Quantum holographic (in)stability of maximally symmetric spacetimes

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Context and aims of the project

- **General relativity** and **quantum field theories** are included into the standard model of cosmology.
- However, one of the less understood aspects of cosmology is the interaction between the geometry of spacetime with quantum field theories, especially in a **non-perturbative** regime.
- Quantum fluctuations of matter act on a background geometry through the semi-classical Einstein equation $G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$.

Aim of the project : study the **back-reaction** of a **strongly coupled QFT** into cosmological spacetimes.

Outline

I The framework of semi-classical gravity

II 2-point correlation functions

III Poles and stability

The framework of semi-classical gravity

Semi-classical gravity with holography

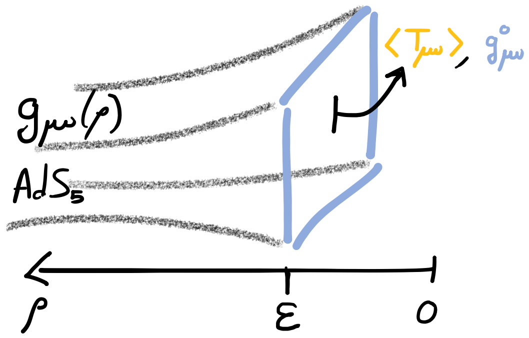
- Some solutions of the semi-classical Einstein equation

$$G_{\mu\nu} = 16\pi G \langle T_{\mu\nu} \rangle$$

are unstable.

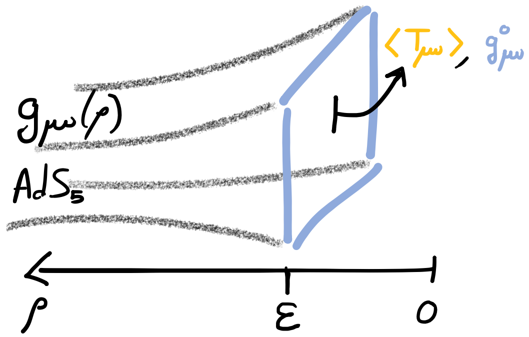
Question : Can instability be an artifact of perturbation theory ?

- The **holographic principle** suggests that the **strongly coupled** regime of a QFT is equivalent to a higher dimensional theory of gravity.



In particular, $\langle T_{\mu\nu} \rangle$ at the boundary of an asymptotically AdS_5 space has the same trace anomaly as a $\mathcal{N} = 4$ super Yang-Mills theory in the weak-coupling limit if $L^3/G_5 = 2N^2/\pi$.

[Henningson, Skenderis JHEP '98], [Duff Class.Quant.Grav. '94]



A non-perturbative generalized Starobinsky model

- *Starobinsky model* : free massless conformally invariant field theory in $d = 4$ FLRW spacetime has the trace anomaly

$$\langle T^{\omega}_{\omega} \rangle = M^{-2}\Box R + H_0^{-2}\left(R_{\alpha\beta}R^{\alpha\beta} - \frac{1}{3}R^2\right)$$

  de Sitter with Hubble rate H_0 is a solution to the EFE with $\Lambda = 0$. A perturbation that **preserves FLRW**,

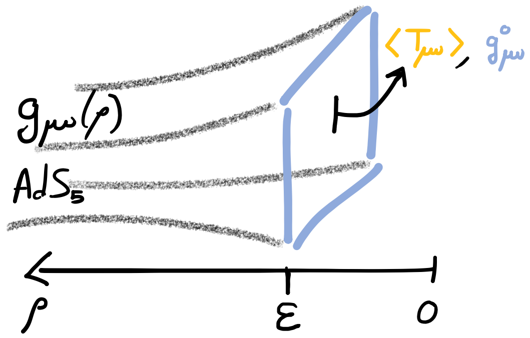
$$H(t) = \frac{\dot{a}}{a} = H_0(1 + \delta(t))$$

- $\delta(t)$ diverges (at linear order) depending on the sign of M^2 only.

  [Starobinsky Phys. Lett. B '80], [Vilenkin Phys. Rev. D '85]

- We extend this idea to a strongly coupled CFT, for the most general metric perturbations, and an arbitrary background curvature H_0.

