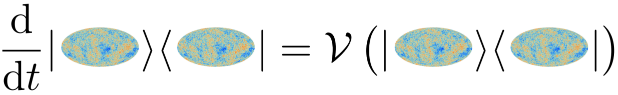
Cosmological master equations



Thomas Colas

TUG

Outline

1. From cosmology to quantum optics
2. Master equations in cosmology
3. Benchmarking cosmological master equations

Can we probe BSM physics using inflation ?

Inflation may:

- probe energy scales far above colliders;
- test quantum gravity in its perturbative regime.

Such an ambitious program needs the conjonction of **high precision data** and **efficient theoretical modelling**.

Cosmology enters a golden era.
New limiting factors:

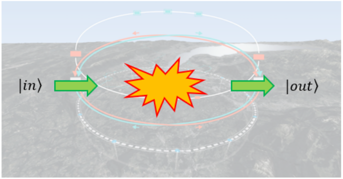
1. Computing times;
2. Systematics.

$\Rightarrow$ *Is our understanding of QFT in curved spacetime enough ?*

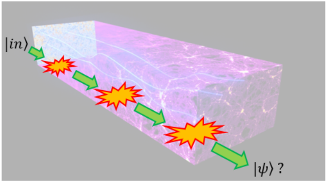
Scattering experiments versus cosmological histories

Two differences between flat and curved spacetime QFT based on [Burgess *et al.*, 2015]:

- Cumulative effects lead to **secular divergences** ⇒ Need late-time resummation.

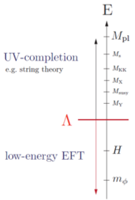


Credits : FCC, CERN

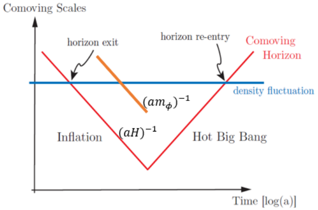


Credits : Planck, ESA

- Dynamical backgrounds **mixes scale hierarchies** ⇒ Need beyond Wilsonian EFT.



Credits : D. Baumann



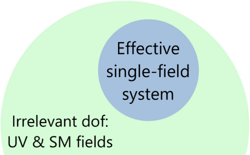
A minimal approach

We look for a framework that:

- go **beyond standard perturbative techniques**;
- explore **systematic extensions** around vanilla inflation.

It must incorporates prior knowledge:

1. **Single-field slow-roll inflation** provides an excellent fit of the data.
2. At some point, inflation must end: couple to **SM fields**.
3. **UV-completions** of inflation often introduce new degrees of freedom.



What is the quantum description of the effective single-field system ?

Outline

An example of extra ingredient: spectator field

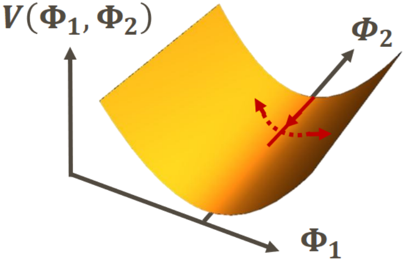
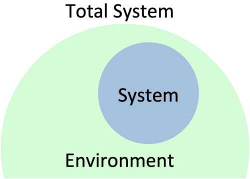


Figure: Adiabatic and entropic perturbations [Credits: L. Pinol]

- *WEFT result*: field stabilised, just a speed of sound at linear order.
- *OQS result*: curvature perturbations may decohere while interacting with isocurvature modes [Prokopec & Rigopoulos, 2007].

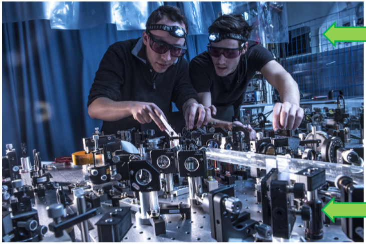
The early universe as an Open Quantum System (OQS)



- By integrating out the environment, the system dynamics becomes non-unitary.
- Cosmological perturbations are described by an OQS with dissipation and decoherence.
- They experience energy exchange and information loss into the environment.

Can we build an effective formalism which encompasses WEFT unitary results and OQS non-unitary evolution ?

The lab-based experiments wisdom



Credits: LKB

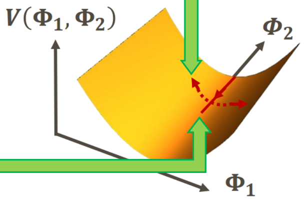
Environment:

- weakly specified

Interactions

System:

- measurable



Modelled by Master Equations

Outline

The master equation zoo

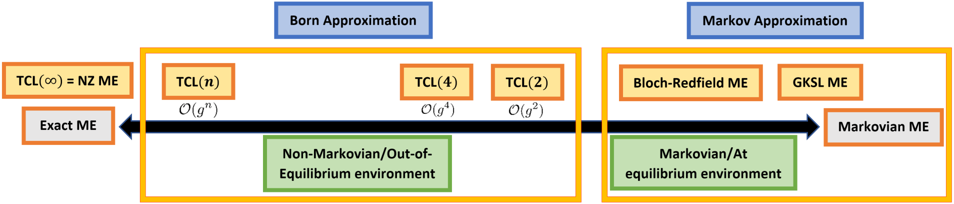
- Fundamental observables are correlators

$$\left\langle \widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \right\rangle (t) \equiv \mathrm{Tr}\left[\widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \widehat{\rho}_{\mathrm{red}}(t)\right]$$

