Muon Anti-muon creation in Photon-Photon collision in Quantum Electrodynamics

Bachelor thesis 2022

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Introduction

- The creation of a pair muon and anti-muon in the photon-photon collision cannot be explained by Classical Electrodynamics.
- We will use QED to explain this, it also gives us some results that can be measure in experiments.
- For leading order, I will apply Wick's theorem and use Feynman rules to reach the scattering amplitude, then we can calculate the cross section and differential cross section.
- For next-to-leading order, I will show all contributing Feynman diagrams and their scattering amplitudes.

Lagrangian for QED

The Lagrangian of Quantum Electrodynamics is:

$$\mathcal{L}_{QED} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - eA_{\mu}\bar{\psi}\gamma^{\mu}\psi \tag{1}$$

The mostly concerned term in this Lagrangian is the interaction term $-eA_{\mu}\bar{\psi}\gamma^{\mu}\psi$, which describes the interaction between fermion field and photon field.

S-matrix

After defining Lagrangian of the process, the next step for us is calculate the S-matrix, which is defined by:

$$S = T\exp\left[-i\int_{-\infty}^{\infty} dt \mathcal{L}_I(t)\right] \tag{2}$$

$$= \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} d^4x_1 \cdots \int_{-\infty}^{\infty} d^4x_n T\{\mathcal{L}_I(x_1)\cdots\mathcal{L}_I(x_n)\} \tag{3}$$

From the calculation result of S-matrix, we can get scattering amplitude by:

$$\langle f|S|i\rangle = i(2\pi)^4\mathcal{M}\delta^4(p_i - p_f) \tag{4}$$

where the delta Dirac function $\delta^4(p_i - p_f)$ show us the conservation of momentum of the process.

Wick's theorem

The problem in calculating S-matrix is the time ordering operation $T\{\mathcal{L}_I(x_1)\cdots\mathcal{L}_I(x_n)$.
Wick's theorem show that the time ordering product can be expressed as a sum of normal ordering of all possible contractions:

$$T\left\{\phi_1(x_1)\cdots\phi_1(x_n)\right\} =: \phi_1(x_1)\cdots\phi_1(x_n) + \text{all possible contractions}: \tag{5}$$

where normal ordering, presented with colons, showing every creation operators are sorted on the left and annihilation operators are sorted on the right.

Feynman rules and definition of fields

Feynman rules for QED:

Initial fermion :$u(s,p)$

Initial anti fermion : $\bar{v}(s,p)$

Initial photon : $\epsilon_\mu(\lambda,p)$

Vertex : $-ie\gamma^\mu$

Photon propagator : $iS_{F\mu\nu} = \frac{-ig_{\mu\nu}}{q^2+i\epsilon}$

Final fermion : $\bar{u}(s,p)$

Final anti fermion : $v(s,p)$

Final photon: $\epsilon^*_\mu(\lambda,p)$

Fermion propagator : $iD_F(x-y) = \frac{i(\not{p}-m)}{p^2-m^2+i\epsilon}$

Fields definition:

Fermion fields:

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_{s=1}^{2} \left[c^s_{\vec{p}} u^s(\vec{p}) e^{-ipx} + b^{s\dagger}_{\vec{p}} v^s(\vec{p}) e^{ipx} \right] \tag{6}$$

Photon field:

$$A_\mu(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_{\lambda=0}^{3} \left(\epsilon^\lambda_{\mu\vec{p}} a^\lambda_{\vec{p}} e^{-ipx} + \epsilon^{\lambda *}_{\mu\vec{p}} a^{\lambda\dagger}_{\vec{p}} e^{ipx} \right) \tag{7}$$

Leading order

The leading order term of the scattering amplitude:

$$\langle f|S|i\rangle = \frac{(-i)^2}{2!}\int_{-\infty}^{\infty} d^4x d^4y \langle f|T\{\mathcal{L}_I(x)\mathcal{L}_I(y)\}|i\rangle \tag{8}$$

The initial and final state of the process are defined by:

$$|i\rangle = 2\sqrt{E_{\vec{p}_1}E_{\vec{p}_2}}\, a_{\vec{p}_1}^{\lambda_1\dagger} a_{\vec{p}_2}^{\lambda_2\dagger}|0\rangle \tag{9}$$

$$|f\rangle = 2\sqrt{E_{\vec{p}_3}E_{\vec{p}_4}}\, c_{\vec{p}_3}^{s_3\dagger} b_{\vec{p}_4}^{s_4\dagger}|0\rangle \tag{10}$$

