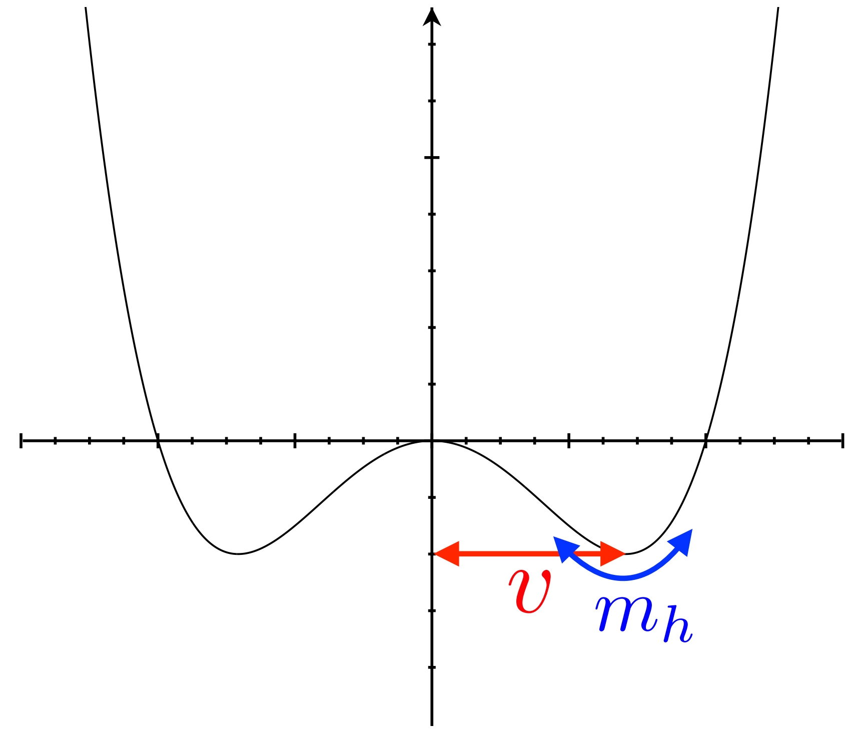
Lecture 5

$$\Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h) \end{pmatrix}$$



Higgs VEV v	Higgs boson h
- vacuum structure	- properties of h
- effective potential	- productions & decays
	- Higgs coupling measurements
@high energy - renormalization group evolution	-> tests of mass generation & symmetry breaking

Higgs boson

The Higgs was discovered in July 2012.

H^0

$J = 0$

PDG2022

Mass $m = 125.25 \pm 0.17$ GeV (S = 1.5)

Full width $\Gamma = 3.2^{+2.8}_{-2.2}$ MeV (assumes equal on-shell and off-shell effective couplings)

H^0 Signal Strengths in Different Channels

Combined Final States $= 1.13 \pm 0.06$

$W W^* = 1.19 \pm 0.12$

$Z Z^* = 1.06 \pm 0.09$

$\gamma\gamma = 1.11^{+0.10}_{-0.09}$

$c\overline{c}$ Final State $= 37 \pm 20$

$b\overline{b} = 1.04 \pm 0.13$

Higgs productions@LHC

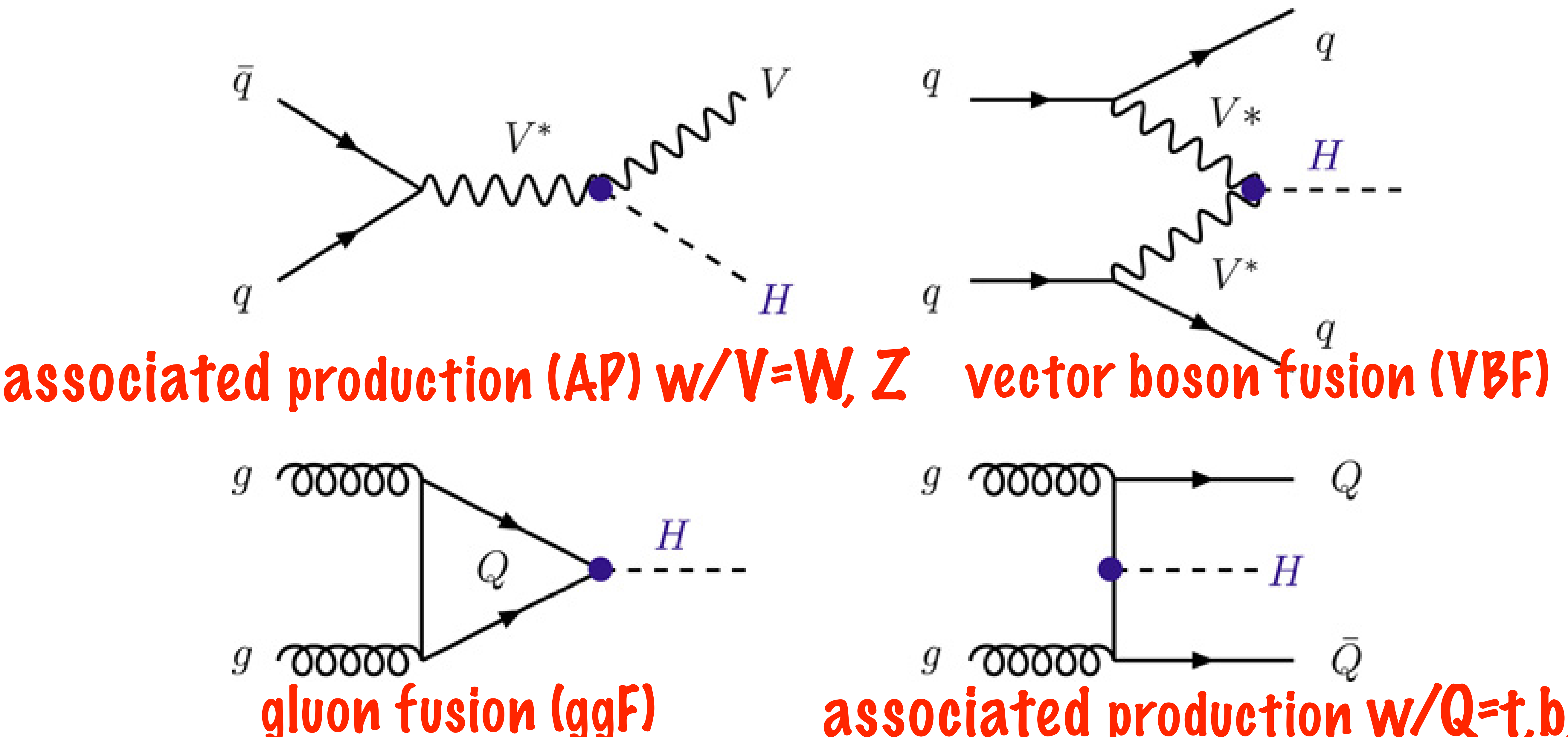
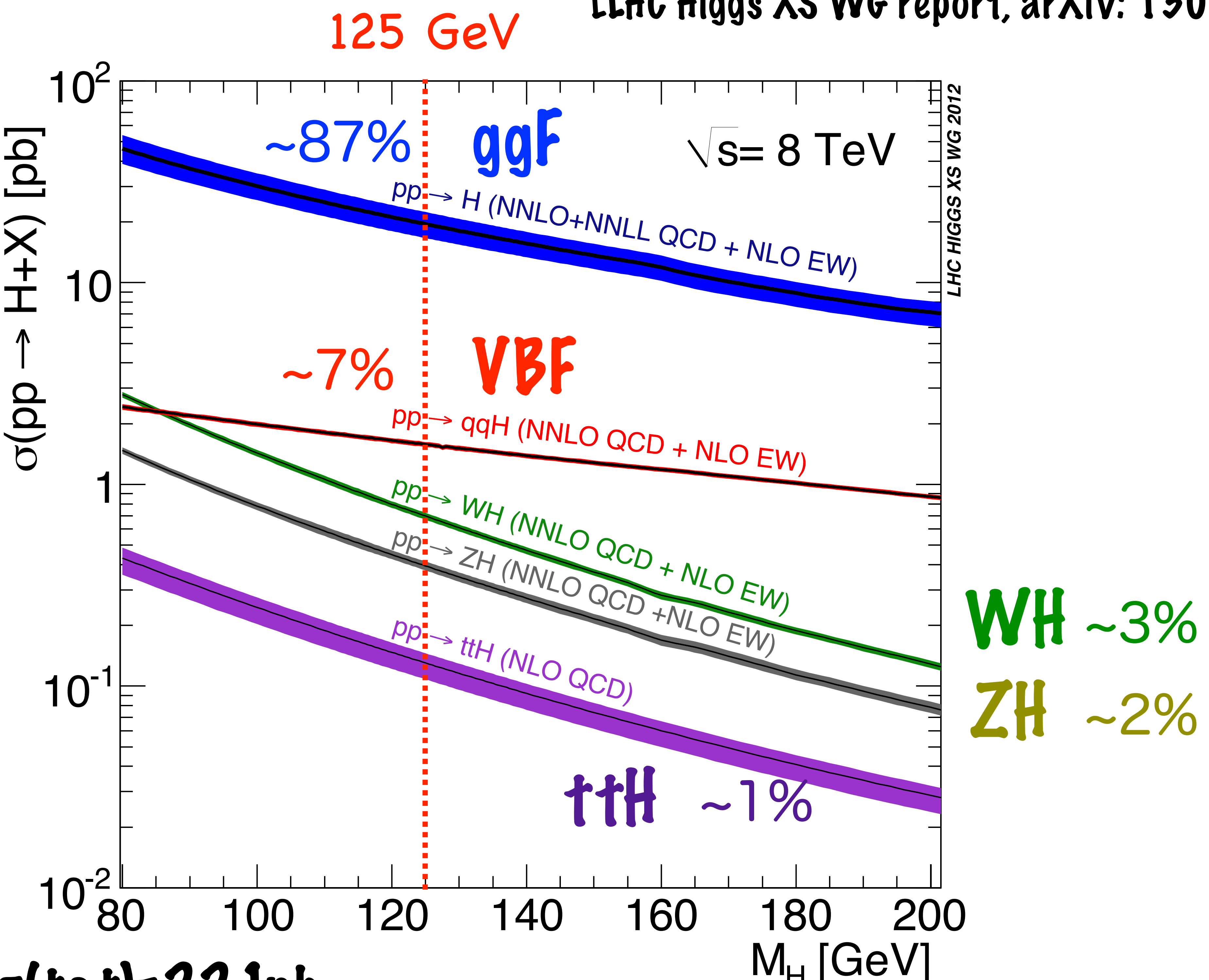


Fig. 3.1. The dominant SM Higgs boson production mechanisms in hadronic collisions.

