CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Shift symmetries of flavorful axions

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w/ C. Grojean and J. Kley

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Theory effort : studies of the UV landscape (light/heavy/decoupled/…) hand in hand with studies of **generic parametrizations** (EFTs)

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? Flavor ?
Field redefinitions ?

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Fermion-induced shift symmetry breaking

$$\frac{a}{f}\left(\bar{Q}\tilde{Y}_u\tilde{H}u + \bar{Q}\tilde{Y}_d H d + \bar{L}\tilde{Y}_e H e + \mathrm{h.c.}\right)$$

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3x3 complex **3x3 hermitian**

$\Longrightarrow$ **constraints on** $\tilde{Y}$

[Chala/Guedes/Ramos/Santiago '20, Bauer/Neubert/Renner/ Schnubel/Thamm '20, Bonilla/Brivio/Gavela/Sanz '21]

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= 0

$(+e)$

$(X\equiv YY^\dagger)$

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$$\tilde{Y}_u=i(Y_uc_u-c_QY_u)$$

$$[c_Q,Y_uY_u^\dagger]=i(\tilde{Y}_uY_u^\dagger+h.c.)$$

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$$\frac{a}{f}\bar{Q}_1\tilde{y}_{uc}\tilde{H}u_2$$ **PQ ?**

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$(+e)$

	$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$	$SU(3)_L$	$SU(3)_e$
Q_L	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Y_u	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Y_d	$\mathbf{3}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$
Y_e	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\bar{\mathbf{3}}$

Fermion-induced shift symmetry breaking

$\frac{a}{f}\bar{Q}_1\tilde{y}_{uc}\tilde{H}u_2$ **PQ ?**

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Fermion-induced shift

Discussion similar for **CPV in the SM** and the **Jarlskog** invariant

[Jarlskog '85]

$$\frac{a}{f}\bar{Q}_1\tilde{y}_{uc}\tilde{H}u_2$$

PQ ?

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What are y_u

Flavor-invariance

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Ex : axiflavon/flaxion model [Ema/Hamaguchi/Moroi/Nakayama '16 Calibbi/Goertz/Redigolo/Ziegler/Zupan '16]

$$-\mathcal{L} = \alpha^d_{ij} \left(\frac{\phi}{M}\right)^{q_{Q_i}-q_{d_j}} \bar{Q}_i H d_j + \alpha^u_{ij} \left(\frac{\phi}{M}\right)^{q_{Q_i}-q_{u_j}} \bar{Q}_i \tilde{H} u_j + \alpha^e_{ij} \left(\frac{\phi}{M}\right)^{q_{L_i}-q_{e_j}} \bar{L}_i H e_j + \text{h.c.}$$

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$$\Longrightarrow I_u^{(1)} = I_u^{(2)} = \ldots = 0$$

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Similar

Applications

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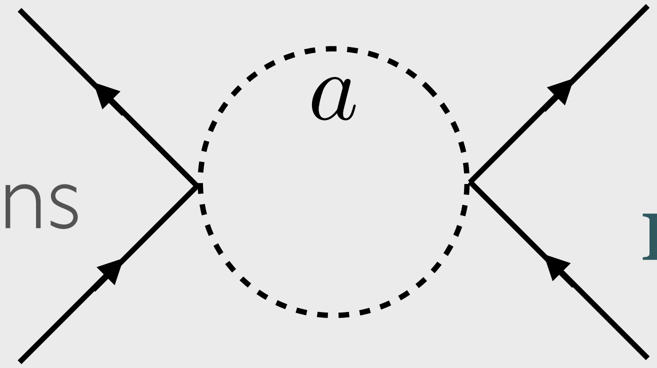
Ex, SMEFT+axion RGEs : $\dfrac{dC_{\bar{L}e\bar{d}Q}}{d\mu}$ contains 

[Galda, Neubert, Renner '21]

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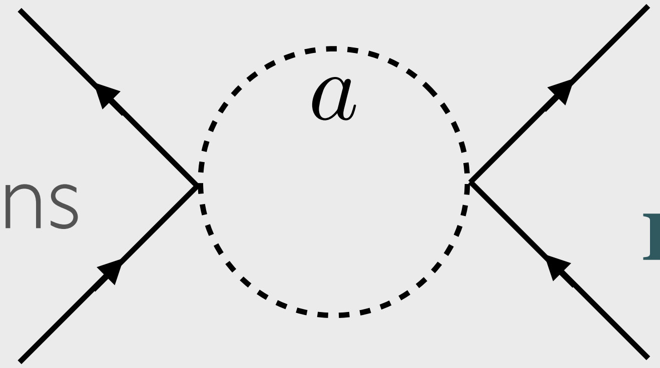
Ex, SMEFT+axion RGEs : $\dfrac{dC_{\bar{L}e\bar{d}Q}}{d\mu}$ contains 

