Nuisances in the fit: the nuclear model uncertainties in the new era of Spectral Function

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ÉCOLE POLYTECHNIQUE | INSTITUT POLYTECHNIQUE DE PARIS

IN2P3 Les deux infinis

Physique des 2 Infinis et des Origines

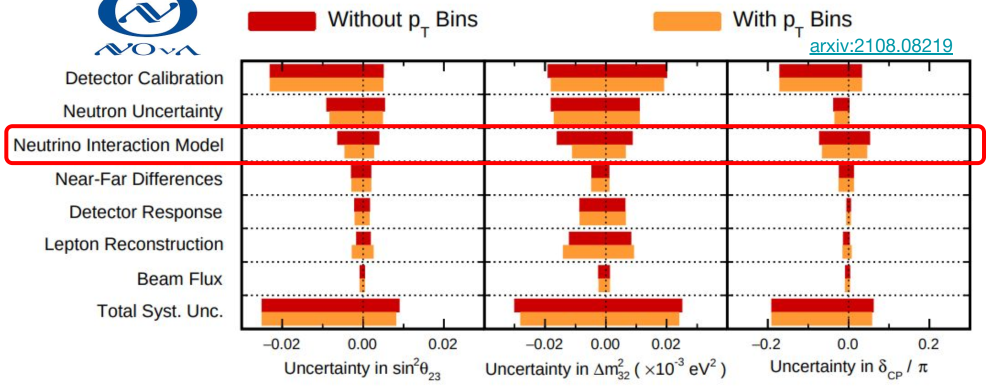
Outline

1. Introduction

Neutrino oscillations: the precision era

- First hints of CP violation
- Currently: NOνA, T2K (w/ ND upgrade)
- Future: DUNE, Hyper-Kamiokande
- Still limited by statistics, but not for long!

DEEP UNDERGROUND NEUTRINO EXPERIMENT

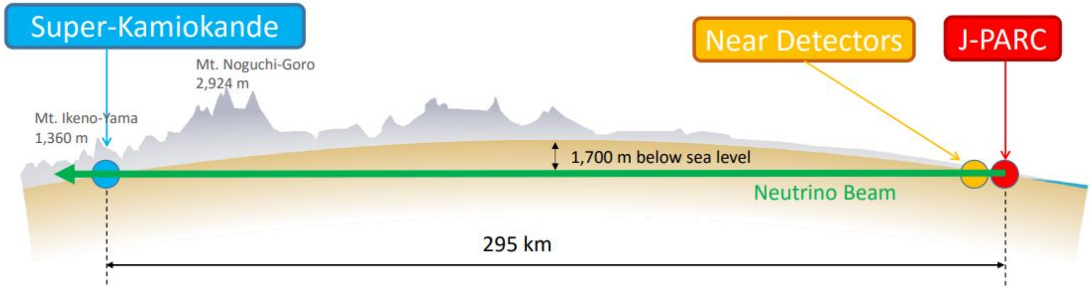


Phys.Rev.D 103, 112008 (2021)

	1-Ring μ		1-Ring e			
Error source	FHC	RHC	FHC	RHC	FHC 1 d.e.	FHC/RHC
SK Detector	2.4	2.0	2.8	3.8	13.2	1.5
SK FSI+SI+PN	2.2	2.0	3.0	2.3	11.4	1.6
Flux + Xsec (ND constrained)	3.3	2.9	3.2	3.1	4.1	2.7
Nucleon Removal Energy	2.4	1.7	7.1	3.7	3.0	3.6
$\sigma(\nu_e)/\sigma(\bar{\nu}_e)$	0.0	0.0	2.6	1.5	2.6	3.0
NC1γ	0.0	0.0	1.1	2.6	0.3	1.5
NC Other	0.3	0.3	0.2	0.3	1.0	0.2
$\sin^2\theta_{23} + \Delta m^2_{21}$	0.0	0.0	0.5	0.3	0.5	2.0
$\sin^2\theta_{13}$ PDG2018	0.0	0.0	2.6	2.4	2.6	1.1
All Systematics	5.1	4.5	8.8	7.1	18.4	6.0

⇒ Neutrino interaction uncertainties must be reduced!

T2K Oscillation Analysis strategy



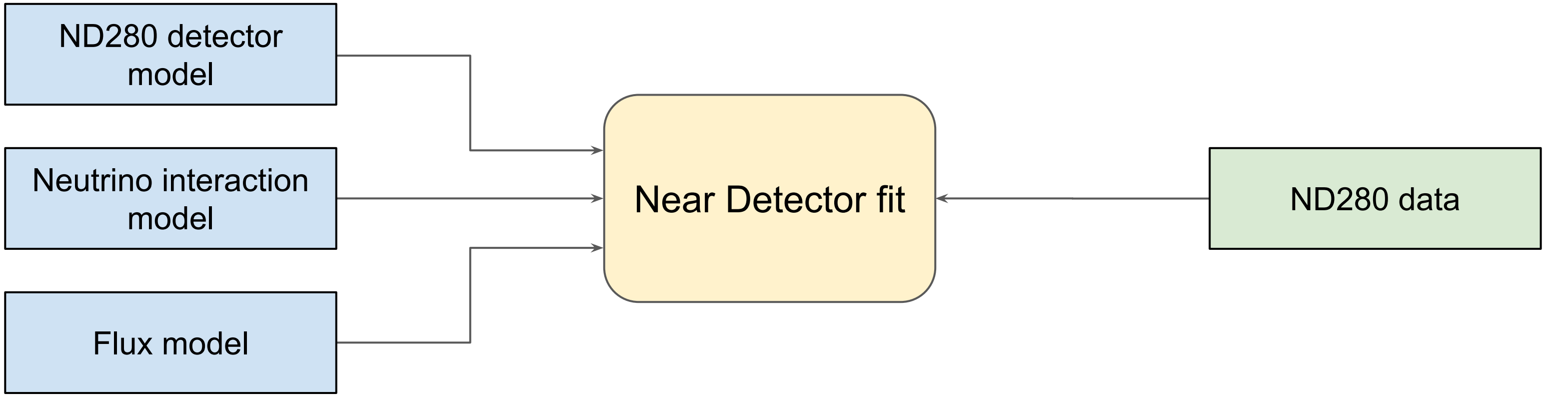
Flux model | Detector model | Neutrino interaction model

$$N_{\nu_\alpha}^{ND}(E_\nu) = \Phi_{\nu_\alpha}^{ND}(E_\nu) \times \epsilon^{ND}(E_\nu) \times \sigma_{\nu_\alpha}^{ND}(E_\nu)$$

$$N_{\nu_\beta}^{FD}(E_\nu) = \Phi_{\nu_\beta}^{FD}(E_\nu) \times \epsilon^{FD}(E_\nu) \times \sigma_{\nu_\beta}^{FD}(E_\nu) \times P_{\nu_\alpha \to \nu_\beta}(E_\nu)$$

- **Far/Near ratio does not fully cancel systematic uncertainties:**
  - Flux model different at ND vs. FD due to geometry and oscillation
  - Different detectors, i.e. different acceptance, efficiencies, targets…
  - Mainly muon neutrinos at ND interacting with CH → use model to infer interactions with electron neutrino interactions and with H2O

T2K Oscillation Analysis strategy



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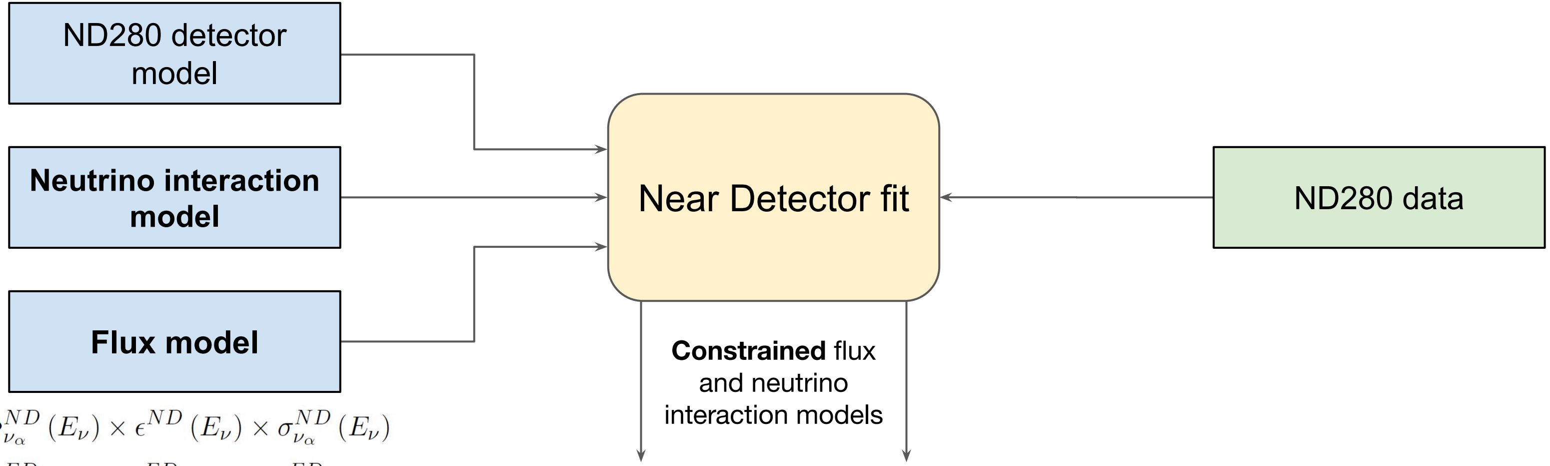

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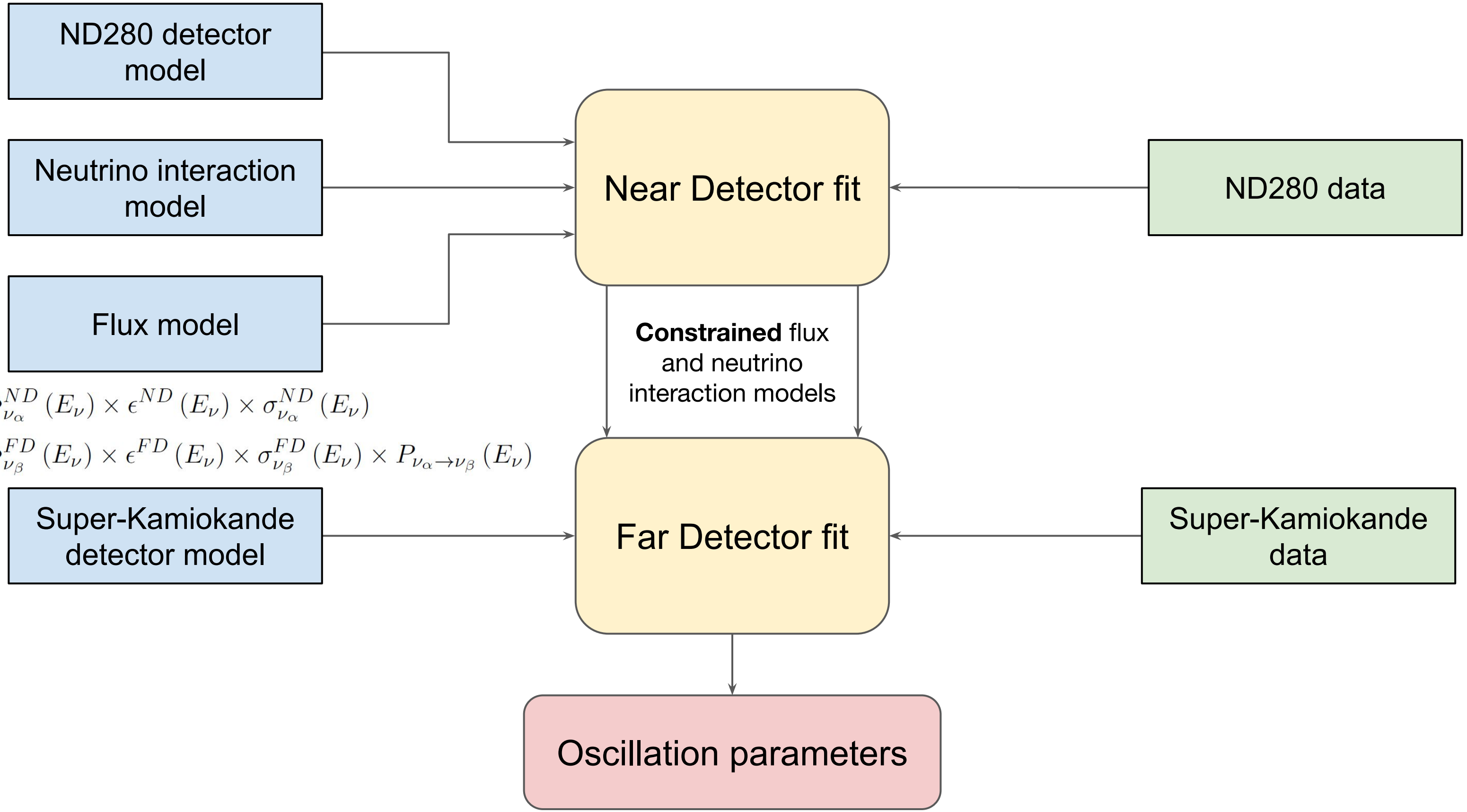

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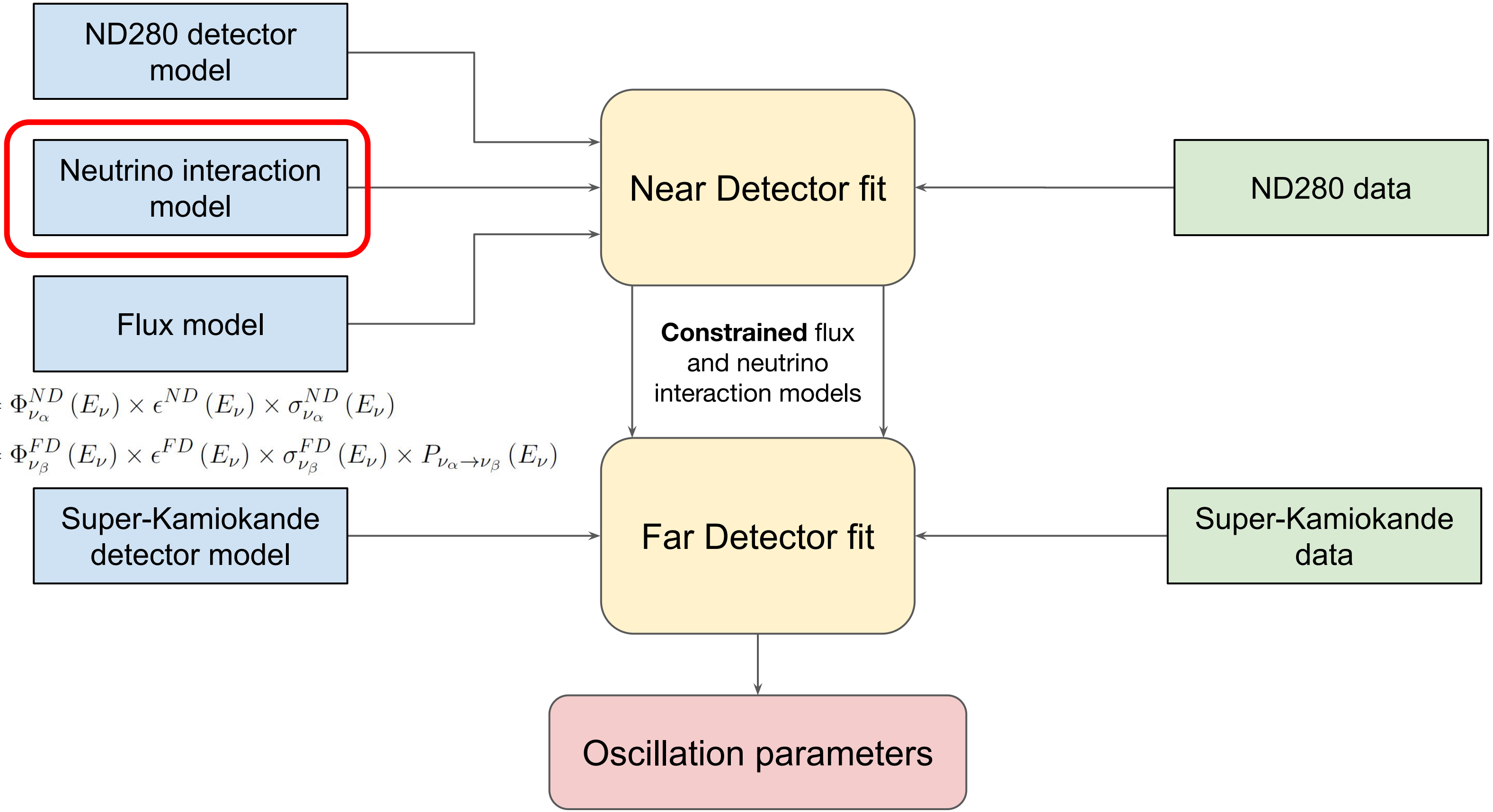
2. Neutrino-nucleus interaction models

