LQC

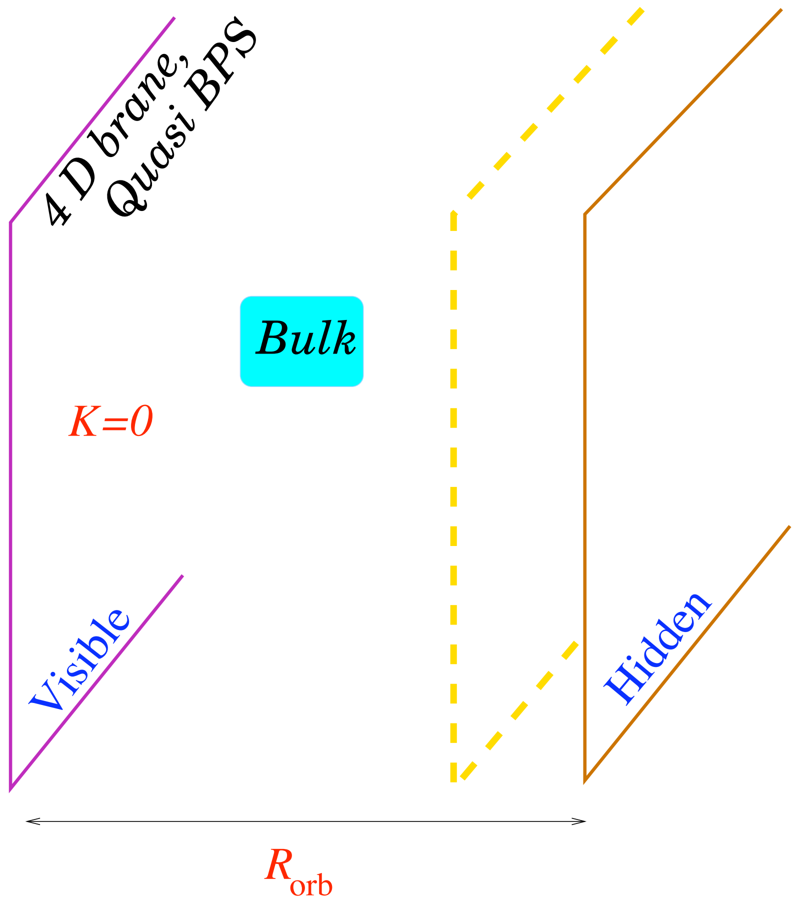
Non relativistic quantum gravity

Canonical quantum gravity (WdW)

String theory

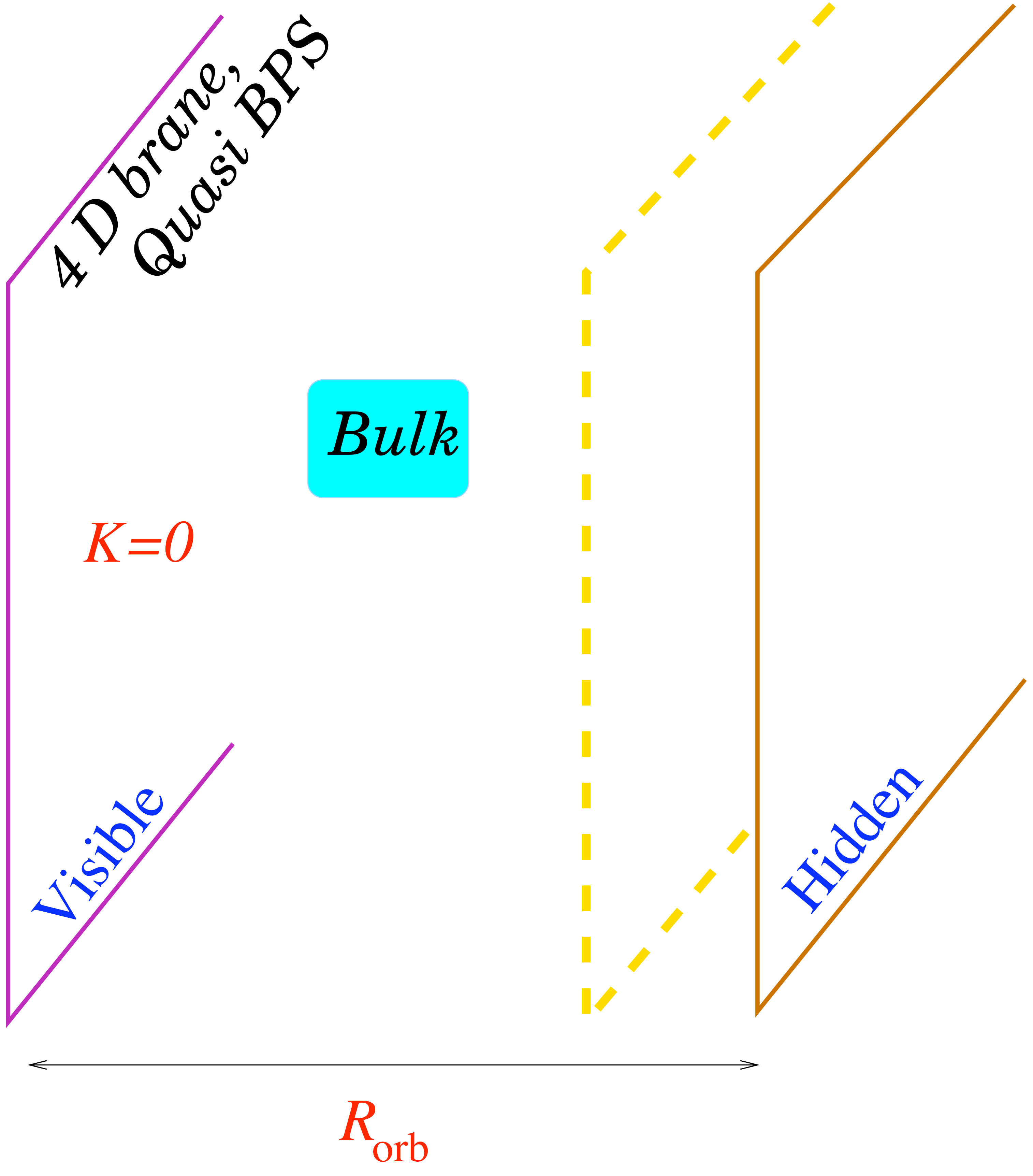
Ekpyrotic & cyclic

Branes



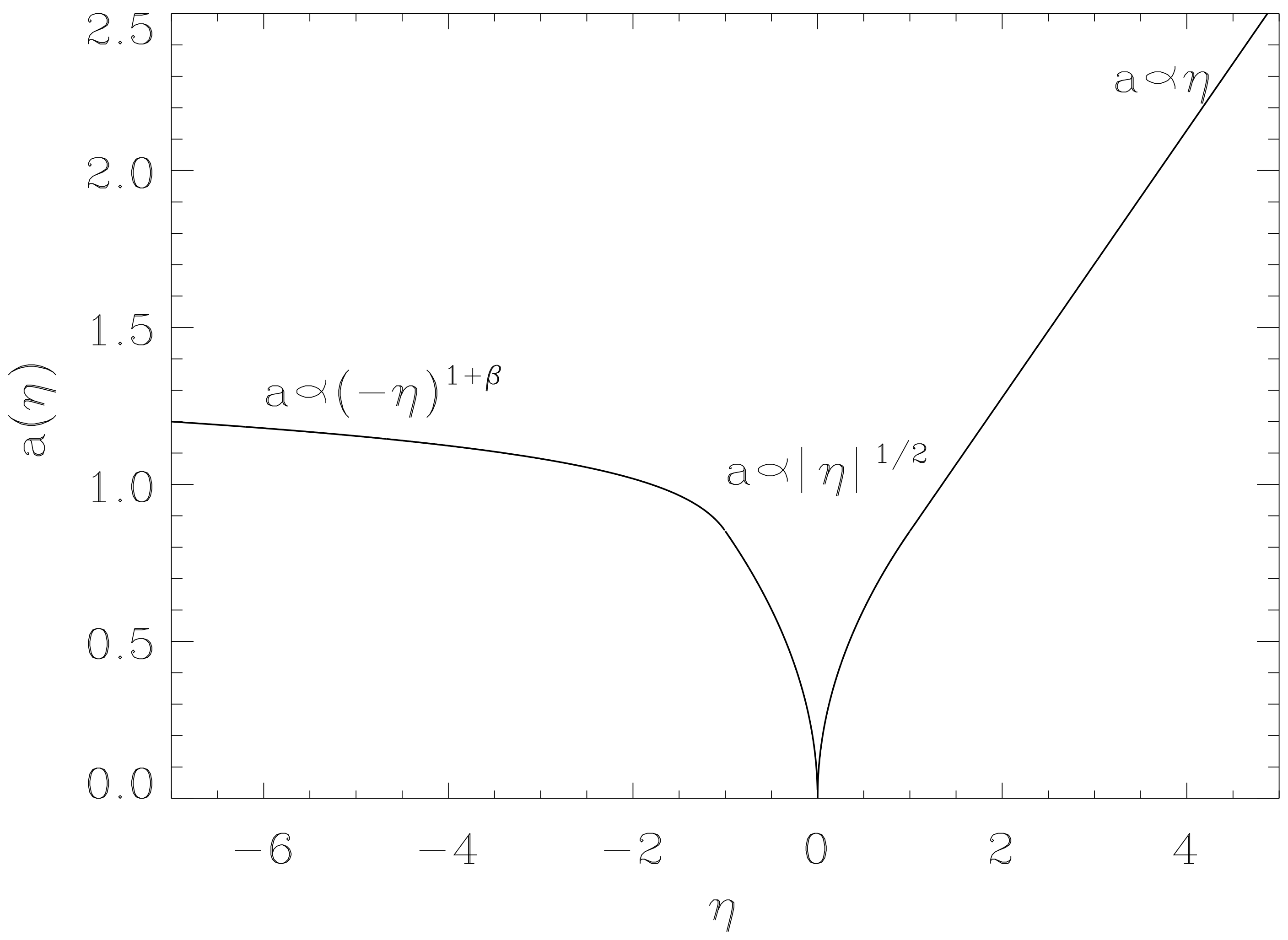
Ekpyrotic scenario:

$$\mathcal{S}_5 \propto \int_{\mathcal{M}_5} \mathrm{d}^5 x \sqrt{-g_5} \left[R_{(5)} - \frac{1}{2} (\partial\varphi)^2 - \frac{3}{2} \frac{\mathrm{e}^{2\varphi} \mathcal{F}^2}{5\,!} \right],$$

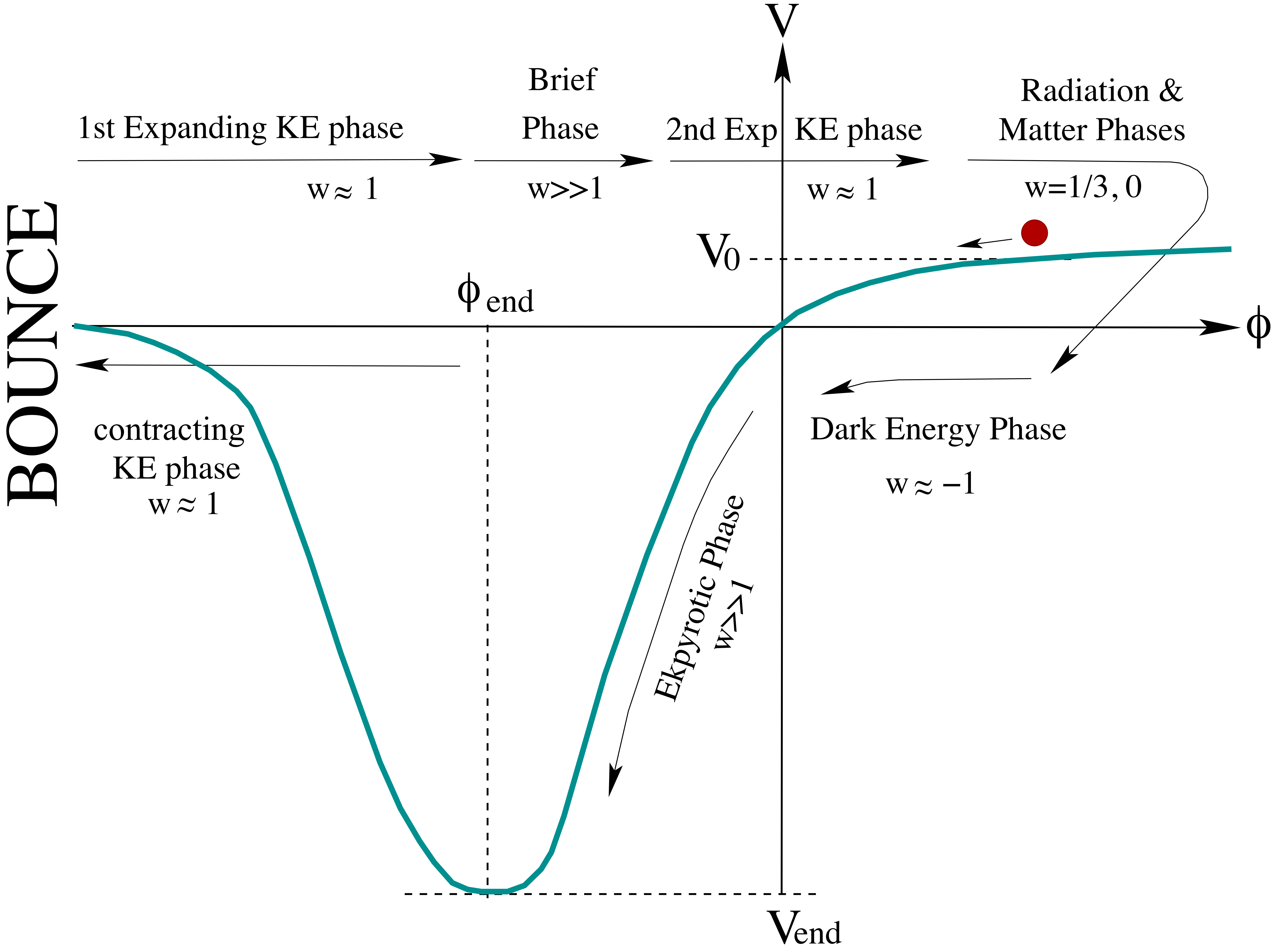


$$\mathcal{S}_4 = \int_{\mathcal{M}_4} \mathrm{d}^4 x \sqrt{-g_4} \left[\frac{R_{(4)}}{2\kappa} - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$

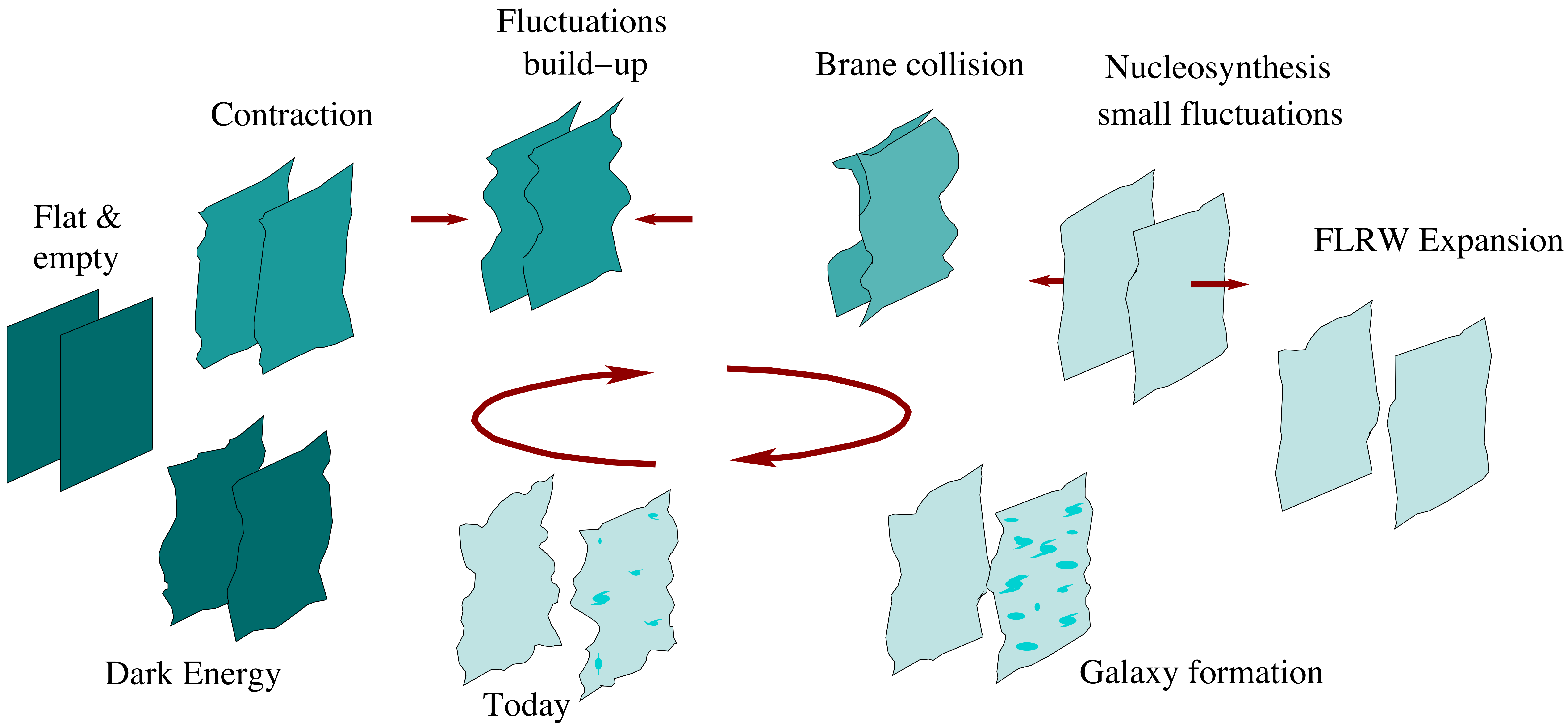
$$V(\varphi) = -V_{\mathrm{i}} \exp\left[-\frac{4\sqrt{\pi\gamma}}{m_{\mathrm{Pl}}} (\varphi - \varphi_{\mathrm{i}}) \right]$$

