
Tests of general relativity with gravitational waves

Sylvain Marsat (L2IT, Toulouse)

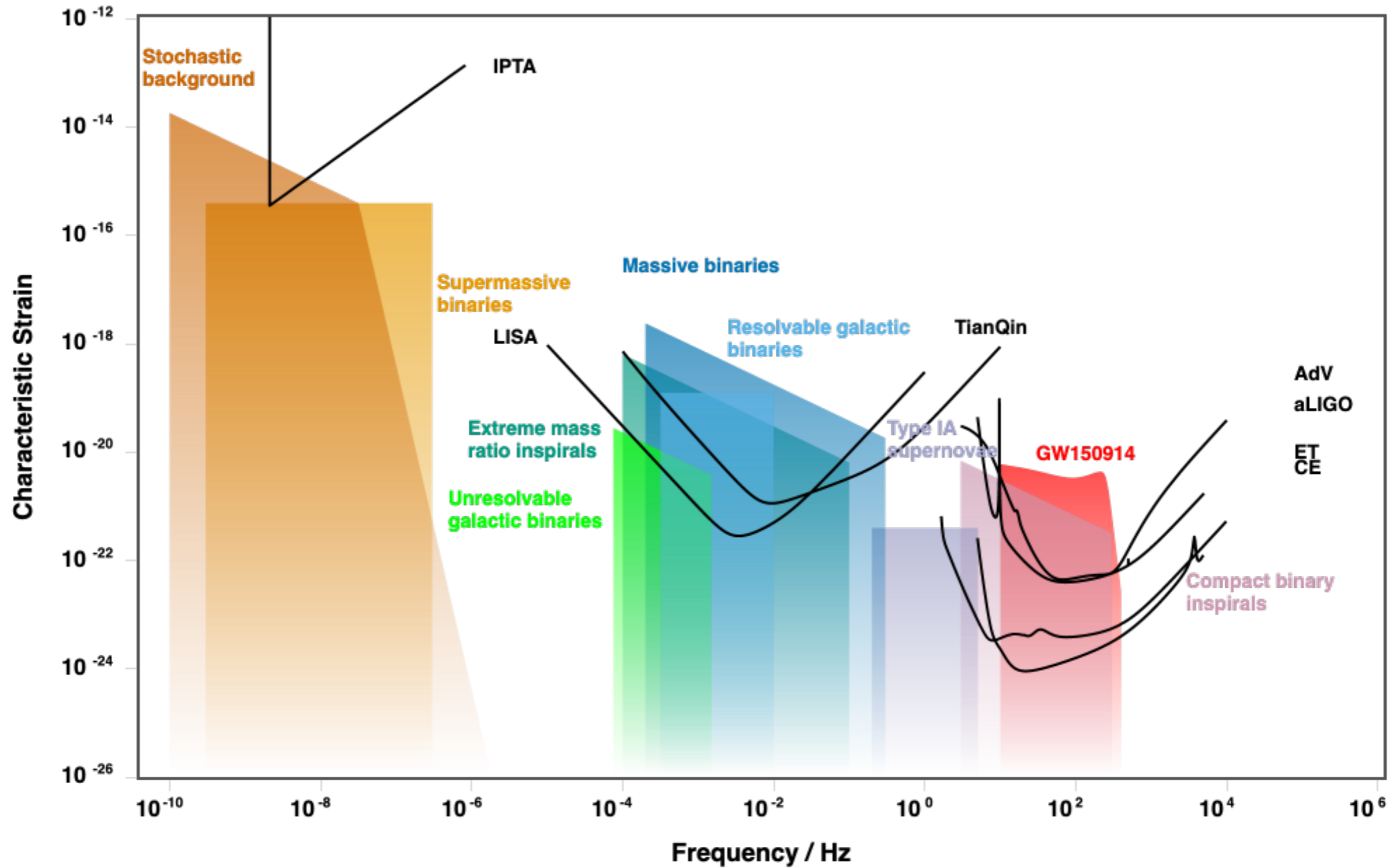


Overview

- Introduction: fundamental physics and gravitational waves
- LIGO-Virgo-Kagra observations
- GW170817 and its consequences
- Tests of GR from LVK GWTC-3 observations
- The future of GW astronomy and TGR

- **Introduction: fundamental physics and gravitational waves**
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The gravitational waves landscape



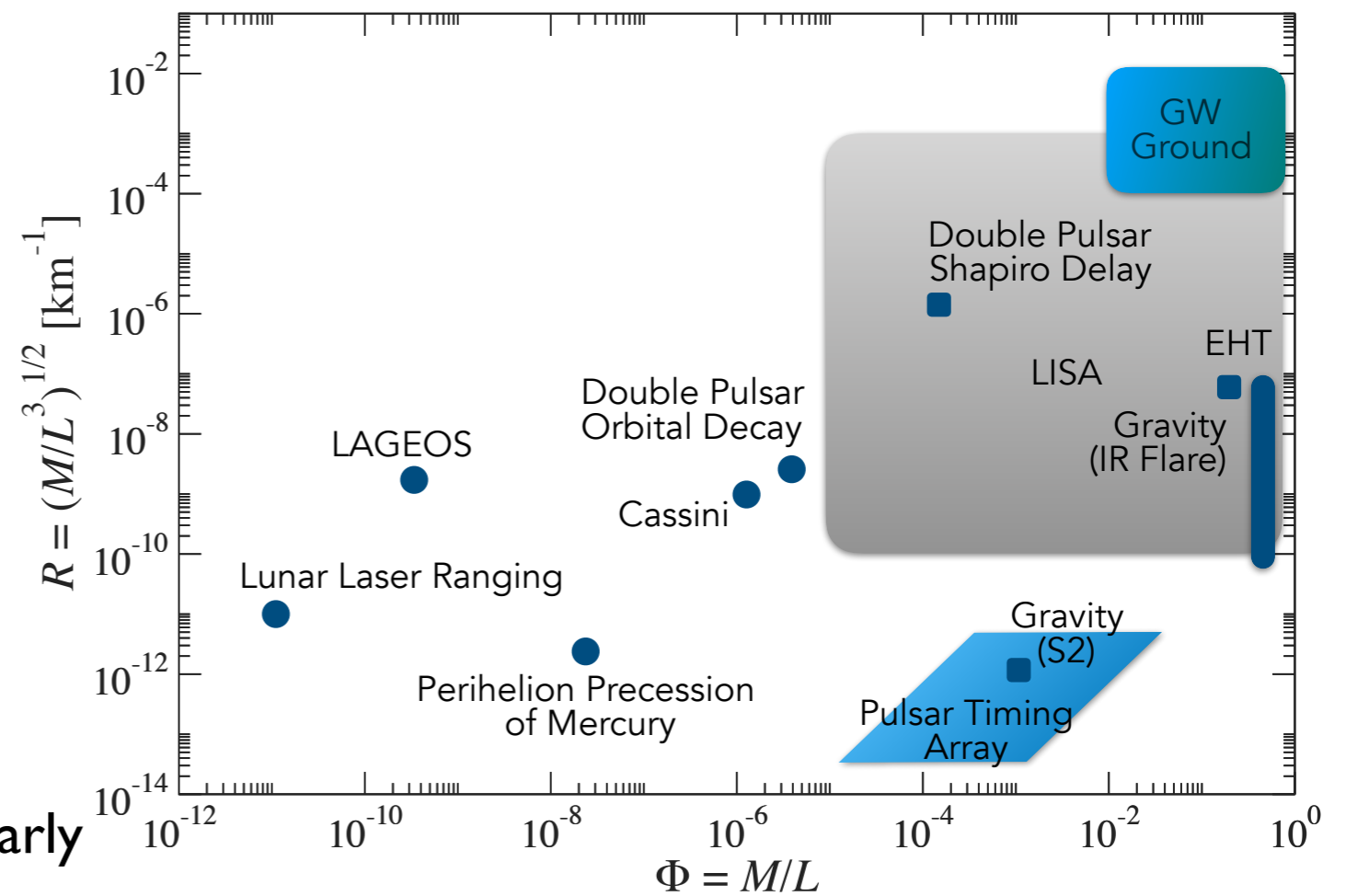
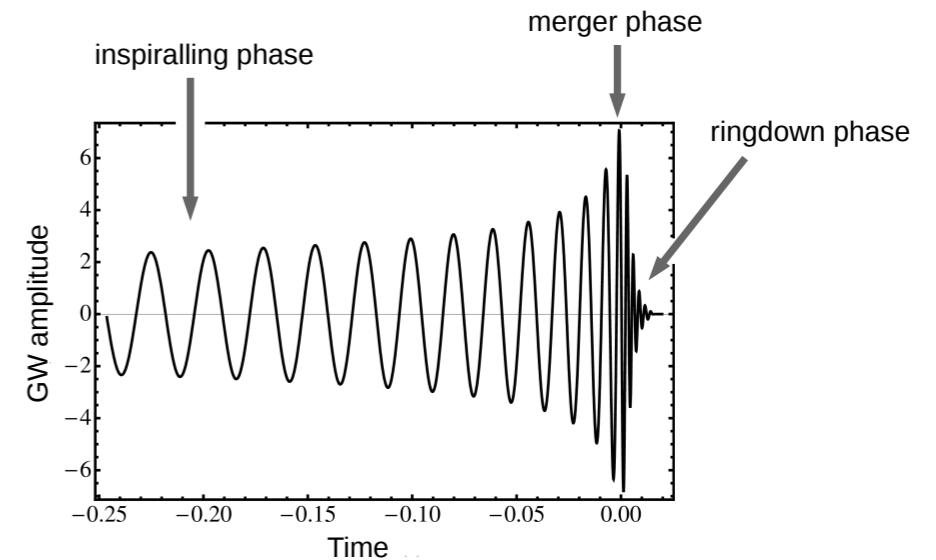
[Moore&al 2014]

Gravitational waves and fundamental physics

Open questions in fundamental physics: dark matter/
dark energy, quantum gravity, nature of compact objects

What gravitational waves science can bring:

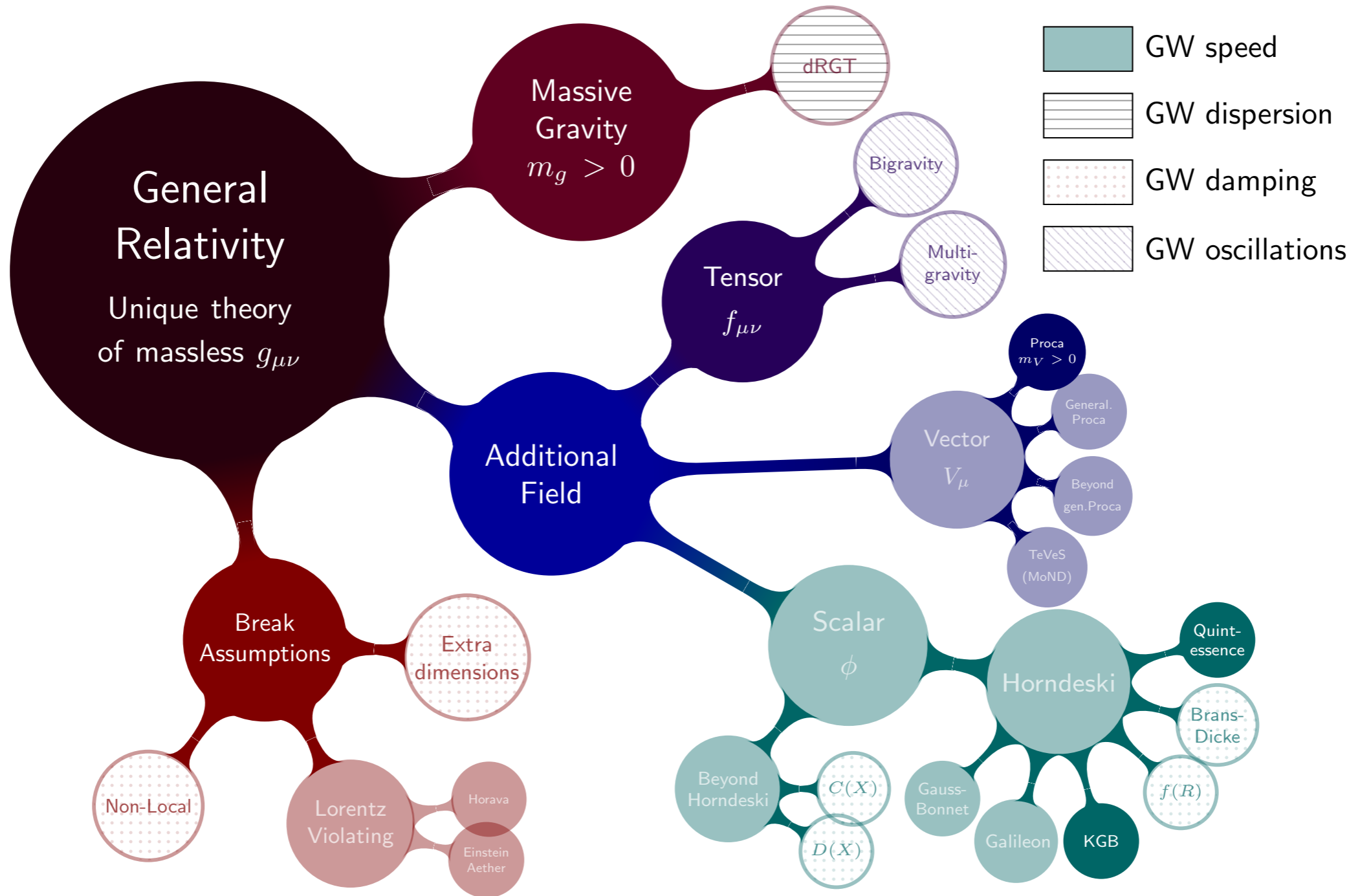
- Confirming the GR prediction in the first place !
- GW inspiral, merger, ringdown probe gravity and compact objects in different regimes
- Explore different scales across the GW spectrum
- Extreme phenomena in strong-field GR
- A new, independent cosmological probe
- Stochastic GW backgrounds from the early universe



[Extreme Gravity White Paper 2020]

Gravitational waves and fundamental physics

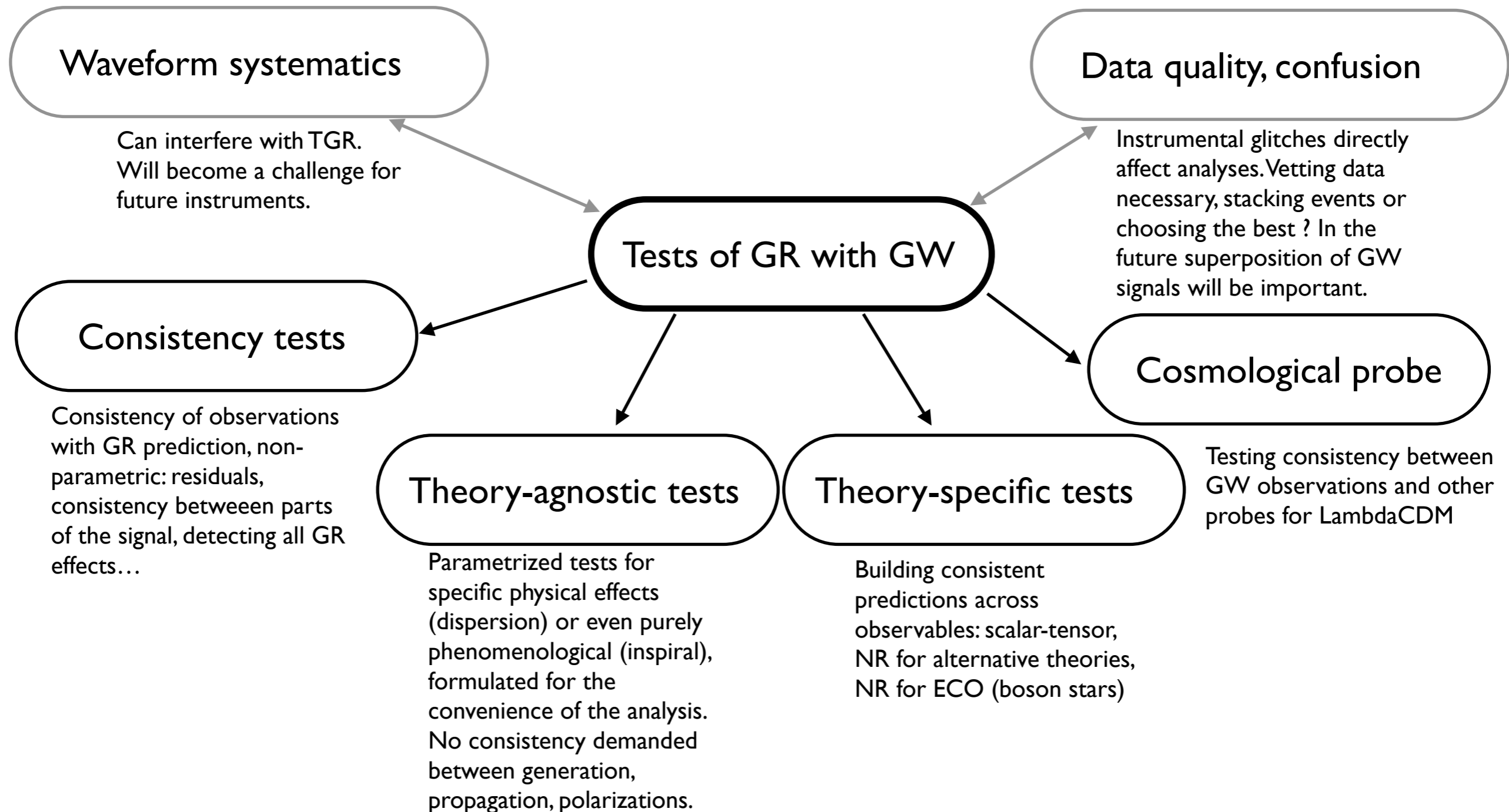
Modified gravity roadmap



[Ezquiaga&al 2018]

Tests of general relativity with gravitational waves

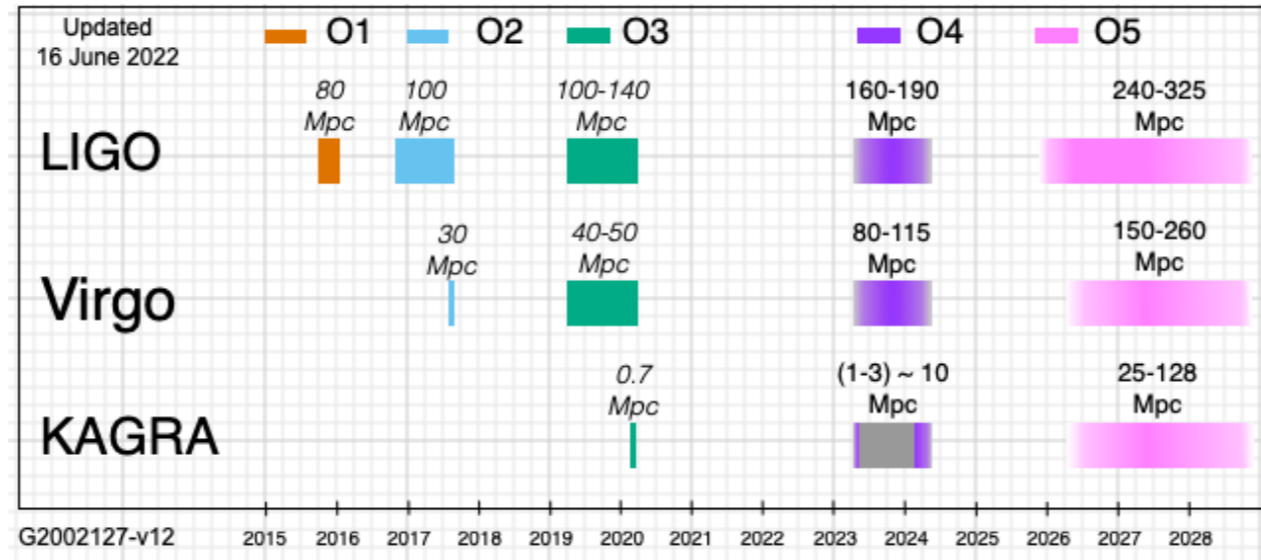
What can we really test for from our observations ? No 1-to-1 mapping between theory space and data analysis space.