T2K Oscillation Analysis strategy



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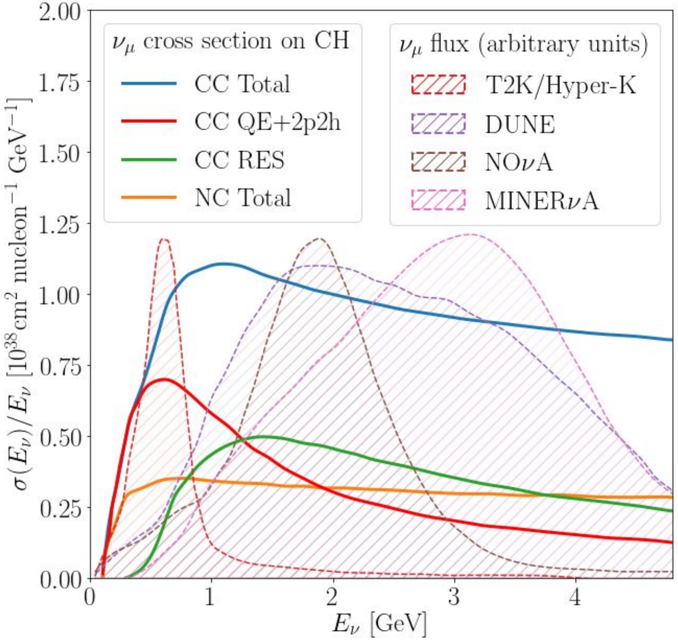
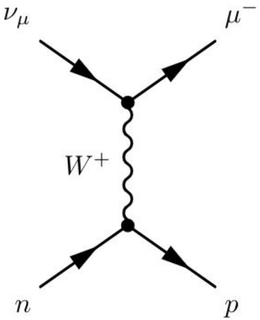
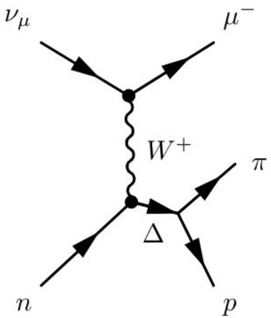
Neutrino interactions



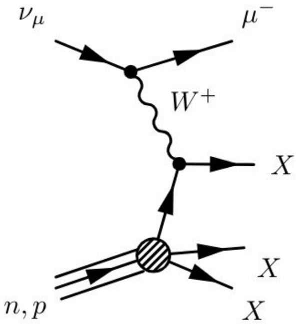
Quasi-elastic (CCQE)



Resonant (CCRES)

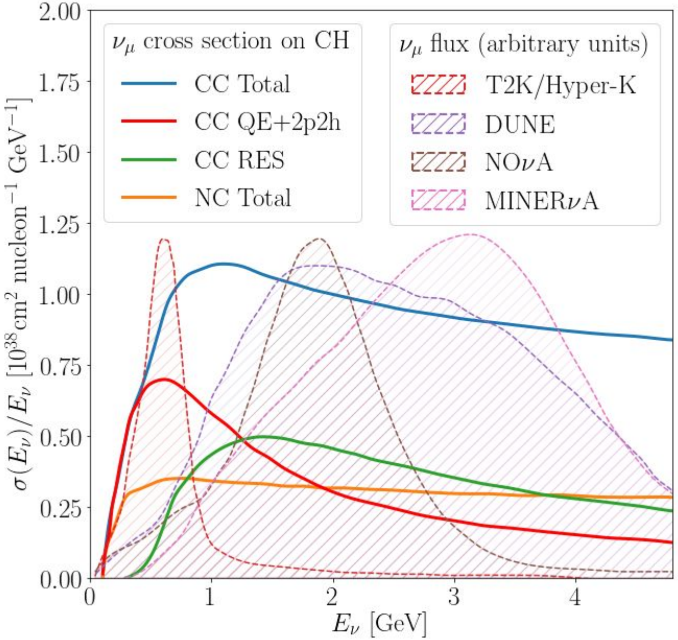


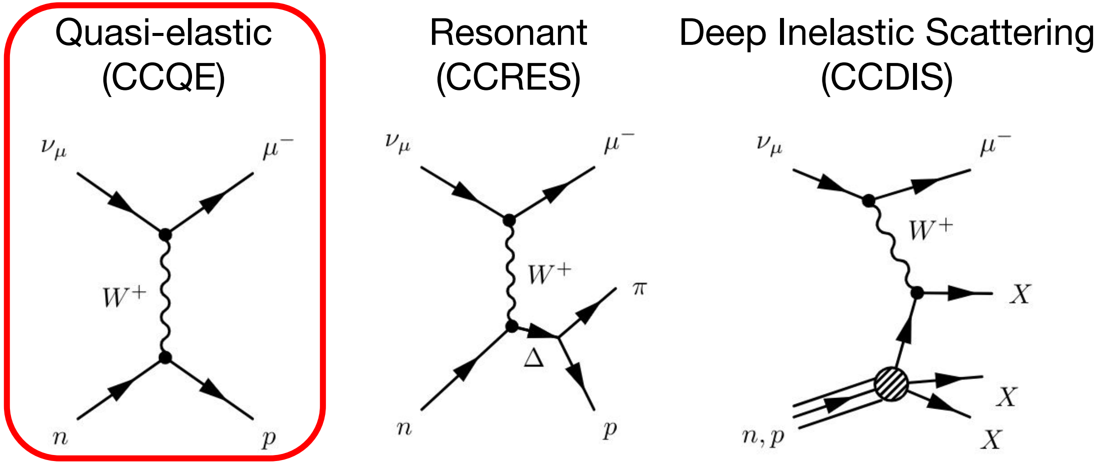
Deep Inelastic Scattering (CCDIS)



- In order to estimate neutrino energy, a good understanding of neutrino-nucleus interactions is necessary
- CCQE is the dominant interaction in T2K/Hyper-Kamiokande, and is a significant mode in NOνA, MINERνA and DUNE

Neutrino interactions





This talk

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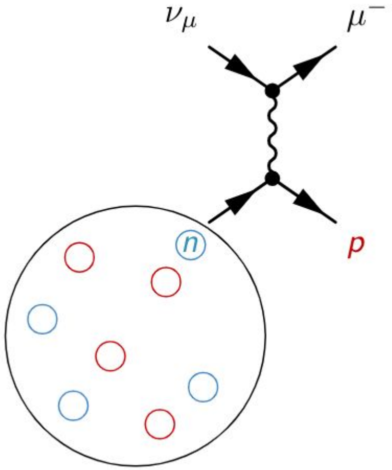
Initial state nucleons

- Neutrinos can interact with nucleons bound within nuclei (Carbon, Oxygen, Argon...)
- Initial state nucleons are non-static: Fermi motion
- How to model this?



Initial state nucleons: Fermi gas models

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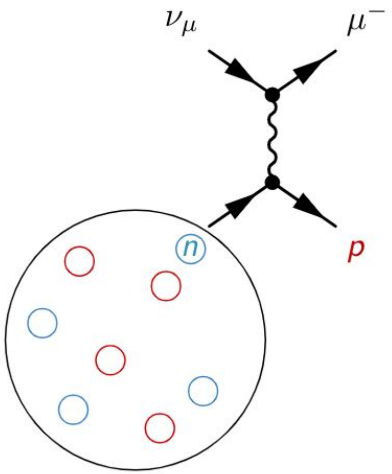


Fermi gas

Relativistic Fermi Gas (RFG)

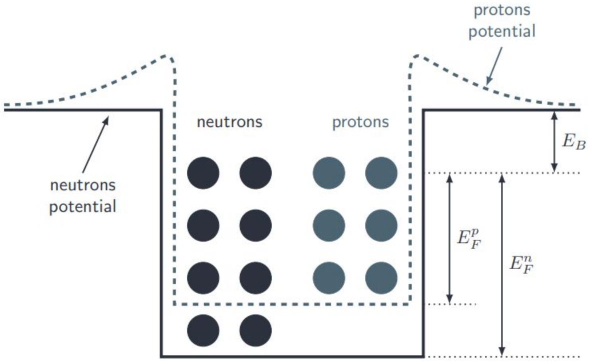
Nucleons move freely in a constant binding energy within the nuclear volume

$$p_F = \left(3\pi^2 \rho \frac{Z}{A}\right)^{1/3}$$

Local Fermi Gas (LFG)

The nucleus is described with the local density approximation

$$p_F(r) = \left(3\pi^2 \rho(r) \frac{Z}{A}\right)^{1/3}$$



T. Golan

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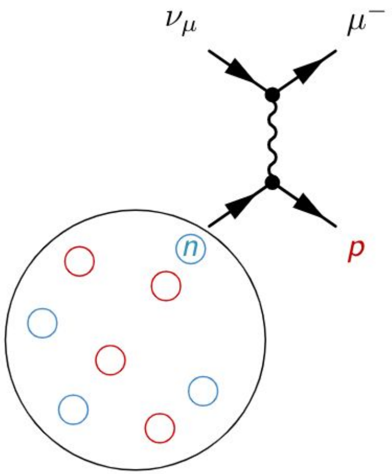


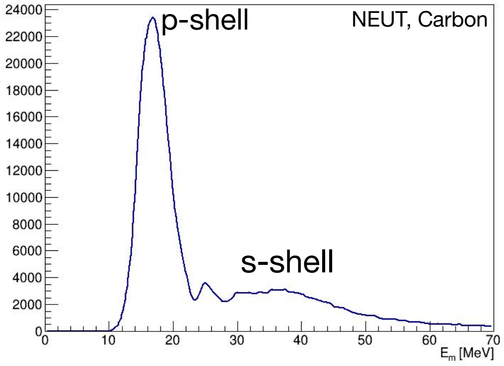
Spectral Function (SF)

The probability of removing of a nucleon with momentum p_m and leaving residual nucleus with excitation energy E_m

$$P(p_m, E_m) = P_{MF}(p_m, E_m) + P_{corr}(p_m, E_m)$$

Independent nucleons, moving in a mean-field potential within the shell-model picture → built from (e,e'p) data (~80%)

→ One outgoing nucleon is produced



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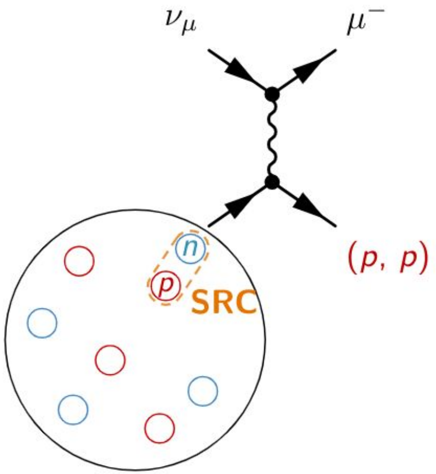

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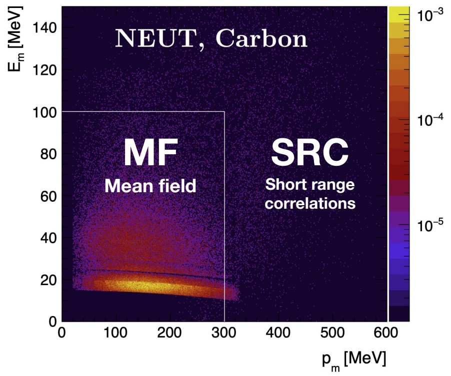

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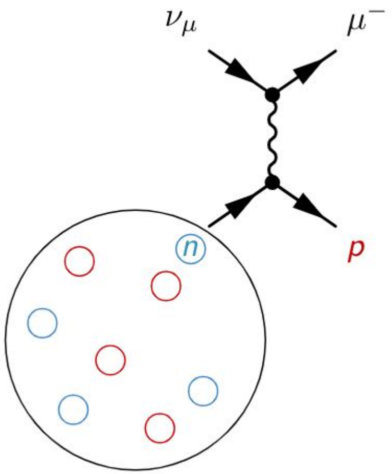

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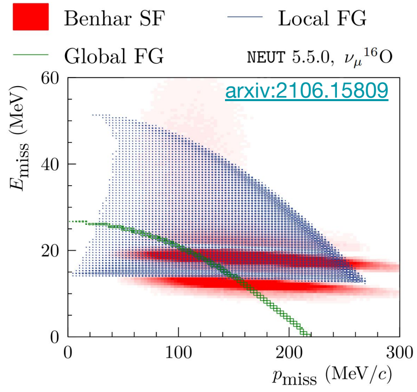

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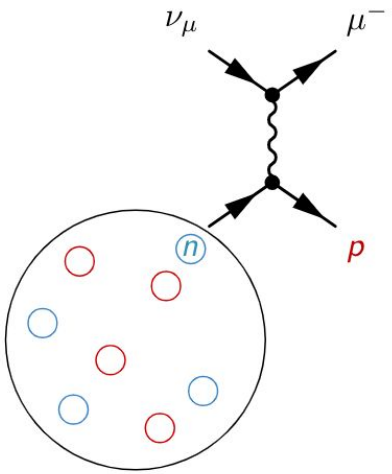

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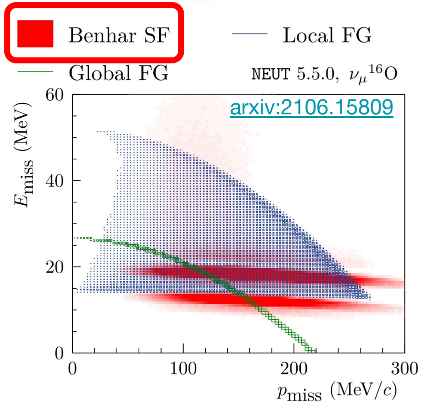
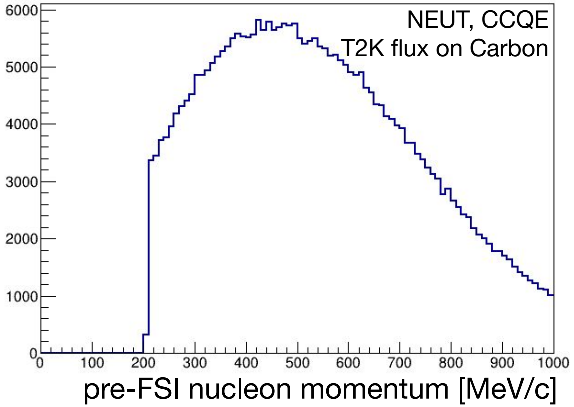