- **Master Equations** (ME) are dynamical equations for $\widehat{\rho}_{\mathrm{red}}(t)$

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{V}\left(\widehat{\rho}_{\mathrm{red}}\right)$$

- There exists a **whole bestiary** of MEs [Breuer & Petruccione, 2002]

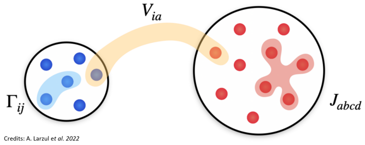


At which level should we work in cosmology ?

Master equations in cosmology

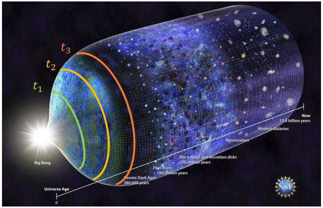
In **quantum optics**, Markovian MEs are ubiquitous:

1. Environments are *large*;
2. Environments are *stationary*;
3. Environments reach *thermal equilibrium*.



In **cosmology**, these assumptions must be reassessed:

1. Background is *dynamical*;
2. There is *no stationary* $|\mathrm{out}\rangle$ *state*;
3. Environments can be *out-of-equilibrium*.



Assessing cosmological master equations

- ME have already been applied in cosmology, see [Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015], [Hollowood & McDonald, 2017], [Martin & Vennin, 2018], [Brahma, Berera & Calderón-Figueroa, 2021], $\cdots$
- ME were **designed in a specific context** and **need some adaptations**
  $\Rightarrow$ Working in curved-space implies to reassess:
  - approximation schemes;
  - regimes of validity.
- We **benchmark the ME program** on an exactly solvable model [Colas, Grain & Vennin, 2022]:
  - We have analytic control on the system dynamics;
  - We compare exact and ME results.

Outline

The curved-space Caldeira-Leggett model

- Action for the field sector:

$$S = -\int \mathrm{d}^4 x \sqrt{-\det g}\left(\overbrace{\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2} m^2\varphi^2\right]}^{\text{System}} + \underbrace{\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2} M^2\chi^2\right]}_{\text{Environment}} + \underbrace{\lambda^2\varphi\chi}_{\text{Interaction}}\right)$$

- **Field redefinition**: rotation in field space of

$$\theta = -\frac{1}{2}\arctan\left(\frac{2\lambda^2}{M^2 - m^2}\right)$$

decouple the two sectors: fully integrable model.

- **Gaussian system**:
  - All information contained in the system covariance $\boldsymbol{\Sigma}_{\varphi\varphi}$;
  - Quantum information properties: $\gamma = \det\left[\boldsymbol{\Sigma}_{\varphi\varphi}\right]^{-1}/4$.

Integrating out the environment

- In the **Fock space**: a master equation, with $\widehat{\boldsymbol{z}} = (\widehat{v}_\varphi, \widehat{p}_\varphi)^{\mathrm{T}}$

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -i\,\overbrace{\left[\widehat{H}_0(\eta) + \widehat{H}^{(\mathrm{LS})}(\eta), \widehat{\rho}_{\mathrm{red}}(\eta)\right]}^{\text{Unitary evolution}} \underbrace{+ \sum_{i,j} \boldsymbol{\mathcal{D}}_{ij}(\eta)\left[\widehat{\boldsymbol{z}}_i\widehat{\rho}_{\mathrm{red}}(\eta)\widehat{\boldsymbol{z}}_j - \frac{1}{2}\left\{\widehat{\boldsymbol{z}}_j\widehat{\boldsymbol{z}}_i, \widehat{\rho}_{\mathrm{red}}(\eta)\right\}\right]}_{\text{Non-unitary evolution}}$$

- In the **phase space**: a Fokker-Planck equation, with $\boldsymbol{z} = (v_\varphi, p_\varphi)^{\mathrm{T}}$

$$\frac{\mathrm{d}W_{\mathrm{red}}}{\mathrm{d}\eta} = \overbrace{\left\{\widetilde{H}_0(\eta) + \widetilde{H}^{(\mathrm{LS})}(\eta), W_{\mathrm{red}}(\eta)\right\}}^{\text{Unitary evolution}} \underbrace{+\boldsymbol{\Delta}_{12}(\eta)\sum_i \frac{\partial}{\partial \boldsymbol{z}_i}\left[\boldsymbol{z}_i W_{\mathrm{red}}(\eta)\right] - \frac{1}{2}\sum_{i,j}\left[\boldsymbol{\omega}\boldsymbol{D}(\eta)\boldsymbol{\omega}\right]_{ij}\frac{\partial^2 W_{\mathrm{red}}(\eta)}{\partial \boldsymbol{z}_i \partial \boldsymbol{z}_j}}_{\text{Non-unitary evolution}}$$

Benchmarking cosmological ME

- For the ME to be an interesting tool, it **needs to do better than standard techniques**.
- Benchmark against standard perturbation theory (SPT) results: **in-in formalism at linear order**:
  1. Write down mode functions equations of motion;
  2. Solve them perturbatively order by order.