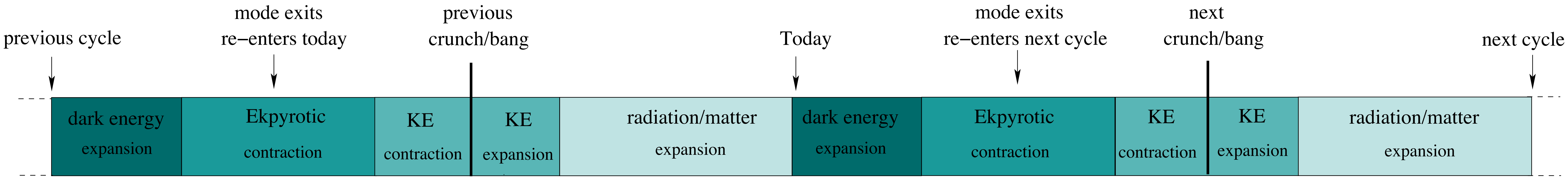


Singular…


V
1st Expanding KE phase
w ≈ 1
Brief
Phase
w>>1
2nd Exp KE phase
w ≈ 1
Radiation &
Matter Phases
w=1/3, 0
V_0
ϕ_{end}
ϕ
BOUNCE
contracting
KE phase
w ≈ 1
Dark Energy Phase
w ≈ −1
Ekpyrotic Phase
w>>1
V_{end}

Cyclic extension





previous cycle

mode exits
re–enters today

previous
crunch/bang

Today

mode exits
re–enters next cycle

next
crunch/bang

next cycle

dark energy expansion	Ekpyrotic contraction	KE contraction	KE expansion	radiation/matter expansion	dark energy expansion	Ekpyrotic contraction	KE contraction	KE expansion	radiation/matter expansion

Model listing:

Quantum gravity

Ekpyrotic & cyclic

Branes

String gas cosmology

Antigravity

Galileon

Massive gravity

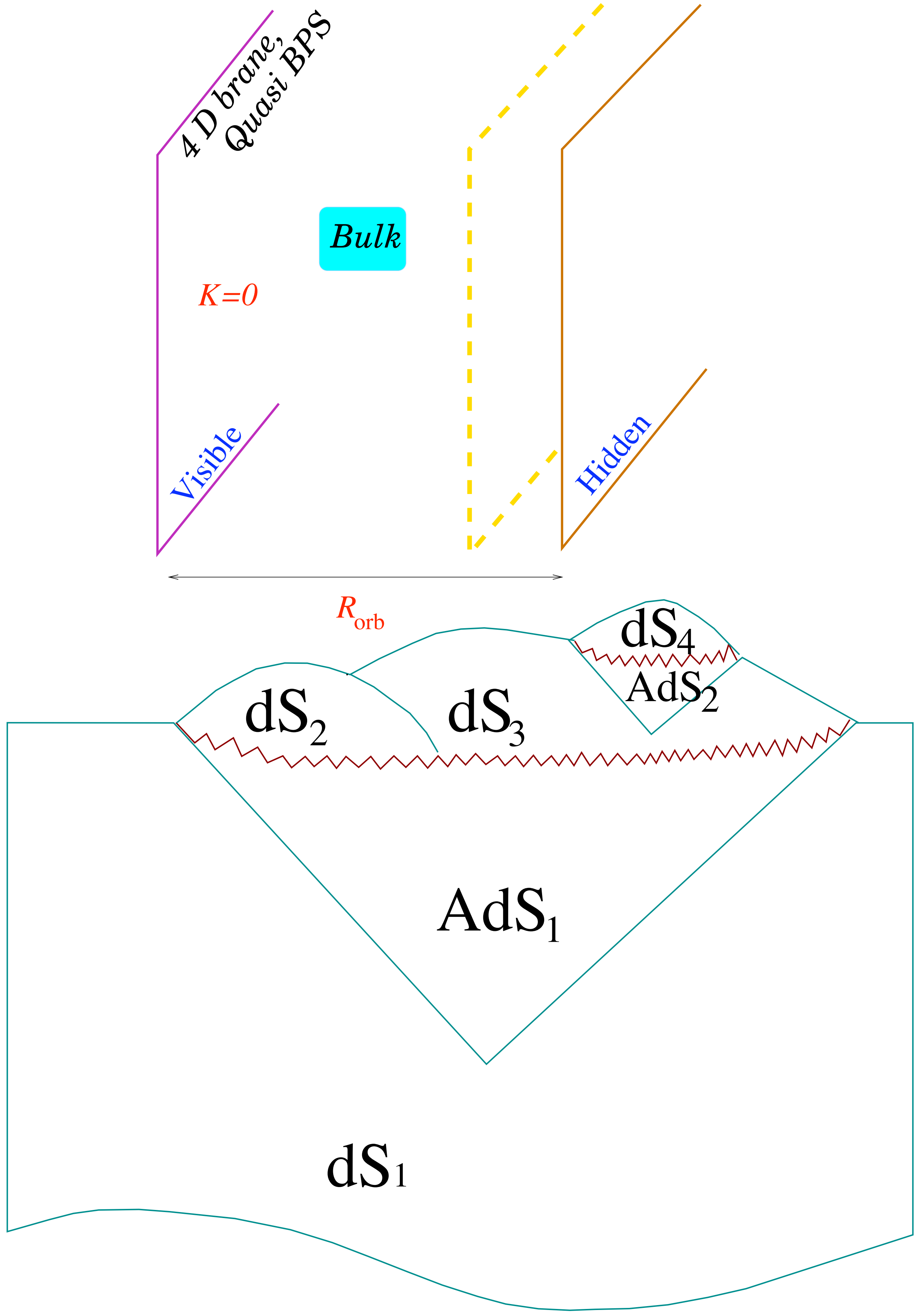
Multiverse models

Strings & AdS/CFT

LQG & LQC

Canonical quantum gravity (WdW)

String theory



Non relativistic quantum gravity

Horava-Lifshitz

Lee-Wick & Quintom

F(R), *f(T)*, Gauss-Bonnet

Mimetic matter

Non-linear electromagnetic action

Spinors & torsion

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(~500 in 2015)

Implementing a bounce = problem with GR!

$$\dot{H} = \frac{\mathcal{K}}{a^2} - \frac{1}{2}(\rho + P)$$

Violation of Null Energy Condition (NEC)

$$\rho + P \leq 0$$

↓

Instabilities for perfect fluids

Implementing a bounce = problem with GR!

$$\dot{H} = \frac{\mathcal{K}}{a^2} - \frac{1}{2}(\rho + P)$$

Violation of Null Energy Condition (NEC)

$$\rho + P \leq 0$$

Positive spatial curvature + scalar field

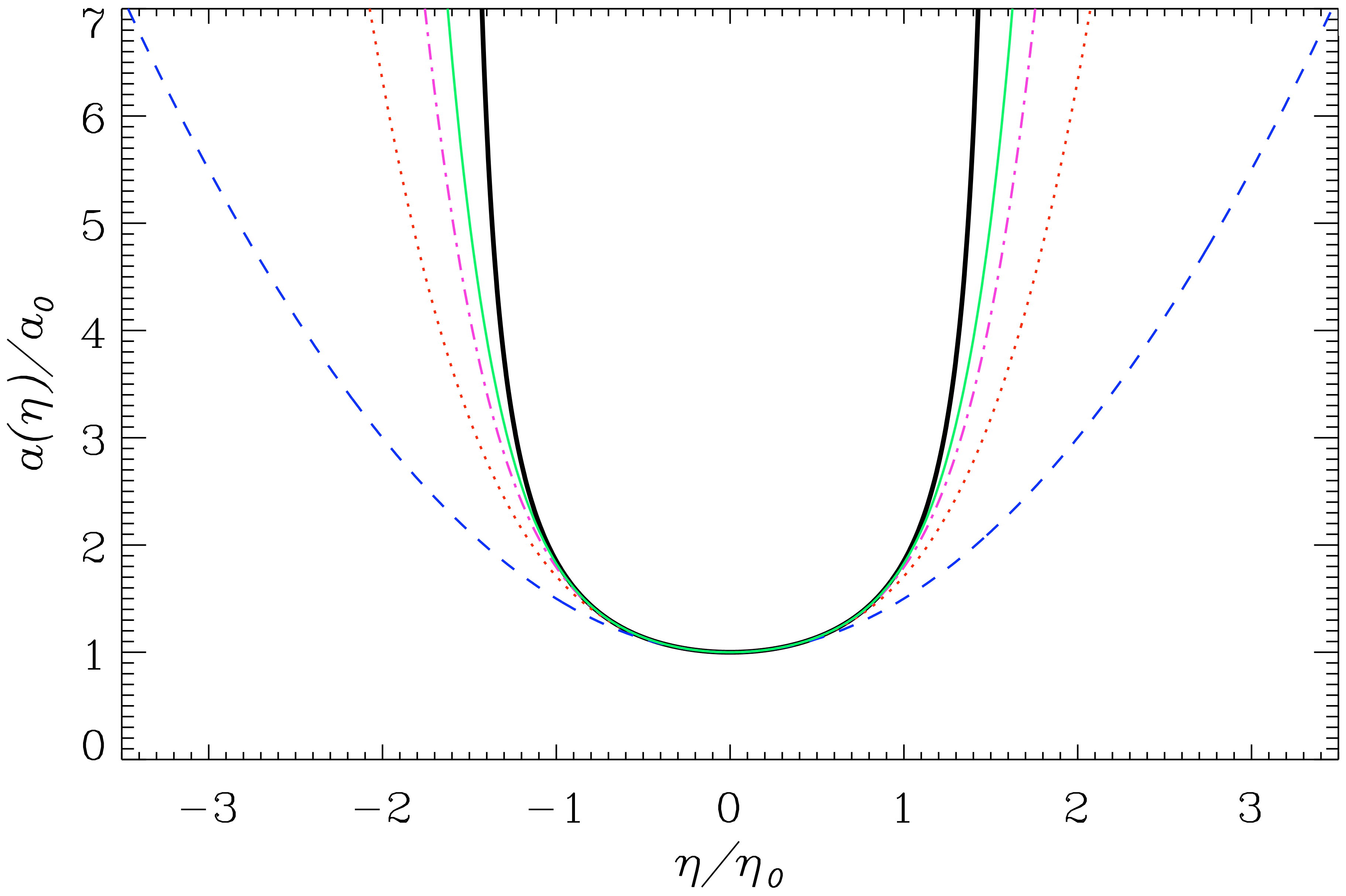
Self consistent bounce:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\mathrm{d}\Omega^2\right)$$

→ One d.o.f. + 4 dimensions G.R.

✹ $$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{6\ell_{\mathrm{Pl}}^2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V\left(\varphi\right)\right]$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) - \frac{\mathcal{K}}{a^2}$$ Positive spatial curvature



J. Martin & PP., *Phys. Rev.* **D68**, 103517 (2003)

Self consistent bounce:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\mathrm{d}\Omega^2\right)$$

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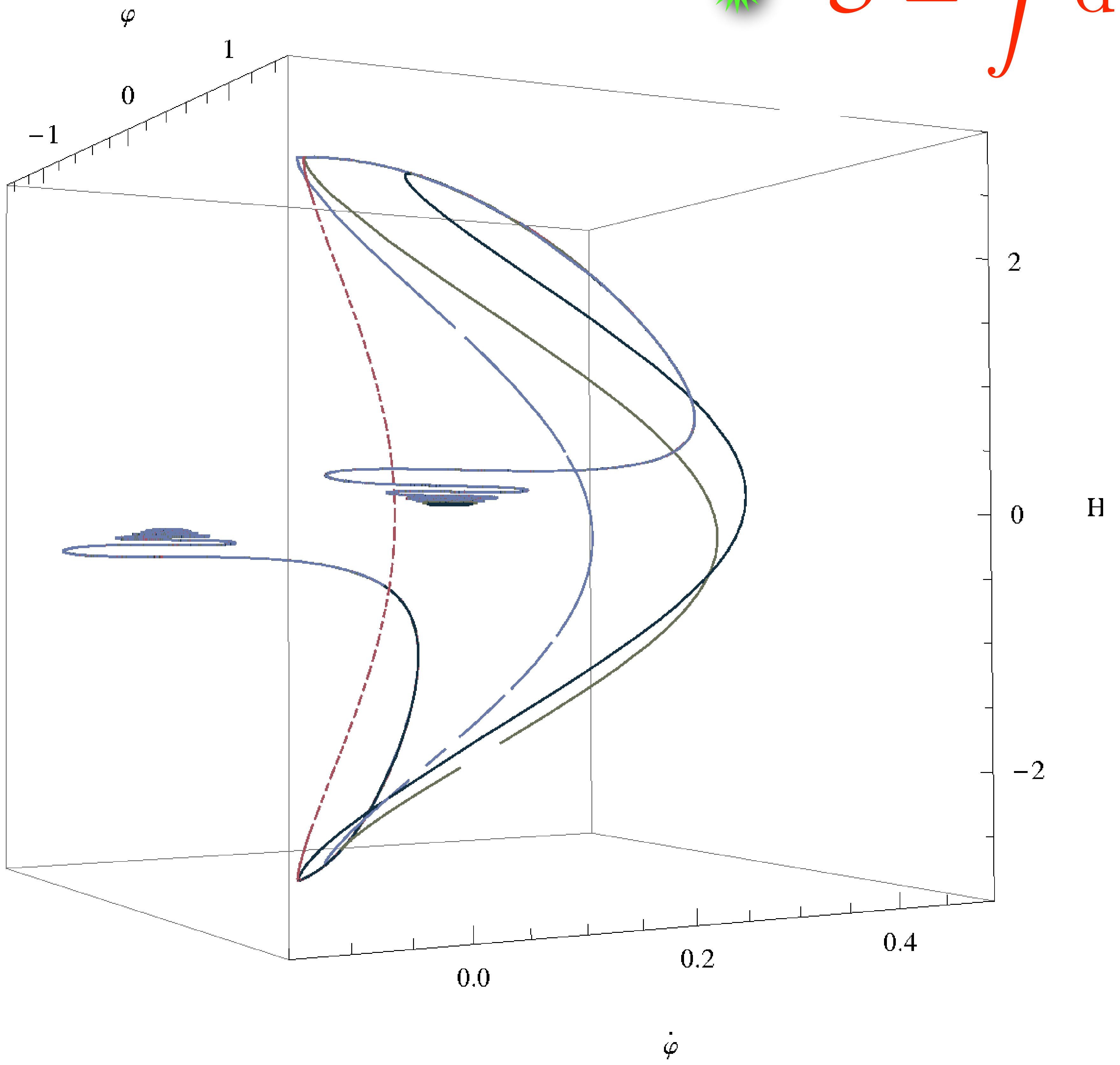
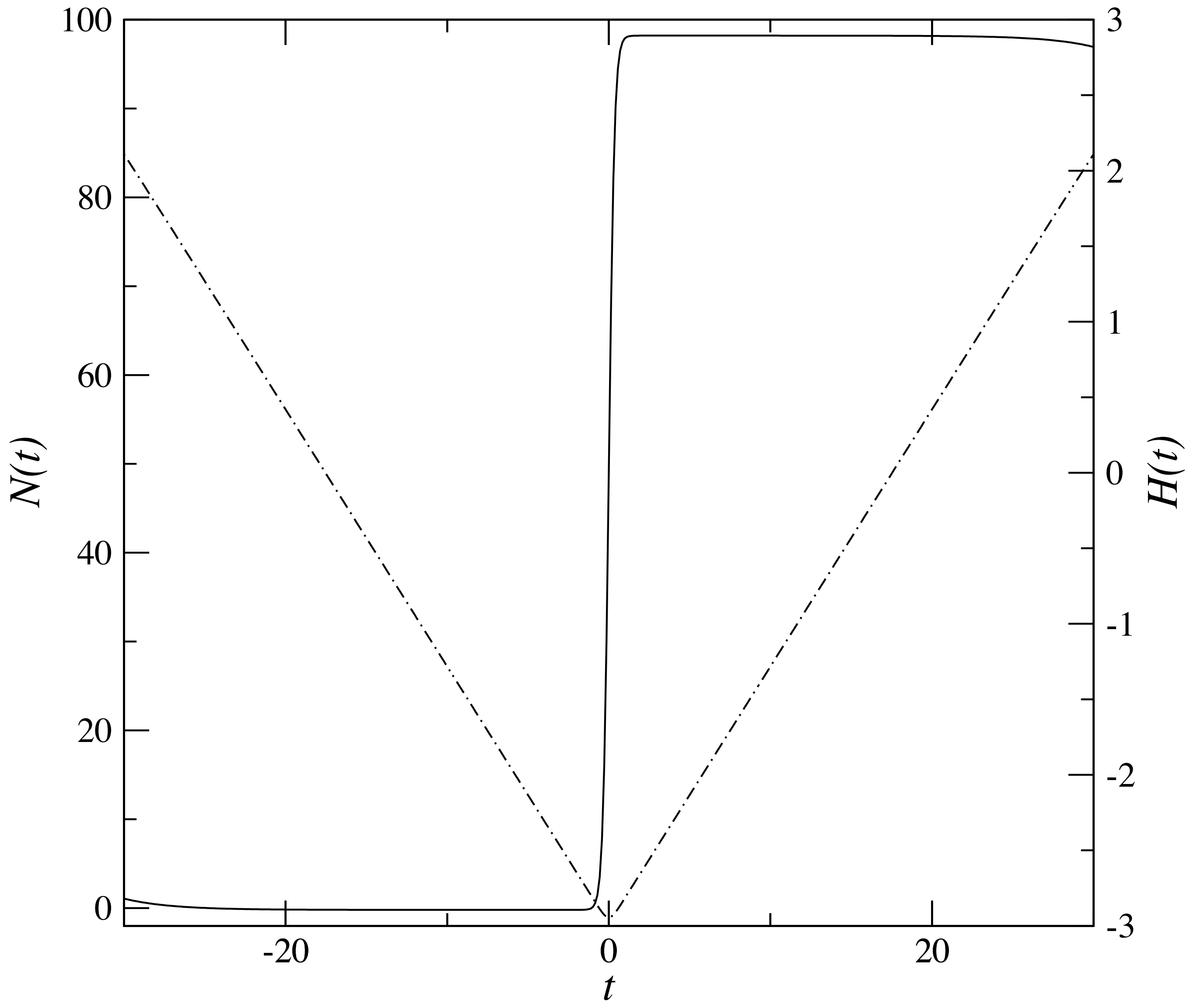
$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V\right) - \frac{\mathcal{K}}{a^2}$$

Positive spatial curvature





F. Falciano, M. Lilley & P. P., *Phys. Rev.* **D77**, 083513 (2008)

Implementing a bounce = problem with GR!

$$\dot{H} = \frac{\mathcal{K}}{a^2} - \frac{1}{2}(\rho + P)$$

Violation of Null Energy Condition (NEC)

$$\rho + P \leq 0$$

Positive spatial curvature + scalar field

Modify GR?