Additional nuclear effects

- Pauli blocking (PB):
  - By Pauli principle, an interaction cannot occur if it leads to the creation of a nucleon in a state that is already occupied
  - Simple model: reject events with outgoing nucleon momentum below Fermi level



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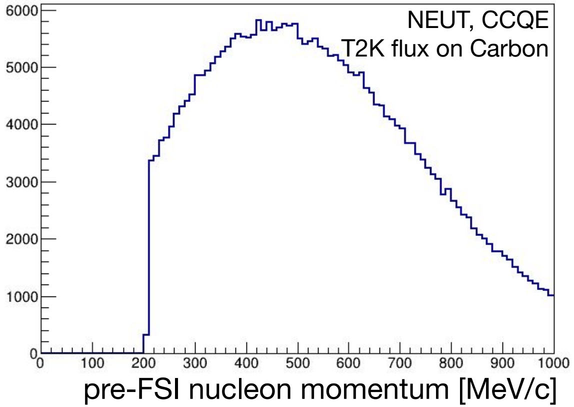

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  - Outgoing particles may re-interact with the nuclear matter
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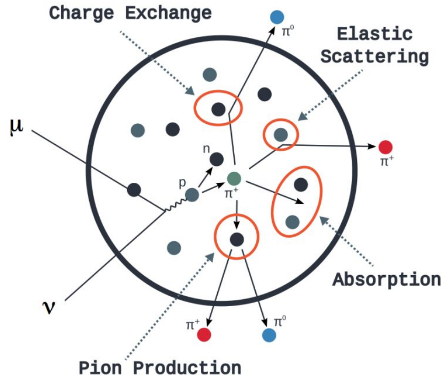

See Anna's presentation

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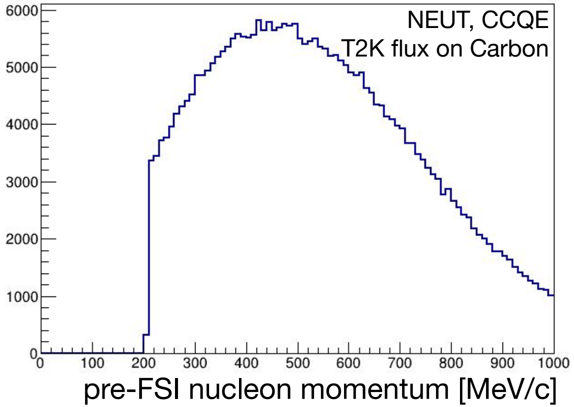

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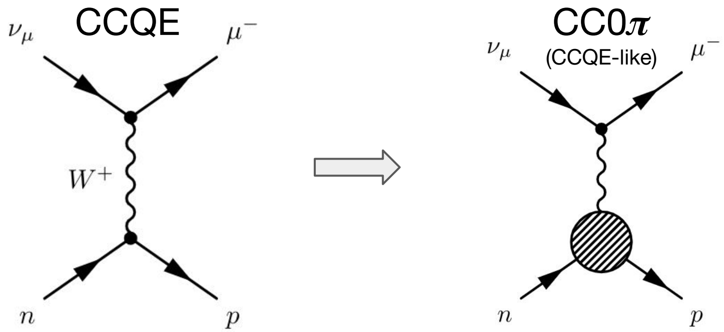
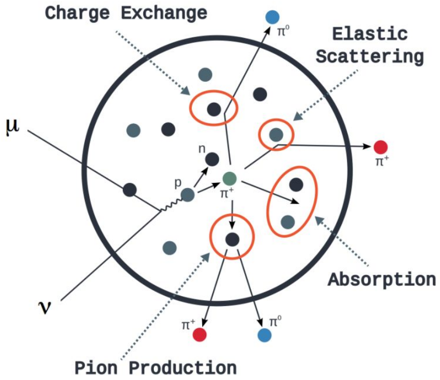


Interaction mode

Interaction topology



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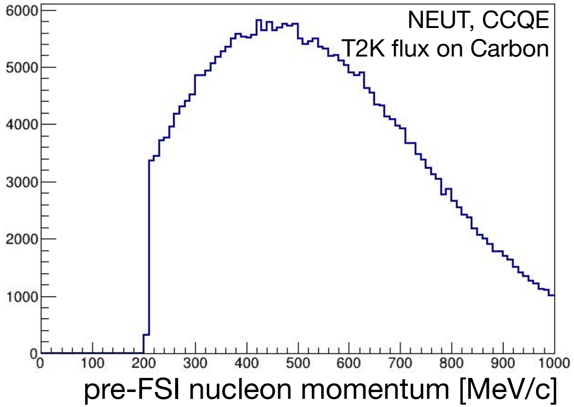

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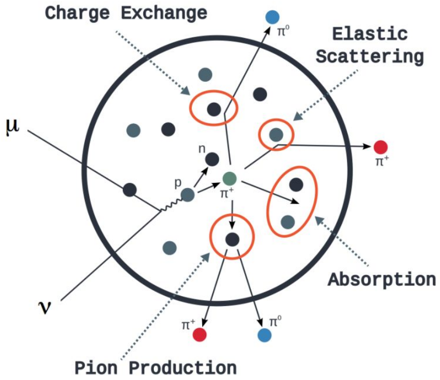
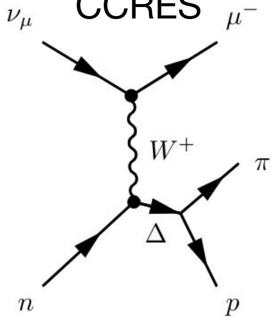


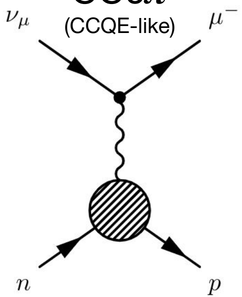
CCRES



Interaction mode

CC0π

(CCQE-like)



Interaction topology

See Anna's presentation

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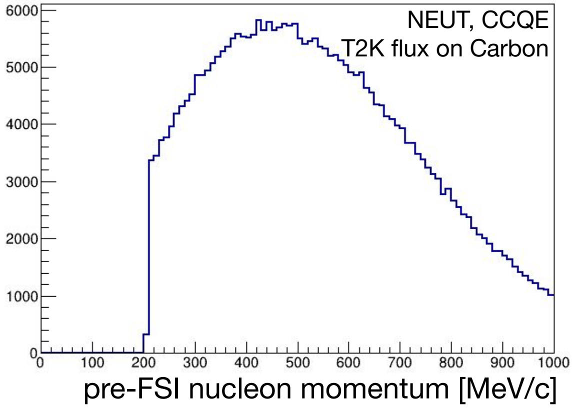

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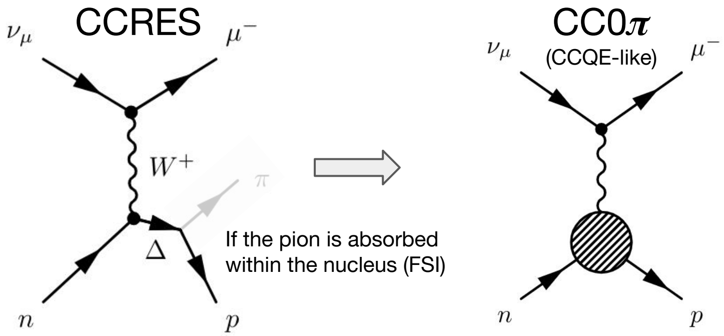

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Interaction mode

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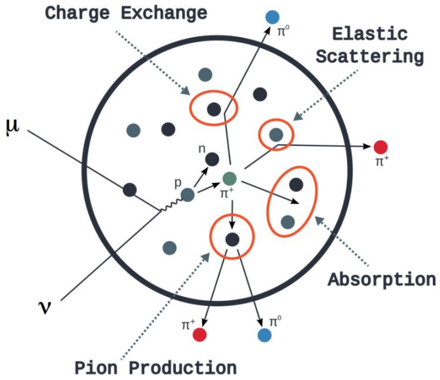
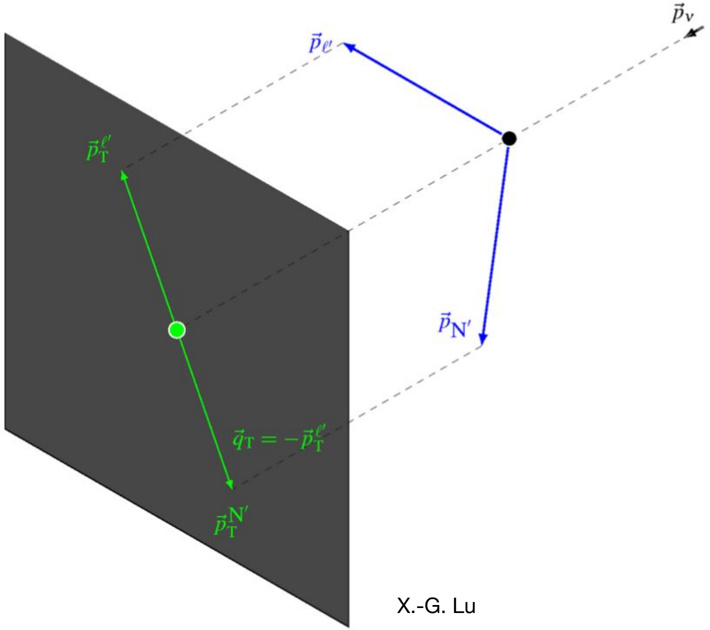


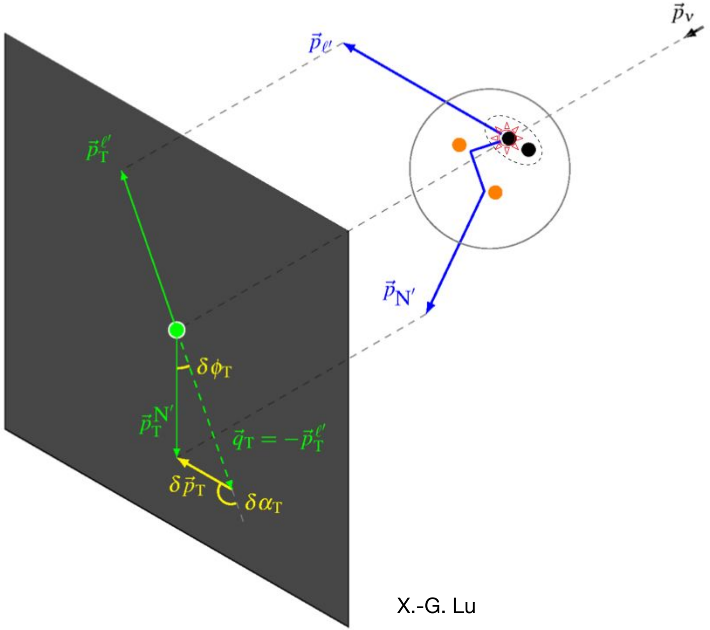
Single Transverse Variables Phys. Rev. C 94, 015503 (2016)

Static nucleon target



X.-G. Lu

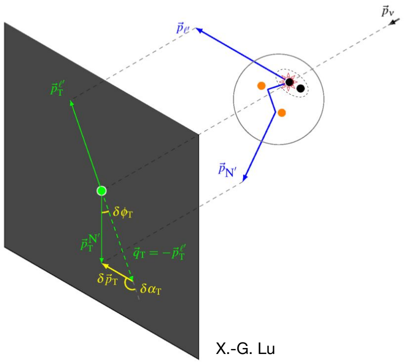
Nucleon bound within nuclear target



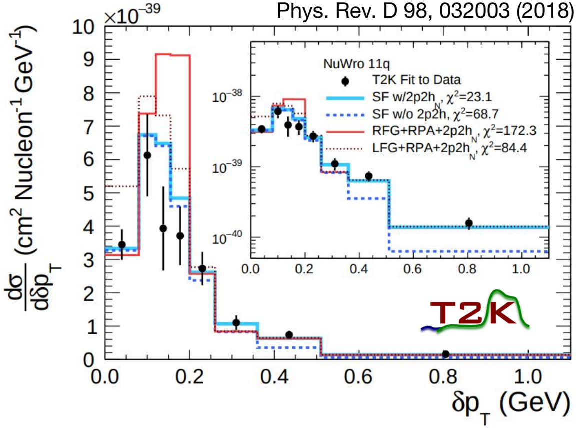
X.-G. Lu

- Need the reconstruction of both muons and nucleons
- Probe nuclear effects (Fermi motion, FSI, ...)

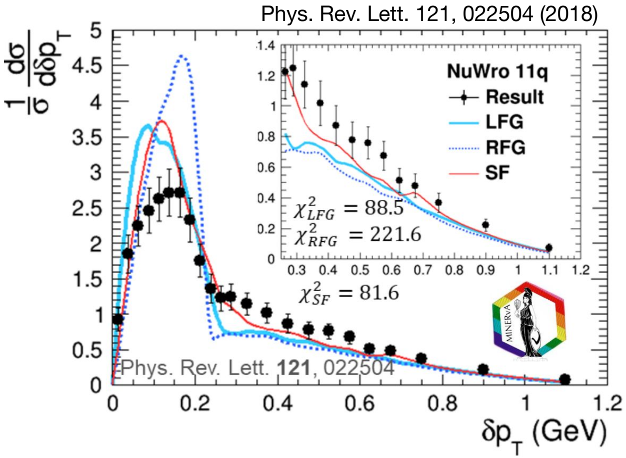
Single Transverse Variables: δp_T Phys. Rev. C 94, 015503 (2016)



T2K



MINERνA



Single Transverse Variables: δp_T Phys. Rev. C 94, 015503 (2016)

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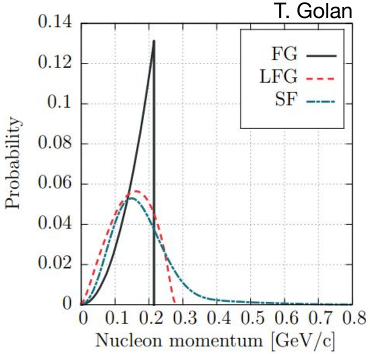
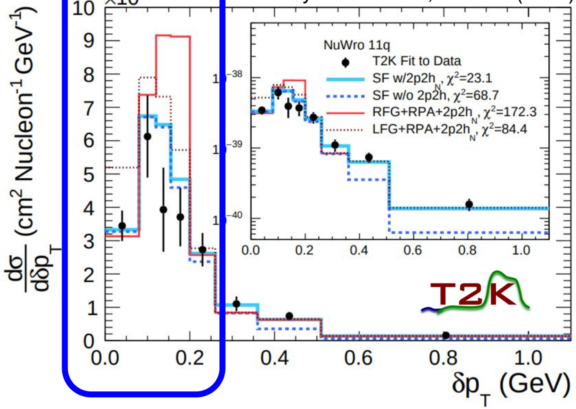
T. Golan