2-point correlation functions



$$ds^2 = \ell^2 \left[\frac{d\rho^2}{4\rho^2} + g^{\omega\sigma}(x,\rho) dx^\omega dx^\sigma \right]$$

$$g_{\omega\sigma}(x,\epsilon) = g^{(0)}_{\omega\sigma} + \epsilon g^{(2)}_{\omega\sigma} + \epsilon^2 \left[g^{(4)}_{\omega\sigma} + \log \epsilon \hat{g}_{\omega\sigma} \right] + \dots$$

[Fefferman, Graham '85]

The bulk gravity action $S_{\text{bulk}}[g]$ diverges on-shell :

$$S_{\text{bulk}}[g^{(0)}] = \epsilon^{-2} S^{(0)}_{\text{bulk}}[g^{(0)}] + \log \epsilon S^{(2)}_{\text{bulk}}[g^{(0)}] + \mathcal{O}(\epsilon^0)$$
$$= S_{\text{div}} + \mathcal{O}(\epsilon^0).$$

[de Haro, Skenderis, Solodukhin '01]

The boundary theory

Divergent terms of the on-shell bulk action are counterterms for the bare boundary theory (Λ^0, G^0,...).

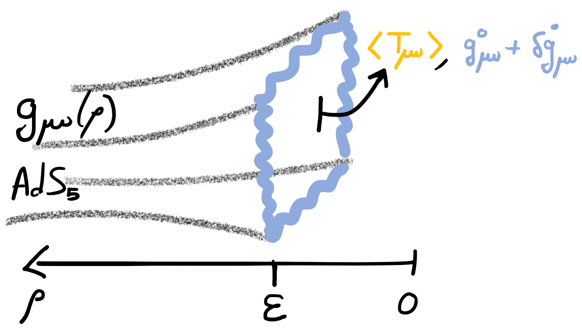
$$S_{\text{renormalized}} = \lim_{\epsilon\to 0}\left[S_{\text{bulk}} + S^0_{\text{boundary}}\right]$$
$$= \underbrace{\lim_{\epsilon\to 0}\left[S_{\text{bulk}} - S_{\text{div}}\right]}_{S_{\text{CFT}}} + \underbrace{\left[S^0_{\text{boundary}} + S_{\text{div}}\right]}_{S_{\text{grav}}}$$

S_{CFT} contains non-local terms coming from the bulk dynamics through $g^{(4)}$. The other piece is the usual quadratic gravity action given by

$$S_{\text{grav}} = S_{\text{EH}} + \frac{\alpha}{384\pi}\int\sqrt{g^{(0)}}R^2 +$$
$$\frac{\beta}{64\pi}\int\sqrt{g^{(0)}}\left[R^{\omega\sigma}R_{\omega\sigma} - \frac{1}{3}R^2\right]$$

The coefficient β can absorb a scheme dependent term coming from S_{CFT}.

Metric perturbations



$$g^{(0)} \to g^{(0)} + \varepsilon \delta g^{(0)}$$

Fixing the gauge :

$$\delta g^{(0)}_{\omega\sigma} = \psi g^{(0)}_{\omega\sigma} + h^{(0)}_{\omega\sigma}$$

- $h_{\omega\sigma}$: 5 TT degrees of freedom, propagates into the bulk AdS_5, evaluates to $h^{(0)}_{\omega\sigma}$ on the 4-dimensional boundary.
- ψ is local on the boundary

$$S = S^{(0)}[g^{(0)}] + \varepsilon S^{(1)}[g^{(0)}] + \varepsilon^2 S^{(2)}[g^{(0)}] + \dots$$

Scalar and tensor 2-point functions

$$\left\langle \delta g^{\omega\sigma} \delta g^{\rho\delta} \right\rangle^{-1} \equiv \frac{\delta^2 S^{(2)}}{\delta g_{\omega\sigma} \delta g_{\rho\delta}}$$

No interaction between scalar and tensor sectors : only 2 independent correlators given by $\left\langle h^{(0)}_{\omega\sigma} h^{(0)\omega\sigma} \right\rangle (\nu)$ and $\langle \Psi\Psi \rangle (\nu)$, where ν is the momentum dual to the radial direction of the slice, defined as the eigenvalue

