Non-minimally assisted chaotic inflation

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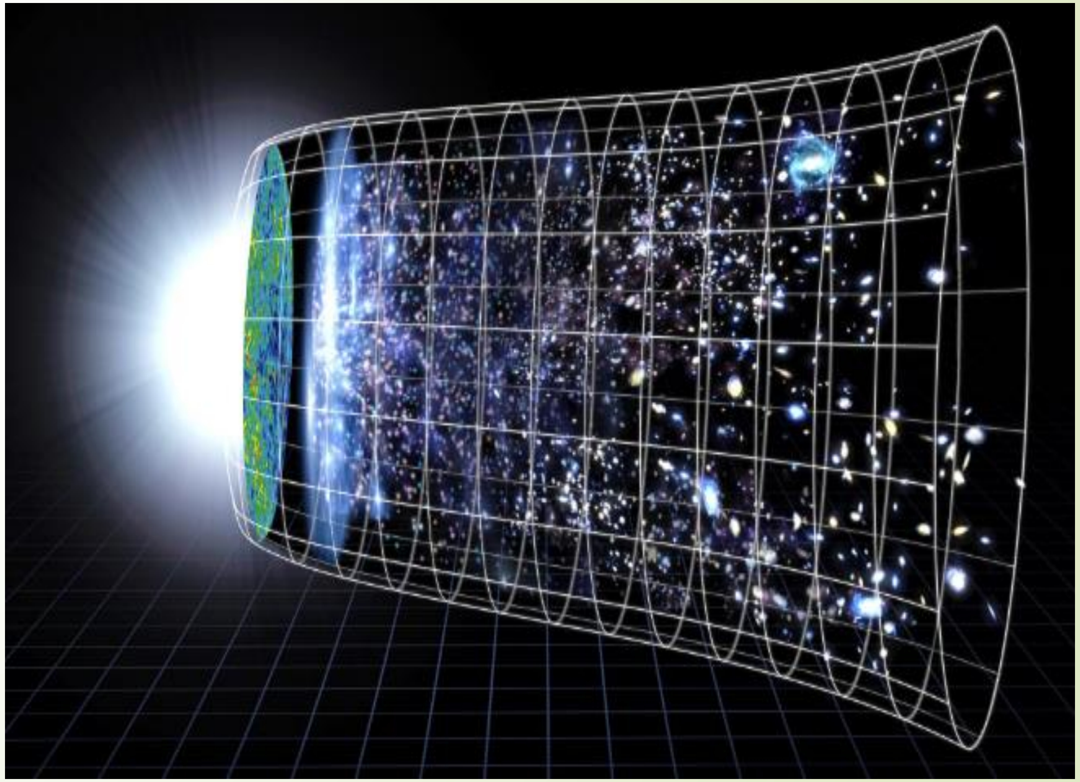
(arXiv : 2203.09201)

Inflationary Cosmology

Cosmic Inflation solves several problems that Big Bang Cosmology originally have.

- Flatness & Horizon Problem
- The Origin of Large-Scale Structure
- Monopole problem



Chaotic Inflation

A. D. Linde (1983)

Model :

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right]$$

$$V(\varphi) = \lambda_\varphi M_{Pl}^4\left(\frac{\varphi}{M_{Pl}}\right)^n$$

Two free parameters

CMB Observables :

Scalar Spectral Index   Tensor-to-Scalar Ratio

$$n_s \simeq 1 - \frac{2(n+2)}{n+4N}, r \simeq \frac{16n}{n+4N} \quad \text{with} \quad \lambda_\varphi \simeq \begin{cases} 10^{-11}, & n=2 \\ 10^{-13}, & n=4 \end{cases}$$

$N = \int_\star^e H dt$: Number of e-folds from horizon crossing point($\star$) to end-of-inflation point(e)

