

Tachyonic Production of Primordial Black Holes and Gravitational Waves in Higgs- R^2 Inflation

“The Inflaton that Could”

Dhong Yeon Cheong (Yonsei University)

(in collab. with Kazunori Kohri, Seong Chan Park)

[DYC, K.Kohri, S.C.Park, arXiv 2205.14813](#)

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June. 23rd, 2022*

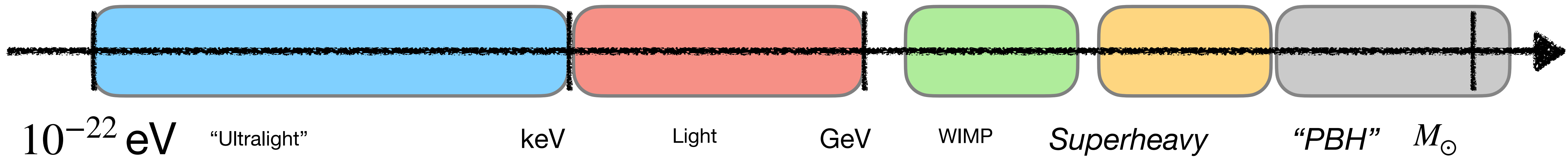


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Content

- ❖ Introduction
- ❖ Higgs- R^2 model
- ❖ Ultra-slow-roll in Higgs- R^2 .
- ❖ Tachyonic Instability in Higgs- R^2
- ❖ Phenomenological Consequences, Primordial Black Holes, Gravitational Waves
- ❖ Summary and Outlooks

Introduction - Dark Matter



Introduction - Primordial Black Holes

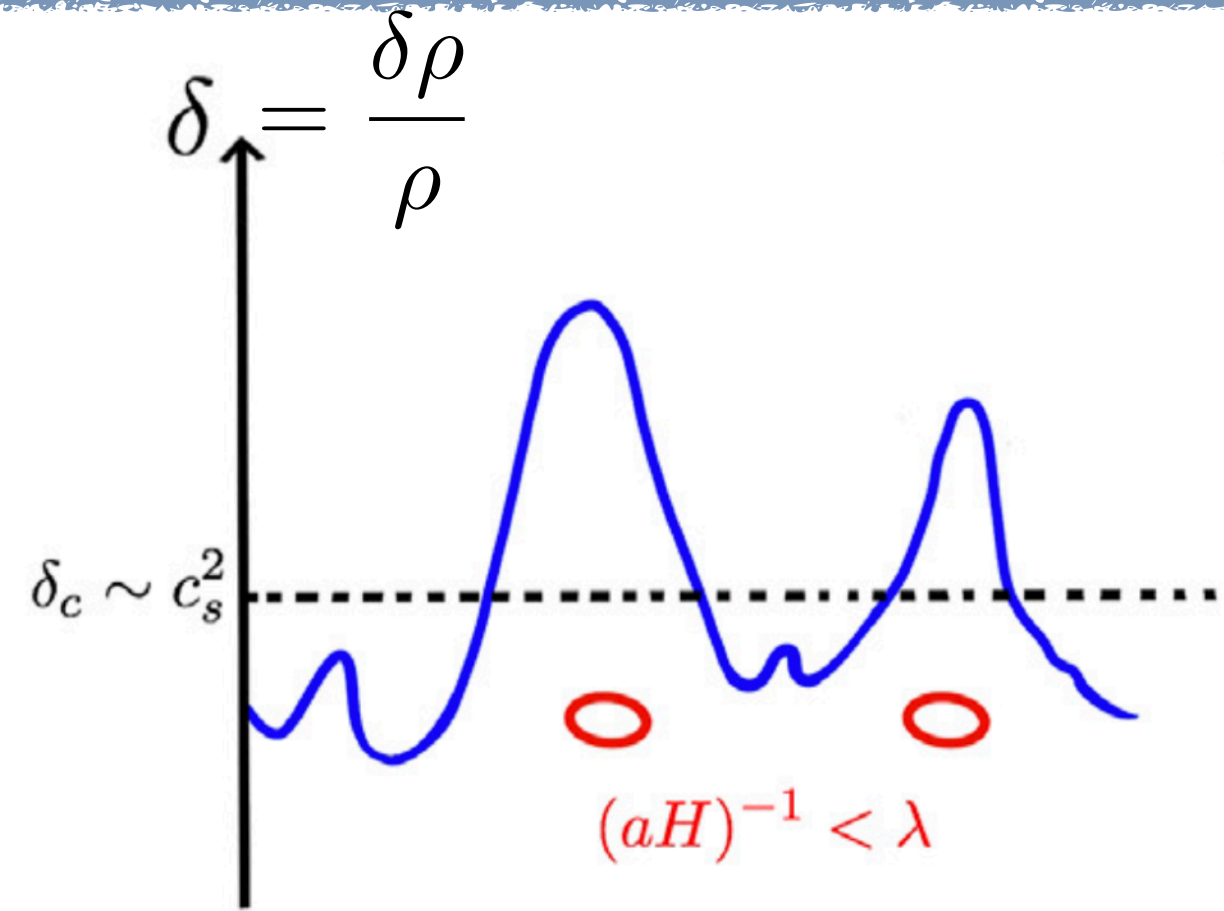


Figure from [P. Villanueva-Domingo *et. al.*, arXiv:2103.12087]

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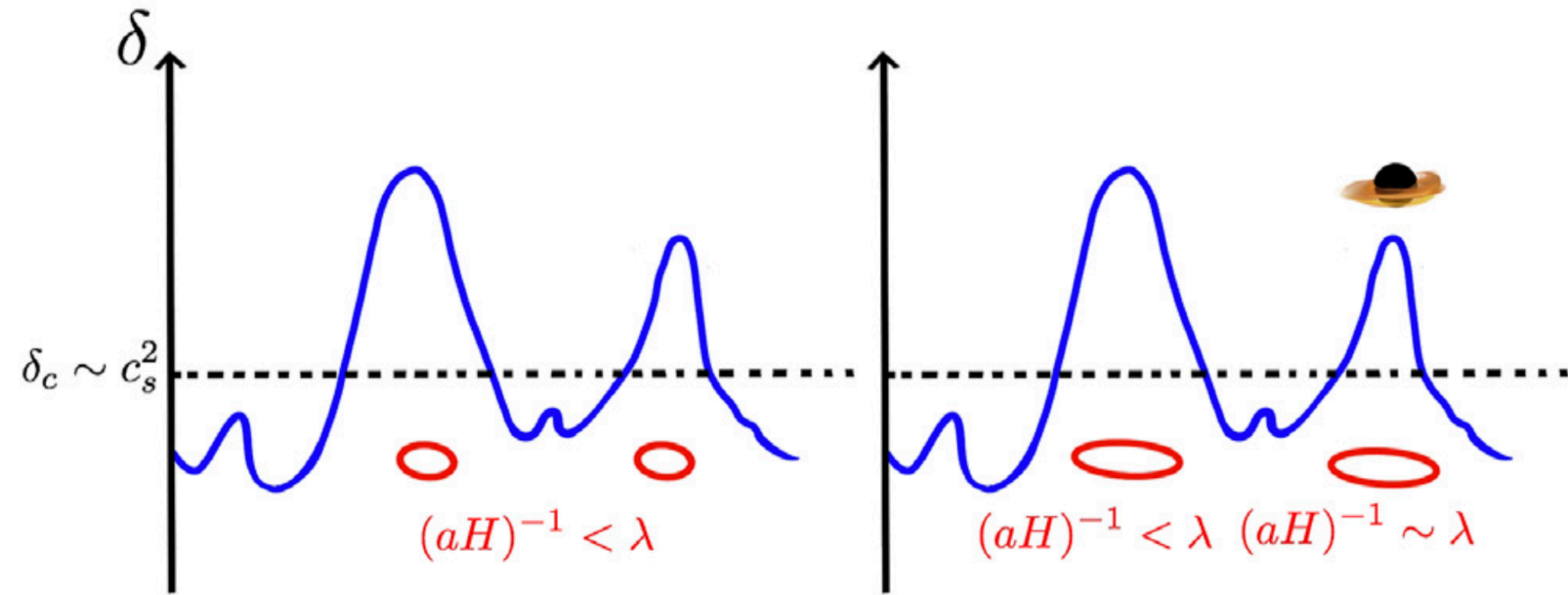


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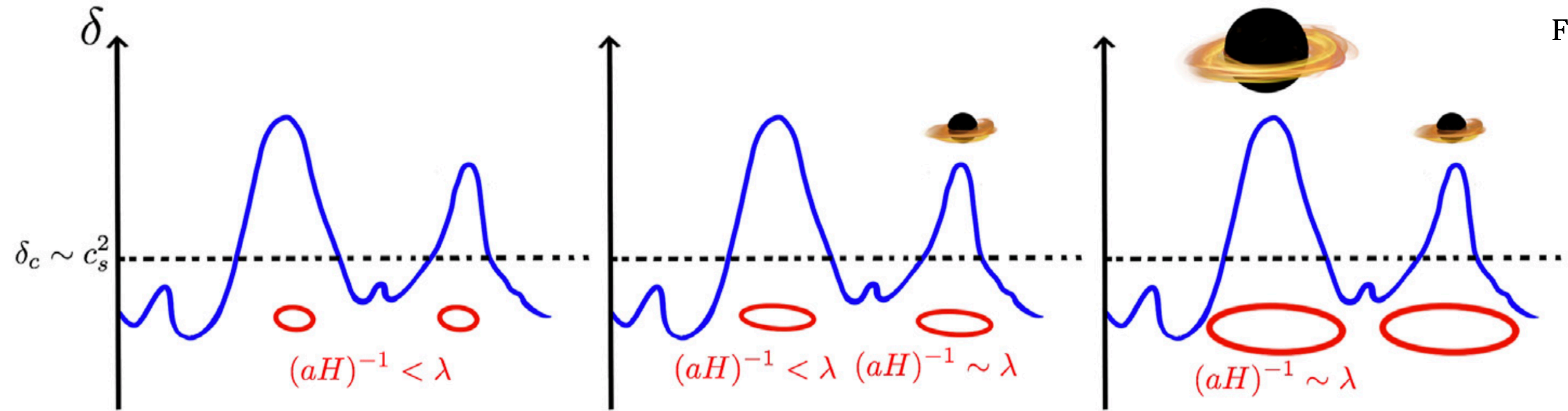


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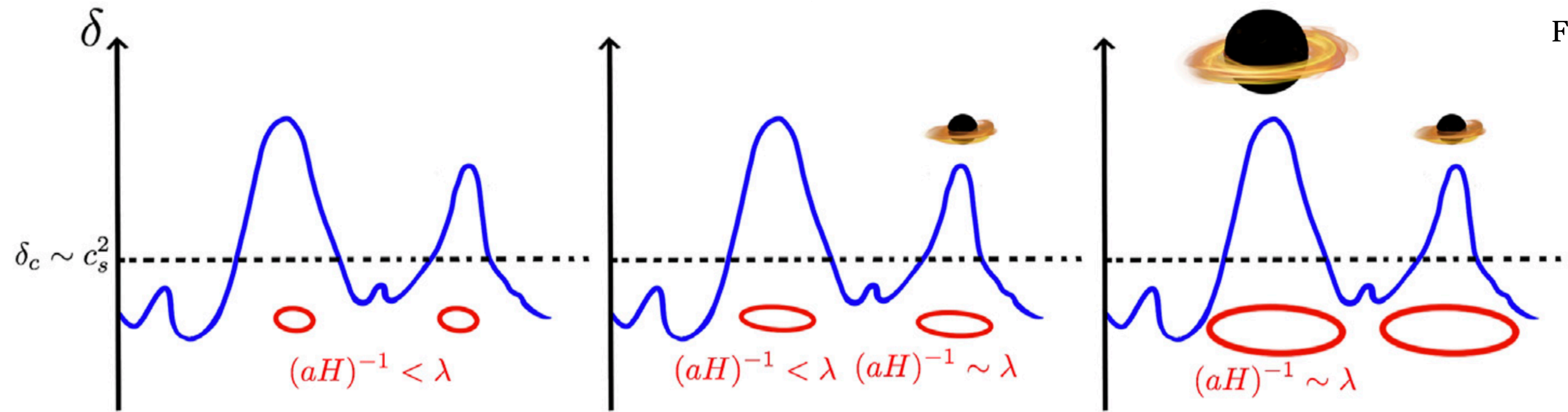


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PBHs form through the collapse of large over-densities in the density perturbation

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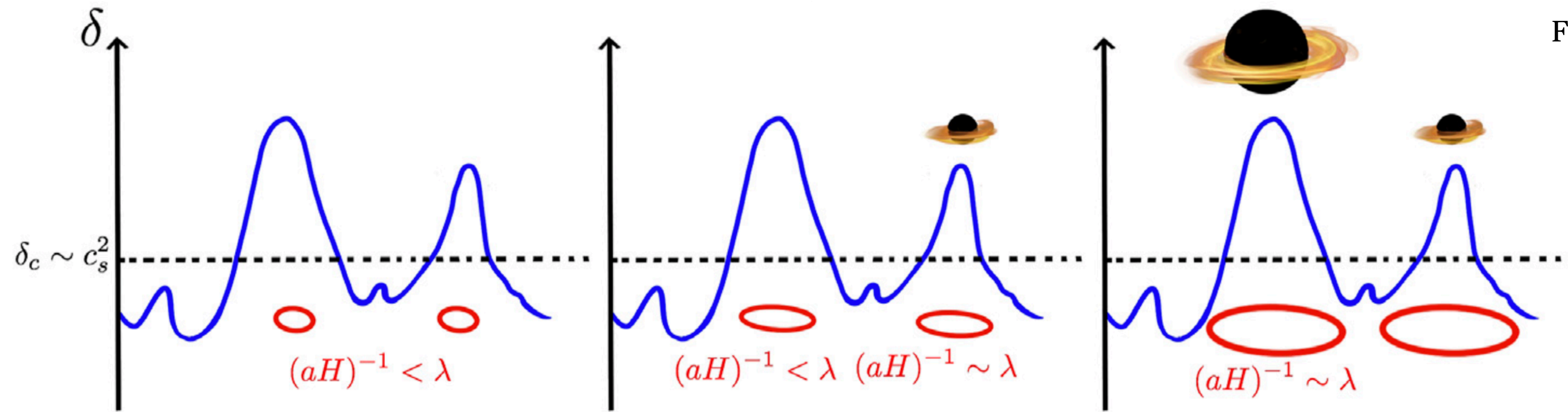


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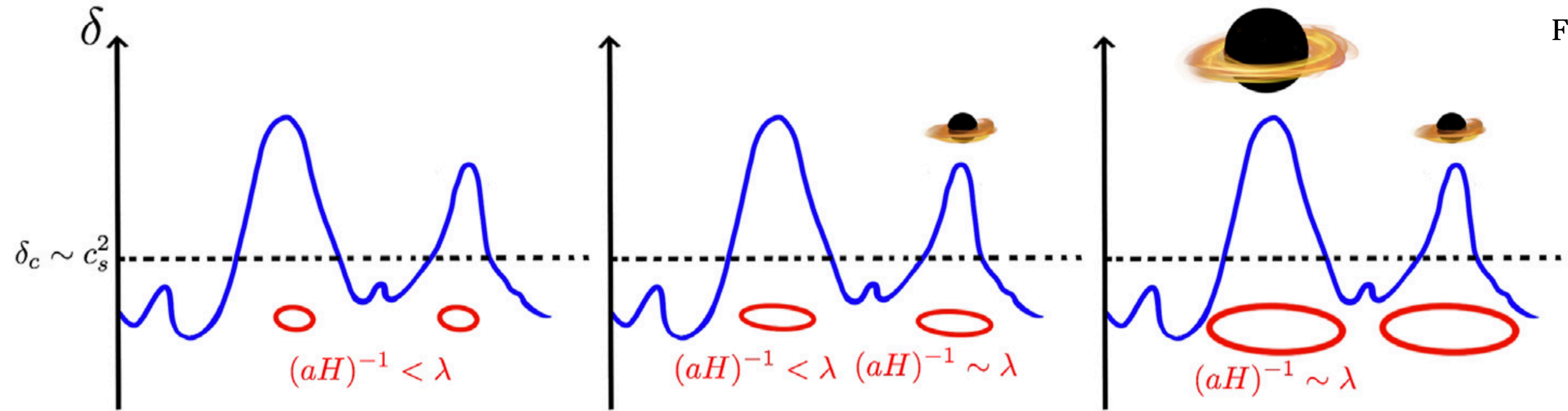


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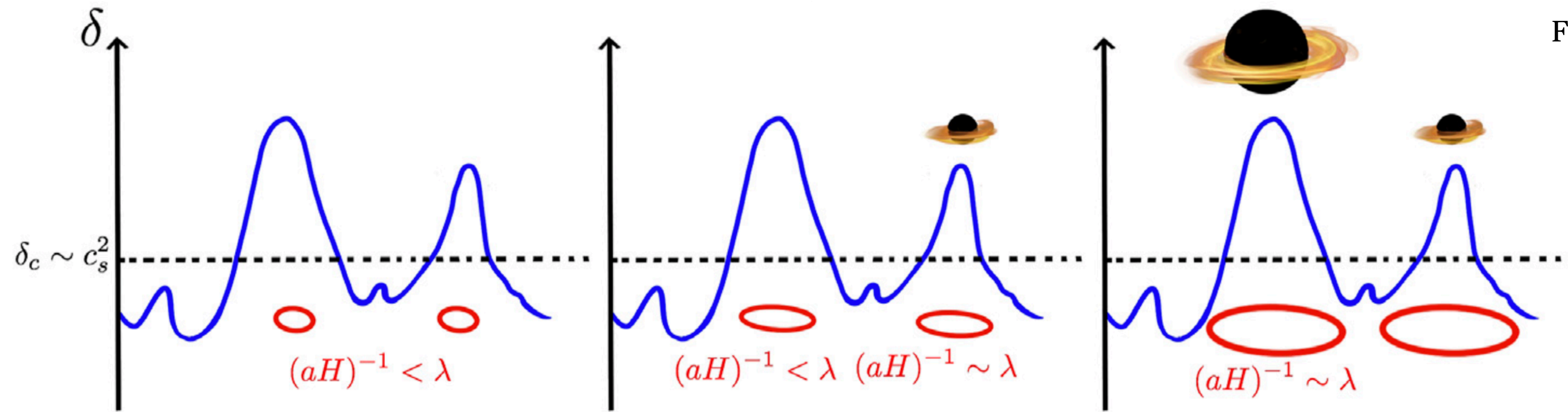


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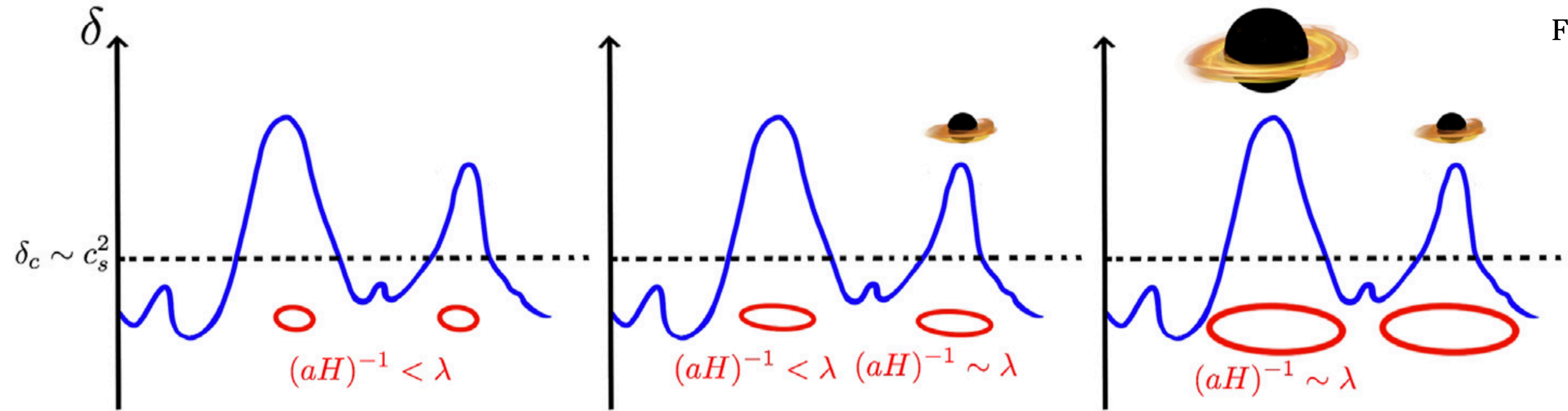


