

SORBONNE

UNIVERSITÉ

# DATRICK DETER (Théorie, Univers et Gravitation



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#### Buveurs tres illustres...







Limitations and puzzles of the standard model



Validity of classical GR?

### Standard paradigm: inflation! Phase of accelerated expansion



THE MAIN IDEA OF THE INFLATIONARY UNIVERSE SCENARIO



## Standard Model Failures and inflationary solutions

Singularity Not solved... actually not addressed! Horizon  $d_{\rm H} \equiv a(t) \int_{t_{\rm H}}^{t} \frac{{\rm d}\tau}{a(\tau)}$  can be made as big as one wishes Flatness  $\frac{d}{dt} |\Omega - 1| = -2\frac{\ddot{a}}{\dot{a}^3}$   $\ddot{a} > 0$  &  $\dot{a} > 0$ 

### Homogeneity & Isotropy Initial Universe = very small patch Accelerated expansion drives the shear to zero...

Perturbations Bonus of the theory: superb predictions!!!

accelerated expansion (inflation)



+ attractor

solves cosmological puzzles •••

Inflation:

- uses GR + scalar fields [(semi-)classical] ••
- can be implemented in high energy theories???
- makes falsifiable predictions ...
- **:** ... consistent with all known observations

#### **Alternative model???**



From R. Brandenberger, in M. Lemoine, J. Martin & PP (Eds.), "Inflationary cosmology", Lect. Notes Phys. 738 (Springer, Berlin, 2007).



- solves cosmological puzzles ••
- uses GR + scalar fields [(semi-)classical] ••
- Inflation: • can be implemented in high energy theories???
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  - **•** ... consistent with all known observations

### **Alternative model???**

singularity, initial conditions & homogeneity string based ideas (PBB, other brane models, string gas, ...) **Quantum gravity / cosmology** provide challengers / new ingredients!

**bouncing cosmology** 



### Non singular bounce



A brief history of bouncing cosmology

R. C. Tolman, "On the Theoretical Requirements for a Periodic Behaviour of the Universe", PRD 38, 1758 (1931) G. Lemaître, "L'Univers en expansion", Ann. Soc. Sci. Bruxelles (1933)

- -> A. A. Starobinsky, "On one non-singular isotropic cosmological model", Sov. Astron. Lett. 4, 82 (1978) V. N. Melnikov, S.V. Orlov, Phys. Lett. A 70, 263 (1979).
- M. Novello & J. M. Salim, Phys. Rev. D20, 377 (1979).
- R. Durrer & J. Laukerman, "The oscillating Universe: an alternative to inflation", Class. Quantum Grav. 13, 1069 (1996)

#### Many new ideas, models...

-> M. Novello & S.E. Perez Bergliaffa, "Bouncing cosmologies", Phys. Rep. 463, 127 (2008) D. Battefeld & PP, "A Critical Review of Classical Bouncing Cosmologies", Phys. Rep. 571, 1 (2015) -> R. Brandenberger & PP, "Bouncing cosmologies: Progress and problems", Found. Phys. (2017)



### PP & N. Pinto-Neto, Phys. Rev. D78, 063506 (2008) Standard Model Failures and bouncing solutions

Singularity Merely a non issue in the bounce case! **Horizon**  $d_{\rm H} \equiv a(t) \int_{t_{\rm H}}^{t} \frac{{\rm d}\tau}{a(\tau)}$  can be made divergent easily if  $t_{\rm i} \to -\infty$ Flatness  $\frac{d}{dt} |\Omega - 1| = -2\frac{\ddot{a}}{\dot{\lambda}^3}$ **Homogeneity** Large & flat Universe + low initial density + diffusion  $\frac{t_{\text{dissipation}}}{t_{\text{Hubble}}} \propto \frac{\lambda}{R_{\text{H}}^{1/3}} \left( 1 + \frac{\lambda}{AR_{\text{H}}^2} \right) \quad \text{enough time to dissipate any wavelength} \\ \longrightarrow \quad \text{quantum vacuum fluctuations...}$ **ISOTROPY** Potentially problematic: model dependent Others dark matter/energy, baryogenesis, ...