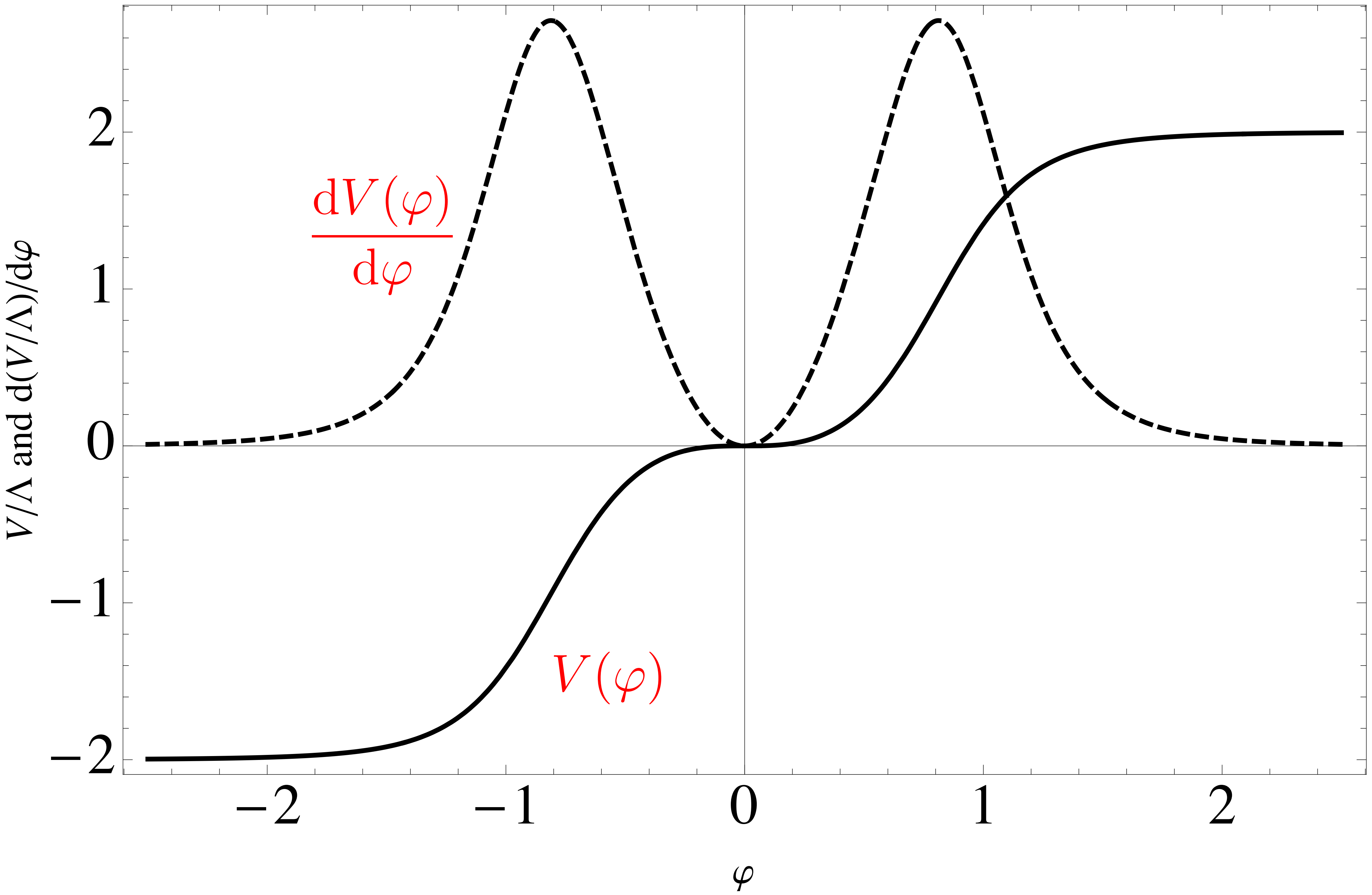
Add new terms?

K-bounce, Ghost condensates, Galileons...?

→ Modify GR to non singular theories (curvature invariants)

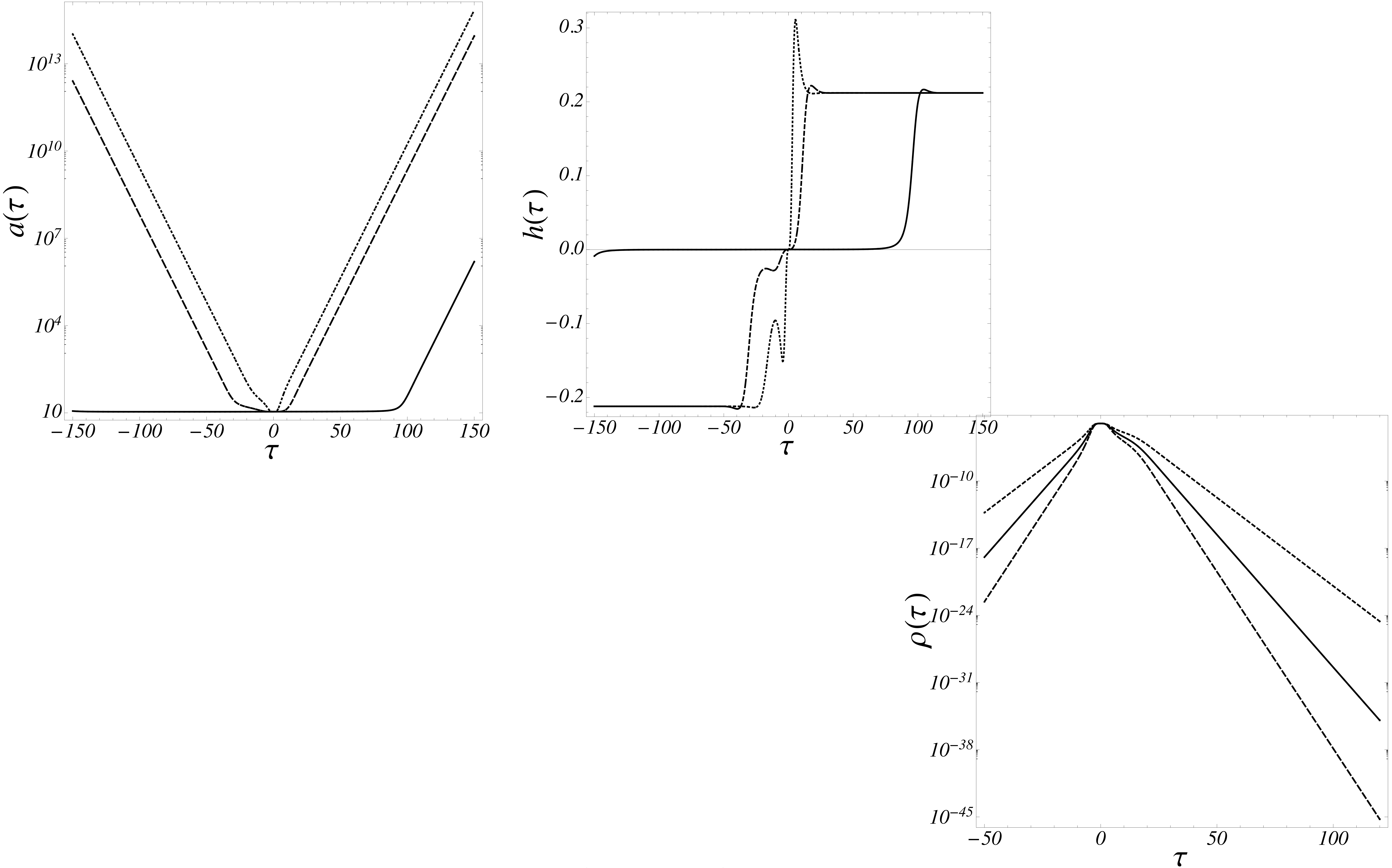
$$\mathcal{S} = \frac{1}{16\pi G_{\mathrm{N}}}\int \mathrm{d}^4x\sqrt{-g}\left[R+\sum_{i=1}^{N}\varphi_i I^{(i)} - V\left(\boldsymbol{\varphi}\right)\right] \qquad \Longrightarrow \frac{\mathrm{d}V}{\mathrm{d}\varphi} = I$$

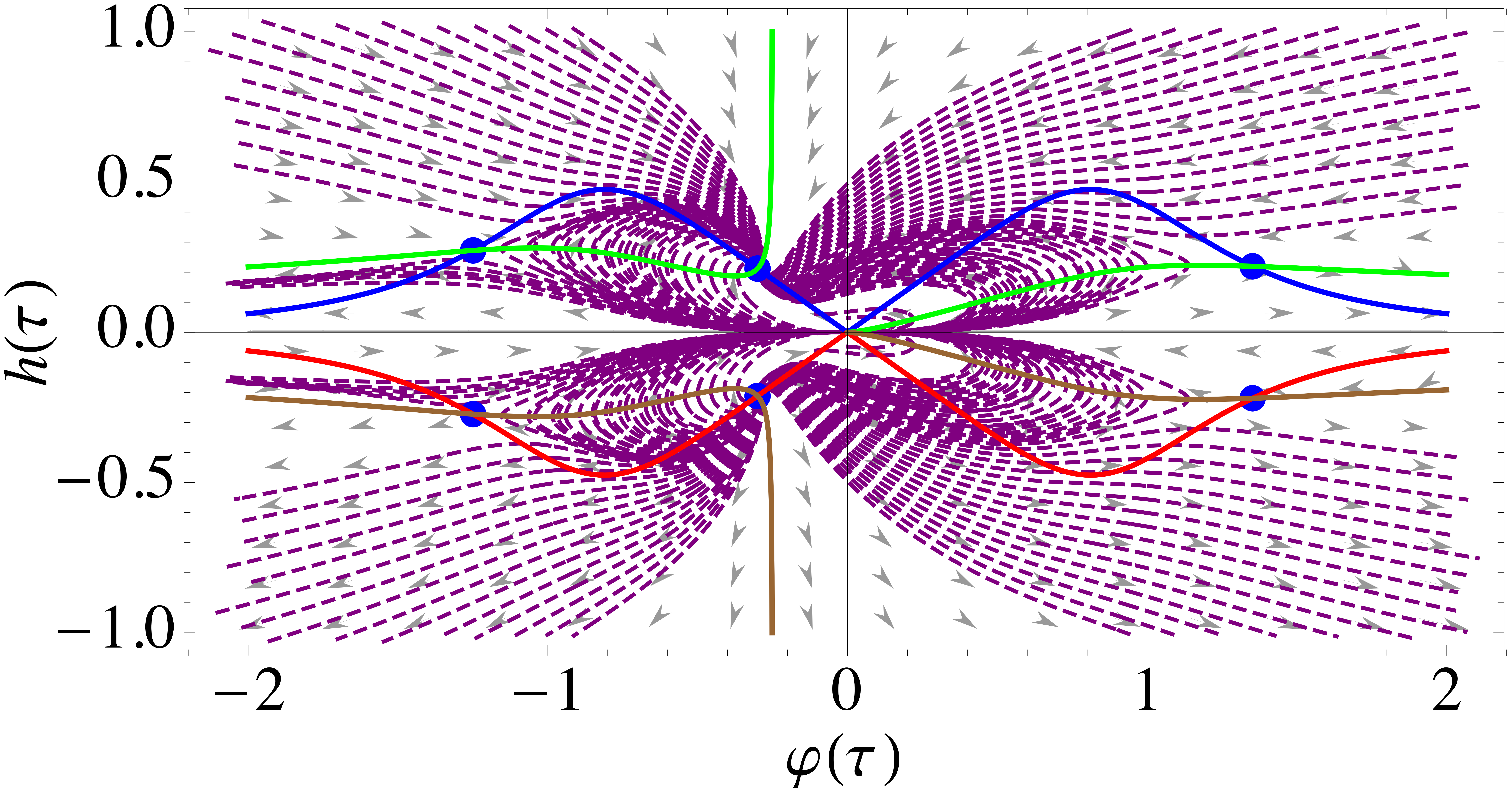
R. Brandenberger, V. F. Mukhanov and A. Sornborger, *Phys. Rev.* **D48**, 1629 (1993)



$$I = R - \sqrt{3\left(4R_{\mu\nu}R^{\mu\nu} - R^2\right)}$$

R. Abramo, P. P. & I. Yasuda, *Phys. Rev.* **D81**, 023511 (2010)


$a(\tau)$
10^{13}
10^{10}
10^{7}
10^{4}
10
−150
−100
−50
0
50
100
150
τ
$h(\tau)$
0.3
0.2
0.1
0.0
−0.1
−0.2
−150
−100
−50
0
50
100
150
τ
$\rho(\tau)$
10^{-10}
10^{-17}
10^{-24}
10^{-31}
10^{-38}
10^{-45}
−50
0
50
100
τ


1.0
0.5
0.0
−0.5
−1.0
$h(\tau)$
−2
−1
0
1
2
$\varphi(\tau)$

✸ K-bounce: $\mathcal{L} = p(X, \varphi)$

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

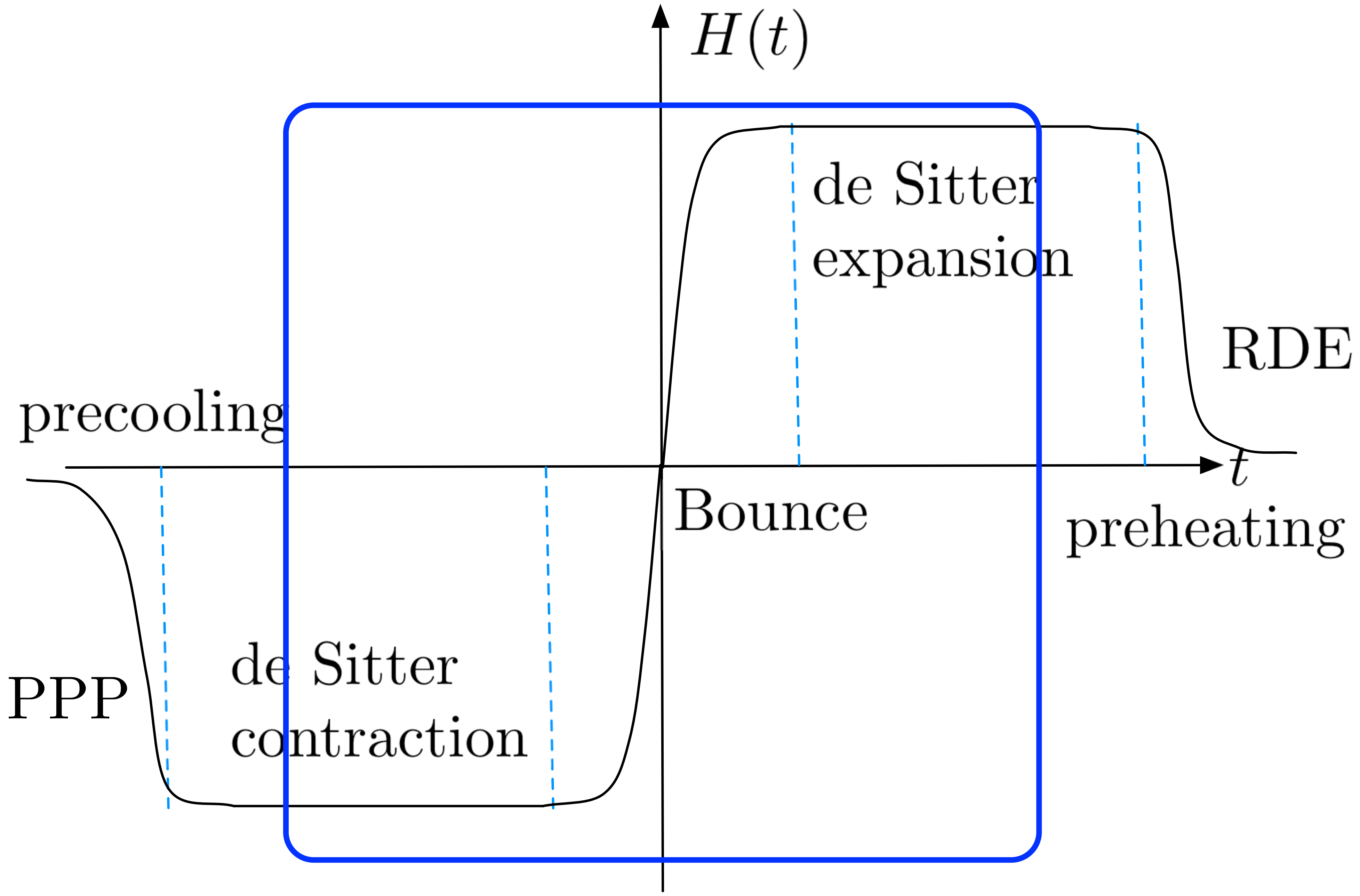
$$\Longrightarrow \quad T^{\mu\nu} = (\rho + p)\, u^\mu u^\nu - p g^{\mu\nu}$$

$$\rho \equiv 2X \frac{\partial p}{\partial X} - p$$

vanishing spatial curvature possible in 4 dimensions G.R.?

$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{2X}}$$

$$\rho\left(t_{\text{bounce}}\right) = 0 \Longrightarrow p\left(t_{\text{bounce}}\right) < 0$$



Implementing a bounce = problem with GR!

$$\dot{H} = \frac{\mathcal{K}}{a^2} - \frac{1}{2}(\rho + P)$$

Violation of Null Energy Condition (NEC)

$$\rho + P \leq 0$$

Positive spatial curvature + scalar field

Modify GR?

Add new terms?

K-bounce, Ghost condensates, Galileons...?