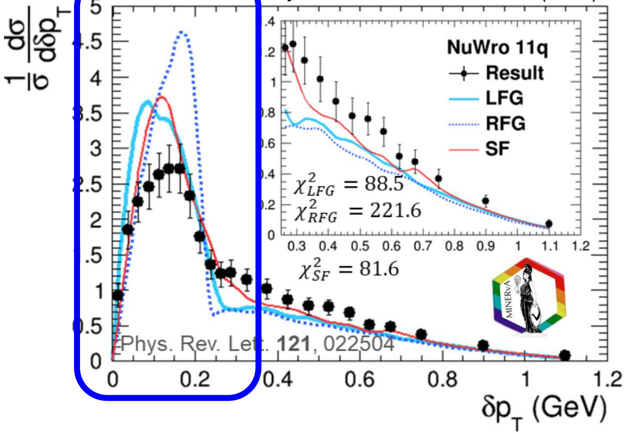
T2K

Phys. Rev. D 98, 032003 (2018)



MINERνA

Phys. Rev. Lett. 121, 022504 (2018)

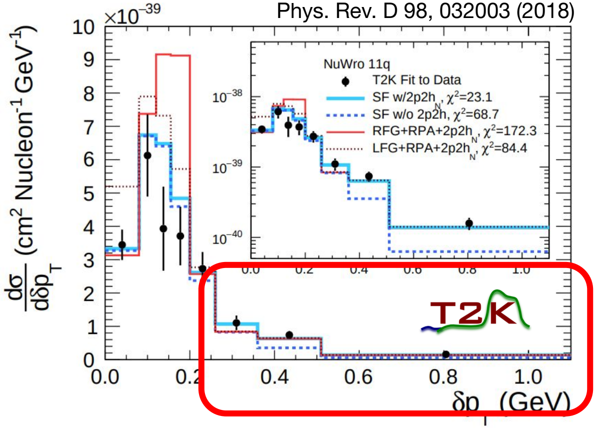


Single Transverse Variables: δp_T Phys. Rev. C 94, 015503 (2016)

- The bulk of the distribution is sensitive to the initial state nucleon momentum
- The tail of the distribution is sensitive to FSI, SRC, 2p2h

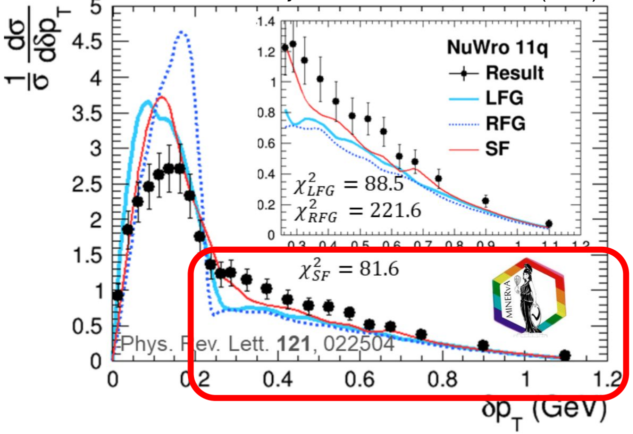
T2K

Phys. Rev. D 98, 032003 (2018)



MINERνA

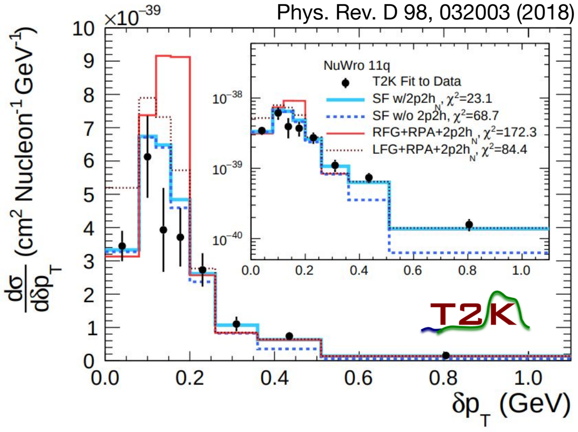
Phys. Rev. Lett. 121, 022504 (2018)



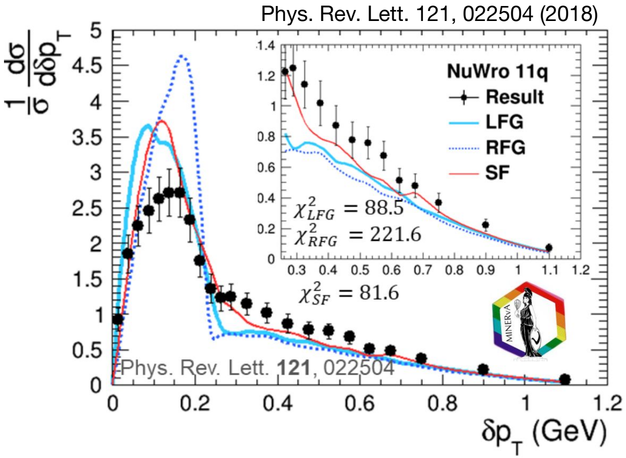
Single Transverse Variables: δp_T Phys. Rev. C 94, 015503 (2016)

- The bulk of the distribution is sensitive to the initial state nucleon momentum
- The tail of the distribution is sensitive to FSI, SRC, 2p2h
- None of the models describe well the data...

T2K



MINERνA

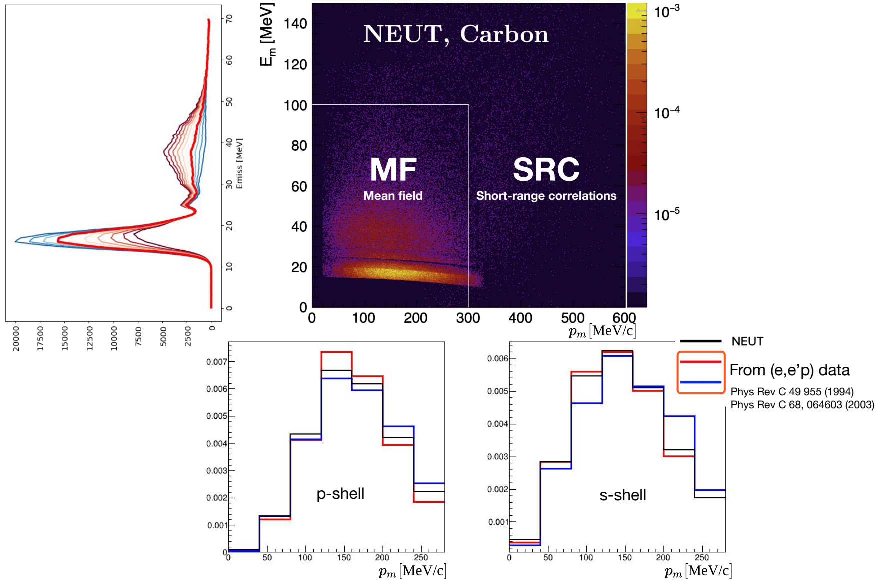


3. Proposed SF systematic uncertainties

Systematic uncertainties in CCQE interactions

PoS(NuFact2021)235

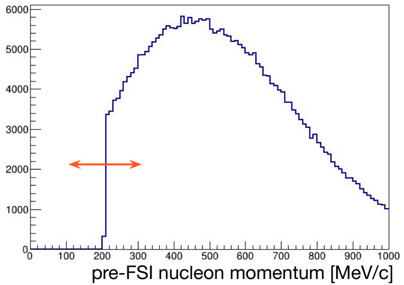
Initial state parameters



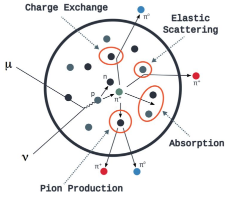
- Change the shape of the missing energy distribution by altering the shell occupancies
- Change the missing momentum distribution shape
- Change the contribution of SRC

Pauli Blocking parameter

- Change Pauli Blocking threshold on the pre-FSI nucleon momentum



FSI parameters



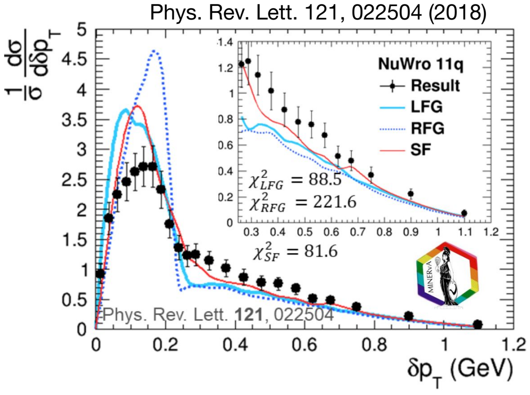
- Outgoing nucleon: Change the amount of events undergoing FSI w.r.t. no FSI *(see Anna's presentation for ways to improve)*
- Outgoing lepton: Change the lepton kinematics with an optical nuclear potential (Phys. Rev. D 91, 033005 (2015))

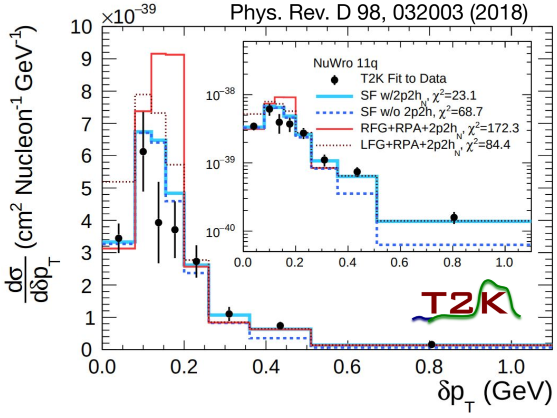
This parameterisation was implemented in NUISANCE and applied on NEUT 5.4.0 neutrino event generator (arxiv:2106.15809)

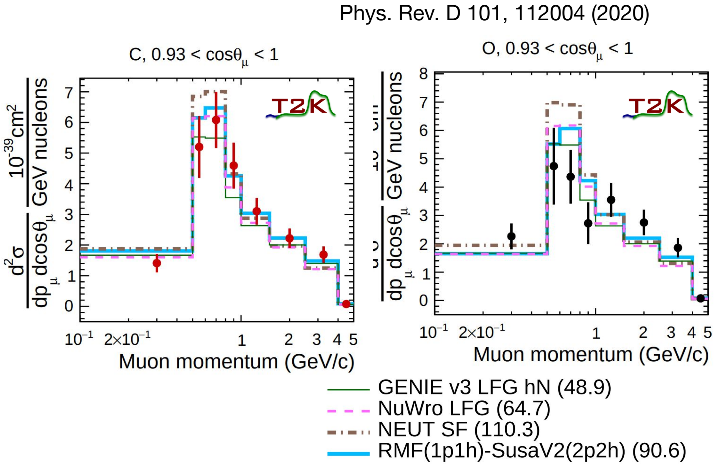
4. Validations with fits to available cross section measurements

Cross section measurements

- Cross section measurements allow to test our neutrino interaction models
- Current models struggle to describe cross section data
- In this talk we explore the uncertainties on the models to see if they can accommodate these discrepancies, and also use the ND data to constrain them







Fits to published data

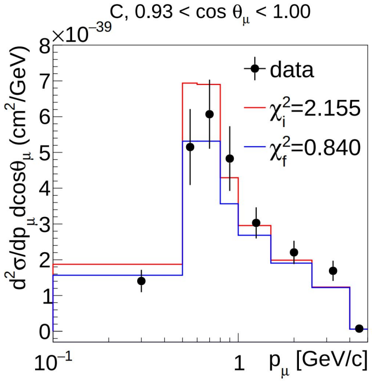
- Ingredients:
    - Model: NEUT with SF model (for Oxygen and Carbon)
    - Parameters: SF model parameters (+ normalisation parameters for other interactions)
    - Data: cross section measurements from T2K and MINERvA
- Chi-square(*): $\chi^2_{\text{data}} = \sum_{1\leq i,j\leq n} \left(B_i - B_i^{MC}\right) \left(M^{-1}\right)_{ij} \left(B_j - B_j^{MC}\right)$
- How is this parameterisation able to improve agreement with the data?
- NUISANCE, which is a framework that aims to provide a coherent framework for comparing different neutrino event generators to external data, is used for this study

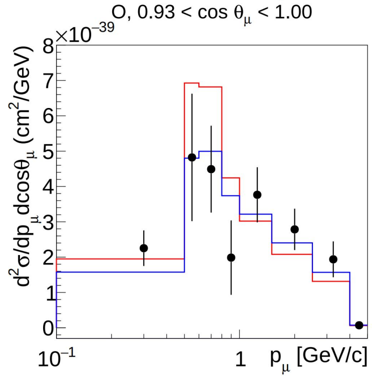
(*) Peelle's Pertinent Puzzle was avoided using a different decomposition of the data histogram and covariance matrix, see back-up

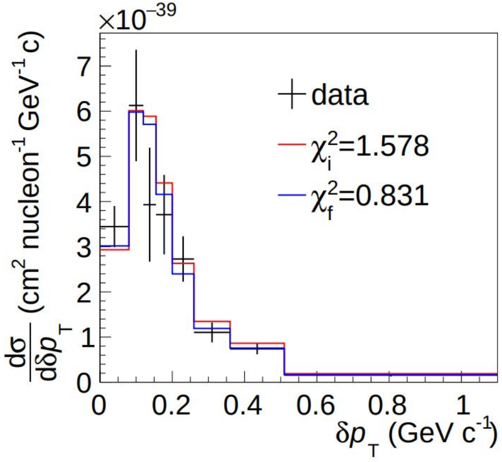
Fits to cross section measurements

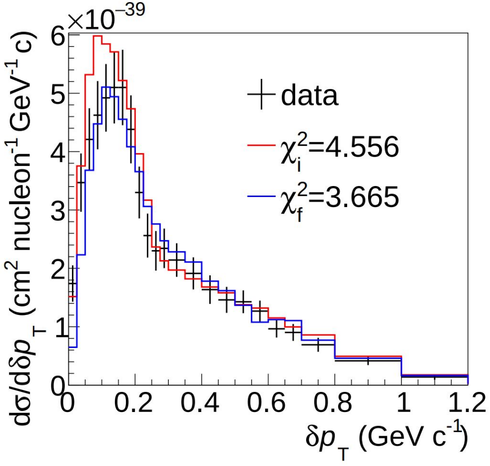
T2K lepton kinematics on O & C (CC0π)	T2K δp_T on C (CC0π+Np)	MINERνA δp_T on C (CC0π+Np)

Only the forward bins are shown here

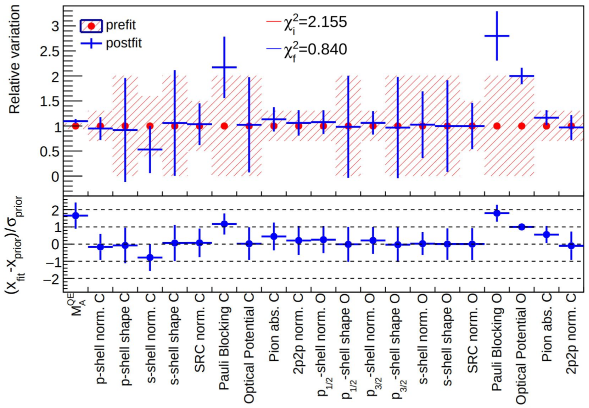
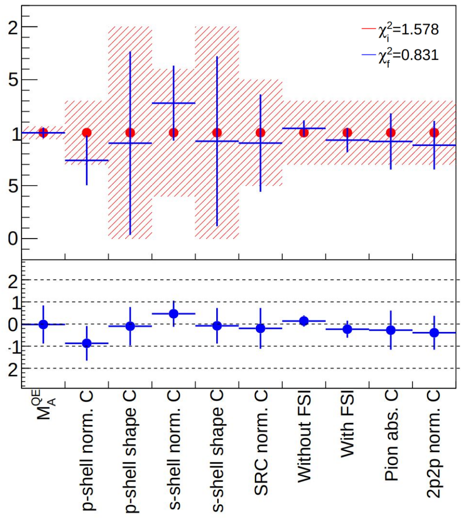
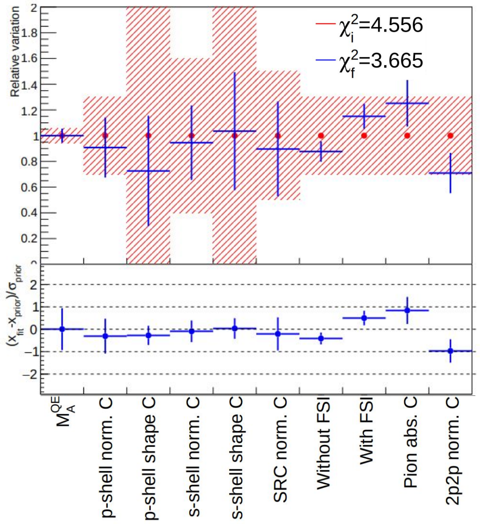






