PARTON MODEL: Intuitive idea

At large transverse momentum scattering, partons behave as quasi free particles.

$$\Rightarrow \quad \sigma = \sum_{ij} \int_0^1 dx_1 dx_2 \, f_i^{H_1}(x_1) f_j^{H_2}(x_2) \, \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2)$$

$P_{1/2} \to$ proton

$p_{1/2} \to$ parton

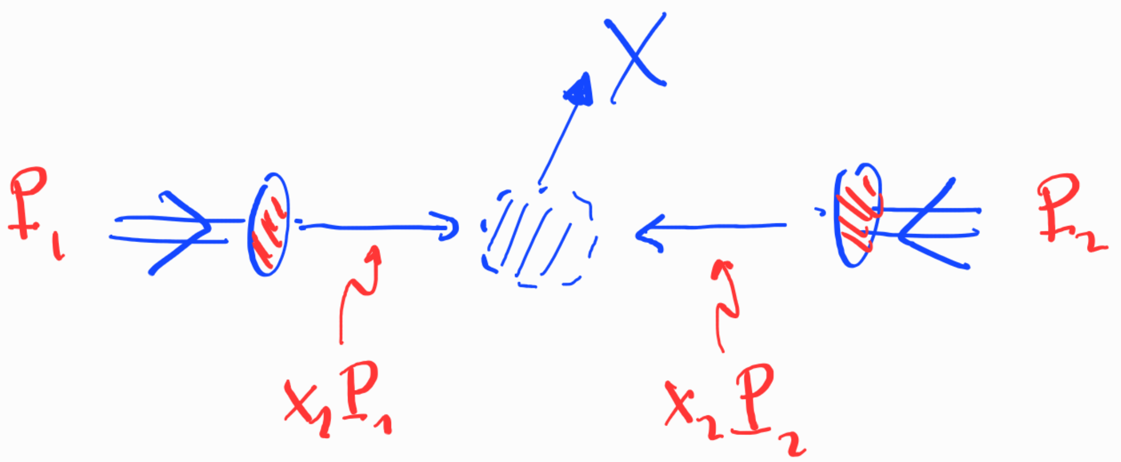
$f_i^H(x)$: probability to find parton i in H with fraction x of the H momentum

PDF: partonic distribution function

- transverse momentum of parton inside H neglected
- $f_i^H(x)$: extracted from data

$\hat{\sigma}_{ij}$ partonic x-section (perturbative QCD)

- How do we extract PDFs from data? DIS/LHC
- Are PDF's universal? Yes!
- Does the above equation survive radiative corrections? No...



Kinematics and PDFs :

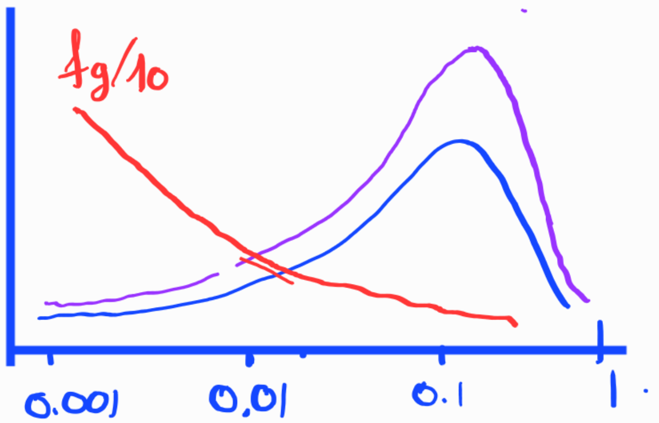
- Production of X, with mass M_X :