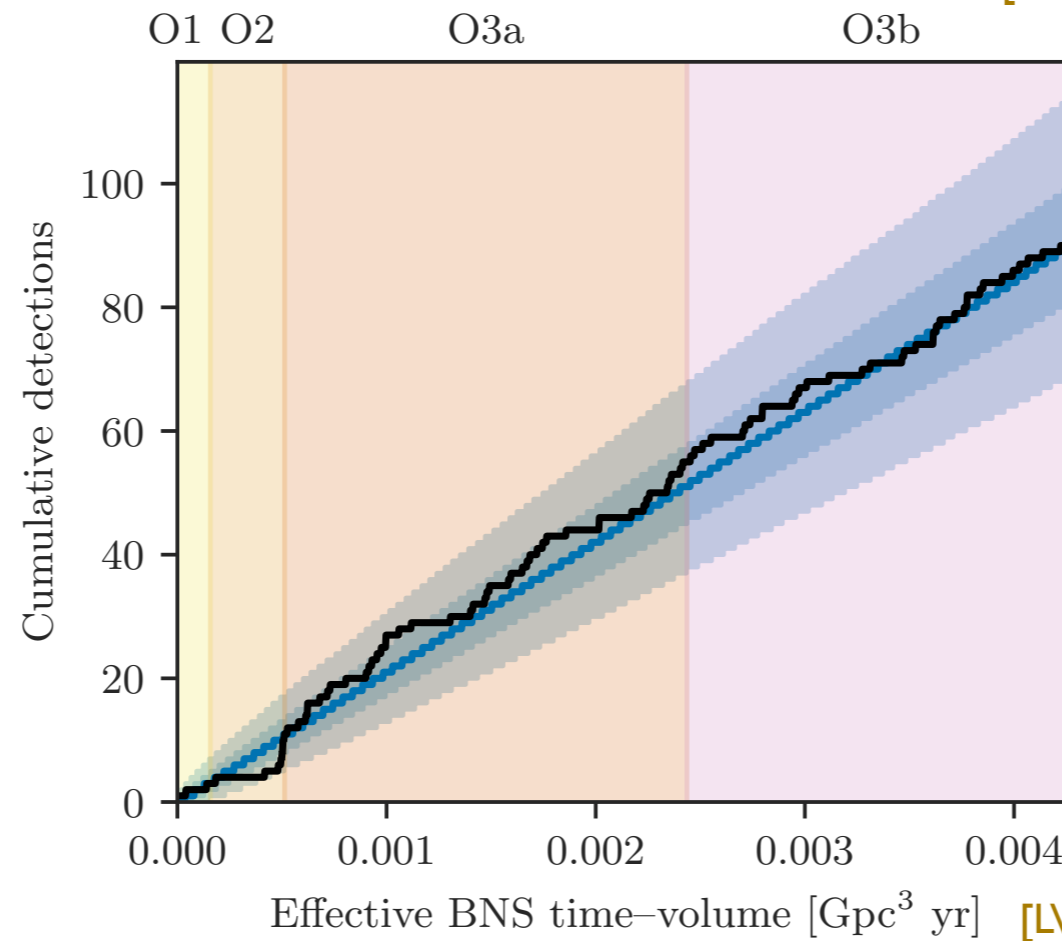
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Ground based detectors: timeline



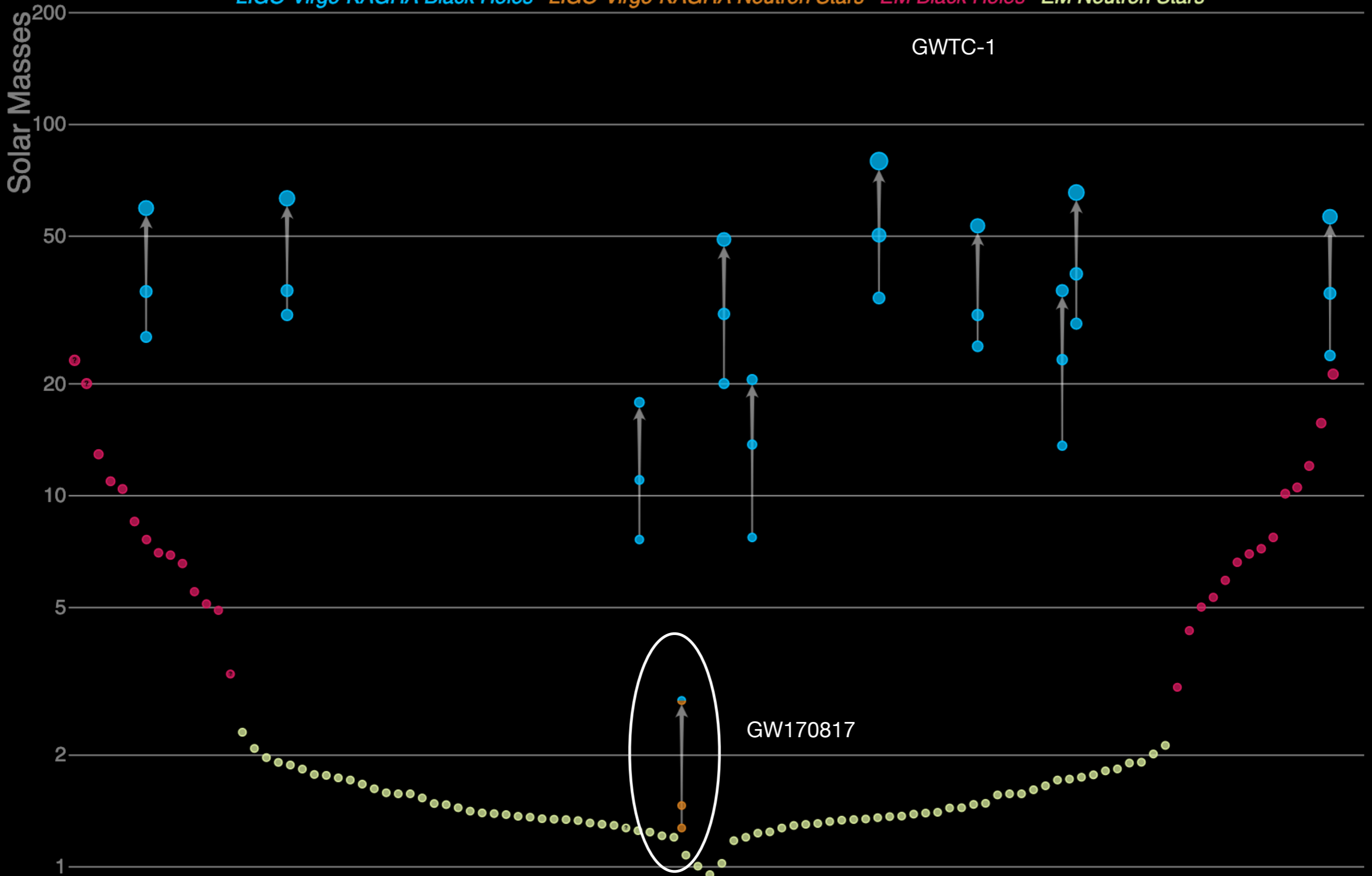
[IGWN 2022]



[LVK 2021]

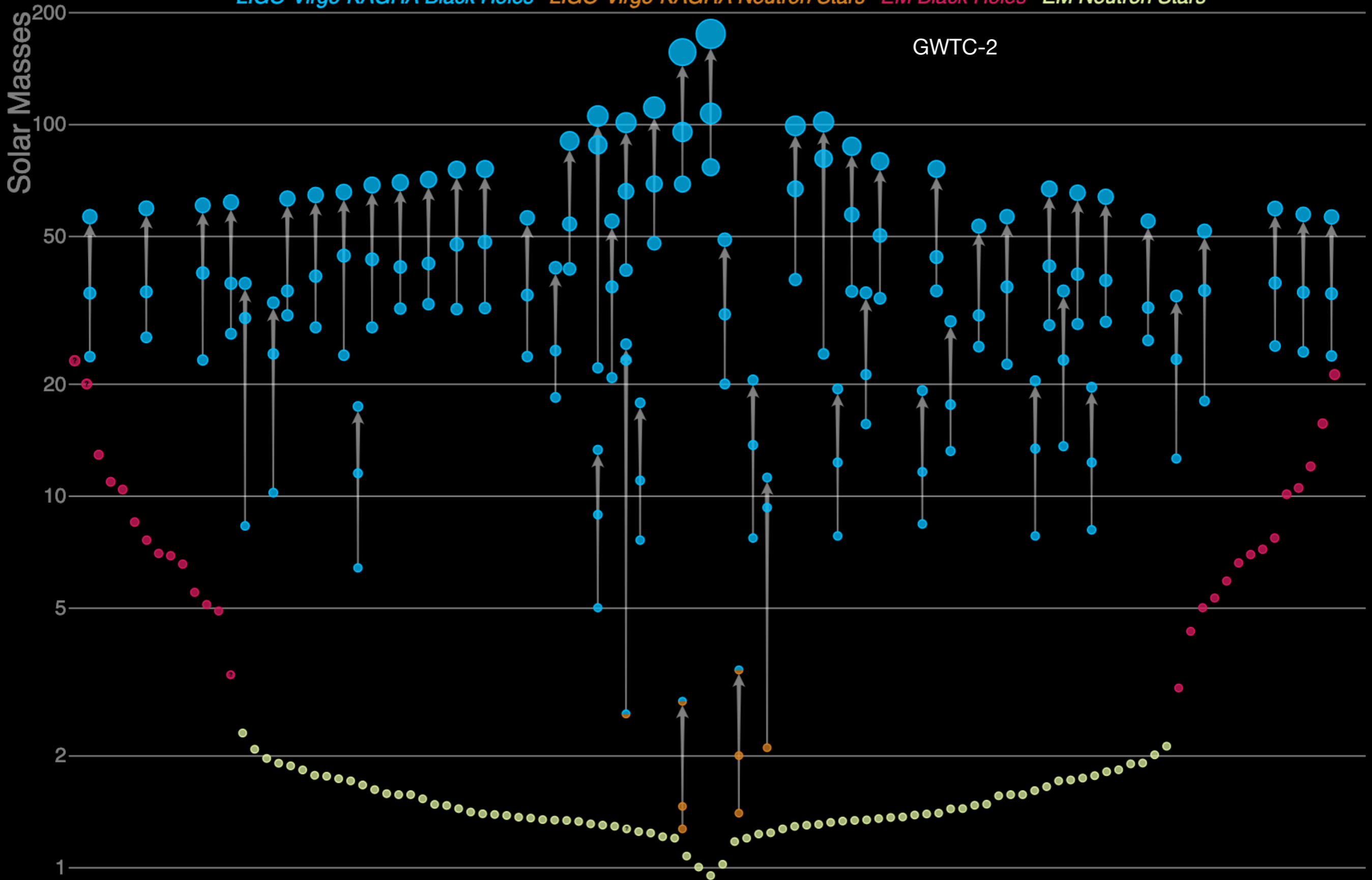
Masses in the Stellar Graveyard

LIGO-Virgo-KAGRA Black Holes *LIGO-Virgo-KAGRA Neutron Stars* *EM Black Holes* *EM Neutron Stars*



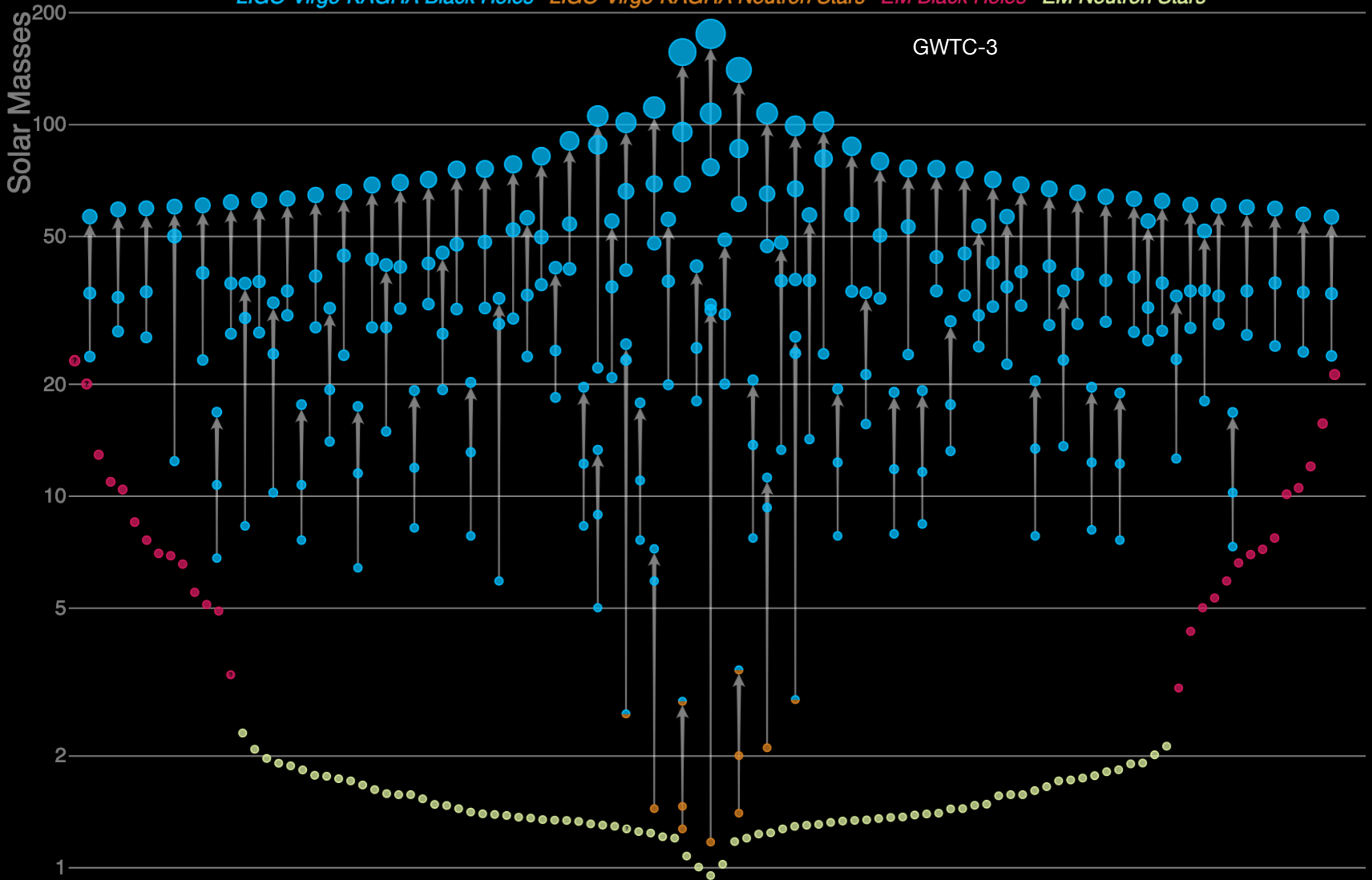
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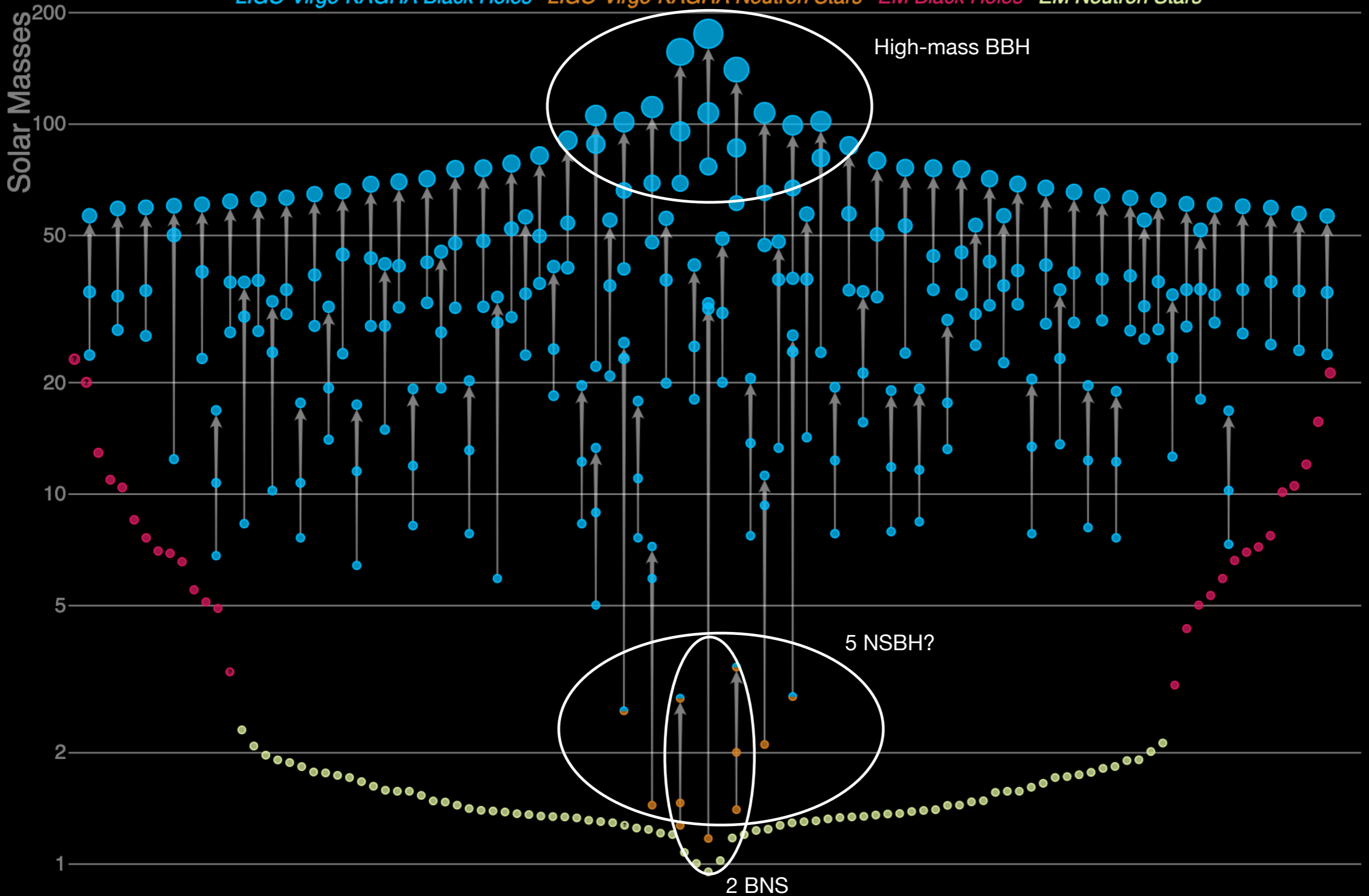
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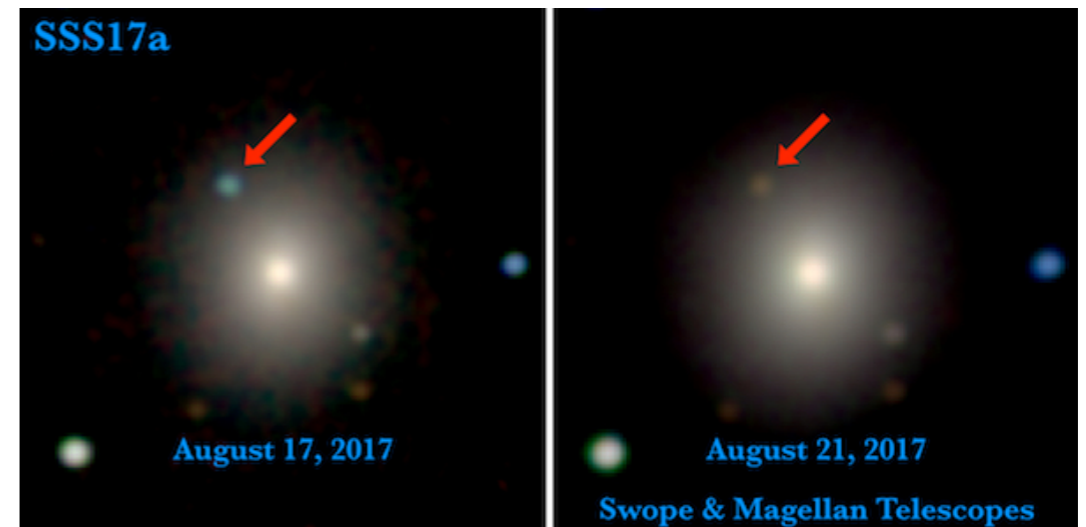
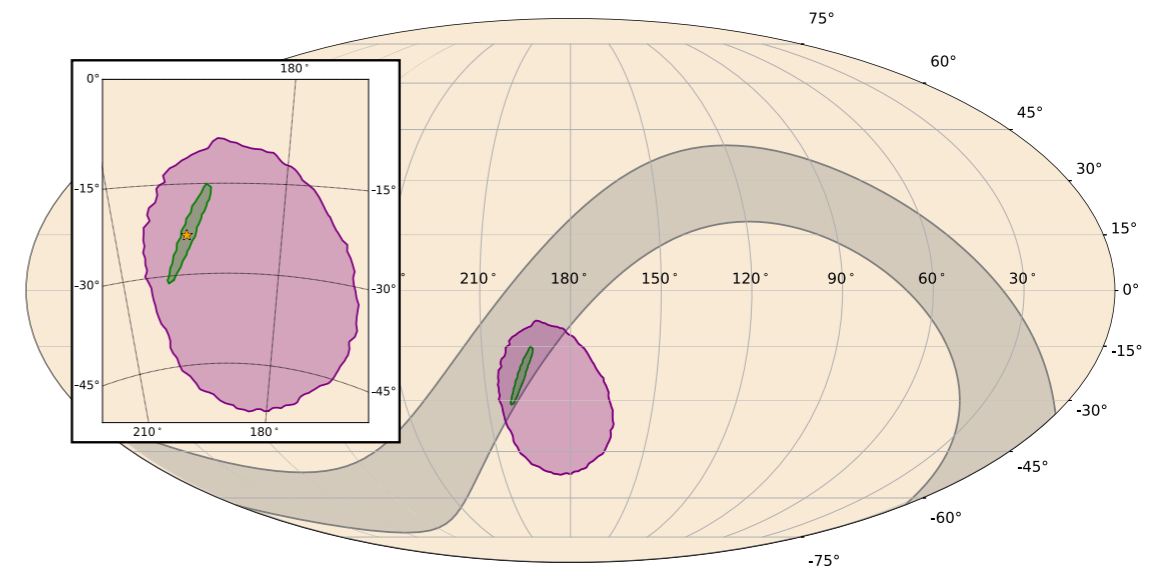
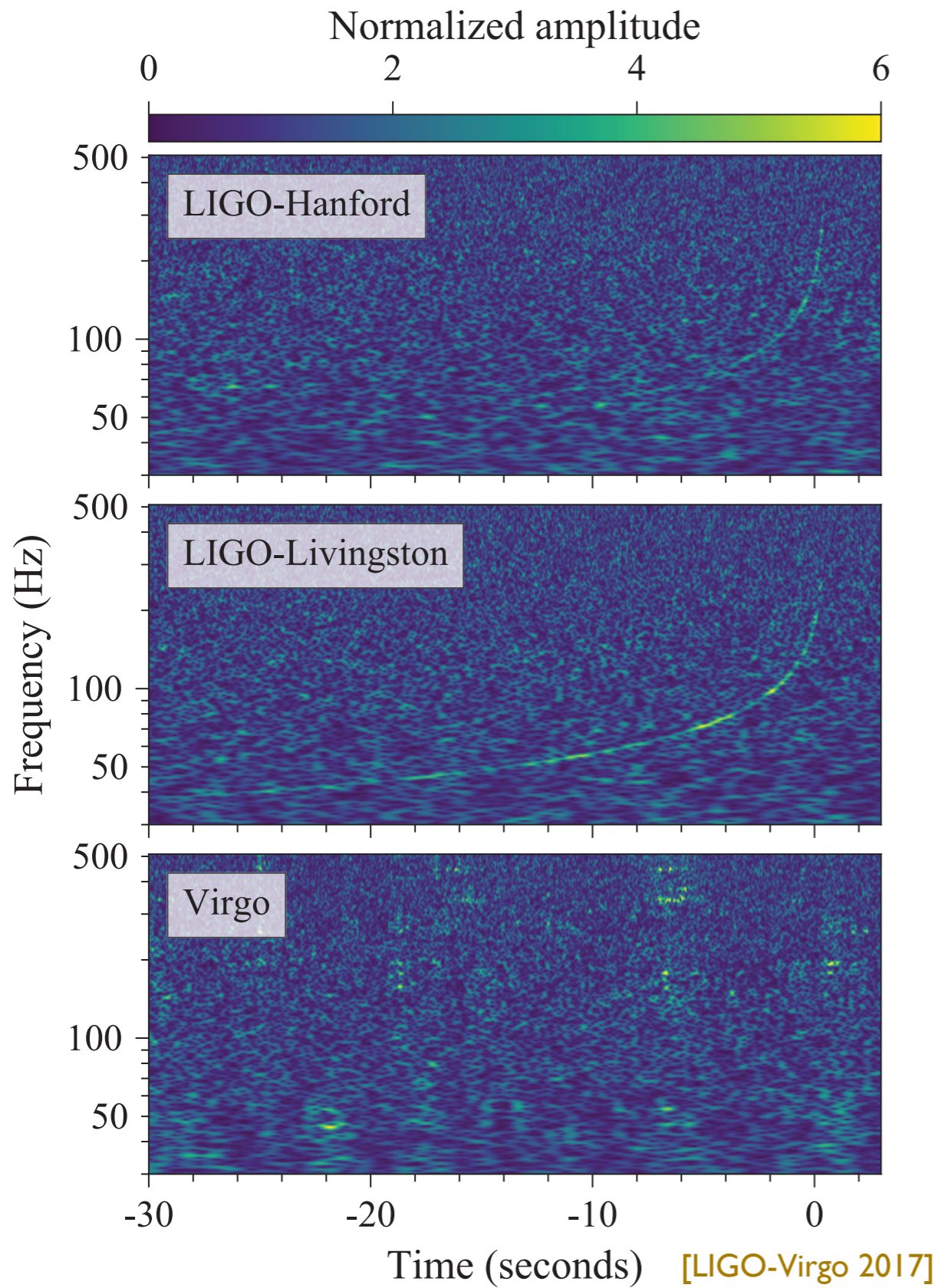
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Overview