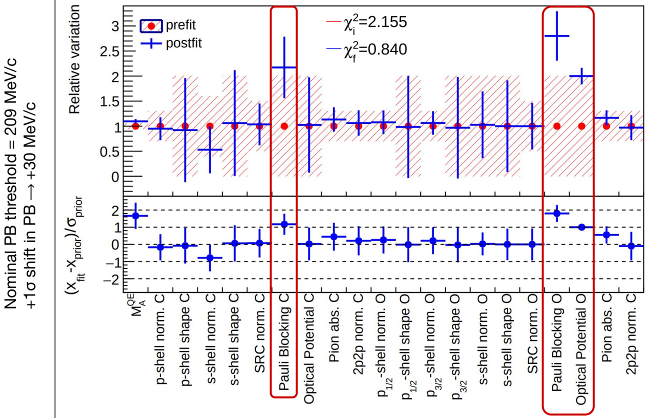


Fits to cross section measurements

T2K lepton kinematics on O & C (CC0π)	T2K δp_T on C (CC0π+Np)	MINERνA δp_T on C (CC0π+Np)
		

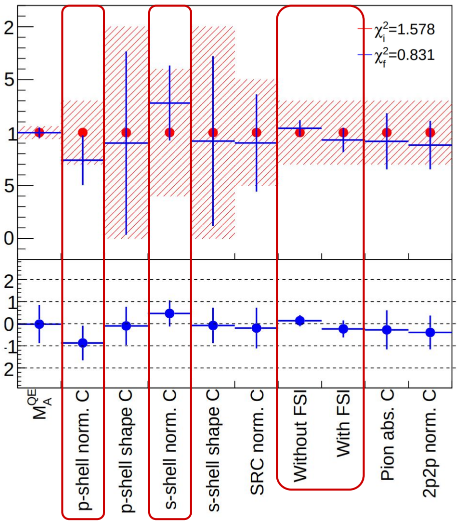
Fits to cross section measurements

T2K lepton kinematics on O & C (CC0π)

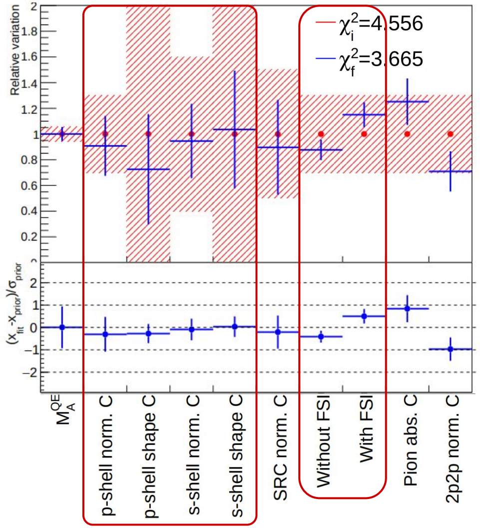


- The agreement with the data is driven by the Pauli Blocking and Optical Potential parameters
- Some sensitivity to the shell parameters can be noticed
- No sensitivity to the missing momentum shape parameters

T2K δp_T on C (CC0π+Np)



MINERνA δp_T on C (CC0π+Np)



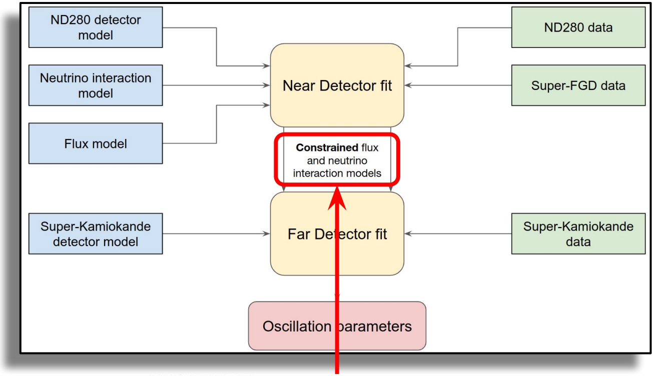
- The transverse momentum imbalance fits show more sensitivity to the initial state parameters as well as the FSI parameters
- The high postfit chi-square in MINERνA data may suggest that the FSI model is insufficient

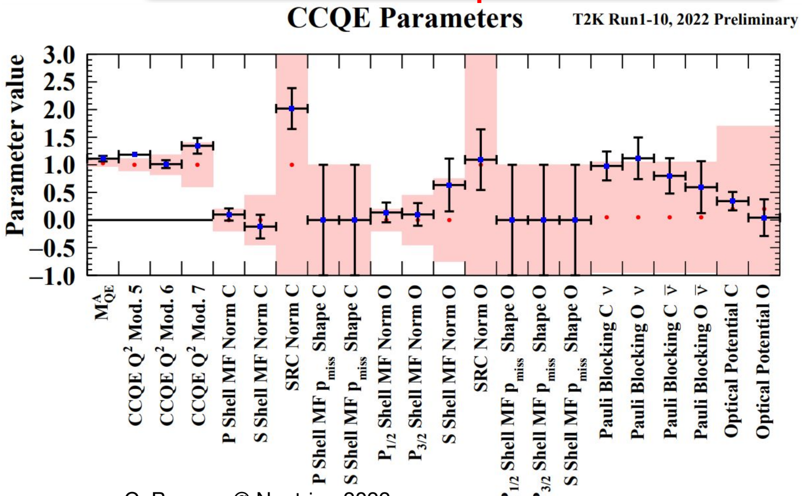
To the T2K 2022 Oscillation Analysis

- Fits to **published cross section measurements** gave sensible results and validated the newly-introduced parameters

To the T2K 2022 Oscillation Analysis

- Fits to **published cross section measurements** gave sensible results and validated the newly-introduced parameters
- This parameterisation was used in the latest **Oscillation Analysis** (OA) presented at Neutrino 2022 (which uses lepton kinematics as fitting variables)
- What we see:
  - All Pauli blocking parameters are pulled to higher values (suppression of low energy transfer region)
  - Some sensitivity to shell normalisation parameters
  - No sensitivity to the missing momentum shape uncertainties





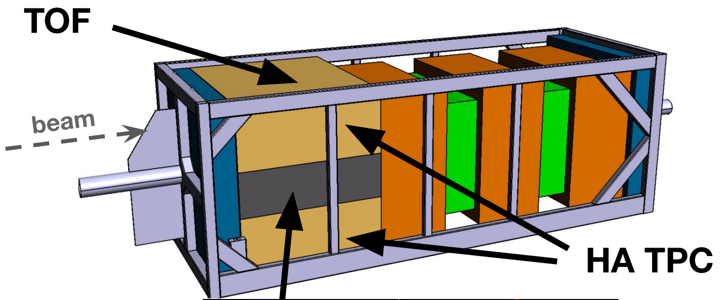
C. Bronner @ Neutrino 2022

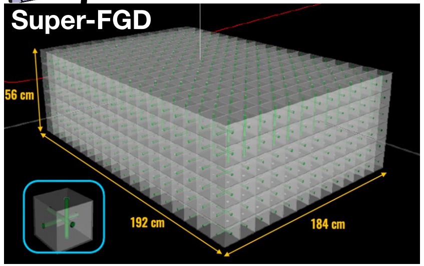
5. Projected constraints with T2K ND Upgrade

T2K Near Detector Upgrade *(See David Henaff's presentation)*

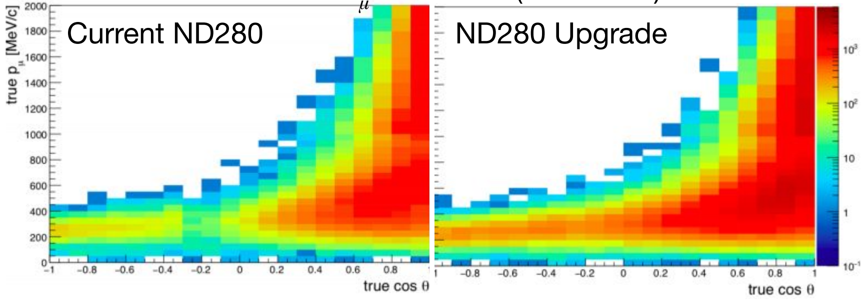
arxiv:1901.03750

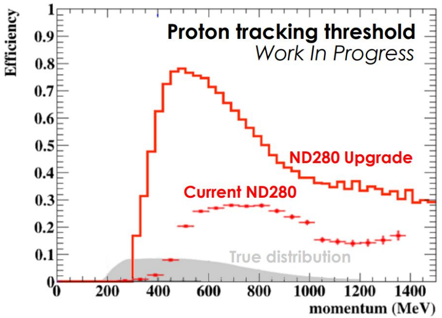
- Improved reconstruction at high and backward angles ⟶ better constraints on the neutrino interaction model
- Increased target mass (x2 current ND280) ⟶ more statistics
- Better reconstruction of outgoing nucleons ⟶ access to new observables

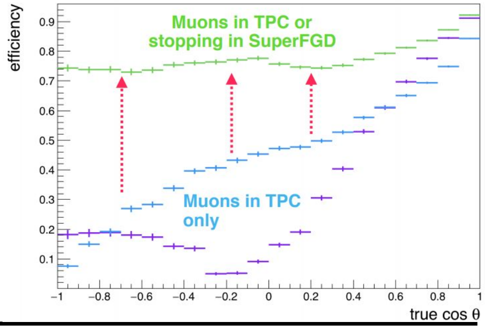




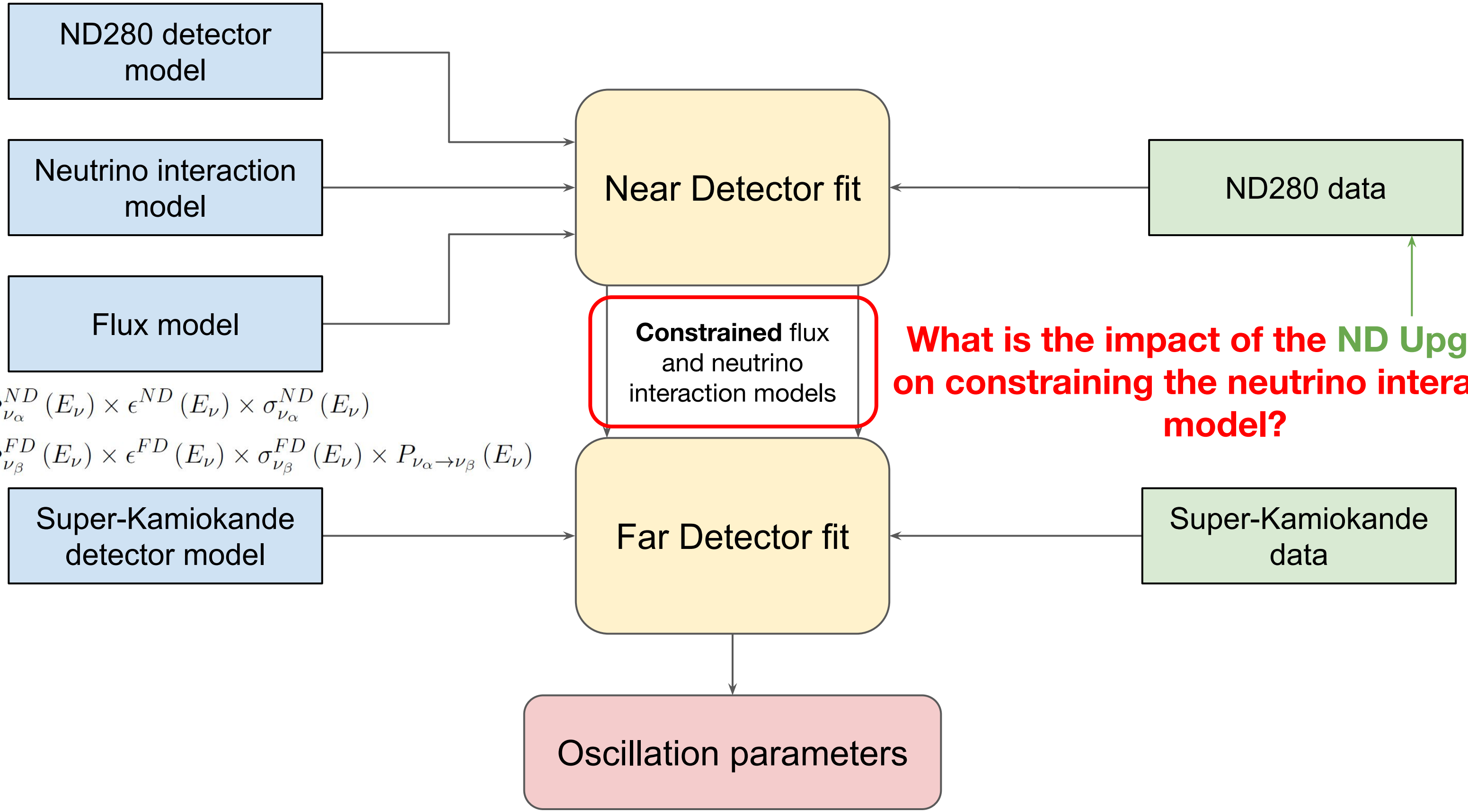
Selected ν_μ CC events (NEUT MC)







T2K Oscillation Analysis strategy



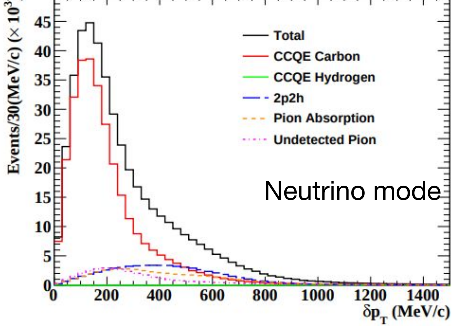
$$N_{\nu_\alpha}^{ND}(E_\nu) = \Phi_{\nu_\alpha}^{ND}(E_\nu) \times \epsilon^{ND}(E_\nu) \times \sigma_{\nu_\alpha}^{ND}(E_\nu)$$

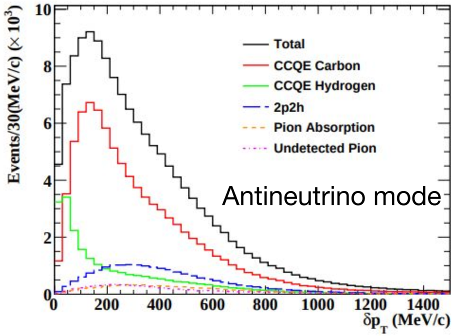
$$N_{\nu_\beta}^{FD}(E_\nu) = \Phi_{\nu_\beta}^{FD}(E_\nu) \times \epsilon^{FD}(E_\nu) \times \sigma_{\nu_\beta}^{FD}(E_\nu) \times P_{\nu_\alpha \to \nu_\beta}(E_\nu)$$

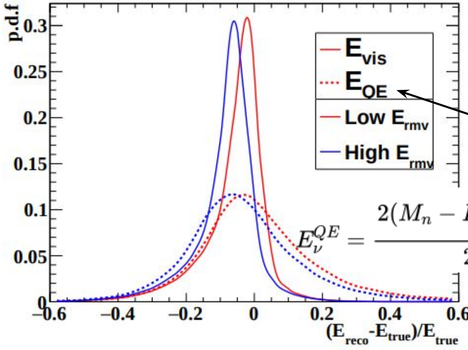
What is the impact of the ND Upgrade on constraining the neutrino interaction model?