Fig. from A. Djouadi, Phys.Rept. 457 (2008) 1

Cross sections

[LHC Higgs XS WG report, arXiv: 1307.1347]



m_h = 125 GeV, σ(tot)=22.1pb

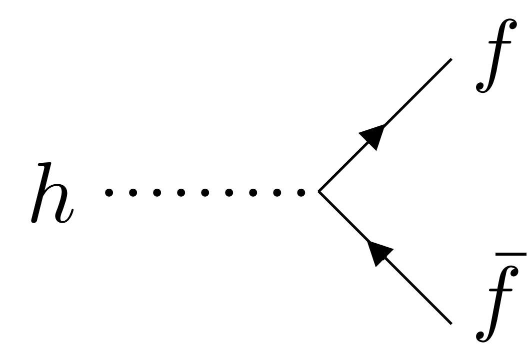
ggF: 19.3pb, VBF: 1.6pb, WH: 0.7pb, ZH: 0.4pb, ttH: 0.13pb

Higgs decays

A. Djouadi, Phys.Rept.457 (2008) 1

h -> f+fbar

$$\Gamma(h \to f\bar{f}) = \frac{N_C m_f^2 m_h}{8\pi v^2}\left(1 - \frac{4m_f^2}{m_h^2}\right)^{3/2}$$



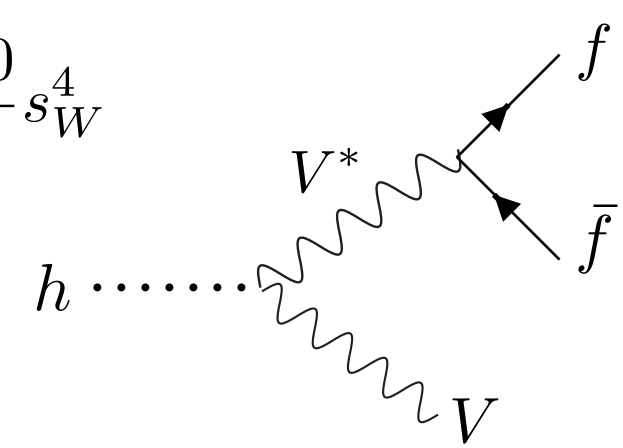
For $m_h \lesssim 130$ GeV, the main decay mode is h->b+bbar.

∵ largest Yukawa coupling and color

h -> VV*

$$\Gamma(h \to VV^*) = \frac{9 m_h m_V^4}{32\pi^3 v^4} c_V I(x), \quad c_W = 1,\ c_Z = \frac{7}{12} - \frac{10}{9}s_W^2 + \frac{40}{9}s_W^4$$

$$I(x) = \frac{(x^2-1)(2-13x^2+47x^4)}{2x^2} - 3(1-6x^2+4x^4)\ln x + \frac{3(1-8x^2+20x^4)}{\sqrt{4x^2-1}}\cos^{-1}\left(\frac{3x^2-1}{2x^3}\right)$$



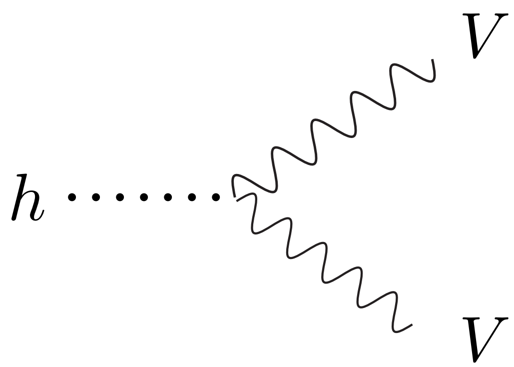
For $m_h \gtrsim 130$ GeV, WW* starts to dominate over h->b+bbar. ∵ $m_b \ll m_V$

h -> V*V* is equally important.

h -> ZZ, WW

$$\Gamma(h \to ZZ) = \frac{m_h^3}{32\pi v^2}\sqrt{1-\frac{4m_Z^2}{m_h^2}}\left[1-\frac{4m_Z^2}{m_h^2}+\frac{12m_Z^4}{m_h^4}\right],$$

$$\Gamma(h \to W^+W^-) = \frac{m_h^3}{16\pi v^2}\sqrt{1-\frac{4m_W^2}{m_h^2}}\left[1-\frac{4m_W^2}{m_h^2}+\frac{12m_W^4}{m_h^4}\right]$$

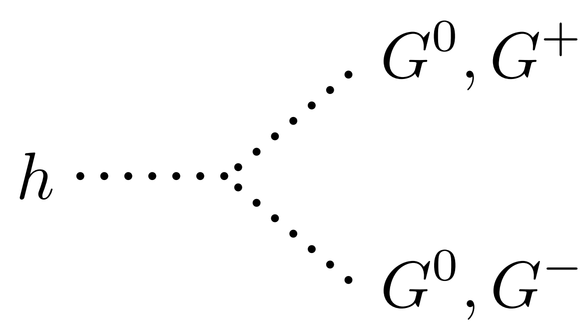


2* Γ (h -> ZZ) ≃ Γ (h -> WW)

Goldstone boson equivalence theorem

$$\Gamma(h \to G^0G^0) = \frac{\lambda_{hG^0G^0}^2}{32\pi m_h} = \frac{m_h^3}{32\pi v^2} \qquad \lambda_{hG^0G^0} = \frac{m_h^2}{v}$$

$$\Gamma(h \to G^+G^-) = \frac{\lambda_{hG^+G^-}^2}{16\pi m_h} = \frac{m_h^3}{16\pi v^2} \qquad \lambda_{hG^+G^-} = \frac{m_h^2}{v}$$



$$\Gamma(h \to ZZ) \underset{m_Z \ll m_h}{\simeq} \Gamma(h \to G^0G^0)$$

$$\Gamma(h \to W^+W^-) \underset{m_W \ll m_h}{\simeq} \Gamma(h \to G^+G^-)$$

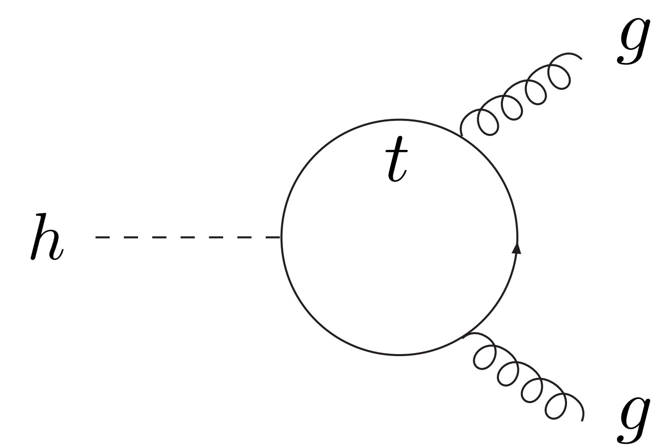
For $m_V \ll m_h$, h->VV (V=Z,W) is dominated by the longitudinal mode.

h -> gg

1-loop induced decay is not always smaller than the tree level decay.

$$\Gamma(h \to gg)_{\rm top-loop} = \frac{\alpha_s^2 m_h^3}{32\pi^3 v^2}\left[\tau_t^2 \left|1 + (1-\tau_t) f(\tau_t)\right|^2\right]$$

$$\alpha_s = \frac{g_3^2}{4\pi}, \;\; \tau_t = \frac{4m_t^2}{m_h^2}$$



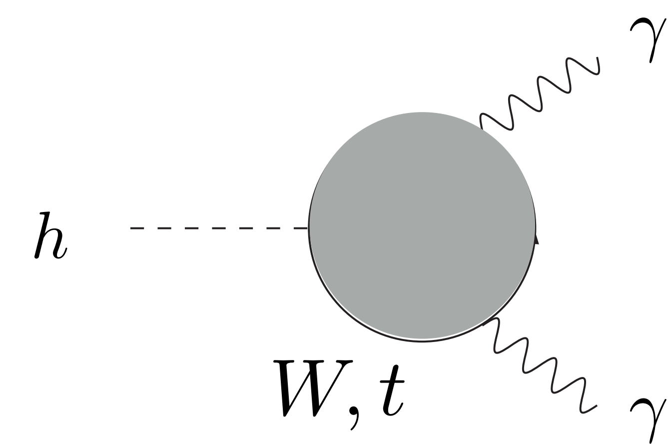
$$f(\tau_f) = -\frac{1}{2}\int_0^1 \frac{dy}{y} \ln\left[\frac{m_h^2 y(y-1) + m_f^2 - i\epsilon}{m_f^2 - i\epsilon}\right]$$

$$= \begin{cases} -\frac{1}{4}\left[\ln\left(\frac{1+\sqrt{1-\tau_f}}{1-\sqrt{1-\tau_f}}\right) - i\pi\right]^2 & \text{for } \tau_f < 1, \\ \left[\arcsin(\sqrt{1/\tau_f})\right]^2 = \left[\arctan\left(1/\sqrt{\tau_f - 1}\right)\right]^2 & \text{for } \tau_f > 1, \\ \frac{\pi^2}{4} & \text{for } \tau_f = 1. \end{cases}$$

h -> γγ

W and top give main contributions.

