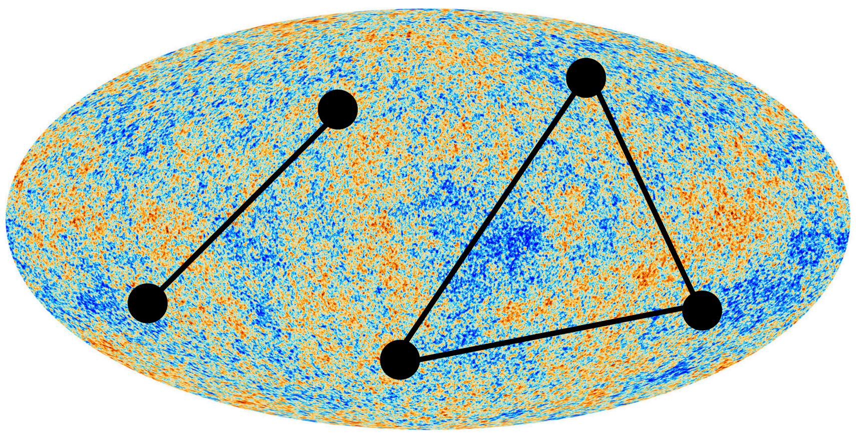
The Cosmological Flow:

A Systematic Approach to Inflationary Correlators

$$\mathcal{L}(\delta\phi^a) \longrightarrow$$



Denis Werth

Based on ArXiv:2210.xxxxx

with Lucas Pinol and Sébastien Renaux-Petel

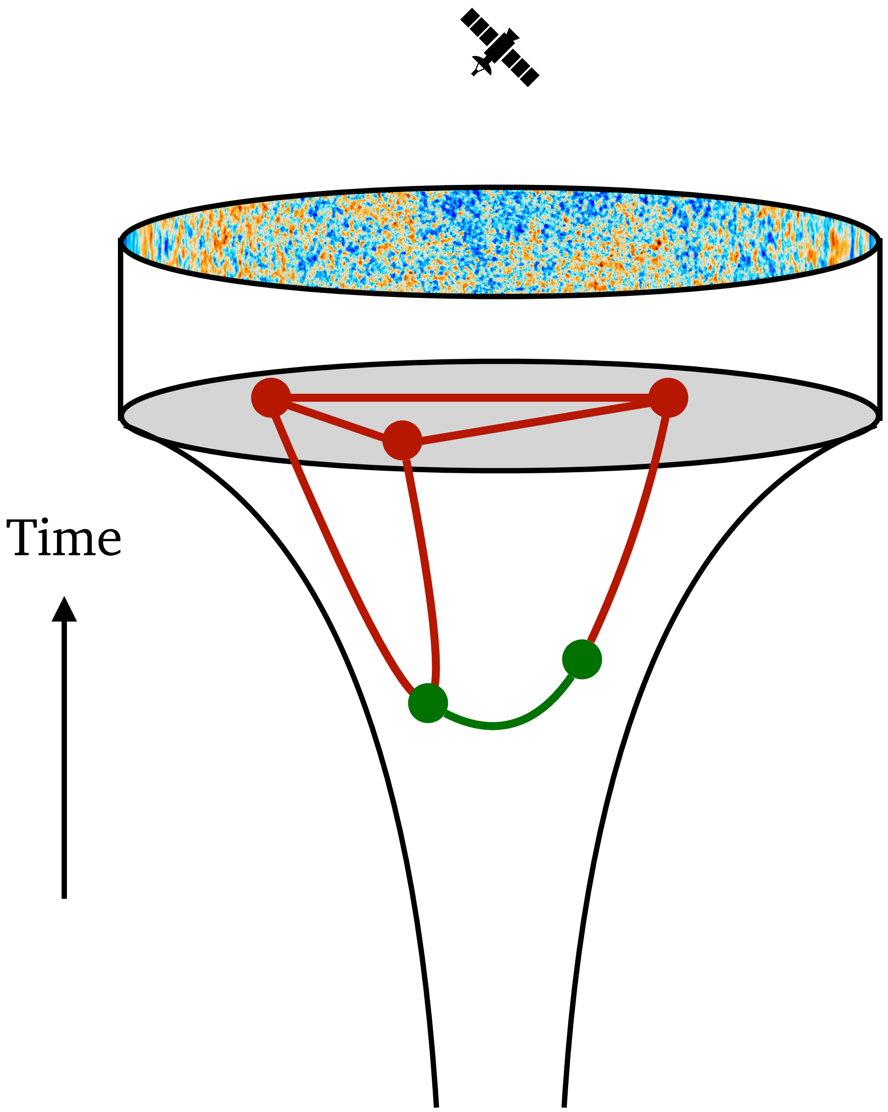
SORBONNE UNIVERSITÉ

IAP

European Research Council
Established by the European Commission

GEODESI

Primordial Fluctuations to Probe High Energy Physics

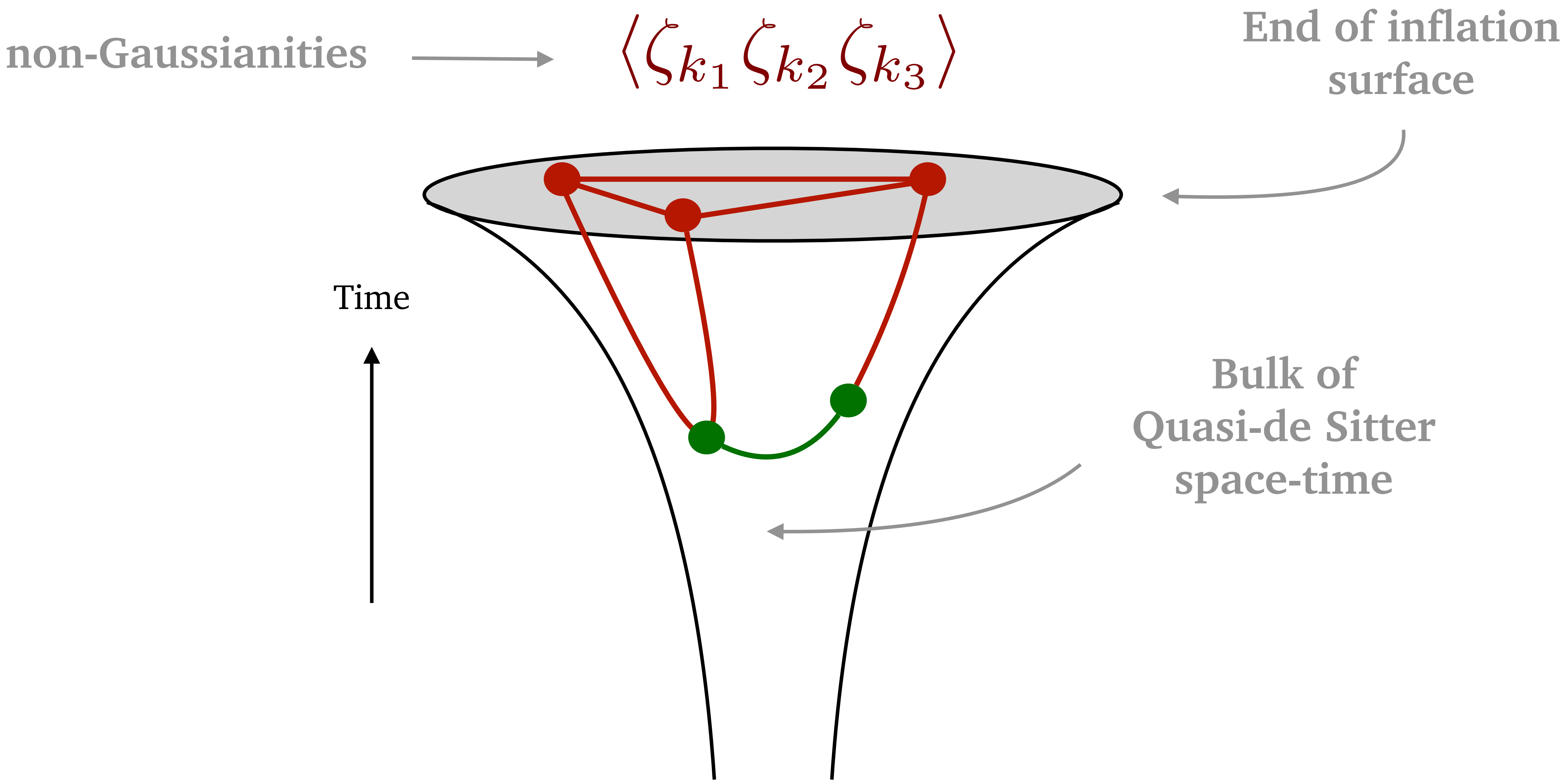


- Information about **primordial fluctuations** are encoded in inflationary correlators.
- **Observational progresses** are expected to be made in the near and far future (LSS, 21cm)
- During inflation, very **massive particles** ($\sim 10^{14}$ GeV) can be produced whose decays lead to observable correlations.

[Planck 2018] [BICEP/Keck 2021]

We want to probe **high energy physics** encoded in **inflationary correlators**.