Future Oscillation Analysis?

- Currently, T2K uses only lepton kinematics for the Oscillation Analysis (OA)
- With the ND280 Upgrade, we expect to obtain more precise measurements of the nucleons coming out from neutrino interactions
  $\longrightarrow$ what will the impact be on the OA?
- With the nucleon information, we can introduce samples with new observables:
  - Transverse momentum imbalance
  - Visible energy:
    - $E_{\text{vis}} = E_\mu + T_p$ for neutrino interactions
    - $E_{\text{vis}} = E_\mu + T_n$ for antineutrino interactions
- We use T2K projections of POT assuming a scenario where nu and anti-nu beam modes are alternated on a yearly basis



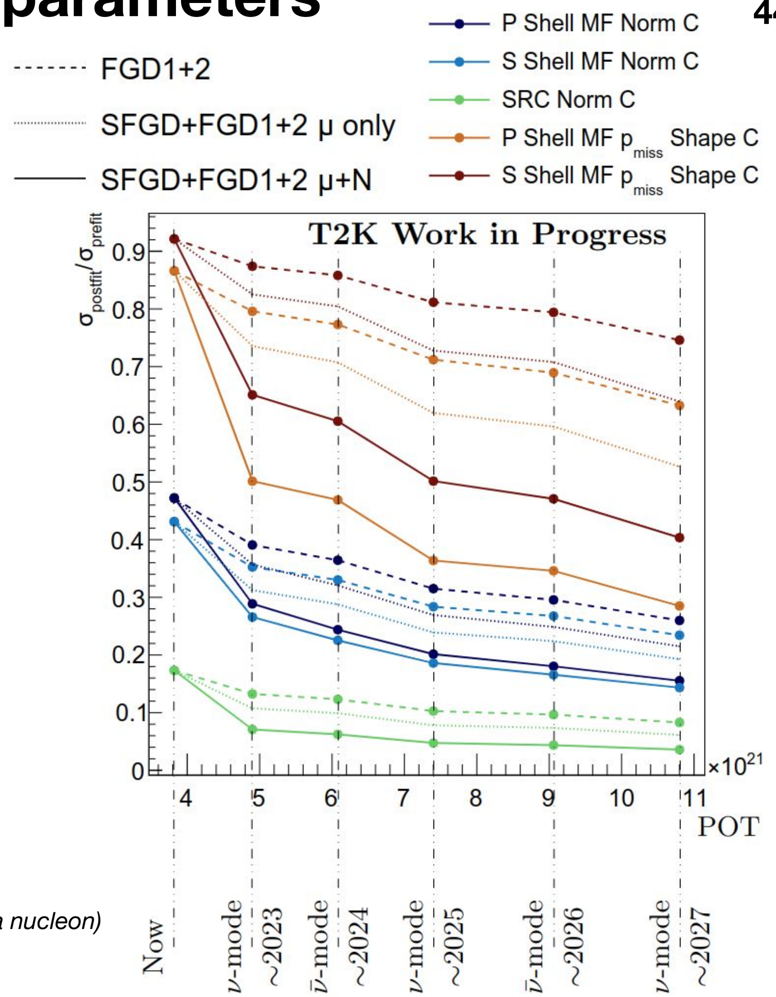




Neutrino energy estimator using lepton kinematics only:

$$E_\nu^{QE} = \frac{2(M_n - E_B)E_l - \left[(M_n - E_B)^2 + m_l^2 - M_p^2\right]}{2[M_n - E_B - E_l + p_l \cos\theta_l]}$$

Expected improvement: Carbon SF parameters



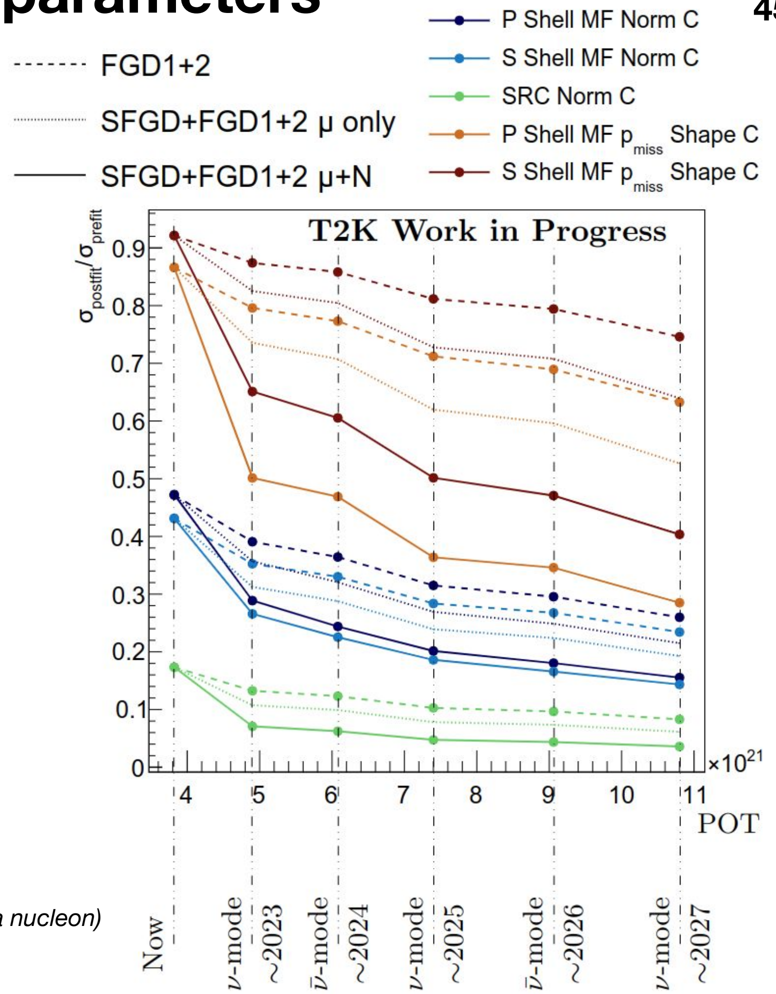
------ FGD1+2 : Current ND fit (no ND280 Upgrade)

·········· SFGD+FGD1+2 μ only : ND280 Upgrade using lepton kinematics only

—— SFGD+FGD1+2 μ+N : ND280 Upgrade using (Evis, δp_T) *(when reconstructing a nucleon)*

Expected improvement: Carbon SF parameters

- Significant improvement with respect to the current ND configuration
- The use of nucleon information with (Evis, δp_T) allows larger constraints especially on the pmiss shape parameters (these parameters are fixed in the current OA due to lack of sensitivity)



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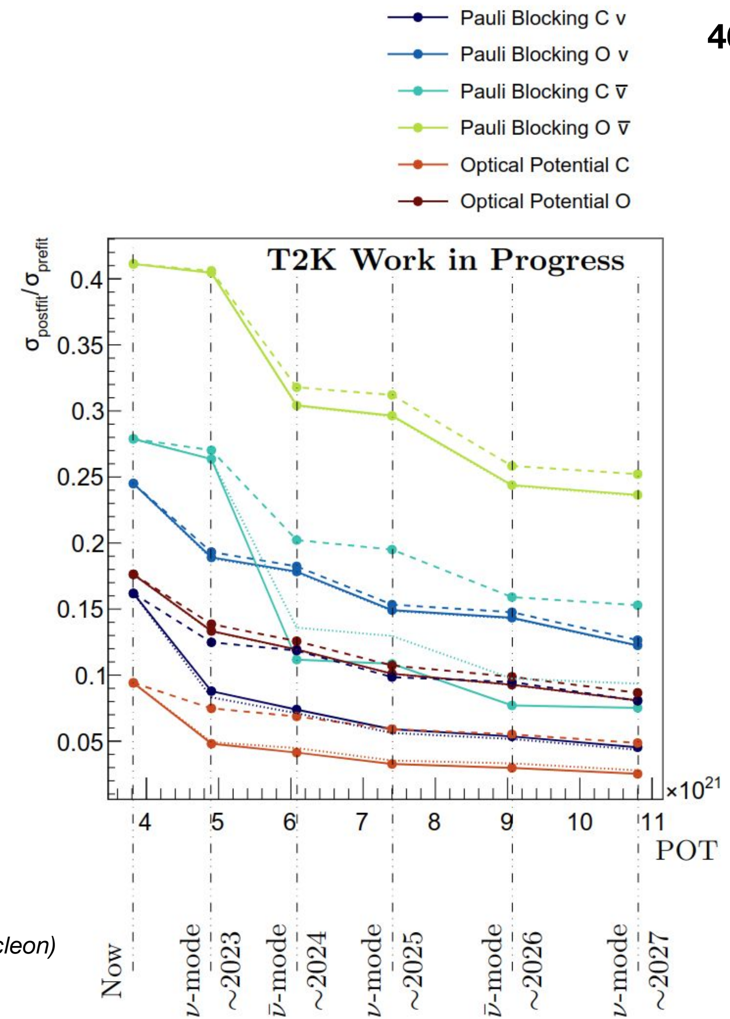
··········· SFGD+FGD1+2 μ only : ND280 Upgrade using lepton kinematics only

—— SFGD+FGD1+2 μ+N : ND280 Upgrade using (Evis, δp_T) *(when reconstructing a nucleon)*

Expected improvement: PB and OP

- The increased statistics and the use of lepton kinematics when the nucleon is not reconstructed allow improved constraints on the Pauli Blocking and Optical Potential parameters (low energy transfer region)
- The upgrade has small impact on the oxygen parameters (SFGD is made of plastic scintillator)



FGD1+2 : Current ND fit (no ND280 Upgrade)

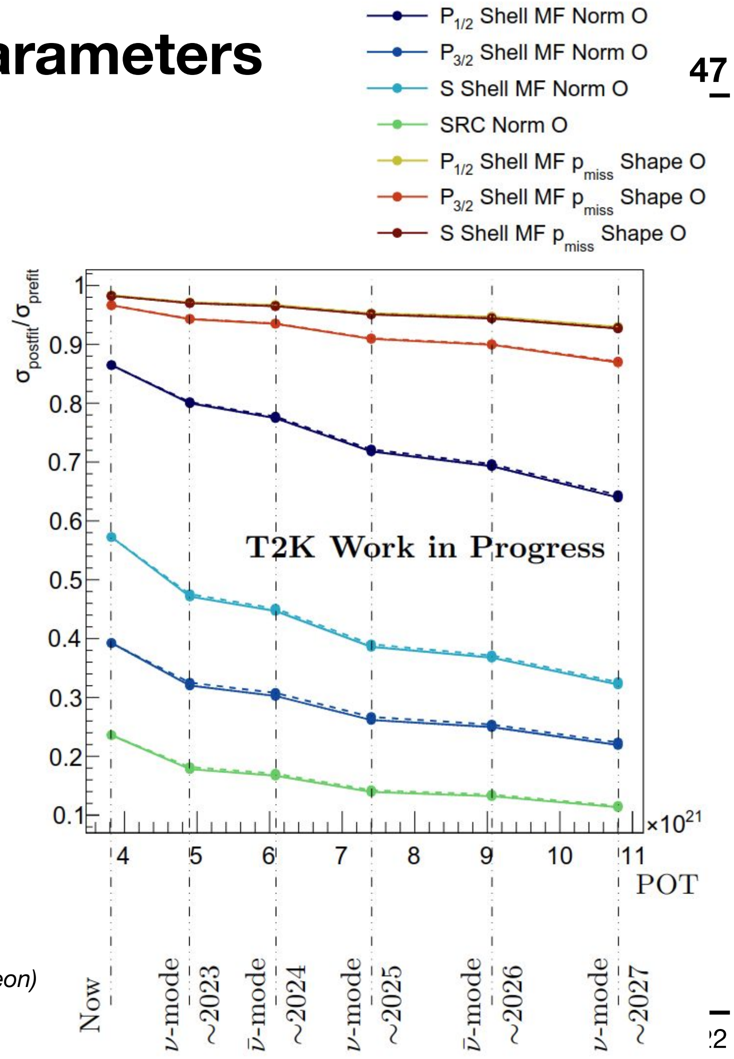
SFGD+FGD1+2 μ only : ND280 Upgrade using lepton kinematics only

SFGD+FGD1+2 μ+N : ND280 Upgrade using (Evis, δp_T) *(when reconstructing a nucleon)*

Expected improvement: Oxygen SF parameters

- With the presented parameterisation, the upgrade has little impact on the constraints of the Oxygen parameters:
    - SF model is built independently from electron scattering data for O and C

- This is still under study



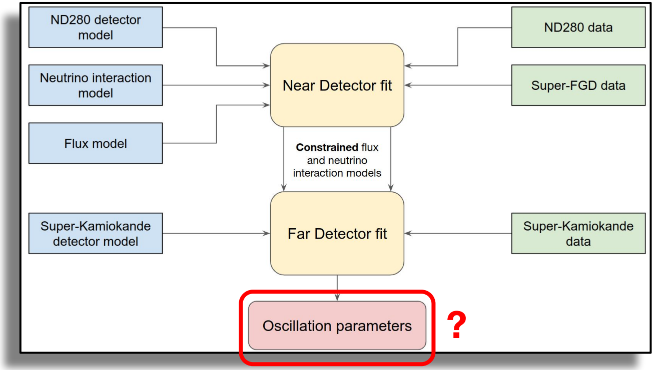
------- FGD1+2 : Current ND fit (no ND280 Upgrade)

.... SFGD+FGD1+2 μ only : ND280 Upgrade using lepton kinematics only

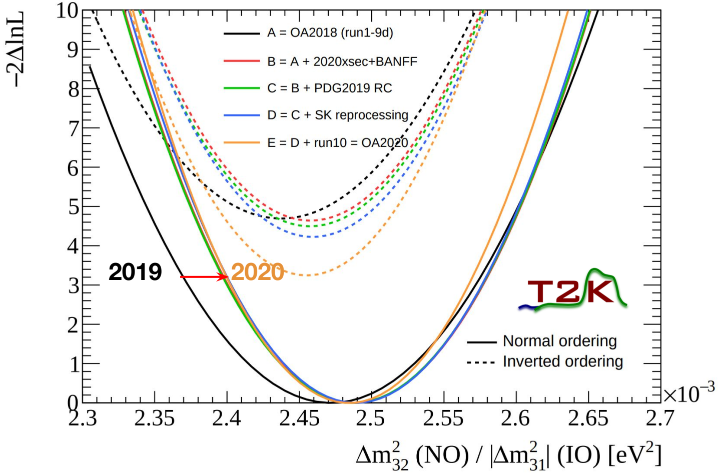
—— SFGD+FGD1+2 μ+N : ND280 Upgrade using (Evis, δp_T) *(when reconstructing a nucleon)*

What will the impact be on the T2K oscillation analysis?