The time ordering product after expressed by Wick's theorem will give us two contributing terms:

$$:\bar{\psi}_{x\alpha}\gamma^{\mu}_{\alpha\beta}\psi_{x\beta}\bar{\psi}_{y\sigma}\gamma^{\nu}_{\sigma\rho}\psi_{y\rho}A_{\mu x}A_{\nu y}: \tag{11}$$

$$:\bar{\psi}_{x\alpha}\gamma^{\mu}_{\alpha\beta}\psi_{x\beta}\bar{\psi}_{y\sigma}\gamma^{\nu}_{\sigma\rho}\psi_{y\rho}A_{\mu x}A_{\nu y}: \tag{12}$$

Leading order

After calculating, we can see that these two contributing terms give us the same result:

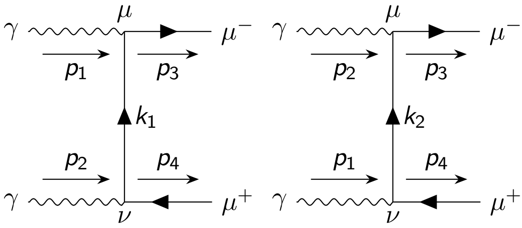
$$
\begin{aligned}
&2\pi^4 ie^2\bar{u}_3\gamma^\mu\left[\frac{\epsilon_{1\mu}(\not{k}_1+m)\epsilon_{2\nu}}{(k_1^2-m^2)}+\frac{\epsilon_{2\mu}(\not{k}_2+m)\epsilon_{1\nu}}{(k_2^2-m^2)}\right]\gamma^\nu v_4\delta^4(p_1+p_2-p_3-p_4)\\
=&2\pi^4 i\mathcal{M}\delta^4(p_i-p_f)\\
\Rightarrow& i\mathcal{M}=ie^2\bar{u}_3\gamma^\mu\left[\frac{\epsilon_{1\mu}(\not{k}_1+m)\epsilon_{2\nu}}{(k_1^2-m^2)}+\frac{\epsilon_{2\mu}(\not{k}_2+m)\epsilon_{1\nu}}{(k_2^2-m^2)}\right]\gamma^\nu v_4 \qquad (13)
\end{aligned}
$$

where

$$
\begin{cases}
k_1 = p_2 - p_4\\
k_2 = p_1 - p_4
\end{cases}
$$

Feynman diagrams for leading order

After applying Wick's theorem, we got two Feynman diagram:



(a) $\mathcal{M}_1$ (b) $\mathcal{M}_2$

$$\mathcal{M}_1 = e^2 \bar{u}_3 \gamma^\mu \epsilon_{\mu 1} \left(\frac{\not{k}_1 + m}{k_1^2 - m^2} \right) \epsilon_{\nu 2} \gamma^\nu v_4, \tag{14}$$

$$\mathcal{M}_2 = e^2 \bar{u}_3 \gamma^\mu \epsilon_{\mu 2} \left(\frac{\not{k}_2 + m}{k_2^2 - m^2} \right) \epsilon_{\nu 1} \gamma^\nu v_4. \tag{15}$$

Leading order result

Now we can find the scattering amplitude square of the process, sum and average over spins and polarizations:

$$\frac{1}{4}\sum_{\text{spins,pols}} |\mathcal{M}|^2 = \frac{1}{4}\sum_{\text{spins,pols}} (\mathcal{M}_1 + \mathcal{M}_2)(\mathcal{M}_1^* + \mathcal{M}_2^*) \tag{16}$$

We have these properties:

- For **particle** and **anti particle** when sum over spinors :

$$\textstyle\sum_{s=1}^{2} u_s(p)\bar{u}_s(p) = \not{p} + m, \tag{17}$$

$$\textstyle\sum_{s=1}^{2} v_s(p)\bar{v}_s(p) = \not{p} - m. \tag{18}$$

- For **photon** when we sum over polarizations:

$$\sum_{\lambda=0}^{3} \epsilon_{p\mu}^{\lambda}\epsilon_{p\nu}^{\lambda *} = -g_{\mu\nu}. \tag{19}$$