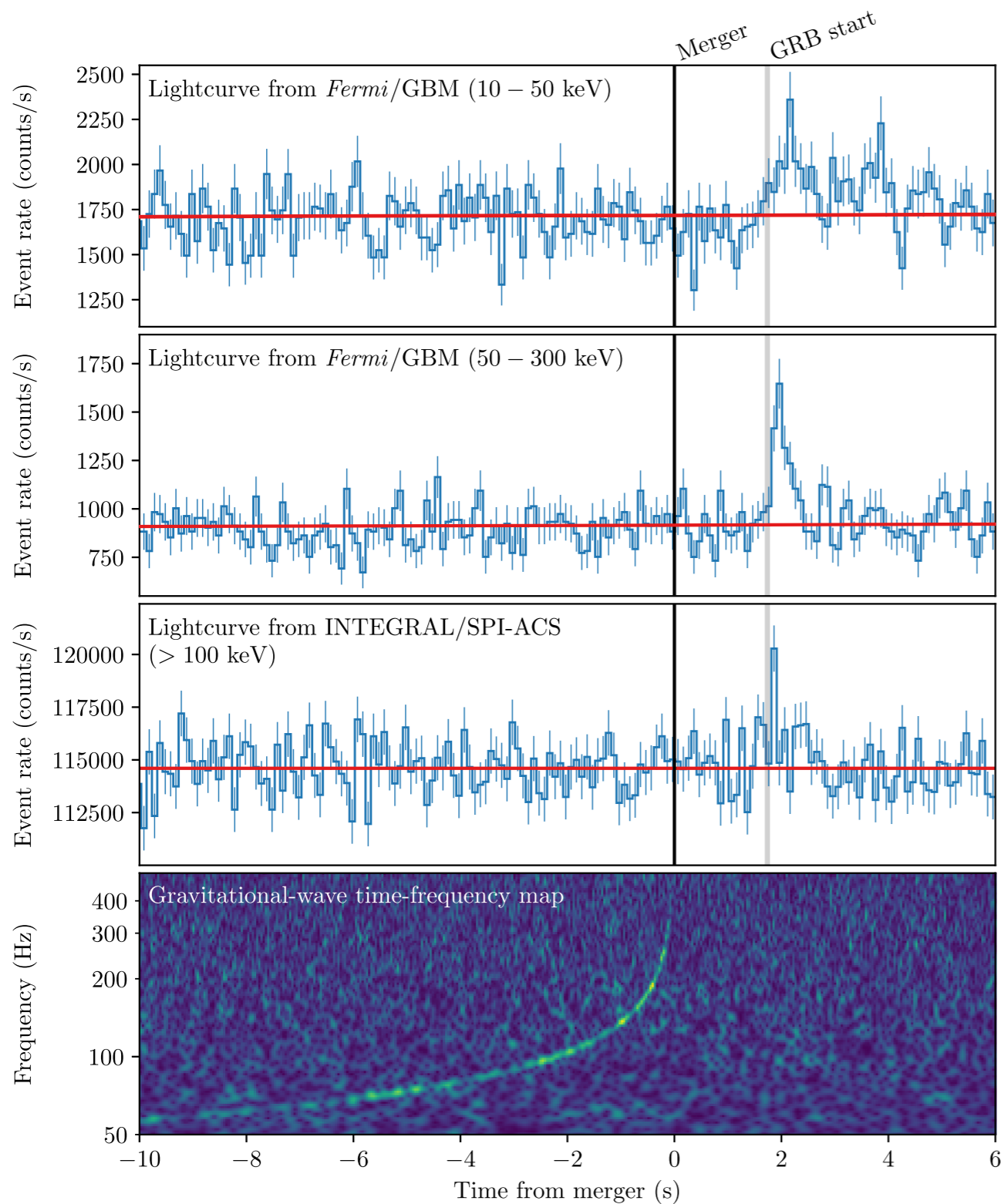
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GW170817: a GW signal with EM counterpart



[Santa Cruz and Carnegie Observatories/Ryan Foley]

GW170817: measuring the speed of gravitational waves



GRB signal received with
~1.7s delay

Assuming a ~10s delay in
the original emission:

$$-3 \times 10^{-15} \leq \frac{\Delta v}{v_{EM}} \leq +7 \times 10^{-16}.$$

(Longer emissions delays are
possible but not favored)

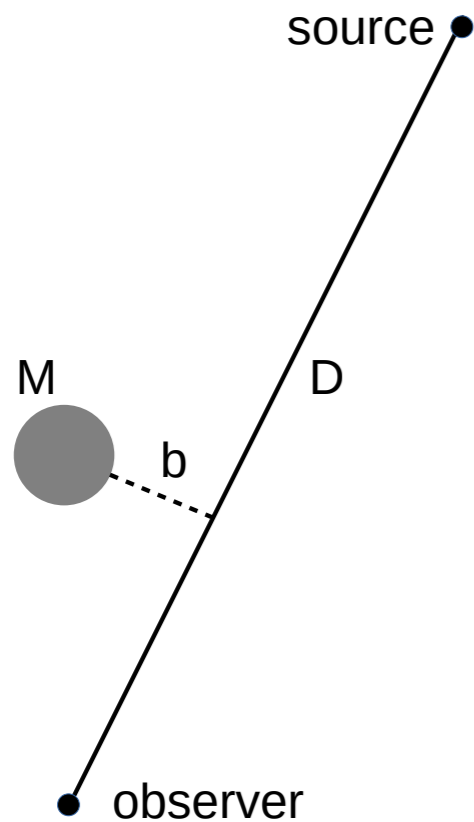
GW170817: consequences of the speed measurement

The constraint $c_g = c$ excludes wide regions of theory space

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/ $f(R)$ [47, 48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)\cdot$ Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

[Ezquiaga&al 2017]

GW170817: Shapiro time delay test



Shapiro time delay:
propagation delay

$$\delta t_S = -\frac{1 + \gamma}{c^3} \int_{r_e}^{r_o} U(\mathbf{r}(l)) dl.$$

For a spherical mass distribution, in terms of the PPN coefficient γ :

$$\Delta t_{\text{Shapiro}}^a = (1 + \gamma_a) \frac{GM}{c^3} \ln \left(\frac{D}{b} \right)$$

For GR, $\gamma_{\text{GW}} = \gamma_{\text{EM}} = 1$

Contribution of the mass of NGC4993 and the Milky Way: [LIGO-Virgo, Fermi, INTEGRAL 2017]

$$-2.6 \times 10^{-7} \leq \gamma_{\text{GW}} - \gamma_{\text{EM}} \leq 1.2 \times 10^{-6}$$

Taking into account dark matter profile: [Boran&al 2017]

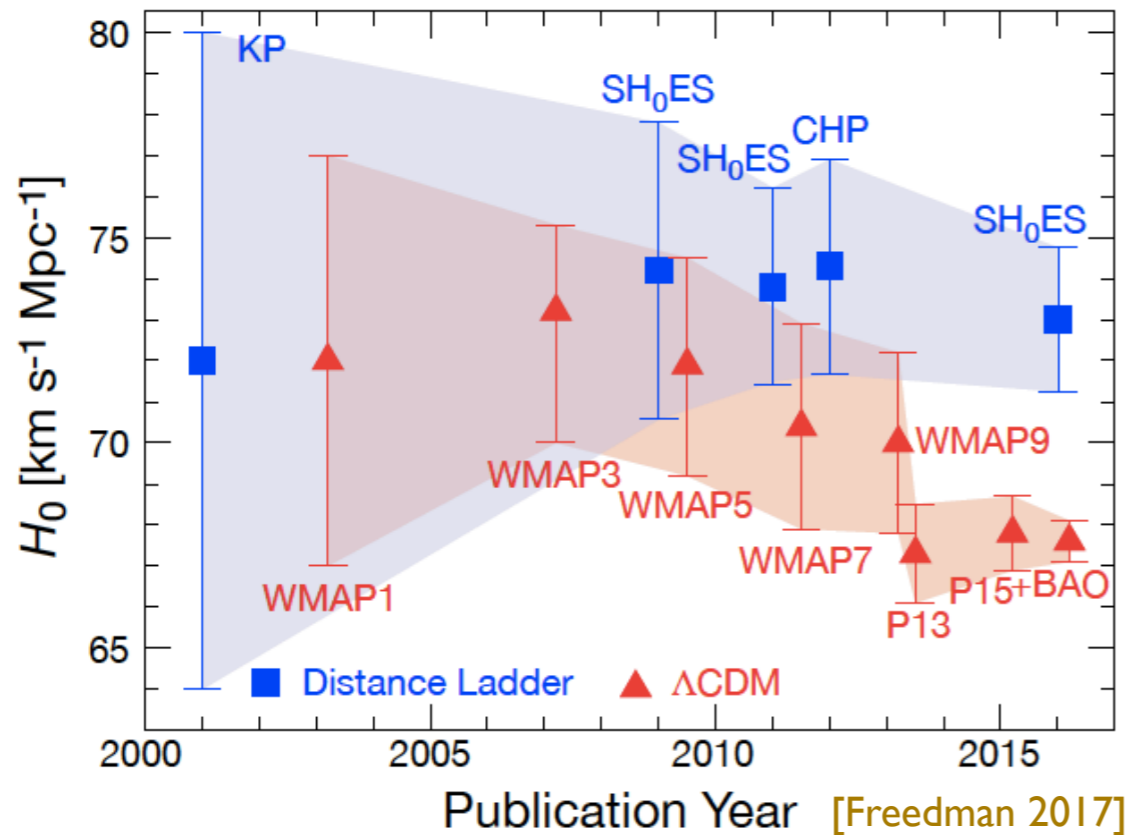
$$|\gamma_{\text{GW}} - \gamma_{\text{EM}}| < 9.8 \times 10^{-8}$$

Best absolute bound so far from Cassini: [Bertotti&al 2003]

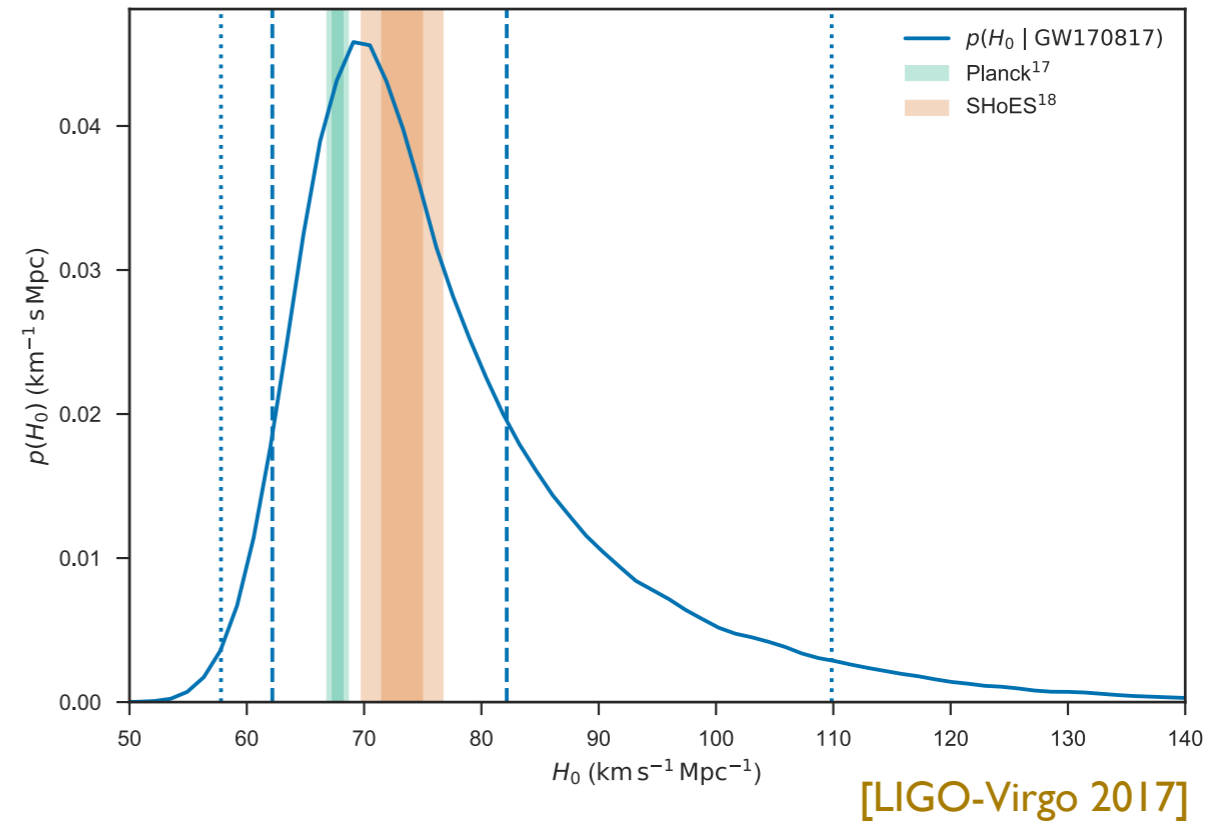
$$\gamma_{\text{EM}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