→ Various instabilities may arise!
(e.g. radiation for matter bounce or curvature perturbations)

The problem with contraction: BKL/shear instability

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\sum_i \mathrm{e}^{2\theta_i(t)}\sigma^i\sigma^i$$

Ricci flat:

$$\sigma^i = \mathrm{d}x^i$$

$$\sum_i \theta_i = 0$$

Average scale factor

$$\frac{\dot{a}}{a}$$ Mean Hubble parameter

$$H_i \equiv \frac{1}{a\mathrm{e}^{\theta_i}}\frac{\mathrm{d}}{\mathrm{d}t}\left(a\mathrm{e}^{\theta_i}\right) = H + \dot{\theta}_i$$

Friedman equations

$$\left.\begin{aligned} H^2 &= \frac{\rho_\mathrm{T}}{3M_\mathrm{Pl}^2} + \frac{1}{6}\sum_i \dot{\theta}_i^2 \\ \dot{H} &= -\frac{\rho_\mathrm{T}+p_\mathrm{T}}{2M_\mathrm{Pl}^2} - \frac{1}{2}\sum_i \dot{\theta}_i^2 \end{aligned}\right\} \ddot{\theta}_i + 3H\dot{\theta}_i = 0$$

$$\rho_\mathrm{shear} \propto a^{-6}$$

slow contraction solution:

$$w_{\mathrm{ekp}} \gg 1 \quad \Longrightarrow \rho_{\mathrm{ekp}} \propto a^{-3\left(1+w_{\mathrm{ekp}}\right)} \gg a^{-6}$$ when $a \to 0$

Problem: regular bounce → $\exists$ phase with $w_{\mathrm{bounce}} < -1$

So finally...

$$\rho_{\mathrm{Shear}} \equiv \frac{M_{\mathrm{Pl}}^2}{2} \sum_i \dot{\theta}_i^2 \propto a^{-6} \gg \rho_{\mathrm{Fluid}}$$

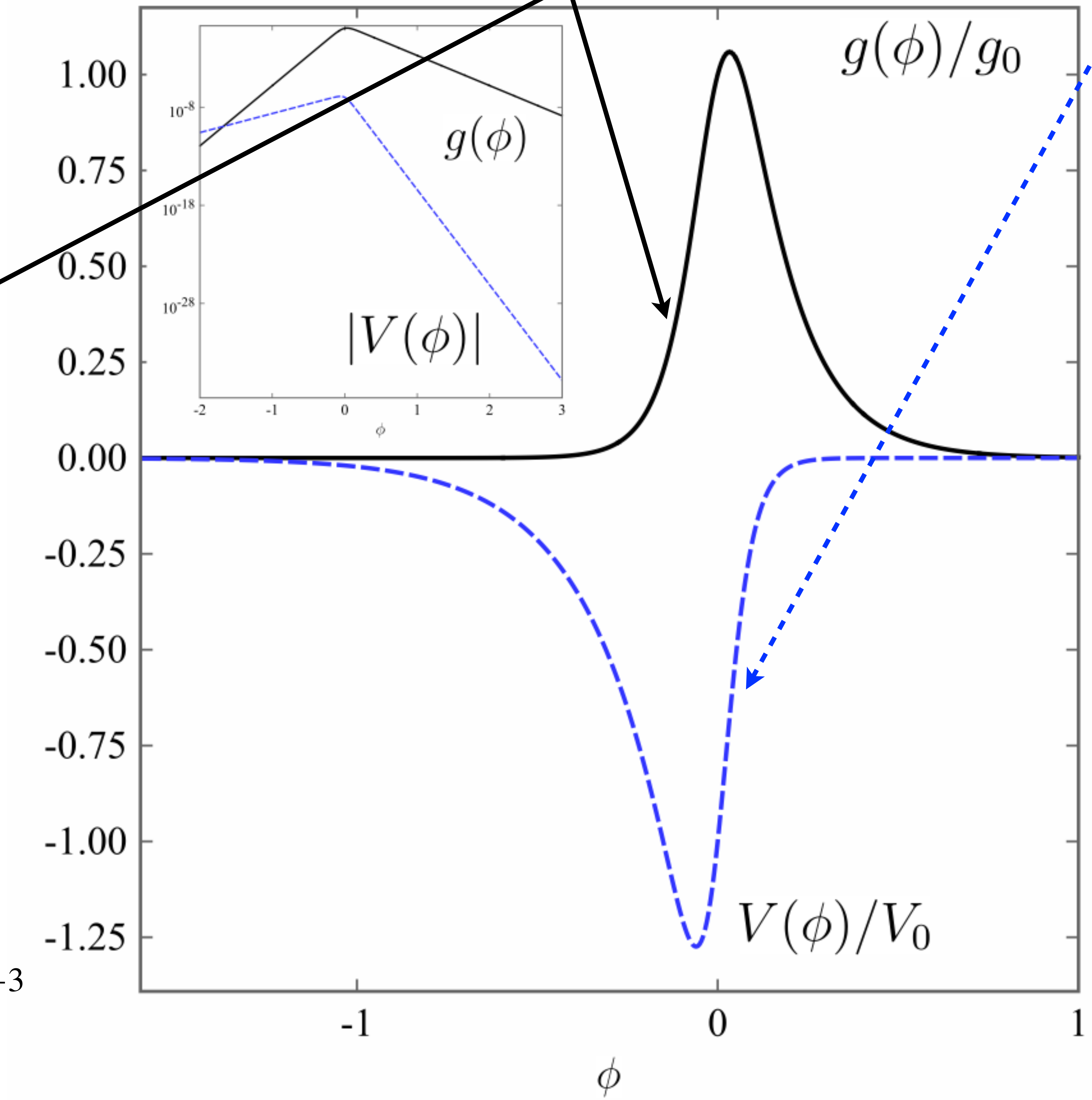
→ Singularity!

Y. Cai, D. Easson & R. Brandenberger JCAP **08,** 020 (2012)

A nonsingular bounce model: ghost condensate & Galileon

$\mathcal{L}\left[\phi\left(x\right)\right] = K(\phi, X) + G(\phi, X)\Box\phi$ with kinetic term $X \equiv \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ + Fluid

Specific choices: $K(\phi, X) = M_{\rm Pl}^2\left[1 - g(\phi)\right]X + \beta X^2 - V(\phi)$ $G(X) = \gamma X$



$$g(\phi) = \frac{2g_0}{e^{-\sqrt{\frac{2}{p}}\phi} + e^{b_g\sqrt{\frac{2}{p}}\phi}}$$

$$V(\phi) = -\frac{2V_0}{e^{-\sqrt{\frac{2}{q}}\phi} + e^{b_V\sqrt{\frac{2}{q}}\phi}}$$

+Bianchi

$V_0 = 10^{-7}, g_0 = 1.1, \beta = 5, \gamma = 10^{-3}$
$b_V = 5, b_g = 0.5, p = 0.01, q = 0.1$

Stress-energy tensor

$$T^{\phi}_{\mu\nu} = \left(-K + 2XG_{,\phi} + G_{,X}\nabla_\sigma X \nabla^\sigma \phi\right) g_{\mu\nu} + (K_{,X} + G_{,X}\Box\phi - 2G_{,\phi})\nabla_\mu\phi\nabla_\nu\phi - G_{,X}\left(\nabla_\mu X \nabla_\nu \phi + \nabla_\nu X \nabla_\mu \phi\right)$$

Energy density & Pressure

$$\rho_\phi = \frac{1}{2}M^2_{\rm Pl}(1-g)\dot{\phi}^2 + \frac{3}{4}\beta\dot{\phi}^4 + 3\gamma H\dot{\phi}^3 + V(\phi)$$

$$p_\phi = \frac{1}{2}M^2_{\rm Pl}(1-g)\dot{\phi}^2 + \frac{1}{4}\beta\dot{\phi}^4 - \gamma\dot{\phi}^2\ddot{\phi} - V(\phi)$$

+ Fluid $p = w\rho$

Einstein equation + $\nabla_\mu T^{\mu\nu}_{\rm Fluid} = 0$

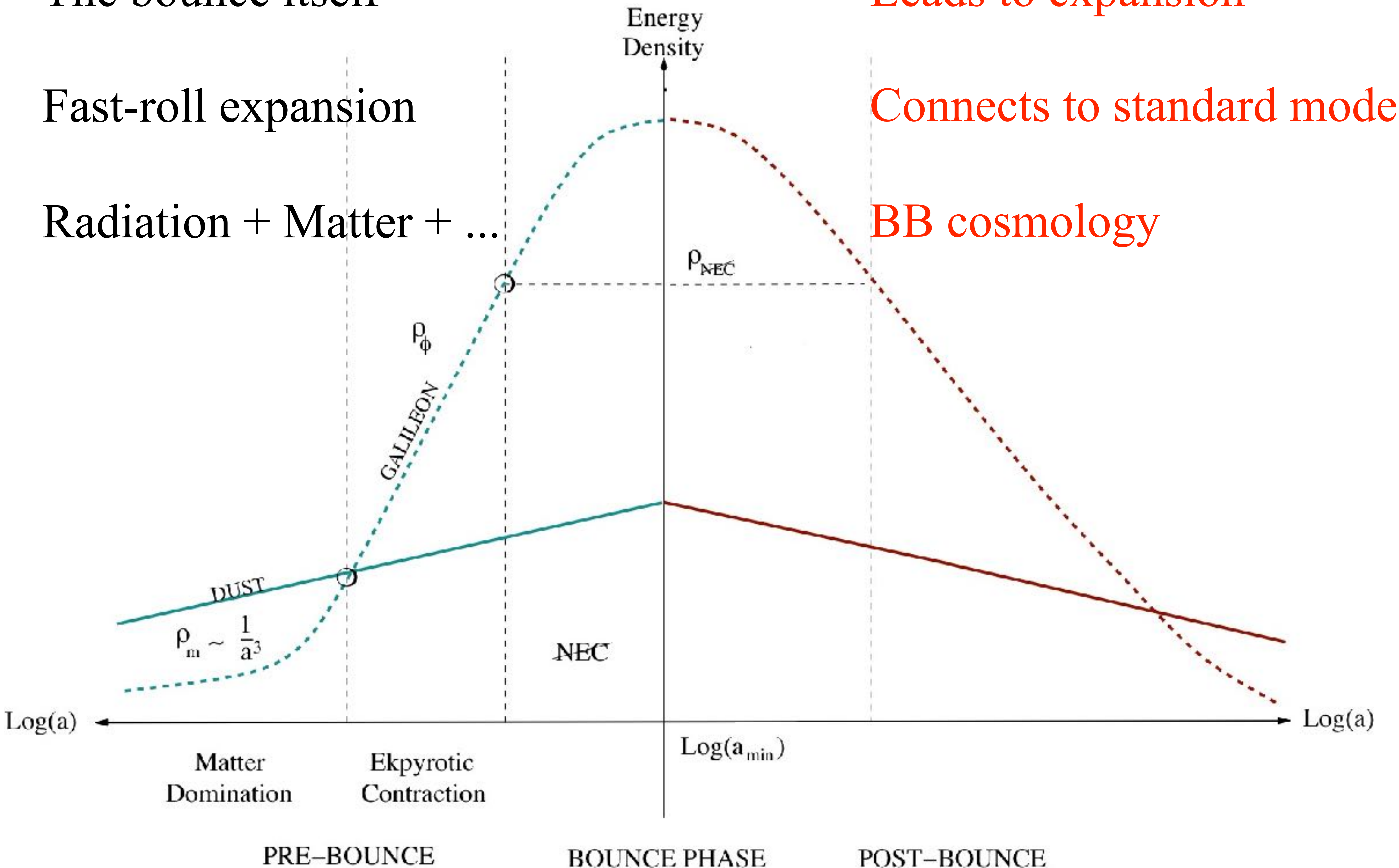
+ modified Klein-Gordon $\mathcal{P}\ddot{\phi} + \mathcal{D}\dot{\phi} + V_{,\phi} = 0$

with...

$$\begin{aligned}
\mathcal{P} &= (1-g)M_{\rm Pl}^2 + 6\gamma H\dot{\phi} + 3\beta\dot{\phi}^2 + \frac{3\gamma^2}{2M_{\rm Pl}^2}\dot{\phi}^4 \\
\mathcal{D} &= 3(1-g)M_{\rm Pl}^2 H + \left(9\gamma H^2 - \frac{1}{2}M_{\rm Pl}^2 g_{,\phi}\right)\dot{\phi} + 3\beta H\dot{\phi}^2 \\
&\quad -\frac{3}{2}(1-g)\gamma\dot{\phi}^3 - \frac{9\gamma^2 H\dot{\phi}^4}{2M_{\rm Pl}^2} - \frac{3\beta\gamma\dot{\phi}^5}{2M_{\rm Pl}^2} \\
&\quad -\frac{3}{2}G_{,X}\sum_i \dot{\theta}_i^2\dot{\phi} - \frac{3G_{,X}}{2M_{\rm Pl}^2}(\rho_{\rm m} + p_{\rm m})\dot{\phi}
\end{aligned}$$

5 phases:

***A*.**	→	Matter contraction	Produces scale invariant perturbations
***B*.**	→	Ekpyrotic contraction	Removes anisotropies
***C*.**	→	The bounce itself	Leads to expansion
***D*.**	→	Fast-roll expansion	Connects to standard model!!
***E*.**	→	Radiation + Matter + …	BB cosmology



explicit example...

$$V_0 = 10^{-7}, \quad g_0 = 1.1, \quad \beta = 5, \quad \gamma = 10^{-3}$$

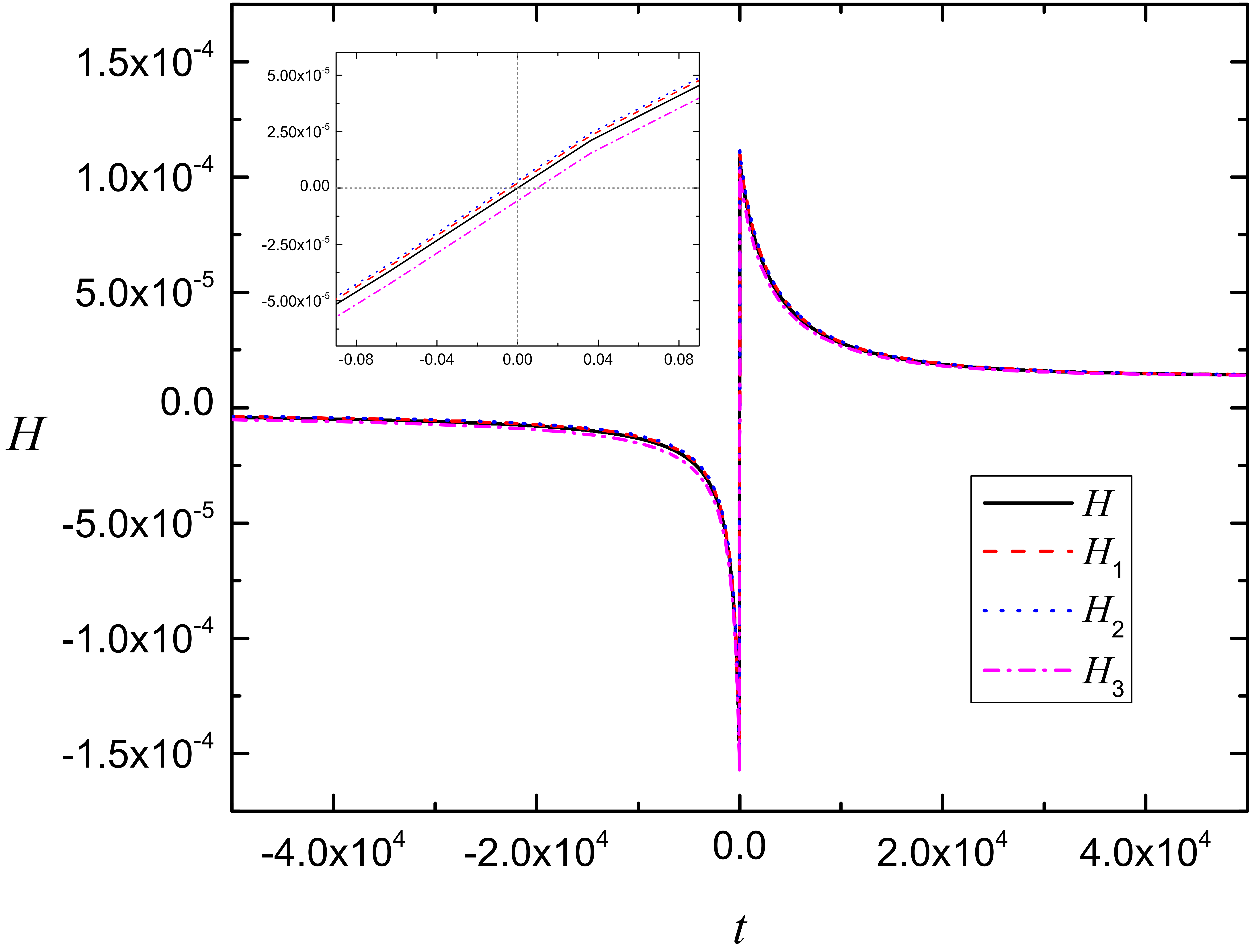
$$b_V = 5, \quad b_g = 0.5, \quad p = 0.01, \quad q = 0.1$$

$$\rho_{\mathrm{m,B}} = 2.8 \times 10^{-10}, \quad M_{\theta,1} = 2.2 \times 10^{-6}$$