- We plan on evaluating the impact of the improved sensitivity with the ND Upgrade on the oscillation parameters using the FD fitter



- The 2020 results, which improved the cross section model, showed better constraints on Δm^2



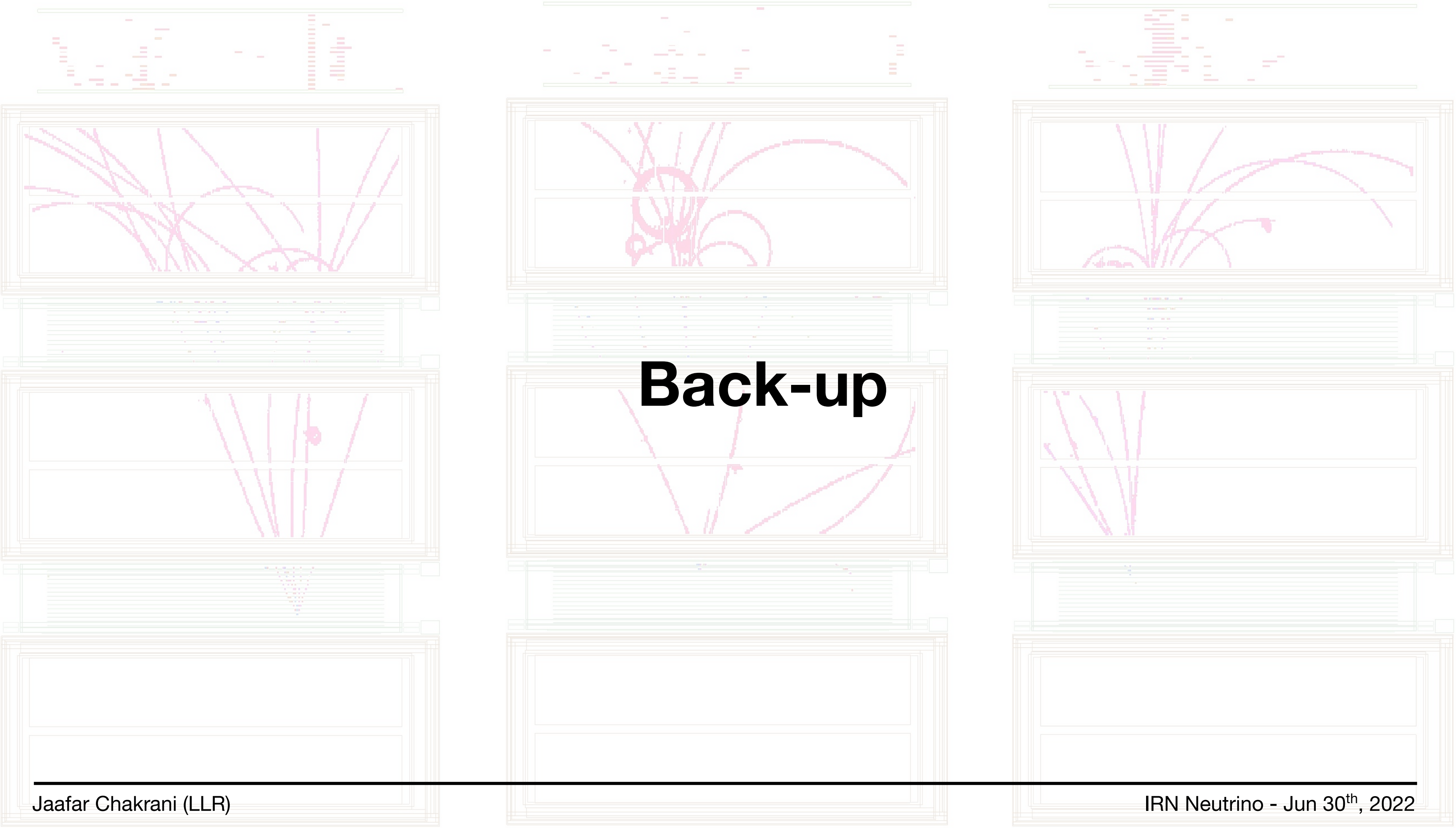
6. Summary and outlook

Summary and outlook

- A new set of physically-motivated uncertainties on the predictions of the CCQE Spectral Function model were introduced
- Fitting to external data shows great improvement for T2K cross section measurements, whereas MINERνA data is sensitive to this parameterisation but shows little improvement in the data/MC agreement
- The impact of adding SFGD samples on the ND fit is studied with:
    - The choice of the binning variables (lepton kinematics vs. (Evis, δpT)) has a significant impact on the constraints of some uncertainties (e.g. SF model parameters, 2p2h, nucleon FSI)
    - The impact on Oxygen parameters is limited since the current model for OA2022 does not account for O vs. C correlations

Next steps (non-exhaustive):

- What is the impact of the added SFGD samples on the oscillation parameters?
- What would the impact be if we add correlations between O and C?
- This parameterisation is a first step to account for the uncertainty on the nucleon production, it still needs further improvement especially for FSI *(see Anna's talk next)*



Back-up

Avoiding Peelle's Pertinent Puzzle (PPP)

- Attempts to fit neutrino cross-section data with a parameterisation of the interaction model gives us seemingly unphysically low normalisations PPP
- The reason this happens is subtle, but is related to the strong correlations in published covariance matrices and the corresponding "type" of Gaussian errors approximation
- In our standard approach we assume the absolute uncertainty on the cross section is independent of its normalisation
    - i.e. our uncertainties state that a 10 fb / GeV uncertainty on some bin remains at 10 fb / GeV even if we had underestimated our flux by 10% (and so the cross section is lower than measured)
    - This implies the relative uncertainty is larger if fitting to models that predict lower normalisations. This is what give us PPP
- We could alternatively suggest that it should be the relative uncertainty that is independent of its normalisation (D'Agostini does)
    - i.e. our uncertainties would state that a 10% uncertainty on some bin remains at 10% even if we had underestimated our flux by 10%
    - This implies the absolute uncertainty is larger if fitting to models that predict lower normalisations

- We can construct a covariance matrix that keeps the relative uncertainties constant when the normalization changes: a "Norm-Shape" covariance where one row contains the normalization of the data and the rest contains the shape

$$H_1 : \{B_1, ..., B_n\} \rightarrow H_2 : \{C_1, ..., C_n\}$$

$$C_i = \begin{cases} \frac{B_i}{\sum_k B_k} & , i < n \\ \sum_k B_k & , i = n \end{cases}$$

- We can obtain $\mathrm{Cov}\left[\{C_i\}\right]$ (norm-shape covariance) directly from the data covariance given by experiments $\mathrm{Cov}\left[\{B_i\}\right]$

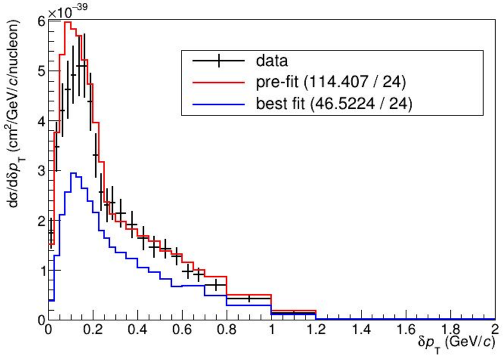
- Perform the fit in this new basis using $N = \mathrm{Cov}[\{C_i\}]$

$$\chi^2_{\mathrm{NS}} = \sum_{1 \leq i,j \leq n} \left(C_i - C_i^{MC}\right) \left(N^{-1}\right)_{ij} \left(C_j - C_j^{MC}\right)$$

Avoiding Peelle's Pertinent Puzzle

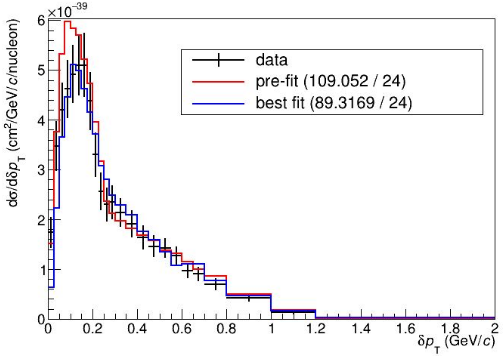
- Example of MINERvA dpt fit with the same parameterisation of slide 41:

Fit minimizing the usual chi2



	Normal chi2	NS chi2
prefit	**114.407**	109.052
postfit	**46.5224**	108.201

Fit minimizing the new chi2

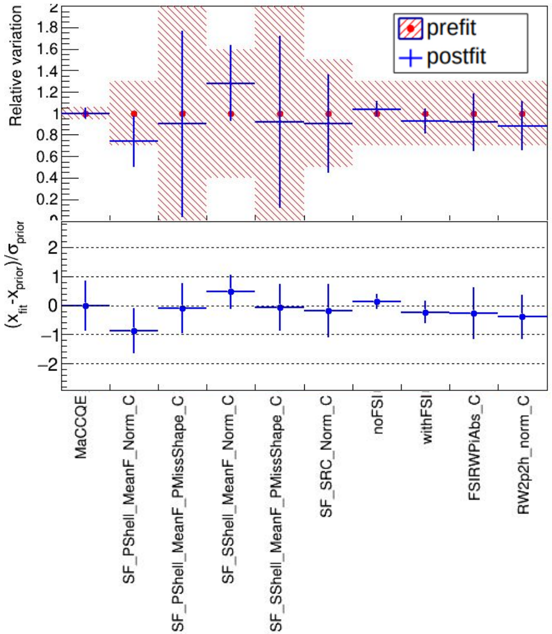


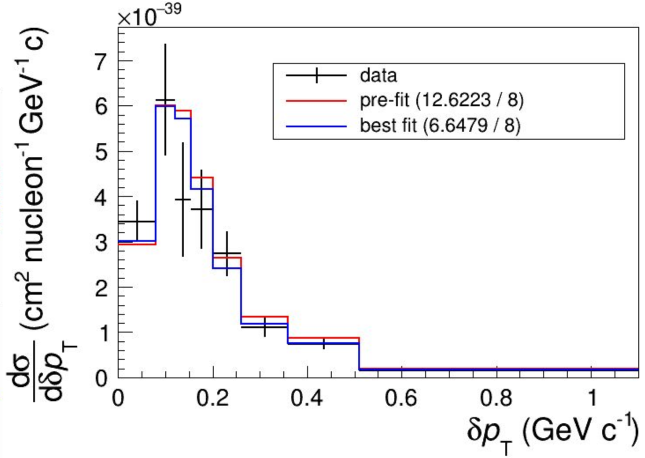
	Normal chi2	**NS chi2**
prefit	114.407	**109.052**
postfit	85.2858	**89.3169**

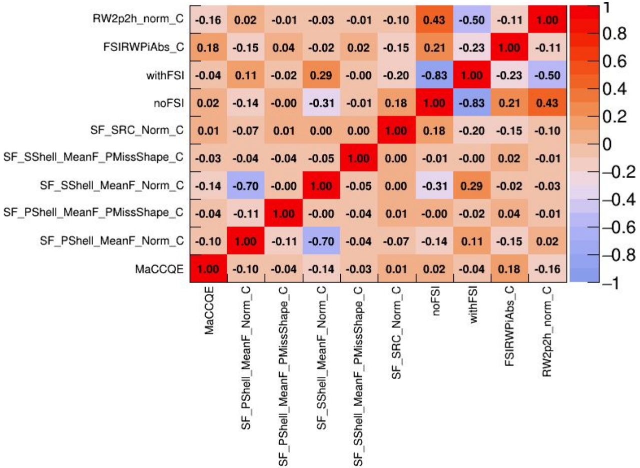
T2K CC0pi δp_T Phys. Rev. D 98, 032003 (2018)

Parameters used in this fit:

- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- No FSI / with FSI (correlated)
- Pion absorption normalization
- 2p2h normalization







MINERvA CC0pi δp_T Phys. Rev. Lett. 121, 022504 (2018)

Parameters used in this fit:

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- Shell occupancies
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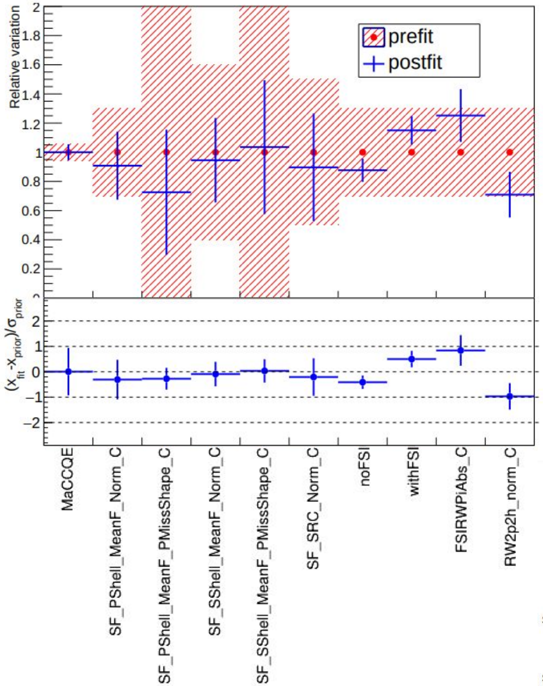
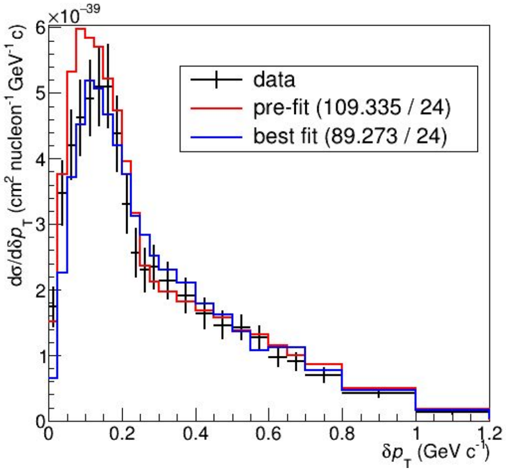
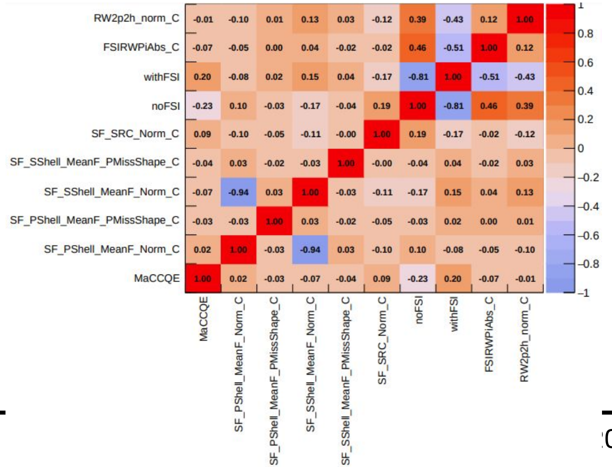
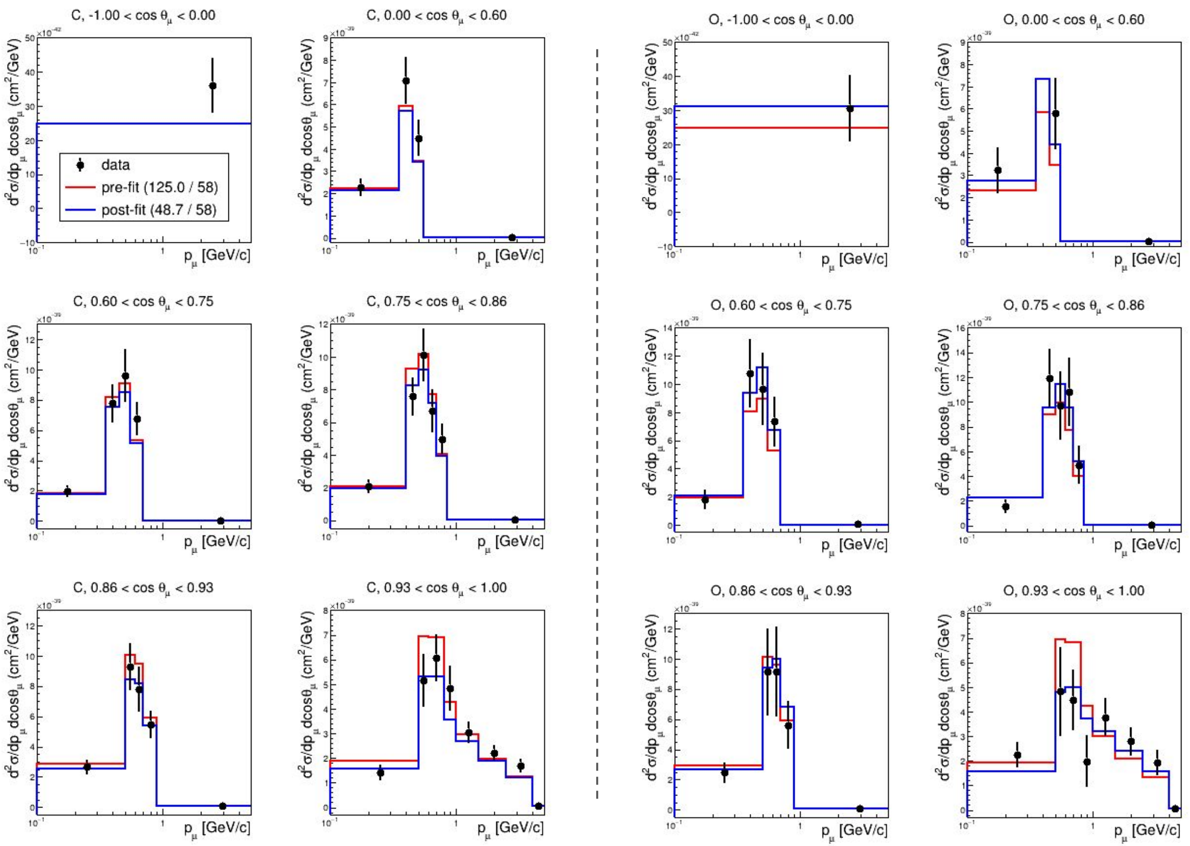