GW170817: a new measure of H_0

The Hubble tension

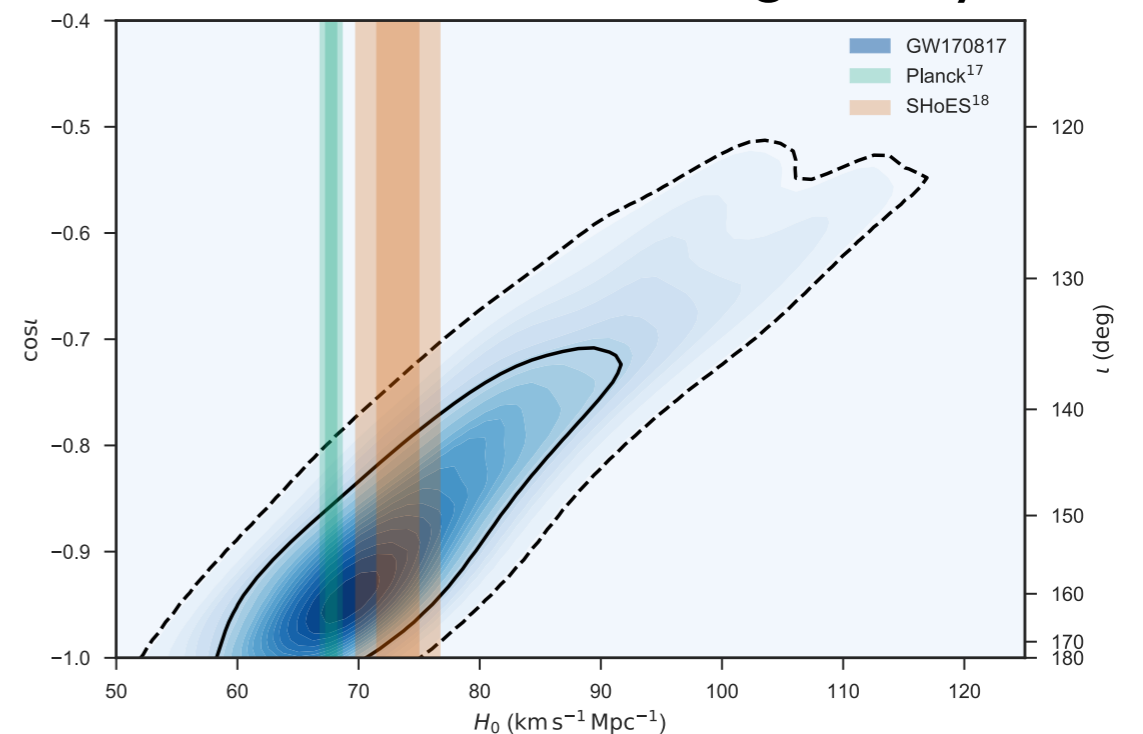


GW170817



Gravitational waves as standard sirens:
 direct access to the luminosity distance.
 Different systematics !
 Bright siren: presence of an EM
 counterpart measuring z

Distance-inclination degeneracy



GW170817: constraining leakage in extra dimensions

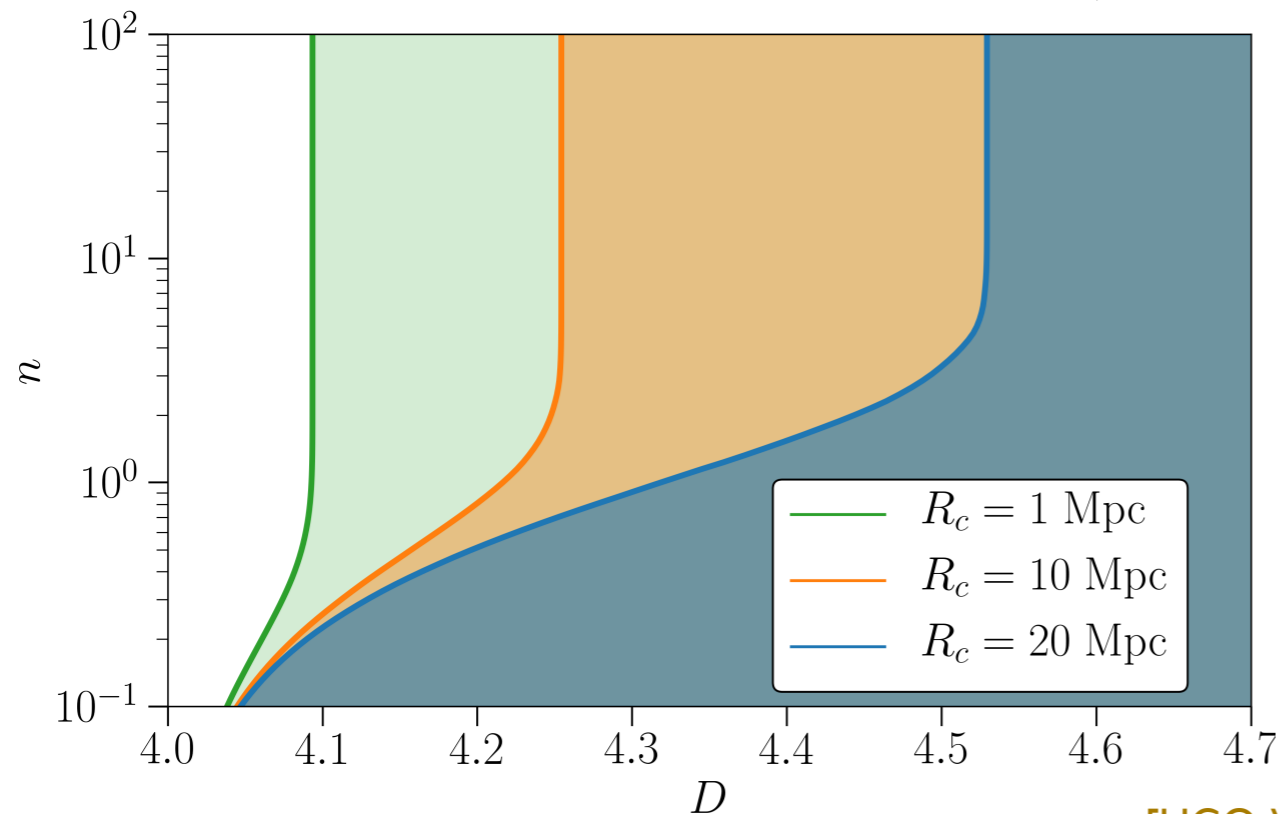
Motivation: consider the propagation of gravitational degrees of freedom in extra dimensions $D > 4$ on large scales

Phenomenological ansatz:

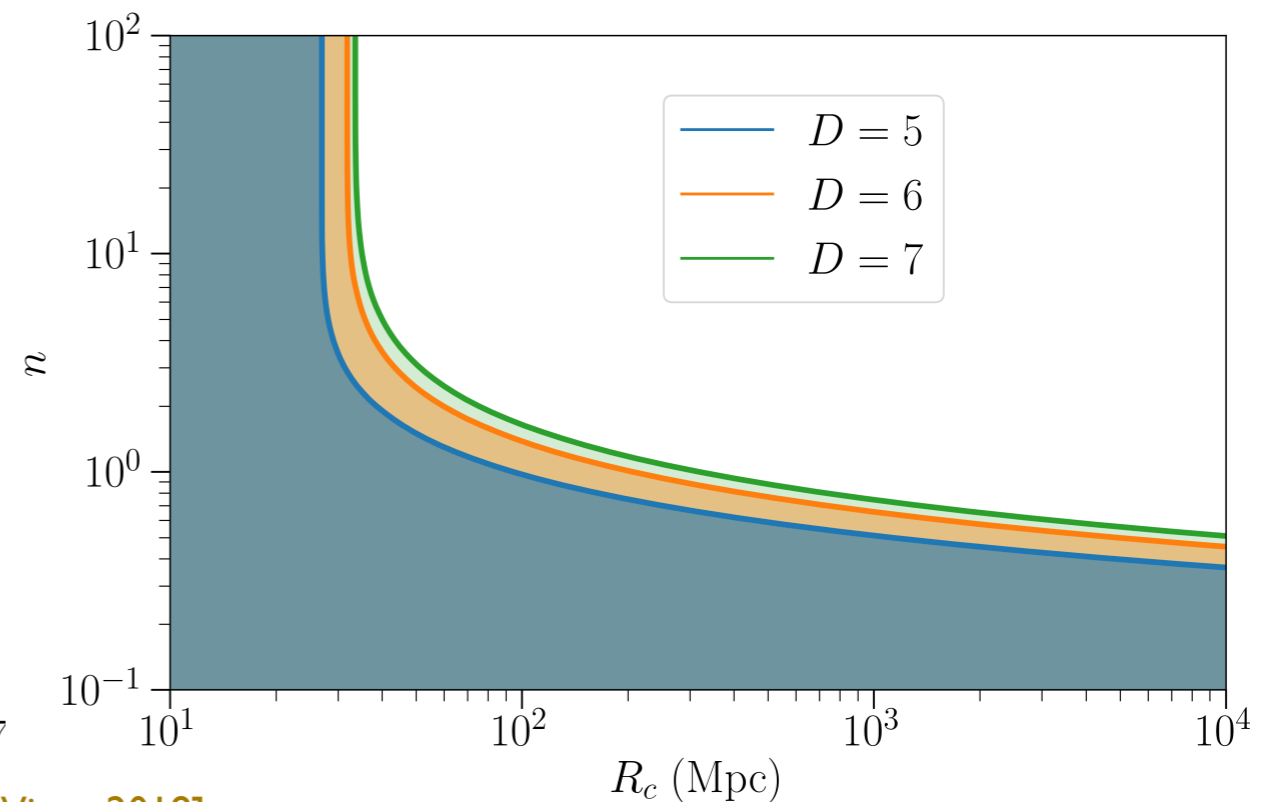
$$h \propto \frac{1}{d_L^{\text{GW}}} = \frac{1}{d_L^{\text{EM}}} \left[1 + \left(\frac{d_L^{\text{EM}}}{R_c} \right)^n \right]^{-(D-4)/(2n)} \quad \begin{array}{l} R_c \text{ transition distance} \\ n \text{ transition index} \end{array}$$

d_L^{GW} , d_L^{EM} both measured by GW170817 obs. (if we assume a cosmology).

GW170817 constraints in D, n



GW170817 constraints in R_c, n



[LIGO-Virgo 2018]

Testing this relation of luminosity distance also constrains scenarios with damping, oscillations.

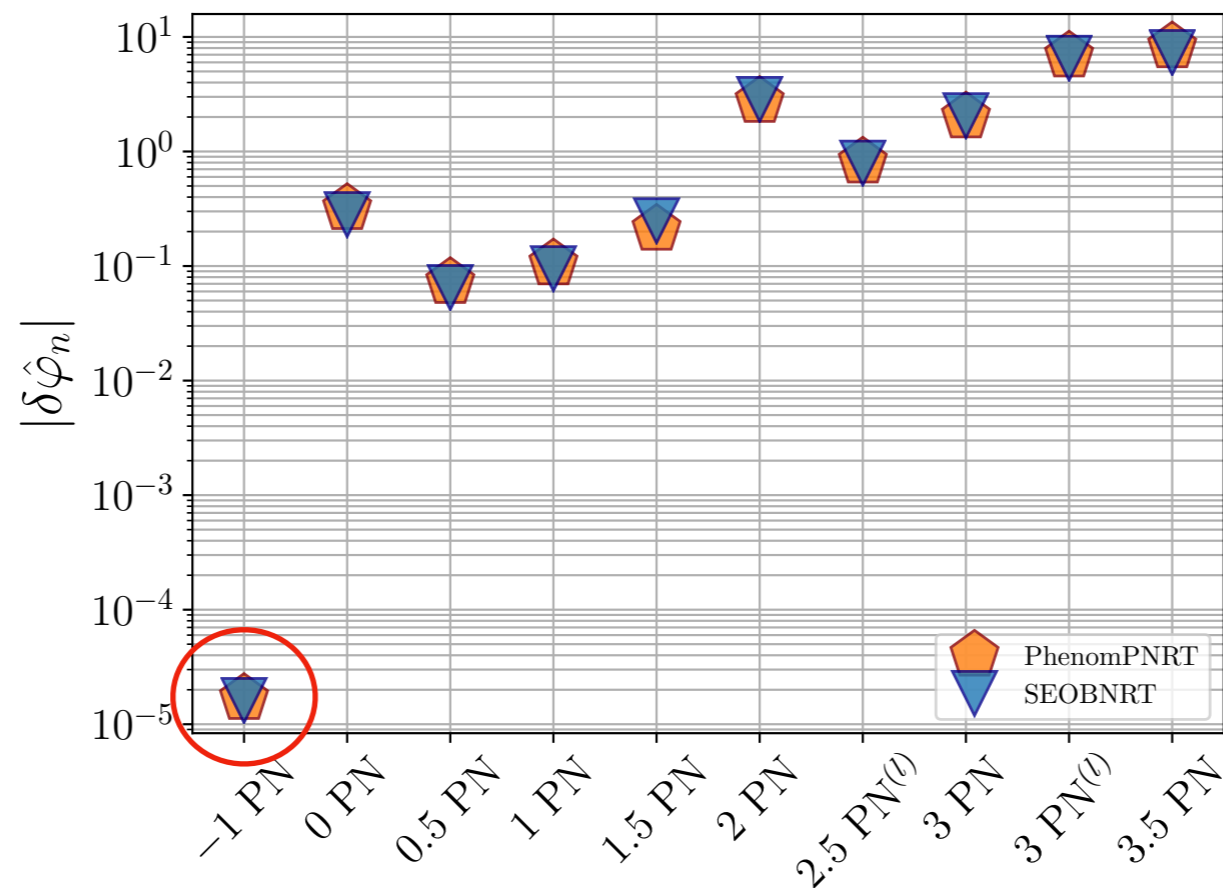
GW170817: constraining dipolar radiation

Dipolar radiation: present in particular for scalar-tensor theories

Fourier-domain phasing modified by the energy flux emitted in dipolar radiation:

$$\Psi(f) = 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128} u^{-5/3} \left(1 - \frac{4}{7} b \eta^{2/5} u^{-2/3} \right) \quad [\text{Will 1994}]$$

-IPN term in the phasing



[LIGO-Virgo 2018]

Binary pulsar constraints: $|\delta\hat{\varphi}_{-2}| < 3.4 \times 10^{-8}$

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GWTC-3: tests of GR from LVK GW detections

Data analysis aspects

Bayesian analysis: $p(\theta|d) = \frac{\mathcal{L}(d|\theta)p_0(\theta)}{p(d)}$

extend waveforms to include deviations, parametrized by new params $\theta \rightarrow (\theta, \alpha)$

Bayesian hypothesis testing: Bayes ratio

$$\mathcal{B} = \frac{p(d|\text{modGR})}{p(d|\text{GR})}$$

$$p(d|A) = \int d\theta p(d|\theta, A)p(\theta|A) \quad \text{evidence}$$

How to combine different events ?

- Fix a common value of the modGR param α for all events
- Hierarchical inference: introduce an underlying distribution, infer its hierarchical params — ex.: $\alpha \sim \mathcal{N}(\mu, \sigma)$, infer (μ, σ)

Events selection

Different tests require different selection thresholds (e.g. enough SNR in inspiral/merger)

Event	Inst.	Tests performed							
		RT	IMR	PAR	SIM	MDR	POL	RD	ECH
GW191109_010717	HL	✓	-	-	-	-	✓	✓	✓
GW191129_134029	HL	✓	-	✓	✓	✓	-	-	✓
GW191204_171526	HL	✓	-	✓	✓	✓	✓	-	✓
GW191215_223052	HLV	✓	-	-	-	✓	✓	-	✓
GW191216_213338	HV	✓	-	✓	✓	✓	✓	-	✓
GW191222_033537	HL	✓	-	-	-	✓	✓	✓	✓
GW200115_042309	HLV	✓	-	✓	-	-	-	-	✓
GW200129_065458	HLV	✓	✓	✓	✓	✓	✓	✓	✓
GW200202_154313	HLV	✓	-	✓	-	✓	-	-	✓
GW200208_130117	HLV	✓	✓	-	-	✓	✓	-	✓
GW200219_094415	HLV	✓	-	-	-	✓	✓	-	✓
GW200224_222234	HLV	✓	✓	-	-	✓	✓	✓	✓
GW200225_060421	HL	✓	✓	✓	✓	✓	✓	-	✓
GW200311_115853	HLV	✓	✓	✓	-	✓	✓	✓	✓
GW200316_215756	HLV	✓	-	✓	✓	-	-	-	✓

[LVK 2021]

Data quality (glitches) is crucial !