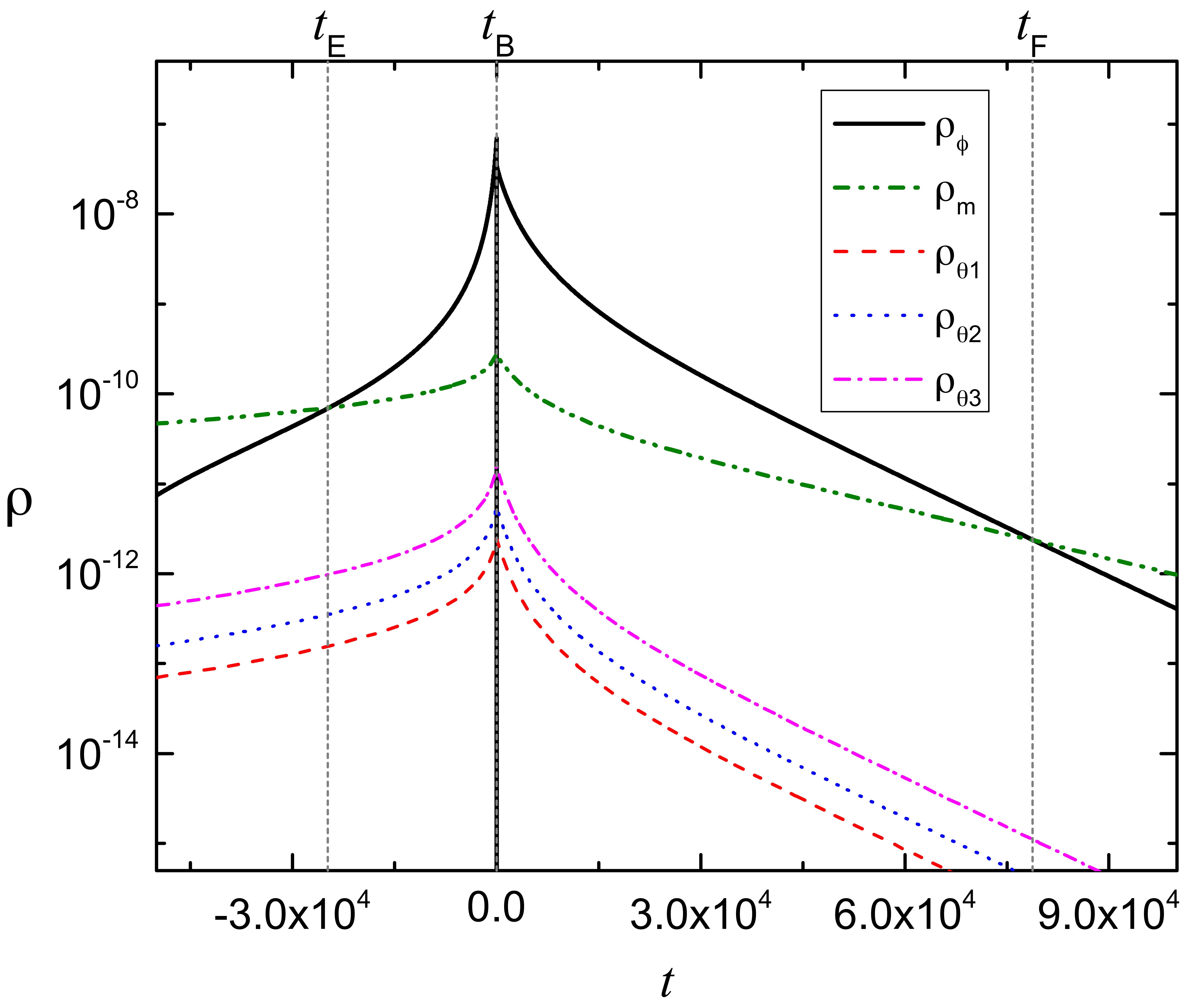
$$M_{\theta,2} = 3.4 \times 10^{-6}, \quad M_{\theta,3} = -5.6 \times 10^{-6}$$

$$\phi_{\mathrm{ini}} = -2, \quad \dot{\phi}_{\mathrm{ini}} = 7.8 \times 10^{-6}.$$

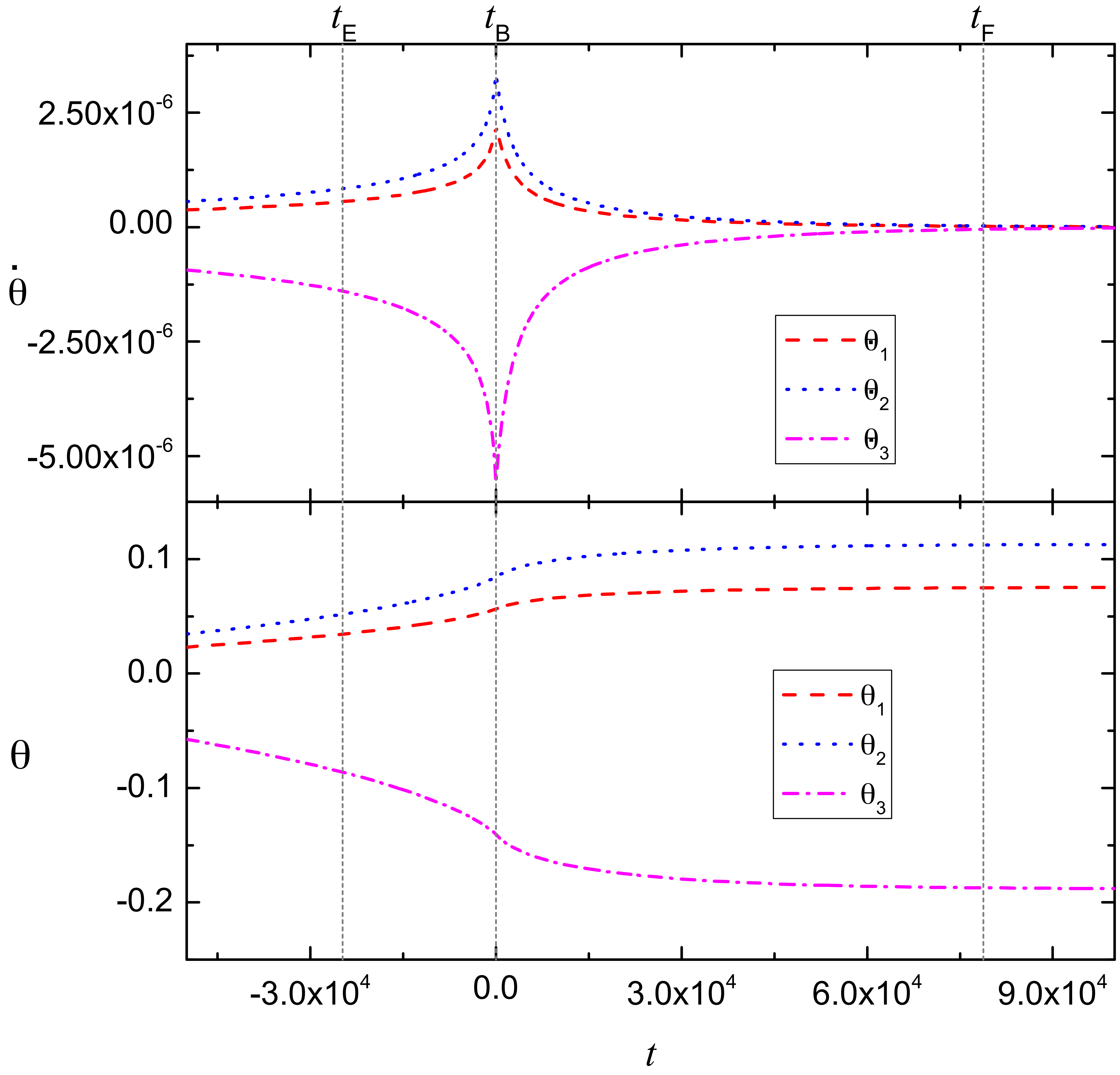
Hubble parameters



Energy densities



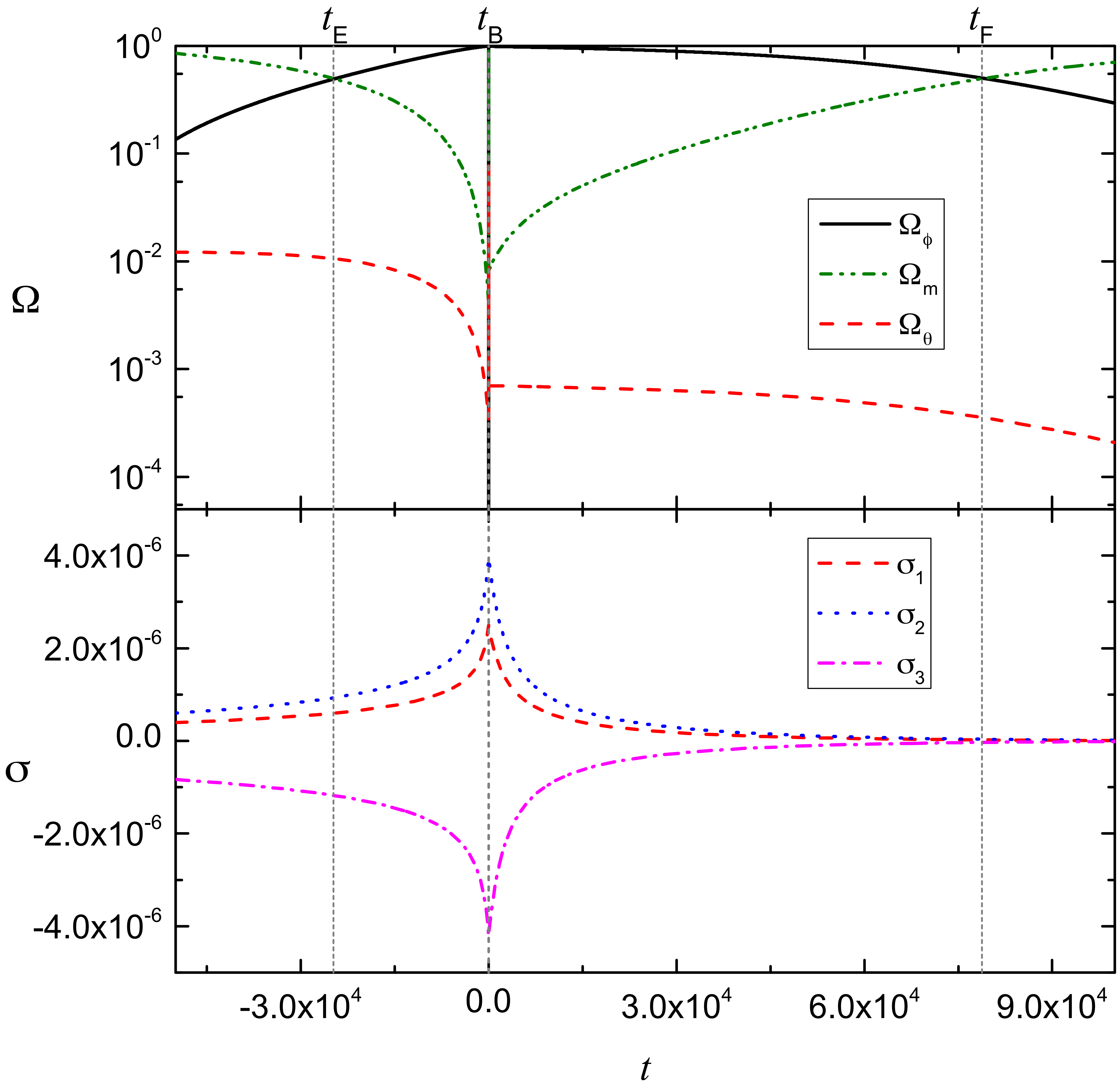
Anisotropies



Density parameters and shears

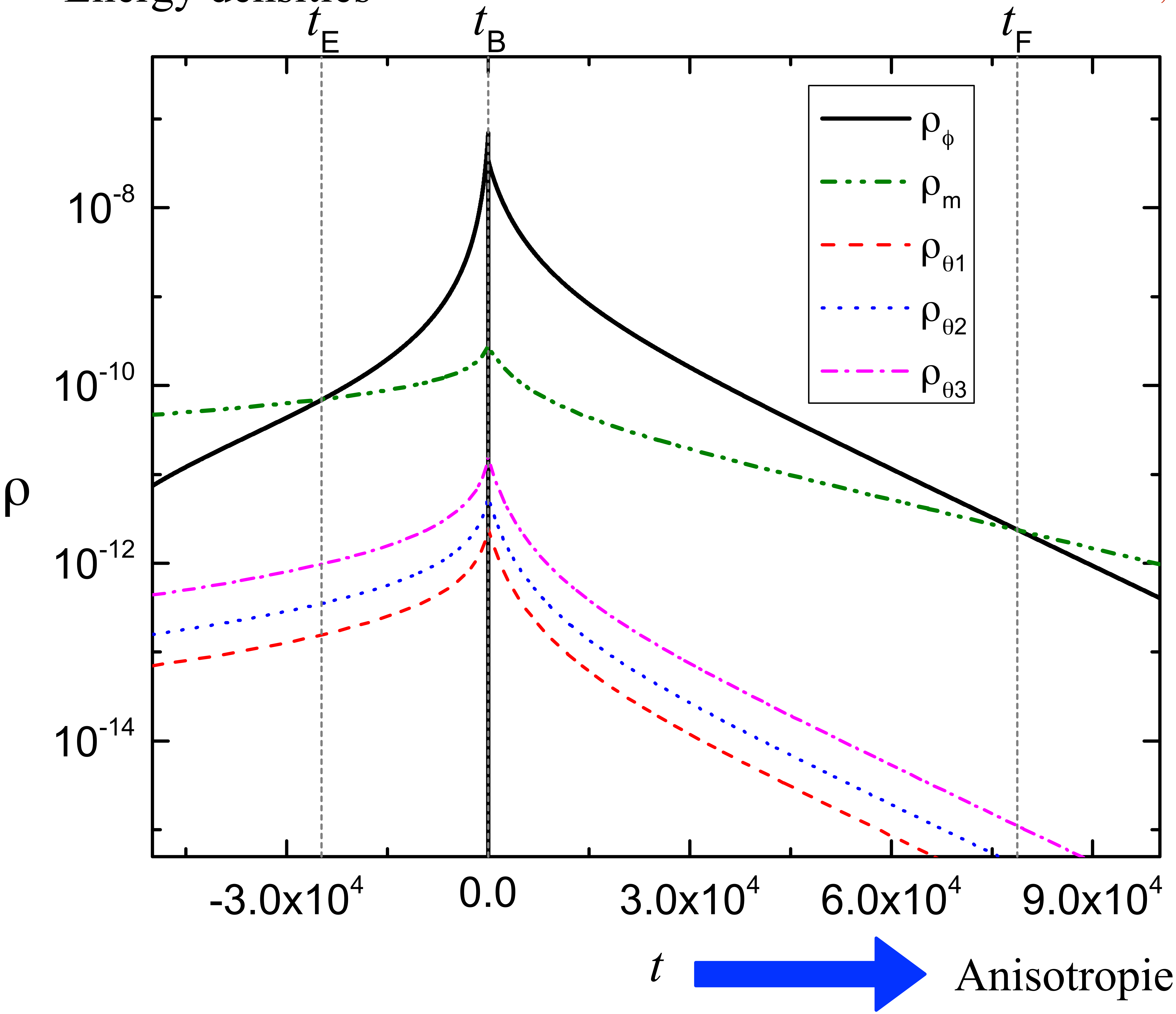
$$\Omega_{\mathrm{I}} \equiv \frac{\rho_{\mathrm{I}}}{\sum_{\mathrm{I}} \rho_{\mathrm{I}}}$$

$$\sigma_i \equiv \dot{\theta}_i \mathrm{e}^{2\theta_i}$$



Energy densities

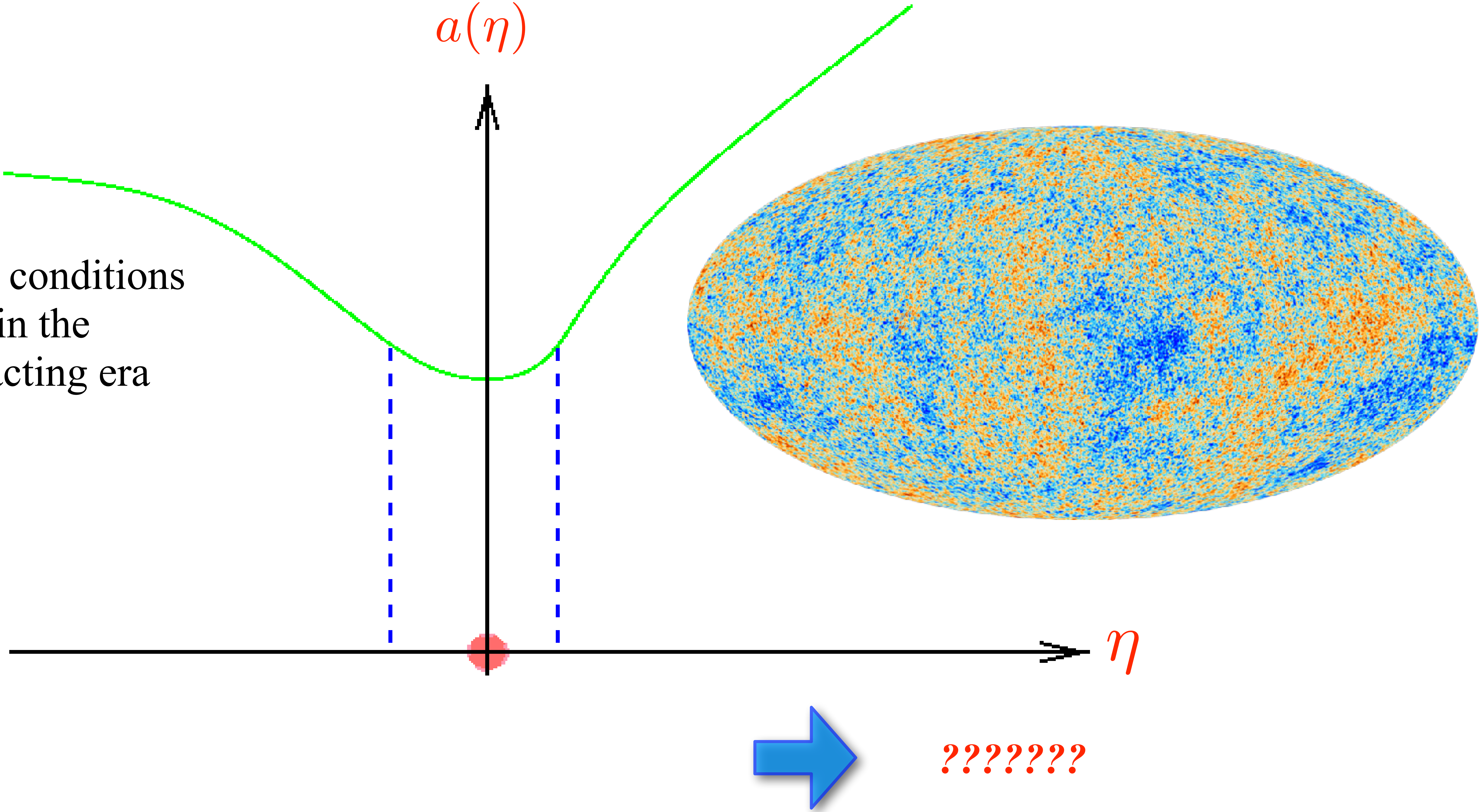
Y. Cai, R. Brandenberger & PP, CQG **30,** 075019 (2013)