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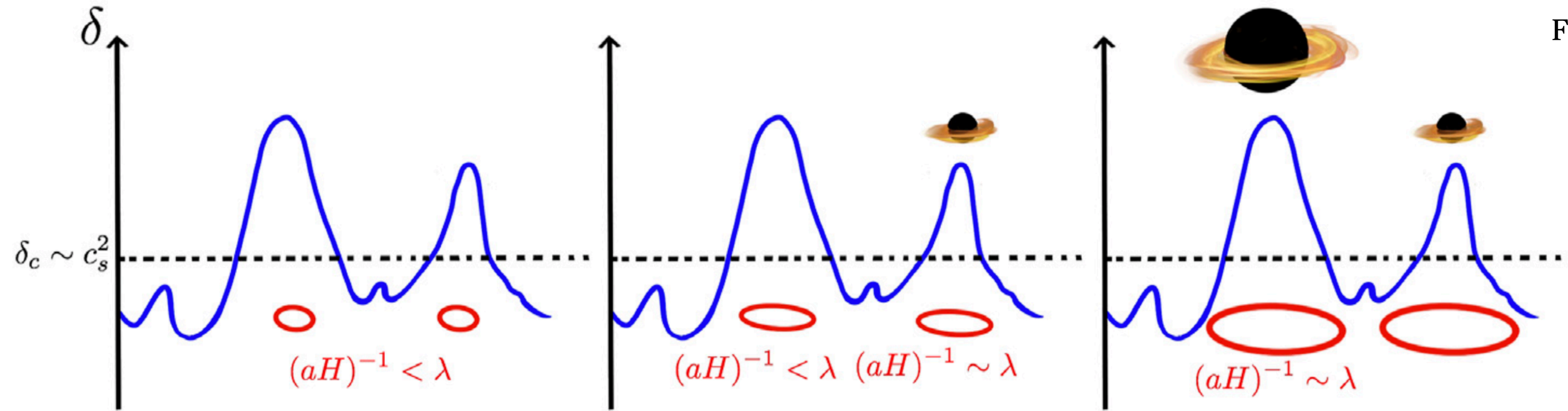


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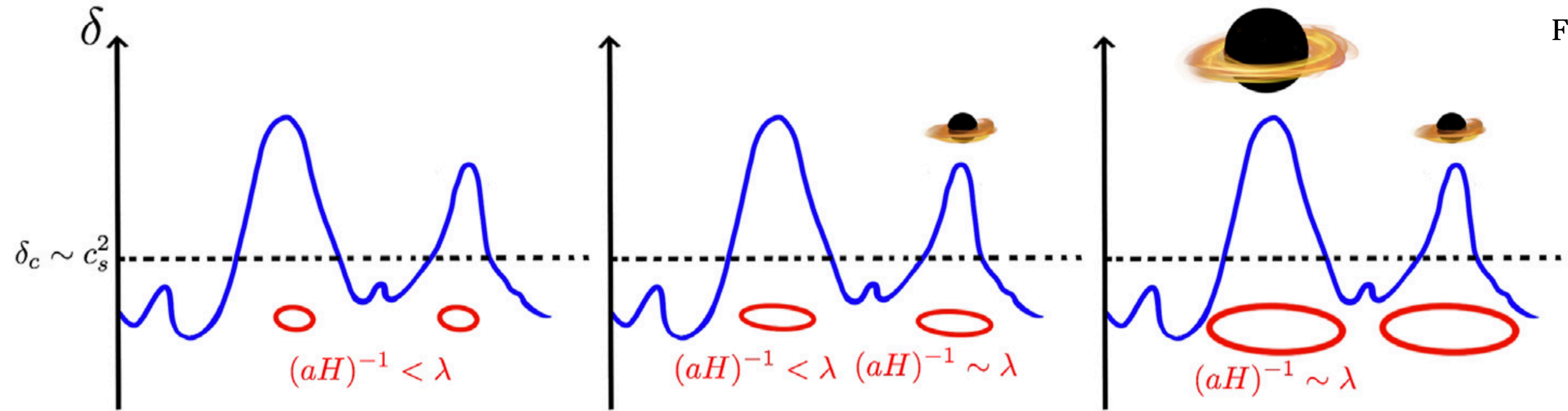


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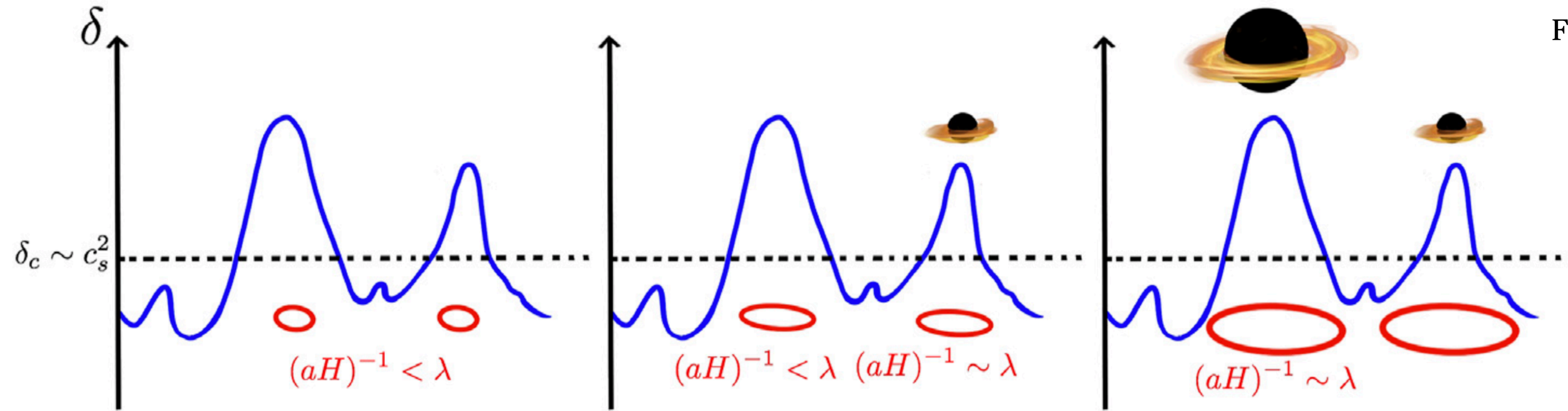


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Many mechanisms to produce large density perturbations —> Inflation is appealing!

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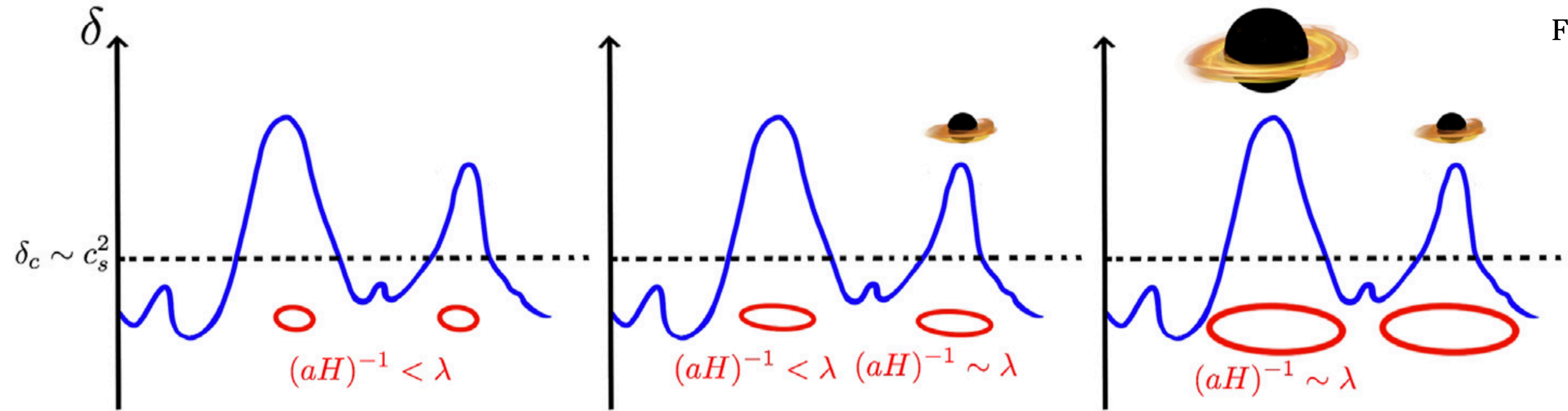


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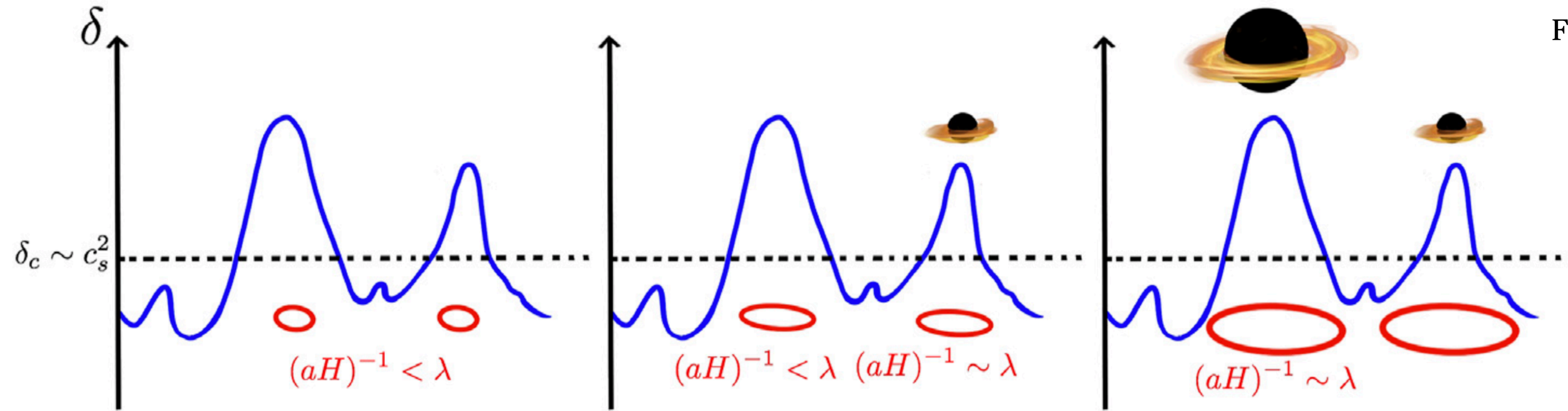


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Inflation, phase transition, cosmic strings, reheating..

Introduction - Primordial Black Holes

Then, how large should the density perturbations / curvature perturbations be?

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$$\beta \sim \frac{\sigma}{\sqrt{2\pi}\delta_c} e^{-\delta_c^2/(2\sigma^2)} \sim e^{-\delta_c^2/\mathcal{P}_{\mathcal{R}}} \leftarrow \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}} \Big|_{\text{form}}$$

$\sigma^2(R) = \int_0^\infty W^2(kR) \mathcal{P}_\delta(k) d \ln k$ Variance, Naive! $\sigma^2 \sim \mathcal{P}_{\mathcal{R}}$ Exponential dependence

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Naive! $\sigma^2 \sim \mathcal{P}_{\mathcal{R}}$ Exponential dependence

$$f_{\text{PBH}} \equiv \frac{\rho_{\text{PBH}}}{\rho_{\text{DM}}} = \left(\frac{a_{\text{eq}}}{a_{\text{form}}} \right) \beta(M) \quad \text{For solar-mass black holes, } a_{\text{eq}}/a_{\text{form}} \sim 10^8 \text{ in RD}$$

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For solar-mass black holes, $a_{\text{eq}}/a_{\text{form}} \sim 10^8$ in RD

Take $f_{\text{PBH}} \sim \mathcal{O}(1) \rightarrow \beta(M) \sim 10^{-8}$ then

$$\mathcal{P}_{\mathcal{R}} \sim \mathcal{O}(10^{-2})$$

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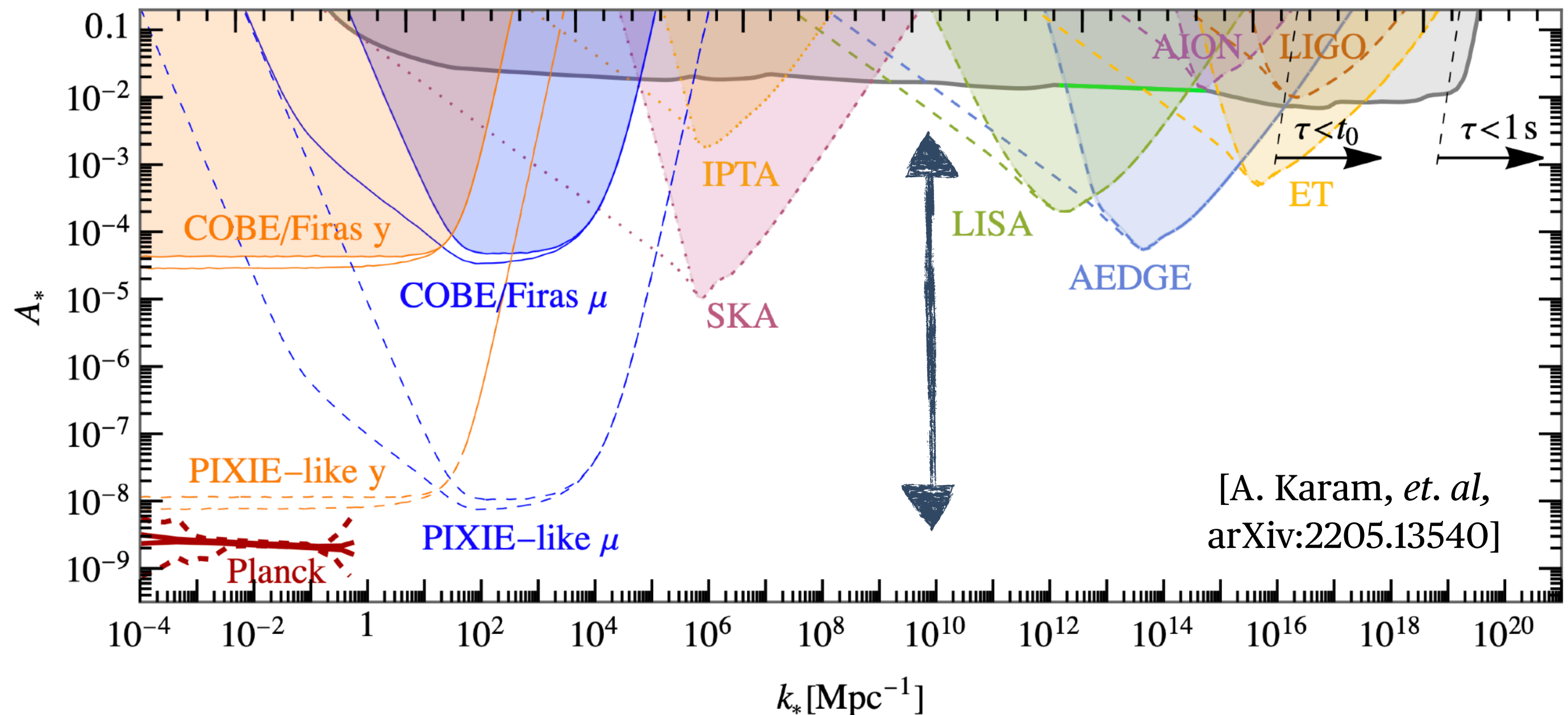
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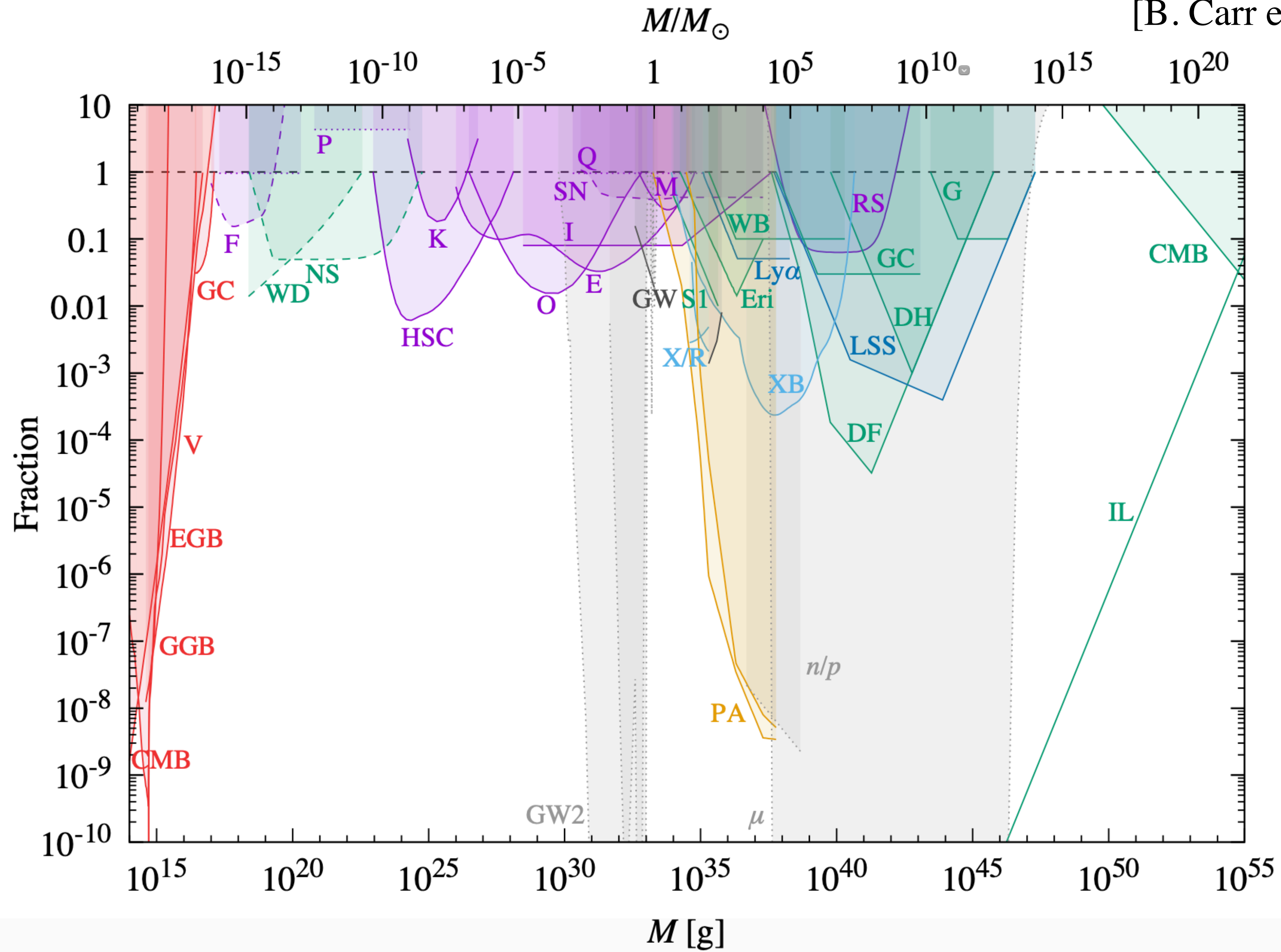
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[A. Karam, et. al, arXiv:2205.13540]