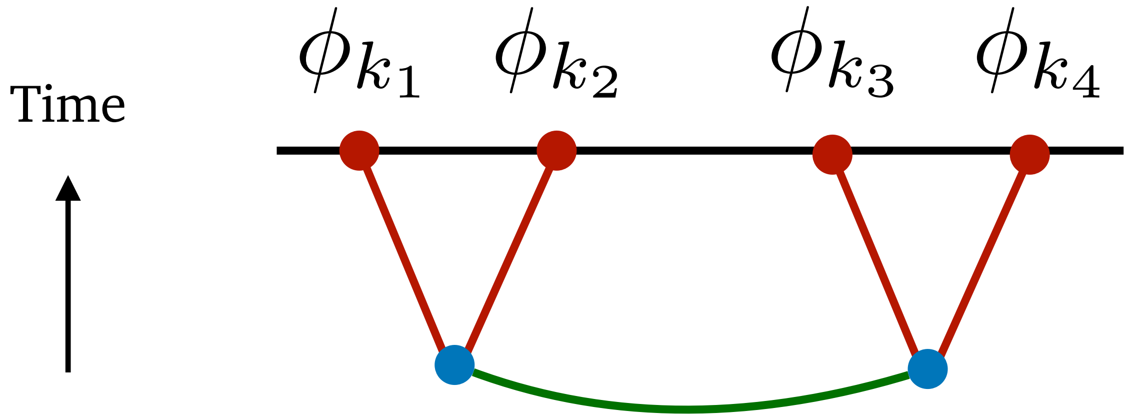
Inflationary Correlators



A detection of non-Gaussianities would give us information about inflation.

Inflationary Correlators and Perturbation Theory

Cosmological correlators are very difficult to compute.



$$\langle \phi^4 \rangle = \int \mathrm{d}t \int \mathrm{d}t' V(t) V(t') \mathcal{G}(k_{12}, t, t') K(k_1, t) K(k_2, t) K(k_3, t') K(k_4, t')$$

- Background is time-dependent
- Algebraic complexity
- Late-time correlators receive contributions from all times
- We cannot use standard techniques from particle physics
- …

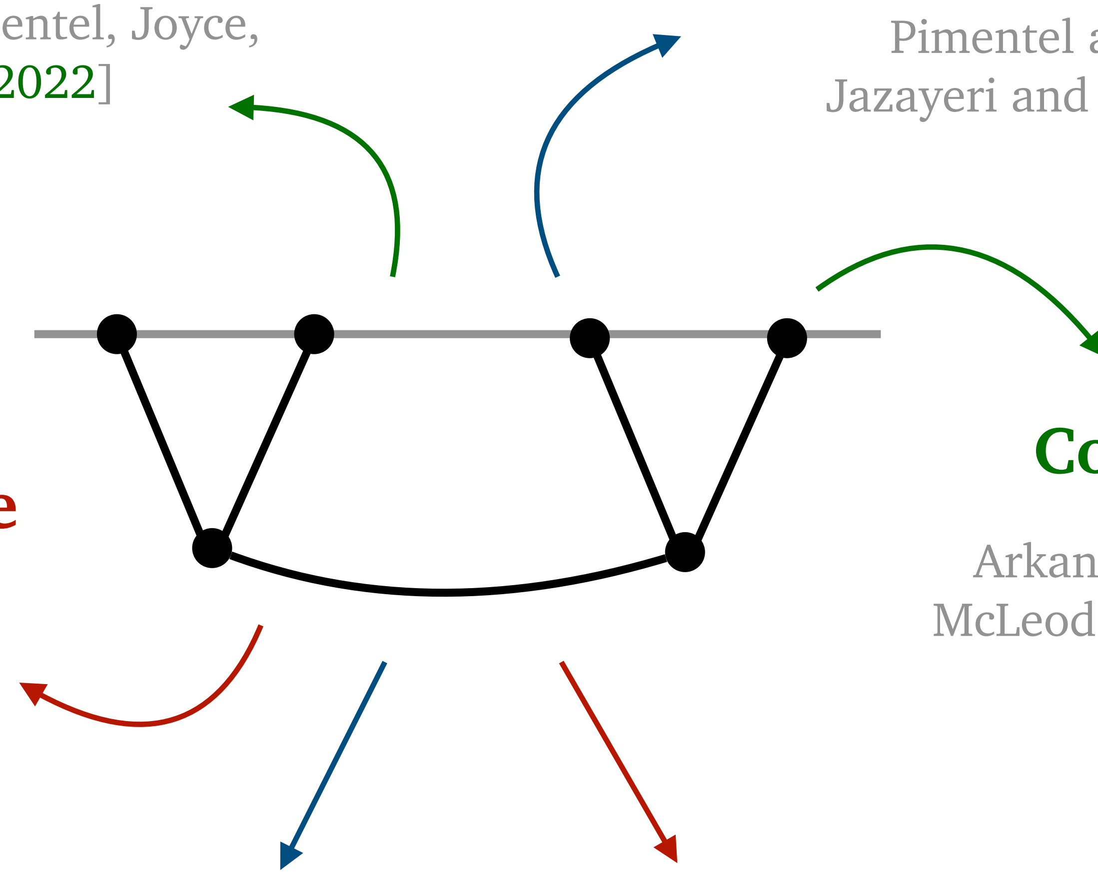
Recent Analytical Developments

Cosmological Bootstrap Program

Arkani-Hamed, Baumann, lee, Pimentel, Joyce,
Duaso Pueyo [2019, 2020, 2022]

Bootstrap Equations for Boost-breaking Interactions

Pimentel and Wang [2022],
Jazayeri and Renaux-Petel [2022]



AdS-inspired Mellin Space

Sleight and Taronna [2019, 2021]

Cosmological Potytopes

Arkani-Hamed, Benincasa, Postnikov,
McLeod [2017, 2018, 2019, 2020, 2022]

Fundamental Principles (Symmetries & Causality & Locality)

Pajer, Stefanyszyn, Supeł, Goodhew, Jazayeri,
Melville, Gordon Lee, Bonifacio, Wang [2020, 2021]

Partial Mellin-Barnes Representation

Qin and Xianyu [2022]

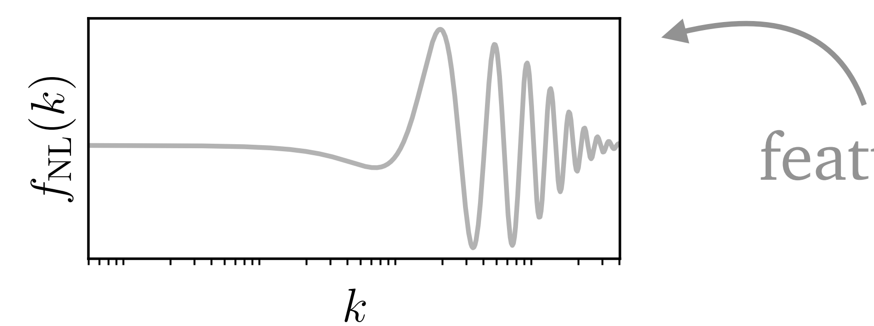
Limitations of Analytical Methods

Weak Quadratic Mixing

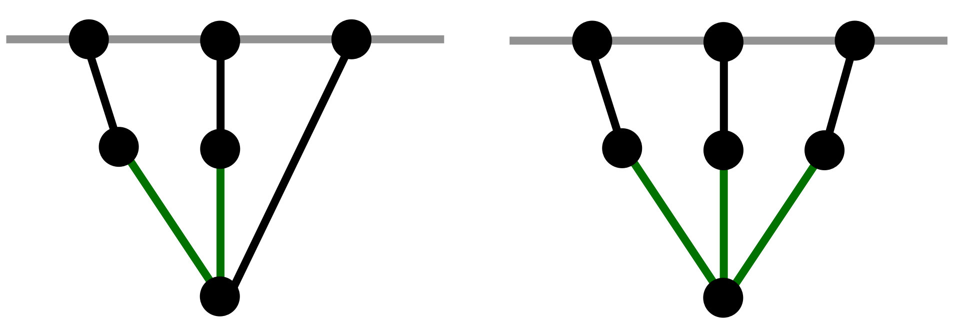
$$\mathcal{L}^{(2)} \supset \rho\dot{\phi}\sigma$$

treated perturbatively

(Near) Scale-Invariance

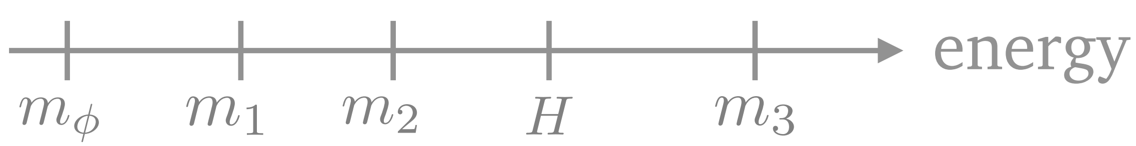


Only Single-Exchange Diagram

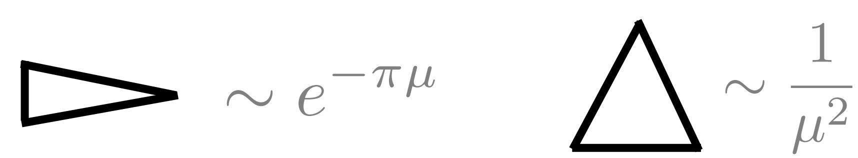


Large hierarchy of masses/couplings but not the intermediate regimes

Often only 1 or 2 Fields

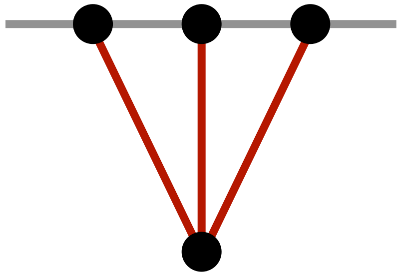


Treatment of Equilateral and Squeezed Configurations Separately



Aside from isolated examples…

Numerical Challenges and Developments



$$\langle\phi^3\rangle(\tau) \sim \int_{-\infty}^{\tau} \mathrm{d}\tau' \, \tau'^n g(\tau') e^{iK(\tau'-\tau)}$$