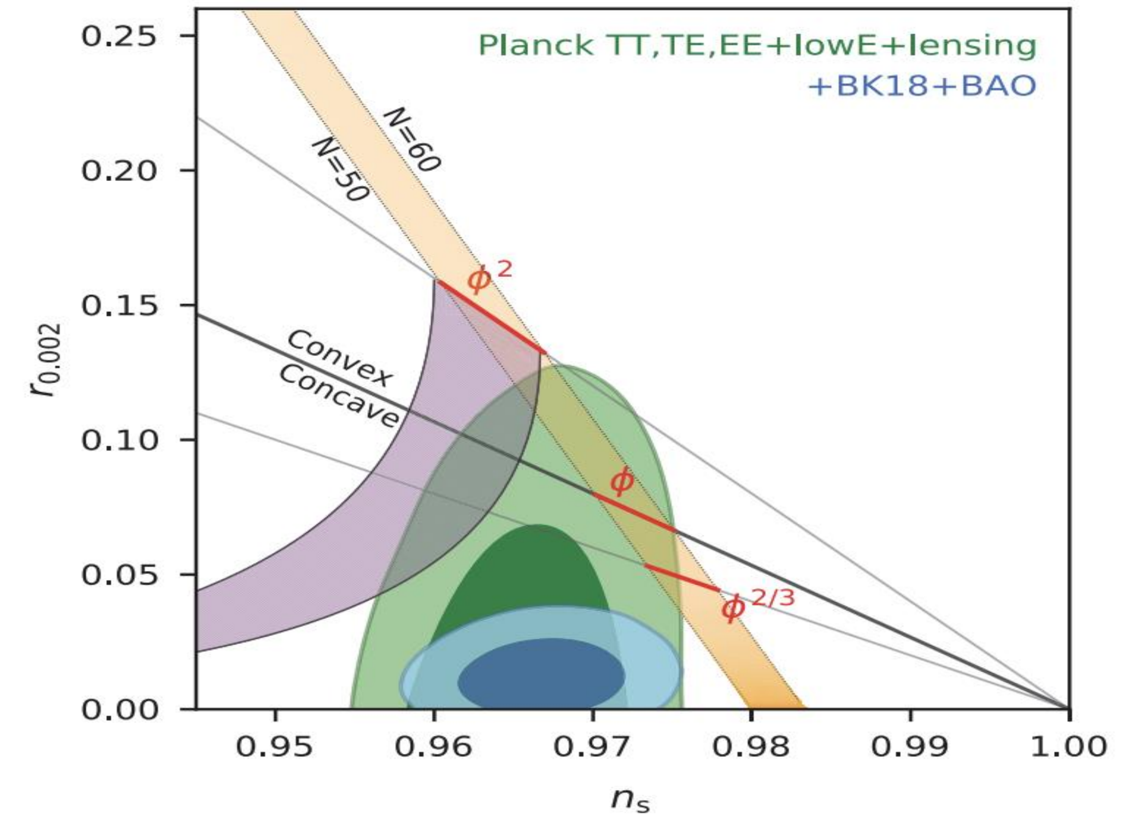
Chaotic Inflation

A. D. Linde (1983)

$$V(\varphi) = \lambda_\varphi M_{Pl}^4 \left(\frac{\varphi}{M_{Pl}}\right)^n$$

$$n_s \simeq 1 - \frac{2(n+2)}{n+4N}, r \simeq \frac{16n}{n+4N}$$

But, chaotic inflation model with any n is ruled out by Planck + BICEP/Keck Array!



· *Y. Akrami et al. (2020), P. A. R. Ade et al. (2021)*

Our idea

Adding additional scalar field s,

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right]$$

$$\rightarrow S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2\Omega^2(s,\varphi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(s,\varphi)\right]$$

where $\Omega^2(s,\varphi) = 1 + g\left(\frac{\varphi}{M_{Pl}}\right) + f\left(\frac{s}{M_{Pl}}\right)$

Our idea

Adding additional scalar field s → "Assistant field",

$$V(s,\varphi) \to V(\varphi)$$
$$\Omega^2(s,\varphi) = 1 + g\left(\frac{\varphi}{M_{Pl}}\right) + f\left(\frac{s}{M_{Pl}}\right) \to 1 + f\left(\frac{s}{M_{Pl}}\right)$$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2\Omega^2(s,\varphi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(s,\varphi)\right]$$

$$\to S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2\left(1 + f\left(\frac{s}{M_{Pl}}\right)\right)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(\varphi)\right]$$