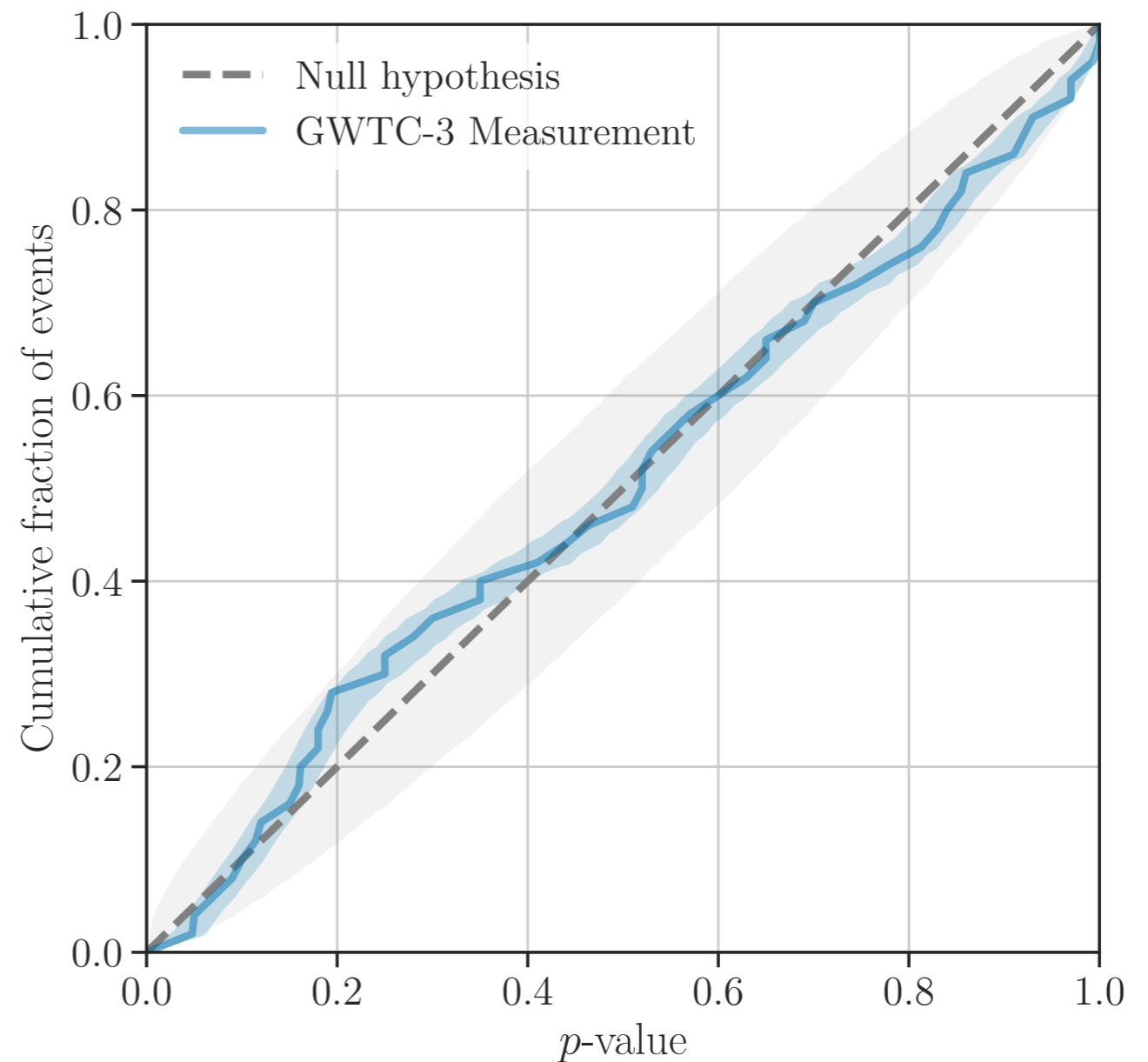
GWTC-3: residuals consistency test

Method:

- Subtract maximum-likelihood waveform (now with IMRPhenomXPHM)
- Build SNR90, measure of the SNR of the residual
- Compute the same SNR90 on noise time segments drawn from 4096s of data
- Check that the residuals p-value with respect to noise is uniform

Likelihood:

$$\mathcal{L}(\hat{p}) = \binom{N}{n} p^n (1 - p)^{N-n}$$



[LVK 2021]

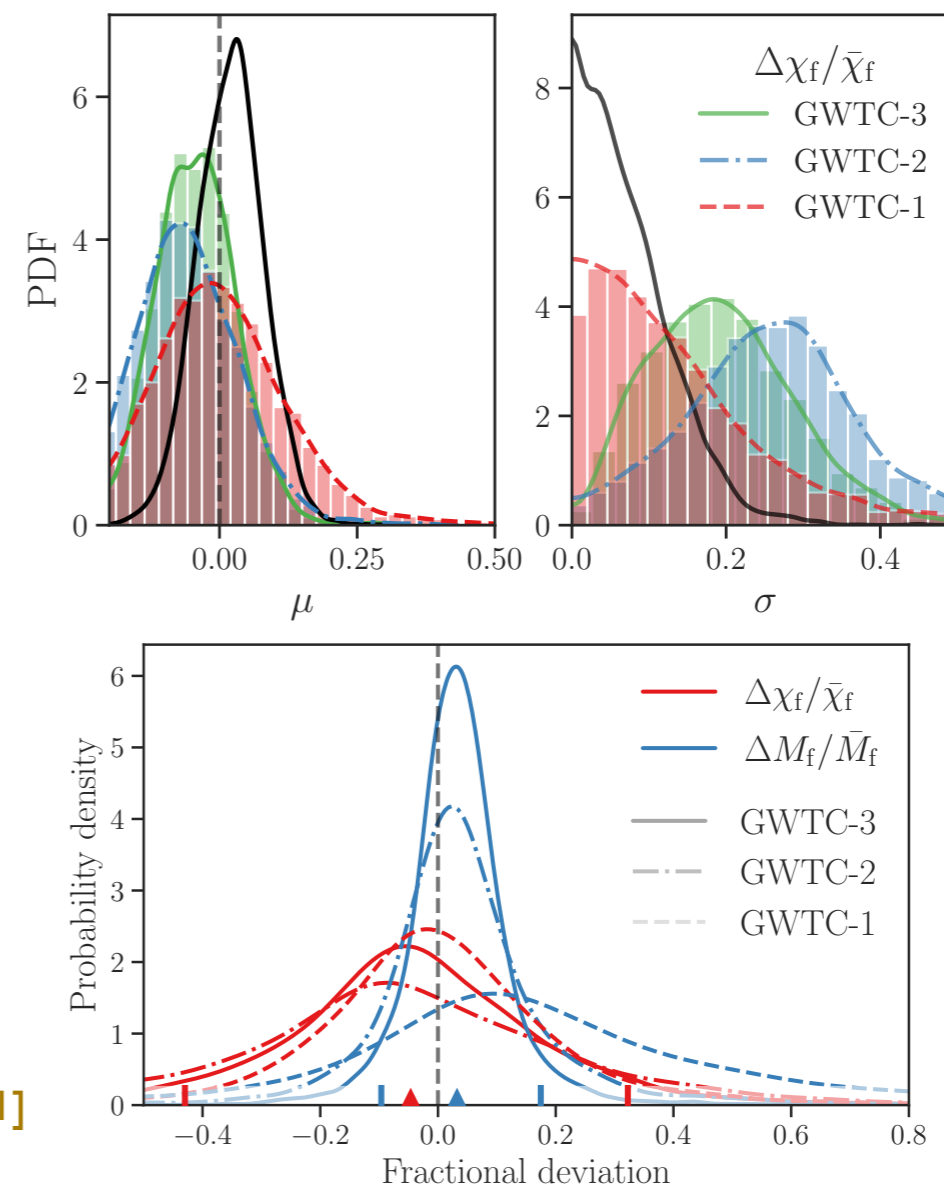
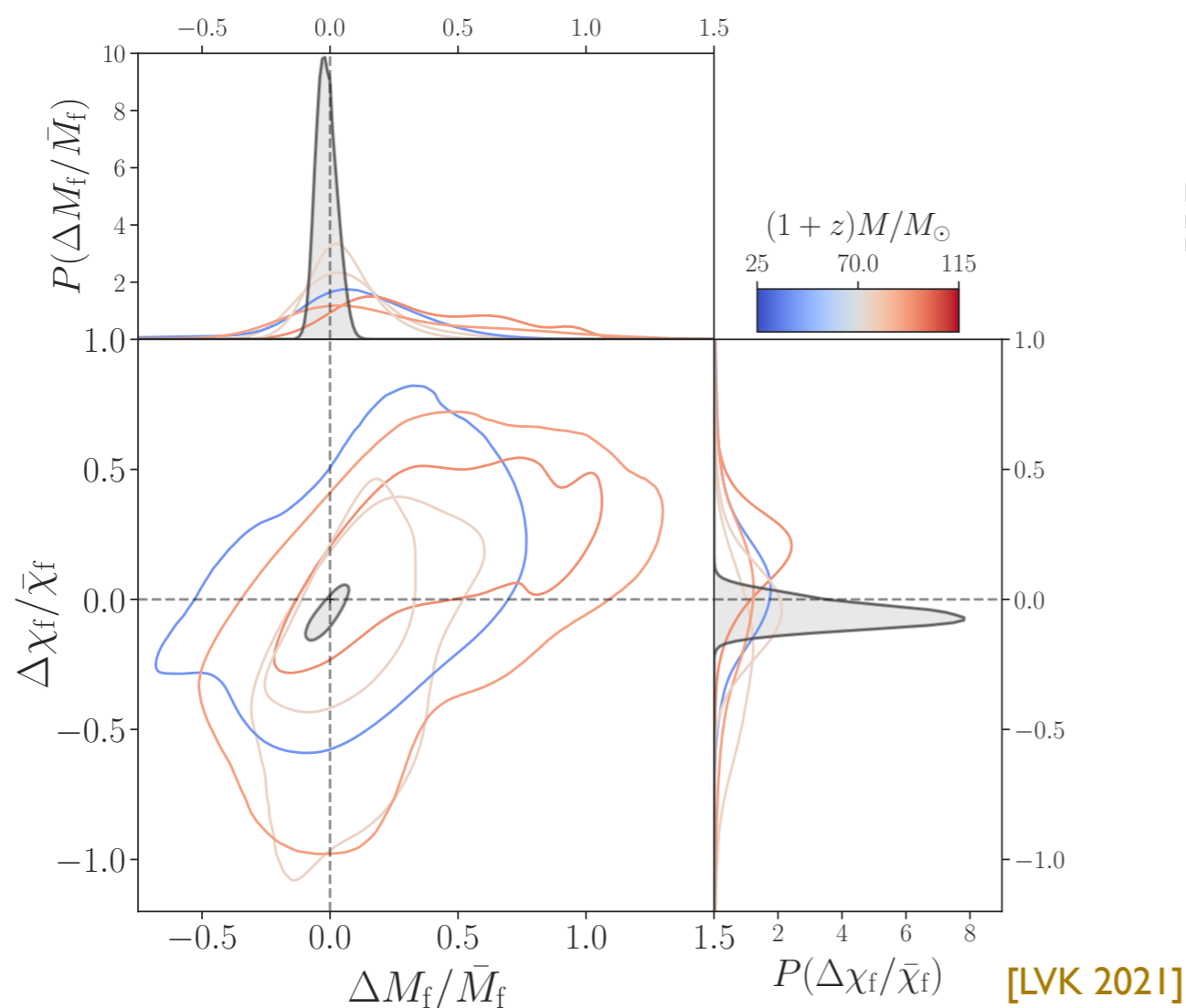
GWTC-3: IMR consistency test

Method:

- Separate inspiral and MR signal after Kerr ISCO (must have enough SNR in both)
- Perform PE on both
- Compute final mass and spin from NR fits for both

Check whether fractional deviations are compatible with 0:

$$\frac{\Delta M_f}{\bar{M}_f} = 2 \frac{M_f^{\text{insp}} - M_f^{\text{postinsp}}}{M_f^{\text{insp}} + M_f^{\text{postinsp}}}, \quad \frac{\Delta \chi_f}{\bar{\chi}_f} = 2 \frac{\chi_f^{\text{insp}} - \chi_f^{\text{postinsp}}}{\chi_f^{\text{insp}} + \chi_f^{\text{postinsp}}}$$



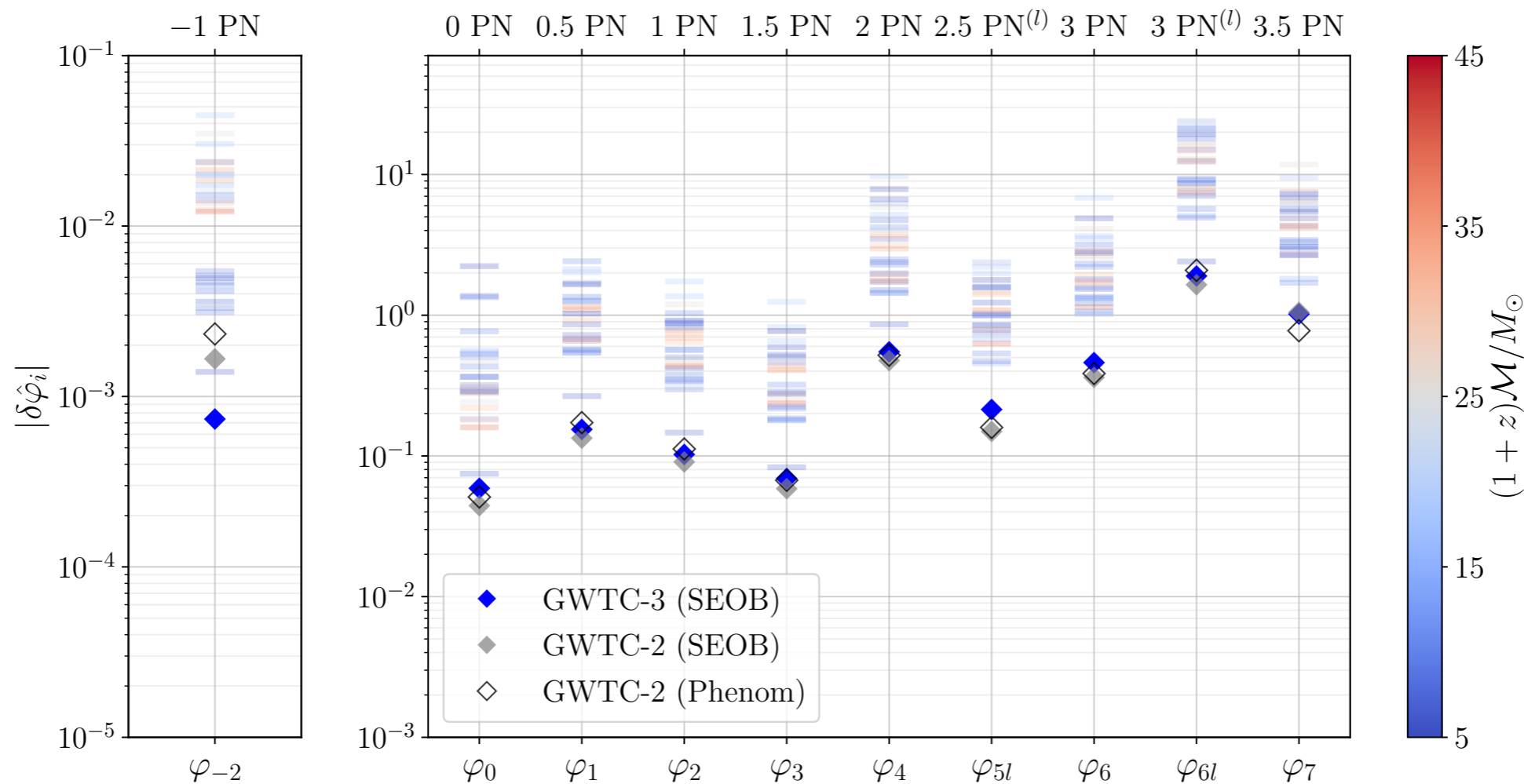
GWTC-3: generic inspiral modification

Modify PN coefficients in the inspiral part of the phasing (up to a where the effect is tapered)

$$\varphi_{\text{PN}}(f) = 2\pi f t_c - \varphi_c - \frac{\pi}{4} + \frac{3}{128\eta} (\pi \tilde{f})^{-5/3} \sum_{i=0}^7 \left[\varphi_i + \varphi_{il} \log(\pi \tilde{f}) \right] (\pi \tilde{f})^{i/3}$$

PN phasing coeffs (GR: functions of mass ratio, spin)

Set of coefficients: known PN coefficients + dipolar term $\{\delta\hat{\varphi}_{-2}, \delta\hat{\varphi}_0, \delta\hat{\varphi}_1, \delta\hat{\varphi}_2, \delta\hat{\varphi}_3, \delta\hat{\varphi}_4, \delta\hat{\varphi}_{5l}, \delta\hat{\varphi}_6, \delta\hat{\varphi}_{6l}, \delta\hat{\varphi}_7\}$



[LVK 2021]

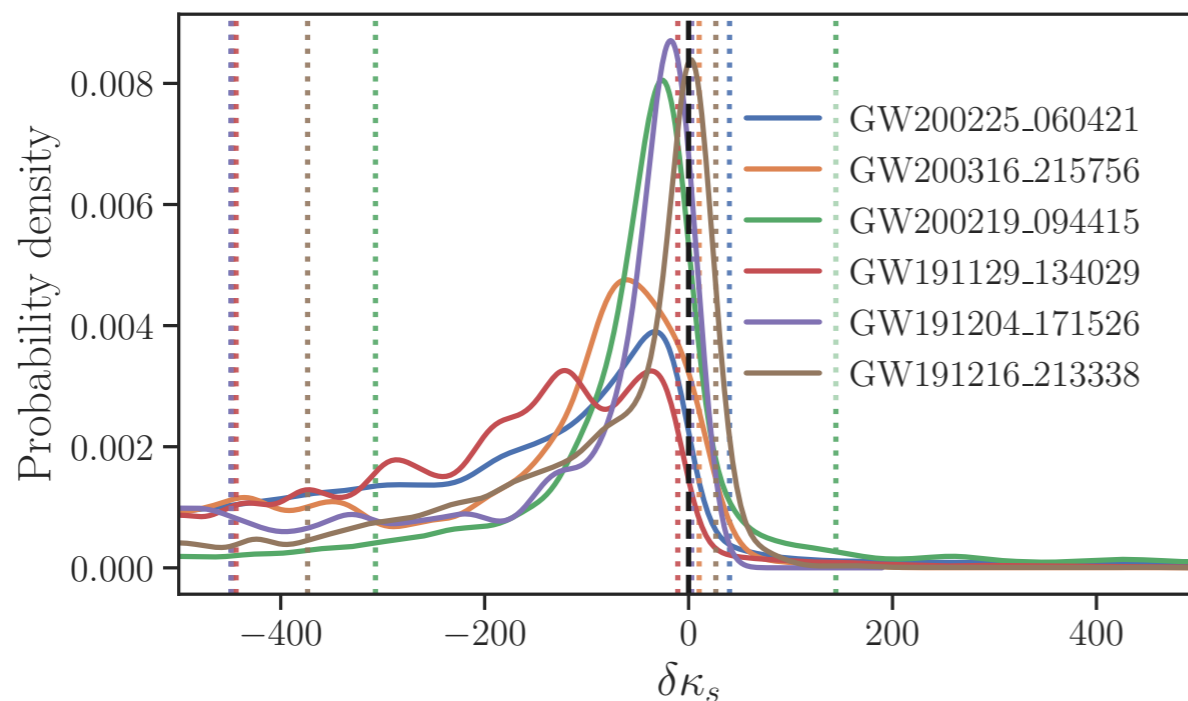
GWTC-3: spin-induced quadrupole test

Test of the multipolar structure of compact objects, as a modification of the inspiral phasing

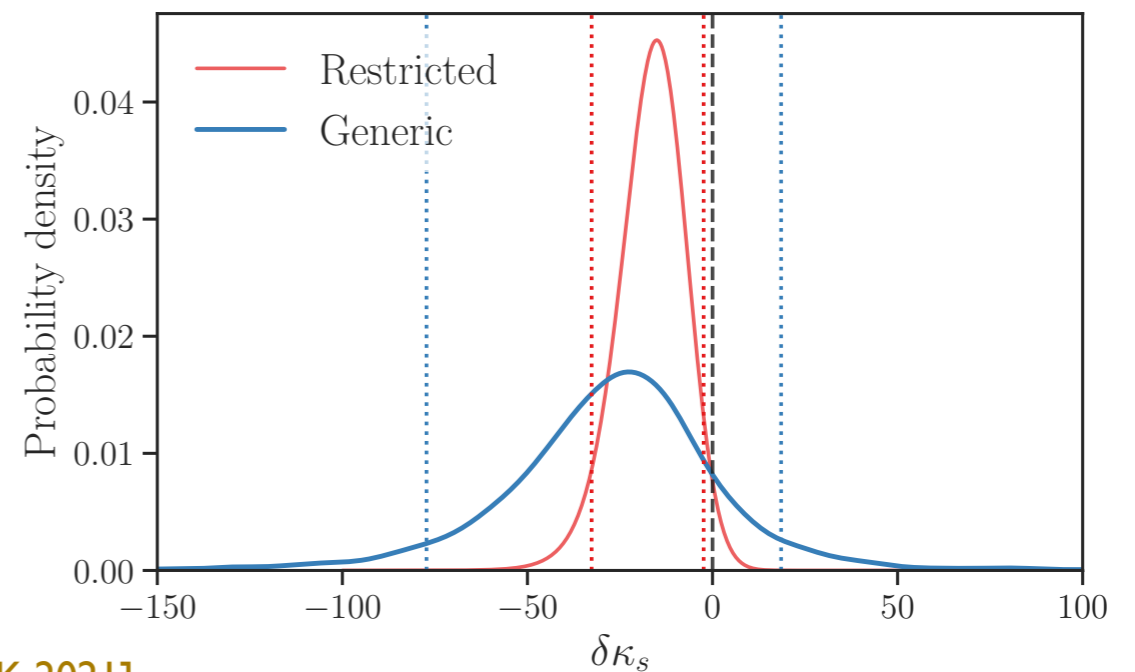
Spin-induced quadrupole: $Q = -\kappa \chi^2 m^3$

- For a Kerr black hole: $\kappa = 1$
- For a neutron star: $\kappa \simeq 2 - 14$
- Exotic compact objects (ECOs) ?