Scattering amplitude square

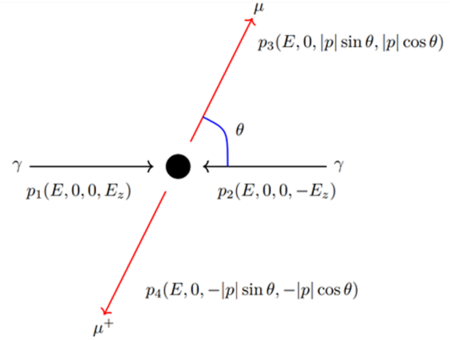
Scattering amplitude square for leading order is:

$$\frac{1}{4}\sum_{\text{spins,pols}} |\mathcal{M}|^2 = 2e^4\left[\frac{p_{14}}{p_{24}} + \frac{p_{24}}{p_{14}} + 2m^2\left(\frac{1}{p_{14}} + \frac{1}{p_{24}}\right) - m^4\left(\frac{1}{p_{14}} + \frac{1}{p_{24}}\right)^2\right]$$

Consider our process in center of mass frame:

$$\begin{aligned}
p_1 &= (E, 0, 0, E_z) \\
p_2 &= (E, 0, 0, -E_z) \\
p_3 &= (E, 0, p\sin(\theta), p\cos(\theta)) \\
p_4 &= (E, 0, -p\sin(\theta), -p\cos(\theta)) \\
|p| &= \sqrt{E^2 - m^2} \\
E &= E_{cm}/2
\end{aligned}$$

Center of mass frame



$$\frac{1}{4}\sum_{spins,pols}|\mathcal{M}|^2 = 2e^4\left[\frac{E^2+E|p|\cos\theta}{E^2-E|p|\cos\theta}+\frac{E^2-E|p|\cos\theta}{E^2+E|p|\cos\theta}\right.$$
$$+2m^2\left(\frac{1}{E(E-E|p|\cos\theta)}+\frac{1}{E(E+E|p|\cos\theta)}\right)$$
$$\left.-m^4\left(\frac{1}{E(E-E|p|\cos\theta)}+\frac{1}{E(E+E|p|\cos\theta)}\right)^2\right].$$

Crossing Symmetry

Crossing symmetry is another way to get scattering amplitude square from a similar process. Applying the crossing symmetry to the result of Compton scattering $\gamma + e^- \to e^- + \gamma$:

$$\frac{1}{4}\sum_{spin,pols.}|\mathcal{M}|^2 = 2e^4\left[\frac{q_{24}}{q_{12}} + \frac{q_{12}}{q_{24}} + 2m^2\left(\frac{1}{q_{12}} - \frac{1}{q_{24}}\right) + m^4\left(\frac{1}{q_{12}} - \frac{1}{p_{24}}\right)^2\right]$$

$$\begin{array}{cccccccc} \gamma & + & e^- & \longrightarrow & e^- & + & \gamma \\ q_1 & & q_2 & & q_3 & & q_4 \\ & & & & & & \\ \gamma & + & \gamma & \longrightarrow & \mu^- & + & \mu^+ \\ p_1 & & p_2 & & p_3 & & p_4 \end{array} \qquad \begin{cases} q_1 = p_1 \\ q_2 = -p_4 \\ q_3 = p_3 \\ q_4 = -q_2 \end{cases}.$$

We also get the result of $\gamma + \gamma \to \mu^- + \mu^+$ scattering process:

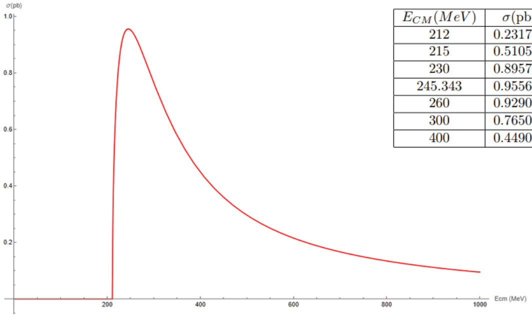
$$\frac{1}{4}\sum_{pols.}|\mathcal{M}|^2 = 2e^4\left[\frac{p_{24}}{p_{14}} + \frac{p_{14}}{p_{24}} + 2m^2\left(\frac{1}{p_{14}} + \frac{1}{p_{24}}\right) - m^4\left(\frac{1}{p_{12}} + \frac{1}{p_{24}}\right)^2\right]$$