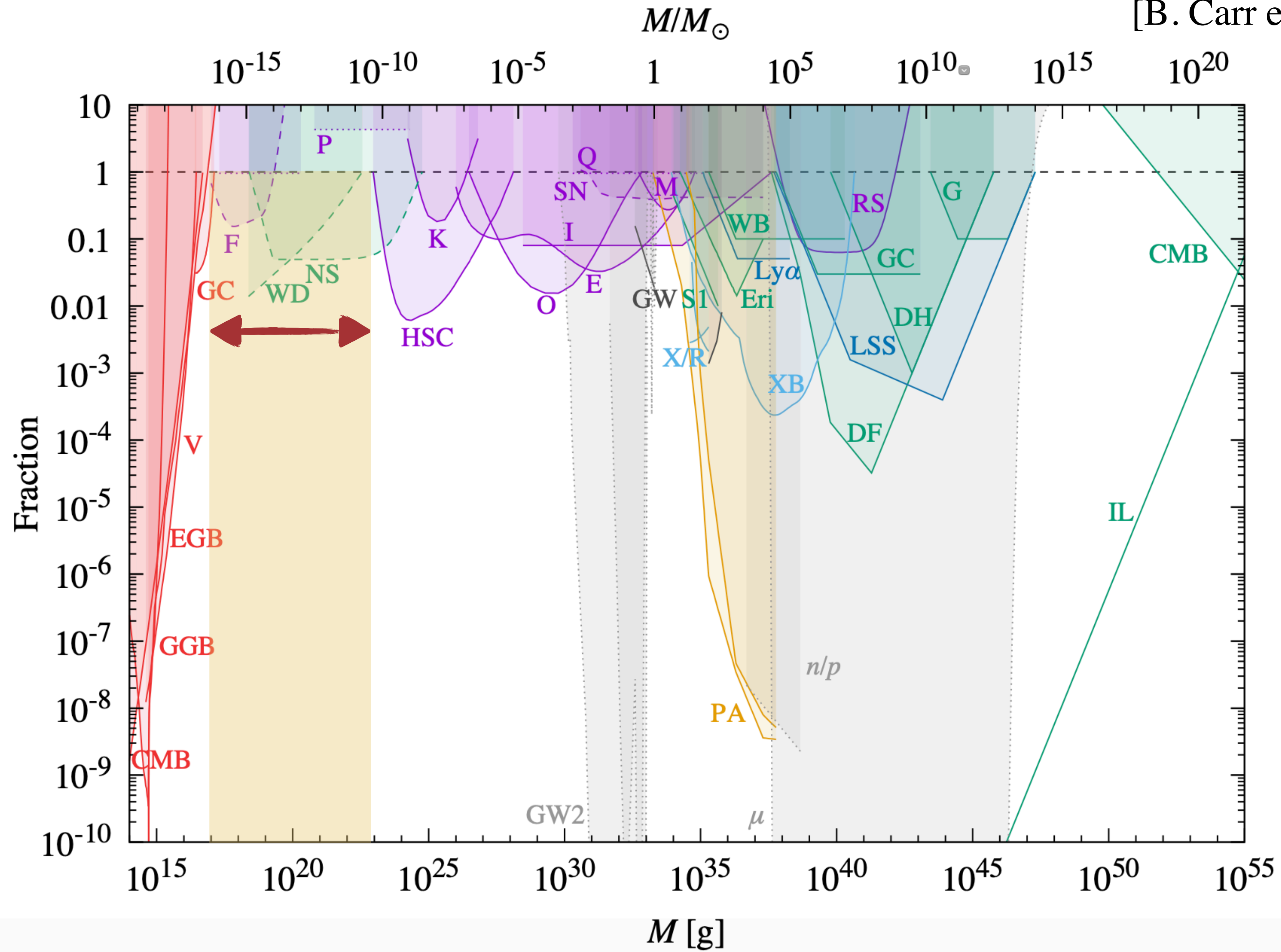
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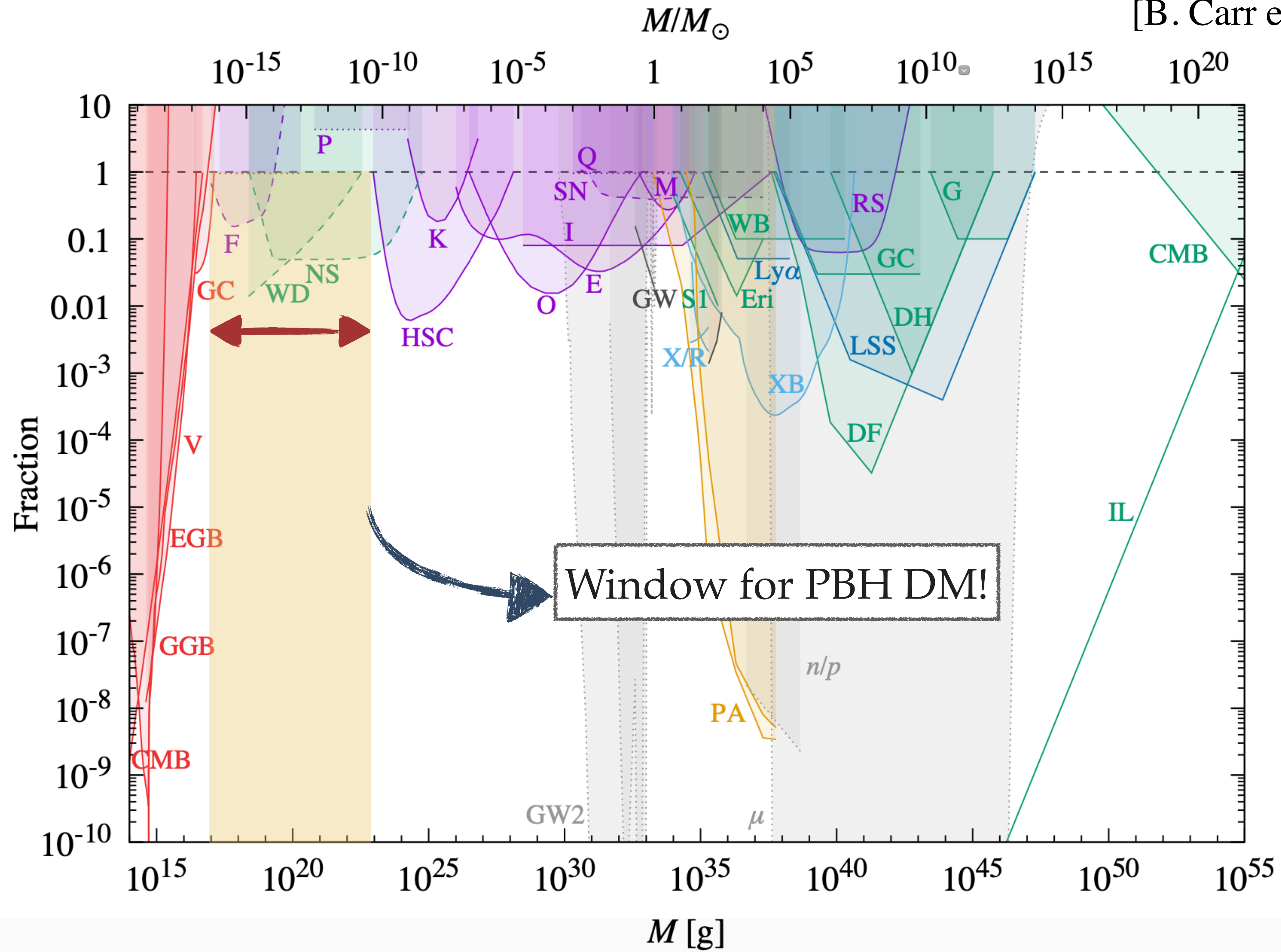
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Introduction - Second Order GWs

Scalar and tensor perturbations couple at “second order metric perturbations”.

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$$\Omega_{\text{GW}}(\eta_0, k) = c_g \frac{\Omega_{r,0}}{6} \int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2 - (1 + v^2 - u^2)^2}{4uv} \right)^2 \overline{\mathcal{I}^2(v, u)} \mathcal{P}_{\mathcal{R}}(kv) \mathcal{P}_{\mathcal{R}}(ku)$$

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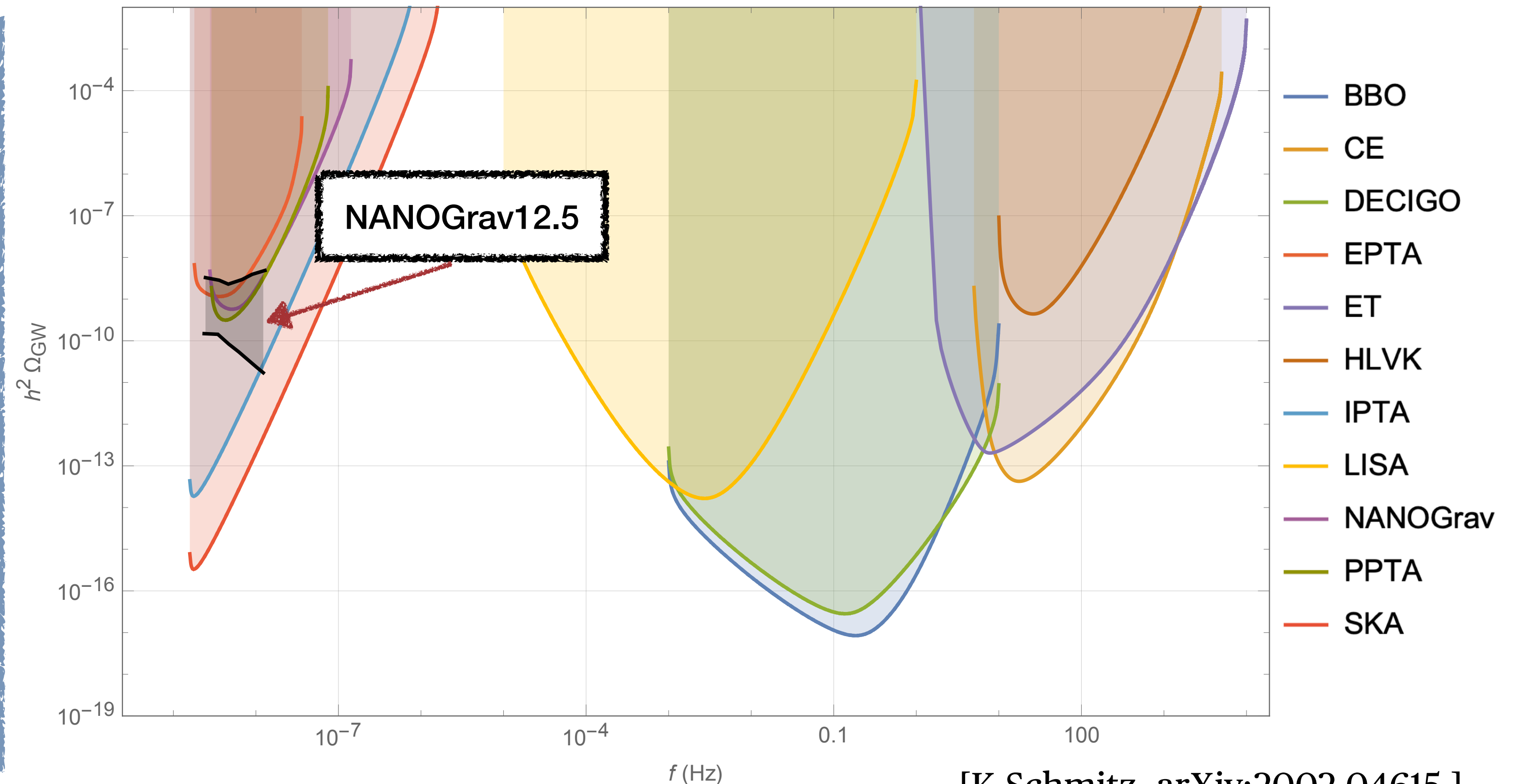
$$\Omega_{\text{GW}} h^2 \sim \frac{1}{12} \Omega_{r,0} h^2 \times \mathcal{P}_{\mathcal{R}}^2 \sim 10^{-6} \mathcal{P}_{\mathcal{R}}^2$$

DECIGO, SKA : $\mathcal{P}_{\mathcal{R}} \sim 10^{-5}$

LISA, CE, ET : $\mathcal{P}_{\mathcal{R}} \sim 10^{-4}$

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Future GW observatories well involved in probing the small scale.



[K.Schmitz, arXiv:2002.04615]
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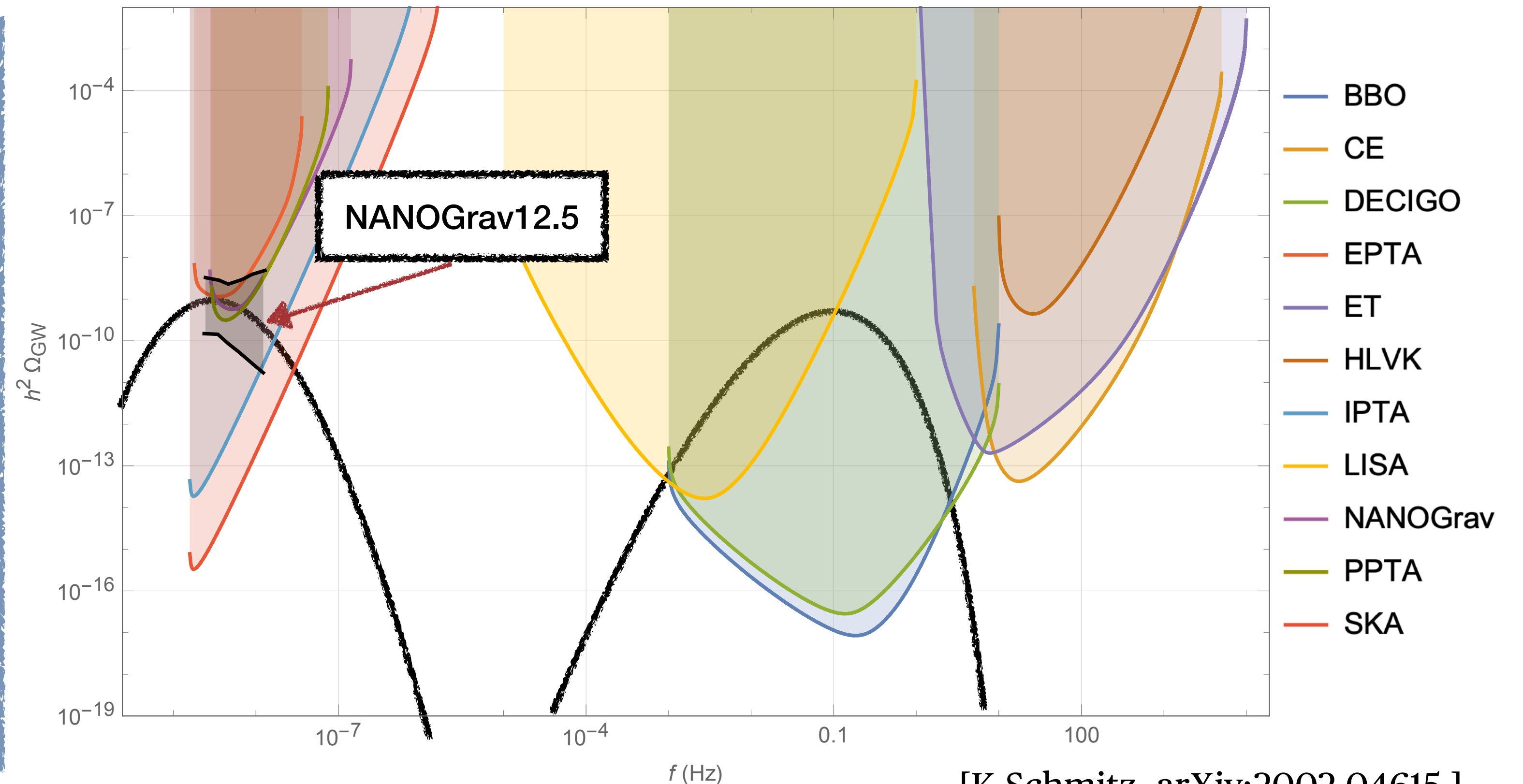
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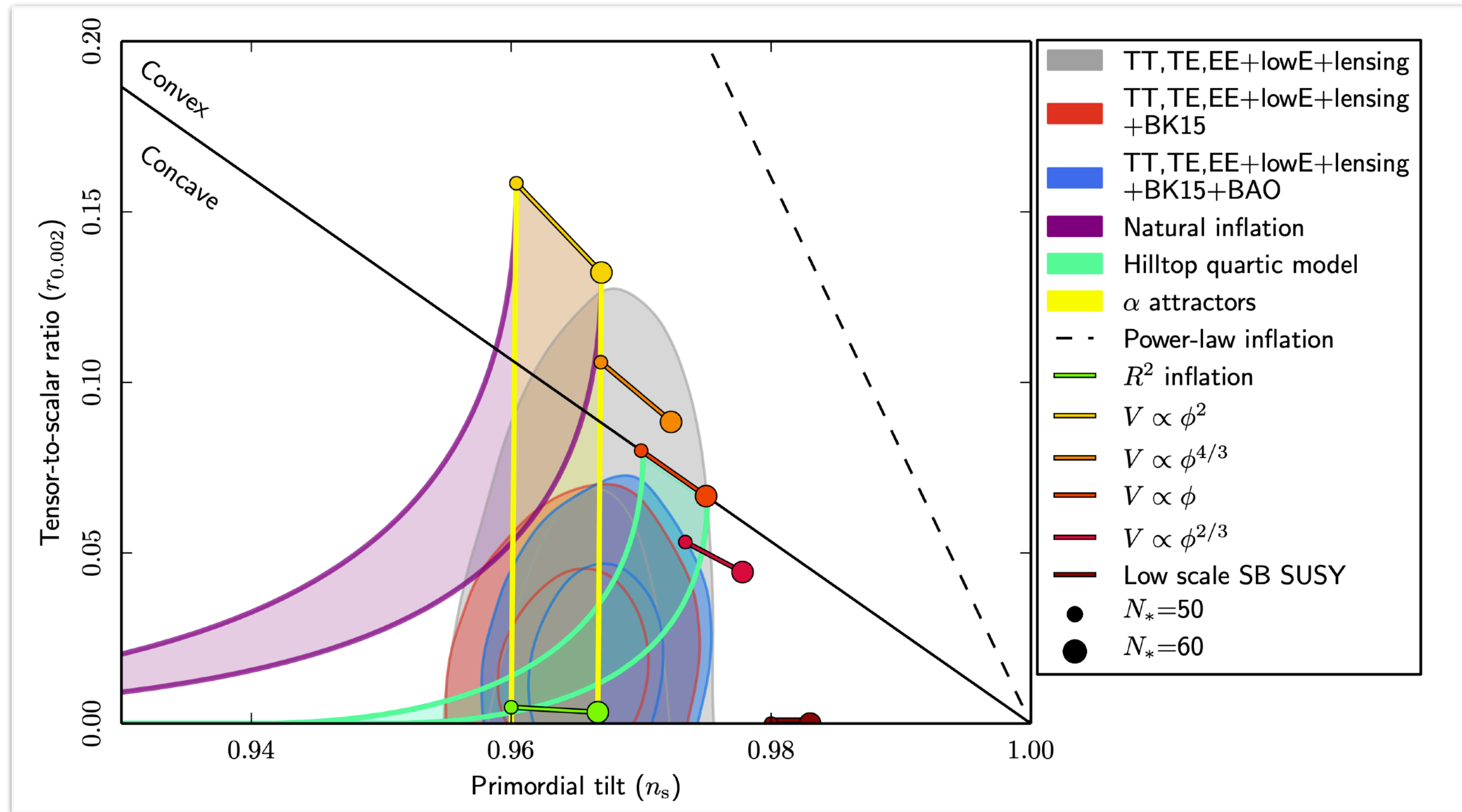


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Introduction

Inflation paradigm very successful! CMB observations attempt constraining inflationary models.

[Planck Collaboration., arXiv:1807.06211]



Characterized by *slow-roll parameters*

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2}$$

$$\eta \equiv -\frac{\ddot{\phi}}{\dot{\phi}H}$$

$$n_s \simeq 1 - 6\epsilon + 2\eta$$

Q1) What can be our inflaton?

Q2) Phenomenology induced by inflation in smaller scales?

Introduction

Q1) What can be our inflaton?