$\phi_{\text{ini}} = -2, \dot{\phi}_{\text{ini}} = 7.8 \times 10^{-6}$

Anisotropies can remain small all throughout

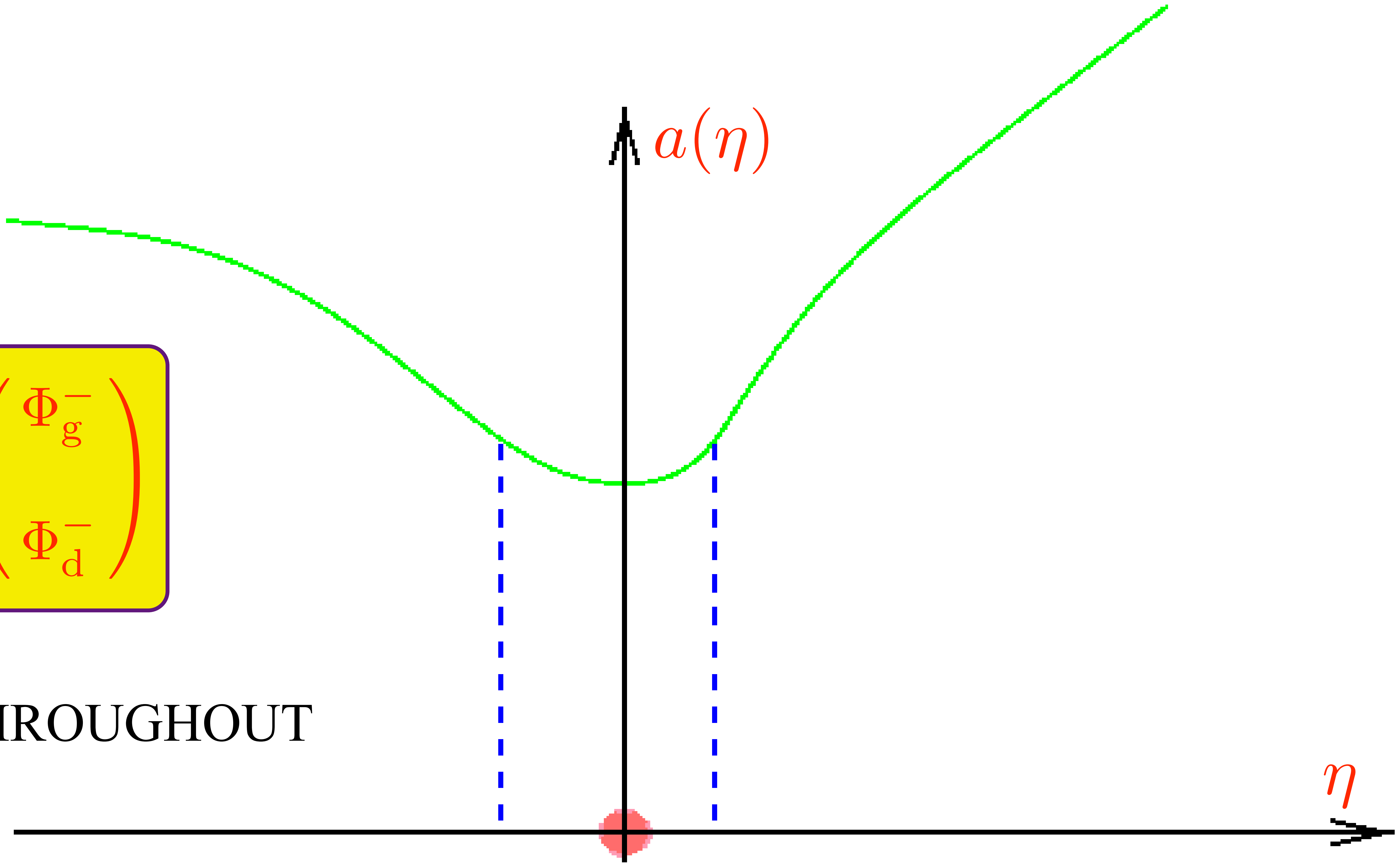
Perturbations: $ds^2 = a^2(\eta)\left\{(1+2\Phi)\,d\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]dx^i dx^j\right\}$



Initial conditions
fixed in the
contracting era

???????

Perturbations: $\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$

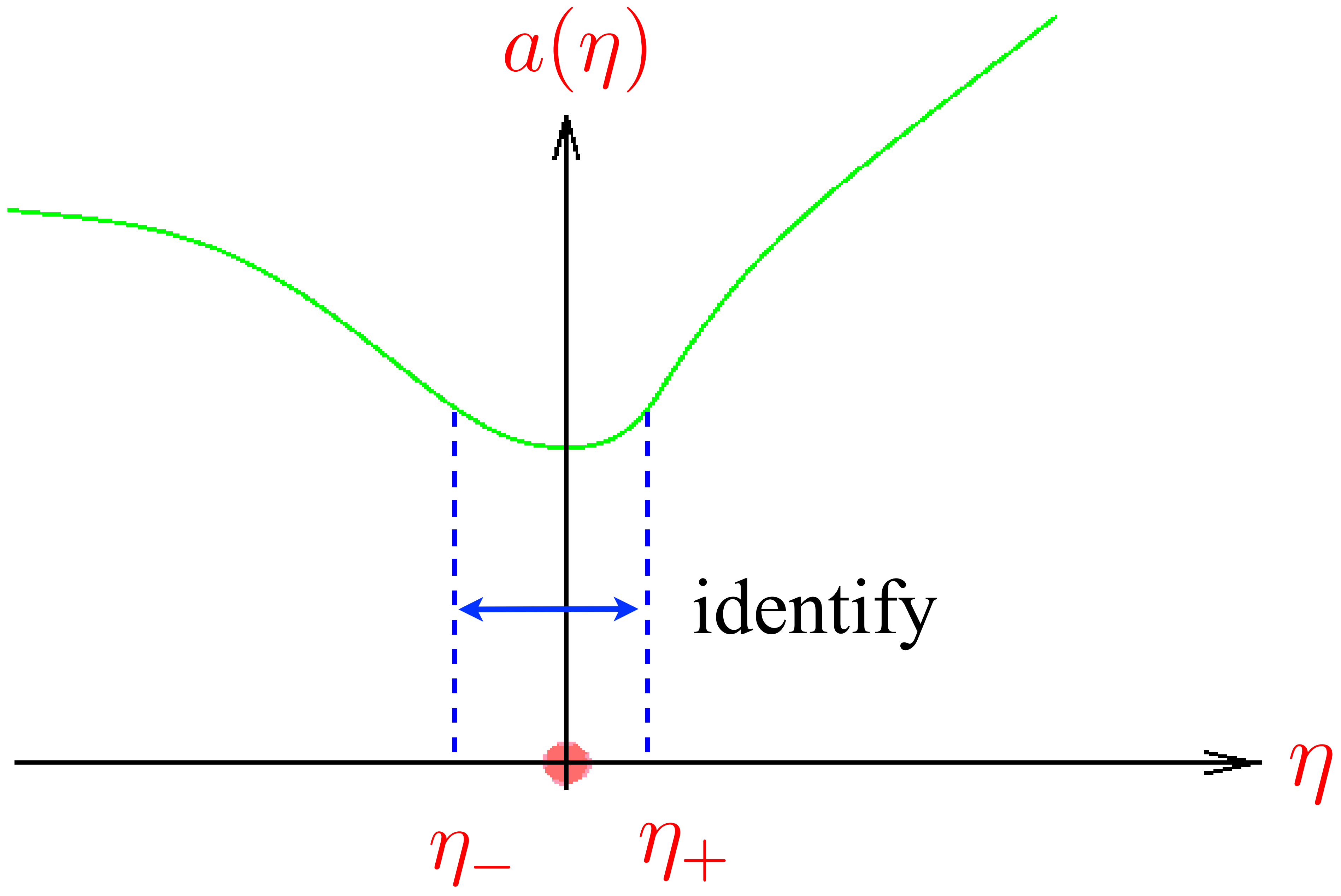


$$\begin{pmatrix}\Phi_{\mathrm{g}}^{+} \\ \\ \Phi_{\mathrm{d}}^{+}\end{pmatrix} = \mathbf{T}_{ij}(k)\begin{pmatrix}\Phi_{\mathrm{g}}^{-} \\ \\ \Phi_{\mathrm{d}}^{-}\end{pmatrix}$$

ASSUME LINEARITY THROUGHOUT

``central feature of bouncing cosmology = the bounce”…

A generic model-independent treatment of the bounce phase?



Geometric matching conditions?

Continuity of metric	$[a]_\pm = 0$	OK
Continuity of extrinsic curvature	$[H]_\pm = 0$	???
Perturbations?	$[\zeta]_\pm = 0$	???

Perturbations: $$\mathrm{d}s^2 = a^2(\eta)\left\{(1+2\Phi)\,\mathrm{d}\eta^2 - \left[(1-2\Phi)\,\gamma_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}$$

$$\Longleftrightarrow \qquad \Phi = \frac{3\mathcal{H}u}{2a^2\theta} \qquad \theta \equiv \frac{1}{a}\sqrt{\frac{\rho_\varphi}{\rho_\varphi + p_\varphi}\left(1 - \frac{3\mathcal{K}}{\rho_\varphi a^2}\right)}$$

$$\boxed{u'' + \left[k^2 - \frac{\theta''}{\theta} - 3\mathcal{K}\left(1 - c_{\mathrm{S}}^2\right)\right]u = 0}$$

$$V_u(\eta) \equiv \frac{\theta''}{\theta} + 3\mathcal{K}(1 - c_{\mathrm{S}}^2) = \frac{P_{24}(\eta)}{Q_{24}(\eta)},$$

Non trivial transfer matrix

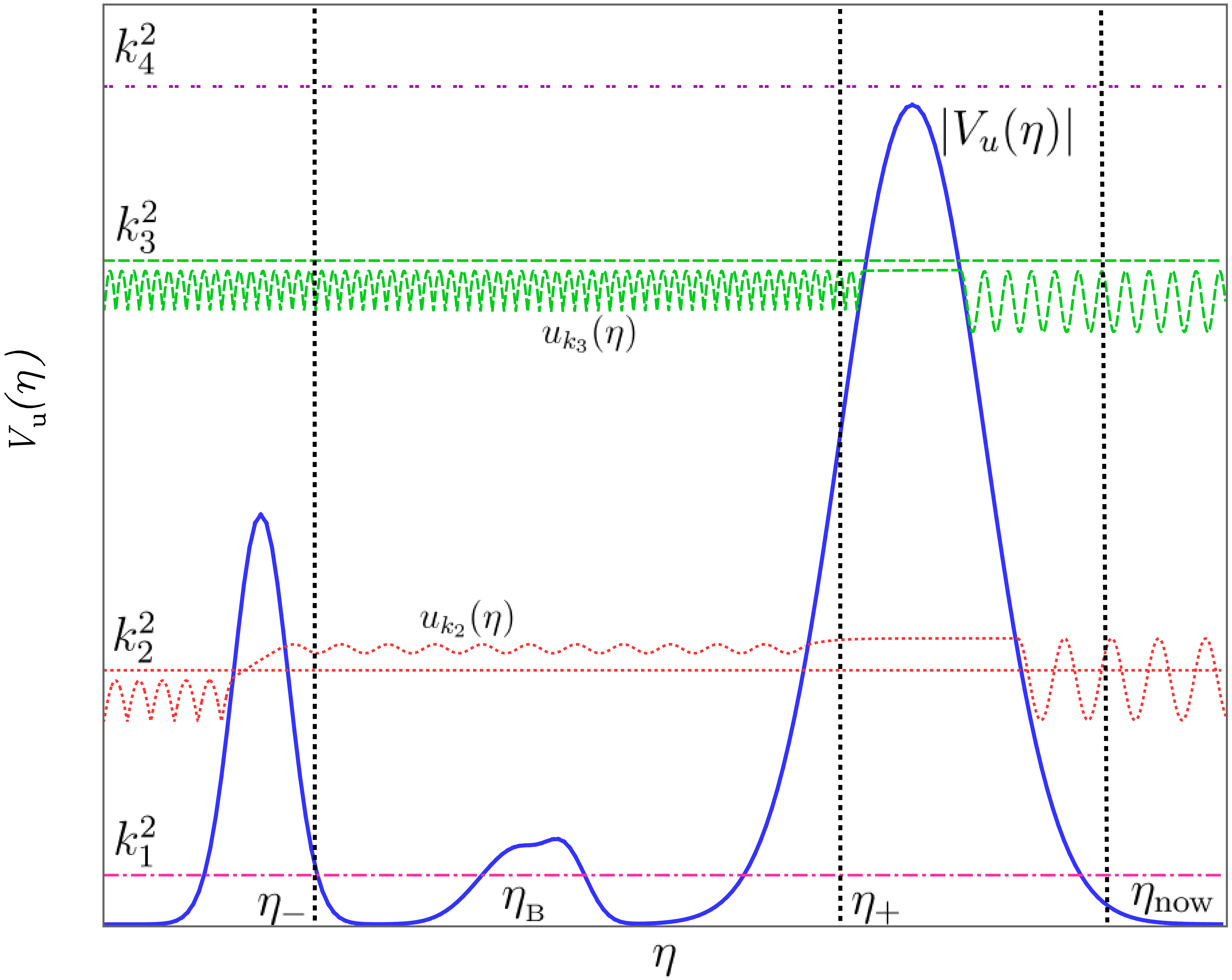
$$\boldsymbol{T}_{ij}(k) = \begin{bmatrix} A(k) & B(k) \\ C(k) & D(k) \end{bmatrix}$$

“Causality” argument...

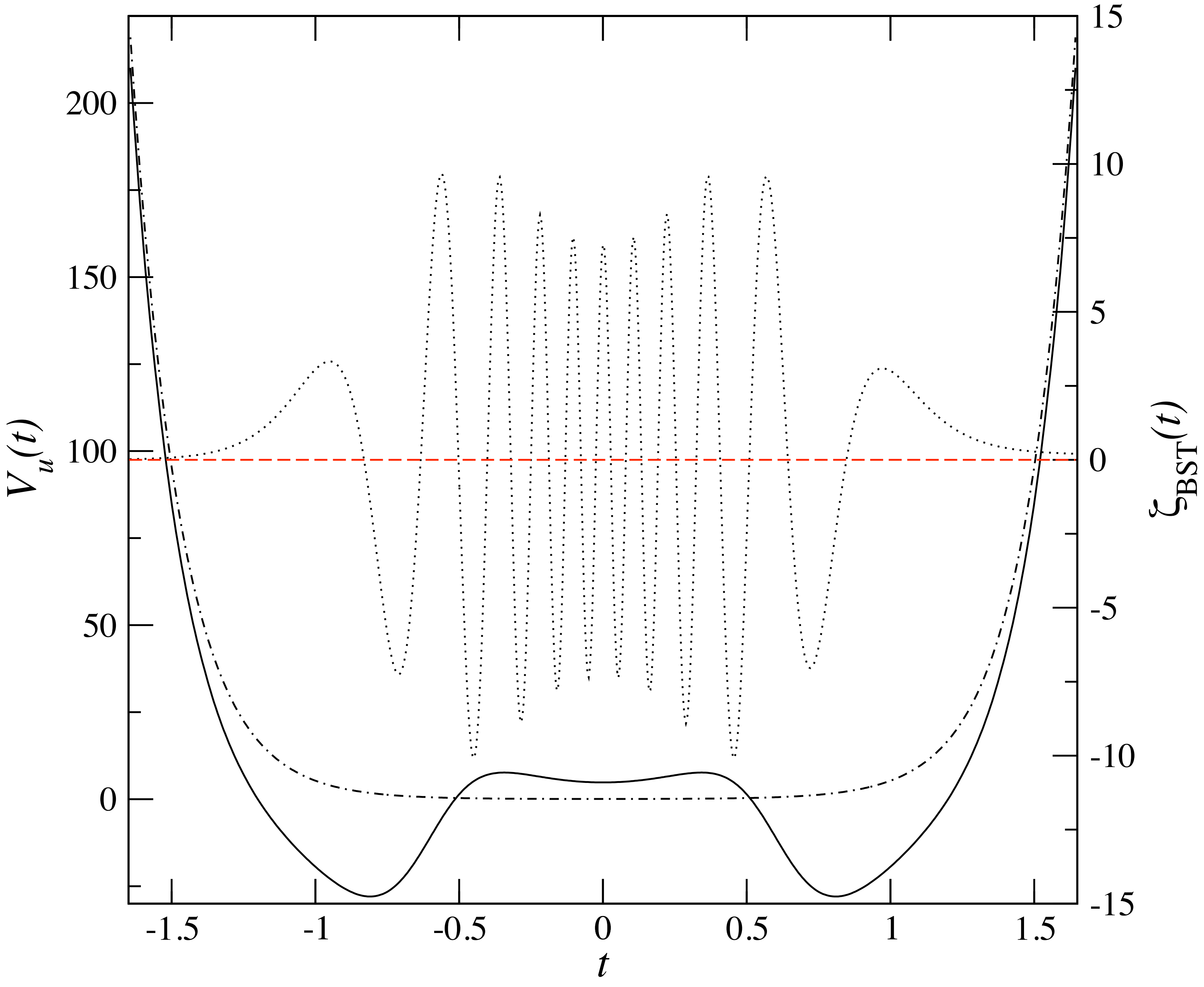
J. Martin & PP, *Phys. Rev. Lett.* **92,** 061301 (2004)