Cross section result

We can calculate cross section analytically by:

$$\sigma = \int \frac{|p|}{32\pi^2 E_{cm}^3} \frac{1}{4} \sum |\mathcal{M}|^2 d\Omega$$

The relation between cross section σ and center of mass energy E_{cm} is showed in figure below:

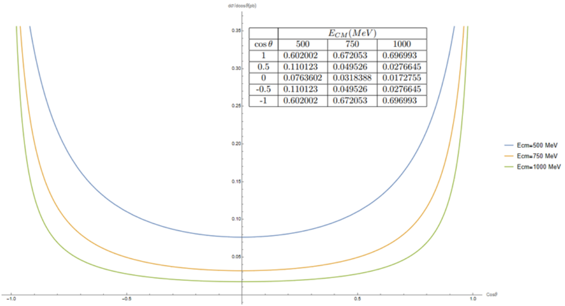


$E_{CM}(MeV)$	σ(pb)
212	0.23177
215	0.51057
230	0.895742
245.343	0.955664
260	0.929066
300	0.765009
400	0.449057

Differential cross section result

The relation between differential cross section $d\sigma/d\cos(\theta)$ and $\cos(\theta)$:

$$\frac{d\sigma}{d\cos\theta} = \frac{|p|}{32\pi E^3}\frac{1}{4}\sum|\mathcal{M}|^2$$



$\cos\theta$	$E_{CM}(MeV)$ 500	750	1000
1	0.602002	0.672053	0.696993
0.5	0.110123	0.049526	0.0276645
0	0.0763602	0.0318388	0.0172755
-0.5	0.110123	0.049526	0.0276645
-1	0.602002	0.672053	0.696993

Next-to-leading order

The next-to-leading order S-matrix is:

$$\langle f|S|i\rangle = \frac{(-i)^2}{4!}\int_{-\infty}^{\infty} d^4x d^4y d^4z d^4t \langle f|T\{\mathcal{L}_I(x)\mathcal{L}_I(y)\mathcal{L}_I(z)\mathcal{L}_I(t)\}|i\rangle \tag{20}$$

Let's pick a contributing term of Wick theorem's expression for time ordering to calculate:

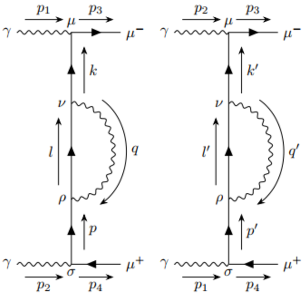
$$:\bar\psi_{x\alpha}\gamma^{\mu}_{\alpha\beta}\psi_{x\beta}\bar\psi_{y\gamma}\gamma^{\mu}_{\gamma\delta}\psi_{y\delta}\bar\psi_{z\epsilon}\gamma^{\rho}_{\epsilon\xi}\psi_{z\xi}\bar\psi_{t\eta}\gamma^{\sigma}_{\eta\theta}\psi_{t\theta}A_{\mu x}A_{\nu y}A_{\rho z}A_{\sigma t}: \tag{21}$$