 $\ddot{a} < 0 \ \& \ \dot{a} < 0$ 

- accelerated expansion (inflation) or decelerated contraction (bounce)





# Standard Failures and bouncing solutions

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 $d_{\rm H}^{\rm cont} = \frac{3(1+w)}{1+3w} t_{\rm end} \left[ 1 - \left(\frac{t_{\rm ini}}{t_{\rm end}}\right)^{(1+3w)/[3(1+w)]} \right]$ 

 $t_{\rm ini} \rightarrow -\infty$ 

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### Implementing a bounce

Quantized scalar field effect model:

Parker & Fulling '73: massive scalar field, if  $\langle a^{\dagger}a \rangle \gg 1$ , then solution ( $\mathcal{K} > 0$ )

$$a(t) = \left(\frac{|B_2|^2 - |B_1|^2}{|m^2|B_2|^2} + \frac{8\pi}{2}\right)$$

Coherence of the quantum state crucial



FIG. 1. Solution with  $a(0) = 0.2m^{-1}$ : time-symmetric expansion from the minimum radius.

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# $\frac{3\pi G m^2 |B_2|^2 t^2}{3} \Big)^{1/2};$



FIG. 2. Solution with  $a(0) = 0.2m^{-1}$  (solid curve): approach to a Friedmann solution (dashed curve). The horizontal and vertical scales are logarithmic, and the time origin has been shifted to the initial singularity of the Friedmann curve, so that the latter becomes a straight line of slope  $\frac{2}{3}$ . (The deviation of the Friedmann solution from the  $a \propto t^{2/3}$ law due to the three-space curvature of the closed universe is negligible in the range of t plotted.)

### **Model listing:**



Quantum gravity

LQG & LQC

Canonical quantum gravity (WdW) String theory

Non relativistic quantum gravity



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### **Model listing:**

Quantum gravity

Ekpyrotic & cyclic Branes

LQG & LQC

Canonical quantum gravity (WdW)

String theory



# Non relativistic quantum gravity







$$\times \int_{\mathcal{M}_5} \mathrm{d}^5 x \sqrt{-g_5} \left[ R_{(5)} - \frac{1}{2} \left( \partial \varphi \right)^2 - \frac{3}{2} \frac{\mathrm{e}^{2\varphi} \mathcal{F}^2}{5!} \right],$$

$$= \int_{\mathcal{M}_4} \mathrm{d}^4 x \sqrt{-g_4} \left[ \frac{R_{(4)}}{2\kappa} - \frac{1}{2} \left( \partial \phi \right)^2 - V(\phi) \right],$$

$$V(\varphi) = -V_{\rm i} \exp\left[-\frac{4\sqrt{\pi\gamma}}{m_{\rm Pl}}(\varphi - \varphi_{\rm i})\right]$$



### Cyclic extension



### Model listing:

Quantum gravity

Ekpyrotic & cyclic Branes

String gas cosmology Antigravity Galileon Massive gravity

Multiverse models Strings & AdS/CFT



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TUG - Montpellier / 5 octobre 2022

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#### Implementing a bounce = problem with GR!

Violation of Null Energy Condition (NEC)



#### Instabilities for perfect fluids



#### Implementing a bounce = problem with (

Violation of Null Energy Condition (NEC)

Positive spatial curvature + scalar field

GR! 
$$\dot{H} = \frac{\mathcal{K}}{a^2} - \frac{1}{2}\left(\rho + P\right)$$

- $\rho + P \leq 0$





J. Martin & PP., Phys. Rev. D68, 103517 (2003)

 $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t) \left(\frac{\mathrm{d}r^2}{1 - \mathcal{K}r^2} + r^2 \mathrm{d}\Omega^2\right)$ 

 $\mathcal{S} = \int \mathrm{d}^4 x \sqrt{-g} \left[ \frac{R}{6\ell_{\mathrm{el}}^2} - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right]$ 

 $H^2 = \frac{1}{3} \left( \frac{1}{2} \dot{\varphi}^2 + V \right) - \frac{\mathcal{K}}{a^2}$  Positive spatial curvature



Self consistent bounce:



F. Falciano, M. Lilley & P. P., Phys. Rev. D77, 083513 (2008)

 $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t) \left(\frac{\mathrm{d}r^2}{1 - \mathcal{K}r^2} + r^2\mathrm{d}\Omega^2\right)$ 



Implementing a bounce = problem with GR!