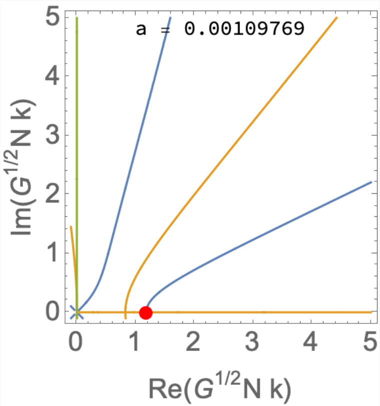
$$\left.\begin{aligned} \left(\tfrac{12}{R}\nabla^2 - 2\right) h^{(0)}_{\omega\sigma} &= -\left(\nu^2 - \tfrac{9}{4}\right) h^{(0)}_{\omega\sigma} \\ \tfrac{12}{R}\nabla^2 \Psi &= -\left(\nu^2 - \tfrac{9}{4}\right) \Psi \end{aligned}\right\} \quad \text{same tachyonic-stability conditions on } \nu.$$

A pole ν of the scalar or tensor propagator contains information about

value of ν	$\rightarrow$	tachyonic instability
residue of the pole	$\rightarrow$	ghost-like instability

Poles and stability

Minkowski tensor instability



Classical solutions :

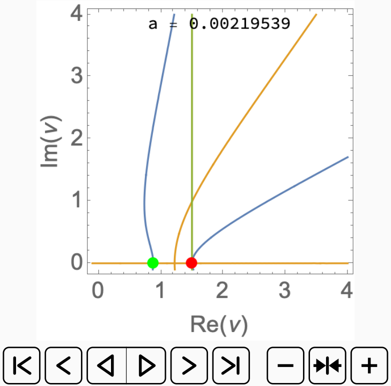
$$\left\langle h^{(0)}_{\omega\sigma} h^{(0)\omega\sigma} \right\rangle^{-1} (\nu \in \mathbb{C}) = 0$$

$$\longrightarrow X(\nu^2) \log X(\nu^2) = a$$

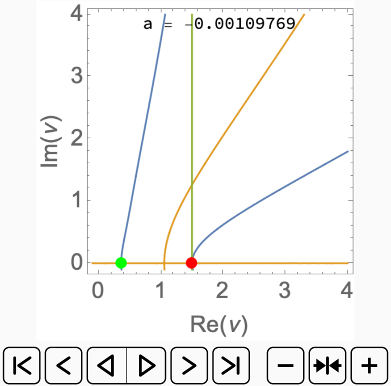
The only non-local propagator is the tensor [Hawking et. al PRD '01]. It is not written as a polynomial of ν^2. However, in the $|\nu| >> 1$ limit, it is still possible to bring the EoM into the form $X \log X = a$.