Leading order contribution at 2PN in the phasing, weak constraints for small spins.



[LVK 2021]



Here using only $\kappa_s = (\kappa_1 + \kappa_2)/2$

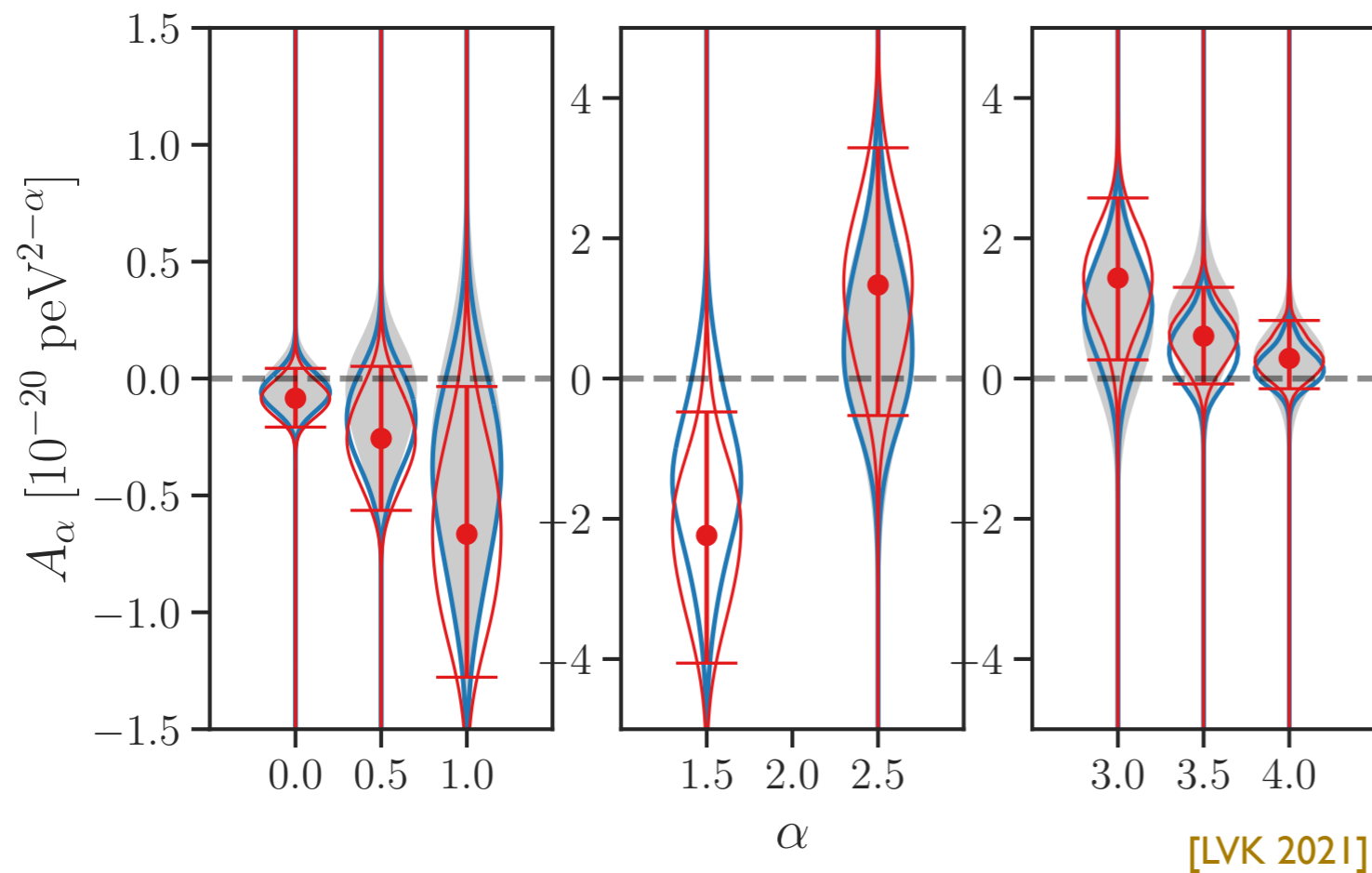
GWTC-3: dispersion relation

Test of the propagation of gravitational waves

Generalized dispersion relation considered: $E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha$

- Massive gravity: $\alpha = 0$ $\sqrt{A_0} = m_g c^2$
- Change in the speed of GW: $\alpha = 2$

Low and high frequencies propagate at different speeds (except $\alpha = 2$), leaves an imprint on the phasing of the wave



Graviton mass bound:

$$m_g \leq 1.27 \times 10^{-23} \text{eV}/c^2$$

Graviton mass bound from solar system tests:

$$m_g \leq 3.16 \times 10^{-23} \text{eV}/c^2$$

GWTC-3: ringdown tests

Test of the nature of coalescence remnants as Kerr black holes through their QNMs

Post-merger ringdown signal: spin-weighted spheroidal modes with overtones

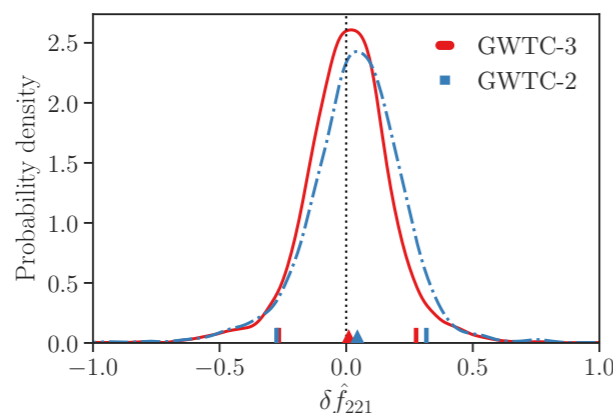
$$h_+(t) - ih_\times(t) = \sum_{\ell=2}^{+\infty} \sum_{m=-\ell}^{\ell} \sum_{n=0}^{+\infty} \mathcal{A}_{\ell mn} \exp\left[-\frac{t-t_0}{(1+z)\tau_{\ell mn}}\right] \exp\left[-\frac{2\pi i f_{\ell mn}(t-t_0)}{1+z}\right] {}_{-2}S_{\ell mn}(\theta, \phi, \chi_f)$$

QNM frequencies and damping times functions of the remnant mass and spin in GR:

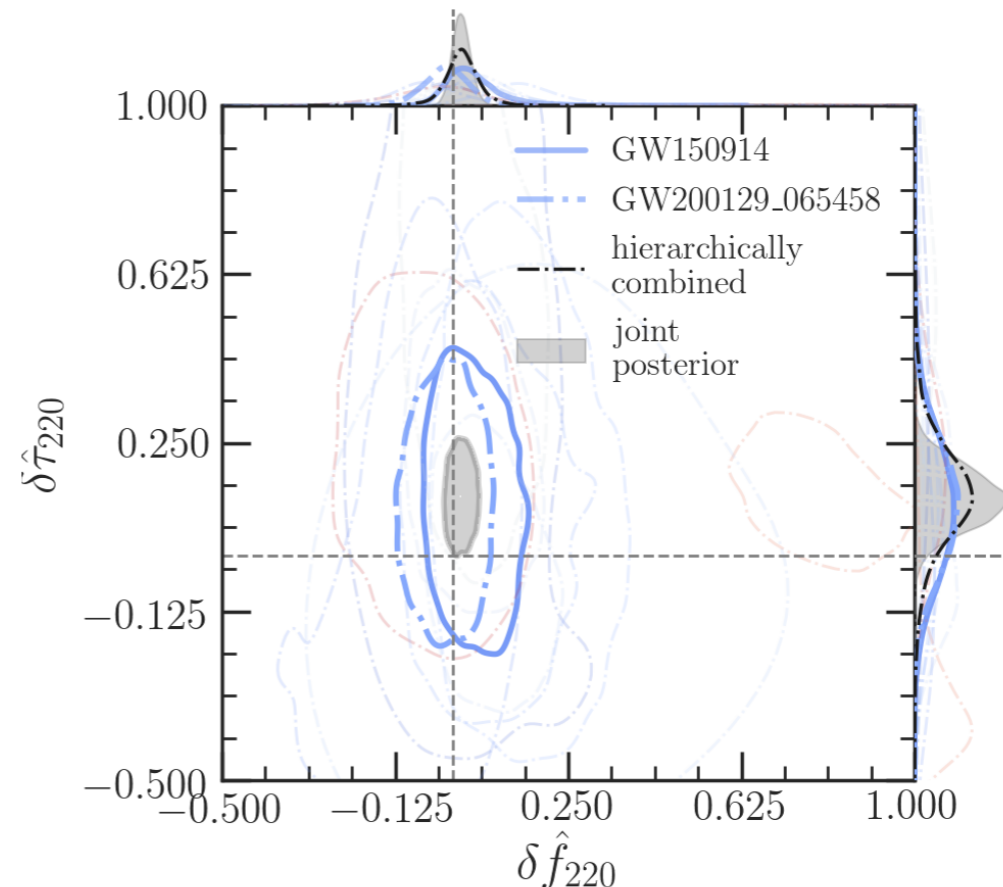
$$f_{\ell mn}(M_f, \chi_f), \tau_{\ell mn}(M_f, \chi_f)$$

- Method:
- Analyze ringdown-only signal, consider HM, overtones, mod GR (pyRing)
 - Modify QNMs inside an IMR waveform model (pSEOBNRv4HM)

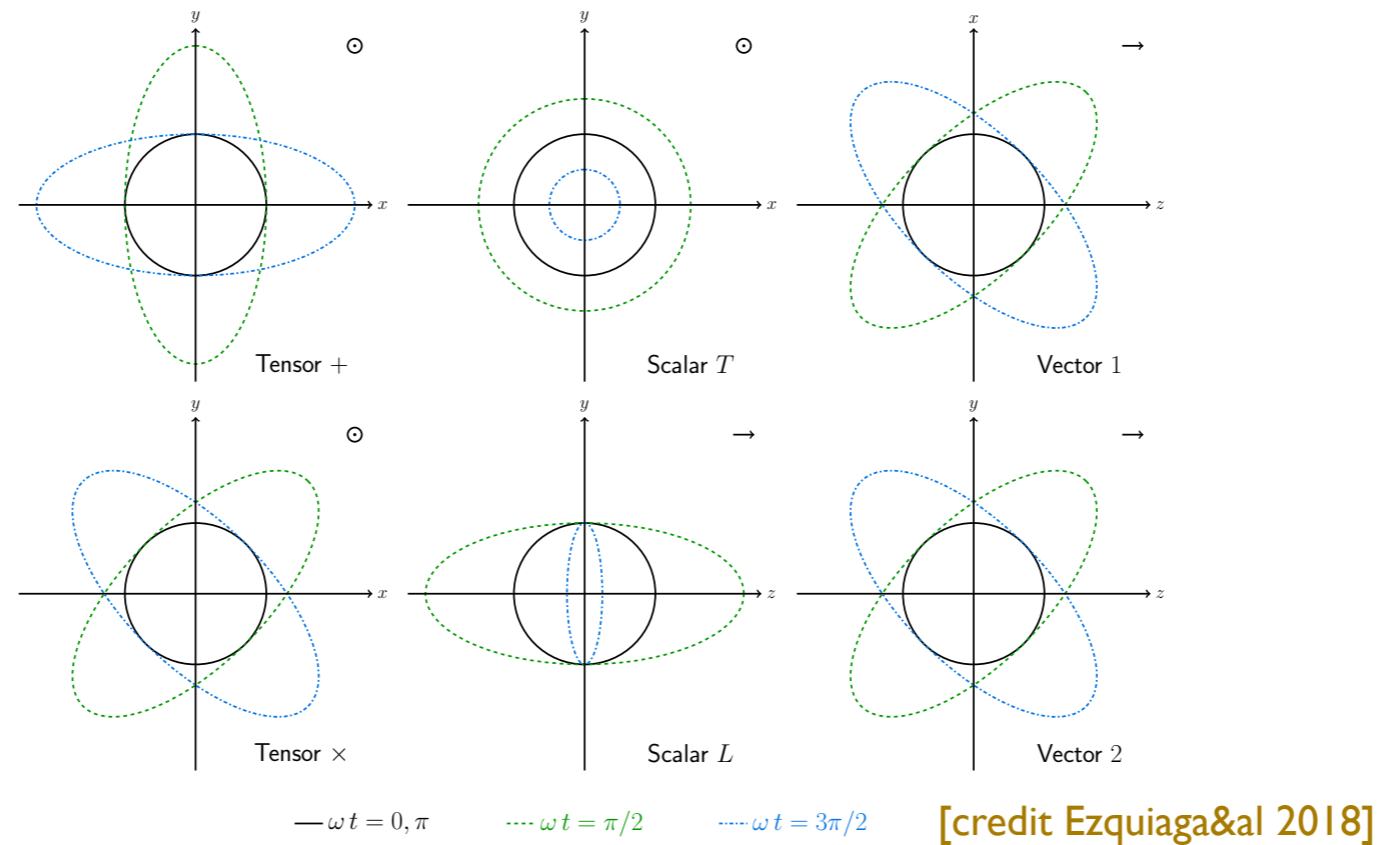
Event	Higher modes	Overtones	
	$\log_{10} \mathcal{B}_{220}^{\text{HM}}$	$\log_{10} \mathcal{B}_{220}^{221}$	$\log_{10} \mathcal{O}_{\text{GR}}^{\text{modGR}}$
GW191109_010717	-0.11	1.03	-0.27
GW191222_033537	0.08	-0.83	-0.20
GW200129_065458	-0.00	-0.47	-0.09
GW200224_222234	0.20	0.95	-0.11
GW200311_115853	0.02	-1.16	-0.15



[LVK 2021]



GWTC-3: polarization content of GW



Different methods:

- GR templates, Bayes ratio for full-vector and full-tensor
- GWTC-2: agnostic null-stream construction
- GWTC-3: extension of the null-stream construction for 2-detector events

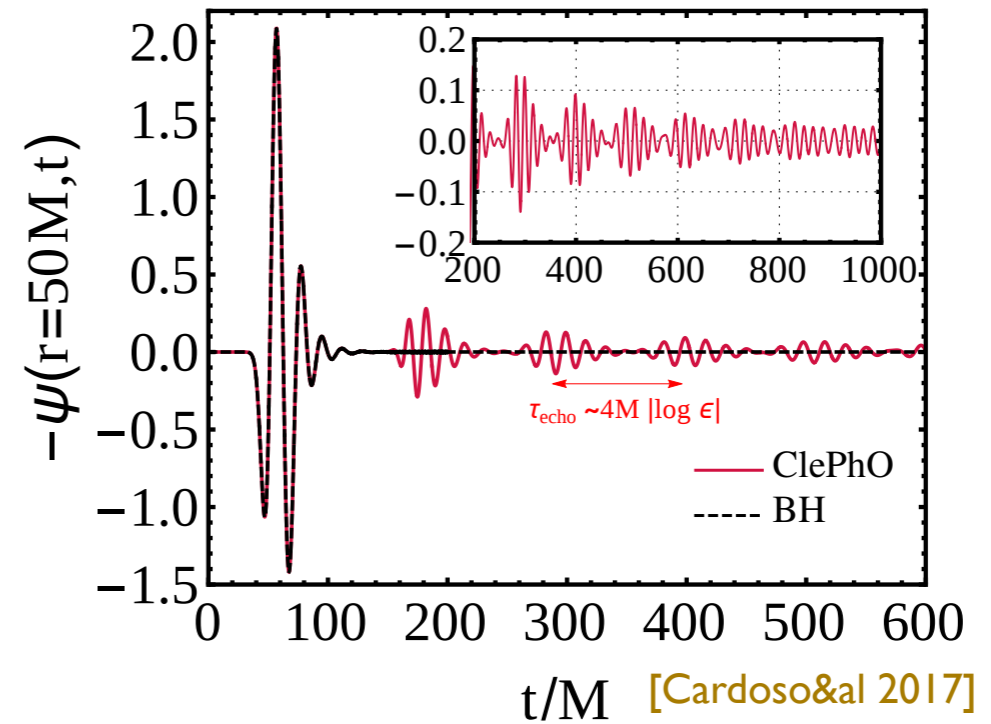
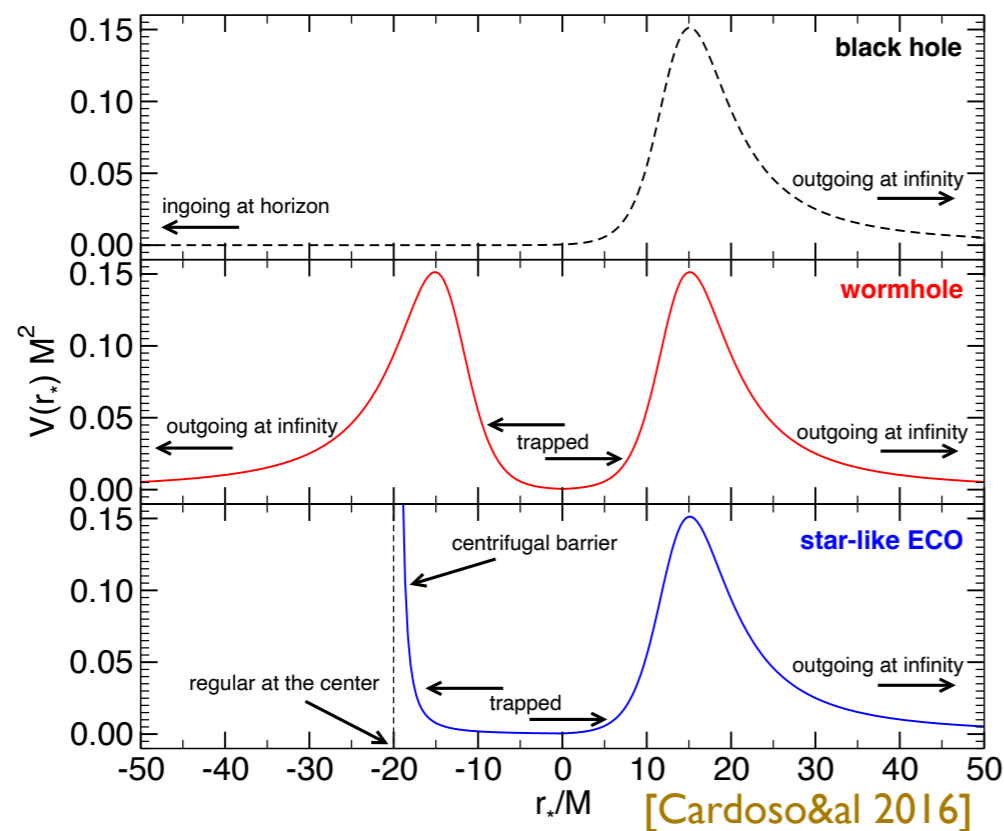
Best constraints: GW170817,
known location

Full-vector and tensor
disfavoured at $\mathcal{B} \sim 20$

Very dependent on the number and
orientation of detectors !