  $\Rightarrow$ Observables are computed at a given order in λ^2.
- We compare ME and SPT against exact results on:
  1. Accuracy on the **system covariance** $\mathbf{\Sigma}_{\varphi\varphi}$;
  2. Ability to recover the **purity** $\gamma = \det\left[\mathbf{\Sigma}_{\varphi\varphi}\right]^{-1}/4$.

Non-perturbative resummation

- ME studied in cosmology for its ability to resum late-time secular effects
  [Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015], [Brahma, Berera & Calderón-Figueroa, 2021]

- Resumation obtained when solving the effective transport equation

$$\frac{\mathrm{d}\boldsymbol{\Sigma}_{\varphi\varphi}}{\mathrm{d}\eta} = \underbrace{\boldsymbol{\omega}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\Sigma}_{\varphi\varphi}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\omega}}_{\text{Unitary evolution}} - \underbrace{2\boldsymbol{\Delta}_{12}\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\omega}\boldsymbol{D}\boldsymbol{\omega}}_{\text{Non-unitary evolution}}$$

  non-perturbatively, considering ME as a *bona fide* dynamical map.

- **General idea** : ME provides the cumulative effect of perturbative corrections.

- **Caveat** : need to first remove spurious terms related to *non-Markovianity* and *nature of the resummation*.

Spurious terms (1)

Effect of the environment encoded in

$$\boldsymbol{D}(\eta) = g^2 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \boldsymbol{G}_\varphi\left(\eta, \eta'\right) \mathrm{Re}\left\{\mathcal{K}^>(\eta, \eta')\right\} = \boldsymbol{F}_{\boldsymbol{D}}(\eta) - \boldsymbol{F}_{\boldsymbol{D}}(\eta_0)$$

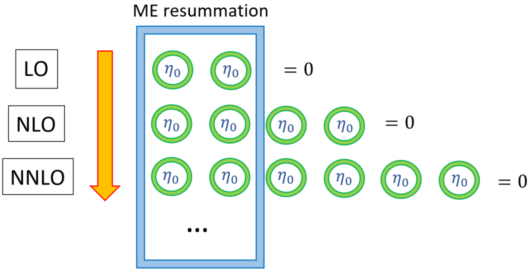
$$\boldsymbol{\Delta}(\eta) = g^2 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \boldsymbol{G}_\varphi\left(\eta, \eta'\right) \mathrm{Im}\left\{\mathcal{K}^>(\eta, \eta')\right\} = \boldsymbol{F}_{\boldsymbol{\Delta}}(\eta) - \boldsymbol{F}_{\boldsymbol{\Delta}}(\eta_0)$$

with the **memory kernel** $\mathcal{K}^>(\eta, \eta') = \langle \widehat{v}_\chi(\eta) \widehat{v}_\chi(\eta') \rangle_{\mathrm{free}}$.

- The η_0-dependent terms cancel each other when computing the observables **perturbatively** at any order.
- If we **include them in the non-perturbative resummation**, ME results diverge from exact ones at late time.

Spurious terms (2)

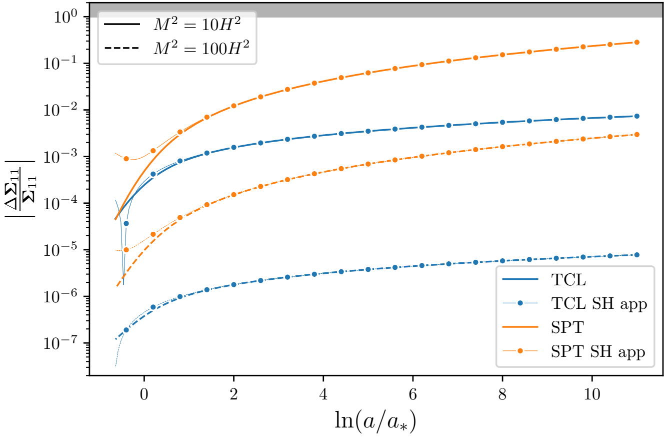
Explanation: partial resummation breaks order-by-order relations:



- TCL_n has **all terms** of order g^n and **some terms** of order $g^{m>n}$;
- The **cancellation requires all η_0-dependent terms** at a given order.