● : ghost-free ● : positive residue ● : complex residue

de Sitter tensor instability depending on the sign of a

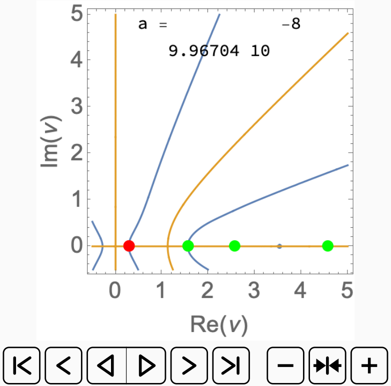


$$GN^2H^2 = 2\pi \; \frac{\pi\alpha}{N^2} = -1$$

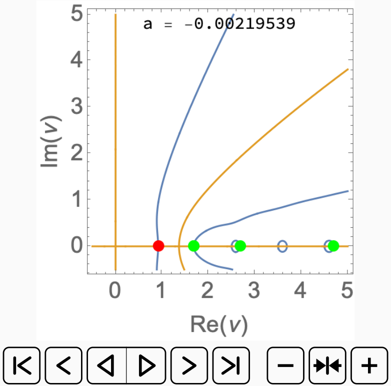


$$GN^2H^2 = 4\pi \; \frac{\pi\alpha}{N^2} = 0$$

Anti de Sitter tensor instability depending on the sign of a

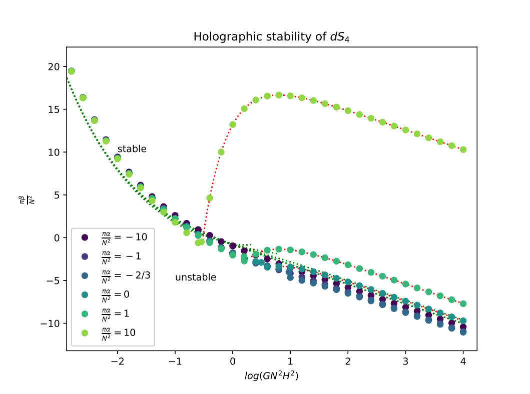


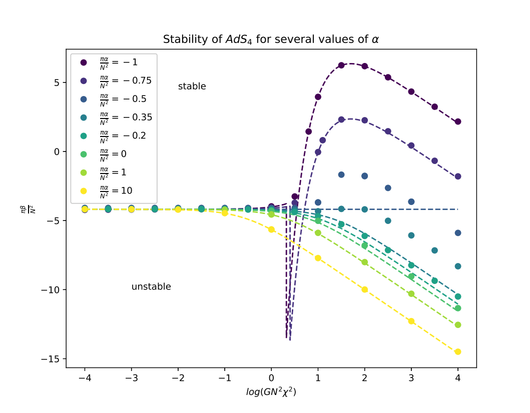
$GN^2\chi^2 = 2\pi \; \frac{\pi\alpha}{N^2} = 0$



$GN^2H^2 = 2\pi \; \frac{\pi\alpha}{N^2} = -2$

Spin-2 tachyon-instability





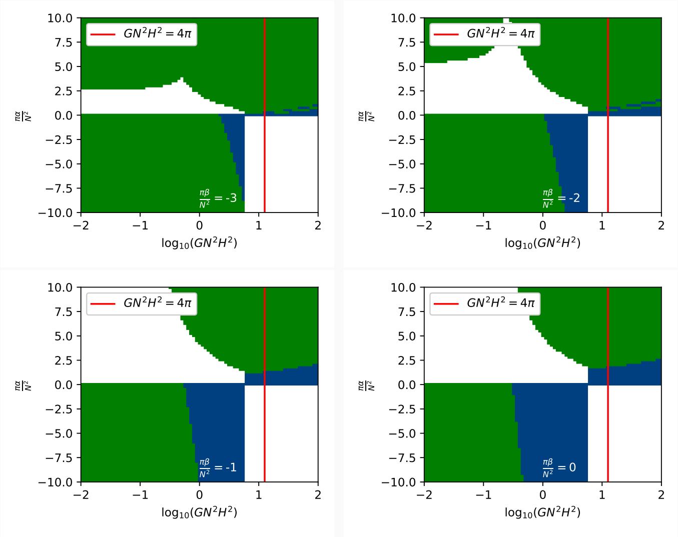
Tensor versus scalar instabilities

The Laplacian operator of (A)dS is diagonalized using scalar and tensor eigen-modes Ψ and $h^{(0)}_{\omega\sigma}$. For de Sitter, their asymptotic time dependence is $e^{\omega t}$, where

$$\omega = H(|\mathrm{Re}(\nu)| - 3/2).$$

The strongest *tachyonic* solution is either a tensor instability, a scalar instability or nothing is unstable. These three different regions can be explored in the parameter space (α, β and GN^2H^2).

Tensor versus scalar tachyonic rates



green : tensor tachyon dominates

white : scalar tachyon dominates

blue : tachyon-free

Conclusion and outlooks

Conclusion and outlooks

Conclusion

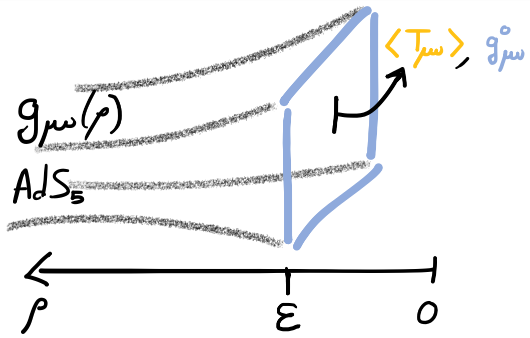
- We have extended the holographic 2-point functions of the stress tensor in a maximally symmetric spacetime with any curvature and determined stability (tachyonic and ghost-like).
- The tensor tachyonic strength can dominate the scalar mode which is usually studied in inflationary models.

Outlooks

- Study the QFT back-reaction by including a running scalar field in the bulk
- Generalize our results to a FLRW background geometry.