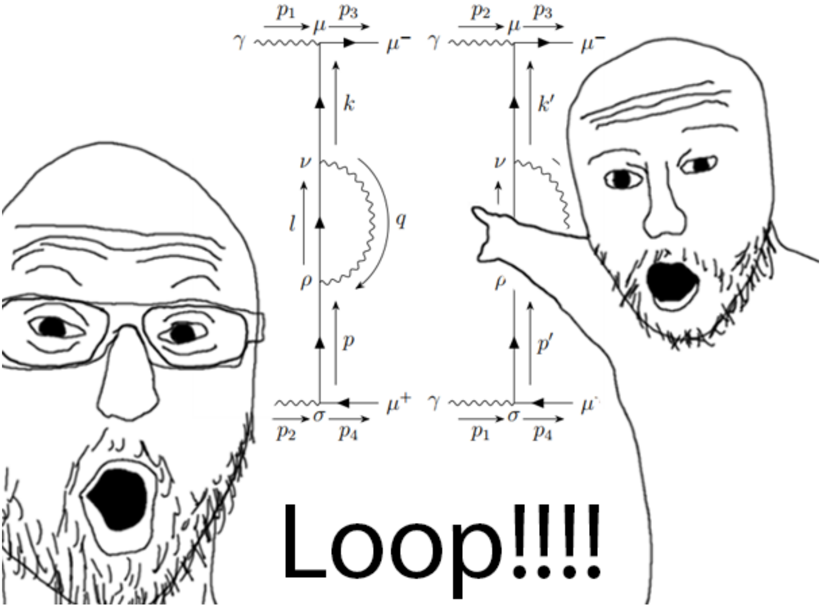
NLO calculation

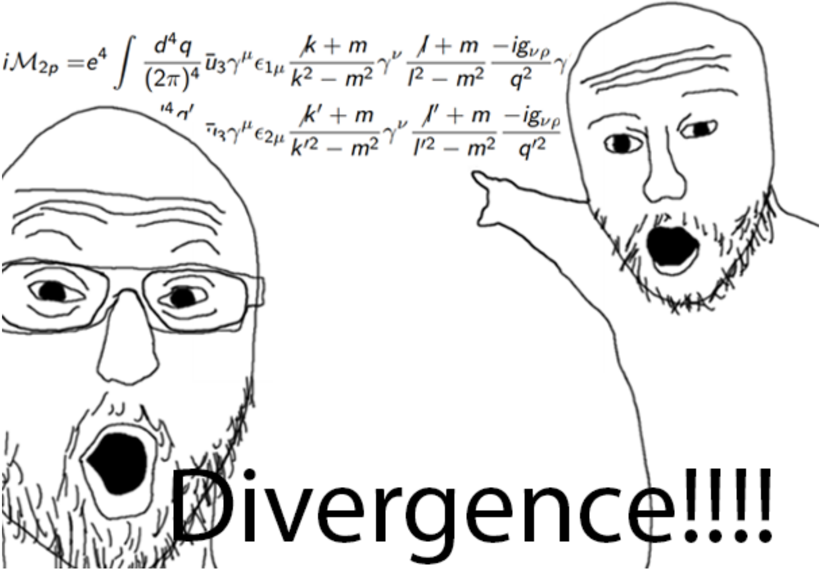
After calculating, we got the result:

$$i\mathcal{M}_{2p} = e^4 \int \frac{d^4 q}{(2\pi)^4} \bar{u}_3 \gamma^\mu \epsilon_{1\mu} \frac{\not{k}+m}{k^2-m^2} \gamma^\nu \frac{\not{l}+m}{l^2-m^2} \frac{-ig_{\nu\rho}}{q^2} \gamma^\rho \frac{\not{p}+m}{p^2-m^2} \epsilon_{2\sigma} \gamma^\sigma v_4$$
$$+ e^4 \int \frac{d^4 q'}{(2\pi)^4} \bar{u}_3 \gamma^\mu \epsilon_{2\mu} \frac{\not{k}'+m}{k'^2-m^2} \gamma^\nu \frac{\not{l}'+m}{l'^2-m^2} \frac{-ig_{\nu\rho}}{q'^2} \gamma^\rho \frac{\not{p}'+m}{p'^2-m^2} \epsilon_{1\sigma} \gamma^\sigma v_4$$

It also gives two Feynman diagrams:




p_1 μ p_3
γ μ^-
k
ν
l q
ρ
p
μ^+
p_2 σ p_4
p_2 μ p_3
γ μ^-
k'
ν
ρ
p'
γ
p_1 σ p_4
Loop!!!!


$i\mathcal{M}_{2p} = e^4 \int \frac{d^4 q}{(2\pi)^4} \bar{u}_3 \gamma^\mu \epsilon_{1\mu} \frac{\not{k} + m}{k^2 - m^2} \gamma^\nu \frac{\not{l} + m}{l^2 - m^2} \frac{-ig_{\nu\rho}}{q^2} \gamma$
$\gamma^\mu \epsilon_{2\mu} \frac{\not{k}' + m}{k'^2 - m^2} \gamma^\nu \frac{\not{l}' + m}{l'^2 - m^2} \frac{-ig_{\nu\rho}}{q'^2}$
Divergence!!!!