Small field approximation

$$\Omega^2(s,\varphi) = 1 + f\left(\frac{s}{M_{Pl}}\right)$$

When $s \ll M_{Pl}$,

$$f\left(\frac{s}{M_{Pl}}\right) = f(0) + f'(0)\frac{s}{M_{Pl}} + \frac{1}{2!}f''(0)\frac{s^2}{M_{Pl}^2} + \cdots$$

$$= \xi_2\frac{s^2}{M_{Pl}^2} + \xi_4\frac{s^4}{M_{Pl}^4} + \cdots = \sum \xi_m \frac{s^m}{M_{Pl}^m}$$

Z_2 symmetry ($s \to -s$)

$$\Omega^2(s) \approx 1 + \xi_2\frac{s^2}{M_{Pl}^2}$$ Quadratic Case $(m=2)$

$$\Omega^2(s) \approx 1 + \xi_4\frac{s^4}{M_{Pl}^4}$$ Quartic Case $(m=4)$

Weyl transformation

Converting Jordan-frame action into Einstein-frame one,

$$S_J = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{Pl}^2\Omega^2(s)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu s\partial_\nu s - V(\varphi)\right]$$

$$g_{E,\mu\nu} = \Omega^2 g_{J,\mu\nu},\ \Omega^2 \equiv 1 + \xi_m(s/M_{Pl})^m$$

$$R = \Omega^2[R_E - 6g^{E,\mu\nu}\nabla_\mu\nabla_\nu\ln\Omega - 6g^{E,\mu\nu}\nabla_\mu\ln\Omega\nabla_\nu\ln\Omega]$$

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2}\mathcal{K}_1 g_E^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\mathcal{K}_2 g_E^{\mu\nu}\partial_\mu s\partial_\nu s - V_E(\varphi, s)\right]$$

Einstein-frame potential

$$V_E(\varphi, s) = \frac{V(\varphi)}{\Omega^4} = \frac{V(\varphi)}{(1+\xi_m s^m/M_{Pl}^m)^2} \qquad \mathcal{K}_1 \equiv \frac{1}{\Omega^2},\quad \mathcal{K}_2 \equiv \frac{\Omega^2 + (3/2)m^2\xi_m^2(s/M_{Pl})^{2m-2}}{\Omega^4}$$

Canonical field

Introducing another scalar field σ to canonically normalize kinetic term of assistant field,

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2}\mathcal{K}_1 g_E^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\mathcal{K}_2 g_E^{\mu\nu}\partial_\mu s\partial_\nu s - V_E(\varphi, s)\right]$$

$$= \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2\Omega^2}(\partial\varphi)^2 - \frac{1}{2}(\partial\sigma)^2 - V_E(\varphi, \sigma(s))\right]$$

Einstein-frame potential

$$V_E(\varphi, s) = \frac{V(\varphi)}{\Omega^4} = \frac{V(\varphi)}{(1+\xi_m s^m/M_{Pl}^m)^2}$$

Normalized field σ

$$\left(\frac{\partial\sigma}{\partial s}\right)^2 = \frac{1+\xi_m s^m/M_{Pl}^m + (3/2)m^2\xi_m^2(s/M_{Pl})^{2m-2}}{(1+\xi_m s^m/M_{Pl}^m)^2}$$

Connecting to Natural Inflation

$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{M_{Pl}^2}{2}R_E - \frac{1}{2\Omega^2}(\partial\varphi)^2 - \frac{1}{2}(\partial\sigma)^2 - V_E(\varphi,\sigma(s))\right]$$

When $m = 2, \xi_m s^m/M_{Pl}^m \ll 1$,

$$V_E(\varphi,\sigma(s)) = \frac{\lambda_\varphi M_{Pl}^4(\varphi/M_{Pl})^n}{(1+\xi_m s^m/M_{Pl}^m)^2} \simeq \lambda_\varphi M_{Pl}^4\left(\frac{\varphi}{M_{Pl}}\right)^n\left(1 - 2\xi_2\frac{s^2}{M_{Pl}^2}\right)$$

$$\simeq \lambda_\varphi M_{Pl}^4\left(\frac{\varphi}{M_{Pl}}\right)^n\left[1+\cos\left(\frac{s}{f}\right)\right]$$