Resulting spectrum: very much model dependent…

J. Martin& PP, *PR***D68,** 103517 (2003)

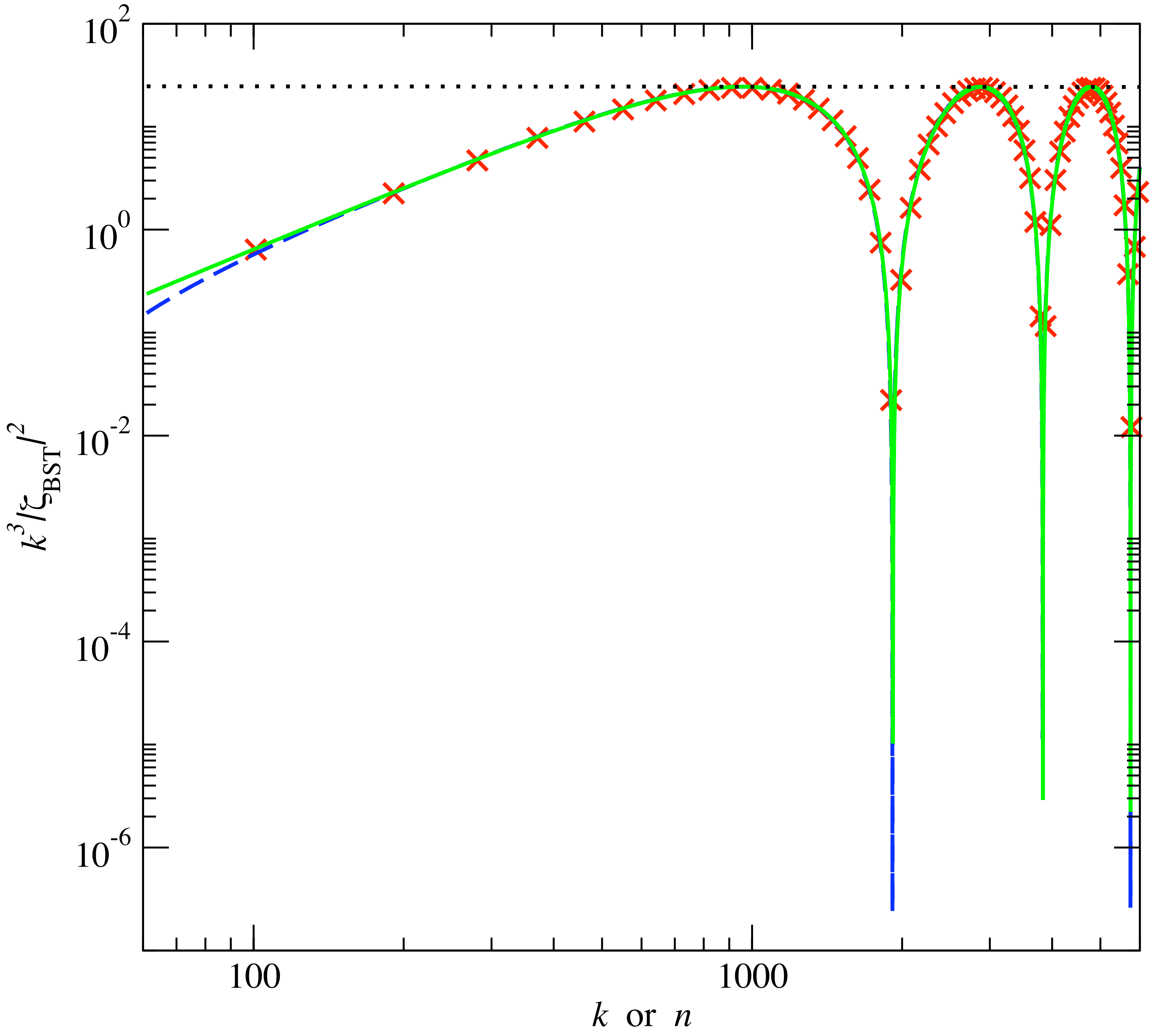


$$\mathcal{P}_\zeta = \mathcal{A} k^{n_{\mathrm{S}}-1} \cos^2\left(\omega \frac{k_{\mathrm{ph}}}{k_\star} + \psi\right)$$



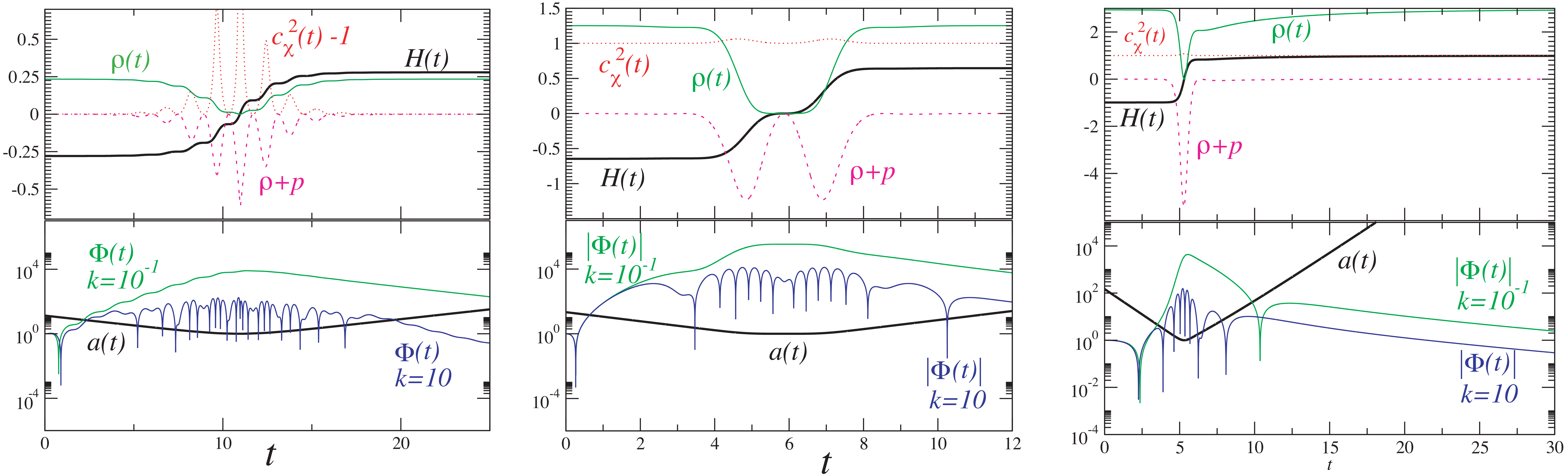
Different parameters

primordial spectrum



Perturbations in the *K*-bounce

$$p = p_0 + p_X(X - X_0) + p_\varphi \varphi + p_{X\varphi}\varphi(X - X0) + \frac{1}{2}p_{XX}(X - X_0)^2 + \frac{1}{2}p_{\varphi\varphi}\varphi^2 + \cdots$$



Slow

Fast

Oscillations + ζ conserved

R. Abramo & P. P., *JCAP* **09**, 001 (2007)

Another issue…

spectral index $n_\mathrm{s} < 1$

Non gaussianities:

phenomenological description $S = -\int \mathrm{d}^4 x \sqrt{-g} \left[R + (\partial\phi)^2 + V(\phi) \right]$

$$a(\eta) = a_0 \left[1 + \frac{1}{2} \left(\frac{\eta}{\eta_\mathrm{c}} \right)^2 + \lambda_3 \left(\frac{\eta}{\eta_\mathrm{c}} \right)^3 + \frac{5}{24} (1 + \lambda_4) \left(\frac{\eta}{\eta_\mathrm{c}} \right)^4 \right]$$

+ scalar field

$$\begin{cases} \dfrac{\phi'^2}{a^2} = \dfrac{2}{a^2} \left(\mathcal{H}^2 - \mathcal{H}' + \mathcal{K} \right) \\ -\dfrac{6}{a^2} \mathcal{H}' = -2V(\phi) \left[1 - \dfrac{\phi'^2}{a^2 V(\phi)} \right] \end{cases}$$

$$\phi'' + 2\mathcal{H}\phi' + a^2 V_{,\phi} = 0$$

$$\Upsilon \equiv \phi_0'^2 / 2 \longrightarrow \eta_c^2 = \frac{1}{1 - \Upsilon}$$

$$\varepsilon_V = \frac{V_0'}{V_0}$$

"slow-roll"

$$\eta_V = \frac{V_0''}{V_0}$$

complete set of parameters

perturbed metric $\mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu = a^2 \left(-\mathrm{e}^{2\Phi} \mathrm{d}\eta^2 + \mathrm{e}^{-2\Psi} \gamma_{ij} \mathrm{d}x^i \mathrm{d}x^j \right)$

X. Gao, M. Lilley & P. P., *JCAP* **07**, 010 (2014) *perturbations up to 2nd order* $X\left(\boldsymbol{x},\eta\right)=X_{(1)}\left(\boldsymbol{x},\eta\right)+\frac{1}{2}X_{(2)}\left(\boldsymbol{x},\eta\right)+\cdots$

$$\mathcal{D}\,\Psi_{(i)}=\mathcal{S}\left[\Psi_{(i-1)}\right]$$

first order

$$\Psi''_{(1)}+F\left(\eta\right)\Psi'_{(1)}-\bar{\nabla}^2\Psi_{(1)}+W\left(\eta\right)\Psi_{(1)}=0$$

$$2\left(\mathcal{H}-\frac{\bar{\phi}''}{\bar{\phi}'}\right)$$

$$2\left(\mathcal{H}'-\mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}-2\mathcal{K}\right)$$

positive spatial curvature: decomposition on the 3-sphere

$$\Psi_{(1)}(\boldsymbol{x},\eta)=\sum_{\ell mn}\Psi_{\ell mn}(\eta)Q_{\ell mn}(\chi,\theta,\varphi)$$

Legendre

$Q_{\ell mn}(\chi,\theta,\varphi)=R_{\ell n}(\chi)Y_{\ell m}(\theta,\varphi)$ *hyperspherical harmonics*

$$R_{n\ell}\left(\chi\right)=\sqrt{\frac{\left(n+1\right)\left(n+\ell+1\right)!}{\left(n-\ell\right)!}}\sqrt{\frac{\mathcal{K}}{f_{\mathcal{K}}\left(\chi\right)}}P^{-\ell-\frac{1}{2}}_{n+\frac{1}{2}}\left[\cos\left(\sqrt{\mathcal{K}}\chi\right)\right]$$

effect of the bounce itself: initial conditions = classical gaussian fields

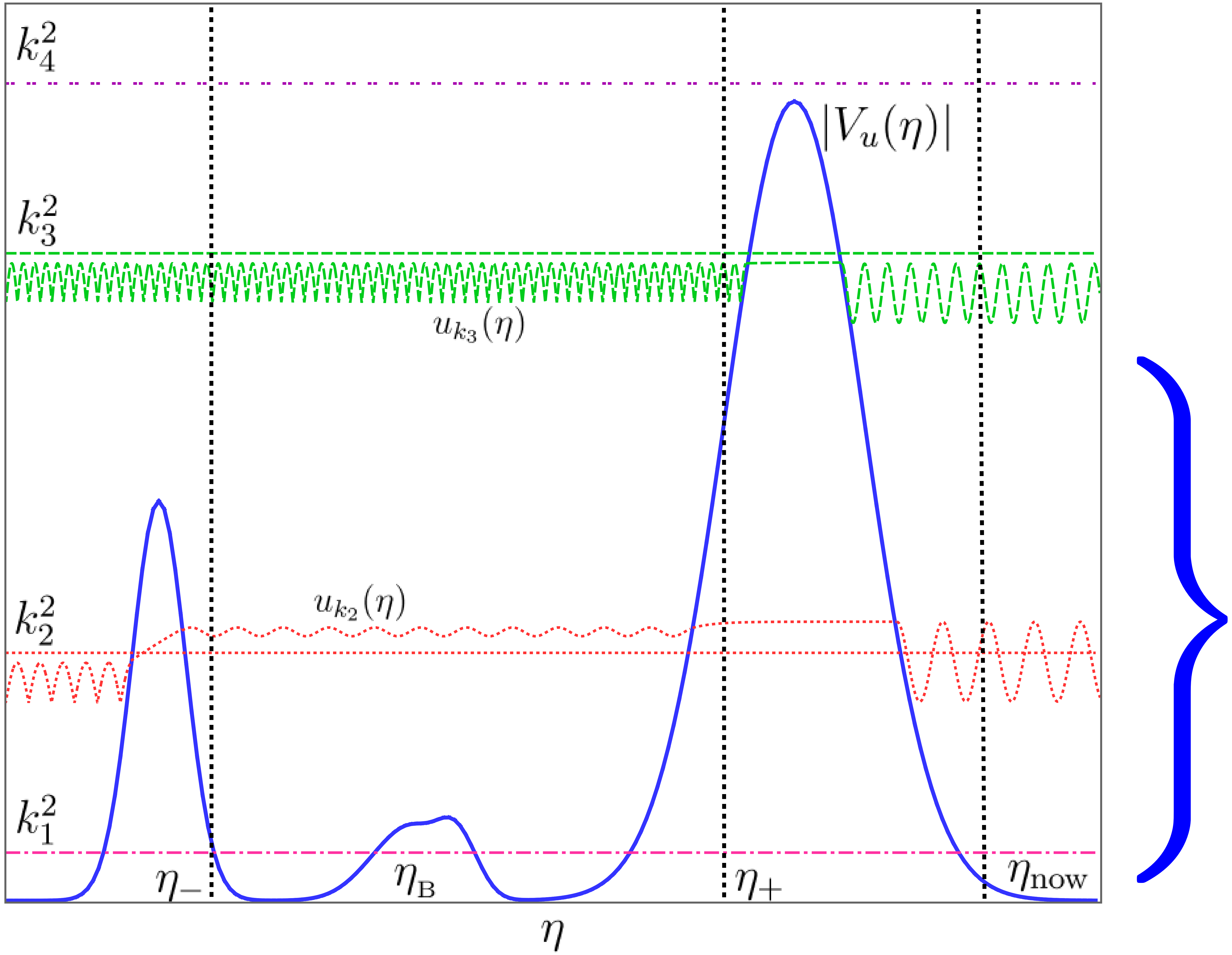
$$\begin{bmatrix}\Psi_{(1)}\left(\boldsymbol{k},\eta_-\right)\\ \Psi'_{(1)}\left(\boldsymbol{k},\eta_-\right)\end{bmatrix}\equiv\begin{bmatrix}\hat{x}_1\left(\boldsymbol{k}\right)\\ \hat{x}_2\left(\boldsymbol{k}\right)\end{bmatrix}$$

spectra

$$\left\langle\hat{x}_i\left(\boldsymbol{k}\right)\hat{x}_j\left(\boldsymbol{k}'\right)\right\rangle\equiv\delta_{\boldsymbol{k},\boldsymbol{k}'}P_{ij}\left(k\right)$$

$$\delta_{nn'}\delta_{\ell\ell'}\delta_{mm'}$$

$$u \propto a\Psi_{(1)}/\phi' \longrightarrow u''_{\boldsymbol{k}} + \left[k^2 - V_u(\eta)\right] u_{\boldsymbol{k}} = 0$$



2nd order

$$\Psi''_{(2)} + 2\left(\mathcal{H} - \frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi'_{(2)} - \bar{\nabla}^2\Psi_{(2)} + 2\left(\mathcal{H}' - 2\mathcal{K} - \mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(2)} = \mathcal{S}_{(2)}$$

$$\begin{aligned}\mathcal{S}_{(2)} = {} & 4\left(2\mathcal{H}^2 - \mathcal{H}' + 2\mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'} + 6\mathcal{K}\right)\Psi^2_{(1)} + 8\Psi'^2_{(1)} + 8\left(2\mathcal{H} + \frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(1)}\Psi'_{(1)} + 8\Psi_{(1)}\bar{\nabla}^2\Psi_{(1)} - \frac{4}{3}\left(\bar{\nabla}_i\Psi_{(1)}\right)^2 \\ & - \left[2\left(2\mathcal{H}^2 - \mathcal{H}'\right) - \frac{\bar{\phi}'''}{\bar{\phi}'}\right]\phi^2_{(1)} - \frac{2}{3}\left(\bar{\nabla}_i\phi_{(1)}\right)^2 - 2\left(\frac{\bar{\phi}''}{\bar{\phi}'} + 2\mathcal{H}\right)\bar{\nabla}^{-2}\bar{\nabla}^i\left(2\Psi'_{(1)}\bar{\nabla}_i\Psi_{(1)} + \phi'_{(1)}\bar{\nabla}_i\phi_{(1)}\right) \\ & + \left[2\left(\mathcal{H}' - \mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}\right) + \frac{1}{3}\bar{\nabla}^2\right]\left[2F\left(\Psi_{(1)}\right) + F\left(\phi_{(1)}\right)\right] + \mathcal{H}\left[2F\left(\Psi_{(1)}\right) + F\left(\phi_{(1)}\right)\right]'\end{aligned}$$

$$F\left(X\right) = \left(\bar{\nabla}^2\bar{\nabla}^2 + 3\mathcal{K}\bar{\nabla}^2\right)^{-1}\left[\bar{\nabla}_i\bar{\nabla}^j\left(3\bar{\nabla}^iX\bar{\nabla}_jX - \delta^i_j\left(\bar{\nabla}_kX\right)^2\right)\right]$$

general solution

$$\mathcal{S}_{(2)}\left(\boldsymbol{k},\eta\right) = \sum_{\boldsymbol{p}_1,\boldsymbol{p}_2}\mathcal{G}_{\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2}\tilde{\Sigma}_{ij}\left(k,p_1,p_2;\eta\right)\hat{a}_i\left(\boldsymbol{p}_1\right)\hat{a}_j\left(\boldsymbol{p}_2\right)$$

$$\Psi_{(2)}\left(\boldsymbol{k},\eta\right) = \Psi^{(0)}_{(2)}\left(\boldsymbol{k},\eta\right) + \sum_{\boldsymbol{p}_1,\boldsymbol{p}_2}\mathcal{G}_{\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2}\Pi_{ij}\left(k,p_1,p_2;\eta\right)\hat{x}_i\left(\boldsymbol{p}_1\right)\hat{x}_j\left(\boldsymbol{p}_2\right)$$

$$\Pi_{ij}\left(k,p_1,p_2;\eta\right) \equiv \int_{\eta_-}^{\eta}\mathrm{d}\eta' G\left(k,\eta,\eta'\right)\Sigma_{ij}\left(k,p_1,p_2;\eta'\right)$$