How to eliminate divergences

UV divergence resolving- Lagrangian for NLO

To eliminate the UV divergence, we will introduce a new Lagrangian for Next-to-leading order, by re-define fields and quantities:

$$\begin{cases} \psi_0 & = & \sqrt{Z_\psi}\psi & = & \sqrt{1+\delta_\psi}\psi \\ A_{0\mu} & = & \sqrt{Z_A}A_\mu & = & \sqrt{1+\delta_A}A_\mu \\ m_0 & = & Z_m \cdot m & = & m+\delta_m \\ e_0 & = & Z_e \cdot e & = & e+\delta_e \end{cases}$$

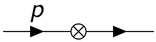
The Lagrangian now can be split into two Lagrangian: Renormalized Lagrangian $\mathcal{L}_R$ and counter term Lagrangian $\mathcal{L}_{ct}$:

$$\begin{aligned} \mathcal{L}_0 =& \mathcal{L}_R + \mathcal{L}_{ct} \\ \mathcal{L}_R =& \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi - \frac{1}{4}F^{\mu\nu}F_\mu\nu - e\bar{\psi}\gamma^\mu A_\mu\psi \\ \mathcal{L}_{ct} =& -\bar{\psi}\delta_m\psi + \delta_\psi\bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi - \frac{1}{4}\delta_A F^{\mu\nu}F_\mu\nu \\ & - e\bar{\psi}\gamma^\mu A_\mu\psi\left(\delta_e + \delta_\psi + \frac{1}{2}\delta_A\right) \end{aligned}$$

Counter term Feynman rules

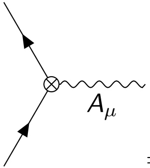
Counter term Lagrangian also give us Feynman diagrams. Rules for these diagrams are:

- Fermion-fermion vertex:



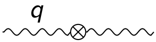
$$= i\delta_\psi(\not{p} - m) - i\delta_m$$

- Fermion-fermion-photon vertex:



$$= -ie\gamma^\mu \left(\delta_e + \delta_\psi + \tfrac{1}{2}\delta_A\right)$$

- Photon-photon vertex:

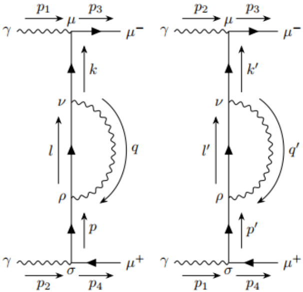


$$= -i\delta_A q^2 g^{\mu\nu}$$

Feynman diagrams for the next-to-leading order that contribute to scattering amplitude include:

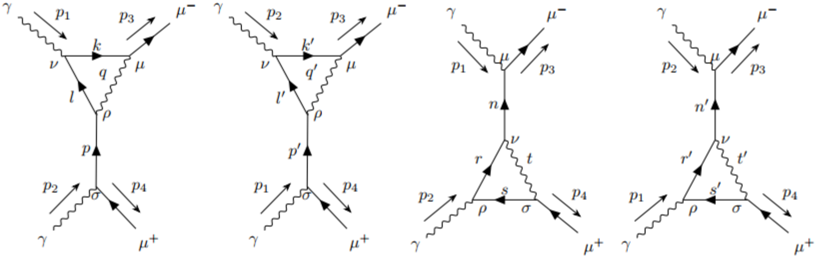
- Two-point diagrams - Self-energy diagrams.
- Three-point diagrams - Vertex correction diagrams.
- Four-point diagrams - Box diagrams.
- Counter term diagrams