PROTON + PROTON → X ----→ PARTON + PARTON → X

$$(x_1 P_1 + x_2 P_2)^2 = M_X^2 \Rightarrow x_1 x_2 S_{had} \simeq M_X^2$$

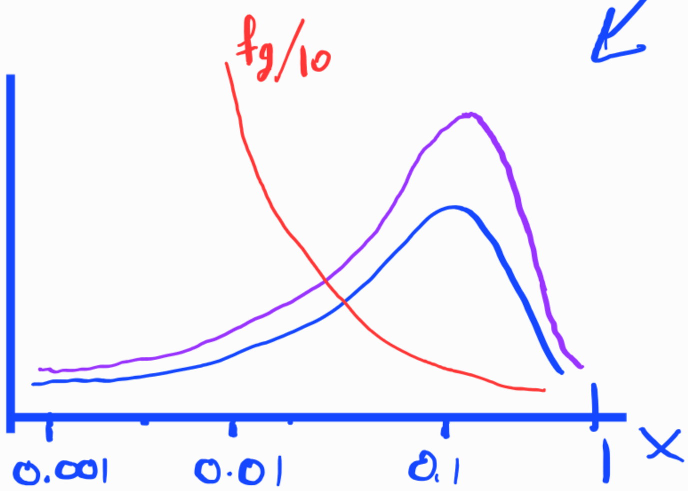
$$x_i = \frac{M_X}{\sqrt{S_{had}}} \qquad (\text{assume } x_1 \simeq x_2)$$

- $M_X \sim 100$ GeV ⟶ SMALL x
- $M_X \sim$ TeV ⟶ LARGE x



$\mu_F \sim 10$ GeV

$f_u(x)$
$f_d(x)$
$\frac{1}{10} f_g(x)$

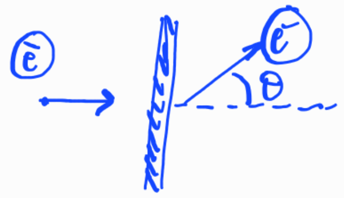


$\mu_F \sim 100$ GeV

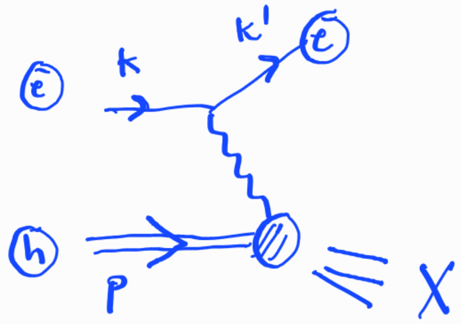
DEEP INELASTIC SCATTERING

- First experiments at SLAC ~ 1968
- Analogous of Rutherford scattering, but to probe structure of nucleons (i.e. proton)

$e^- p \to e^- X$



E.M. interaction



- Kinematics in lab frame

proton ~ at rest

$q = k - k'$

$Q^2 = -q^2 > 0$

DEEP: $Q^2 \gg m^2$ needed to probe internal structure

INELASTIC: $P_X^2 \gg m^2$ (if $P_X^2 = m^2 \Rightarrow$ elastic scattering)

$$\boxed{x = \frac{Q^2}{2pq} \quad \text{"Bjorken } x\text{"} \qquad y = \frac{pq}{pk}}$$

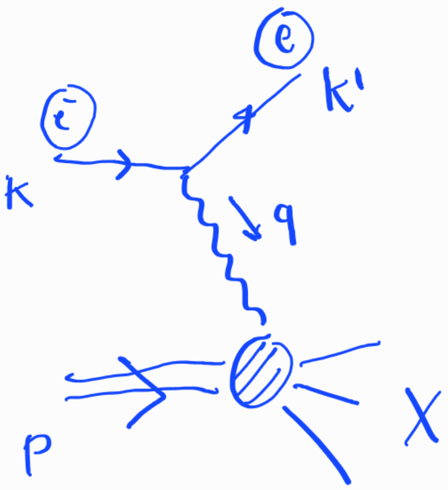
$0 \leq x \leq 1$

$0 \leq y \leq 1$

k and k' measurable $\Rightarrow$ x and y perfectly measurable

[Ex: $x = \frac{EE'(1-\cos\theta)}{m(E-E')}$, $y = 1 - \frac{E'}{E}$, $x \simeq 1$ is elastic scattering, $P_X^2 = (p+q)^2 = m^2 + \frac{Q^2(1-x)}{x}$]