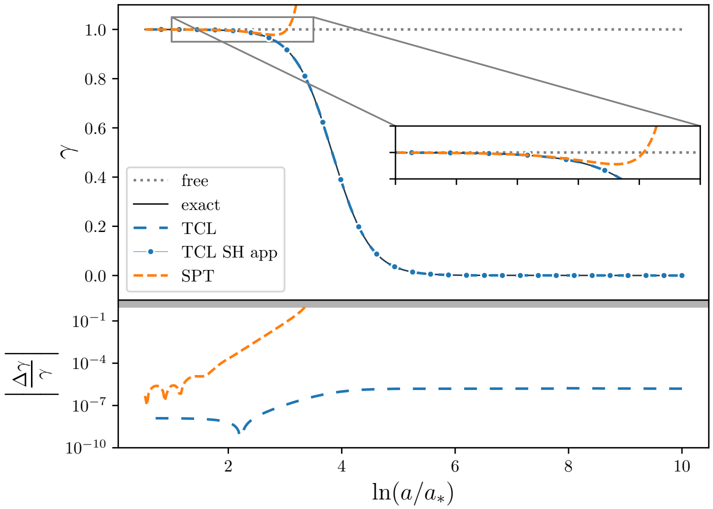
$\Rightarrow$ Need to impose the broken relation by hand before resumming.

Improved late-time results



Error: SPT $\sim \frac{\lambda^8}{M^4} \ln^2 a$; ME $\sim \frac{\lambda^8}{M^6} \ln a$

Access quantum information properties



Summary and outlook

- Probing new physics with cosmology requires efficient and reliable tools.
  - **OQS** useful to parametrize systematic extensions to **SM**.
- **ME** may allow us to go beyond perturbative techniques.
  - Approximation schemes et regime of validity must be **reassessed**.
- We benchmarked cosmological ME on an integrable model:
  1. Remove **spurious terms** to obtain **non-perturbative resummation**.
  2. ME provides **cumulative effect** of **perturbative corrections**.
  3. Results on **observables** and **QI** properties **strongly improved**.

Future directions:

- **Non-linear theory**: quasi-single field inflation [Chen & Wang, 2010]
  $\Rightarrow$ Which relation with PNG generation ? [Assassi *et al.*, 2013]

Thank you for your attention !

More details in arXiv:2209.01929

Outline

Flat vs curved-space Caldeira-Leggett model

Flat space

- System: a harmonic oscillator of frequency

$$\omega^2 = k^2 + m^2$$

- Environment : large number of harmonic oscillators
- Linear interaction:

$$\widehat{H}_{\boldsymbol{k}}^{\mathrm{int}} = \sum_{\boldsymbol{q}} \lambda_{\boldsymbol{q}}^2 \widehat{v}_{\boldsymbol{k}}^{(S)} \widehat{v}_{\boldsymbol{q}}^{(E)}$$

$\Rightarrow$ system interacts with infinitely many dof.

Curved space

- System: a parametric oscillator of frequency

$$\omega^2 = k^2 + m^2 a^2 - a''/a$$

- Environment : large number of parametric oscillators BUT
- Linear interaction + symmetries

$$\widehat{H}_{\boldsymbol{k}}^{\mathrm{int}} = \lambda^2 a^2 \widehat{v}_{\boldsymbol{k}}^{(S)} \widehat{v}_{-\boldsymbol{k}}^{(E)}$$

$\Rightarrow$ system only interacts with ONE environmental dof.

Analytic results on the covariance

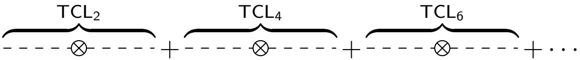
Integrating the transport equation:

$$\begin{aligned}\boldsymbol{\Sigma}_{\varphi\varphi}(\eta) = {} & e^{-2\int_{\eta_0}^{\eta}\mathrm{d}\eta' \boldsymbol{\Delta}_{12}(\eta')}\boldsymbol{g}_{\mathrm{LS}}(\eta,\eta_0)\boldsymbol{\Sigma}_{\varphi\varphi}(\eta_0)\boldsymbol{g}_{\mathrm{LS}}^{\mathrm{T}}(\eta,\eta_0) \\ & -\int_{\eta_0}^{\eta}\mathrm{d}\eta' e^{-2\int_{\eta'}^{\eta}\mathrm{d}\eta'' \boldsymbol{\Delta}_{12}(\eta'')}\boldsymbol{g}_{\mathrm{LS}}(\eta,\eta')\left[\boldsymbol{\omega}\boldsymbol{D}(\eta')\boldsymbol{\omega}\right]\boldsymbol{g}_{\mathrm{LS}}^{\mathrm{T}}(\eta,\eta').\end{aligned}$$

What is being resummed ?

- In the **exact theory**, there is only one 1PI:

- In the **effective theory**, there is an infinite tower of 1PI:



one for each of the TCL cumulant.

- Moreover, there are **non-unitary contributions** from diffusion and dissipation which do not have diagrammatic representation.
- Hence, the question of knowing **which diagram has been resumed** is **ill-posed**. This feature is shared with WEFT and the DRG.