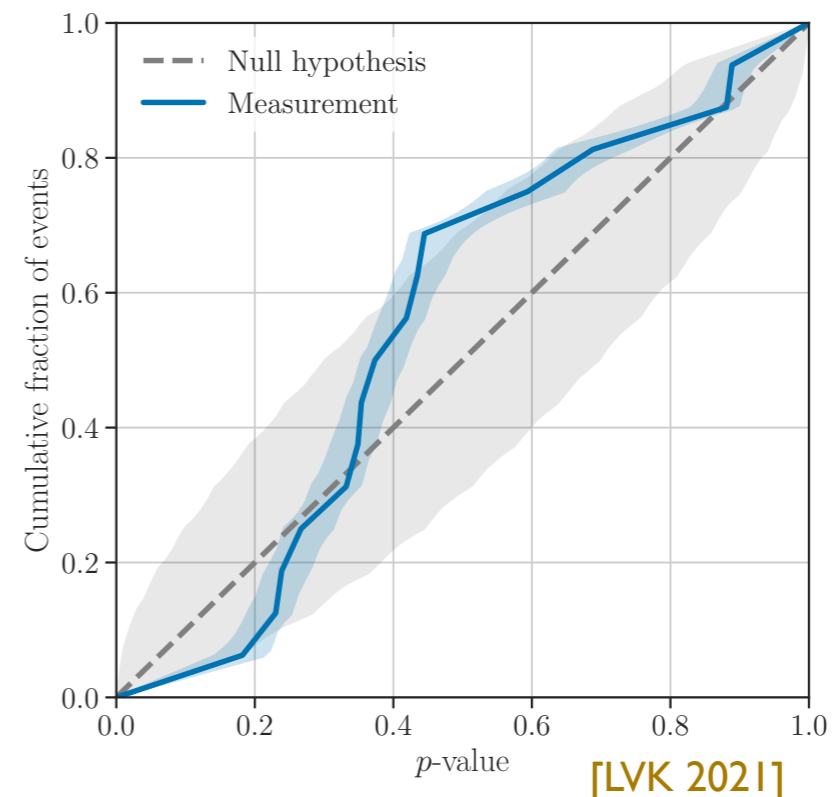
GWTC-3: echoes

GW echoes are generic for horizonless compact objects, have been proposed as smoking gun for Planck-scale structures at the horizon, wormholes, ECOs



Method:

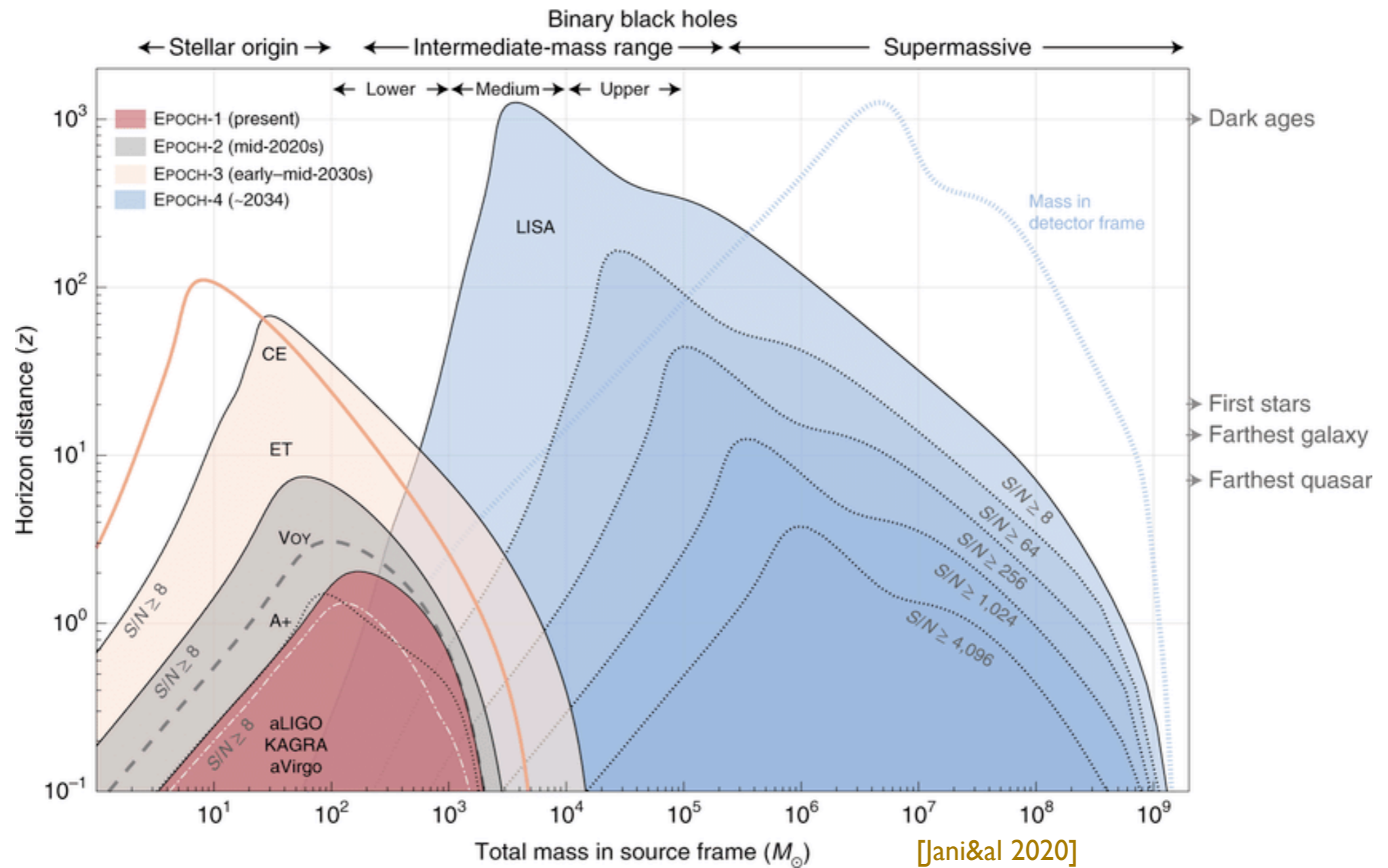
- GWTC-2: morphology-dependent as in [Abedi&al 2017]
- GWTC-3: morphology-independent, superposition of decaying repeating sine-Gaussians, compare the Bayes ratio of signal vs noise to the background



Overview

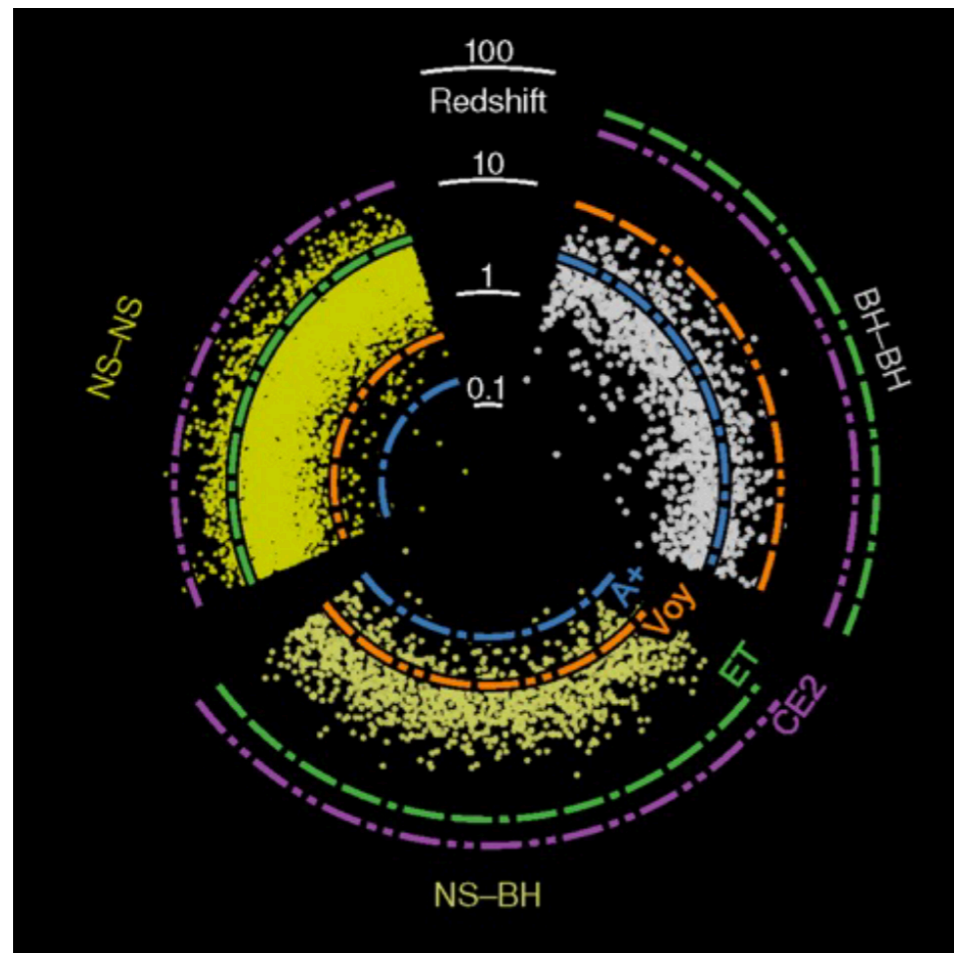
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The future of GW science

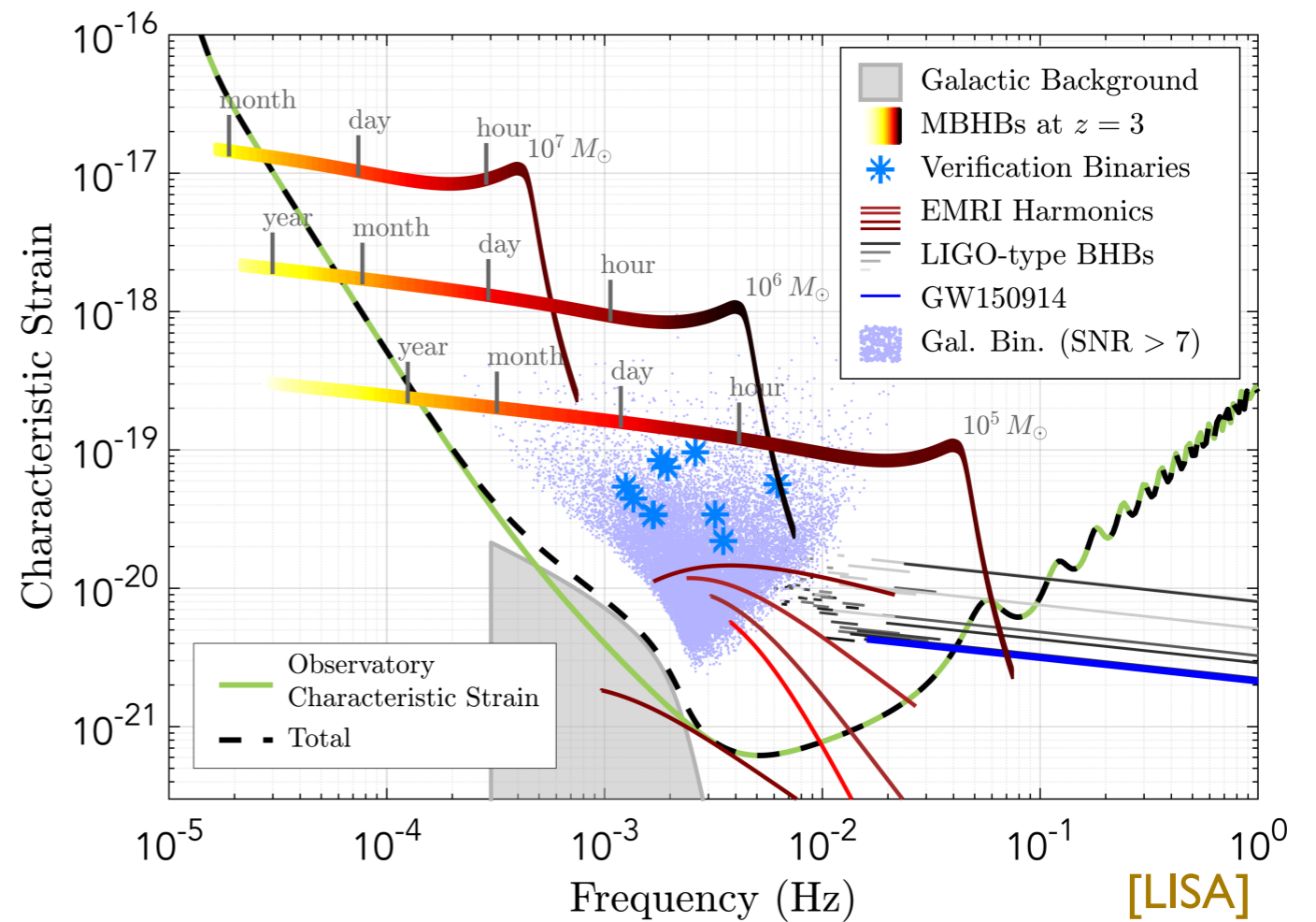


High SNRs → Great precision science !
 High SNRs → Systematics become crucial...

The future of GW science



[Cosmic Explorer]

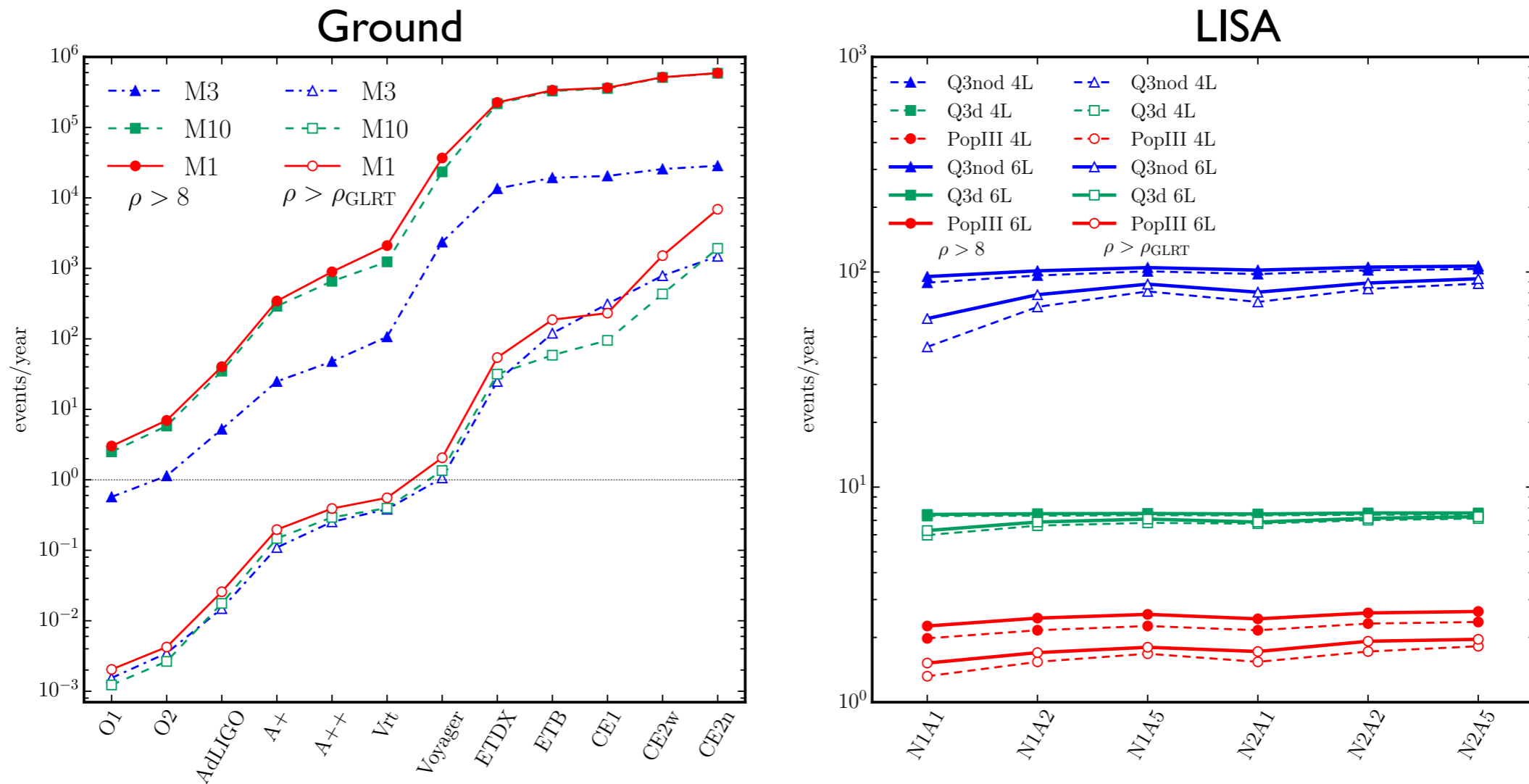


[LISA]

Large number of sources → Great statistical power !
 → Confusion/global fit become crucial...

Measuring multi-mode ringdown signals

Event rates with ringdown and multiple QNMs detectable



[Berti&al 2016]

Black hole spectroscopy with multiple detected QNMs

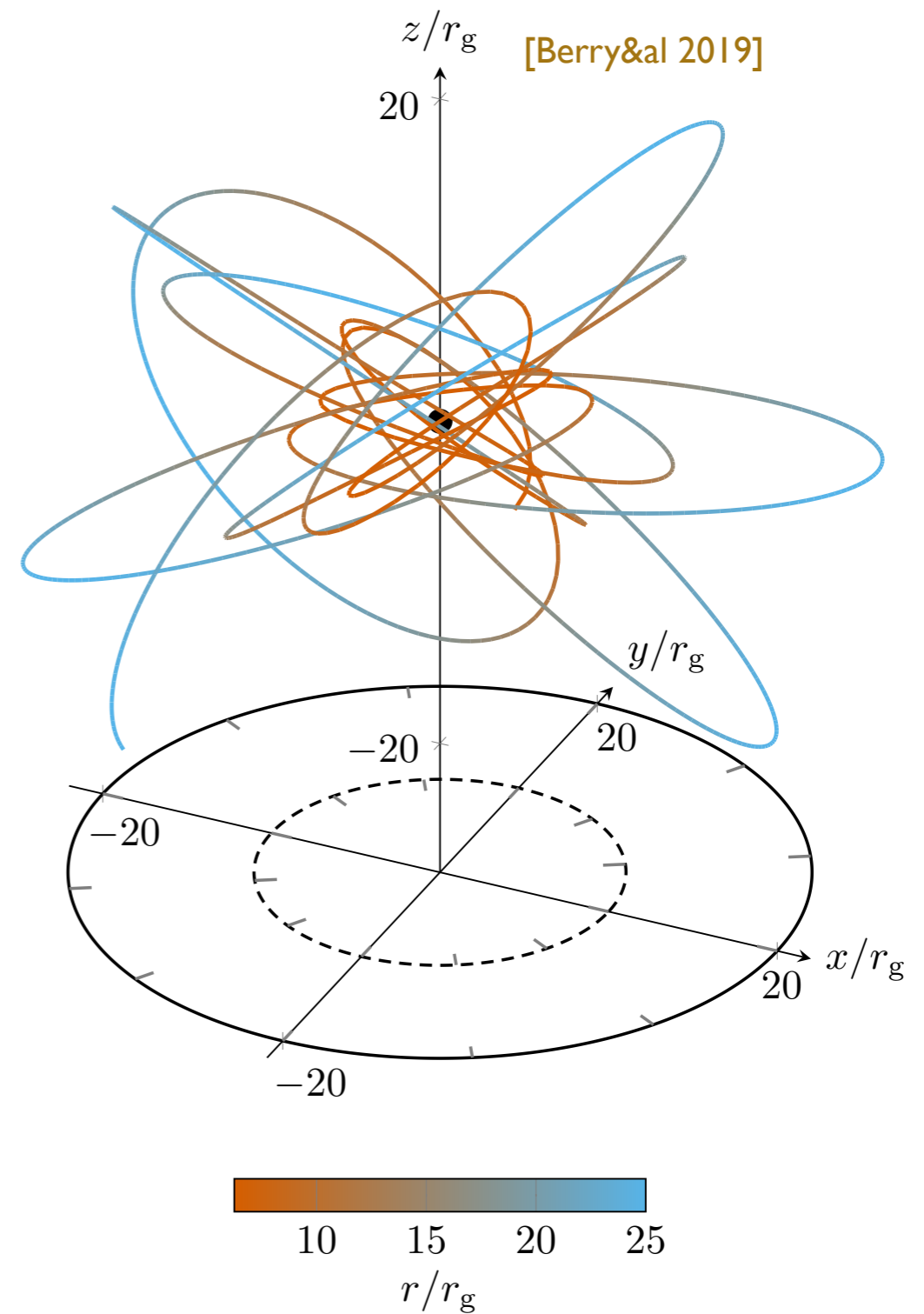
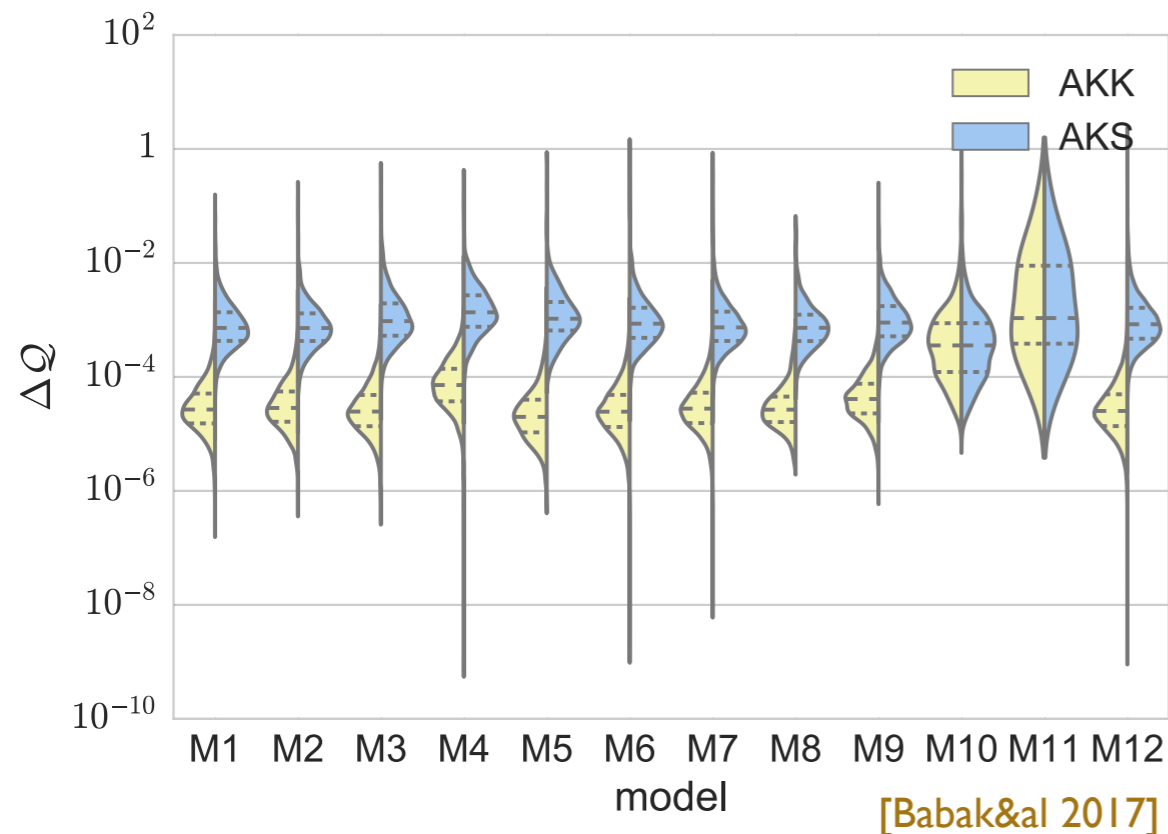
LISA: Extreme Mass Ratio Inspirals