• Amplitude & cross section



Amplitude

$$iM = \bar{u}(k')\,(ie\gamma^{\mu})\,u(k)\,\frac{-ig_{\mu\nu}}{q^2}\langle X|\,eJ^{\nu}_{Em}\,|p,s\rangle$$

We want to express the amplitude squared as a product of a leptonic and an hadronic part

$$d\sigma = \frac{1}{2S}\sum_X \sum_{\lambda\lambda's} |M|^2 \frac{d^3k'}{(2\pi)^3 2E'}\, d\Pi_X\,(2\pi)^4\,\delta^4(p+k-k'-p_X)$$

$$= \frac{1}{2S}\left(\frac{1}{2}\,\frac{1}{2}\right)\sum_{\lambda\lambda'} L^*_{\mu} L_{\nu} \sum_{X,s} \langle X|\,J^{\mu}|p,s\rangle^* \langle X|J^{\nu}|p,s\rangle \times$$

$$\times \frac{e^4}{Q^4}[dk']\cdot d\Pi_X\;(2\pi)^4\,\delta^4(q+p-p_X)$$

leptonic tensor :

$$L_{\nu} = \bar{u}(k')\gamma_{\nu}u(k)$$

$$\Rightarrow L_{\mu\nu} = \frac{1}{2}\sum_{\lambda\lambda'} L^*_{\mu}L_{\nu} = 2\left(k_{\mu}k'_{\nu} + k'_{\mu}k_{\nu} - g_{\mu\nu}(k k')\right)$$

gauge invariance : $q^\mu L_{\mu\nu} = q^\nu L_{\mu\nu} = 0$

hadronic tensor

$$W_{\mu\nu} = \frac{1}{2}\,\frac{1}{4\pi}\sum_{X,s} \langle p,s|J^\dagger_\mu|X\rangle\langle X|J_\nu|p,s\rangle (2\pi)^4\delta^4(q+p-p_X)\,\cdot$$
$$\cdot\, d\Pi_X$$

- $(4\pi)^{-1}$ is a convention. $d\Pi_X = \prod_{j\in X}[dp_j]$
- $W_{\mu\nu}$ can only depend on p and q
- J_μ is an e.m. current $\Rightarrow q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$
- At this stage $W_{\mu\nu}$ parametrizes our ignorance on the details of the photon/proton interaction

flux factor

$$\frac{1}{2s} = \frac{1}{4kp}$$

The x-section for DIS can then be written as

$$d\sigma = \frac{1}{2s}\,\frac{e^4}{Q^4}(4\pi)\,L_{\mu\nu}W^{\mu\nu}\,\frac{d^3k'}{(2\pi)^3 2E'} \qquad \text{(DIS, HAD)}$$

Notice that $W^{\mu\nu}$ has dimension $\emptyset$

$$\text{Ex: } W^{\mu\nu}(p,q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)W_1 + \frac{1}{pq}\left(p^\mu - \frac{pq}{q^2}q^\mu\right)\left(p^\nu - \frac{pq}{q^2}q^\nu\right)W_2$$

using $W^{\mu\nu}$ symmetric, gauge invariant, parity conserving (QED)

Ex: $[dk'] = \frac{1}{2(2\pi)^2} \frac{(s-m^2)y}{2} dx\, dy$

Ex: In the assumption $p^2 = m^2 \ll Q^2$, show that

$$W_{\mu\nu} L^{\mu\nu} = 2Q^2 \left[W_1(p,q) + \frac{W_2(p,q)}{2x} \frac{2(1-y)}{y^2} \right]$$

Putting everything together, one gets

$$\frac{d\sigma}{dx\,dy} = 4\pi \alpha_{em}^2 \frac{s}{Q^4} \left[x y^2 W_1 + (1-y) W_2 \right]$$

Notation.