Perturbative ME is SPT

- ME generates an effective transport equation:

$$\frac{\mathrm{d}\boldsymbol{\Sigma}_{\varphi\varphi}}{\mathrm{d}\eta} = \underbrace{\boldsymbol{\omega}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\Sigma}_{\varphi\varphi}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\omega}}_{\text{Unitary evolution}} \underbrace{- 2\boldsymbol{\Delta}_{12}\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\omega}\boldsymbol{D}\boldsymbol{\omega}}_{\text{Non-unitary evolution}}$$

- If we solve the TCL_2 ME **perturbatively** at LO $\mathcal{O}(\lambda^4)$, we recover exactly SPT results.

- We explicitly checked at **NLO** $\mathcal{O}(\lambda^8)$ **that SPT and TCL_4 match**. The proof is generalisable at any order:

$$\widetilde{\rho}^{(n)}_{\mathrm{red}}(\eta) = (-i)^n \lambda^{2n} \mathrm{Tr}_E \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 \cdots \int_{\eta_0}^{\eta_{n-1}} \mathrm{d}\eta_n \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta_1), \left[\cdots \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta_n), \widehat{\rho}_{\mathrm{i}}\right]\right]\right]$$

$$\left\langle \widehat{O}(\eta)\right\rangle^{(n)} = i^n \lambda^{2n} \left\langle \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 \cdots \int_{\eta_0}^{\eta_{n-1}} \mathrm{d}\eta_n \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta_n), \left[\cdots \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta_1), \widetilde{O}(\eta)\right]\right]\right] \right\rangle_{\widehat{\rho}_{\mathrm{i}}}$$

SPT results (1)

Mode function decomposition

$$\widehat{v}_\varphi(\eta) = v_{\varphi\varphi}(\eta)\widehat{a}_\varphi + v^*_{\varphi\varphi}(\eta)\widehat{a}^\dagger_\varphi + v_{\varphi\chi}(\eta)\widehat{a}_\chi + v^*_{\varphi\chi}(\eta)\widehat{a}^\dagger_\chi$$
$$\widehat{v}_\chi(\eta) = v_{\chi\varphi}(\eta)\widehat{a}_\varphi + v^*_{\chi\varphi}(\eta)\widehat{a}^\dagger_\varphi + v_{\chi\chi}(\eta)\widehat{a}_\chi + v^*_{\chi\chi}(\eta)\widehat{a}^\dagger_\chi$$

which obey **equations of motion**

$$v''_{\varphi\varphi} + \omega^2_\varphi(\eta)v_{\varphi\varphi} = -\lambda^2 a^2(\eta)v_{\chi\varphi}$$
$$v''_{\chi\varphi} + \omega^2_\chi(\eta)v_{\chi\varphi} = -\lambda^2 a^2(\eta)v_{\varphi\varphi}$$

and

$$v''_{\chi\chi} + \omega^2_\chi(\eta)v_{\chi\chi} = -\lambda^2 a^2(\eta)v_{\varphi\chi}$$
$$v''_{\varphi\chi} + \omega^2_\varphi(\eta)v_{\varphi\chi} = -\lambda^2 a^2(\eta)v_{\chi\chi}$$

SPT results (2)

Solution **order by order**

- Zeroth order:

$$
\begin{aligned}
v^{(0)}_{\varphi\varphi}(\eta) &= v_\varphi(\eta) \\
v^{(0)}_{\chi\chi}(\eta) &= v_\chi(\eta)
\end{aligned}
$$

  and $v^{(0)}_{\varphi\chi}(\eta) = v^{(0)}_{\chi\varphi}(\eta) = 0$.

- First order:

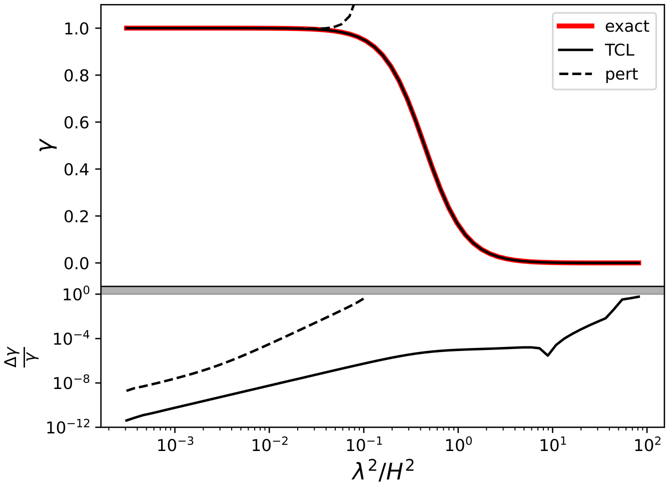
$$
\begin{aligned}
v^{(1)}_{\varphi\chi}(\eta) &= -2\lambda^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \, a^2(\eta_1) \, \mathrm{Im}\left\{ v_\varphi(\eta) v^*_\varphi(\eta_1) \right\} v_\chi(\eta_1) \\
v^{(1)}_{\chi\varphi}(\eta) &= -2\lambda^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \, a^2(\eta_1) \, \mathrm{Im}\left\{ v_\chi(\eta) v^*_\chi(\eta_1) \right\} v_\varphi(\eta_1).
\end{aligned}
$$