Violation of Null Energy Condition (NEC)

Positive spatial curvature + scalar field



- $\rho + P \leq 0$
- Modify GR?
- Add new terms?
- *K*-bounce, Ghost condensates, Galileons...?





$$\varphi_i I^{(i)} - V\left(\boldsymbol{\varphi}\right)$$

 $\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}\varphi} = I$ 

R. Brandenberger, V. F. Mukhanov and A. Sornborger, Phys. Rev. D48, 1629 (1993)

### $I = R - \sqrt{3} \left( 4R_{\mu\nu}R^{\mu\nu} - R^2 \right)$

R. Abramo, P. P. & I. Yasuda, *Phys. Rev.* D81, 023511 (2010)







### vanishing spatial curvature possible in 4 dimensions G.R.?



Implementing a bounce = problem with GR!

Violation of Null Energy Condition (NEC)

Positive spatial curvature + scalar field

Various instabilities may arise!



- $\rho + P \leq 0$
- Modify GR?
- Add new terms?
- K-bounce, Ghost condensates, Galileons...?
- (e.g. radiation for matter bounce or curvature perturbations)

### The problem with contraction: BKL/shear instability









#### Stress-energy tensor

 $T^{\phi}_{\mu\nu} = \left(-K + 2XG_{,\phi} + G_{,X}\nabla_{\sigma}X\nabla^{\sigma}\phi\right)g_{\mu\nu} + \left(K_{,X} + G_{,X}\Box\phi - 2G_{,\phi}\right)\nabla_{\mu}\phi\nabla_{\nu}\phi - G_{,X}(\nabla_{\mu}X\nabla_{\nu}\phi + \nabla_{\nu}X\nabla_{\mu}\phi)$ 

Energy density & Pressu  

$$\rho_{\phi} = \frac{1}{2} M_{P_{1}}^{2} (1-g) \dot{\phi}^{2} + \frac{3}{4} \beta \dot{\phi}^{4} + 3\gamma H \dot{\phi}^{3} + V(\phi)$$

$$p_{\phi} = \frac{1}{2} M_{P_{1}}^{2} (1-g) \dot{\phi}^{2} + \frac{1}{4} \beta \dot{\phi}^{4} - \gamma \dot{\phi}^{2} \ddot{\phi} - V(\phi)$$

+ Fluid  $p = w\rho$ 

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#### lre

 $\phi)$ 

Einstein equation +  $\nabla_{\mu} T^{\mu\nu}_{\text{Fluid}} = 0$ + modified Klein-Gordon  $\mathcal{P}\ddot{\phi} + \mathcal{D}\dot{\phi} + V_{,\phi} = 0$  with...

 $\mathcal{P} = (1-g)M_{\rm Pl}^2 + 6\gamma H\dot{\phi} + 3\beta\dot{\phi}^2 + \frac{3\gamma^2}{2M^2}\dot{\phi}^4$  $\mathcal{D} = 3(1-g)M_{_{\mathrm{Pl}}}^2H + \left(9\gamma H^2 - \frac{1}{2}M_{_{\mathrm{Pl}}}^2g_{,\phi}\right)\dot{\phi} + 3\beta H\dot{\phi}^2$  $-rac{3}{2}(1-g)\gamma\dot{\phi}^{3}-rac{9\gamma^{2}H\dot{\phi}^{4}}{2M_{
m Pl}^{2}}-rac{3eta\gamma\dot{\phi}^{5}}{2M_{
m Pl}^{2}}$  $-\frac{3}{2}G_{,X}\sum_{i}\dot{\theta}_{i}^{2}\dot{\phi} - \frac{3G_{,X}}{2M_{-i}^{2}}(\rho_{\rm m} + p_{\rm m})\dot{\phi}$ 



PRE-BOUNCE

BOUNCE PHASE POST-BOUNCE

Produces scale invariant perturbations **Removes anisotropies** 

Log(a)

Leads to expansion

Connects to standard model!!