Green

Bispectrum

$$\langle \Psi\left(\boldsymbol{k}_1,\eta\right)\Psi\left(\boldsymbol{k}_2,\eta\right)\Psi\left(\boldsymbol{k}_3,\eta\right)\rangle \equiv \frac{1}{2}\mathcal{G}_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3}\mathcal{B}_\Psi\left(k_1,k_2,k_3;\eta\right)$$

$$\delta\left(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3\right)$$

$$\mathcal{B}_\Psi(k_1,k_2,k_3)=\frac{6}{5}f_{\rm NL}\left[P_{\Psi\Psi}(k_1)P_{\Psi\Psi}(k_2)+P_{\Psi\Psi}(k_2)P_{\Psi\Psi}(k_3)+P_{\Psi\Psi}(k_3)P_{\Psi\Psi}(k_1)\right]$$

$$7P_{\Psi\Psi}(k)+11P_{\Psi\Psi'}(k)+4P_{\Psi'\Psi'}(k)$$

$$f_{\rm NL} = -\frac{5(k_1+k_2+k_3)}{3\Upsilon K_3\left(k_1,k_2,k_3\right)}\left(\left[\prod_{\sigma(i,j,\ell)}(k_i+k_j-k_\ell)\right]\left\{\sum_{\sigma(i,j,\ell)}\frac{K_1(k_i)K_1(k_j)}{k_\ell^2}-4\left[\frac{K_1(k_i)K_2(k_j)}{k_j^2k_\ell^2}+\frac{K_1(k_j)K_2(k_i)}{k_i^2k_\ell^2}\right]\right\}\right.$$
$$\left.-\sum_{\sigma(i,j,\ell)}\left[\frac{7}{3}+\frac{2}{3}\left(\frac{k_i^2+k_j^2}{k_\ell^2}\right)-3\left(\frac{k_i^2-k_j^2}{k_\ell^2}\right)^2\right]K_1(k_i)K_1(k_j)\right)+\cdots,$$

$$6P_{\Psi\Psi}(k)+7P_{\Psi\Psi'}(k)+2P_{\Psi'\Psi'}(k)$$

$$81\sum_{\sigma(i,j)}P_{\Psi\Psi}(k_i)P_{\Psi\Psi}(k_j)+108\sum_{\sigma(i,j)}P_{\Psi\Psi}(k_i)P_{\Psi\Psi'}(k_j)+36\sum_{\sigma(i,j)}P_{\Psi\Psi}(k_i)P_{\Psi'\Psi'}(k_j)+$$
$$144\sum_{\sigma(i,j)}P_{\Psi\Psi'}(k_i)P_{\Psi\Psi'}(k_j)+48\sum_{\sigma(i,j)}P_{\Psi\Psi'}(k_i)P_{\Psi'\Psi'}(k_j)+16\sum_{\sigma(i,j)}P_{\Psi'\Psi'}(k_i)P_{\Psi'\Psi'}(k_j)$$

equilateral $k_1 = k_2 = k_3 = k$

$$f_{\rm NL}^{\rm equi} = -\frac{15k^2}{\Upsilon}\frac{K_1^2(k)}{K_3(k,k,k)}$$

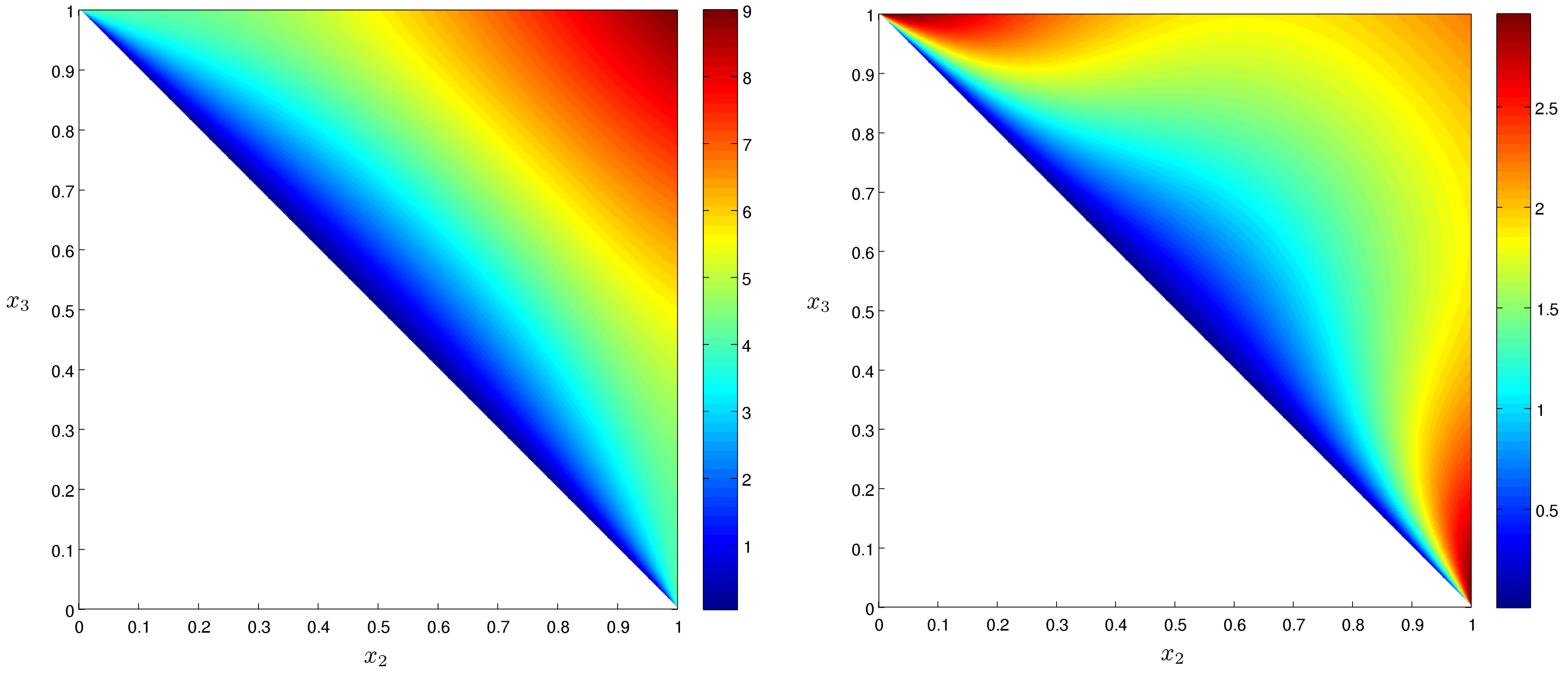
squeezed $k_i = k_j = k$ & $k_\ell = p \ll k$

$$f_{\rm NL}^{\rm sq} = -\frac{20k^2}{3\Upsilon}\frac{K_1^2(k)+K_1(k)K_1(p)}{K_3(k,k,p)}$$

folded $k_2 = k_3 = \frac{1}{2}k_1$

$$f_{\rm NL}^{\rm fold} = \frac{40}{9\Upsilon}\frac{K_1(k)\left[K_1(k)-16K_1(2k)\right]}{K_3(k,k,2k)}$$

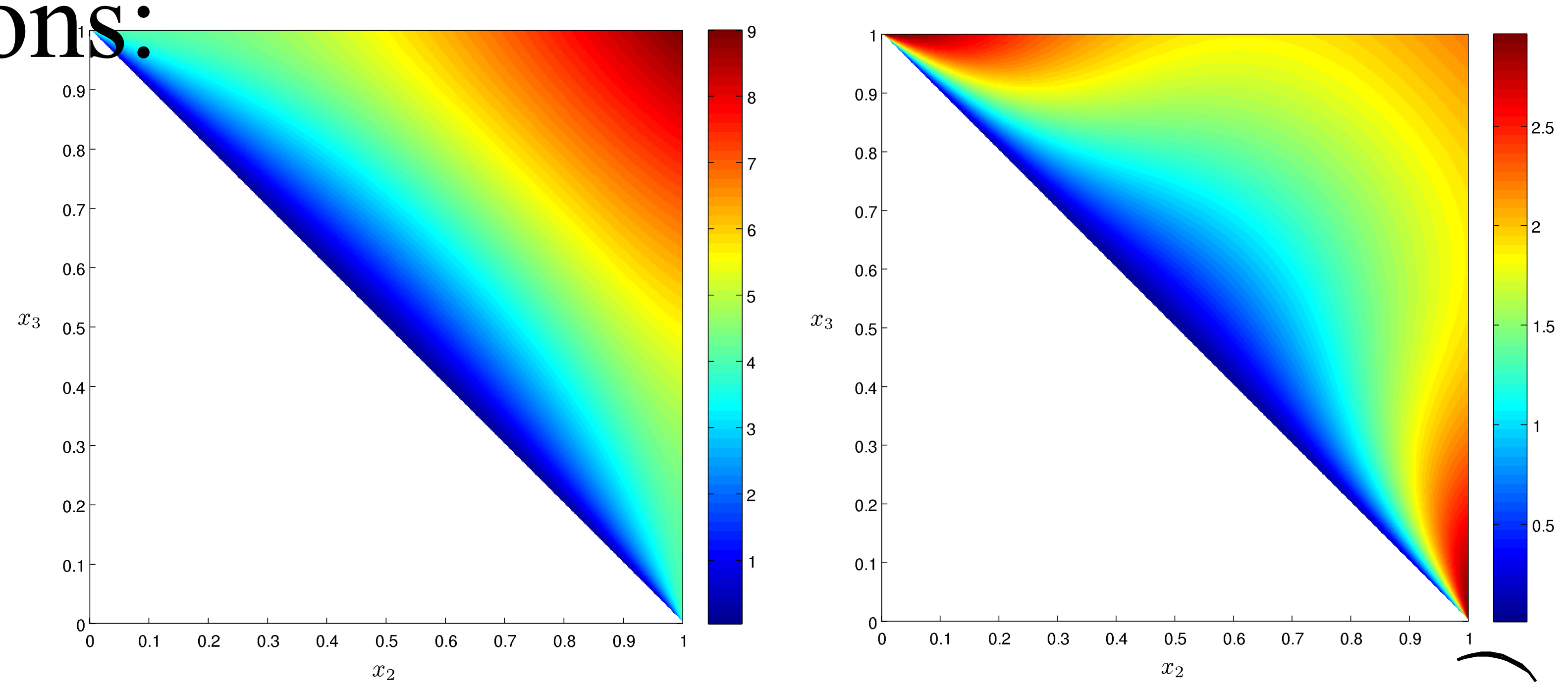
$$f_{\rm NL} \propto (k_1^2/\Upsilon) \times \mathcal{S}(x_2,x_3)$$



$$10^{-2}h^{-1}\,\mathrm{Mpc} \leq k_{\rm phys}^{-1} \leq 10^{3}h^{-1}\,\mathrm{Mpc} \longrightarrow 10^2 \lesssim k \lesssim 10^8 \ \textit{with} \ \ \Omega_\mathcal{K} \leq 10^{-2}$$

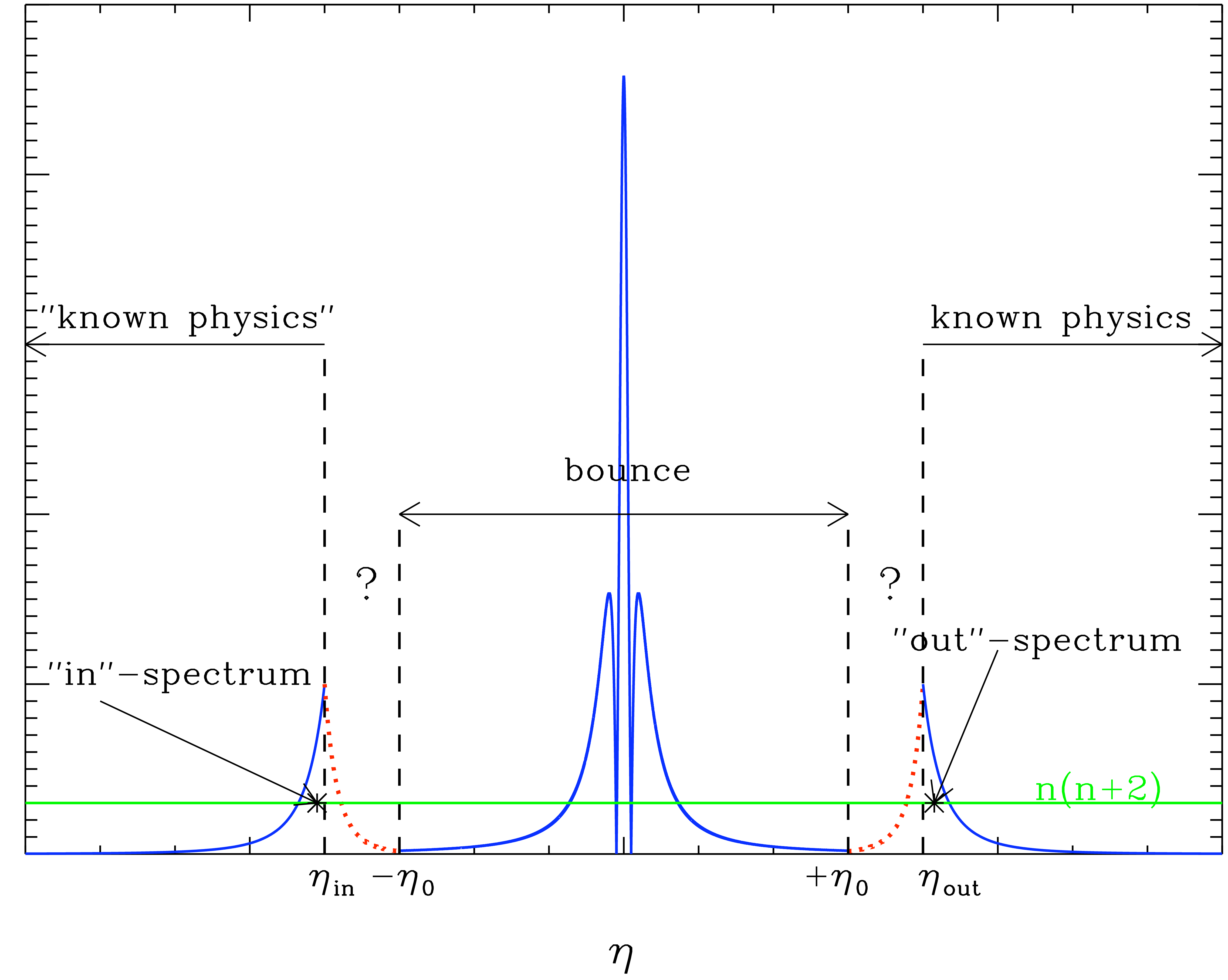
X. Gao, M. Lilley & P. P., *1406.4119*

Conclusions:

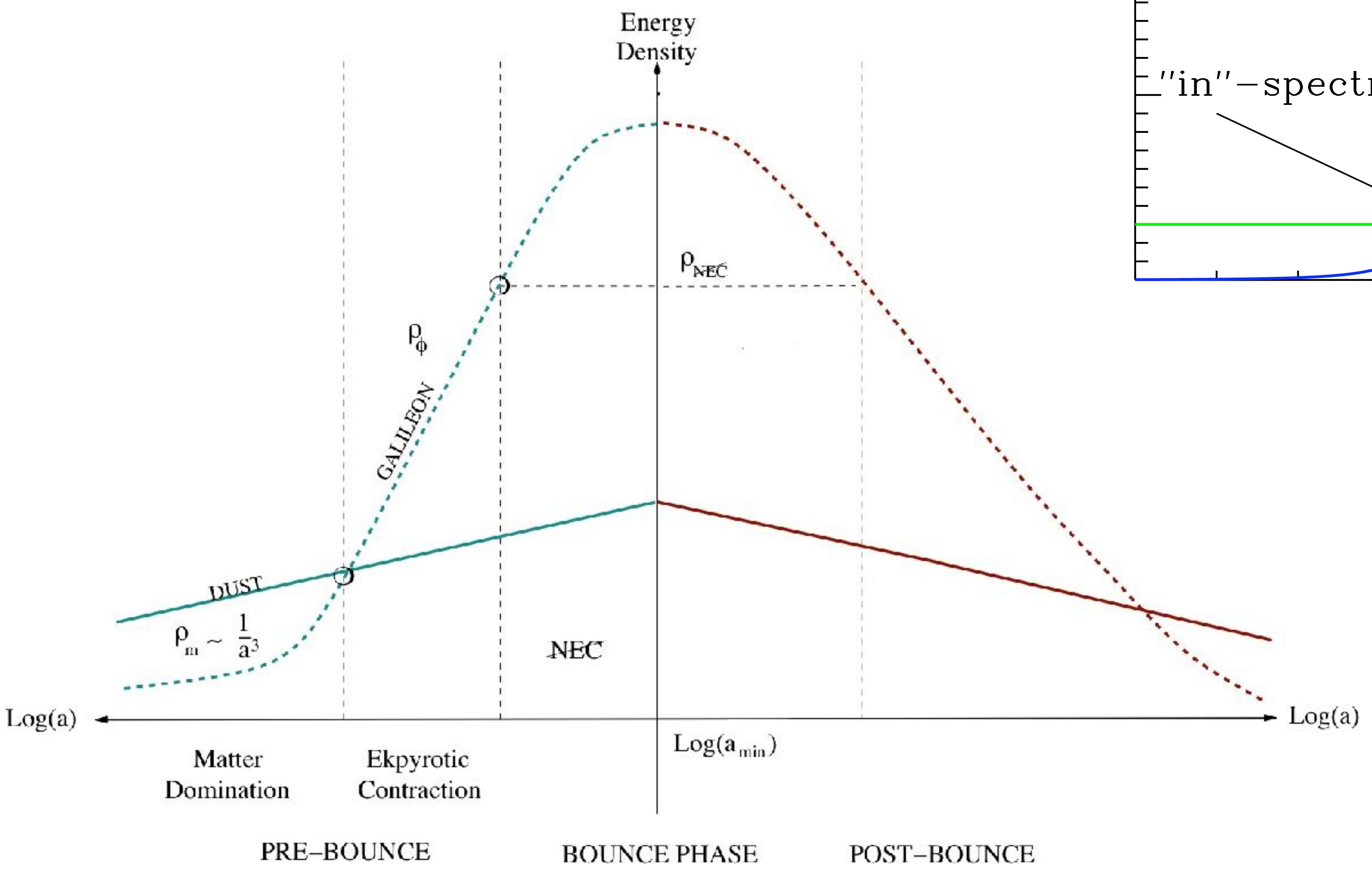


not easy

$$\mathcal{L}\left[\phi\left(x\right)\right]=K(\phi,X)+G(\phi,X)\Box\phi$$



not predictive?



QUANTUM BOUNCE?