[F. Bezrukov, M. Shaposhnikov, 0710.3755]

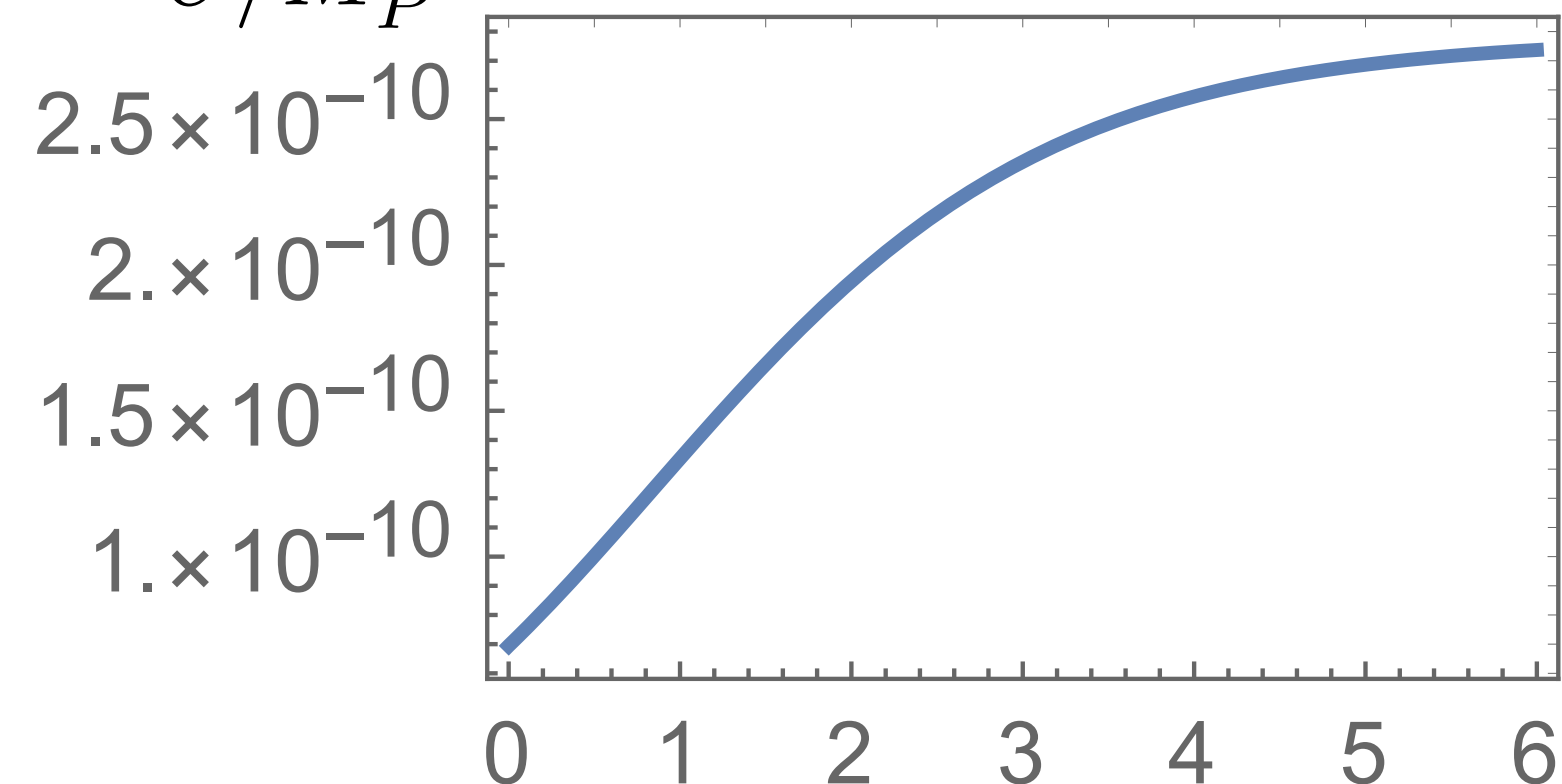
—> Sole scalar particle in our Standard Model, the Higgs (with a non-minimal coupling to gravity)

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{1}{2} \left(M_P^2 + \xi h_J^\dagger h_J \right) R_J - \frac{1}{2} |\partial_\mu h_J|^2 - V(h_J) \right]$$

$$g_{\mu\nu} = \Omega(h_J)^2 g_{J\mu\nu} \quad \Omega(h_J)^2 = 1 + \frac{\xi h_J^2}{M_P^2}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R - \frac{1}{2} |\partial_\mu h|^2 - U(h) \right] \quad U(h) \simeq \frac{\lambda M_P^4}{4\xi^2} \left(1 + e^{-\sqrt{\frac{2}{3}} \frac{h}{M_P}} \right)^{-2}$$

U/M_P^4



- Theoretical issues reside? : $\frac{\lambda}{\xi^2} \sim 10^{-10}$
—> $\lambda \ll 1$ achievable when considering SM running
- Naive cutoff scale appears at $\Lambda \sim \frac{M_P}{\xi}$ for $\xi \sim \mathcal{O}(10^3)$
—> Issues in the (p)reheating era, unitarity problem?

Introduction

Q1) What can be our inflaton?

Considering dim-4 operators, R^2 present as well!

[Y. Ema, Phys. Lett. B770:403-411, 2017]

[Y-C. Wang, T. Wang, Phys. Rev. D96(12):123506, 2017]

[M.He, A. A. Starobinsky, J. Yokoyama, JCAP, 1805(05):064, 2018]

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(R_J + \frac{\xi h^2}{M_P^2} R_J + \frac{R_J^2}{6M^2} \right) - \frac{1}{2} g^{\mu\nu} \nabla_\mu h \nabla_\nu h - \frac{\lambda}{4} h^4 \right],$$

non minimal coupling
 R^2 term
Higgs

Conformal transformation into Einstein frame

$$S_E = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \nabla_\mu s \nabla_\nu s - \frac{1}{2} e^{-\sqrt{\frac{2}{3}} \frac{s}{M_P}} g^{\mu\nu} \nabla_\mu h \nabla_\nu h - U(s, h) \right]$$

$$U(s, h) \equiv e^{-2\sqrt{\frac{2}{3}} \frac{s}{M_P}} \left\{ \frac{3}{4} M_P^2 M^2 \left(e^{\sqrt{\frac{2}{3}} \frac{s}{M_P}} - 1 - \frac{\xi h^2}{M_P^2} \right)^2 + \frac{\lambda_{\text{eff}}(\mu)}{4} h^4 \right\}$$

Introduction

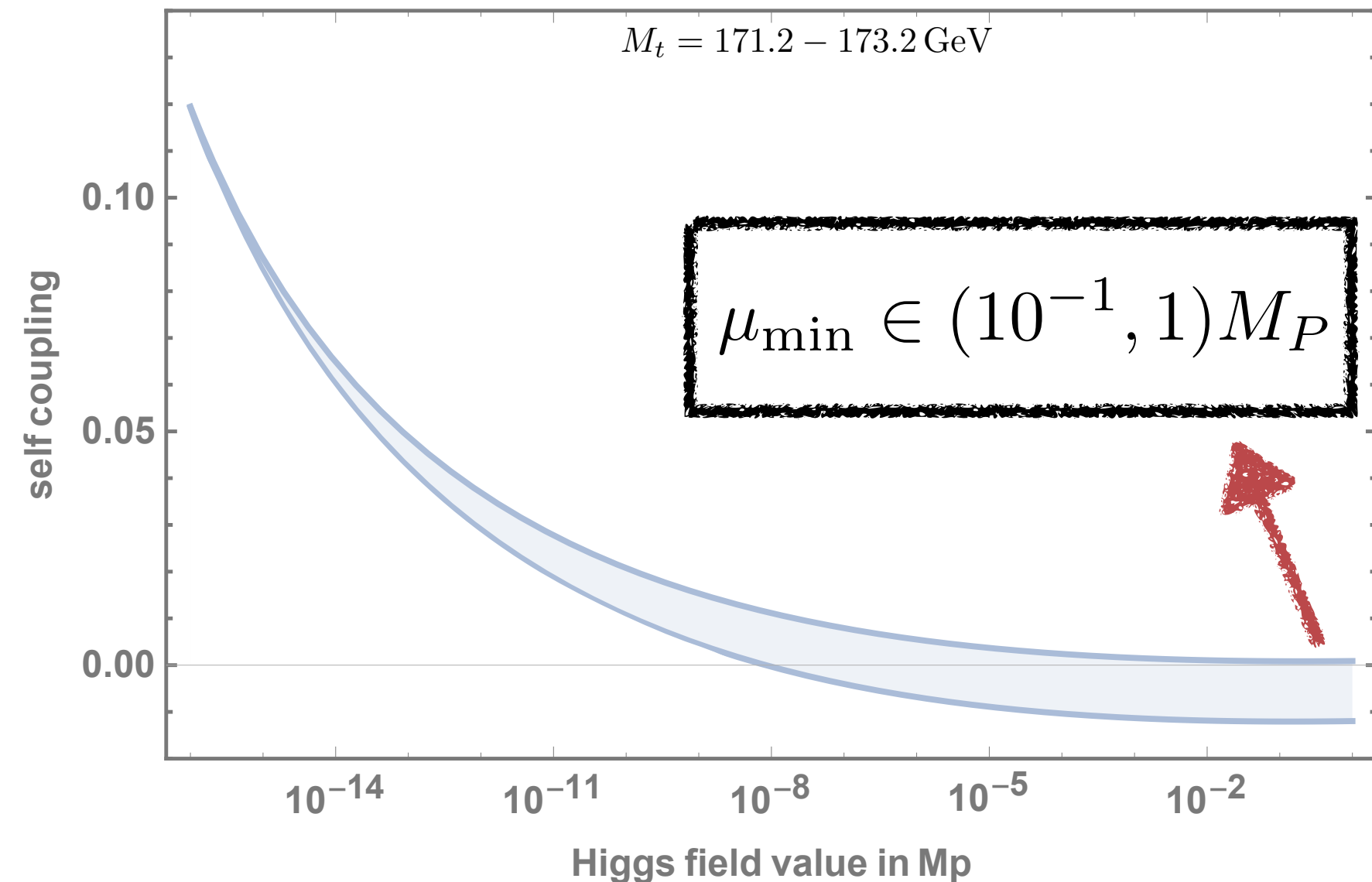
Q1) What can be our inflaton? \rightarrow Higgs + R^2

$$U(s, h) \equiv e^{-2\sqrt{\frac{2}{3}}\frac{s}{M_P}} \left\{ \frac{3}{4}M_P^2 M^2 \left(e^{\sqrt{\frac{2}{3}}\frac{s}{M_P}} - 1 - \frac{\xi h^2}{M_P^2} \right)^2 + \frac{\lambda}{4}h^4 \right\}$$

$(s, h) \simeq (0, 0)$

$$\simeq \frac{\lambda}{4}h^4 + \frac{3\xi^2 M^2}{4M_P^2}h^4 + \frac{1}{2}M^2 s^2 + \dots - \frac{\lambda}{\sqrt{6}M_P}sh^4 - \frac{M^2}{6\sqrt{6}M_P^3}s^5 + \left(\frac{\lambda}{3M_P^2} + \frac{\xi^2 M^2}{M_P^4} \right) h^4 s^2 + \dots$$

$$\Lambda \sim \mathcal{O}\left(\frac{M_P^2}{\xi^2 M^2}\right) M_P > M_P \quad \text{Theory unitarized through the scalaron!}$$



Choosing $\mu \simeq \sqrt{h^2}$

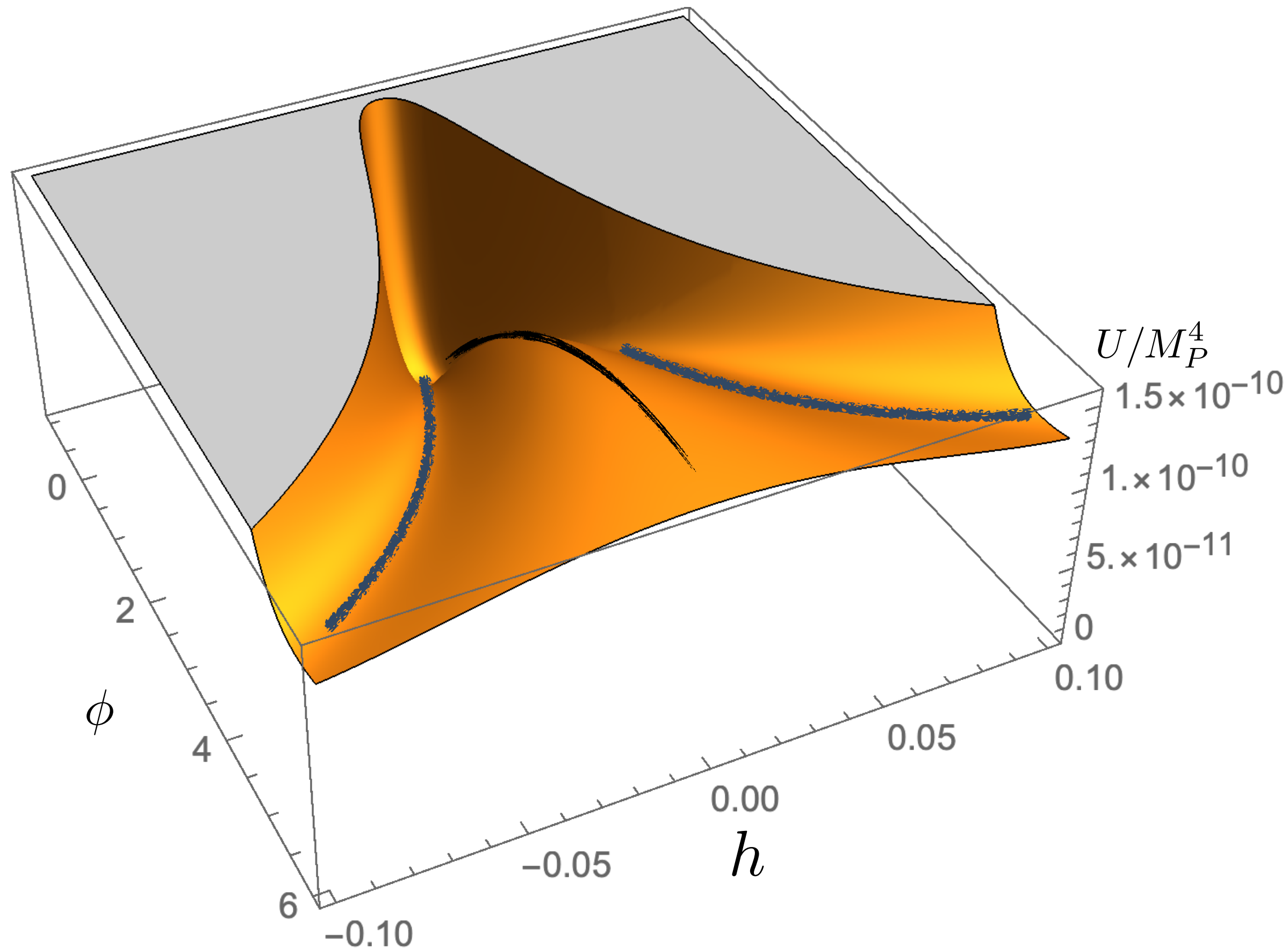
$$\lambda(\mu)|_{\mu=h} = \lambda_m + \frac{\beta_2^{\text{SM}}}{(16\pi^2)^2} \ln^2 \left(\sqrt{\frac{h^2}{h_m^2}} \right) = \lambda_m + b \ln^2 \left(\sqrt{\frac{h^2}{h_m^2}} \right)$$

$$\beta_2^{\text{SM}} \simeq 0.5$$

Critical values where $\lambda_m \sim \mathcal{O}(10^{-6})$ possible

Higgs- R^2 Inflation

- Shape of the potential? Focus on constant λ for simplicity



- “Valley” structure, trajectory (initially) follows,

$$\frac{\partial U(s, h)}{\partial h} = 0 \quad h_v^2 = \frac{e^{\sqrt{\frac{2}{3}} \frac{\phi}{M_P}} - 1}{\frac{\xi}{M_P^2} + \frac{\lambda}{3\xi M^2}} \quad \text{for } \phi > 0$$

➔ Isocurvature mass heavier than the Hubble scale, suppression in perturbations

➔ Flat plateau induced in the large s limit.

- “Tachyonic Direction” at $h = 0$ for $s > 0$

➔ Studies focused on a “tachyonic instability” during (p)reheating

- Previous studies focused on inflationary dynamics *inside the valley, but rich phenomena also on the hill.*

[M. He, *et. al.*, *PLB*, 791, 36-42 (2019).]

[F. Bezrukov, *et. al.*, *PLB*, 795, 657-665 (2019).]

[M. He, *et. al.*, *JCAP* 01 (2021) 066.]