Large number of orbital cycles in the strong-field regime:
map of the central object's spacetime

Multipolar structure of Kerr: [Hansen 1974]

$$\mathcal{M}_\ell^{\text{BH}} + i\mathcal{S}_\ell^{\text{BH}} = \mathcal{M}^{\ell+1} (i\chi)^\ell$$

Constraints on the quadrupole moment

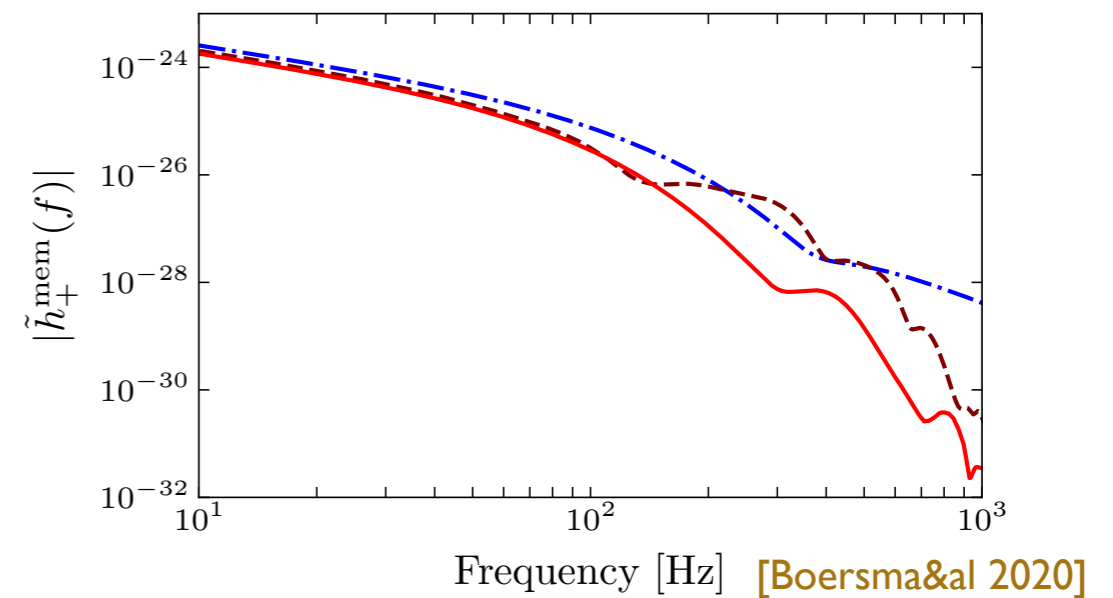
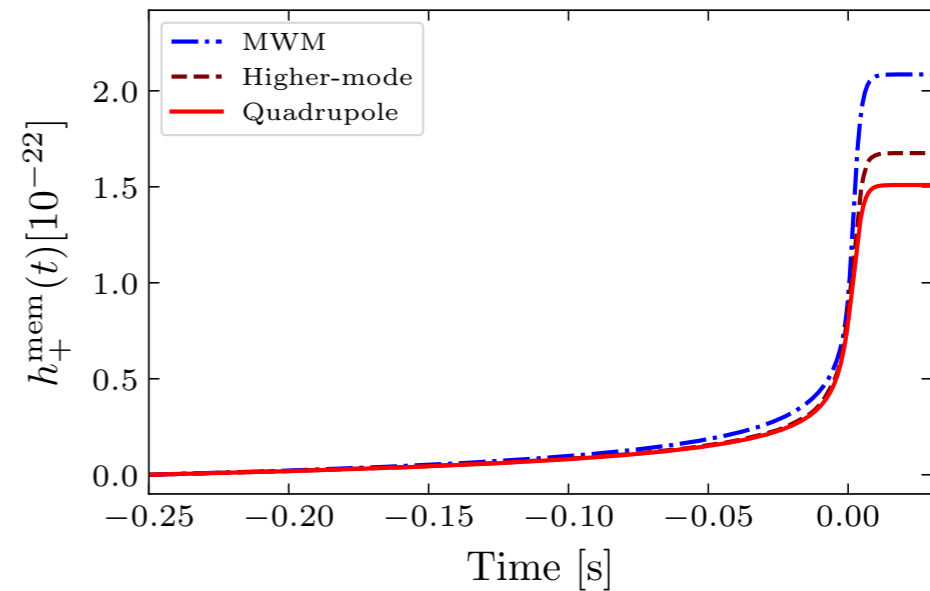


Detecting the nonlinear memory effect

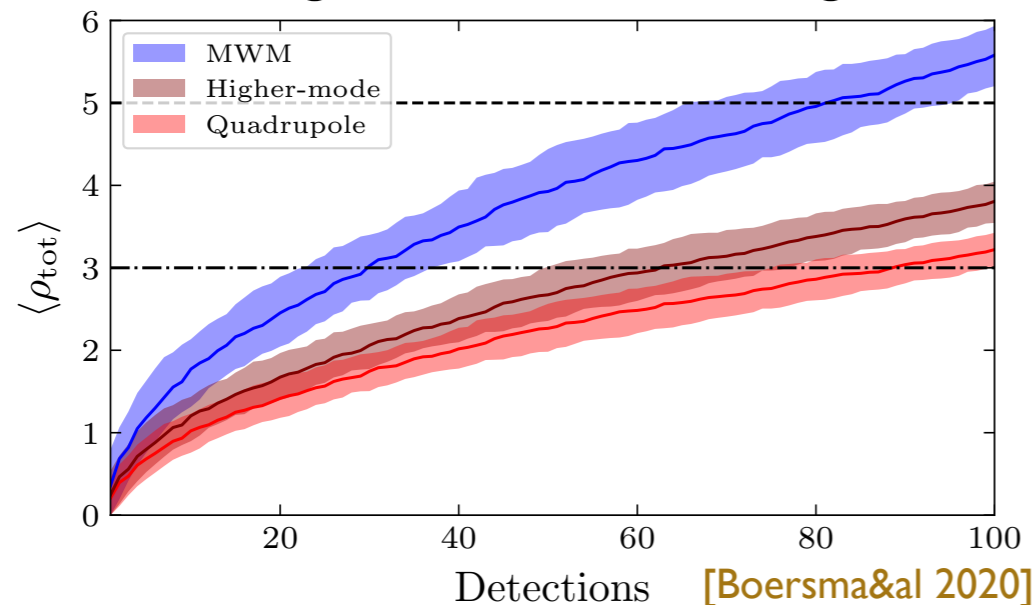
Nonlinear memory: secular term in the gravitational waves [Christodoulou 1991]

$$h_{jk}^{\text{TT,mem}} = \frac{4}{r} \int_{-\infty}^u du' \left[\int \frac{dE}{d\Omega' du} \frac{n'_j n'_k}{1 - \mathbf{n}' \cdot \mathbf{n}} d\Omega' \right]^{\text{TT}}$$

Challenging to detect, needs stacking of a large number of signals

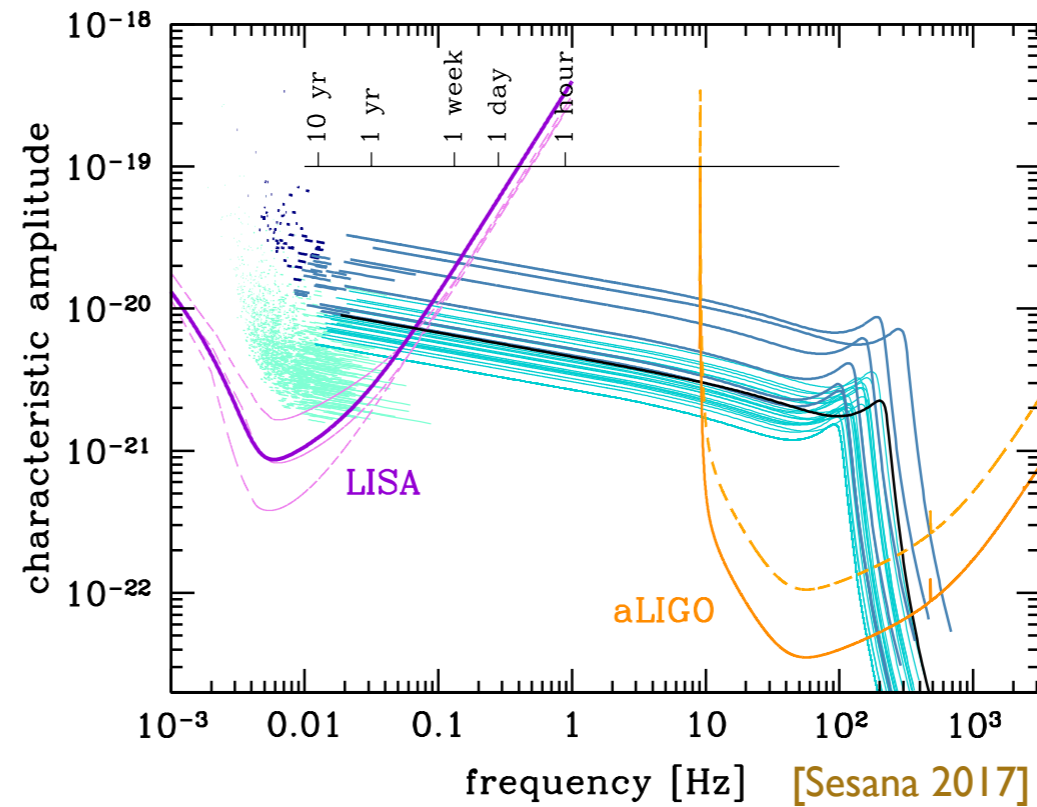


Stacking GW I90514-like signals

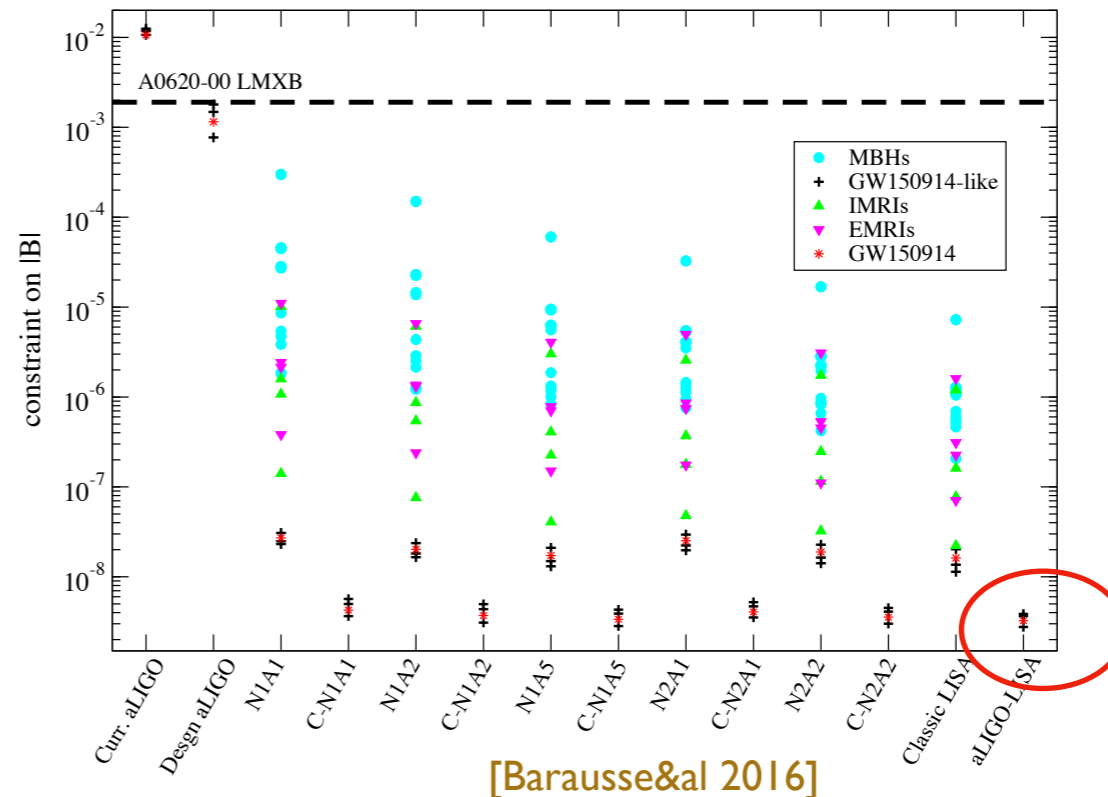


Multiband GW observations and TGR

- Challenging detections for LISA on its own
- Archival searches possible from detections on the ground



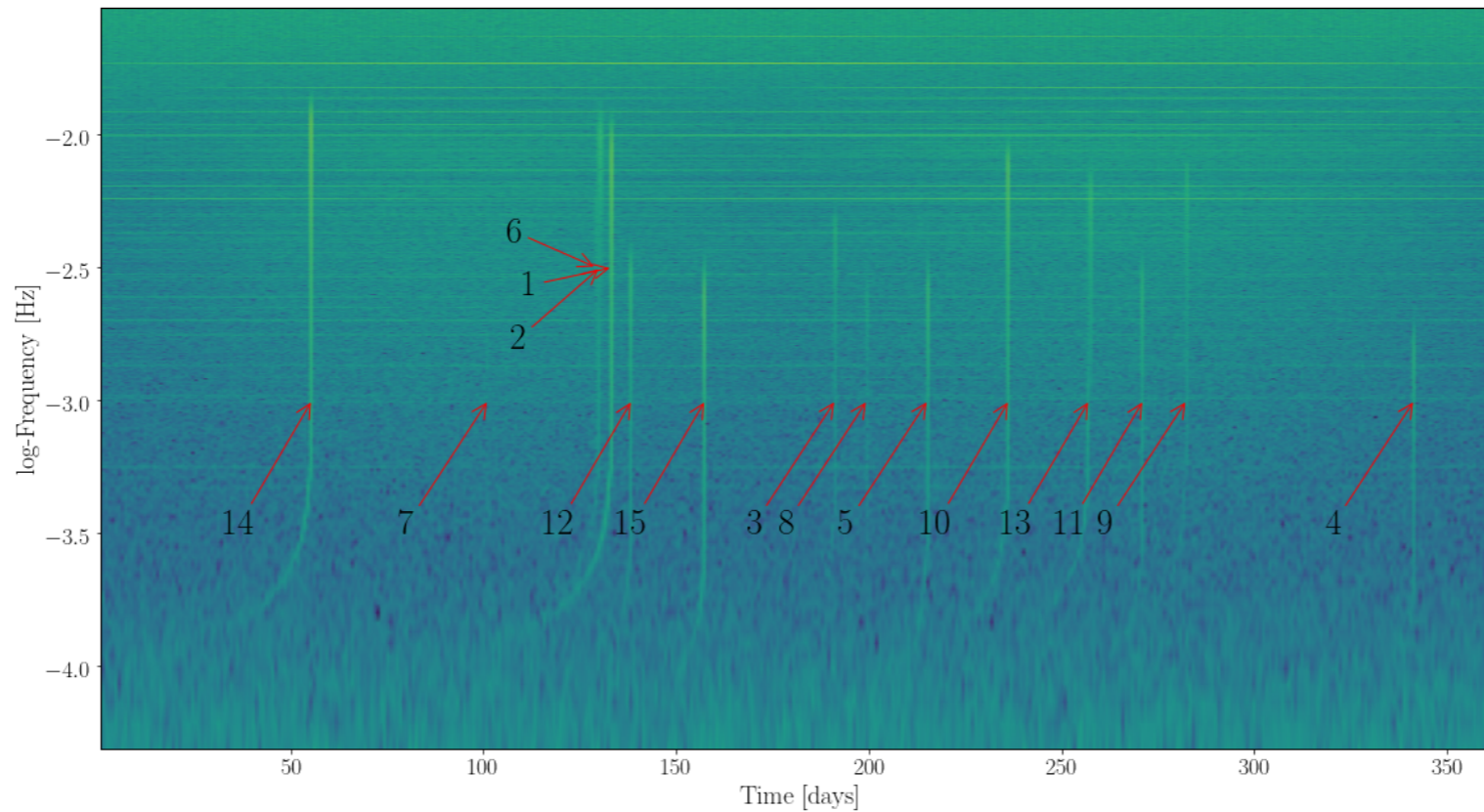
Constraints on dipolar emission from multiband observations:





Thank you !

Future instruments: source superposition and confusion



Superposition of signals for LISA

GW signals for ground, all of them