Direct Calculations (not systematic)

- Wick rotation [Chen and Wang 2010]
- Numerical mode functions [Assassi et al. 2013]
- Holder summation [Junaid et al. 2015]
- Cesaro/Riesz summation [Tran et al. 2022]
- …

Indirect Calculations

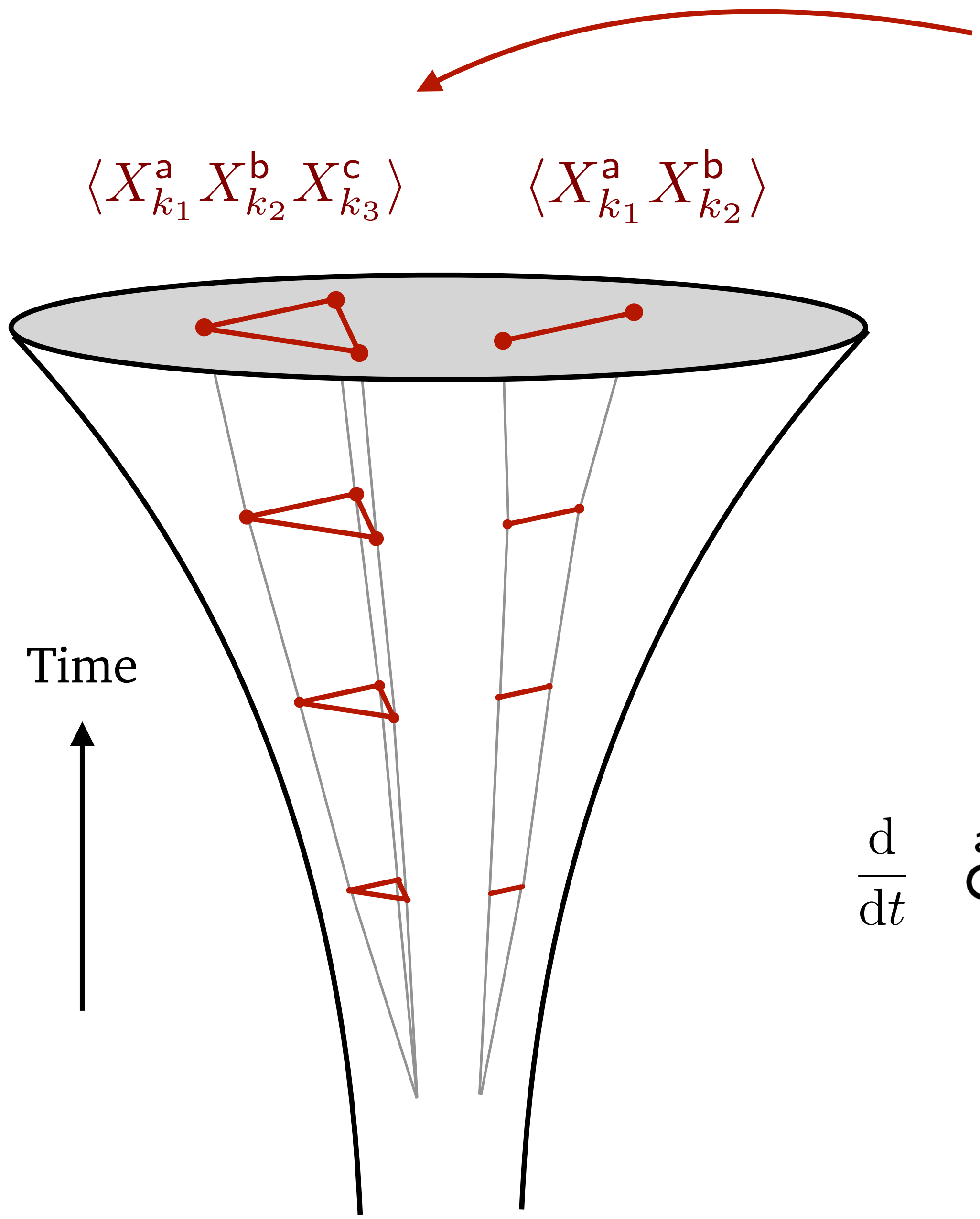
$$\frac{\mathrm{d}}{\mathrm{d}\tau}\langle\phi^3\rangle = g - iK\langle\phi^3\rangle$$

- Translate the problem of computing Feynman-type integrals to **solving differential equations in time**

Systematic framework to study inflationary correlators :
the **transport approach**

[Dias, Fazer, Mulryne and Seery 2016]