$W_1 \to F_1$
$W_2 \to F_2$ } structure function

their role was to parameterize the structure of the proton, when probed at virtuality Q^2

$F_j(q^2, pq, \Lambda^2)$

Λ^2: mass scale associated with constituents. We expect it to be of order $m^2 \sim \left(\frac{1}{r}\right)^2$

F_j are dimensionless $\Rightarrow F_j\left(\frac{pq}{Q^2}, \frac{\Lambda^2}{Q^2}\right) = F\left(x, \frac{\Lambda^2}{Q^2}\right)$

- BJORKEN LIMIT (1969, Bjorken)

Measure F_i in the limit

- Q^2 large
- fixed x ($\hookrightarrow$ also pq is large)

EXP OBSERVATIONS:

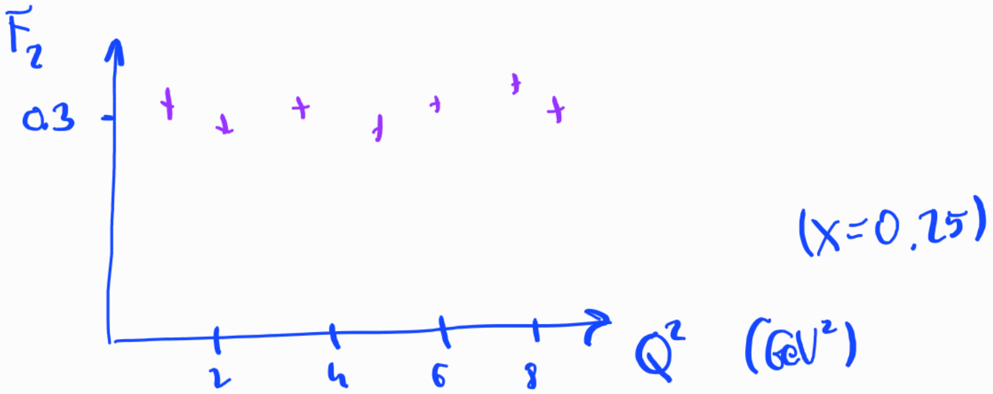
1. BJORKEN SCALING:

$$F_i\left(x, \frac{\Lambda^2}{Q^2}\right) \xrightarrow[\text{Bj limit}]{} F_i(x)$$

(NO Q^2 DEPENDENCE)

("SCALE INVARIANCE")

[violated by log. corrections]



2. CALLAN-GROSS RELATION

$$F_2 \simeq 2xF_1 \qquad \text{(in the Bjorken limit)}$$

From ① one also concludes that, when the proton is probed at large energies, there is no evidence of the typical scale/size of the proton.

For example, in models where the size of the proton is considered, one would have terms like

$$\sim e^{-R^2 Q^2}$$

$\Rightarrow$ scatter off pointlike constituents ||

Similarly, if constituents were interacting, one would expect to see the dependence on a scale of the order of the typical quark-quark / quark-gluon interaction

$\Rightarrow$ hypothetical constituents are assumed to be free, when Q^2 is large! ||

(NAME) PARTON MODEL (BJORKEN-PASCHOS '69 FEYNMAN '72)

- When probed at high energy, the proton is made of elementary pointlike constituents acting as free particles (NOT INTERACTING)
- Proton momentum carried by partons, with given "PARTONIC DISTRIBUTION FUNCTION"

$$\hat{p} = xP \qquad \mathrm{Prob}\left(xP < \hat{p} < (x+dx)P\right) = f(x)\,dx$$

[transverse momentum of partons is negligible]

- the elementary process is an ELASTIC SCATTERING between the electron and the parton
- the hadronic x-section is obtained by an incoherent sum of elastic partonic x-sections

$$d\sigma = \sum_i \int_0^1 f_i(x)\, d\hat{\sigma}(xP)\, dx$$

- ULTIMATELY ONE HAS TO CHECK IF IT WORKS: scaling ✓, Callan-Gross ✓

Comments:

① heuristic justification for summing at the level of probabilities and not coherently : characteristic scale is $\mu \sim Q$ $(Q \to \infty)$

$$m \ll Q$$

scale of (strong) soft interactions

scale of hard interaction (EM)

$$\Delta\tau_{\text{strong}} \gg \Delta\tau_{\text{EM}}$$

- No time for partons to feel the interaction with other constituents
- EXP evidence is that SCALE INVARIANCE shows up also if $m \lesssim Q$

- free constituents $\leftrightarrow$ consistent with the fact that $\alpha(Q) \to 0$ if $Q^2 \to \infty$

② Parton model traditionally formulated in the frame where

$p = (E\ 0\ 0\ E)$, with $E \gg m$.