- $\cdots$

Correlators are evaluated in the Heisenberg picture

$$
\boldsymbol{\Sigma}(\eta) = \frac{1}{2} \mathrm{Tr}\left[\left\{ \widehat{\boldsymbol{z}}(\eta), \widehat{\boldsymbol{z}}^{\mathrm{T}}(\eta) \right\} \widehat{\rho}_0 \right]
$$

Purity and coupling



Outline

The environment as noises

Classical Brownian motion

- Langevin equation

$$\mathrm{d}\mathcal{O} = \{H, O\}\mathrm{d}t + \mathrm{d}\xi$$

- Fokker-Planck equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}_{\mathrm{FP}}[P]$$

- Wiener path integral [Wiener, 1923]

$$P = \int \mathcal{D}q e^{-S_0(q)}$$

Quantum Brownian motion

- Stochastic Schrödinger equation

$$|\mathrm{d}\psi\rangle = -i\left[\widehat{H}, \widehat{\mathcal{O}}\right]\mathrm{d}t + \mathrm{d}\widehat{\xi}$$

- Master equation

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{V}[\widehat{\rho}_{\mathrm{red}}]$$

- Influence functional [Vernon, 1959]

$$\widehat{\rho}_{\mathrm{red}} = \int \mathcal{D}\phi_0^{\pm}\mathcal{I}\left[\phi_0^{\pm}\right]\widehat{\rho}_{\mathrm{red},0}$$

Outline

Example 1: an exact ME

Master equation: dynamical equation for the quantum state of the system.

1. Start with Liouville-von Neumann equation

$$\frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}\eta} = -ig\left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta), \widetilde{\rho}(\eta)\right] \equiv g\mathcal{L}(\eta)\widetilde{\rho}(\eta)$$

2. Introduce projectors $\widetilde{\rho} \mapsto \mathcal{P}\widetilde{\rho} = \widetilde{\rho}_{\mathsf{red}} \otimes \rho_{\mathrm{E}}$ and $\mathcal{Q}\widetilde{\rho} = \widetilde{\rho} - \mathcal{P}\widetilde{\rho}$

3. Rewrite dynamics as

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \mathcal{K}(\eta, \eta')\mathcal{P}\widetilde{\rho}(\eta)$$

$\mathcal{K}(\eta, \eta')$: **memory kernel** which depends on **the coupling** and **the environment**.

Example 2: an effective ME

1. Expand in powers of **the coupling** constant

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = \sum_{n=0}^{\infty} g^n \mathcal{K}_n(\eta)\mathcal{P}\widetilde{\rho}(\eta)$$

2. Lowest order leads to the non-Markovian ME

$$\frac{\mathrm{d}\widetilde{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \, \mathrm{Tr}_E \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta), \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta'), \widetilde{\rho}_{\mathrm{red}}(\eta) \otimes \rho_{\mathrm{E}}\right]\right]$$

3. Error function $e_r^{(2)} \sim g^2 ||\mathcal{K}_4(\eta)||/||\mathcal{K}_2(\eta)||$.

Example 3: a Markovian ME

1. When the environment is a **bath** (large number of dof, thermal equilibrium), the dynamics is **Markovian**, the system admits a **semi-group evolution**

$$V(\eta_1)V(\eta_2) = V(\eta_1 + \eta_2)$$

.

2. It implies a specific form for the ME [Lindblad 1976]

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -i\left[\widehat{H}(\eta), \widehat{\rho}_{\mathrm{red}}(\eta)\right] + \sum_k \gamma_k \left[\widehat{\boldsymbol{L}}_k \widehat{\rho}_{\mathrm{red}}(\eta)\widehat{\boldsymbol{L}}_k^\dagger - \frac{1}{2}\left\{\widehat{\boldsymbol{L}}_k^\dagger \widehat{\boldsymbol{L}}_k, \widehat{\rho}_{\mathrm{red}}(\eta)\right\}\right]$$

3. It relies on a fast decay of temporal correlations in the environment.

Question: At which level should we work in cosmology ?

Cosmological environments

In cosmology, we have **background symmetries**: *homogeneity* and *isotropy*

- At *linear order*, there is **no mode coupling**:

$$\left\langle \widehat{\mathcal{O}}_{\boldsymbol{k}} \widehat{\mathcal{O}}_{\boldsymbol{q}} \right\rangle \propto \delta(\boldsymbol{k}+\boldsymbol{q})$$

  $\Rightarrow$ only way to get a large number of dof: infinitely many environmental fields.

- At *non-linear order*, **environmental self-interactions** may relax the constraint on the number of fields.

  $\Rightarrow$ induce primordial non-Gaussianities (PNG): observationally constrained [Assassi *et al.*, 2014].

Conclusion: not so common to have a Markovian ME in cosmology.