The Transport Formalism

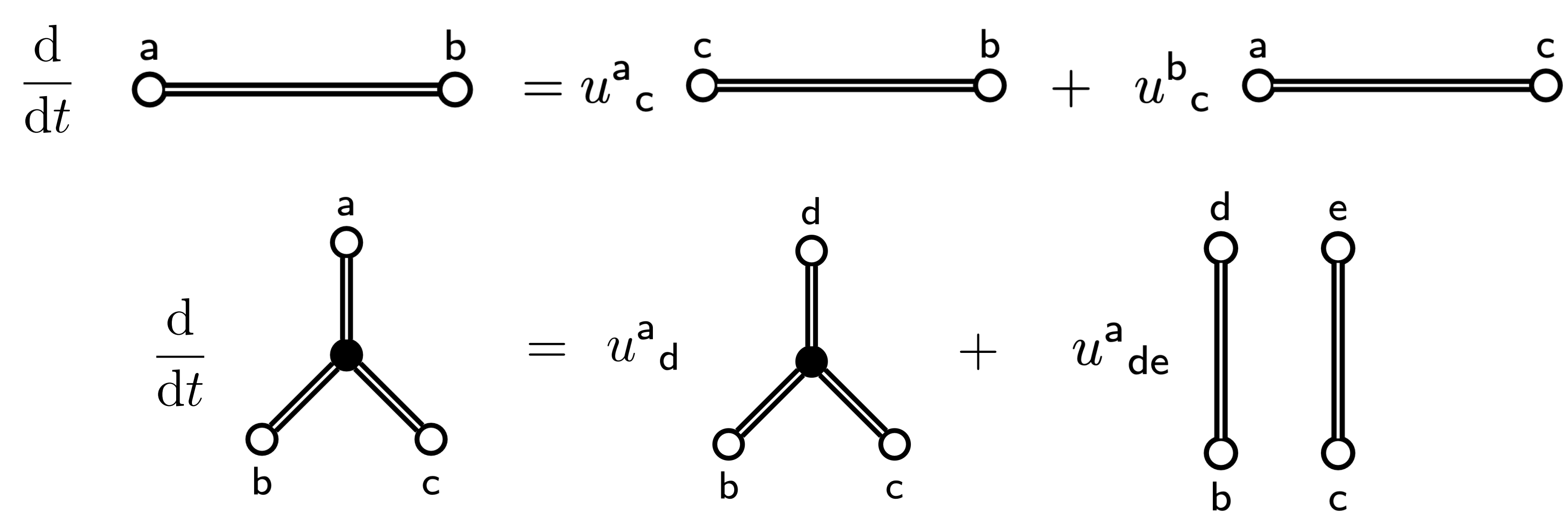


EFT at the level of the fluctuations

Exact Equations of Motion using the canonical operator formalism

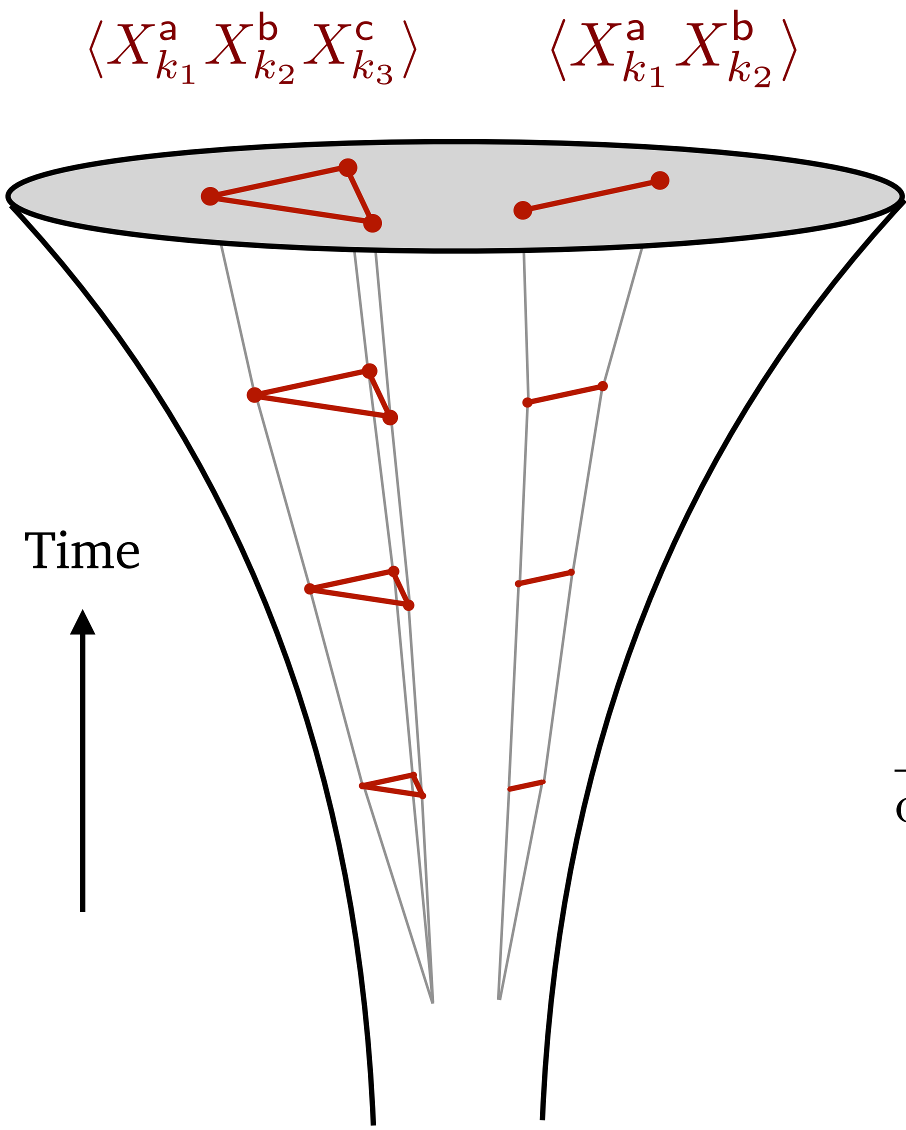
$$\frac{\mathrm{d}X^{a}}{\mathrm{d}t} = i[H(X^{b}), X^{a}]$$

Transport Equations



a, b, c, … run over phase-space variables

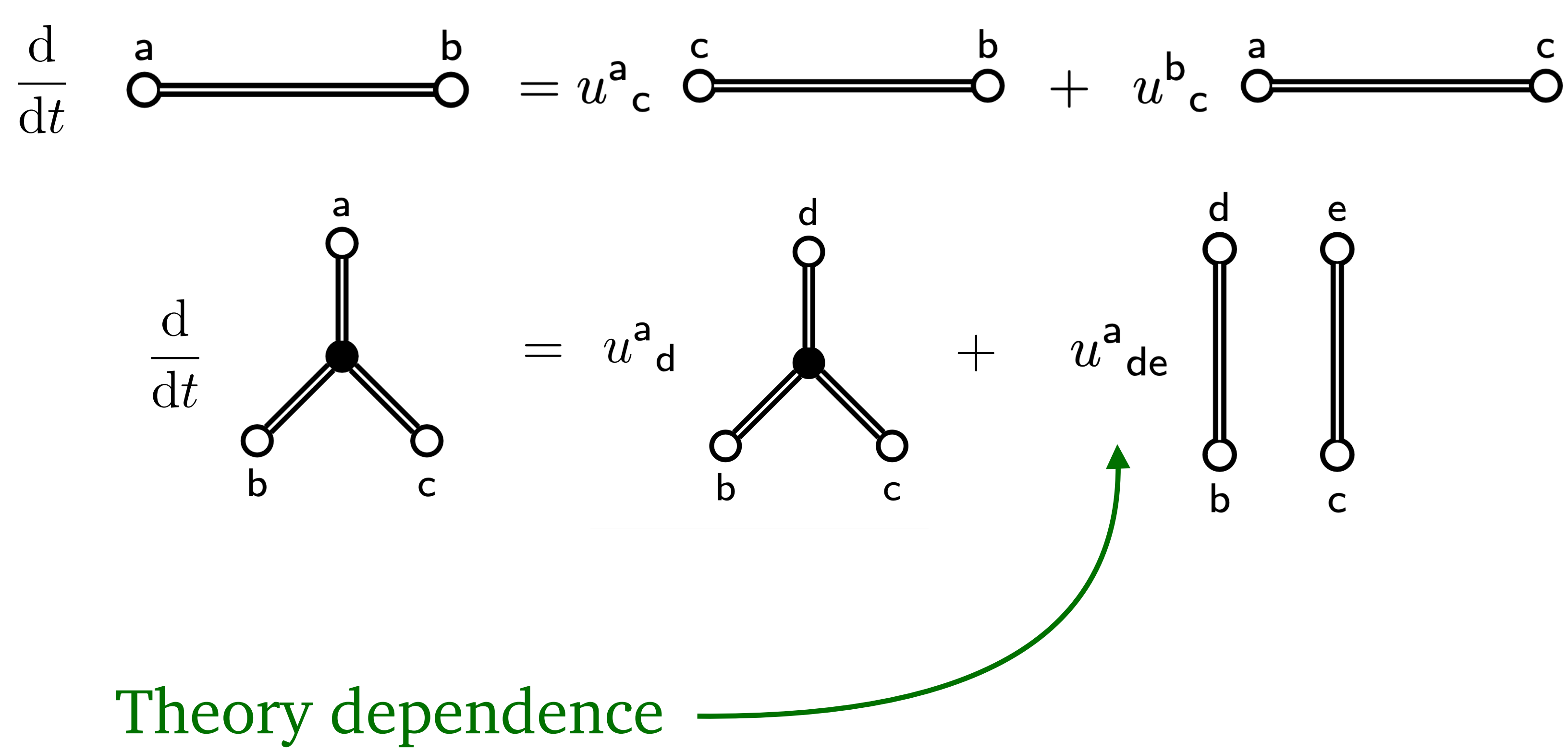
The Transport Formalism



Initial Conditions

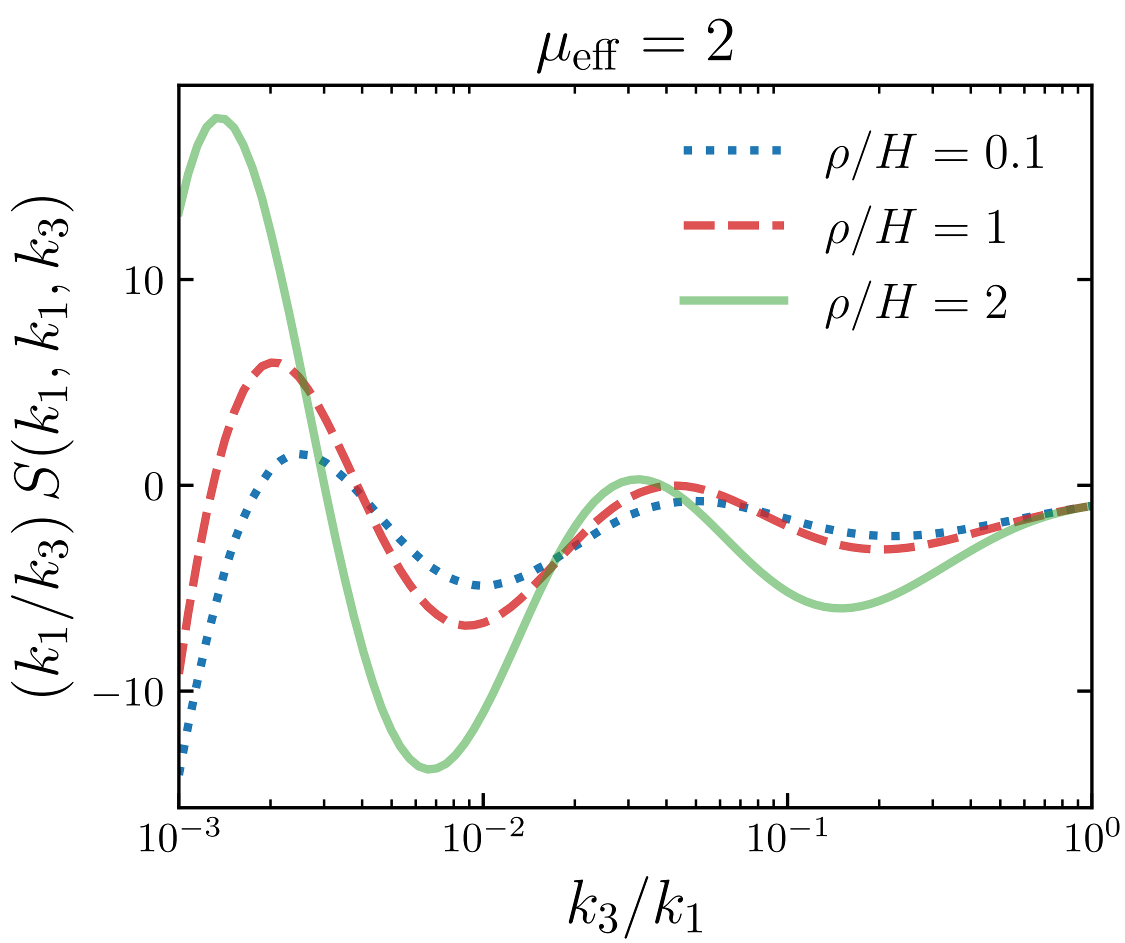
- In the far past, modes do not feel the effect of spacetime curvature
- Set of **uncoupled dofs**
- Analytical approximations become both **tractable** and **accurate**