BB cosmology

explicit example...  

$$V_0 = 10^{-7}, g_0 = 1.1, \beta = 5, \gamma = 10^{-3}$$
  
 $b_V = 5, b_g = 0.5, p = 0.01, q = 0.1$   
 $\rho_{m,B} = 2.8 \times 10^{-10}, M_{\theta,1} = 2.2 \times 10^{-6}$   
 $M_{\theta,2} = 3.4 \times 10^{-6}, M_{\theta,3} = -5.6 \times 10^{-6}$   
 $\phi_{ini} = -2, \phi_{ini} = 7.8 \times 10^{-6}$ 

#### Hubble parameters



### Energy densities



### Anisotropies









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Perturbations:



### **ASSUME LINEARITY THROUGHOUT**

``central feature of bouncing cosmology = the bounce''...



#### A generic model-independent treatment of the bounce phase?



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### Geometric matching conditions?

 $[a]_{\pm} = 0$ OK Continuity of metric ??? Continuity of extrinsic curvature  $[H]_{\pm} = 0$  $[\zeta]_{\pm} = 0$ Perturbations? ???

Perturbations:

$$\longleftrightarrow \qquad \Phi = \frac{3\mathcal{H}u}{2a^2\theta}$$
$$u'' + \left[k^2 - \frac{\theta''}{\theta} - 3\mathcal{K}\left(1 - c_{\rm s}^2\right)\right]u = 0$$

$$V_{u}(\eta) \equiv \frac{\theta''}{\theta} + 3\mathcal{K}(1 - c_{\rm S}^{2}) = \frac{P_{24}(\eta)}{Q_{24}(\eta)},$$

#### Non trivial transfer matrix

$$\boldsymbol{T}_{ij}(k) = \begin{bmatrix} A(k) & B(k) \\ C(k) & D(k) \end{bmatrix}$$

"Causality" argument...

J. Martin & PP, Phys. Rev. Lett. 92, 061301 (2004)

 $ds^{2} = a^{2}(\eta) \left\{ (1+2\Phi) d\eta^{2} - \left[ (1-2\Phi) \gamma_{ij} + h_{ij} \right] dx^{i} dx^{j} \right\}$ 

 $\theta \equiv \frac{1}{a} \sqrt{\frac{\rho_{\varphi}}{\rho_{\varphi} + p_{\varphi}}} \left(1 - \frac{3\mathcal{K}}{\rho_{\varphi}a^2}\right)$ 

#### Resulting spectrum: very much model dependent...



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J. Martin& PP, *PR***D68**, 103517 (2003)



primordial spectrum

 $\mathcal{P}_{\zeta} = \mathcal{A}k^{n_{\rm S}-1}\cos^2\left(\omega\frac{k_{\rm ph}}{k_{\star}} + \psi\right)$ 



#### Perturbations in the K-bounce



Oscillations +  $\zeta$  conserved

R. Abramo & P. P., *JCAP* **09**, 001 (2007)





Another issue...

spectral index  $n_s < 1$ phenomenologi Non gaussianities:  $a(\eta) = a_0 \left| 1 + \frac{1}{2} \left( \frac{\eta}{\eta_c} \right)^2 + \right|$  $\int \frac{\phi'^2}{a^2} = \frac{2}{a^2} \left( \mathcal{H}^2 - \mathcal{H}' + \mathcal{K} \right)$  $- \frac{6}{a^2} \mathcal{H}' = -2V(\phi) \left[ 1 - \frac{\phi'^2}{a^2 V(\phi)} \right]$  $\varepsilon_V = \frac{V'_0}{V_0}$  "slow-roll"  $\eta_V = \frac{V_0''}{V}$ 

perturbed metric  $ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = a^2 \left(-e^{2\Phi}d\eta^2 + e^{-2\Psi}\gamma_{ii}dx^i dx^j\right)$ 

X. Gao, M. Lilley & P. P., JCAP 07, 010 (2014)

ical description 
$$S = -\int d^4x \sqrt{-g} \left[ R + (\partial \phi)^2 + V(\phi) \right]$$
  