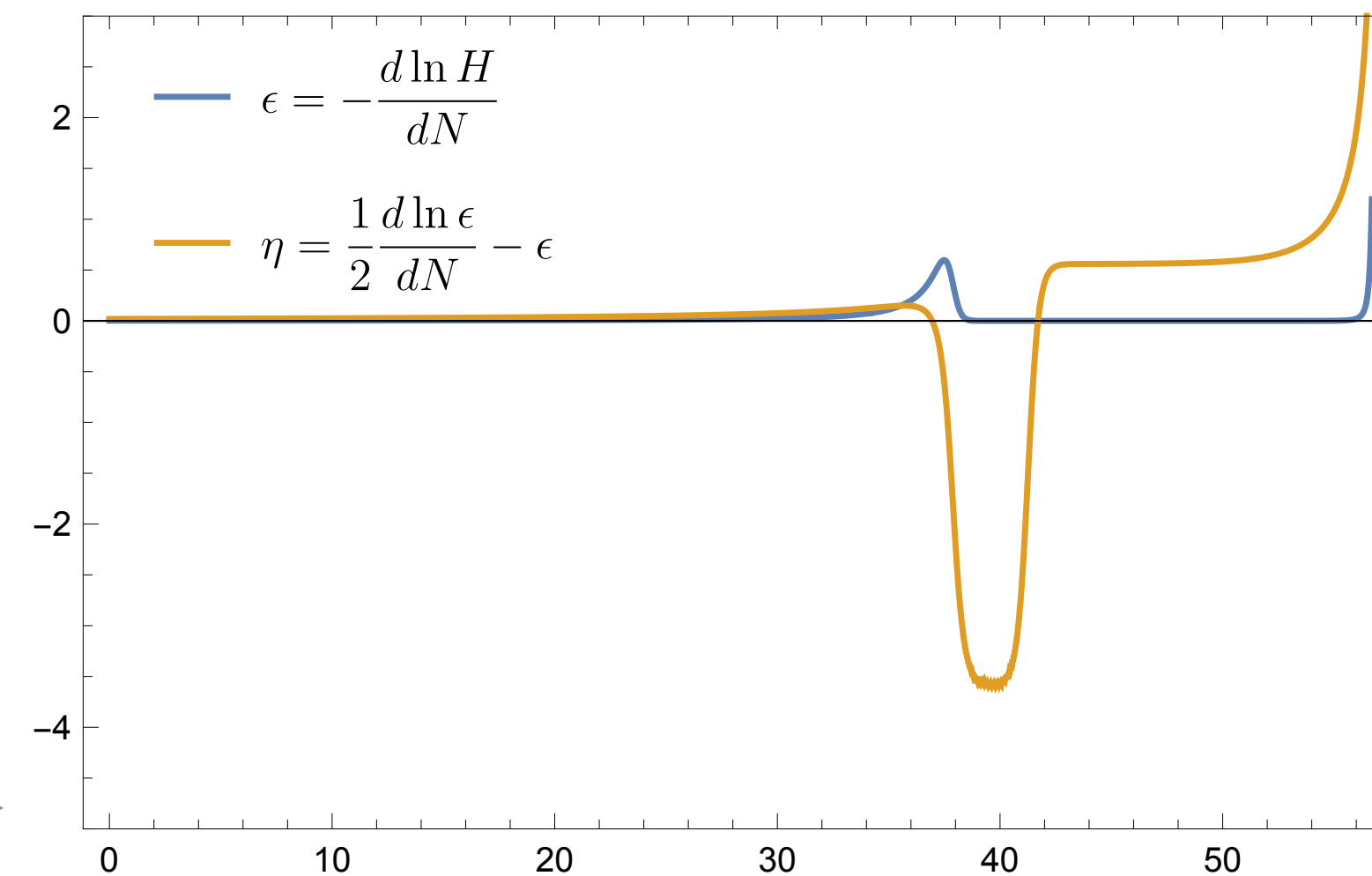
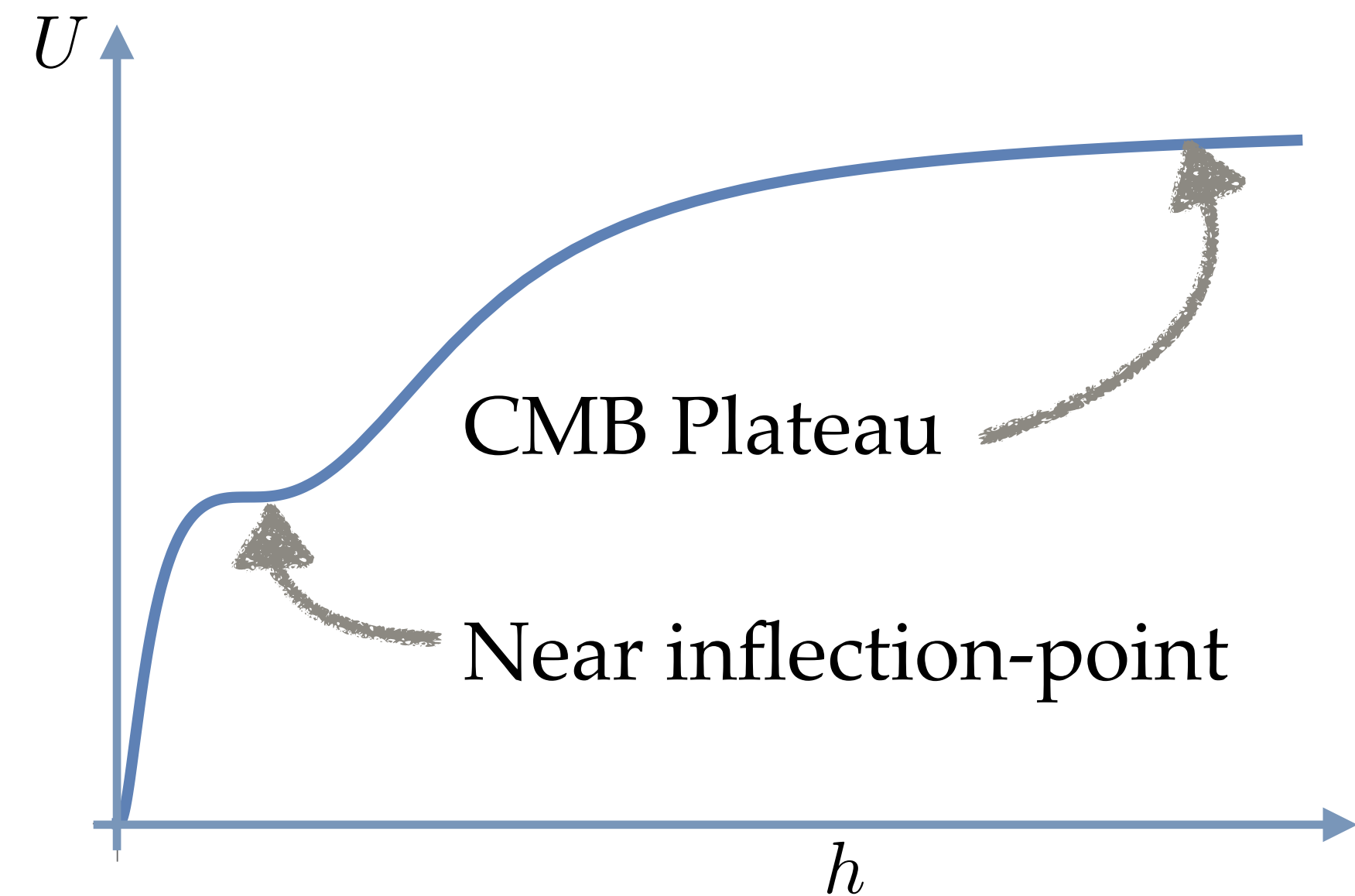
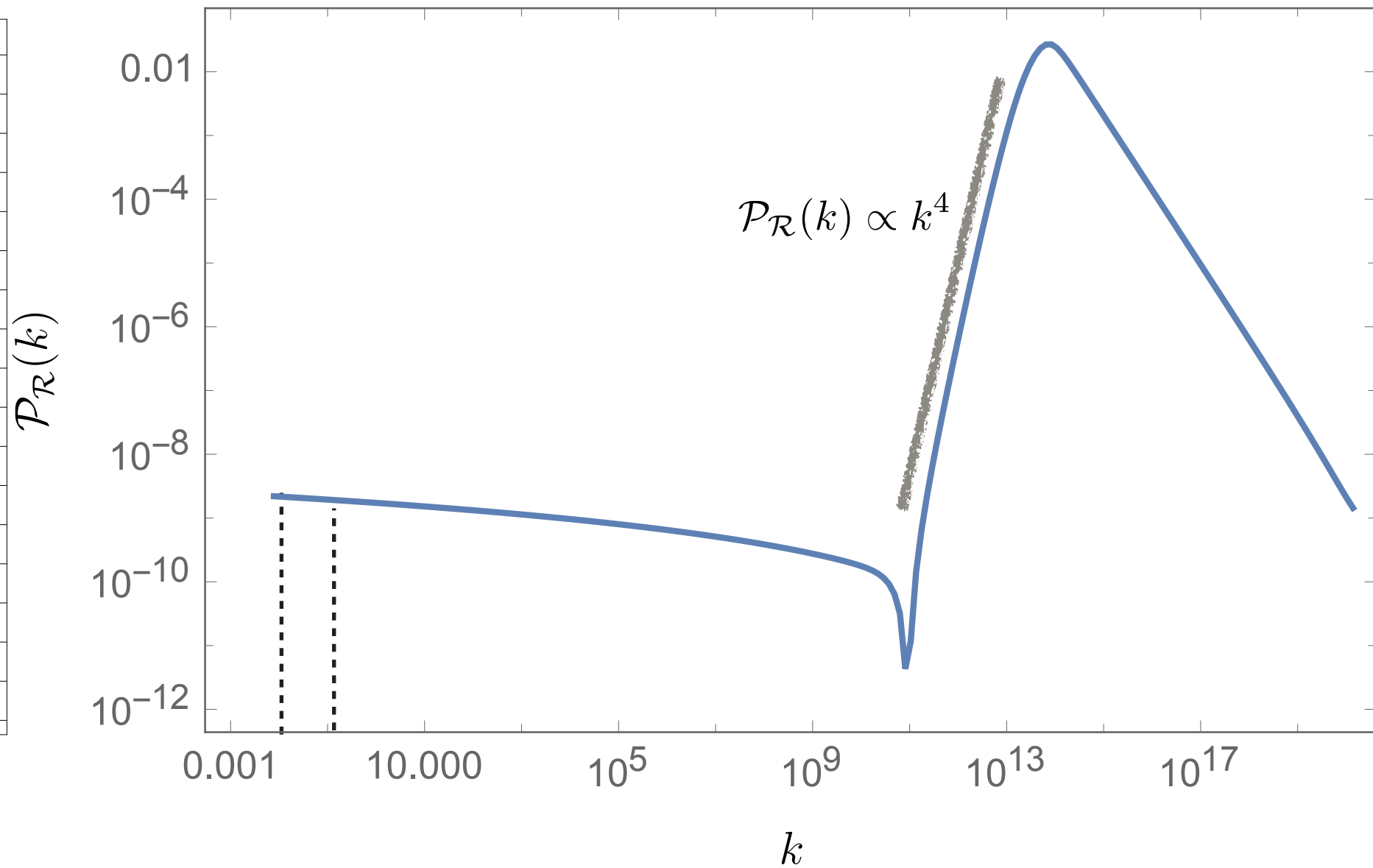
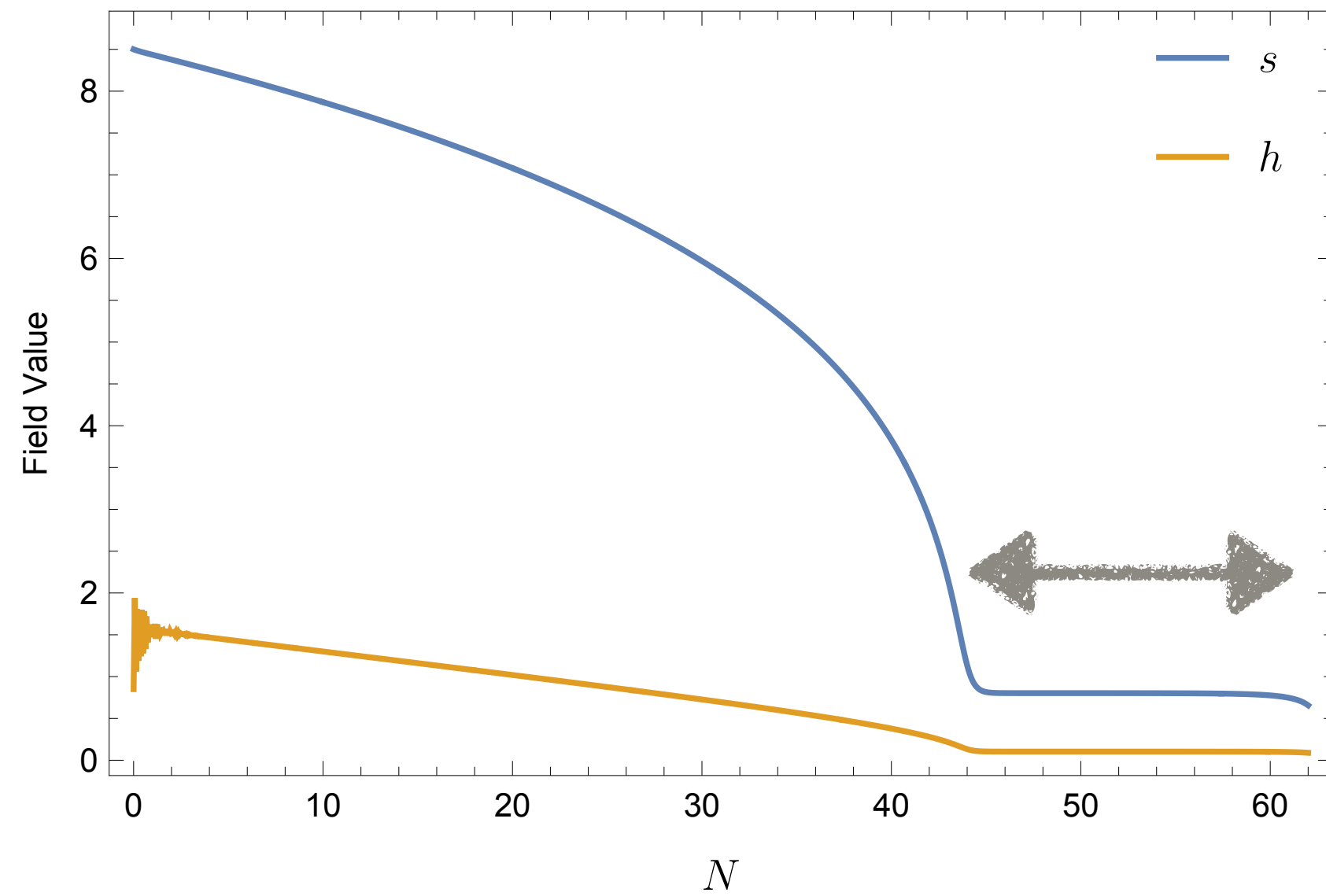
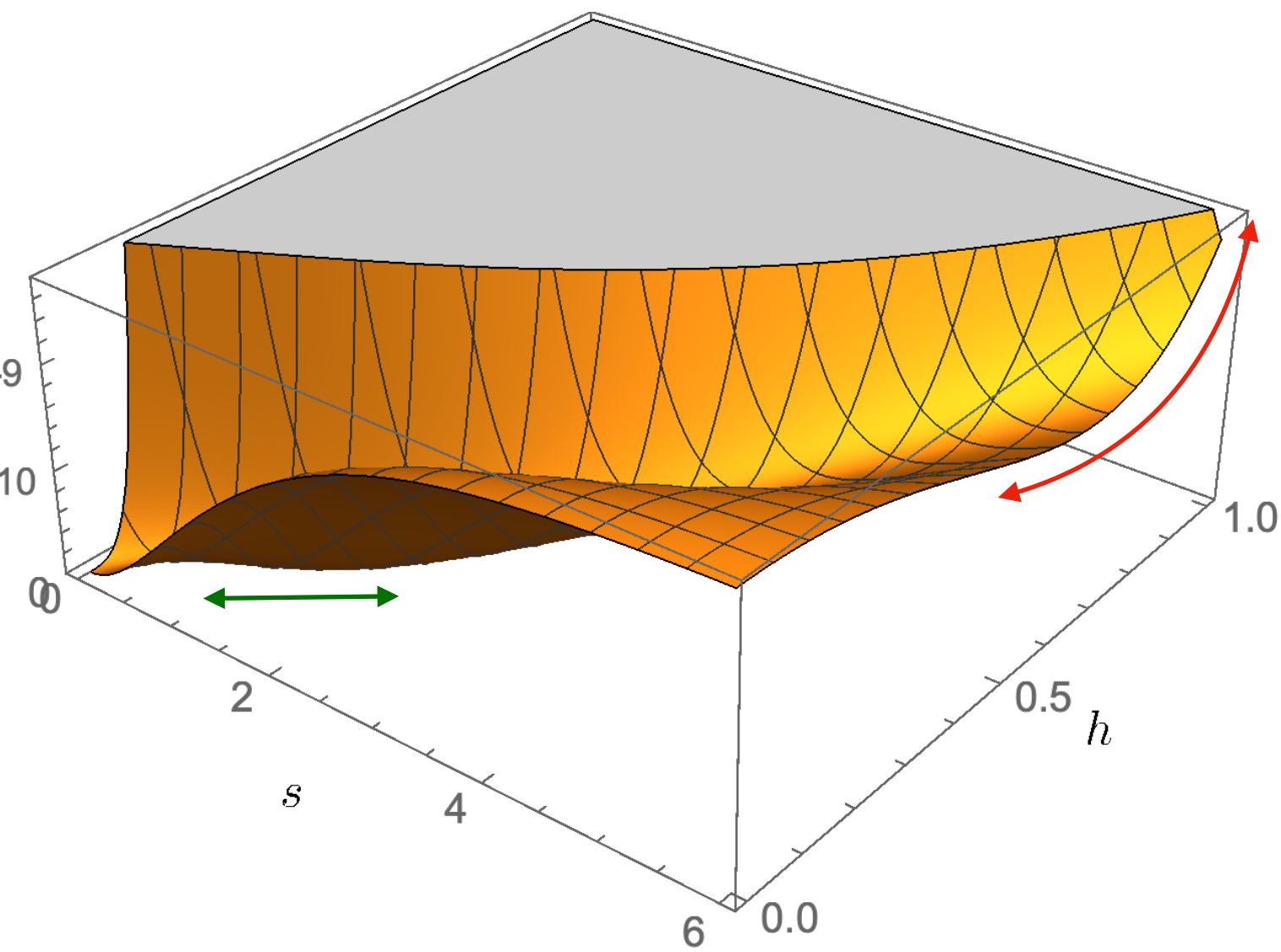
[F. Bezrukov, C. Shepherd *JCAP* 12 (2020) 028.]

[S. Aoki *et. al.*, arXiv:2202.13063], and many more

Higgs- R^2 Inflation - Ultra Slow-Roll

[DYC, S.M. Lee, S.C. Park, JCAP 01 (2021), 032]

$$M = 4.2 \times 10^{-5} M_P \quad \xi = 79 \quad h_{\min} = 0.15 M_P \quad b = 2 \times 10^{-5} \quad \lambda_{\min} = 4.11087 \times 10^{-6}$$