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha_{\rm em}^2 m_h^3}{256\pi^3 v^2} |\mathcal{A}_W + \mathcal{A}_t|^2$$



$$\mathcal{A}_W = F_1(\tau_W),\ \mathcal{A}_t = N_C Q_t^2 F_{1/2}(\tau_t) \qquad \tau_i = \frac{4m_i^2}{m_h^2}$$

different loop functions

$$F_1(\tau) = 2 + 3\tau + 3\tau(2-\tau)f(t) \underset{\tau \gg 1}{\to} 7,$$

$$F_{1/2}(\tau) = -2\tau\left[1 + (1-\tau)f(\tau)\right] \underset{\tau \gg 1}{\to} -\frac{4}{3}$$

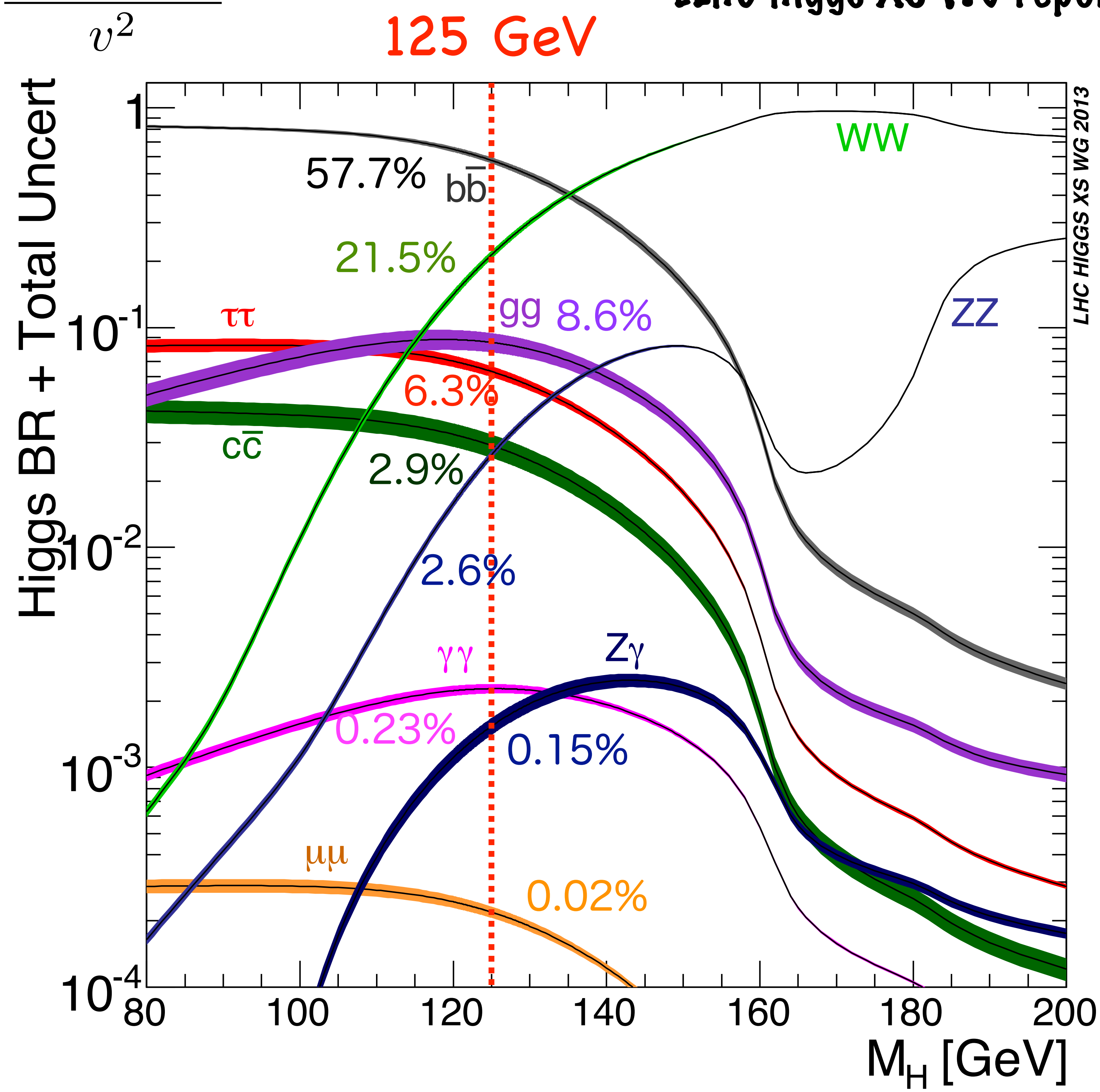
opposite sign

- W loop is the dominant.
- W loop and top have the opposite signs.

Branching ratios

$$\Gamma(h \to f\bar{f}) \propto \frac{N_C m_f^2 m_h}{v^2}$$

[LHC Higgs XS WG report, arXiv: 1307.1347]



$$\Gamma(h \to VV) \propto \frac{m_h^3}{v^2}$$

m_h = 125 GeV

$$b\bar{b} : 57.5\%,\ WW^* : 21.5\%,\ gg : 8.6\%,\ \tau\tau : 6.3\%,\ c\bar{c} : 2.9\%$$

$$ZZ^* : 2.6\%,\ \gamma\gamma : 0.23\%,\ Z\gamma : 0.15\%,\ \mu\mu : 0.02\%$$

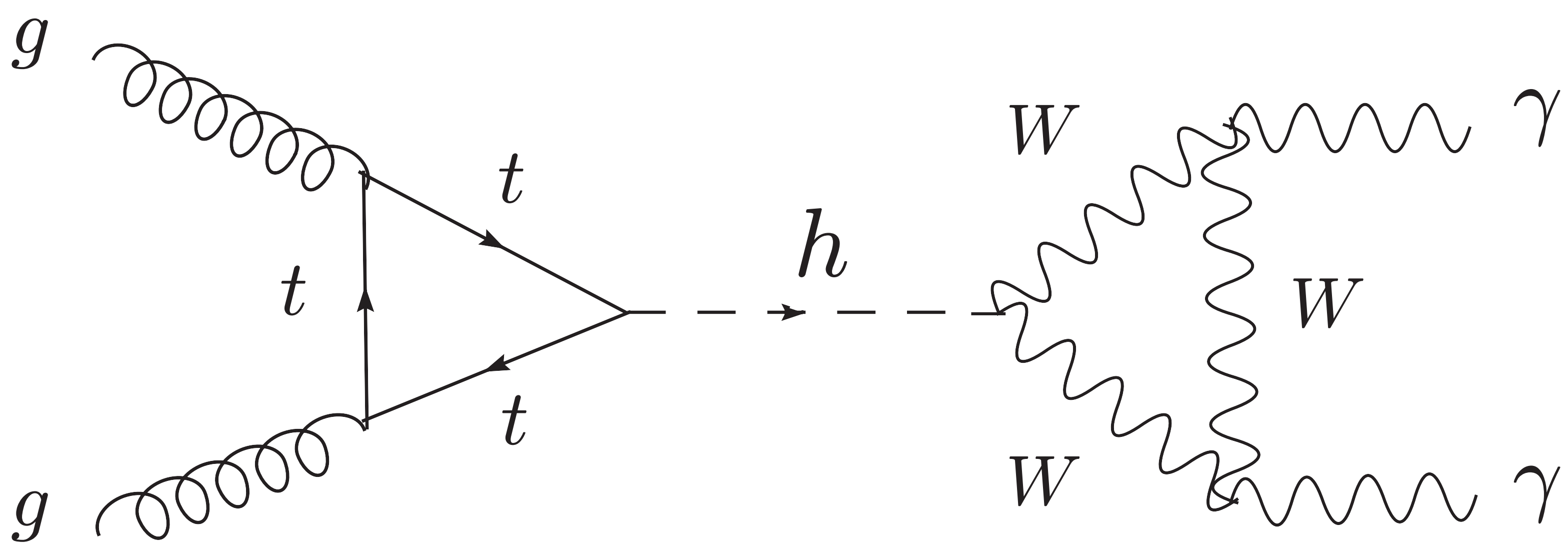
Signal strengths

$$\mu_{i,X} = \frac{\sigma_i \cdot \mathrm{Br}(h \to X)}{\sigma_i^{\mathrm{SM}} \cdot \mathrm{Br}^{\mathrm{SM}}(h \to X)}$$