 $\lambda_3 \left( \frac{\eta}{\eta_c} \right)^3 + \frac{5}{24} \left( 1 + \lambda_4 \right) \left( \frac{\eta}{\eta_c} \right)^4 \right] + scalar field$ 



complete set of parameters

perturbations up to 2nd order  $X(\boldsymbol{x},\eta) = X_{(1)}(\boldsymbol{x},\eta) + \frac{1}{2}$ 

first order 
$$\Psi_{(1)}'' + F(\eta) \Psi_{(1)}' - \overline{\nabla}^2 \Psi_{(1)} + W(\eta) \Psi_{(1)} = 0$$
  
 $2\left(\mathcal{H} - \frac{\overline{\phi}''}{\overline{\phi}'}\right) \qquad 2\left(\mathcal{H}' - \mathcal{H}\frac{\overline{\phi}''}{\overline{\phi}'} - 2\mathcal{K}\right)$ 

positive spatial curvature: decomposition on the 3-sph

$$\Psi_{(1)}(\boldsymbol{x},\eta) = \sum_{\ell m n} \Psi_{\ell m n}(\eta) Q_{\ell n}$$

 $Q_{\ell mn}(\chi,\theta,\varphi) = R_{\ell n}(\chi)Y_{\ell m}(\theta,\varphi)$  hyperspherical



$$\frac{1}{2}X_{(2)}\left(\boldsymbol{x},\eta\right)+\cdots$$

$$\mathcal{D}\Psi_{(i)} = \mathcal{S}\left[\Psi_{(i-1)}\right]$$

 $_{mn}(\chi, heta,arphi)$ 

*Legendre*  
*harmonics*  

$$R_{n\ell}(\chi) = \sqrt{\frac{(n+1)(n+\ell+1)!}{(n-\ell)!}} \sqrt{\frac{\mathcal{K}}{f_{\mathcal{K}}(\chi)}} P_{n+\frac{1}{2}}^{-\ell-\frac{1}{2}} \left[ \cos\left(\sqrt{\mathcal{K}}\chi\right) \right].$$

$$\langle \hat{x}_{i} (\mathbf{k}) \hat{x}_{j} (\mathbf{k}') \rangle \equiv \delta_{\mathbf{k},\mathbf{k}'} P_{ij} (\mathbf{k})$$

$$\delta_{nn'} \delta_{\ell\ell'} \delta_{mm'}$$

<u>obre 2022</u>

 $u \propto a \Psi_{(1)}/\phi'$   $u_{\mathbf{k}}^{\prime\prime} + \left[k^2 - V_u(\eta)\right] u_{\mathbf{k}} = 0$ 



$$\begin{aligned} 2nd \ order \quad \Psi_{(2)}'' + 2\left(\mathcal{H} - \frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(2)}' - \bar{\nabla}^{2}\Psi_{(2)} + 2\left(\mathcal{H}' - 2\mathcal{K} - \mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(2)} = \mathcal{S}_{(2)} \\ \mathcal{S}_{(2)} = 4\left(2\mathcal{H}^{2} - \mathcal{H}' + 2\mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'} + 6\mathcal{K}\right)\Psi_{(1)}^{2} + 8\Psi_{(1)}'^{2} + 8\left(2\mathcal{H} + \frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(1)}\Psi_{(1)}' + 8\Psi_{(1)}\bar{\nabla}^{2}\Psi_{(1)} - \frac{4}{3}\left(\bar{\nabla}_{i}\Psi_{(1)}\right)^{2} \\ &- \left[2\left(2\mathcal{H}^{2} - \mathcal{H}'\right) - \frac{\bar{\phi}'''}{\bar{\phi}'}\right]\phi_{(1)}^{2} - 2\left(\frac{\bar{\phi}''}{\bar{\phi}'} + 2\mathcal{H}\right)\bar{\nabla}^{-2}\bar{\nabla}^{i}\left(2\Psi_{(1)}'\bar{\nabla}_{i}\Psi_{(1)} + \phi_{(1)}'\bar{\nabla}_{i}\phi_{(1)}\right) \\ &+ \left[2\left(\mathcal{H}' - \mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}\right) + \frac{1}{3}\bar{\nabla}^{2}\right]\left[2F\left(\Psi_{(1)}\right) + F\left(\phi_{(1)}\right)\right] + \mathcal{H}\left[2F\left(\Psi_{(1)}\right) + F\left(\phi_{(1)}\right)\right]' \\ &F\left(X\right) = \left(\bar{\nabla}^{2}\bar{\nabla}^{2} + 3\mathcal{K}\bar{\nabla}^{2}\right)^{-1}\left[\bar{\nabla}_{i}\bar{\nabla}^{j}\left(3\bar{\nabla}^{i}X\bar{\nabla}_{j}X - \delta_{j}^{i}\left(\bar{\nabla}_{k}X\right)^{2}\right)\right] \\ general \ solution \qquad \mathcal{S}_{(2)}\left(\mathbf{k},\eta\right) = \sum \mathcal{G}_{\mathbf{k},\mathbf{p}_{1},\mathbf{p}_{2}}\tilde{\Sigma}_{ij}\left(\mathbf{k},\mathbf{p}_{1},\mathbf{p}_{2};\eta\right)\hat{a}_{i}\left(\mathbf{p}_{1}\right)\hat{a}_{j}\left(\mathbf{p}_{2}\right) \end{aligned}$$