Parameters used in this fit:

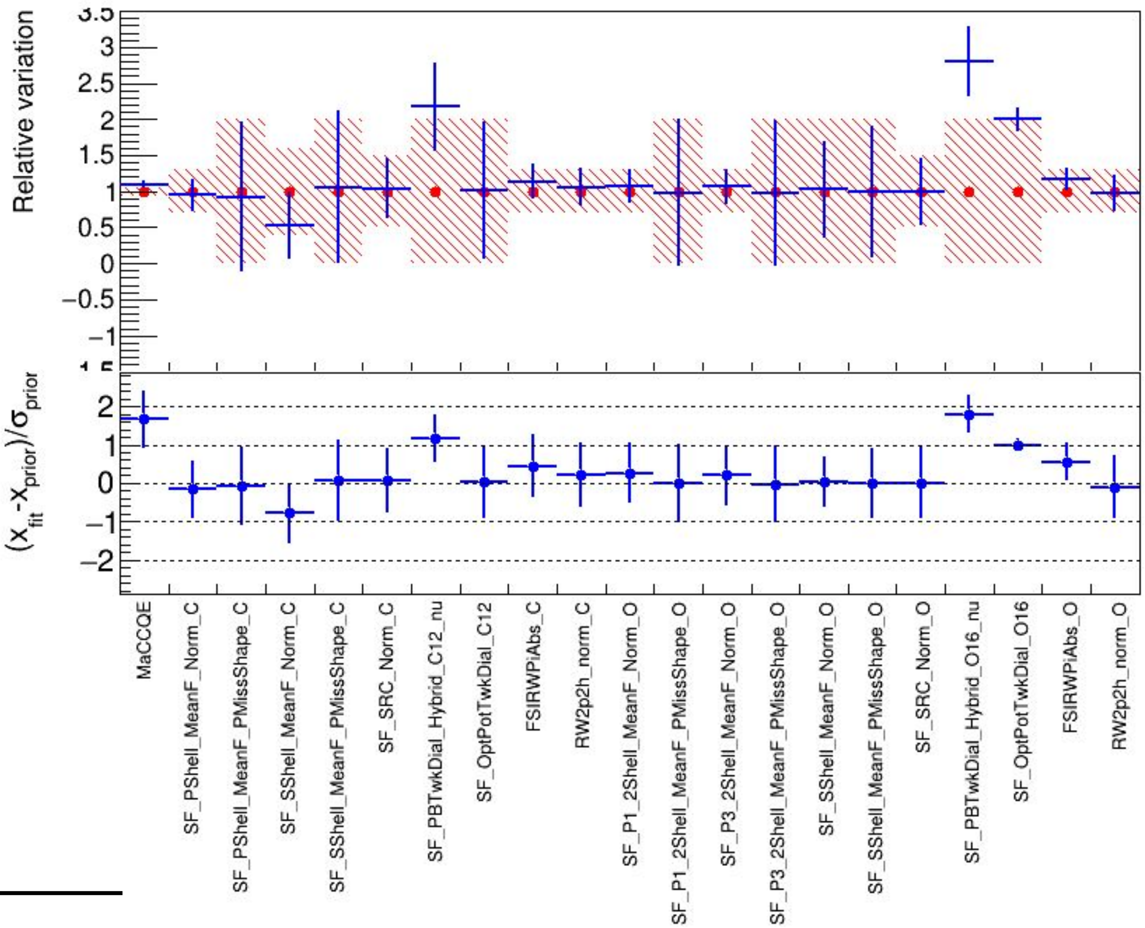
- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- Pauli blocking
- Optical potential correction
- Pion absorption normalization
- 2p2h normalization



T2K CC0pi O & C measurement Phys. Rev. D 101, 112004 (2020)

Parameters used in this fit:

- M_A^{QE}
- Shell occupancies
- p_m shape
- SRC norm
- Pauli blocking
- Optical potential correction
- Pion absorption normalization
- 2p2h normalization



T2K CC0pi O & C measurement

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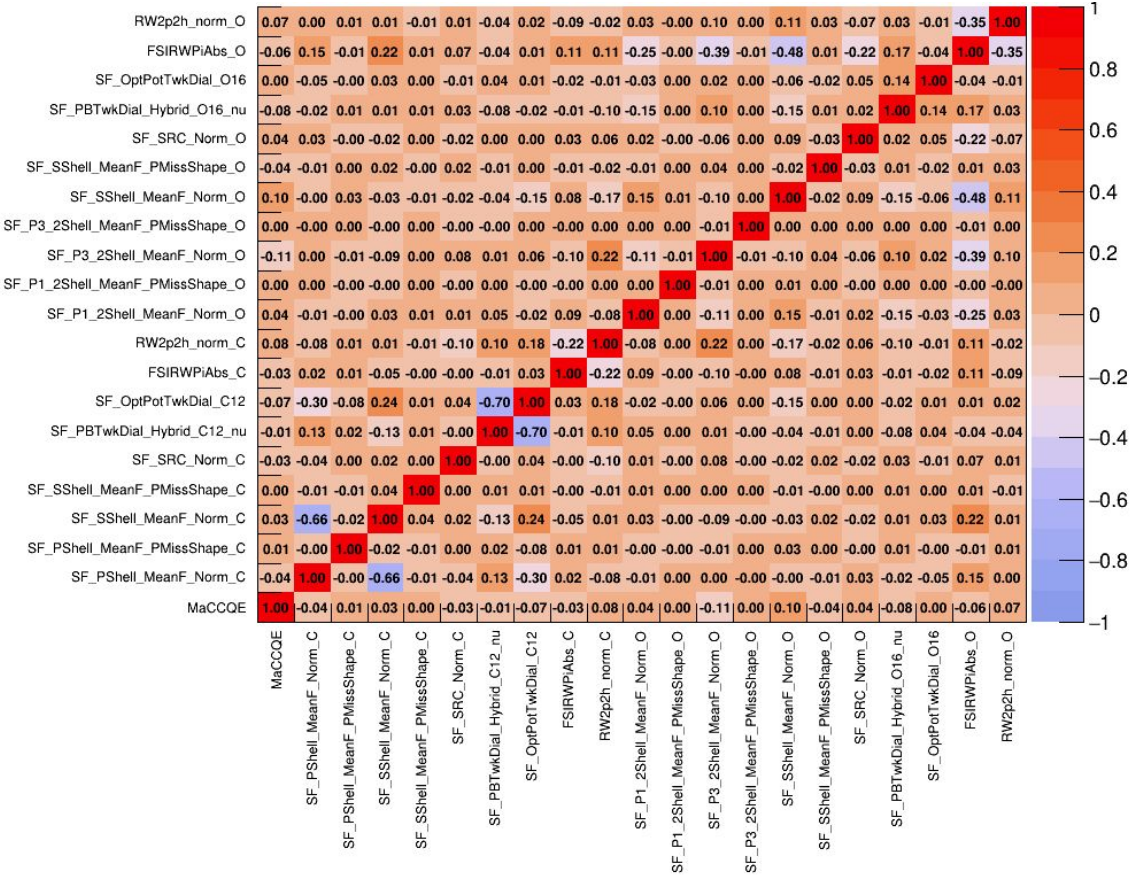
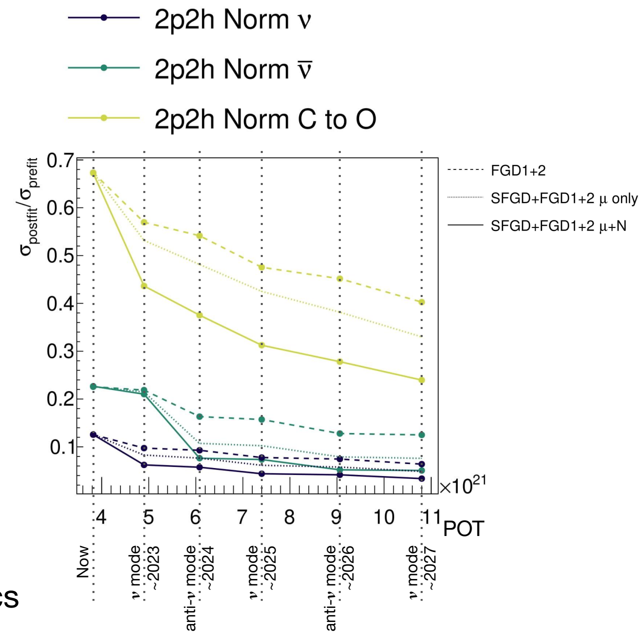


Expected improvement: 2p2h norm

- 2p2h norm parameters also benefit from the Upgrade and the added nucleon information



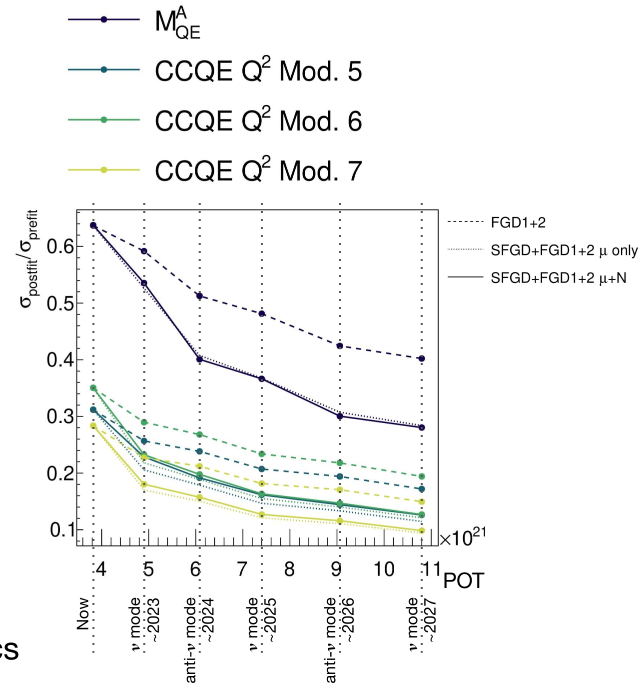
FGD1+2 : Current ND fit, no additional samples

SFGD+FGD1+2 μ only : Add to current ND fit SFGD samples binned in lepton kinematics

SFGD+FGD1+2 μ+N : Add to current ND fit SFGD samples binned in (Evis, δp_T)

Expected improvement: other CCQE parameters

- MAQE has tight prefit constraints, the improvement comes mainly from the additional statistics
- SFGD+FGD1+2 μ only shows slightly better constraints for some of the high-Q^2 parameters



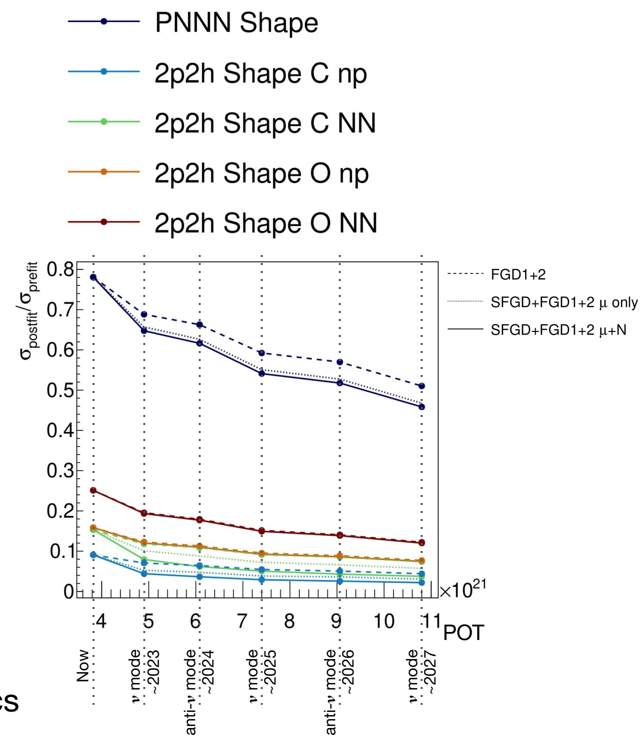
FGD1+2 : Current ND fit, no additional samples

SFGD+FGD1+2 μ only : Add to current ND fit SFGD samples binned in lepton kinematics

SFGD+FGD1+2 μ+N : Add to current ND fit SFGD samples binned in (Evis, δp_T)

Expected improvement: 2p2h shape

- The Upgrade has no significant impact on the Oxygen parameters
- The SFGD adds little more constraint on the 2p2h shape parameters → these parameters are designed to impact lepton kinematics only



FGD1+2 : Current ND fit, no additional samples

SFGD+FGD1+2 μ only : Add to current ND fit SFGD samples binned in lepton kinematics

SFGD+FGD1+2 μ+N : Add to current ND fit SFGD samples binned in (Evis, δp_T)

Expected improvement: Flux parameters