Self energy diagrams



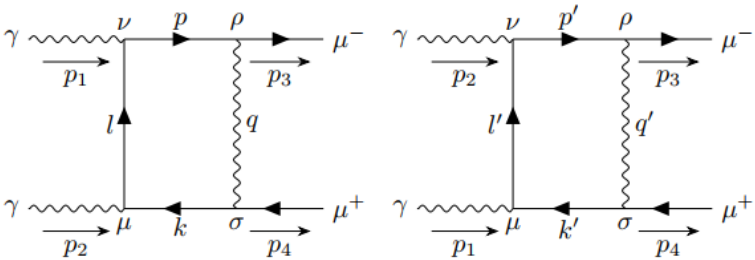
$$
\begin{aligned}
i\mathcal{M}_{2p} = & e^4 \int \frac{d^4 q}{(2\pi)^4} \bar{u}_3 \gamma^\mu \epsilon_{1\mu} \frac{\not{k} + m}{k^2 - m^2} \gamma^\nu \frac{\not{l} + m}{l^2 - m^2} \frac{-ig_{\nu\rho}}{q^2} \gamma^\rho \frac{\not{p} + m}{p^2 - m^2} \epsilon_{2\sigma} \gamma^\sigma v_4 \\
& + e^4 \int \frac{d^4 q'}{(2\pi)^4} \bar{u}_3 \gamma^\mu \epsilon_{2\mu} \frac{\not{k}' + m}{k'^2 - m^2} \gamma^\nu \frac{\not{l}' + m}{l'^2 - m^2} \frac{-ig_{\nu\rho}}{q'^2} \gamma^\rho \frac{\not{p}' + m}{p'^2 - m^2} \epsilon_{1\sigma} \gamma^\sigma v_4
\end{aligned}
$$

Vertex correction diagrams



$$
\begin{aligned}
i\mathcal{M}_{3p} = {} & e^4 \int \frac{d^4p}{(2\pi)^4} \bar{u}_3 \gamma^\mu \frac{\not{k}+m}{k^2-m^2} \epsilon_{1\mu} \gamma^\nu \frac{\not{l}+m}{l^2-m^2} \gamma^\rho \frac{-ig_{\mu\rho}}{q^2} \frac{\not{p}+m}{p^2-m^2} \epsilon_2^\sigma \gamma^\sigma v_4 \\
& + e^4 \int \frac{d^4p'}{(2\pi)^4} \bar{u}_3 \gamma^\mu \frac{\not{k}'+m}{k'^2-m^2} \epsilon_{2\mu} \gamma^\nu \frac{\not{l}'+m}{l'^2-m^2} \gamma^\rho \frac{-ig_{\mu\rho}}{q'^2} \frac{\not{p}'+m}{p'^2-m^2} \epsilon_{1\sigma} \gamma^\sigma v_4 \\
& + e^4 \int \frac{d^4s}{(2\pi)^4} \bar{u}_3 \gamma^\mu \frac{\not{n}+m}{n^2-m^2} \epsilon_{1\mu} \gamma^\nu \frac{\not{r}+m}{r^2-m^2} \gamma^\rho \frac{-ig_{\nu\sigma}}{t^2} \frac{\not{s}+m}{s^2-m^2} \epsilon_2^\sigma \gamma^\sigma v_4 \\
& + e^4 \int \frac{d^4s'}{(2\pi)^4} \bar{u}_3 \gamma^\mu \frac{\not{n}'+m}{n'^2-m^2} \epsilon_{2\mu} \gamma^\nu \frac{\not{r}'+m}{r'^2-m^2} \gamma^\rho \frac{-ig_{\nu\sigma}}{t'^2} \frac{\not{s}'+m}{s'^2-m^2} \epsilon_{1\sigma} \gamma^\sigma v_4 .
\end{aligned}
\tag{22}
$$

Box diagrams



$$
\begin{aligned}
i\mathcal{M}_{4p} = & \int \frac{d^4 k}{(2\pi)^4} \bar{u}_3 \gamma^\sigma \left(\frac{i}{\not{k} - m + i\epsilon} \right) \gamma^\mu \epsilon_{1\mu} \left(\frac{-i g_{\rho\sigma}}{q^2 + i\epsilon} \right) \\
& \times \left(\frac{i}{\not{k} + \not{p}_1 + - m + i\epsilon} \right) \gamma^\nu \epsilon_{2\nu} \left(\frac{i}{\not{p}_2 - \not{p}_4 - m + i\epsilon} \right) \gamma^\rho v_4 \\
& + \int \frac{d^4 k'}{(2\pi)^4} \bar{u}_3 \gamma^\sigma \left(\frac{i}{\not{k}' - m + i\epsilon} \right) \gamma^\mu \epsilon_{2\mu} \left(\frac{-i g_{\rho\sigma}}{q' + i\epsilon} \right) \\
& \times \left(\frac{i}{\not{k}' + \not{p}_1 + - m + i\epsilon} \right) \gamma^\nu \epsilon_{1\nu} \left(\frac{i}{\not{p}_2 - \not{p}_4 - m + i\epsilon} \right) \gamma^\rho v_4 . \qquad (23)
\end{aligned}
$$