Appendix

The renormalized CFT stress tensor



$$ds^2 = \ell^2 \left[\frac{d\rho^2}{4\rho^2} + g^{\omega\sigma}(x,\rho) dx^\omega dx^\sigma \right]$$

$$g^{\omega\sigma}(x,\epsilon) = g^{(0)}_{\omega\sigma} + \epsilon g^{(2)}_{\omega\sigma}$$
$$+\epsilon^2 \left[g^{(4)}_{\omega\sigma} + \ln \epsilon \hat{g}_{\omega\sigma} \right] + \ldots$$

[de Haro, Skenderis, Solodukhin '01]

- If $g^{(0)}$ is the de Sitter metric, then $ds^2 = du^2 + (\sinh u)^2 ds^2_{dS}$ and $\rho \propto e^{-2u}$.
- If $g^{(0)}$ is arbitrary, $\langle T_{\omega\sigma} \rangle$ is a function of $g^{(0)}$ and $g^{(4)}$, while only $\mathrm{Tr} g^{(4)}$ is known in terms of $g^{(0)}$.

Scalar 2-point function

- Linearizing the trace of EFE :

$$\left[1+\frac{\alpha G}{4}\Box-\frac{GN^2R}{24\pi}\right](3\Box+R)\psi=0$$

Unphysical solution $3\Box+R=0$. [L. Alvarez-Gaume et. al Fortsch. Phys. 64 (2016)]

- Constraints from the non-diagonal components of the EFE ? We then need to know how $g^{(4)}$ varies with Ψ.
$\delta_\Psi g^{(4)}_{\mu\nu}$ is obtained by a bulk diffeomorphism which preserves Fefferman-Graham and evaluate as $\psi g^{(0)}_{\omega\sigma}$ on the boundary. [Imbimbo et. al Class. Quant. Grav. 17 (2000)]

The near-boundary tensor mode

- Near-boundary expansion of the bulk tensor perturbation

$$h_{\omega\sigma}(x,\rho) \underset{\rho\to 0}{=} h^{(0)}_{\omega\sigma}(x) + \rho h^{(2)}_{\omega\sigma}(x) + \rho^2 \left[h^{(4)}_{\omega\sigma}(x) + \ln \rho \hat{h}_{\omega\sigma}(x) \right] + \ldots$$

- The vev term of the metric expansion is expressed in terms of the source term by solving the bulk equation of motion, which is separable in ρ and x^ω. Solutions are then given in the space of eigentensors of the Lichnerowicz operator of the slice :

$$\left(H^{-2}\nabla^2 - 2\right) h^{(0)}_{\omega\sigma} = -\left(\nu^2 - \frac{9}{4}\right) h^{(0)}_{\omega\sigma}$$

ν is the momentum dual to the radial direction of the slice coordinates. In Poincaré coordinates, the 3 other directions possess translational symmetry.

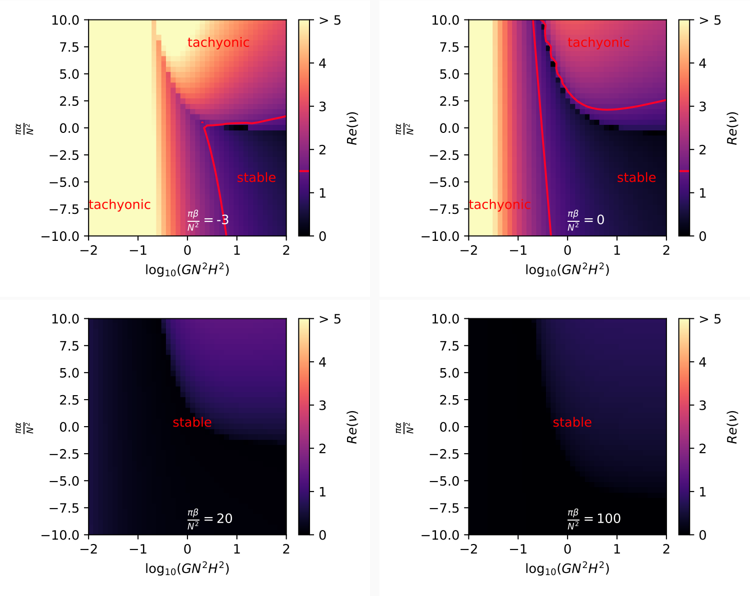
Stability conditions

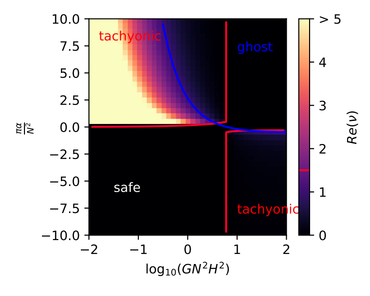
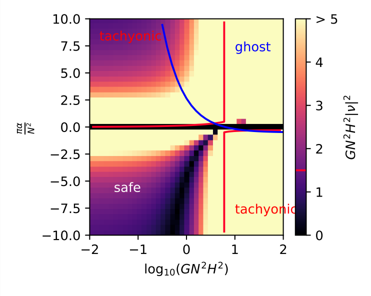
A spacetime is unstable iff the **retarded Green function** of metric perturbations diverges with time.