$i = \mathrm{ggF},\ \mathrm{VBF},\ \mathrm{VH},\ \mathrm{ttH},$

$X = \gamma\gamma,\ VV^*,\ \tau\tau,\ b\bar{b},$

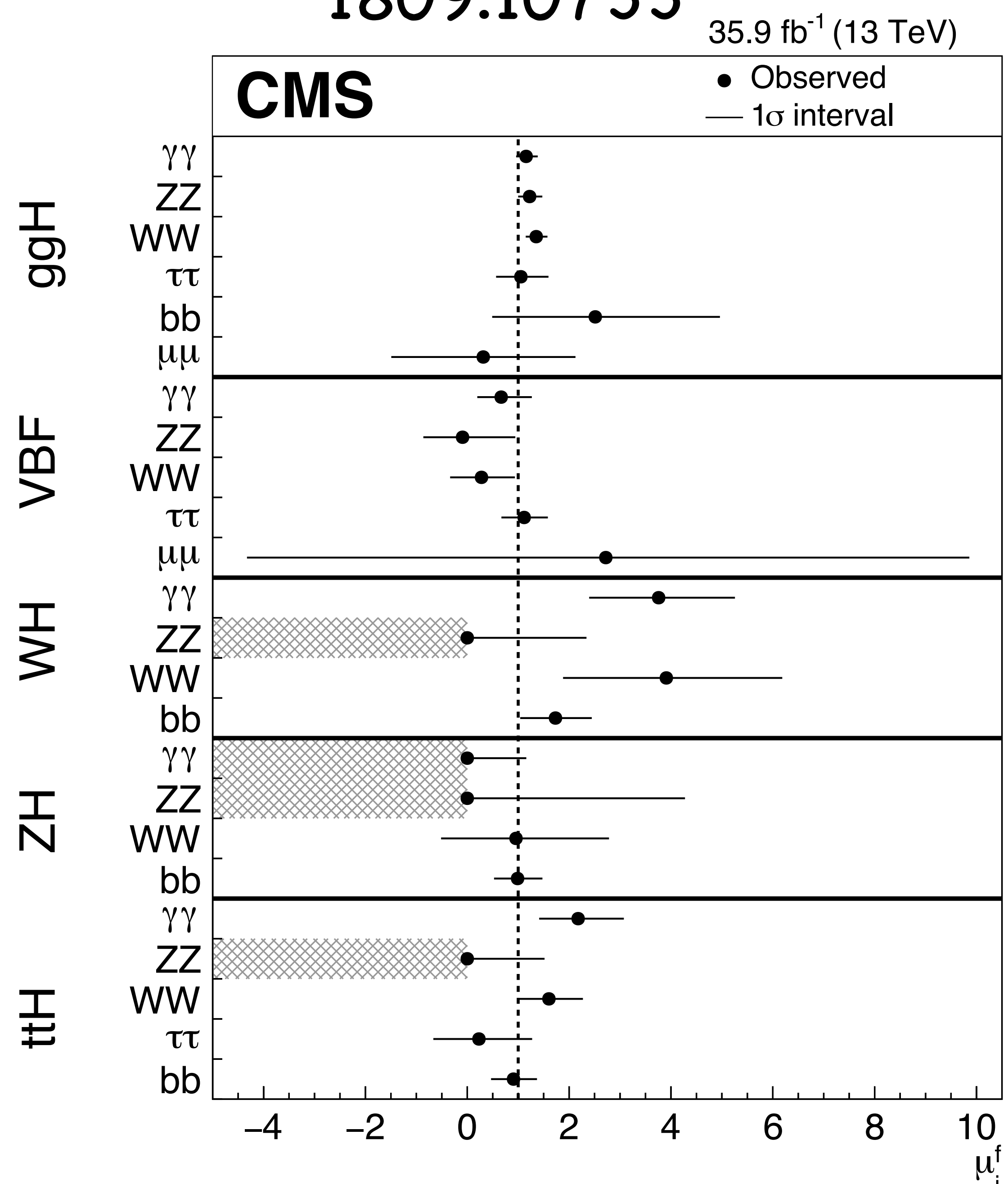
$\mu_{\mathrm{ggF},\gamma\gamma} =$



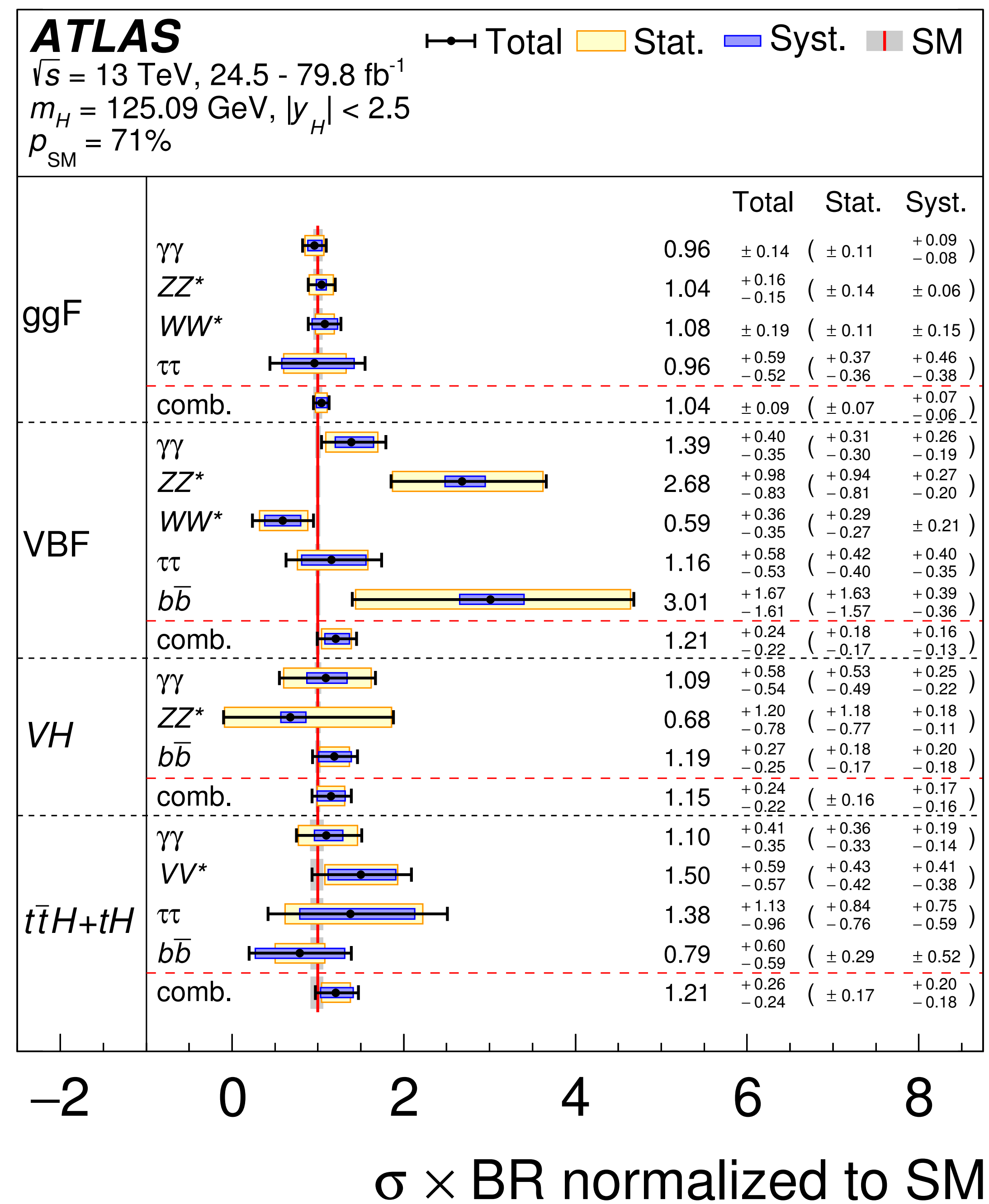
Various channels have been measured.

Current status

1809.10733



1909.02845

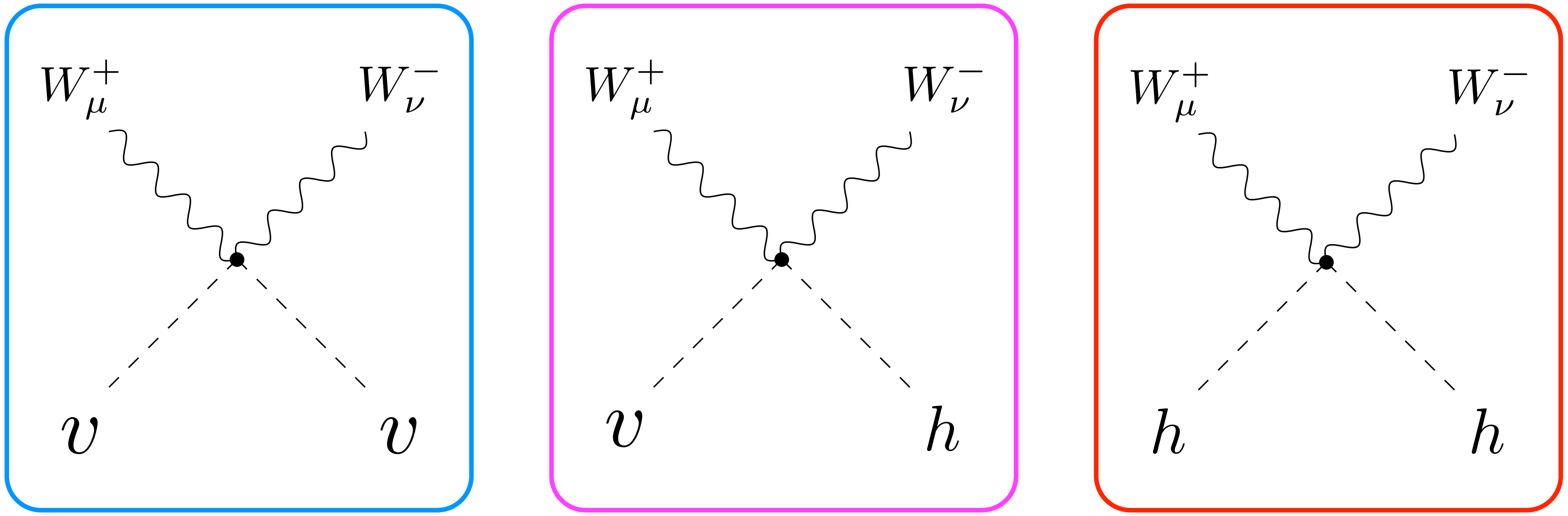


Consistent with the SM.