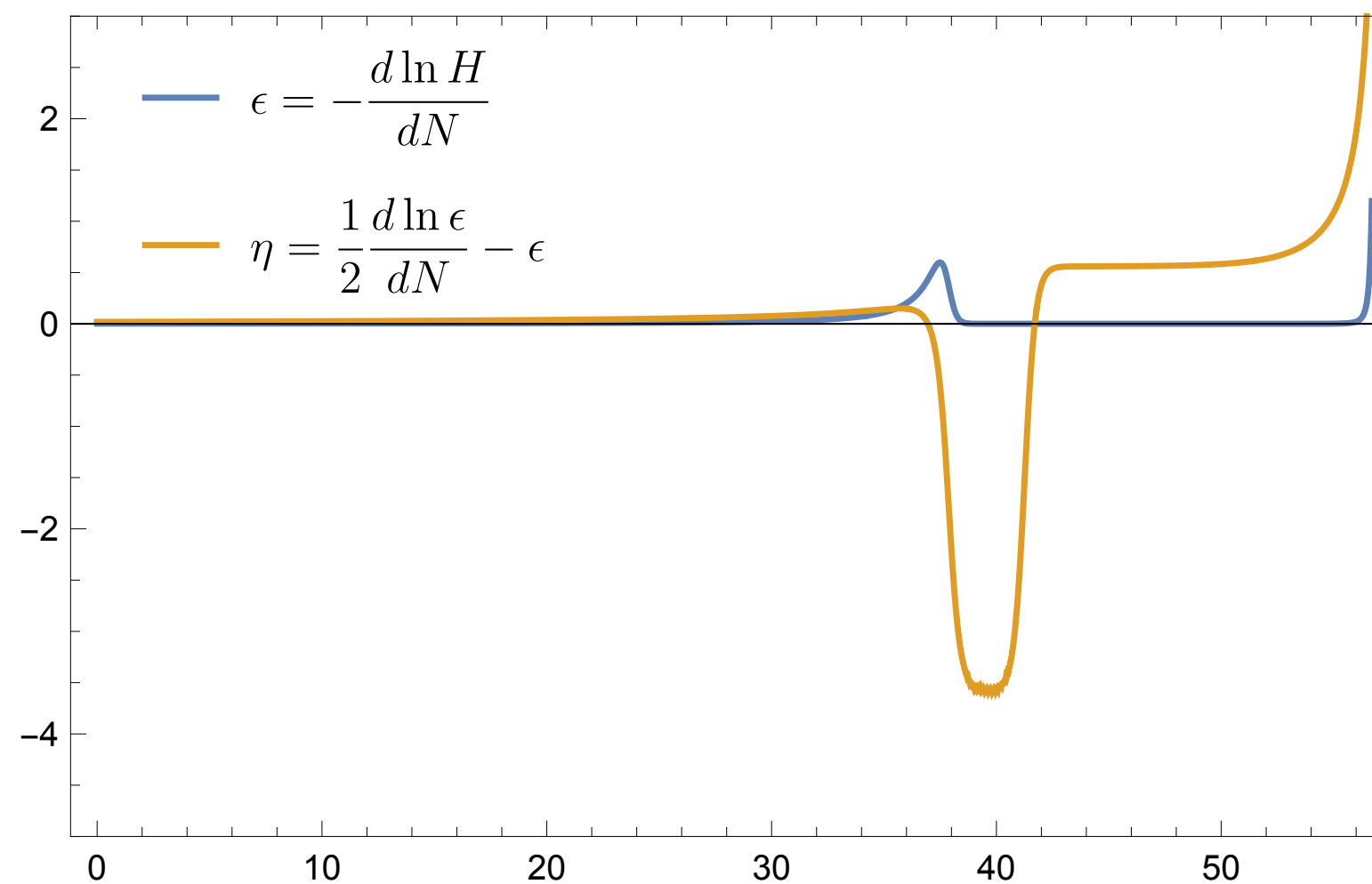
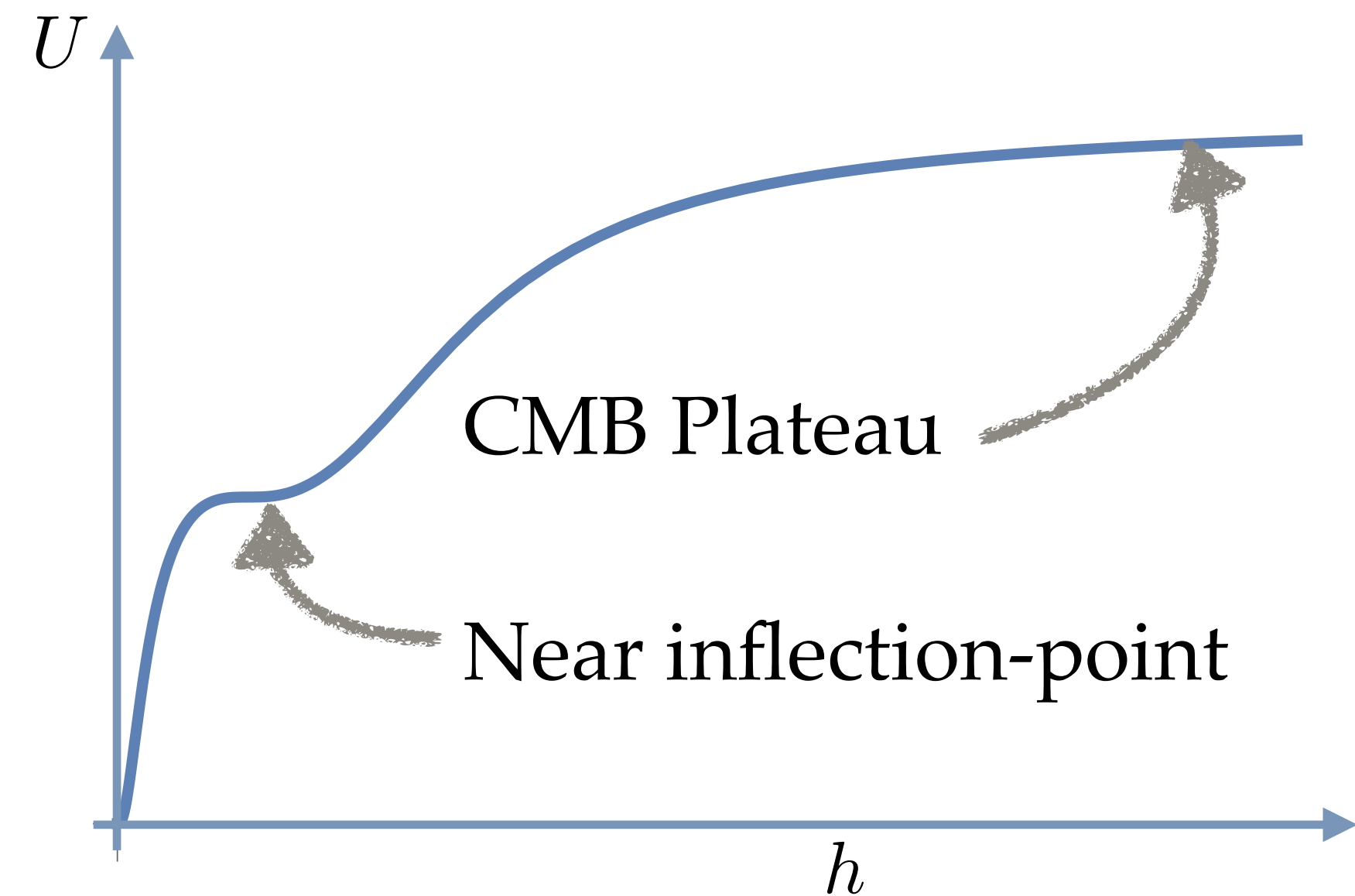
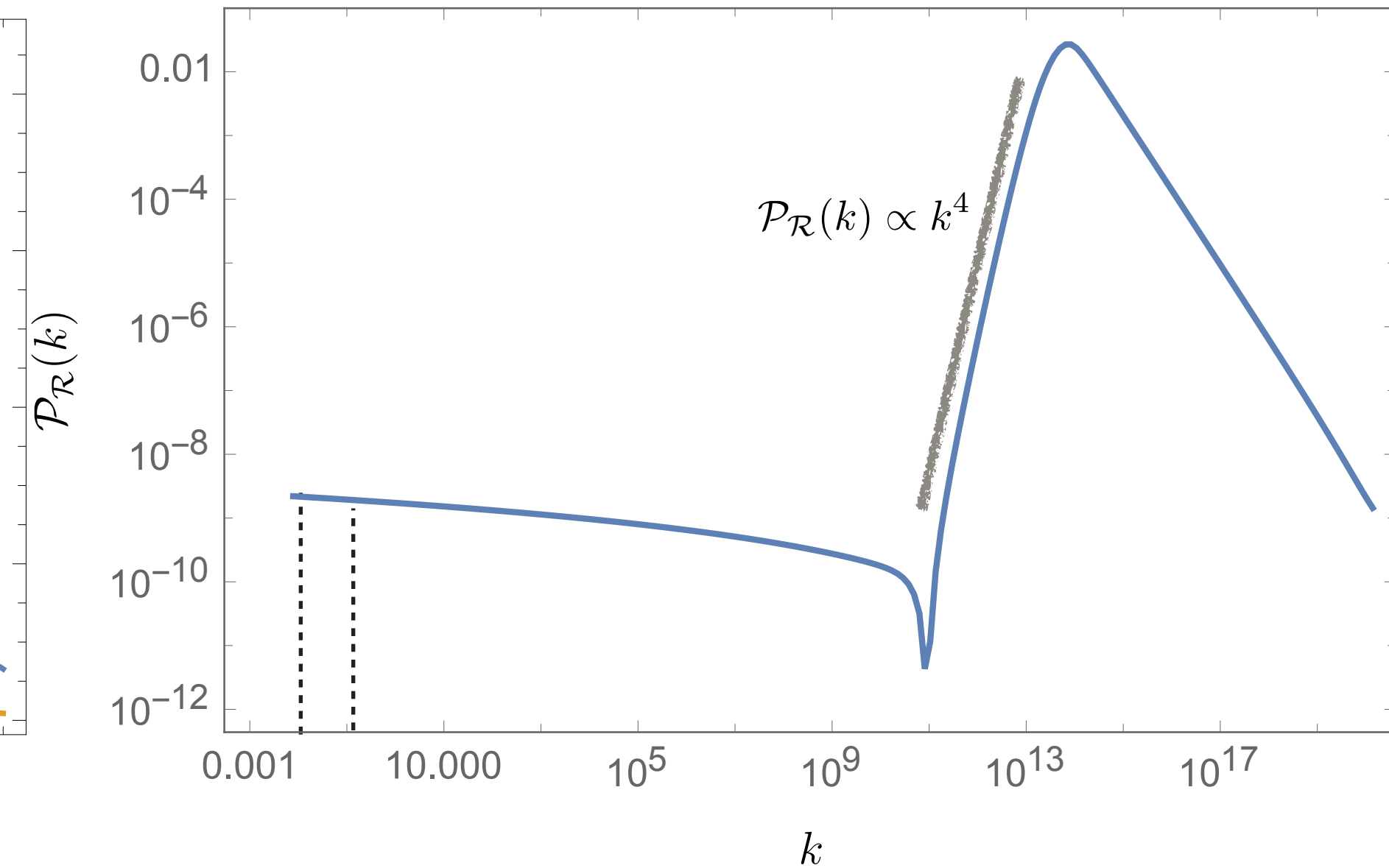
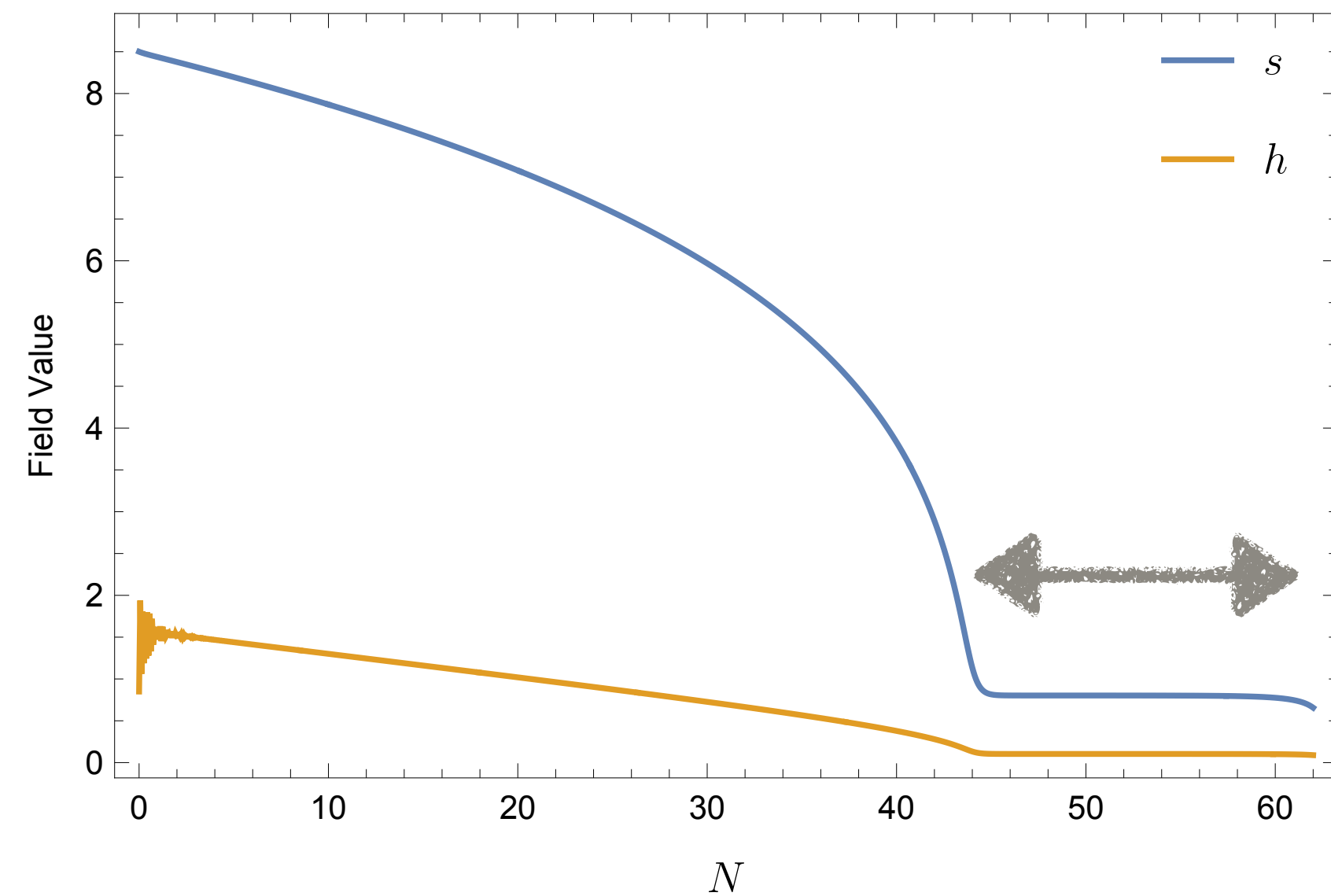
- USR phase induced by $\lambda(h)$. Inflaton *always* rolls down the well defined valley
- $\eta > 3$, Growth in the curvature perturbation
- Growth proportional to $\propto k^4$

Higgs- R^2 Inflation - Ultra Slow-Roll

[DYC, S.M. Lee, S.C. Park, JCAP 01 (2021), 032]