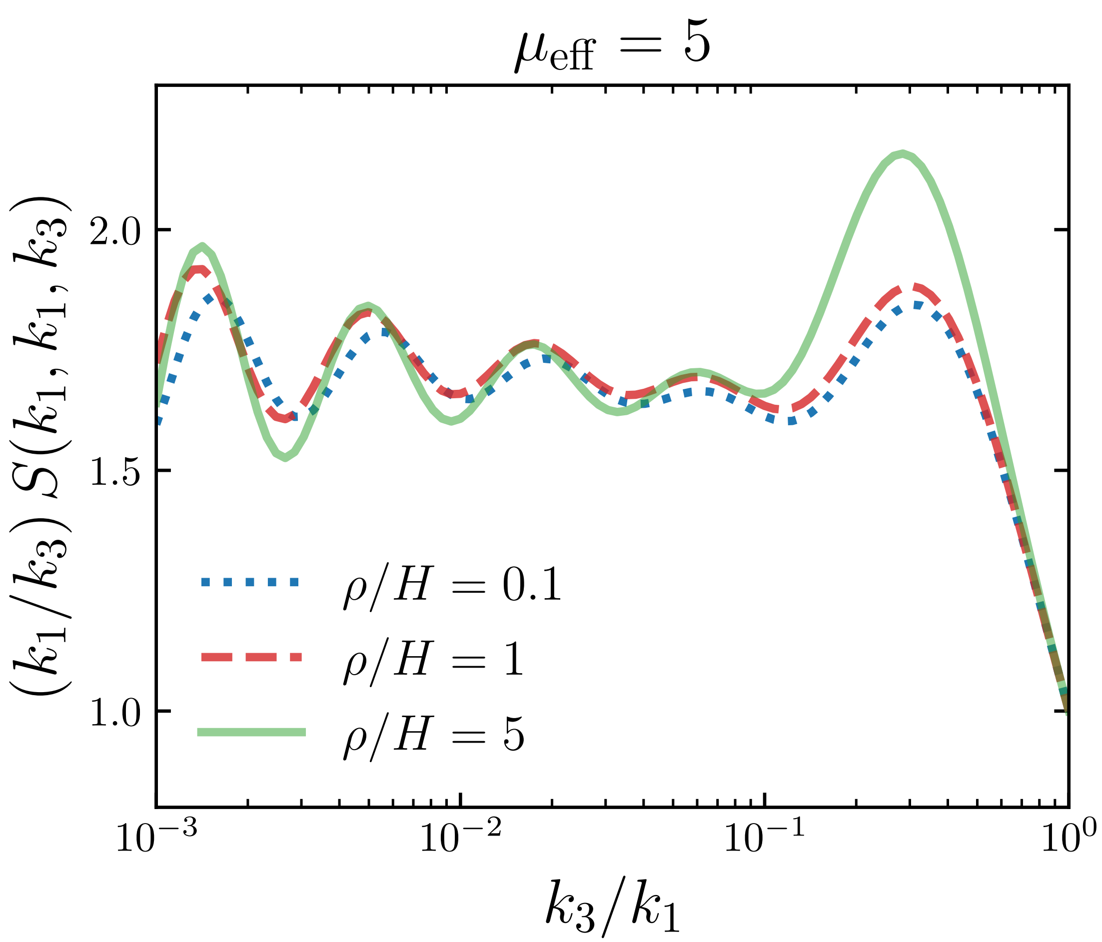
Transport Equations

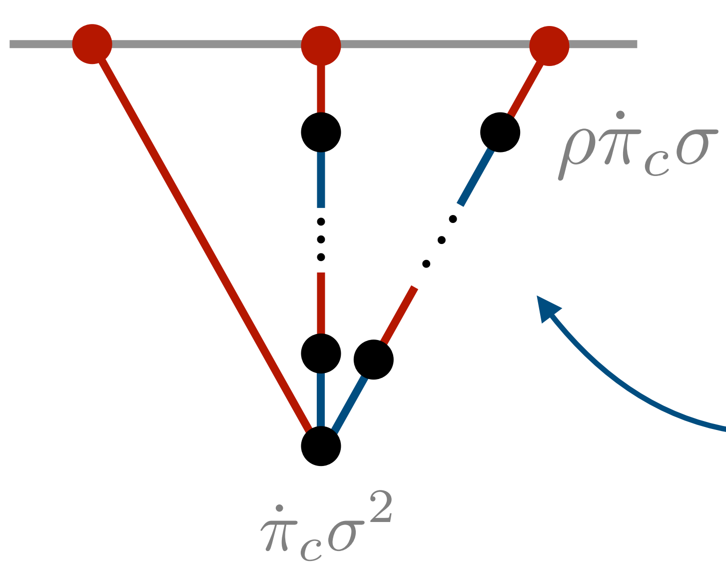


Applications

Cosmological Collider Signal at Strong Mixing





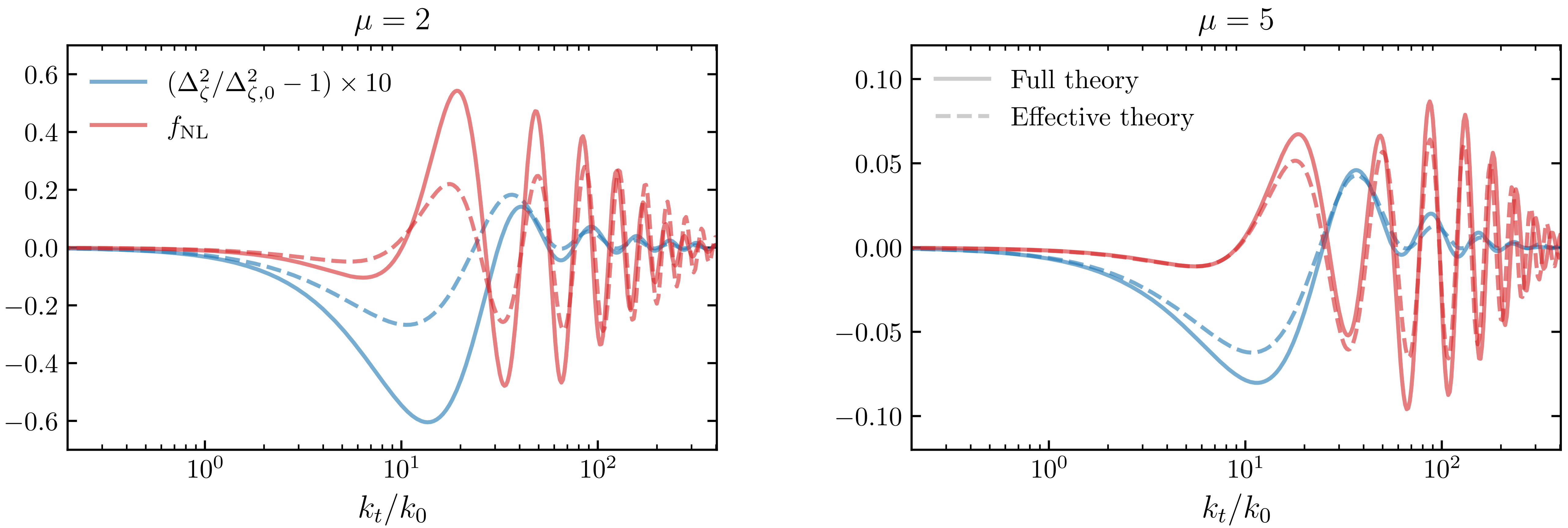


Effective mass for the heavy field

$$m^2 \to m_{\rm eff}^2 = m^2 + \rho^2$$

Resummation of quadratic mixings

Primordial Features



Full Theory :

$$\mathcal{L}_{\pi-\sigma}/a^3 = \rho(t)\dot{\pi}_c\sigma + \frac{\rho(t)}{2f_\pi^2}(\partial_\mu\pi_c)^2\sigma + \frac{\dot{\rho}(t)}{f_\pi^2}\pi\dot{\pi}_c\sigma$$