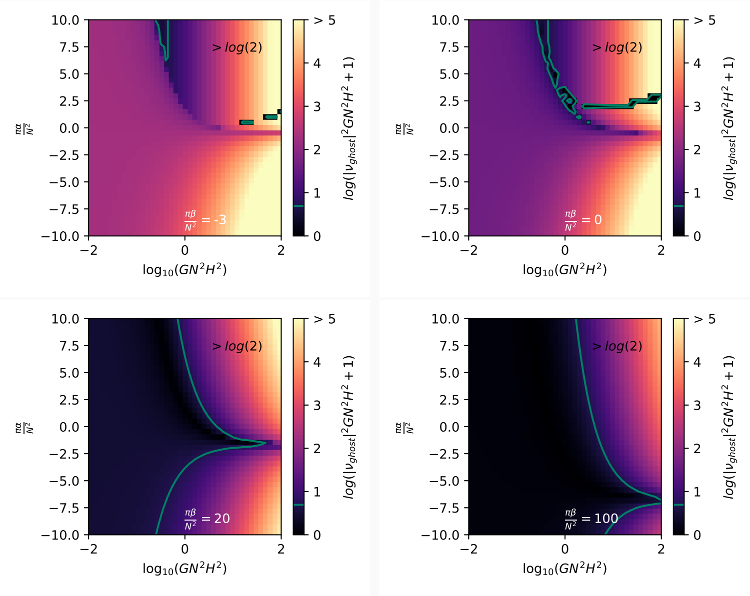
- Minkowski spacetime is unstable if there is a $k \in \mathbb{C}$ such that

$$(\nabla^2 + k^2) h^{(0)}_{\omega\sigma} = 0 \qquad \text{where} \qquad \mathrm{Re}(k) \neq 0. \tag{1}$$

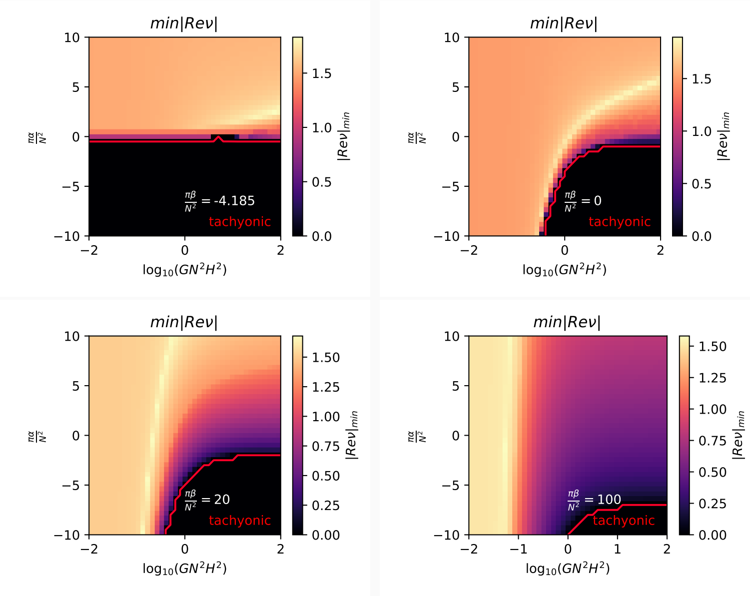
- dS is unstable if $|\mathrm{Re}(\nu)| > 3/2$ (saturated by the massless graviton)
- AdS is unstable if $\mathrm{Re}(\nu) = 0$ (saturated by the BF bound) [Breitenlohner, Freedman Annals of Phys. '82] But time is not the radial direction of AdS ! This bound is equivalent to asking if a solution which diverges with time is normalizable.