Chaotic Inflation   Natural Inflation

$$f \equiv (2\sqrt{2\xi_2})^{-1}M_{Pl}$$

Einstein-frame potential

$$V_E(\varphi,s) = \frac{V(\varphi)}{\Omega^4} = \frac{V(\varphi)}{(1+\xi_m s^m/M_{Pl}^m)^2}$$

Normalized field σ

$$\left(\frac{\partial\sigma}{\partial s}\right)^2 = \frac{1+\xi_m s^m/M_{Pl}^m + (3/2)m^2\xi_m^2(s/M_{Pl})^{2m-2}}{(1+\xi_m s^m/M_{Pl}^m)^2}$$

Slow-roll analysis & δN formalism

- We transformed action in Jordan frame to Einstein frame and earned approximated equation of motions by using slow-roll assumption.

Slow-roll assumption : $\{\epsilon^i, |\eta^{ij}|, \epsilon^b\} \ll 1$

$$\epsilon^\sigma \equiv \frac{M_{Pl}^2}{2}\left(\frac{V_{,\sigma}}{V}\right)^2, \epsilon^\varphi \equiv \frac{M_{Pl}^2}{2}\left(\frac{V_{,\varphi}}{V}e^{-b}\right)^2, \eta^{\sigma\sigma} \equiv M_{Pl}^2\frac{V_{,\sigma\sigma}}{V},$$

$$\eta^{\varphi\varphi} \equiv M_{Pl}^2\frac{V_{,\varphi\varphi}}{V}e^{-2b}, \eta^{\varphi\sigma} \equiv M_{Pl}^2\frac{V_{,\varphi\sigma}}{V}e^{-b}, \epsilon^b \equiv 8M_{Pl}^2 b_{,\sigma}^2$$

- We calculated three CMB observables : spectral index n_s, tensor-to-scalar ratio r and local-type nonlinearity parameter $f_{NL}^{(local)}$ to match with latest constraints. We used δN formalism to calculate these and plotted them numerically.

· *A. A. Starobinsky, PLB117, 175. (1982),*
· *D. S. Salopek, J. R. Bond, PRD42, 3936 (1990),*
· *M. Sasaki and E. D. Stewart, PTP 95, 71. (1996)*

Cosmological observables

Expressions for the cosmological observables in the δN formalism ($N_{,i} \equiv \frac{\partial N}{\partial \varphi^i} \quad (\varphi^i = \{\sigma, \varphi\})$)