Effective single-field Theory :

$$\mathcal{L}/a^3 = \frac{1}{2\tilde{c}_s^2(t)}\dot{\pi}_c^2 - \frac{1}{2}\frac{(\partial_i\pi_c)^2}{a^2} + \frac{1}{2f_\pi^2}\left(\frac{1}{\tilde{c}_s^2(t)} - 1\right)\dot{\pi}_c(\partial_\mu\pi_c)^2 - \frac{\dot{\tilde{c}}_s(t)}{f_\pi^2\tilde{c}_s(t)}\pi_c\dot{\pi}_c^2$$

Conclusion

Inflation is fascinating as it allows us to probe the **laws of physics** at the **highest reachable energies**

Present a **complete formalism** to numerically follow the time evolution evolution of **all 2- and 3-pt correlation functions** in a systematic way

Develop a code that automatically computes observables from an **EFT for fluctuations**

Used to study the physics of inflation in **non-trivial setups** (e.g. in the strong quadratic mixing regime or features)

Thank you

Goldstone Boson Coupled to a Massive Scalar Field

Quadratic theory

Quadratic mixing

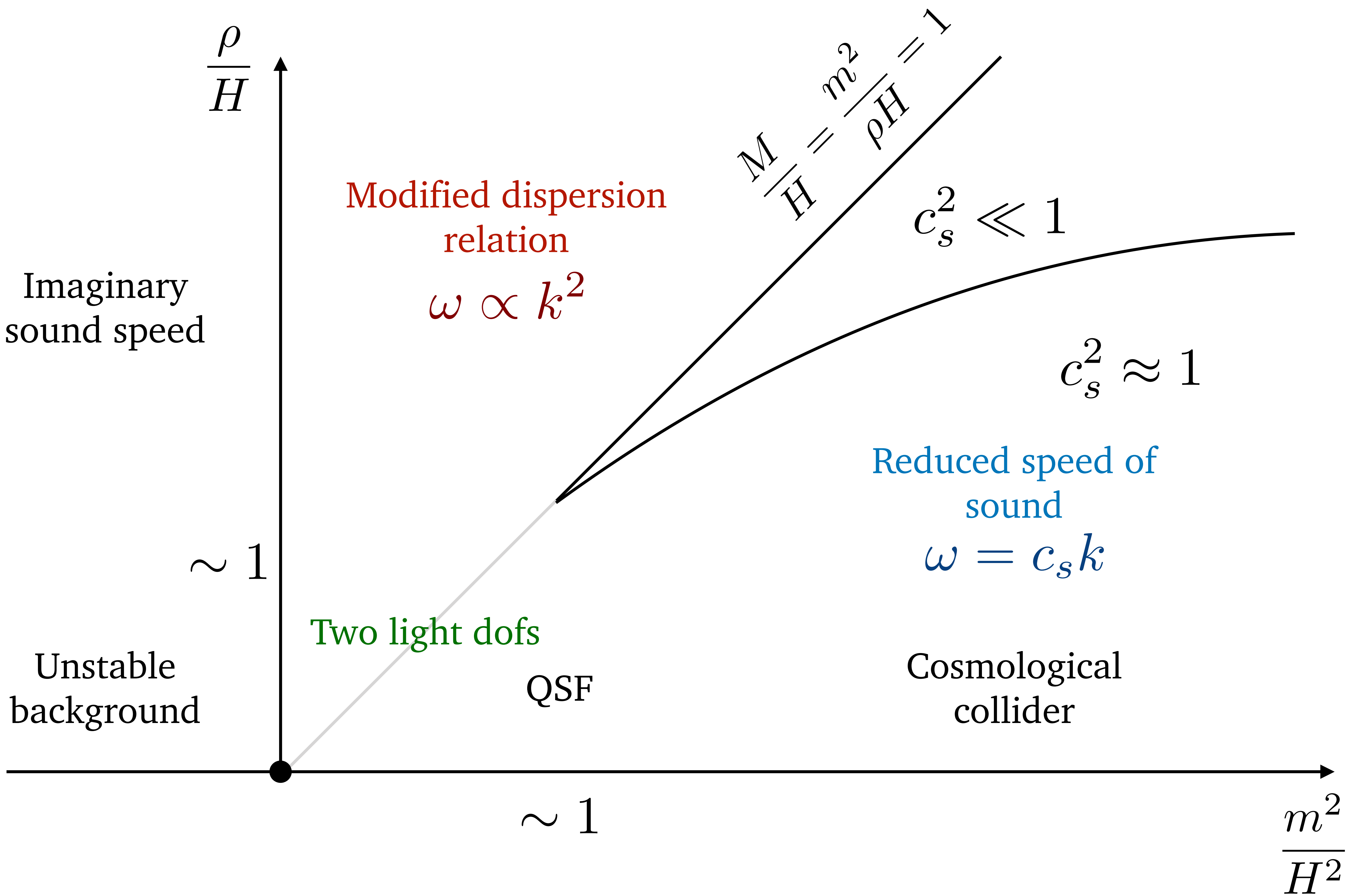
$$\mathcal{L}/a^3 = -\frac{1}{2}\left[-\dot{\pi}_c^2 + c_s^2\frac{(\partial_i\pi_c)^2}{a^2}\right] - \frac{1}{2}\left[(\partial_\mu\sigma)^2 + m^2\sigma^2\right] + \rho\dot{\pi}_c\sigma$$

$$-\lambda_1\dot{\pi}_c\frac{(\partial_i\pi_c)^2}{a^2} - \lambda_2\dot{\pi}_c^3 - \mu\sigma^3 - \frac{1}{2}\alpha\dot{\pi}_c\sigma^2 - \frac{1}{2\Lambda_1}\frac{(\partial_i\pi_c)^2}{a^2}\sigma - \frac{1}{2\Lambda_2}\dot{\pi}_c^2\sigma$$