Higgs couplings

Since the gauge bosons get their masses from $|D\Phi|^2$, hVV coupling inevitably appears.

$$(D_\mu\Phi)^\dagger D^\mu\Phi \underset{\Phi^0\to v+h}{\to} m_W^2 W_\mu^+ W^{-\mu} + \frac{2m_W^2}{v} h W_\mu^+ W^{-\mu} + \frac{m_W^2}{v^2} h^2 W_\mu^+ W^{-\mu} + (W \to Z)$$

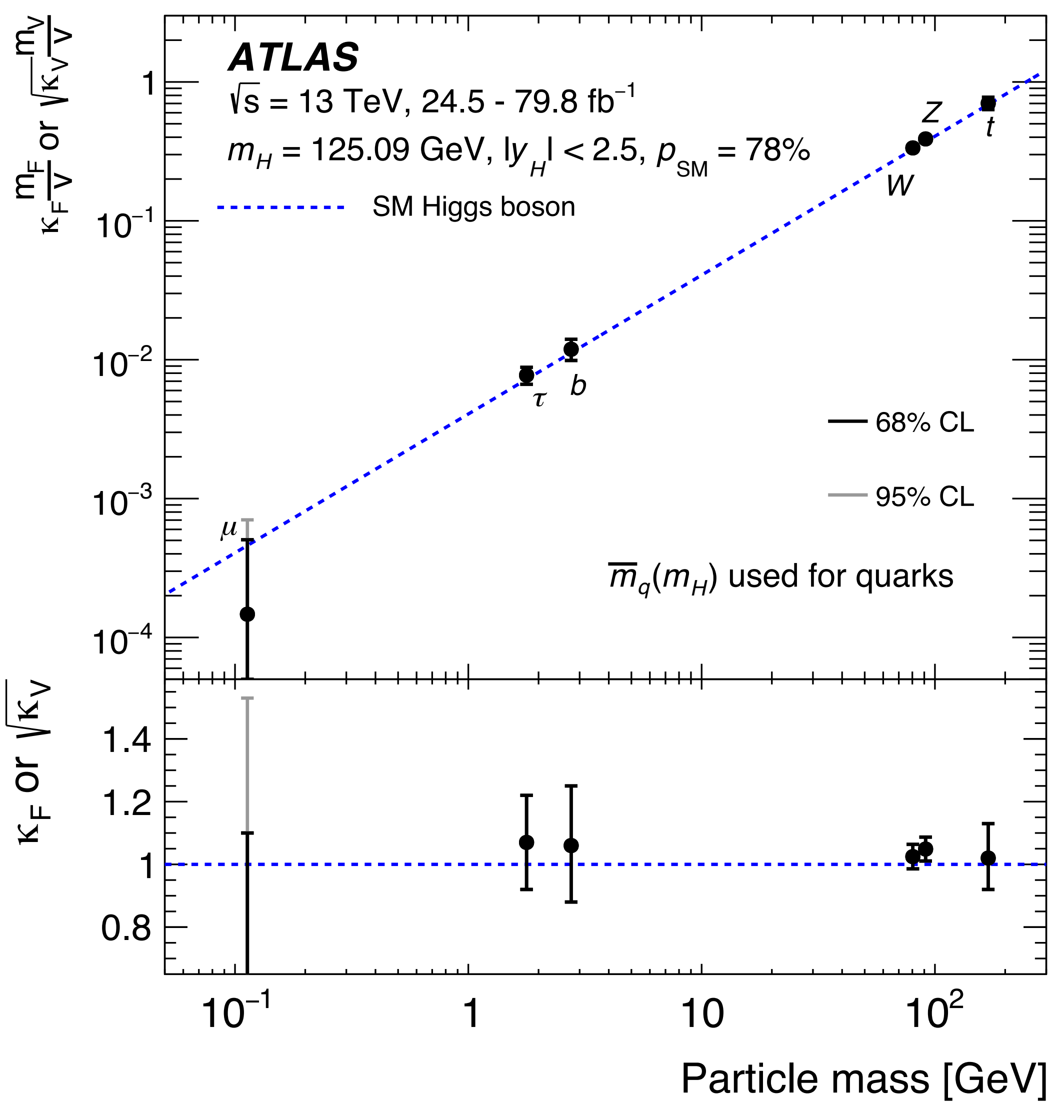


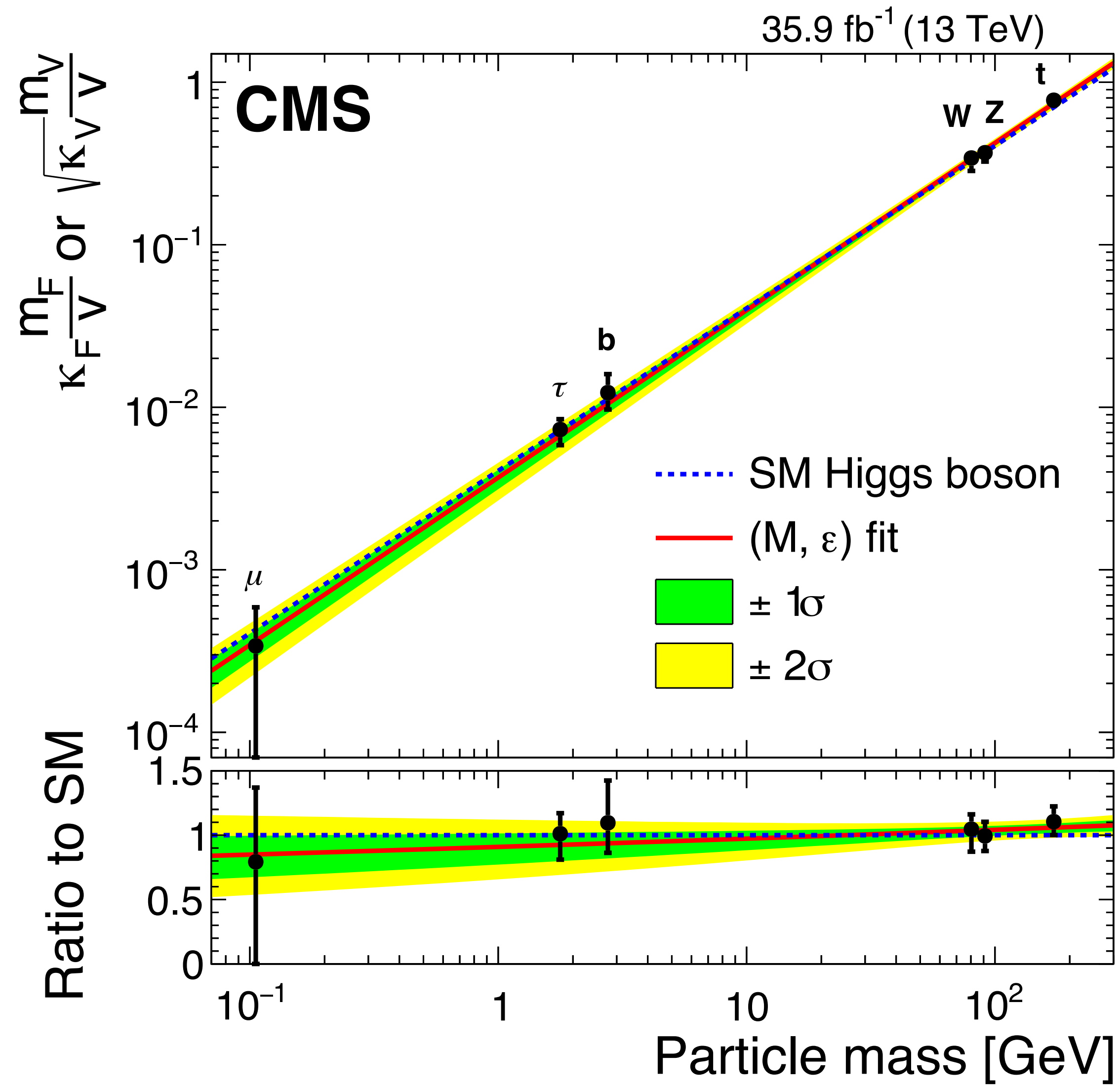
Likewise, for fermions $y_f \bar{f}_L \Phi f_R + \text{h.c.} \underset{\Phi^0\to v+h}{\to} m_f \bar{f} f + \frac{m_f}{v} h \bar{f} f$

In the SM, coupling ∝ mass. $g_{hVV} = m_V/v, \quad g_{h\bar{f}f} = m_f/v$

mass vs. coupling

- Measure Higgs couplings and see the relations btw couplings and masses.





- Dotted line = SM; consistent with the SM.
- mass-coupling relations could differ in BSM models.

ρ parameter

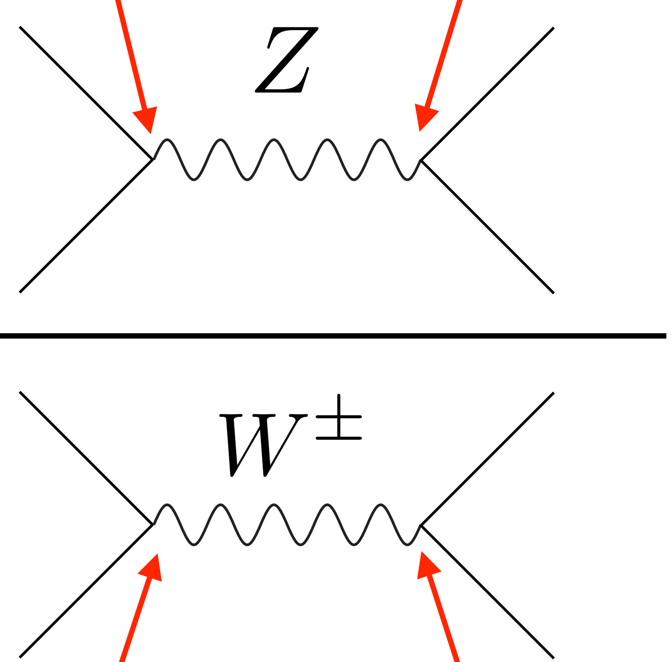
ρ parameter is defined as $\rho = \dfrac{m_W^2}{m_Z^2 \cos^2\theta_W}$

- neutral and charged current ratio
- depends on representation of Φ

SM

Φ:doublet

$$g_Z \times \frac{1}{m_Z^2} \times g_Z = \frac{4}{v^2} \qquad g_Z = \sqrt{g_2^2 + g_1^2}$$

$\rho =$ 

$$= \frac{m_W^2}{m_Z^2}\frac{g_Z^2}{g_2^2} = \frac{m_W^2}{m_Z^2}\frac{1}{\cos\theta_W^2} = 1$$

$$g_2 \times \frac{1}{m_W^2} \times g_2 = \frac{4}{v^2}$$

Experimental data

$$\rho = 1.0004^{+0.0003}_{-0.0004} \quad (95\%\ \text{C.L.})$$

ρ parameter puts a constraint on representation of Φ.

m_W and m_Z depend on the representation of Higgs field.

$$\phi_{(T,Y)} = \begin{pmatrix} \vdots \\ \phi^0_{(T,Y)} \\ \vdots \end{pmatrix}$$

isospin hypercharge

$$m_W^2 = c_{T,Y} g_2^2 \sum_i \left[T_i(T_i+1) - Y_i^2\right] |\langle \phi_i^0 \rangle|^2,$$

$$m_Z^2 = 2(g_2^2 + g_1^2) \sum_i Y_i^2 |\langle \phi_i^0 \rangle|^2.$$

$$c_{T,Y\neq 0} = 1 \text{ and } c_{T,Y=0} = 1/2$$

- In SM $\phi_{(1/2,|1/2|)}$.
- Singlet field ($\phi_{(0,0)}$) does not contribute to m_W and m_Z.
- Fields with Y=0 do not contribute to m_Z.