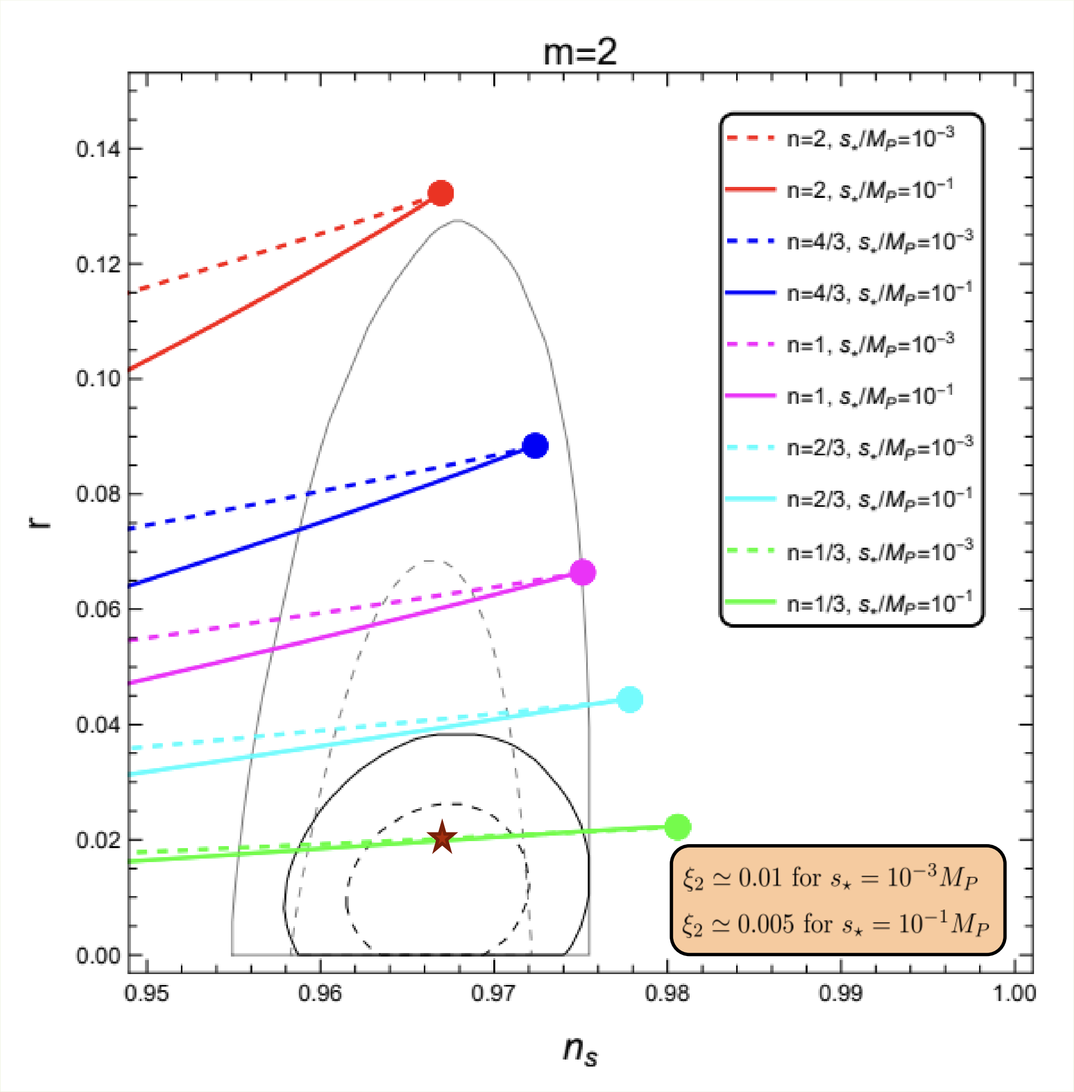
- Curvature power spectrum : $\mathcal{P}_\zeta = \left(\frac{H}{2\pi}\right)^2 G^{ij} N_{,i} N_{,j}$
- Scalar Spectral Index : $n_s = 1 + 2\frac{\dot{H}}{H^2} - 2\frac{1 + N_{,k}\left(\frac{M_{Pl}^6}{3} R^{kmnl} V_{,m} V_{,n}/V^2 - M_{Pl}^4 V^{;kl}/V\right) N_{,l}}{G^{ij} N_{,i} N_{,j} M_{Pl}^2}$
- Tensor-to-Scalar Ratio : $r = \frac{8}{M_{Pl}^2 G^{ij} N_{,i} N_{,j}}$
- Local-type nonlinearity parameter : $f_{NL}^{local} = -\frac{5}{6}\frac{G^{ij} G^{mn} N_{,i} N_{,m} N_{,jn}}{(G^{kl} N_{,k} N_{,l})^2}$
  - There exist three independent parameters : $n, s_\star, \xi_m$.
  - We set the number of e-folds from horizon crossing point($\star$) to end-of-inflation point(e) to be equal to 60. ($N = \int_\star^e H dt = 60$)

Field Space metric

$$G_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & e^{2b(s)} \end{pmatrix}, b(s) \equiv \frac{1}{2}\ln \mathcal{K}_1 = \frac{1}{2}\ln\left(\frac{1}{1 + \xi_m s^m / M_{Pl}^m}\right)$$