- makes it easier to neglect target mass effect
- results can be written with LORENTZ INVARIANTS
- the fact that constituents are free DOES NOT DEPEND ON THE FRAME, but on the fact that $\alpha(Q) \to 0$ if $Q \to \infty$

[more comments in Manohar 9204208]

③ PDF

We introduced them "phenomenologically", but they can be also properly defined as exp-values of certain hadronic operators [OPE analysis of DIS, see e.g. Peskin or Schwartz]

Similarly, an equation like

$$d\sigma = \sum_i \int dx\, f_i(x)\, d\hat{\sigma}(xp)$$

can be derived more formally

$\to$ At hadron collisions

$$d\sigma = \sum_{i,j} \int dx_1\, f_i^{H_1}(x_1)\, f_j^{H_2}(x_2)\, d\hat{\sigma}(x_1 P_1, x_2 P_2)$$

$\Rightarrow$ PDFs enter in all LHC predictions

④ PDF's universality and determination [DKS Mangano lectures]

How do we obtain $f(x)$?

- $f(x)$ depends on internal proton wave function ⇝ non perturbative Physics

- $f(x)$ will depend on soft gluon exchange inside proton

- because of the typical interaction times, PROTON STRUCTURE has been "determined" much before the probe (γ^*) hits

⇒ if probed with $Q^2 \gg m^2$, structure of proton DOES NOT DEPEND ON HARD INTERACTION

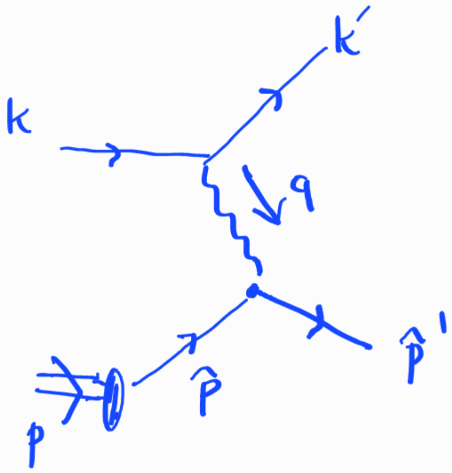
$\boxed{\text{PDF's are universal}}$

• extract in EXP A ⇝ use to predict in EXP B

We'll see in the final lecture that $f(x)$ have a dependence on Q^2 ("scaling violation")

PARTON MODEL: phenomenology

a) scale invariance of structure functions and Callan-Gross relation are PREDICTED



$$\hat{p} = z p \qquad \hat{s} = (k + \hat{p})^2 = z s$$

$$d\hat{\sigma}_i = \frac{1}{2\hat{s}} [dk'][d\hat{p}'] (2\pi)^4 \delta^4(\hat{p} + k - \hat{p}' - k') \times \frac{e^4}{Q^4} L_{\mu\nu} \widetilde{W}_i^{\mu\nu}$$

- Assuming partons are fermions

$$\widetilde{W}_i^{\mu\nu} = \frac{1}{2} Q_i^2 \, \mathrm{Tr}[\hat{p}\gamma^\mu \hat{p}' \gamma^\nu]$$

- $$(2\pi)^4 [d\hat{p}'] = (2\pi)^4 \frac{d^4\hat{p}'}{(2\pi)^3} \delta_+(\hat{p}'^2) = (2\pi) d^4\hat{p}' \frac{\delta(z-x)}{2pq}$$

using $\hat{p}' = q + \hat{p} = q + zp$

Notice that Bjorken $x = z$

• Ex: Compute

$$\tilde{W}_i^{\mu\nu} = Q_i^2\, 2\left\{ 2z p^\mu p^\nu + z\left(p^\mu q^\nu + q^\mu p^\nu\right) - z\, g^{\mu\nu}\, pq \right\}$$