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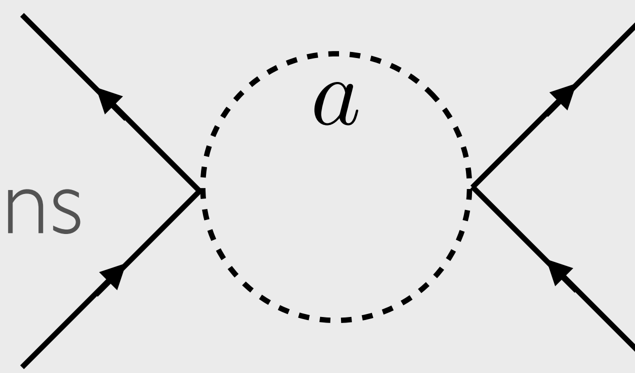

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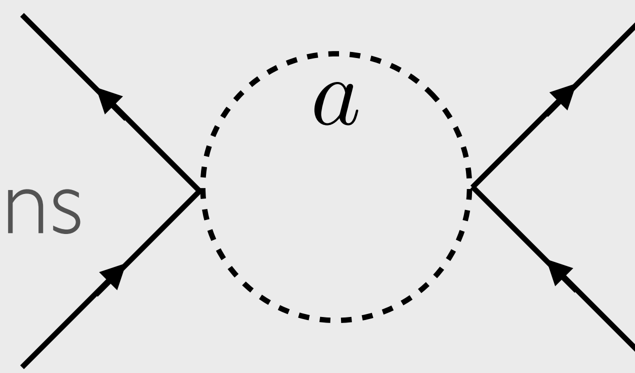

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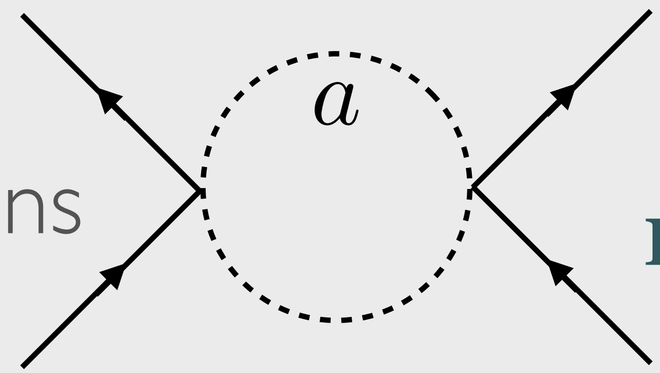

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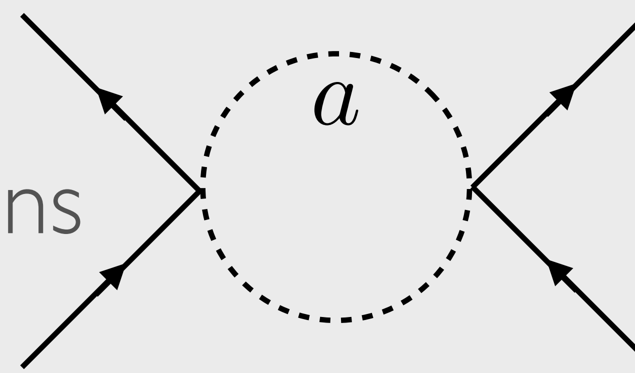

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Ex, axion-induced EDMs : $d_{\mathrm{Hg}} \approx 4 \times 10^{-4} d_n$ for a GB

[Di Luzio, Gröber, Paradisi '20]

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Perspectives :

- CPV
- flavor assumptions VS ~~PQ~~
- EFTs with non-linearly realized EW symmetry
- …

THANK YOU

Non-perturbative ~~PQ~~

$$\frac{a}{f}\left(\bar{Q}\tilde{Y}_u\tilde{H}u+\bar{Q}\tilde{Y}_dHd+\bar{L}\tilde{Y}_eHe+\text{h.c.}\right)-\frac{C_g g_3^2}{16\pi^2}\frac{a}{f}G\tilde{G}$$

For a non-anomalous GB :

$$\boxed{I_g = C_g - i\text{Tr}\left(Y_u^{-1}\tilde{Y}_u + (u \leftrightarrow d)\right) = 0}$$

Collective effect

Ex : axiflavon/flaxion model, $I_g =$ anomaly polynomial

RG running : $\frac{dI_g}{d\mu} = 0$ at one-loop