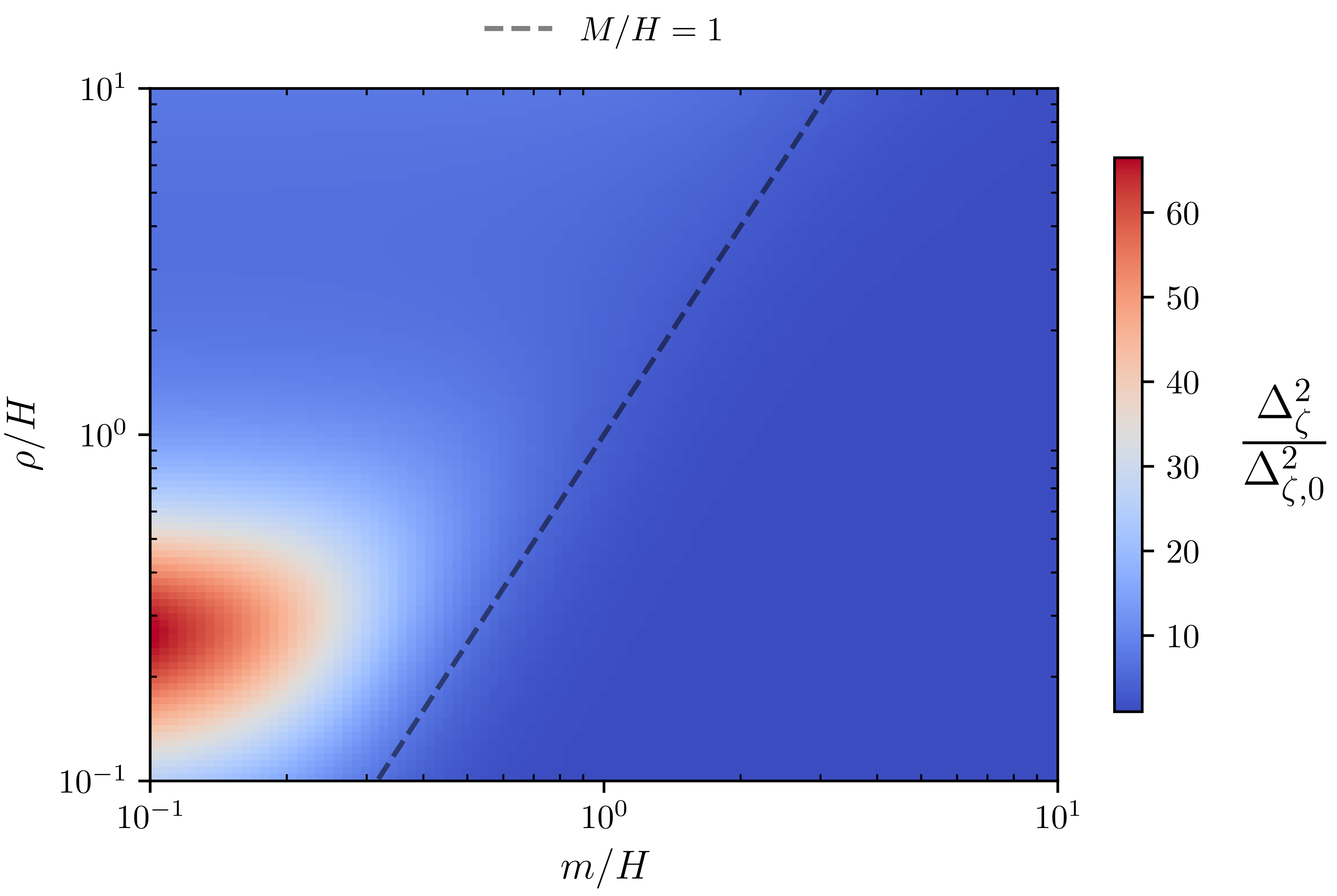
Goldstone boson self-interactions

Mixing interactions

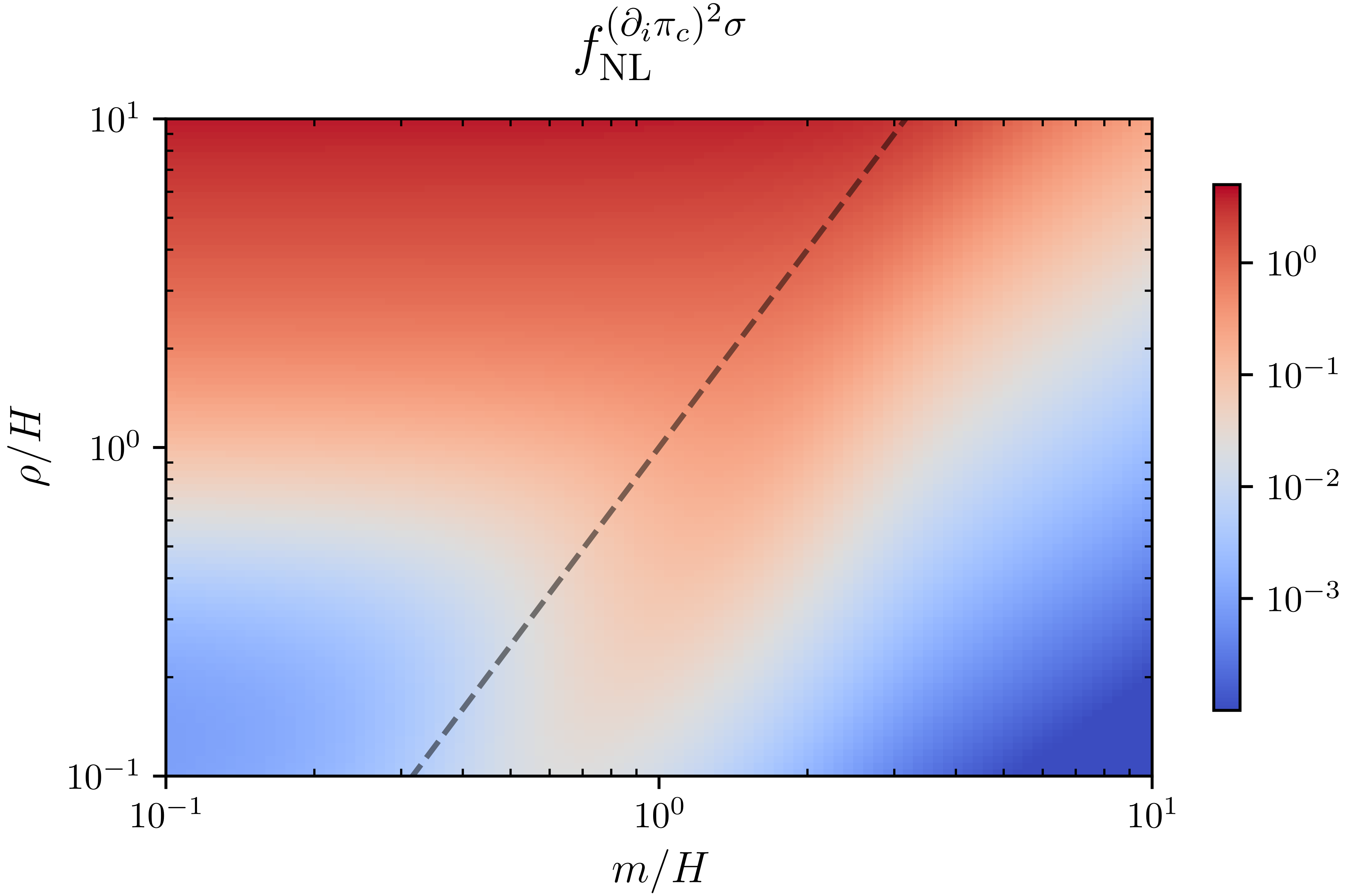
Phase Diagram



Quadratic Theory Phase Diagram



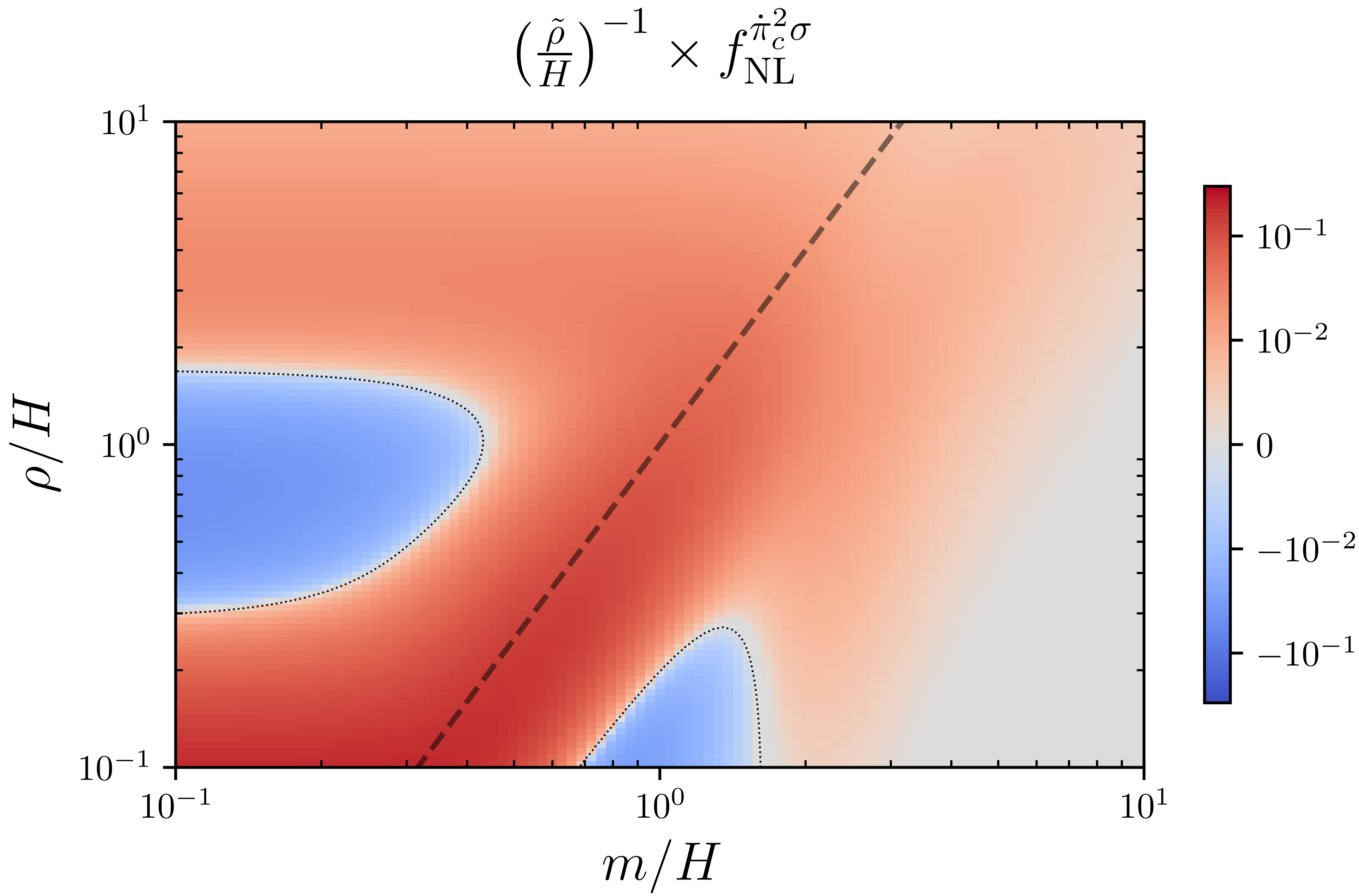
Single-Exchange Diagram Phase Diagram



Weak mixing : $\rho/H \lesssim c_s^{-1/2}$

Strong mixing : $\rho/H \lesssim c_s^{3/4}\dfrac{\kappa^{1/2}}{\Delta_\zeta}$

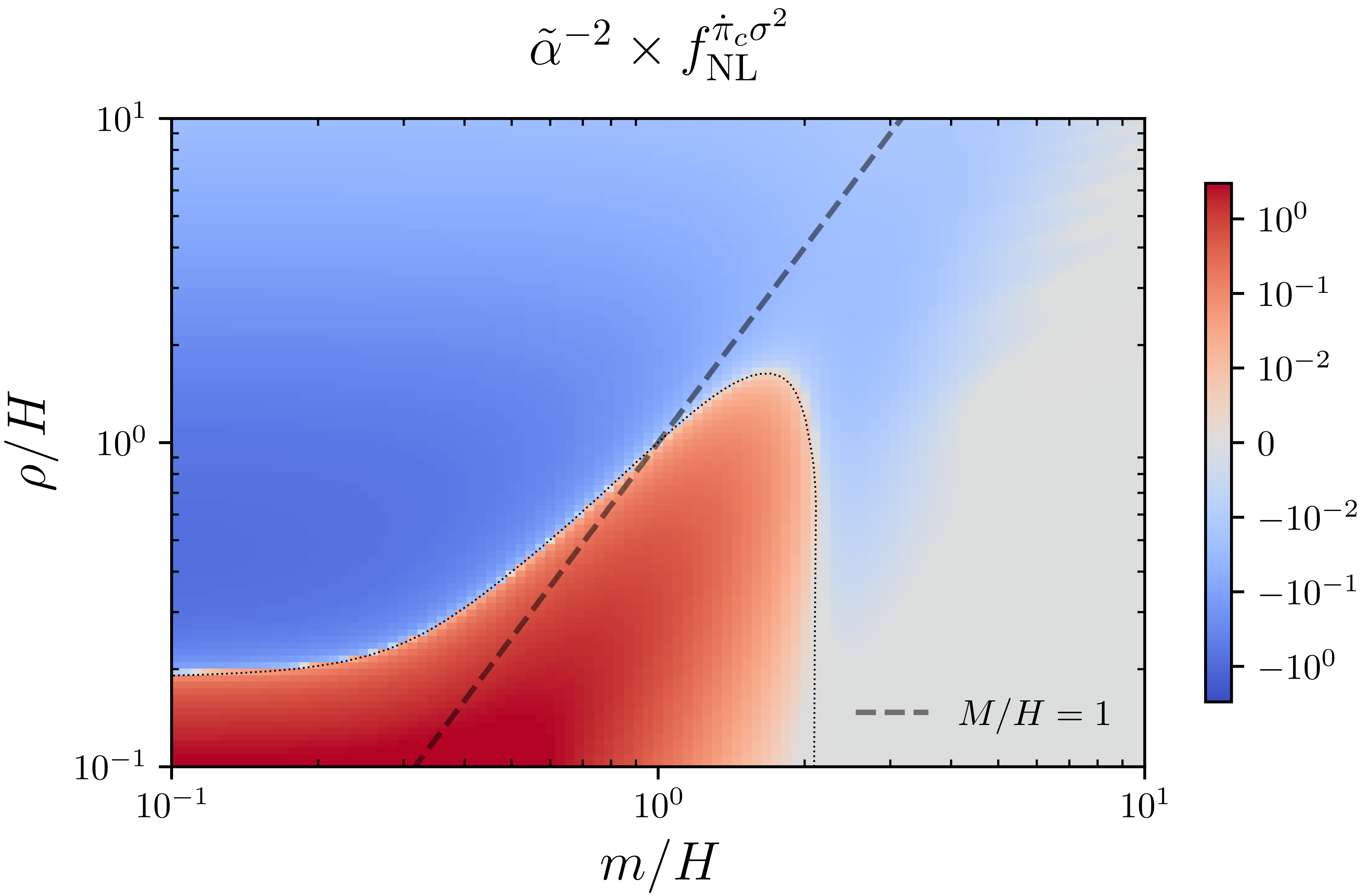
Single-Exchange Diagram Phase Diagram



Weak mixing : $\quad \tilde{\rho}/H \lesssim \dfrac{c_s^{-1/2}}{2\pi\Delta_\zeta}$

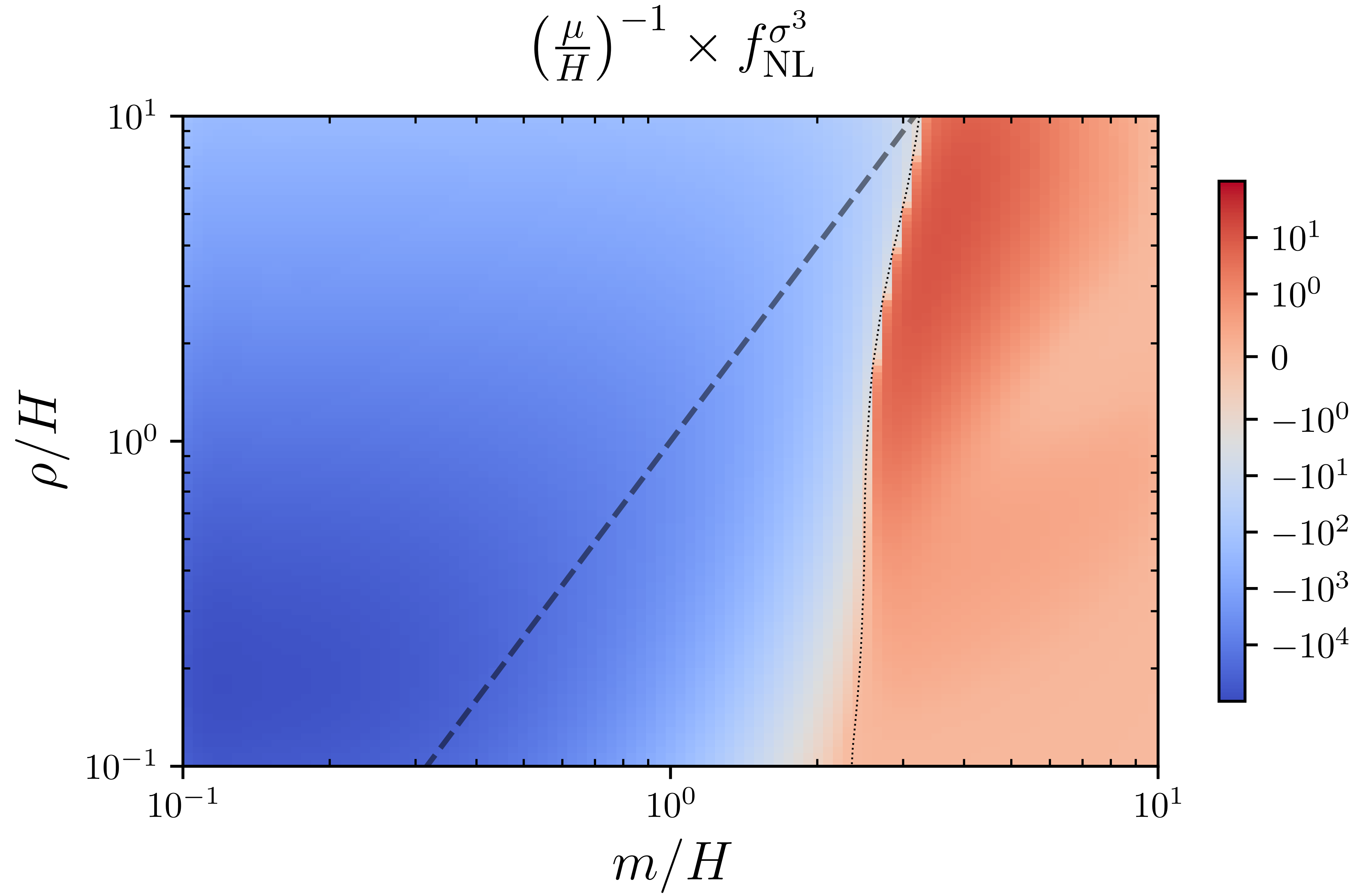
Strong mixing : $\quad \tilde{\rho}/H \lesssim \dfrac{\rho}{H}\dfrac{\kappa^{1/2}}{c_s^{1/4}\Delta_\zeta}$

Double-Exchange Diagram Phase Diagram



Weak mixing : $\tilde{\alpha} \lesssim \frac{c_s^{-1/2}}{2}\frac{1}{(2\pi\Delta_\zeta)^{1/2}}$ Strong mixing : $\tilde{\alpha} \lesssim \left(\frac{\rho\Delta_\zeta}{16c_s^{5/2}H\kappa}\right)^{1/4}$

Triple-Exchange Diagram Phase Diagram



Weak mixing : $\mu/H \lesssim 1$

Strong mixing : $\mu/H \lesssim c_s^{-3/4} \left(\rho/H\right)^{3/4}$

Codes Available for Inflationary Calculations

Two-point function solvers:

- FieldInf
- ModeCode & MultiModeCode
- PyFlation

Our code:

- Decouple from a specific background
- EFT at the level of the fluctuations

Three-point function solvers:

- BINGO (single-field inflation)

Transport approach:

- CppTransport
- PyTransport

Ringeval, Brax, van de Bruck, Davis, Martin [2006]
Price, Frazer, Xu, Peiris, Easther [2015]
Huston, Malik [2009, 2011]
Hazra, Sriramkumar, Martin [2013]
Dias, Fazer, Seery [2015]
Mulryne [2016]