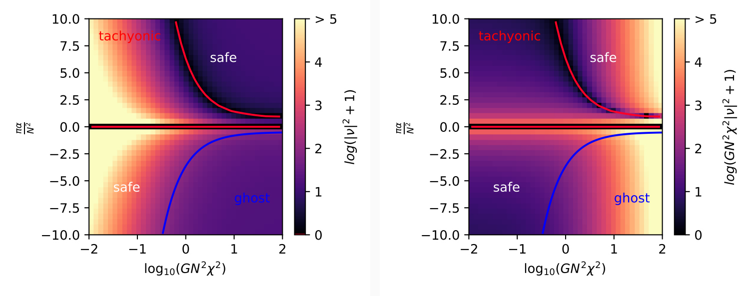

tachyonic
stable
tachyonic
$\frac{\pi\beta}{N^2}$ = -3
tachyonic
stable
tachyonic
$\frac{\pi\beta}{N^2}$ = 0
stable
$\frac{\pi\beta}{N^2}$ = 20
stable
$\frac{\pi\beta}{N^2}$ = 100
$\frac{\pi\alpha}{N^2}$
$\log_{10}(GN^2H^2)$
$Re(\nu)$
> 5

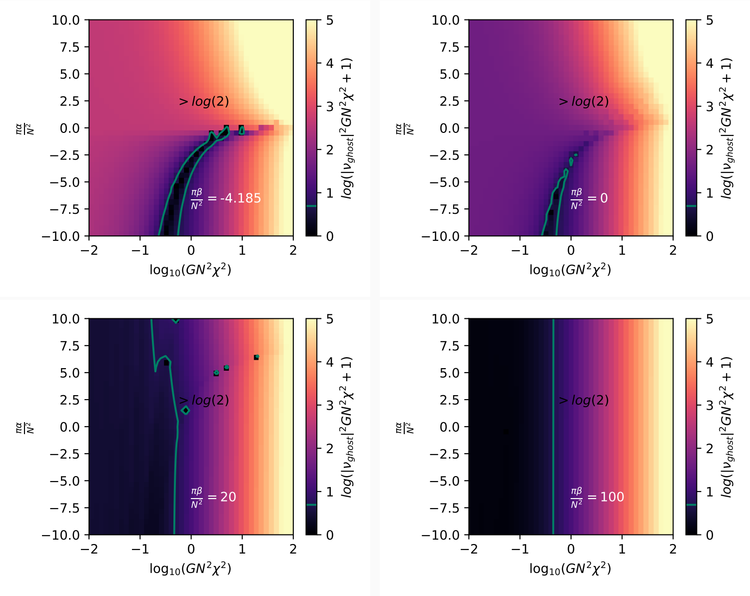

tachyonic
ghost
safe
tachyonic
$\frac{\pi\alpha}{N^2}$
$\log_{10}(GN^2H^2)$
$Re(\nu)$
> 5

tachyonic
ghost
safe
tachyonic
$\frac{\pi\alpha}{N^2}$
$\log_{10}(GN^2H^2)$
$GN^2H^2|\nu|^2$
> 5



> log(2)
πβ/N² = -3
πβ/N² = 0
πβ/N² = 20
πβ/N² = 100
log₁₀(GN²H²)
πα/N²
log(|ν_ghost|²GN²H² + 1)
> 5


min|Reν|
πα/N²
log₁₀(GN²H²)
|Reν|_min
πβ/N² = -4.185
tachyonic
πβ/N² = 0
tachyonic
πβ/N² = 20
tachyonic
πβ/N² = 100
tachyonic


10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−2
−1
0
1
2
$\log_{10}(GN^2\chi^2)$
$\frac{\pi\alpha}{N^2}$
tachyonic
safe
safe
ghost
> 5
4
3
2
1
0
$log(|\nu|^2+1)$
tachyonic
safe
safe
ghost
$\log_{10}(GN^2\chi^2)$
$log(GN^2\chi^2|\nu|^2+1)$



> log(2)
$\frac{\pi\beta}{N^2}$ = -4.185
$\frac{\pi\beta}{N^2}$ = 0
$\frac{\pi\beta}{N^2}$ = 20
$\frac{\pi\beta}{N^2}$ = 100
$\frac{\pi\alpha}{N^2}$
$\log_{10}(GN^2\chi^2)$
$log(|\nu_{ghost}|^2GN^2\chi^2+1)$
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−2
−1
0
1
2
3
4
5