The emergence of Markovianity

- Fast decay of environmental correlations

$$\mathcal{K}^{>}(\eta,\eta') \xrightarrow[\text{graining}]{\text{coarse-}} \delta(\eta-\eta')$$

ME reduces to a GKSL equation for which the dynamical map reads

$$\mathcal{L}\left[\widehat{\rho}_{\text{red}}\right] = -i\left[\widehat{\mathcal{H}},\widehat{\rho}_{\text{red}}\right] + \gamma\left(\widehat{L}\widehat{\rho}_{\text{red}}\widehat{L}^{\dagger} - \frac{1}{2}\left\{\widehat{L}^{\dagger}\widehat{L},\widehat{\rho}_{\text{red}}\right\}\right)$$

GKSL equation is CPTP: physical consistency of the solutions ensured.

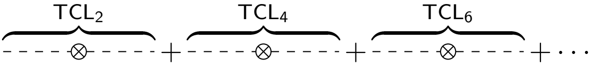
- Non-Markovian evolution/non-semigroup dynamical map implies dissipator matrix non-positive semi-definite.
- Non-positive semi-definite dissipator matrix is a generic feature of Non-Markovian OQS: not directly related to CPTP properties.
- Curved-space Caldeira-Leggett model ME belongs to the class of Gaussian non-Markovian ME $\Rightarrow$ CPTP ensured by [Diósi & Ferialdi, 2014].

Outline

What has been resummed ?

- In the **exact theory**, there is only one 1PI:

- In the **effective theory**, there is an infinite tower of 1PI:



  one for each of the TCL cumulant.

- Moreover, there are **non-unitary contributions** from diffusion and dissipation which do not have diagrammatic representation.
- Hence, the question of knowing **which diagram has been resumed** is **ill-posed**. This feature is shared with WEFT and the DRG.

Late-time resummation technique

Following [Boyanovsky, 2015], [Brahma *et al.*, 2021],

$$\langle\widetilde{v}_\varphi(\eta)\widetilde{v}_\varphi(\eta)\rangle = v_-(\eta)v_-(\eta)\left\langle\widehat{P}_\varphi^2\right\rangle + v_-(\eta)v_+(\eta)\left\langle\widehat{Q}_\varphi\widehat{P}_\varphi + \widehat{P}_\varphi\widehat{Q}_\varphi\right\rangle$$
$$+v_+(\eta)v_+(\eta)\left\langle\widehat{Q}_\varphi^2\right\rangle \to v_+(\eta)v_+(\eta)\left\langle\widehat{Q}_\varphi^2\right\rangle$$

with

$$\frac{\mathrm{d}\left\langle\widehat{Q}_\varphi^2\right\rangle}{\mathrm{d}\eta} = \Gamma(\eta)\left\langle\widehat{Q}_\varphi^2\right\rangle$$

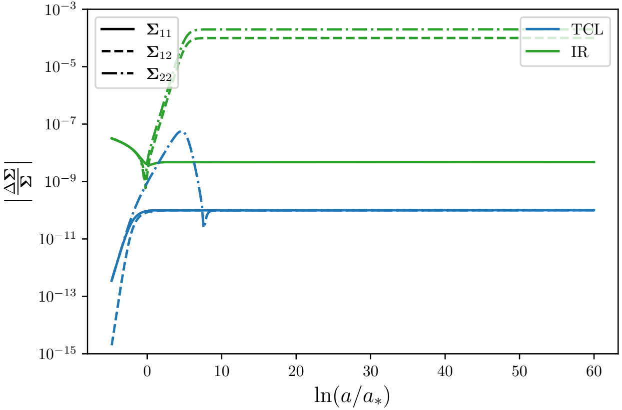
obtained from the TCL_2 ME.

In the *curved-space Caldeira-Leggett model*, leads to

$$\boldsymbol{\Sigma}_{\varphi\varphi}^{\mathrm{TCL}} \supset e^{-\frac{1}{\nu_\varphi}\frac{H^2}{M^2-m^2}\frac{\lambda^4}{H^4}\ln(-k\eta)}\boldsymbol{\Sigma}_{\varphi\varphi}^{(0)}$$

where late-time secular effects have been resummed.

Late-time resummation and the DRG



This resummation technique shares many features with the DRG [Burgess *et al.*, 2009].

Are they equivalent ?