• Ex:

$$\int [d\hat{p}'] (2\pi)^4 \delta^4(\hat{p}+q-\hat{p}')\, \tilde{W}^{\mu\nu} =$$

$$= (2\pi) z \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\hat{W}_1 + \frac{1}{pq}\left(p^\mu - \frac{pq}{q^2} q^\mu\right)\left(\quad\right)^\nu \hat{W}_2 \right\}$$

where $\hat{W}_1 = Q_i^2\ \delta(z-x)$

$$\hat{W}_2 = Q_i^2\, 2z\, \delta(z-x)$$

• Ex: Using

$$d\sigma = \sum_i \int_0^1 dz\, f_i(z)\, d\hat{\sigma}_i =$$

$$= \frac{2\pi}{s}\frac{e^4}{Q^4}[dk'] \times L_{\mu\nu} \times$$

$$\times \left\{ \left(-g^{\mu\nu} + \ldots\right) \frac{1}{2}\sum_i Q_i^2 f_i(x) + \frac{1}{pq}\left(\quad\right)^\mu\left(\quad\right)^\nu \sum Q_i^2 x f_i \right\}$$

and comparing with (DIS, HAD)

$$\Rightarrow \boxed{\begin{aligned} F_1(x) &= \frac{1}{2}\sum_i Q_i^2 f_i(x) \\ F_2(x) &= 2x F_1(x) \end{aligned}}$$

• CALLAN-GROSS $\longleftrightarrow$ FERMIONS

• F_i DON'T DEPEND ON Q^2

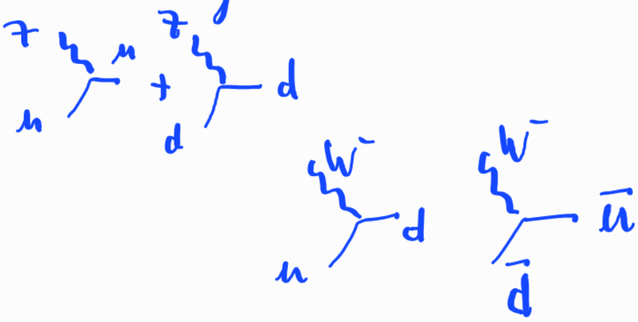
b) From Callan-Gross relation, one gets that

$$\frac{d\sigma}{dx\,dy} \sim [xy^2 F_1 + (1-y) W_2] \sim x[(1-y)^2+1] F_1(x)$$

$\Rightarrow$ scanning $d\sigma$ for different values of x one extracts F_1 (which is a combination of $f_i(x)$)

c) by means of other scatterings

- Z exchange
- $W^{\pm}$ exchanges



one can extract $f_u, f_{\bar{u}}, f_d, f_{\bar{d}}, \dots$

d) Sum rules

- momentum sum rule

$$p = \langle p_u \rangle + \langle p_d \rangle + \langle p_s \rangle + \dots$$

$$= \sum_i \int dx\, f_i(x)\, xp \qquad \Rightarrow \sum_i \int dx\, x\, f_i(x) = 1$$

From DIS in naive parton model $\sum_i (\ ) \approx 0.5$

$\Rightarrow$ 50% of long. momentum from GLUONS!

- flavour sum rule

IN A PROTON: $\begin{cases} \langle N_u \rangle = 2 \\ \langle N_d \rangle = 1 \end{cases}$