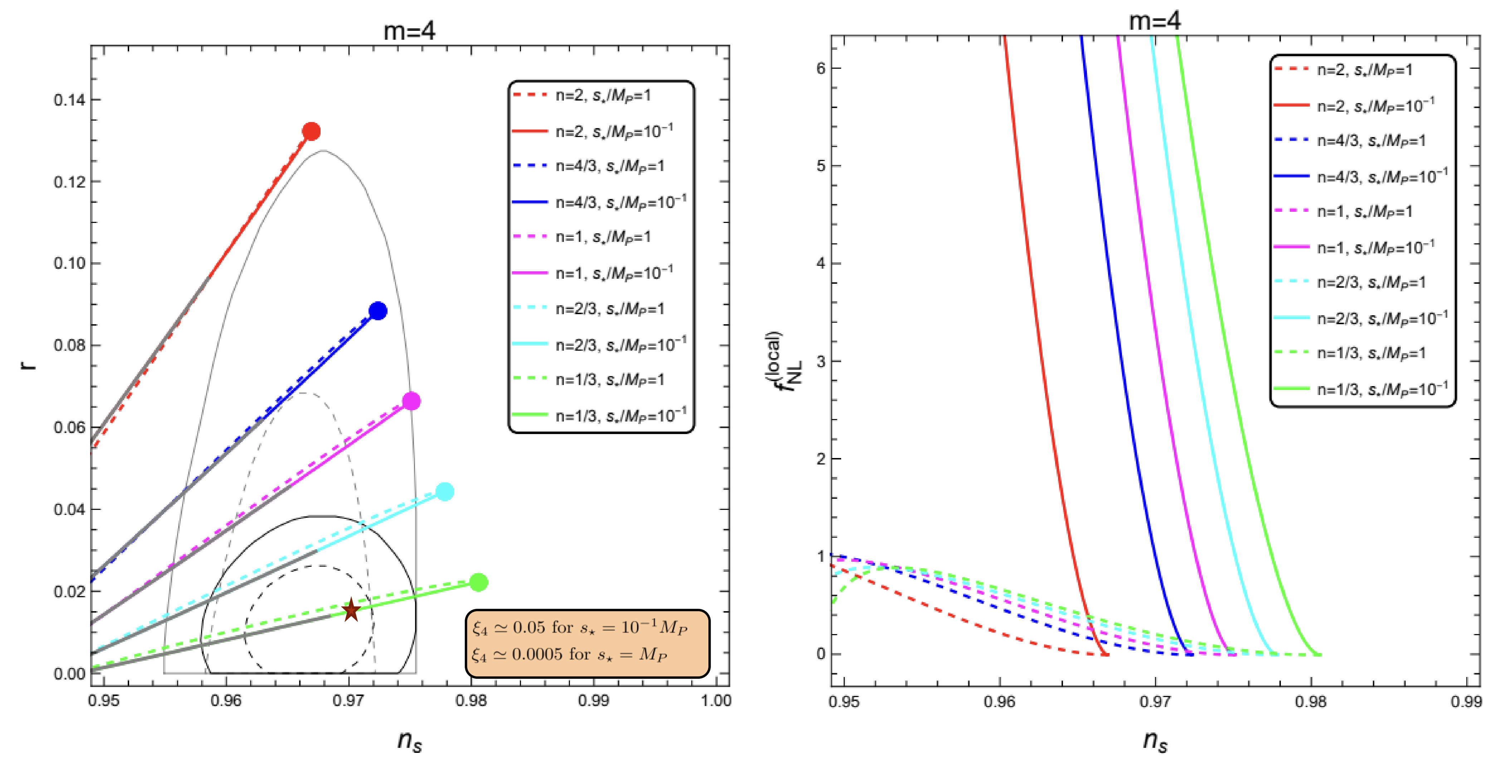
See δN formalism for 2 fields :
J. Kim, Y. Kim and S. C. Park, CQG31, 135004 (2014)

Results (m=2)



· *Planck (2020), Planck + BICEP/Keck (2021)*

Results (m=4)



· *Planck (2020), Planck + BICEP/Keck (2021)*

2σ bound for $f_{NL}^{(local)}$: $-11.1 < f_{NL}^{(local)} < 9.3$

Summary & Future Works

- Chaotic inflation with any power $V \sim \varphi^n$ is ruled out by the recent Planck-BICEP/Keck constraints.
- In order to rescue this problem, we introduced "assistant field" s with original inflaton φ, which couples to gravity non-minimally.
- Both $m = 2$ and $m = 4$ cases, it is possible to rescue chaotic inflation with certain value of n and $s_\star$, just by effect coming from non-minimal coupling, without changing potential.
- $m \geq 6$ is unacceptable since too large f_{NL}^{local}.

Future Studies : (1) Including $V(s)$, (2) s as dark matter, …

Thank you!