$$2\left(\mathcal{H}-\frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi'_{(2)}-\bar{\nabla}^{2}\Psi_{(2)}+2\left(\mathcal{H}'-2\mathcal{K}-\mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(2)}=\mathcal{S}_{(2)}$$

$$\mathcal{S}_{(2)}=4\left(2\mathcal{H}^{2}-\mathcal{H}'+2\mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}+6\mathcal{K}\right)\Psi^{2}_{(1)}+8\Psi^{\prime 2}_{(1)}+8\left(2\mathcal{H}+\frac{\bar{\phi}''}{\bar{\phi}'}\right)\Psi_{(1)}\Psi'_{(1)}+8\Psi_{(1)}\bar{\nabla}^{2}\Psi_{(1)}-\frac{4}{3}\left(\bar{\nabla}_{i}\Psi_{(1)}\right)^{2}$$

$$-\left[2\left(2\mathcal{H}^{2}-\mathcal{H}'\right)-\frac{\bar{\phi}''}{\bar{\phi}'}\right]\phi^{2}_{(1)}-\frac{2}{3}\left(\bar{\nabla}_{i}\phi_{(1)}\right)^{2}-2\left(\frac{\bar{\phi}''}{\bar{\phi}'}+2\mathcal{H}\right)\bar{\nabla}^{-2}\bar{\nabla}^{i}\left(2\Psi'_{(1)}\bar{\nabla}_{i}\Psi_{(1)}+\phi'_{(1)}\bar{\nabla}_{i}\phi_{(1)}\right)$$

$$+\left[2\left(\mathcal{H}'-\mathcal{H}\frac{\bar{\phi}''}{\bar{\phi}'}\right)+\frac{1}{3}\bar{\nabla}^{2}\right]\left[2F\left(\Psi_{(1)}\right)+F\left(\phi_{(1)}\right)\right]+\mathcal{H}\left[2F\left(\Psi_{(1)}\right)+F\left(\phi_{(1)}\right)\right]'$$

$$F\left(X\right)=\left(\bar{\nabla}^{2}\bar{\nabla}^{2}+3\mathcal{K}\bar{\nabla}^{2}\right)^{-1}\left[\bar{\nabla}_{i}\bar{\nabla}^{j}\left(3\bar{\nabla}^{i}X\bar{\nabla}_{j}X-\delta^{i}_{j}\left(\bar{\nabla}_{k}X\right)^{2}\right)\right]$$

$$\mathcal{S}_{(2)}\left(\mathbf{k},\eta\right)=\sum\mathcal{G}_{\mathbf{k},\mathbf{p}_{1},\mathbf{p}_{2}}\tilde{\Sigma}_{ij}\left(\mathbf{k},\mathbf{p}_{1},\mathbf{p}_{2};\eta\right)\hat{a}_{i}\left(\mathbf{p}_{1}\right)\hat{a}_{j}\left(\mathbf{p}_{2}\right)$$

$$\mathcal{S}_{(2)}\left(\boldsymbol{k},\eta\right) = \sum_{\boldsymbol{p}_{1},\boldsymbol{p}_{2}} \mathcal{G}_{\boldsymbol{k},\boldsymbol{p}_{1},\boldsymbol{p}_{2}} \tilde{\Sigma}_{ij}\left(\boldsymbol{k},p_{1},p_{2};\eta\right) \hat{a}_{i}\left(\boldsymbol{p}_{1}\right)$$

$$\Psi_{(2)}\left(\boldsymbol{k},\eta\right) = \Psi_{(2)}^{\left(0\right)}\left(\boldsymbol{k},\eta\right)$$