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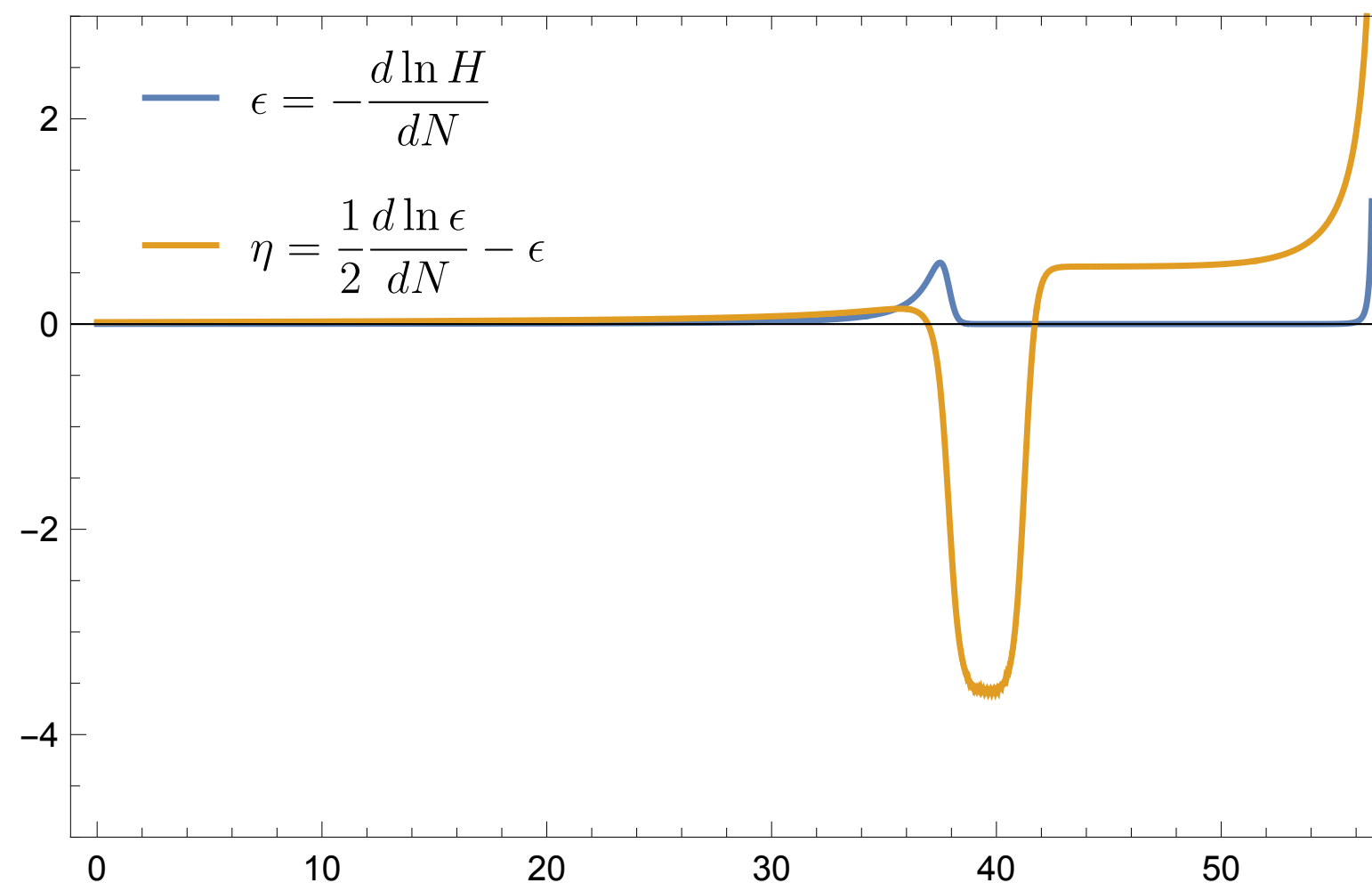
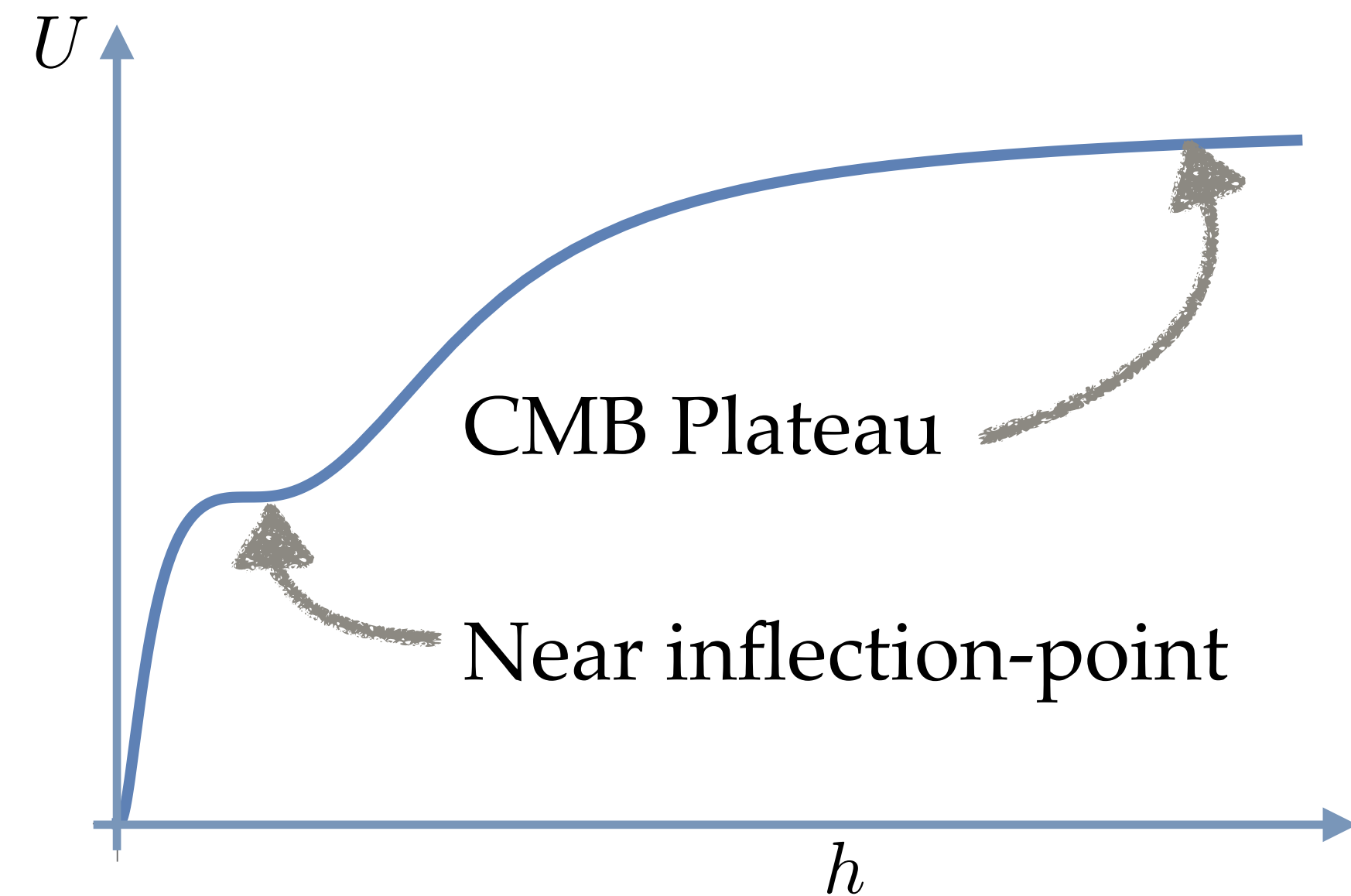
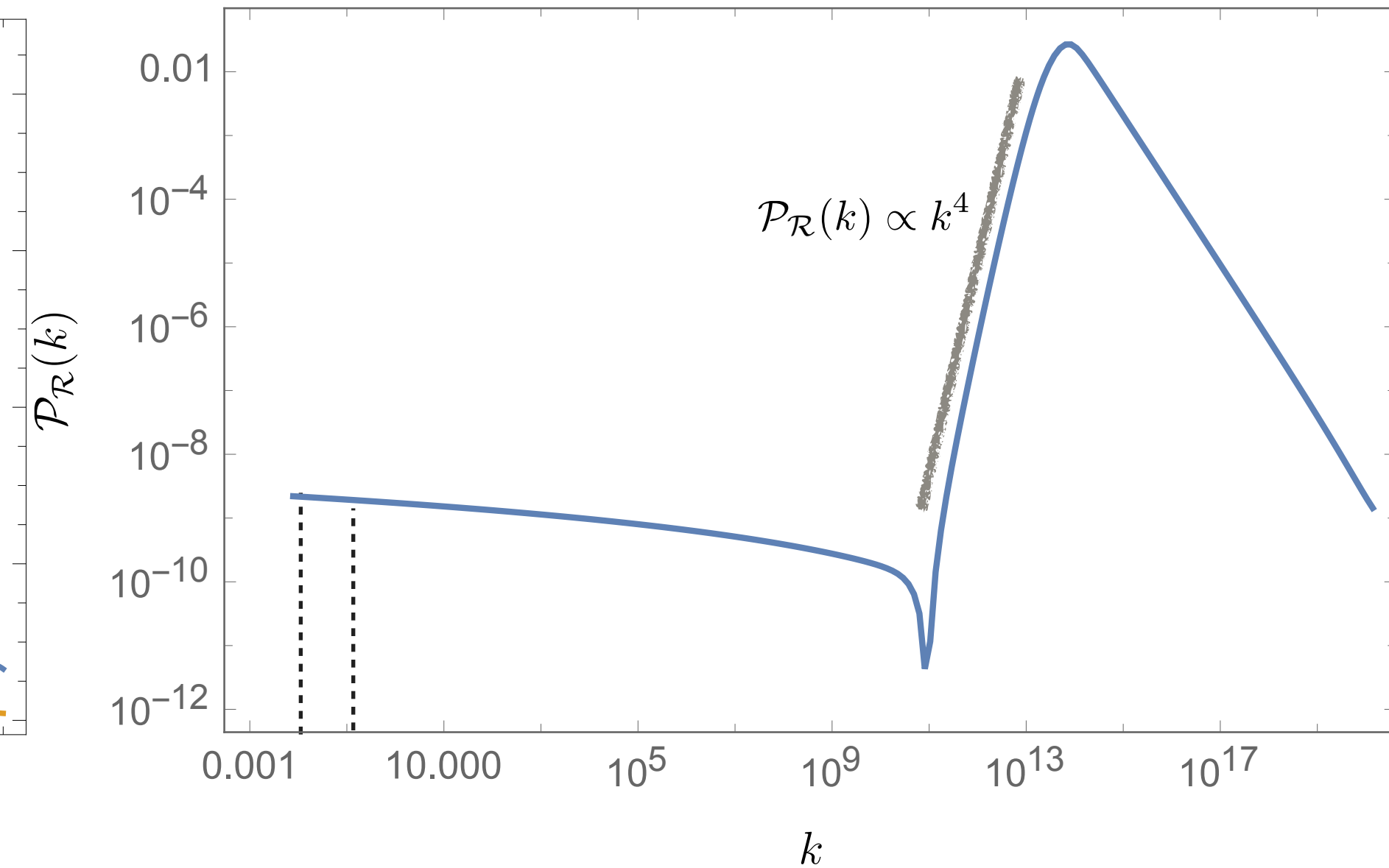
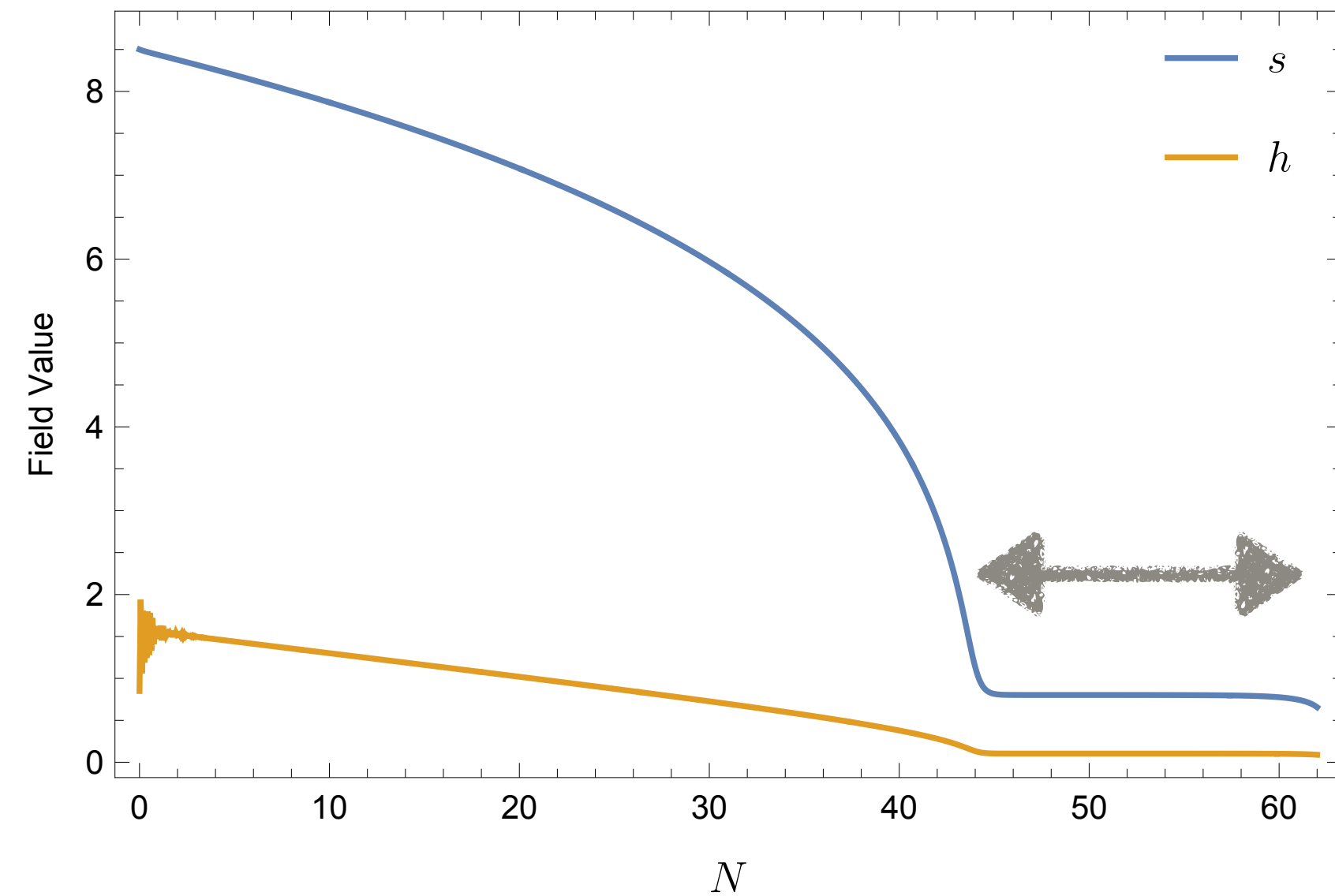
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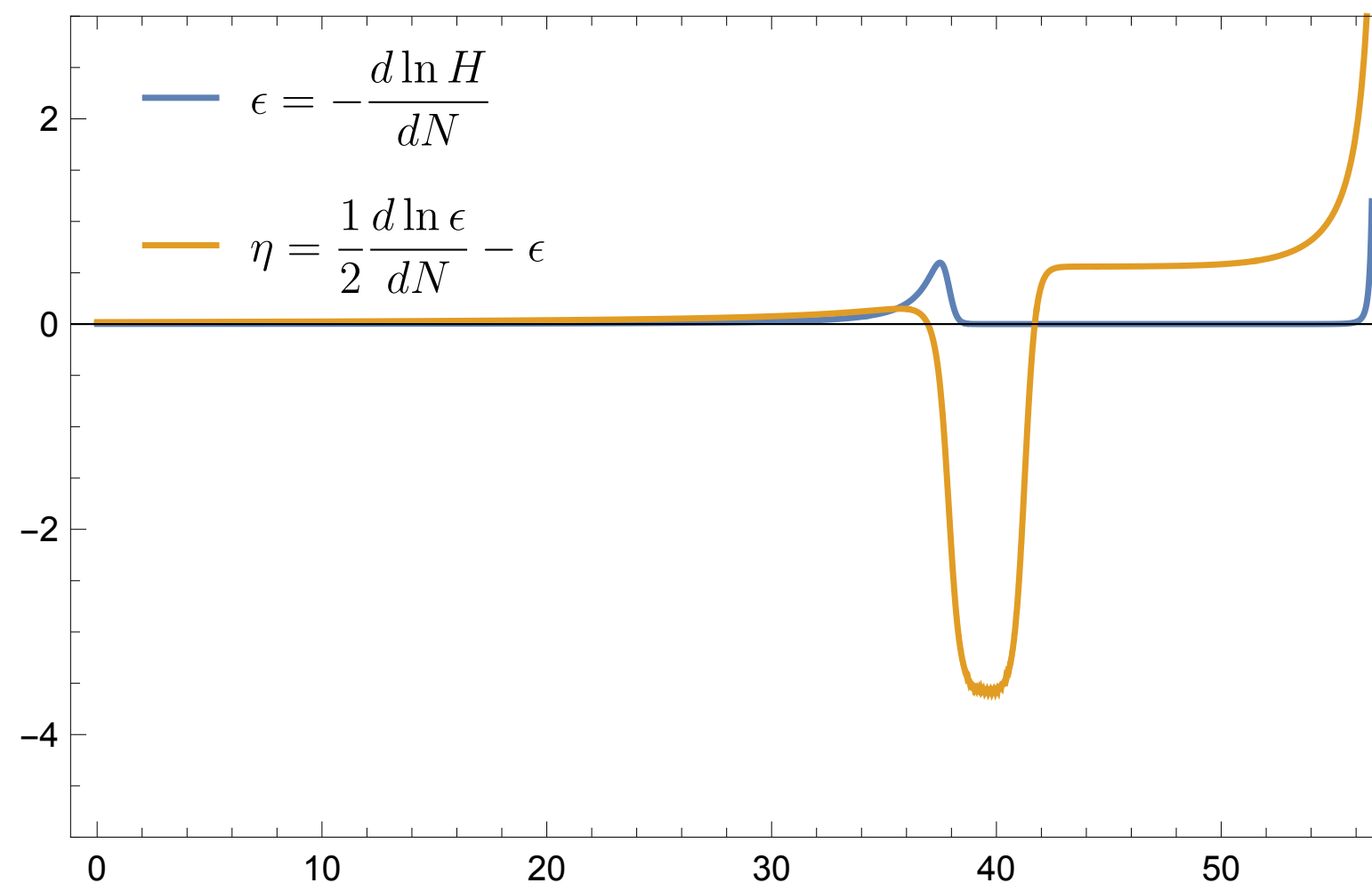
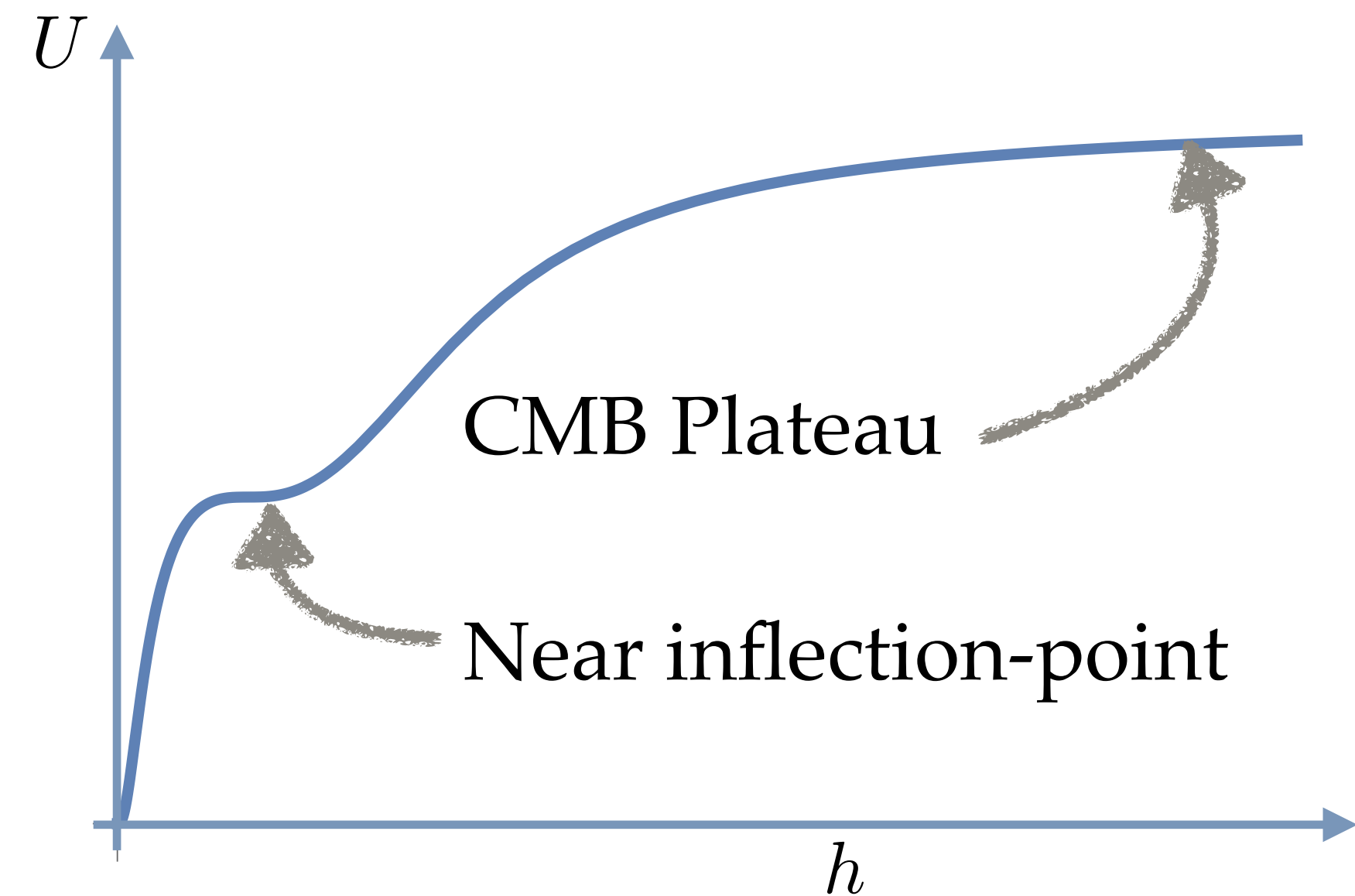
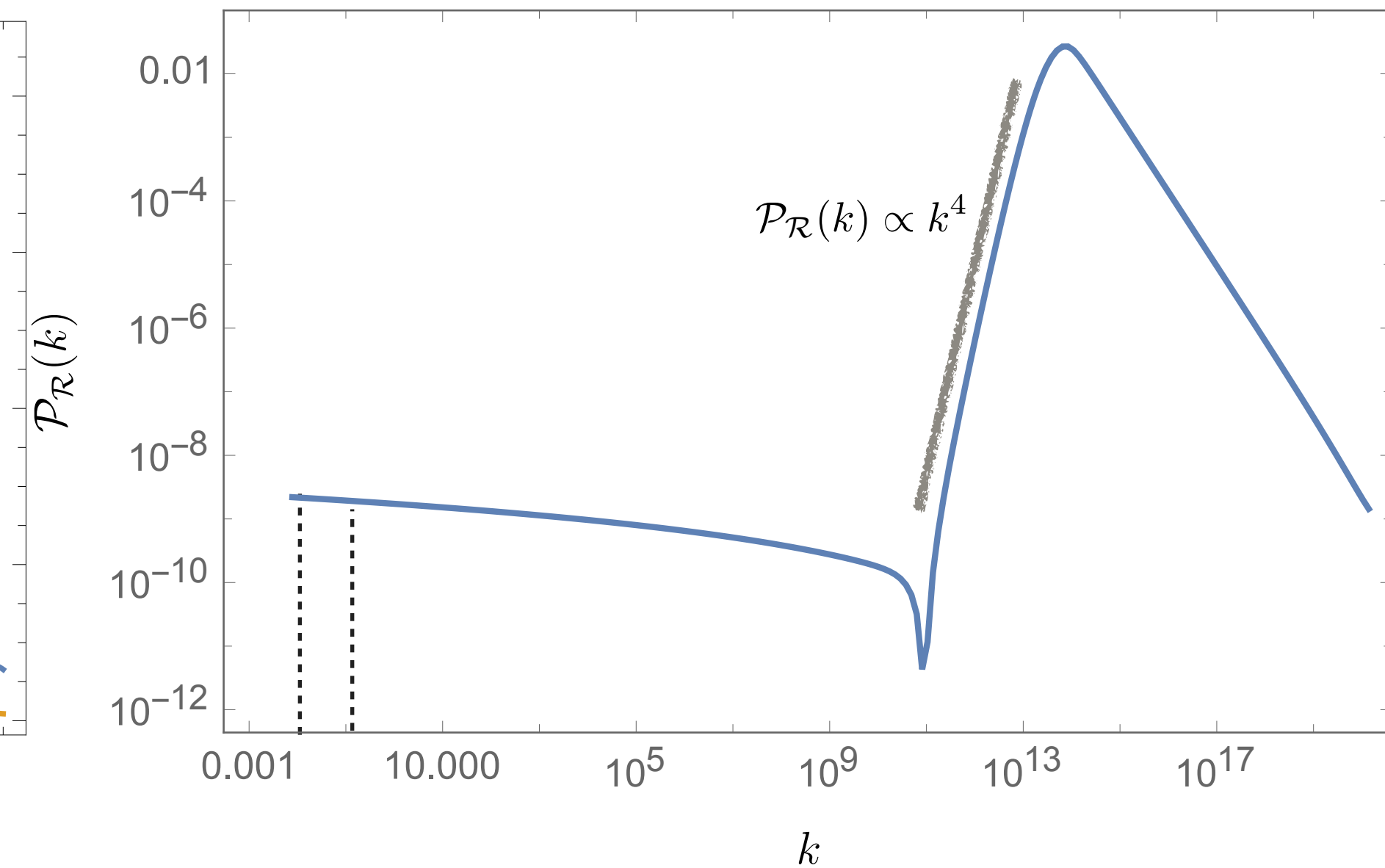
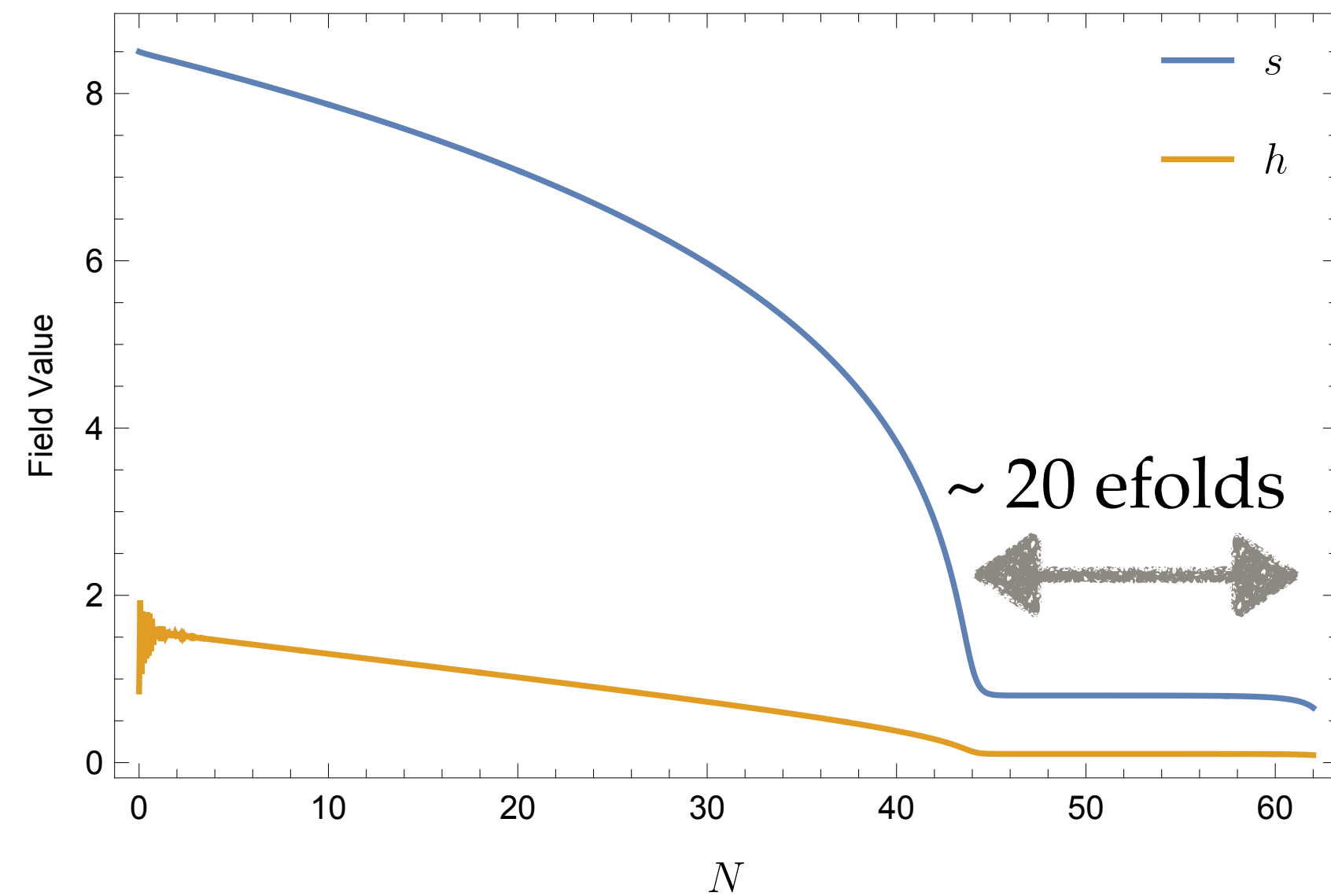
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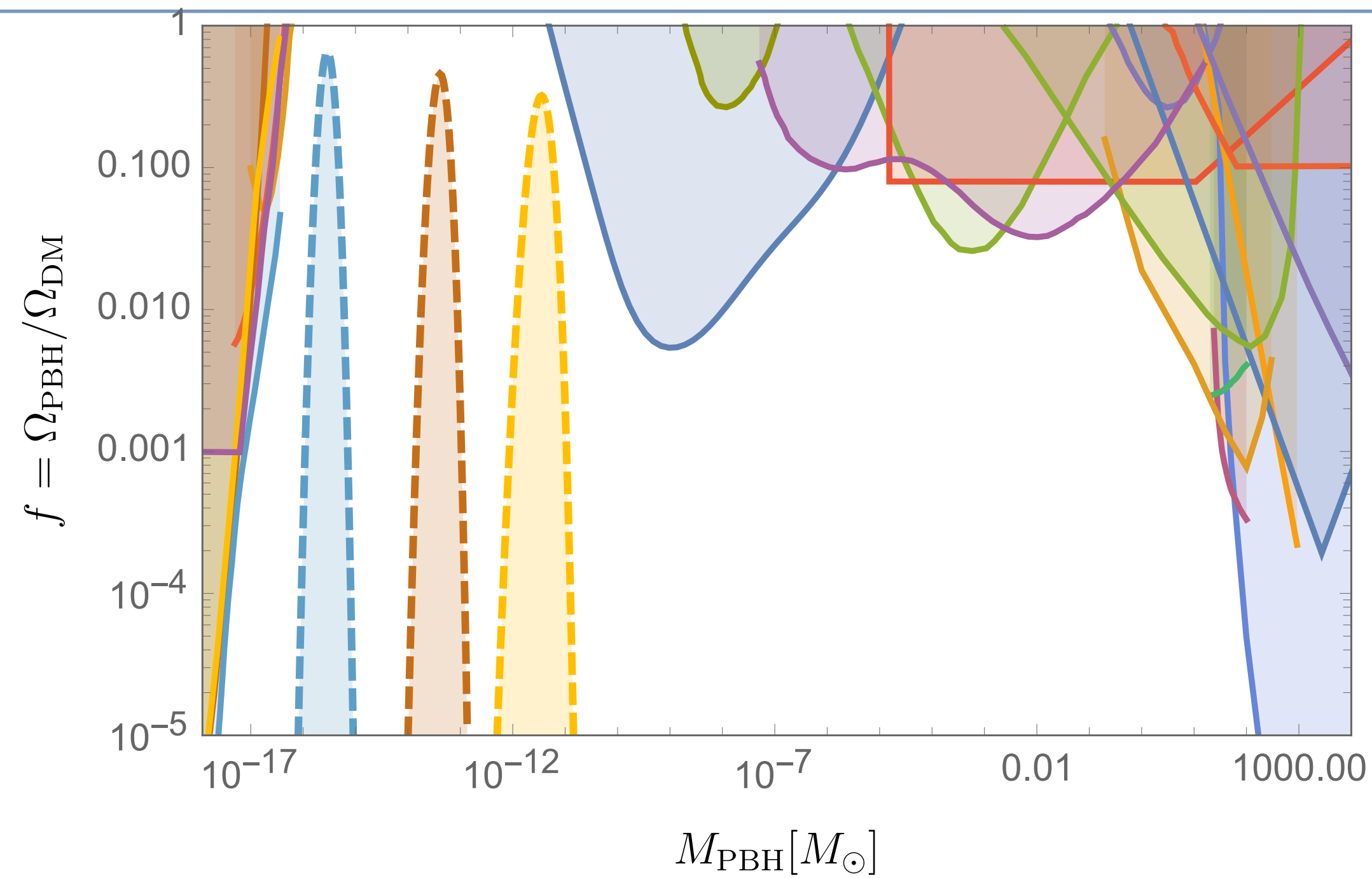
$$\dot{\phi} \simeq 0 \quad \ddot{\phi} \sim \dot{\phi} \simeq 0$$

$$M = 4.2 \times 10^{-5} M_P \quad \xi = 79 \quad h_{\min} = 0.15 M_P \quad b = 2 \times 10^{-5} \quad \lambda_{\min} = 4.11087 \times 10^{-6}$$