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{c_{T,Y} \sum_i \left[T_i(T_i+1) - Y_i^2\right] |\langle \phi_i^0 \rangle|^2}{\sum_i 2Y_i^2 |\langle \phi_i^0 \rangle|^2},$$

If only one scalar field exists,

$$\rho = 1 \longrightarrow \phi_{(T,Y)} \text{ with } T(T+1) - 3Y^2 = 0$$

$\phi_{(1/2,\|1/2\|)}$	doublet
$\phi_{(3,\|2\|)}$	7-plet
$\phi_{(25/2,\|15/2\|)}$	26-plet

ρ parameter is useful for constraining new physics.

Lecture 6

Effective potential

Generating functional

external field

$$Z[J] = e^{iW[J]} = N \int [d\phi] \exp\left[i \int d^4x \Big(\mathcal{L}(\phi) + J(x)\phi(x)\Big)\right], \quad N = Z^{-1}[J=0]$$

VEV w/ J

$$\phi_c(x) = \frac{\delta W[J]}{\delta J(x)} = N \int [d\phi] \phi(x) \exp\left[i \int d^4x \Big(\mathcal{L}(\phi) + J(x)\phi(x)\Big)\right] = \langle 0|\phi(x)|0\rangle_J,$$

VEV w/ J=0 $\varphi = \lim_{J\to 0} \lim_{V\to\infty} \langle 0|\phi(x)|0\rangle\Big|_J$ <- constant field

presumed

J=0 -> translational invariance

$$\varphi = \langle 0|e^{iPx}\phi(0)e^{-iPx}|0\rangle = \langle 0|\phi(0)|0\rangle$$

J=0 eventually. So, why can't we take J=0 at the beginning?

$$\varphi = N \int [d\phi] \phi(x) e^{iS[\phi]}$$

$$\phi(x) \to -\phi(x), \quad S[-\phi] = S[\phi], \quad [d(-\phi)] = [d\phi]$$

$$\varphi = N \int [d\phi]\big(-\phi(x)\big)e^{iS[\phi]} = -\varphi \qquad \therefore \quad \varphi = 0$$

Effective action

functional Legendre trf. of W[J]

$$\Gamma[\phi_c] = W[J] - \int d^4x \; J(x)\phi_c(x), \quad \frac{\delta\Gamma[\phi_c]}{\delta\phi_c(x)} = -J(x)$$

* Generating functional for 1PI diagrams

Effective action can be expanded as

$$\Gamma[\phi_c] = \int d^4x \left[\frac{Z(\phi_c(x))}{2}\Big(\partial_\mu\phi_c(x)\Big)^2 - V\big(\phi_c(x)\big) + \cdots\right]$$

@tree level $\quad \Gamma[\phi_c] = S[\phi_c] = \int d^4x \; \mathcal{L}(\phi_c)$ $\quad$ classical action

If $\phi_c(x)$ is a constant field φ,

$$\Gamma[\varphi] = -V_{\text{eff}}(\varphi)\int d^4x = -V_{\text{eff}}(\varphi)(2\pi)^4\delta^4(0)$$

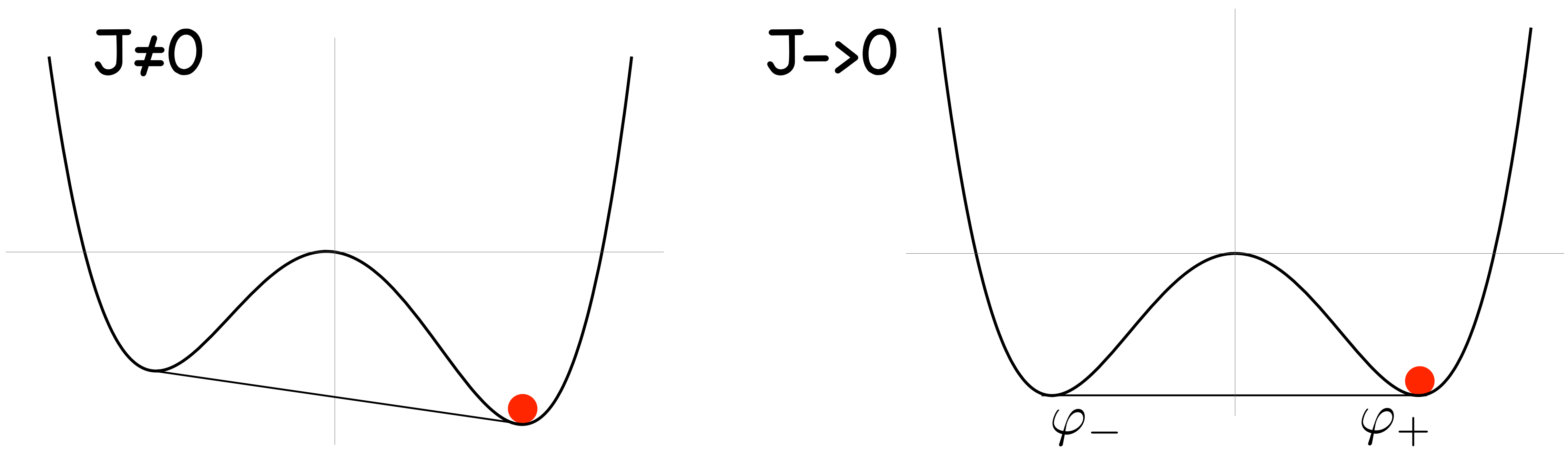
effective potential

Usefulness of V_{eff}

We can search for vacuum at quantum level by $\dfrac{dV_{\text{eff}}(\varphi)}{d\varphi} = 0.$

it can cope with models with radiative SSB (no SSB at tree level)

Convexity



$$\varphi = \lim_{J\to 0}\lim_{V\to\infty}\langle 0|\phi(x)|0\rangle\Big|_J$$

Legendre transformed function is mathematically convex.

- Potential with flat vacuum (convex) = genuine V_{eff}.

  flatness <- mixed configuration $\varphi_{\rm mix} = x\varphi_- + (1-x)\varphi_+, \quad 0\le x\le 1$

- Double-well potential (we use) (*we still call it V_{eff}.)

  no mixed configuration -> non-convex potential

Exercise

Show that vacuum expectation values of fields except for scalars vanish if the vacuum respect the Poiancare symmetry.

1-loop V_{eff}

After summing all 1PI diagrams and regularize them by the MS-bar scheme, one 1-loop V_{eff} as

$$V_1^{(\text{scalar})}(\varphi) = \frac{\bar{m}^4}{64\pi^2}\left(\log\frac{\bar{m}^2}{\bar{\mu}^2} - \frac{3}{2}\right) \qquad \bar{m}^2 = \frac{\partial^2 V_0}{\partial\varphi^2}$$

$\bar{\mu}$: renormalization scale in the $\overline{\text{MS}}$ scheme

statistics

$$V_1^{(\text{fermion})}(\varphi) = -4\cdot\frac{\bar{m}^4}{64\pi^2}\left(\log\frac{\bar{m}^2}{\bar{\mu}^2} - \frac{3}{2}\right)$$

dof

$$V_1^{(\text{gauge})}(\varphi) = 3\cdot\frac{\bar{m}^4}{64\pi^2}\left(\log\frac{\bar{m}^2}{\bar{\mu}^2} - \frac{5}{6}\right) \quad (\xi = 0)$$

We can obtain the Higgs mass at 1-loop level using 1-loop V_{eff}.

Higgs mass at 1-loop

$$V_{\text{eff}}(\varphi) = V_0(\varphi) + V_1(\varphi)$$

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{\varphi}{\sqrt{2}} \end{pmatrix}$$

$$m_h^2 = \left.\frac{\partial^2 V_{\text{eff}}}{\partial \varphi^2}\right|_{\varphi = v} = 2\lambda v^2 - \frac{3 m_t^4}{2\pi^2 v^2} \ln \frac{m_t^2}{\bar{\mu}^2}$$

However

Higgs mass obtained by the potential ≠ Physical Higgs mass

$$\frac{1}{p^2 - M_h^2}$$ physical mass is defined by pole of the propagator

$$M_h^2 = m_h^2 + \text{Re}\bar{\Sigma}_h(M_h) - \text{Re}\bar{\Sigma}_h(0) = m_h^2 + \underline{\frac{3 m_t^2 M_h^2}{4\pi^2 v^2}\left(\frac{1}{3} + \frac{1}{2}\ln\frac{m_t^2}{\bar{\mu}^2}\right)}$$

↑ physical Higgs mass

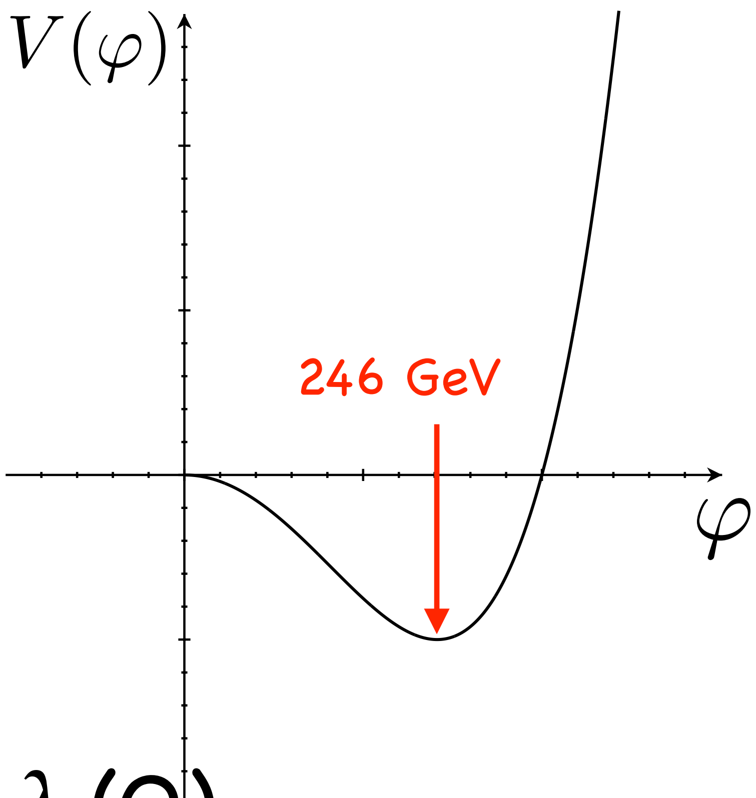
This correction is small for $\bar{\mu} = m_t$

In the SM, the Higgs mass obtained by the potential is close to the physical Higgs mass. (* this is not always true in BSM models)

Vacuum stability

- Is vacuum stable?

$$V(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$$



- It looks stable since $\lambda = m_h^2/2v^2 > 0$.