$$\Rightarrow \int_0^1 dx \left(f_u(x) - f_{\bar{u}}(x) \right) = 2$$

$$\int_0^1 dx \left(f_d(x) - f_{\bar{d}}(x) \right) = 1$$

$$\int_0^1 dx \left(f_s(x) - f_{\bar{s}}(x) \right) = 0$$

- often useful to introduce

"VALENCE QUARKS" = quarks that carry all the proton quantum numbers

"SEA QUARKS" = remainder

$$u(x) = u_{VAL}(x) + u_{SEA}(x)$$

$$\bar{u}(x) = \bar{u}_{SEA}(x)$$

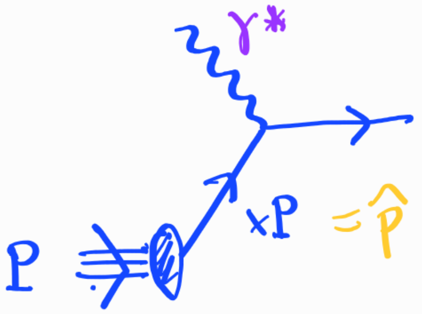
with

$$\int_0^1 dx \, u_v(x) = 2$$

$$\int_0^1 dx \left[u_s(x) - \bar{u}_s(x) \right] = 0$$

"IMPROVED" PARTON MODEL / factorization scale

- It turns out that "naive" parton model DOES NOT SURVIVE radiative corrections:



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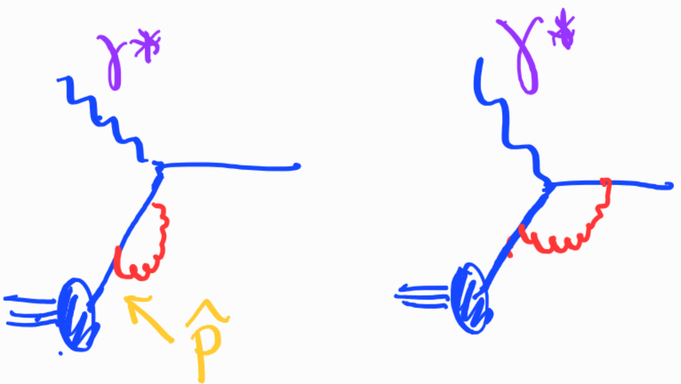
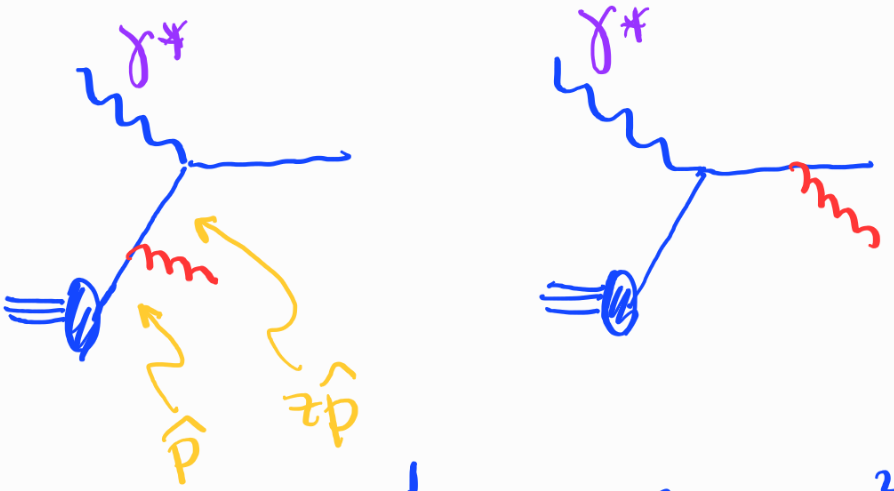


NLO (σ_V)



NLO (σ_R)

$$\sigma_R + \sigma_V \sim \int_0^1 dz \int_0 \frac{dk_T^2}{k_T^2} \frac{1+z^2}{1-z} \left(\sigma_B(z\hat{p}) - \sigma_B(\hat{p})\right)$$

COLLINEAR DIVERGENCE DOES NOT CANCEL

$\Rightarrow$ can be reabsorbed redefining PDF's

- "renormalized" PDF's acquire a new scale dependence

"FACTORIZATION" SCALE $f_i(x, \mu_F)$

LOW ENERGY DYNAMICS $\leftrightarrow$ PERTURBATIVE HARD CROSS SECTION

- Redefinition of PDF's is
  - UNIVERSAL
  - PROCESS - INDEPENDENT

- Dependence upon μ_F is calculable

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu^2\right)$$

- DGLAP EQUATION
- predicts scaling violation effects observed in DIS
- central for LHC