Π

$$\begin{aligned} \mathbf{x}, \eta \end{pmatrix} + \sum_{\mathbf{p}_{1}, \mathbf{p}_{2}} \mathcal{G}_{\mathbf{k}, \mathbf{p}_{1}, \mathbf{p}_{2}} \prod_{ij} \left( k, p_{1}, p_{2}; \eta \right) \hat{x}_{i} \left( \mathbf{p}_{1} \right) \hat{x}_{j} \left( \mathbf{p}_{2} \right) \\ \mathbf{p}_{1} \\ \Pi_{ij} \left( k, p_{1}, p_{2}; \eta \right) \equiv \int_{\eta_{-}}^{\eta} \mathrm{d}\eta' G \left( k, \eta, \eta' \right) \Sigma_{ij} \left( k, p_{1}, p_{2}; \eta' \right) \\ \mathbf{Green} \end{aligned}$$

$$\begin{array}{ll} \textit{Bispectrum} & \langle \Psi\left(\boldsymbol{k}_{1},\eta\right)\Psi\left(\boldsymbol{k}_{2},\eta\right)\Psi\left(\boldsymbol{k}_{3},\eta\right)\rangle\equiv\frac{1}{2}\mathcal{G}_{\boldsymbol{k}_{1}\boldsymbol{k}_{2}\boldsymbol{k}_{3}}\mathcal{B}_{\Psi}\left(\boldsymbol{k}_{1}\right)\\ & \delta\left(\boldsymbol{k}_{1}+\boldsymbol{k}_{2}+\boldsymbol{k}_{3}\right)\mathcal{A}^{T}$$

$$\begin{aligned} f_{\rm NL} &= -\frac{5(k_1 + k_2 + k_3)}{3\Upsilon K_3(k_1, k_2, k_3)} \left( \left[ \prod_{\sigma(i,j,\ell)} (k_i + k_j - k_\ell) \right] \left\{ \sum_{\sigma(i,j,\ell)} \frac{K_1(k_i)K_1(k_j)}{k_\ell^2} - 4 \left[ \frac{K_1(k_i)K_2(k_j)}{k_j^2 k_\ell^2} + \frac{K_1(k_j)K_2(k_i)}{k_i^2 k_\ell^2} \right] \right\} \\ &- \sum_{\sigma(i,j,\ell)} \left[ \frac{7}{3} + \frac{2}{3} \left( \frac{k_i^2 + k_j^2}{k_\ell^2} \right) - 3 \left( \frac{k_i^2 - k_j^2}{k_\ell^2} \right)^2 \right] K_1(k_i)K_1(k_j) \right) + \cdots, \\ 6 P_{\Psi\Psi}(k) + 7 P_{\Psi\Psi'}(k) + 2 P_{\Psi'\Psi'}(k) \\ \\ 81 \sum_{\sigma(i,j)} P_{\Psi\Psi'}(k_i) P_{\Psi\Psi'}(k_j) + 108 \sum_{\sigma(i,j)} P_{\Psi\Psi'}(k_i) P_{\Psi\Psi'}(k_j) + 36 \sum_{\sigma(i,j)} P_{\Psi\Psi'}(k_i) P_{\Psi'\Psi'}(k_j) + \\ 144 \sum_{\sigma(i,j)} P_{\Psi\Psi'}(k_i) P_{\Psi\Psi'}(k_j) + 48 \sum_{\sigma(i,j)} P_{\Psi\Psi'}(k_i) P_{\Psi'\Psi'}(k_j) + 16 \sum_{\sigma(i,j)} P_{\Psi'\Psi'}(k_i) P_{\Psi'\Psi'}(k_j) \\ \end{aligned}$$

 $(k_1,k_2,k_3;\eta)$ 

 $P_{\Psi\Psi}(k_2)P_{\Psi\Psi}(k_3) + P_{\Psi\Psi}(k_3)P_{\Psi\Psi}(k_1)]$ 