- However, λ can change with energy. $\lambda \to \lambda(Q)$.

  $\lambda(Q) > 0$ even for high energy (large ϕ)?

$$V(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \boxed{\frac{\lambda}{4}\varphi^4} \xrightarrow[\varphi \gg v]{} \frac{\lambda(Q)}{4}\varphi^4$$

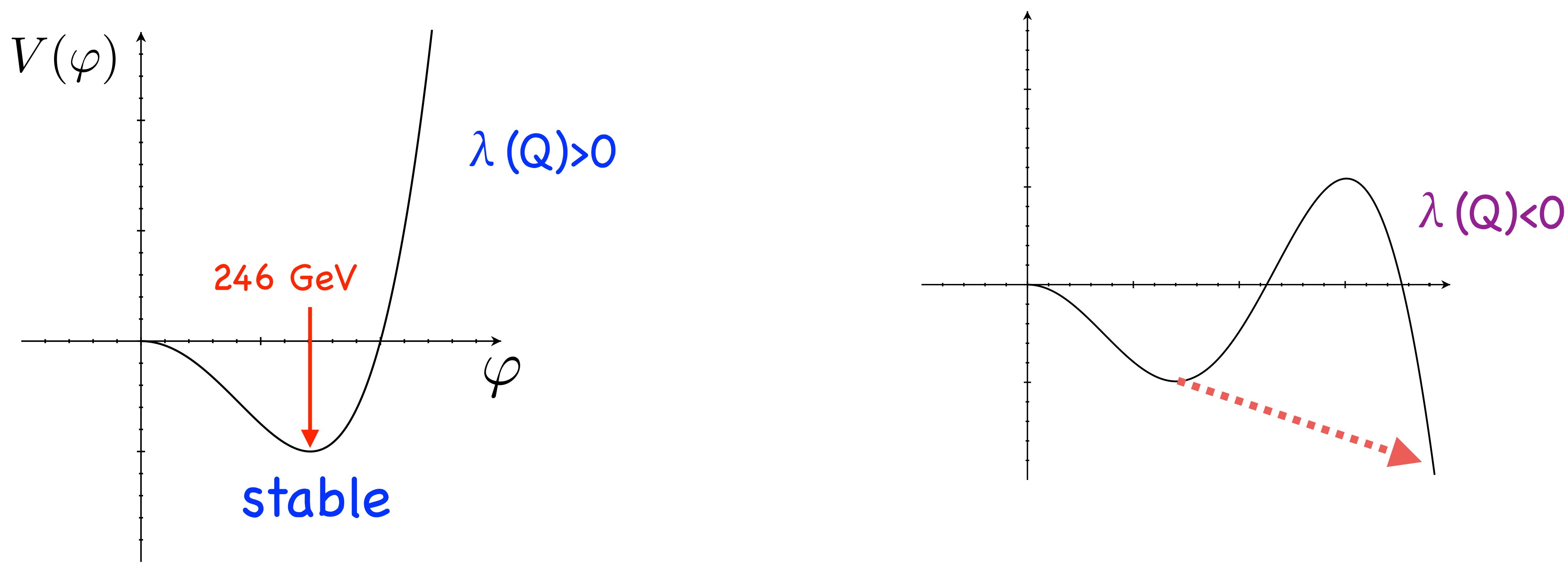
- Quadratic term can be neglected for large ϕ.

Vacuum stability

- λ (Q) obeys renormalization group equation.

$$\frac{d\lambda(t)}{dt} = \frac{1}{16\pi^2}\left[24\lambda^2 - 6y_t^4 + 12\lambda y_t^2 + \cdots\right], \quad t = \ln(Q/v)$$

- vacuum stability largely depends on λ & y_t (& α_s).



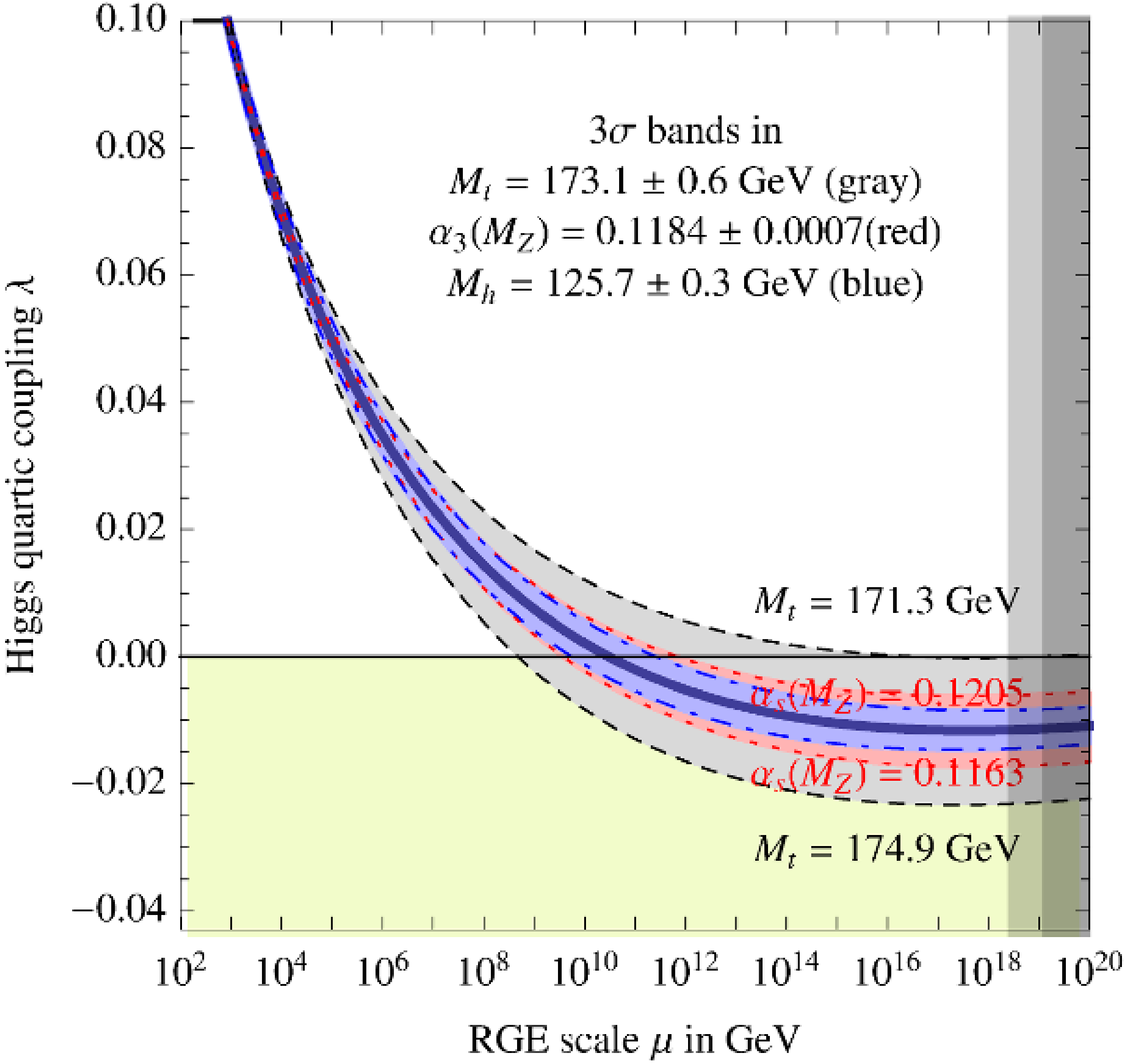
lifetime of the vacuum > age of the Universe -> meta-stable.