Outline

TCL$_4$ generator

$$\mathcal{K}_4(\eta) = \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 \int_{\eta_0}^{\eta_2} \mathrm{d}\eta_3 \Big[\mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_1)\mathcal{L}(\eta_2)\mathcal{L}(\eta_3)\mathcal{P} - \mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_1)\mathcal{P}\mathcal{L}(\eta_2)\mathcal{L}(\eta_3)\mathcal{P} - \mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_2)\mathcal{P}\mathcal{L}(\eta_1)\mathcal{L}(\eta_3)\mathcal{P} - \mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_3)\mathcal{P}\mathcal{L}(\eta_1)\mathcal{L}(\eta_2)\mathcal{P}\Big]$$

TCL$_4$ master equation

$$
\begin{aligned}
\frac{\mathrm{d}\widetilde{\rho}_{\mathrm{red}}^{\mathrm{TCL}_4}}{\mathrm{d}\eta} = \frac{\mathrm{d}\widetilde{\rho}_{\mathrm{red}}^{\mathrm{TCL}_2}}{\mathrm{d}\eta} - 4\lambda^8 a^2(\eta)\int_{\eta_0}^{\eta}\mathrm{d}\eta_1 a^2(\eta_1)\int_{\eta_0}^{\eta_1}\mathrm{d}\eta_2 a^2(\eta_2)\int_{\eta_0}^{\eta_2}\mathrm{d}\eta_3 a^2(\eta_3) \\
\times\Bigg\{ \mathrm{Im}\Big\{v_\chi(\eta)v_\chi^*(\eta_2)\Big\}\,\mathrm{Re}\Big\{v_\chi(\eta_1)v_\chi^*(\eta_3)\Big\}\,\mathrm{Im}\Big\{v_\varphi(\eta_1)v_\varphi^*(\eta_2)\Big\}\,[\widetilde{v}_\varphi(\eta),[\widetilde{v}_\varphi(\eta_3),\widetilde{\rho}_{\mathrm{red}}(\eta)]] \\
+ i\,\mathrm{Im}\Big\{v_\chi(\eta)v_\chi^*(\eta_2)\Big\}\,\mathrm{Im}\Big\{v_\chi(\eta_1)v_\chi^*(\eta_3)\Big\}\,\mathrm{Im}\Big\{v_\varphi(\eta_1)v_\varphi^*(\eta_2)\Big\}\,[\widetilde{v}_\varphi(\eta),\{\widetilde{v}_\varphi(\eta_3),\widetilde{\rho}_{\mathrm{red}}(\eta)\}] \\
+ \mathrm{Im}\Big\{v_\chi(\eta)v_\chi^*(\eta_3)\Big\}\,\mathrm{Re}\Big\{v_\chi(\eta_1)v_\chi^*(\eta_2)\Big\}\,\mathrm{Im}\Big\{v_\varphi(\eta_1)v_\varphi^*(\eta_3)\Big\}\,[\widetilde{v}_\varphi(\eta),[\widetilde{v}_\varphi(\eta_2),\widetilde{\rho}_{\mathrm{red}}(\eta)]] \\
+ i\,\mathrm{Im}\Big\{v_\chi(\eta)v_\chi^*(\eta_3)\Big\}\,\mathrm{Im}\Big\{v_\chi(\eta_1)v_\chi^*(\eta_2)\Big\}\,\mathrm{Im}\Big\{v_\varphi(\eta_1)v_\varphi^*(\eta_3)\Big\}\,[\widetilde{v}_\varphi(\eta),\{\widetilde{v}_\varphi(\eta_2),\widetilde{\rho}_{\mathrm{red}}(\eta)\}] \\
+ \Bigg[- \mathrm{Re}\Big\{v_\chi(\eta)v_\chi^*(\eta_3)\Big\}\,\mathrm{Im}\Big\{v_\chi(\eta_1)v_\chi^*(\eta_2)\Big\} + \mathrm{Im}\Big\{v_\chi(\eta)v_\chi^*(\eta_3)\Big\}\,\mathrm{Re}\Big\{v_\chi(\eta_1)v_\chi^*(\eta_2)\Big\}\Bigg] \\
\mathrm{Im}\Big\{v_\varphi(\eta_2)v_\varphi^*(\eta_3)\Big\}\,[\widetilde{v}_\varphi(\eta),[\widetilde{v}_\varphi(\eta_1),\widetilde{\rho}_{\mathrm{red}}(\eta)]] \Bigg\}.
\end{aligned}
$$

Equivalence between perturbative TCL and in-in formalism

Cosmologists are used to compute correlators using the **in-in formalism**.

- At linear order, it is similar to the perturbative results presented above.

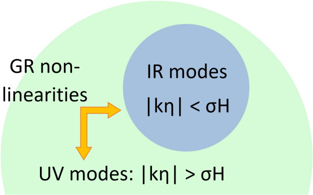
We have shown that:

- Perturbative TCL_2 is equivalent to $\mathcal{O}(\lambda^4)$ in-in.
- Perturbative TCL_4 is equivalent to $\mathcal{O}(\lambda^8)$ in-in.

Probably the proof **extend at all order**. Indeed, from the TCL cumulant expansion, all terms at a given order are included. It should ensure the matching with the in-in formalism at a given order.

Outline

The OpenEFT formalism



In [Brahma *et al.*, 2020], the leading cubic contribution is

$$H_{\text{int}} = \frac{M_{\text{Pl}}^2}{2} \int \mathrm{d}^3 x \varepsilon_H^2 a \zeta^2 \partial^2 \zeta$$

- UV modes backreact on the IR dynamics.
- They induce decoherence of the IR sector.