Two-point counter term diagrams

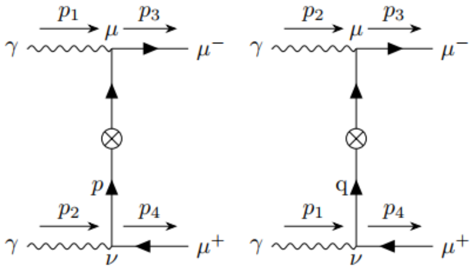


Figure: Two-point counter term diagrams

$$
\begin{aligned}
i\mathcal{M}_{c2p} = &\frac{ie^2}{p^2 - m^2}\bar{u}\gamma^\mu \epsilon_{1\mu}\left(i\delta_\psi(\not{p} - m) - i\delta_m\right)(\not{p} - m)\,\epsilon_{2\nu}\gamma^\nu v_4 \\
&+ \frac{ie^2}{q^2 - m^2}\bar{u}\gamma^\mu \epsilon_{2\mu}\left(i\delta_\psi(\not{q} - m) - i\delta_m\right)(\not{q} - m)\,\epsilon_{1\nu}\gamma^\nu v_4 . \qquad (24)
\end{aligned}
$$

Three-point counter term diagrams

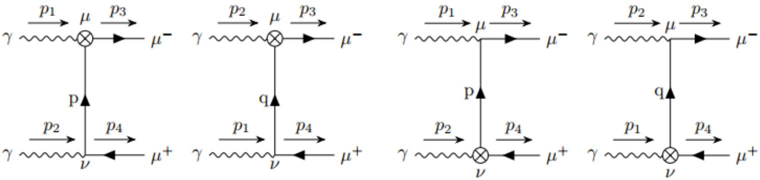


Figure: Three-point counter term diagrams

$$
\begin{aligned}
\mathcal{M}_{c3p} = & \frac{ie^2}{p^2 - m^2} \bar{u}_3 \left[-ie\gamma^\mu (\delta_e + \delta_\psi + \frac{1}{2}\delta_A) \right] \epsilon_{1\mu} \left(\not{p} - m \right) \epsilon_{2\nu} \left(-ie\gamma^\nu \right) v_4 \\
& + \frac{ie^2}{q^2 - m^2} \bar{u}_3 \left[-ie\gamma^\mu (\delta_e + \delta_\psi + \frac{1}{2}\delta_A) \right] \epsilon_{2\mu} \left(\not{q} - m \right) \epsilon_{1\nu} \left(-ie\gamma^\nu \right) v_4 \\
& + \frac{ie^2}{p^2 - m^2} \bar{u}_3 \left(-ie\gamma^\mu \right) \epsilon_{1\mu} \left(\not{p} - m \right) \epsilon_{2\nu} \left[-ie\gamma^\nu (\delta_e + \delta_\psi + \frac{1}{2}\delta_A) \right] v_4 \\
& + \frac{ie^2}{q^2 - m^2} \bar{u}_3 \left(-ie\gamma^\mu \right) \epsilon_{2\mu} \left(\not{q} - m \right) \epsilon_{1\nu} \left[-ie\gamma^\nu (\delta_e + \delta_\psi + \frac{1}{2}\delta_A) \right] v_4 .
\end{aligned}
\tag{25}
$$

Total scattering amplitude for NLO-Virtual

The total scattering amplitude square for Next-to-leading order- virtual is sum of all scattering amplitude of amputated diagrams and their counter term diagrams :

$$i\mathcal{M}_{NLO} = i\mathcal{M}_{2p} + i\mathcal{M}_{3p} + i\mathcal{M}_{4p} + i\mathcal{M}_{c2p} + i\mathcal{M}_{c3p}.$$

Outlook

- The next step is solving the divergence integrals of NLO scattering amplitude, however counter term diagrams method can only eliminate the UV divergence, we need to reconsider our process as $\gamma + \gamma \to \mu^- + \mu^+ + n\gamma$ to eliminate the IR divergence.
- A computer program is needed to help us calculate the scattering amplitude square up to NLO level.
- We can consider other kinds of interaction such as weak interaction to make our calculation more accurate.

Thanks for listening