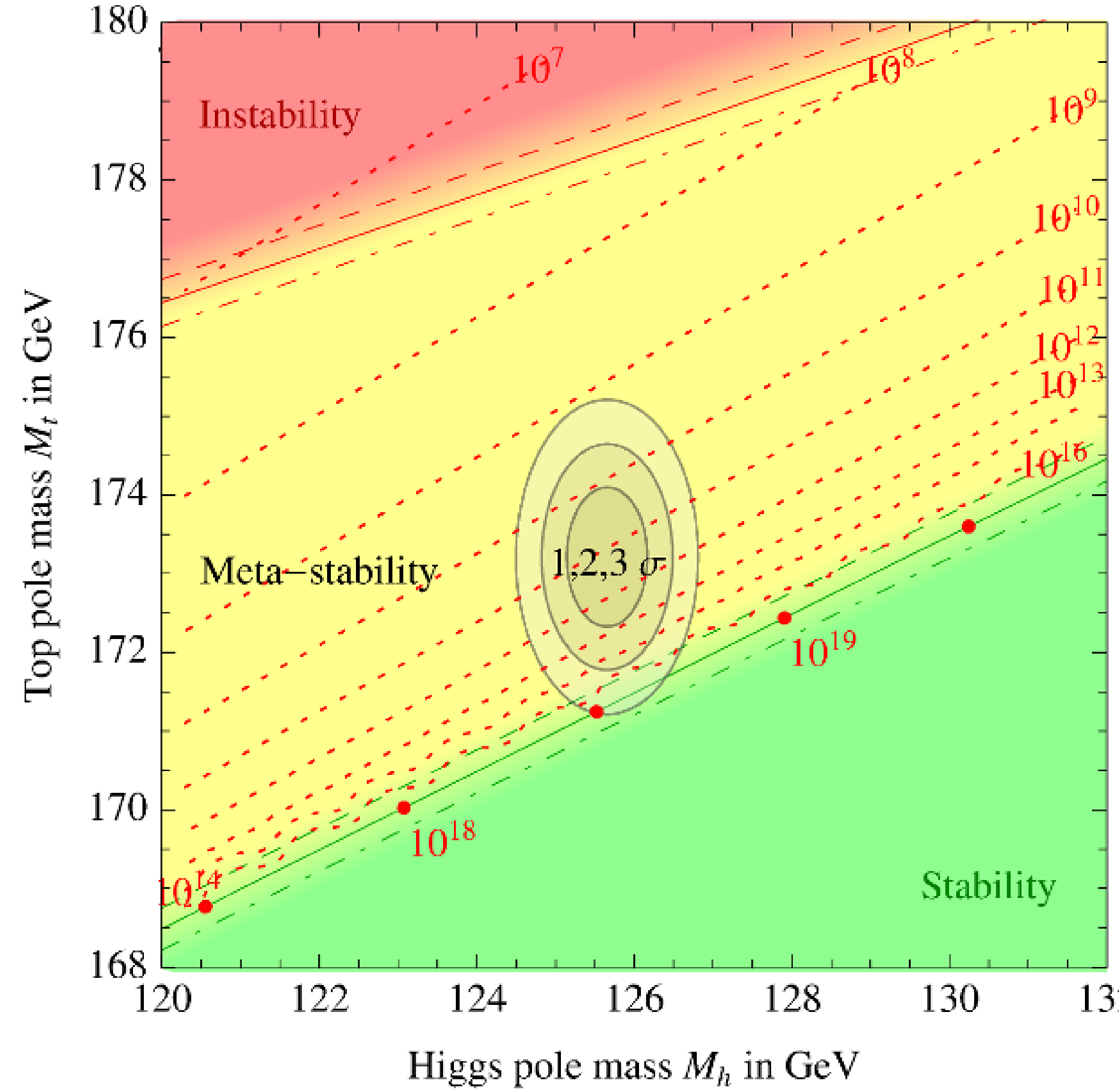
lifetime of the vacuum < age of the Universe -> unstable.

What is the current status?

Vacuum stability

JHEP12 (2013) 089 (1307.3536)





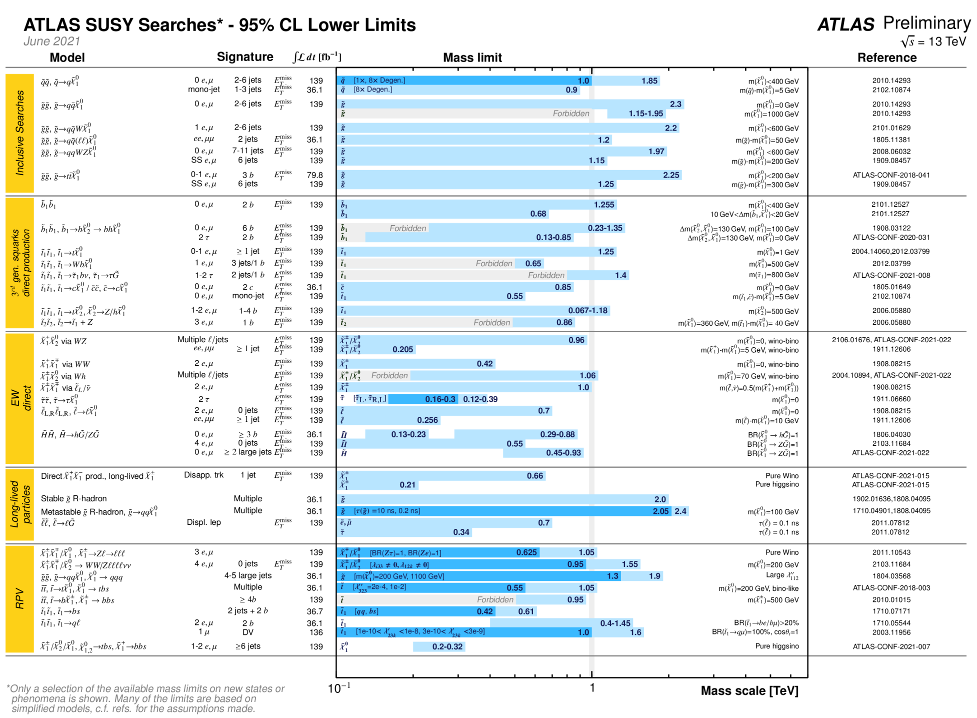
- "stable" or "meta-stable" depends on top mass.
- Δm_t=0.5 GeV@HL-LHC, Δm_t=0.1 GeV@ILC

Summary

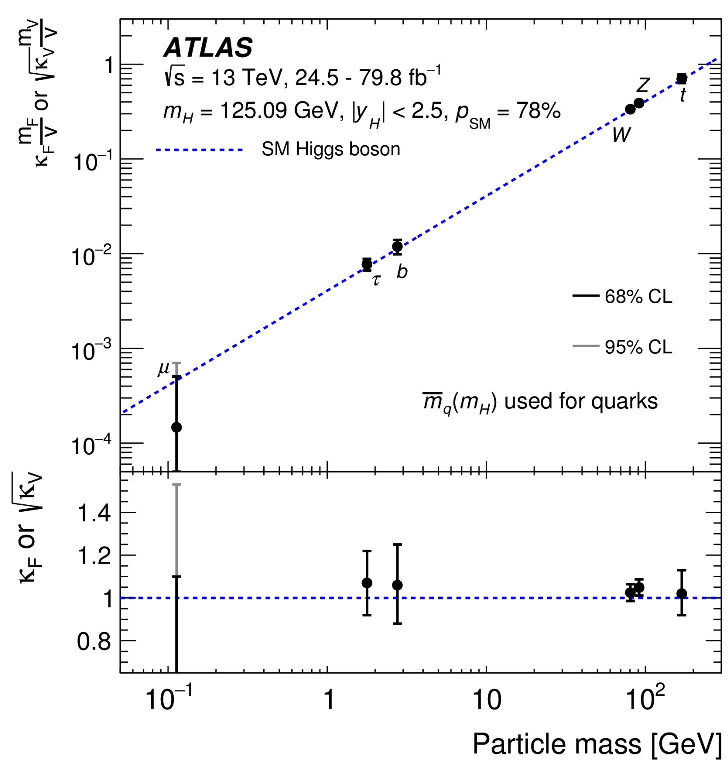
Summary

- SM is $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory.
- All elementary particles get their masses via Higgs mechanism
- All experiments so far are consistent with SM!!

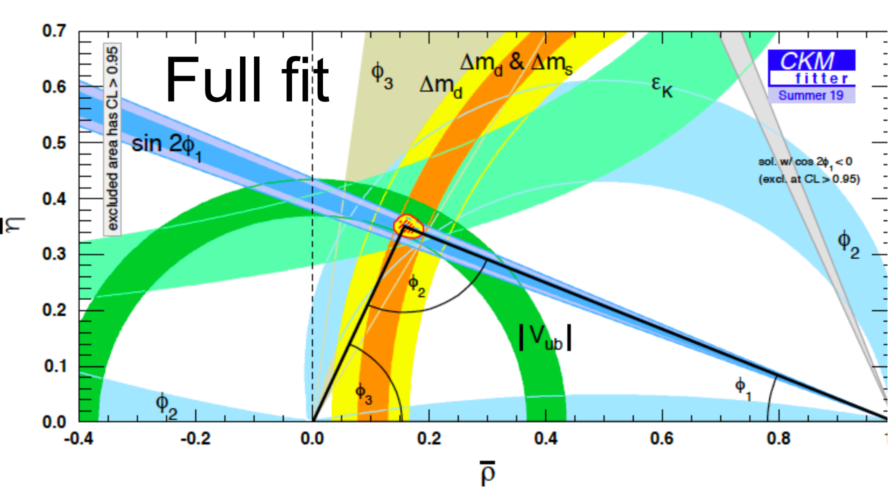
Supersymmetric particles searches at LHC



Higgs coupling measurements at LHC



Constraints from B meson physics



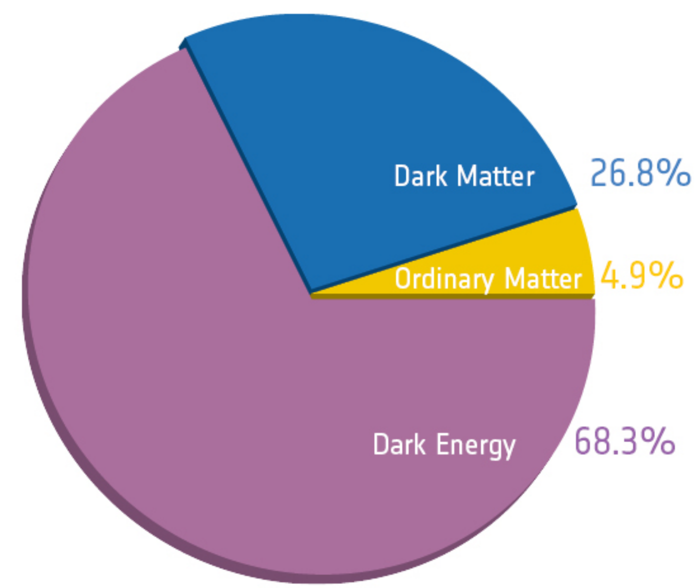
Nevertheless, SM cannot be considered as final theory. Because, we know that neutrinos have masses, moreover

Beyond the SM

SM cannot explain cosmological observations.

- Matter-antimatter (baryon) asymmetry of the Universe
- Dark Matter
- Dark Energy
- Quantum gravity
- etc

Energy budget of the Universe



ESA and the Planck Collaboration

The Universe is NOT made of atoms but made of unknown objects.

So, SM must be extended to solve some (hopefully many) of them.