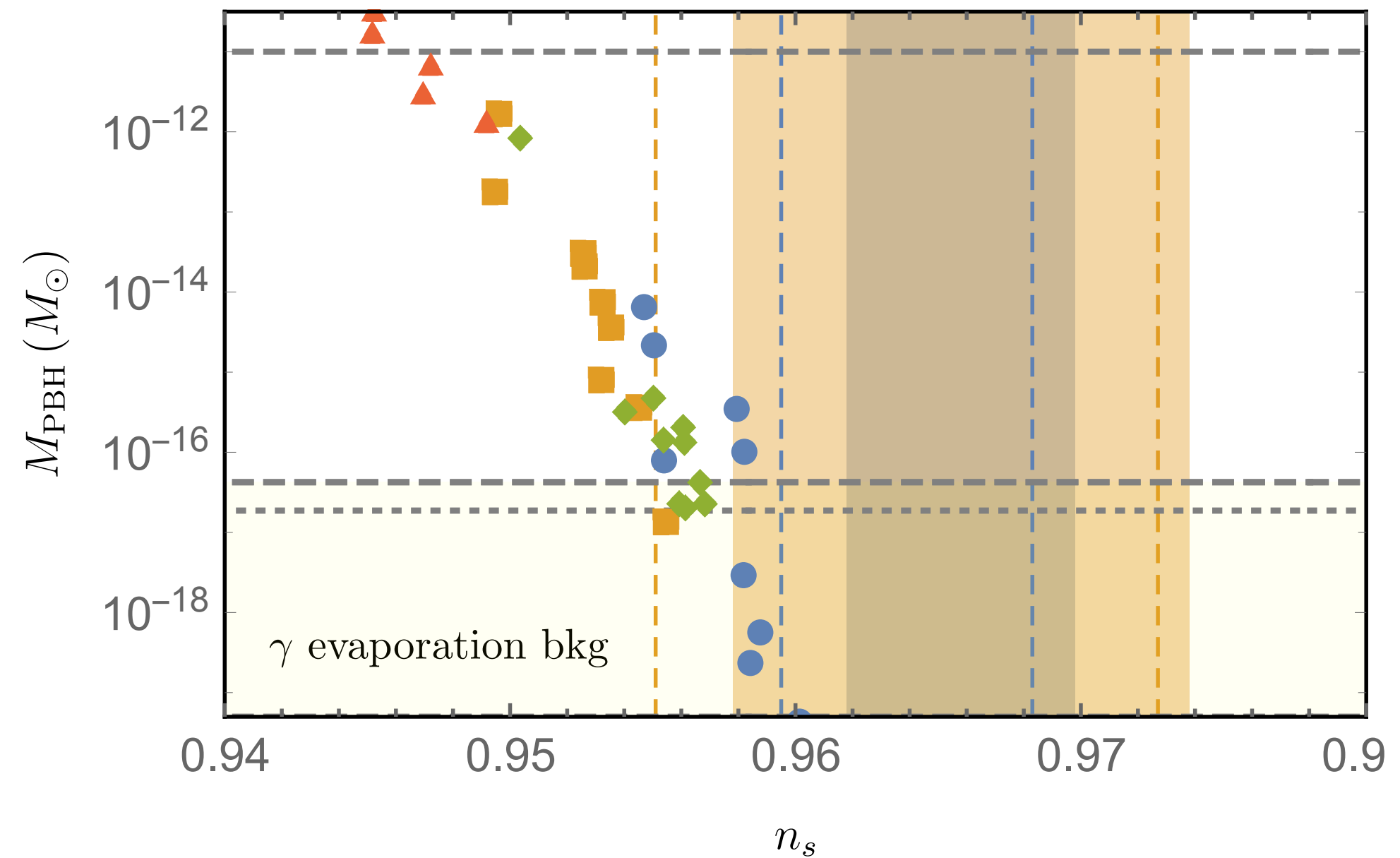


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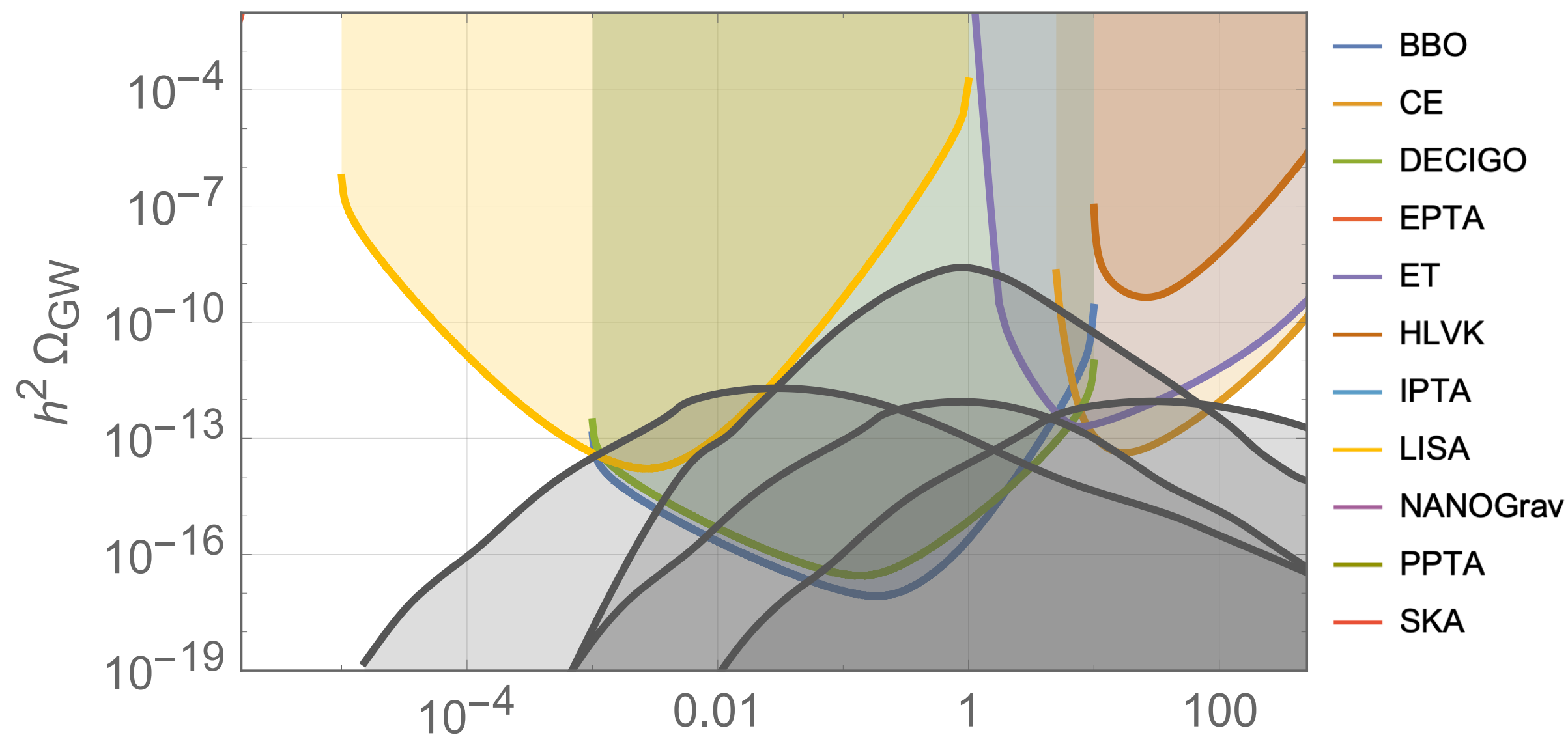
Higgs- R^2 Inflation - Ultra Slow-Roll



● $h_{\min} = 0.15$
◆ $h_{\min} = 0.16$
■ $h_{\min} = 0.17$
▲ $h_{\min} = 0.18$



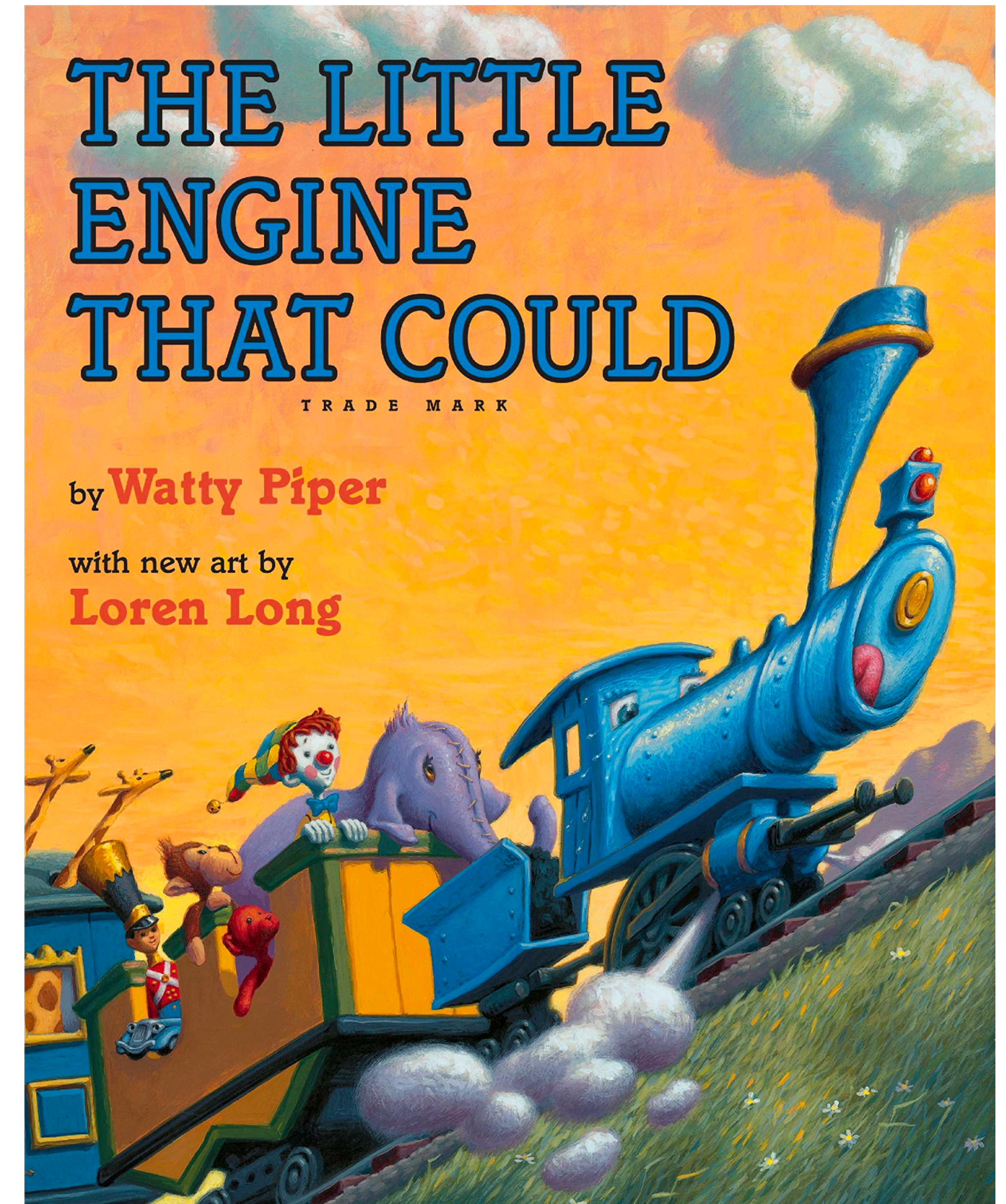
[DYC, S.M. Lee, S.C. Park, JCAP 01 (2021), 032]



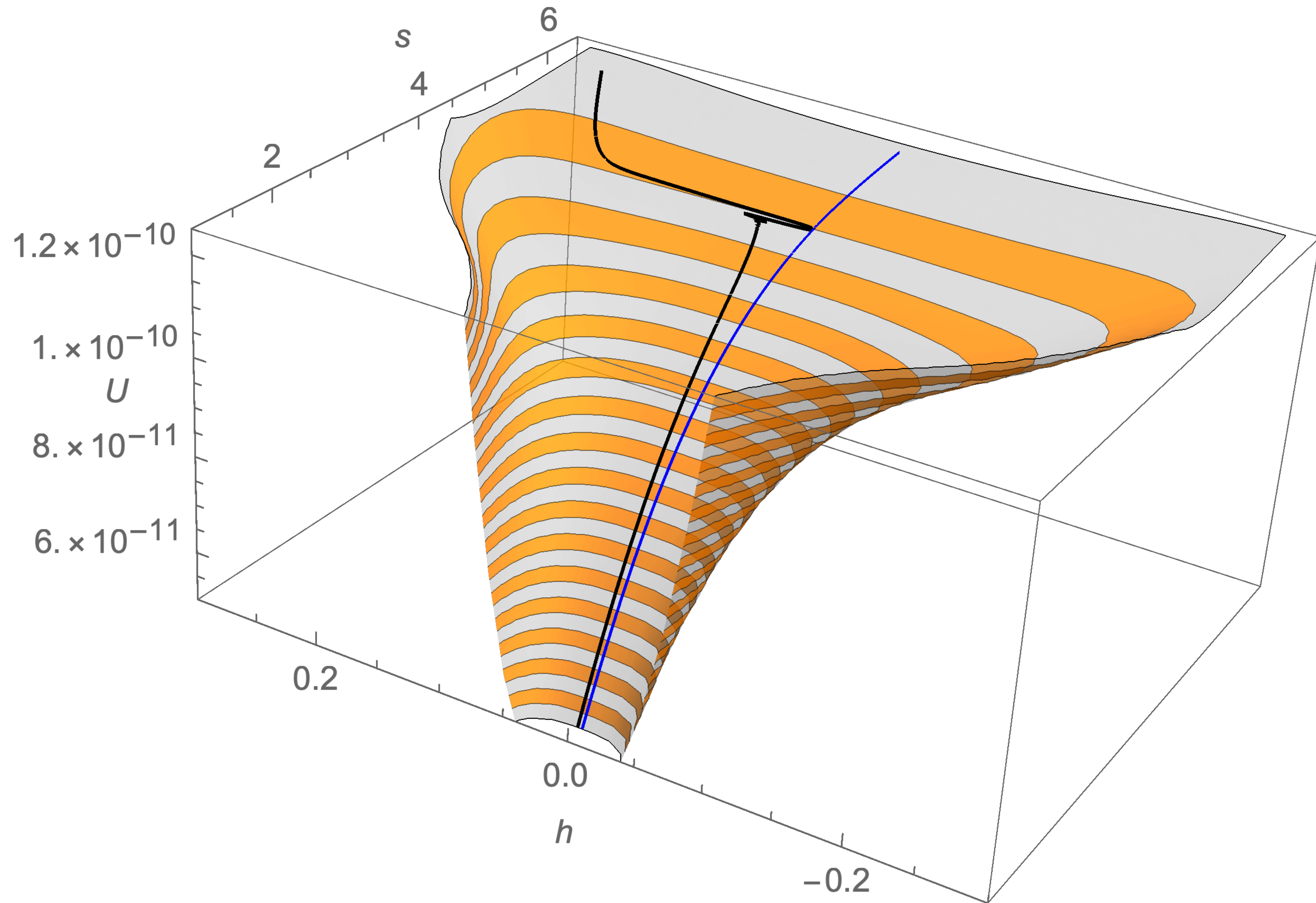
- PBHs compatible for a suitable amount of DM.
 - Slight tension with CMB due to the prolonged USR.
 → can be resolved with additional R^n
- [DYC, H.M. Lee, S.C. Park, Phys. Lett. B 805 (2020) 135453]
- LISA, DECIGO, CE, ET available to probe wider parameter range compared to PBHs.

So, is this the end of the story?

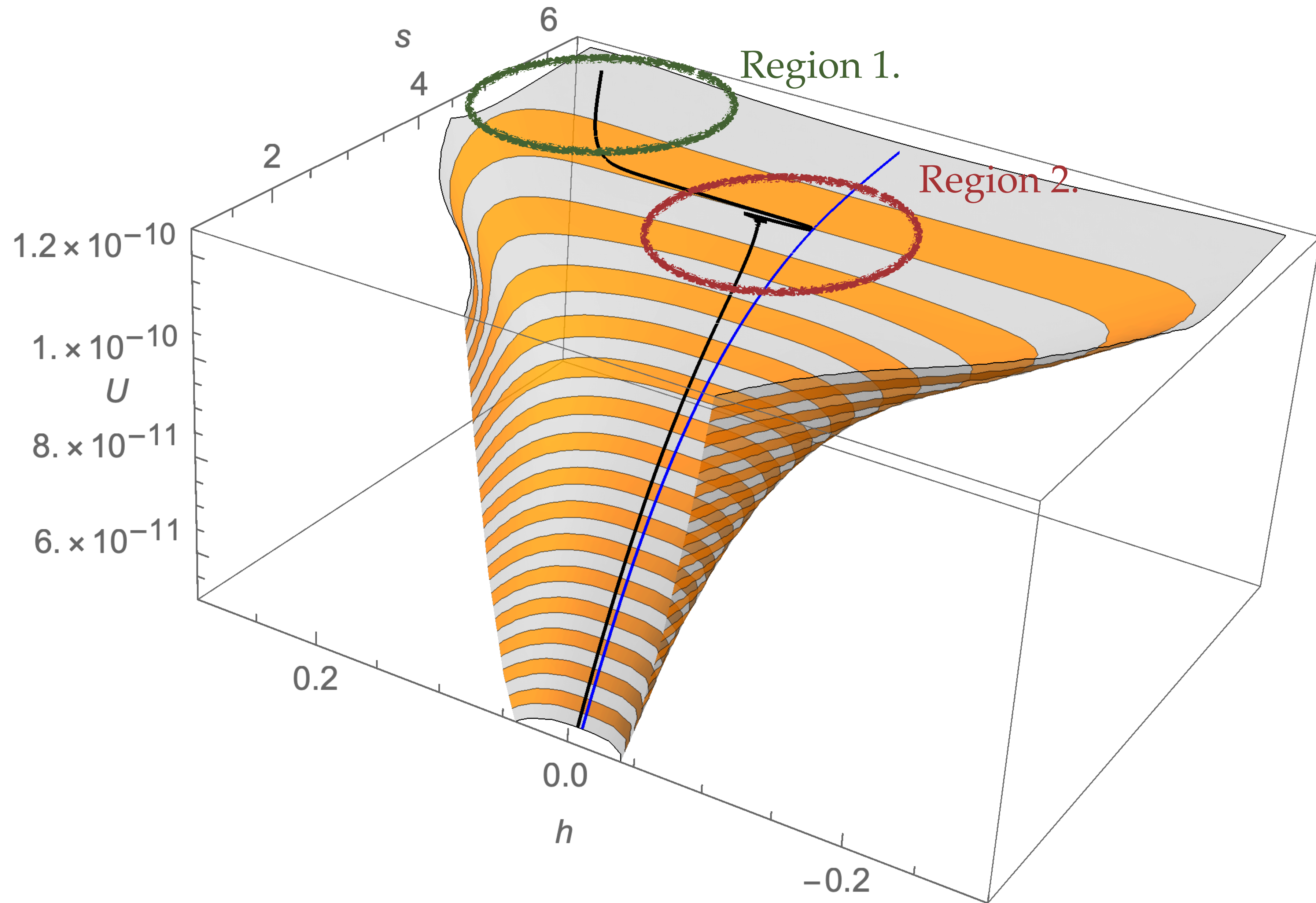
- Apparently, this is not the end of the phenomenology the critical Higgs- R^2 inflation can exhibit.
- Critical Higgs- R^2 inflation can exhibit *turns in the trajectory*.
- Inflaton *rides* towards the hill at $h = 0$, leading to *tachyonic perturbation growth!*



Higgs- R^2 Inflation, Trajectory



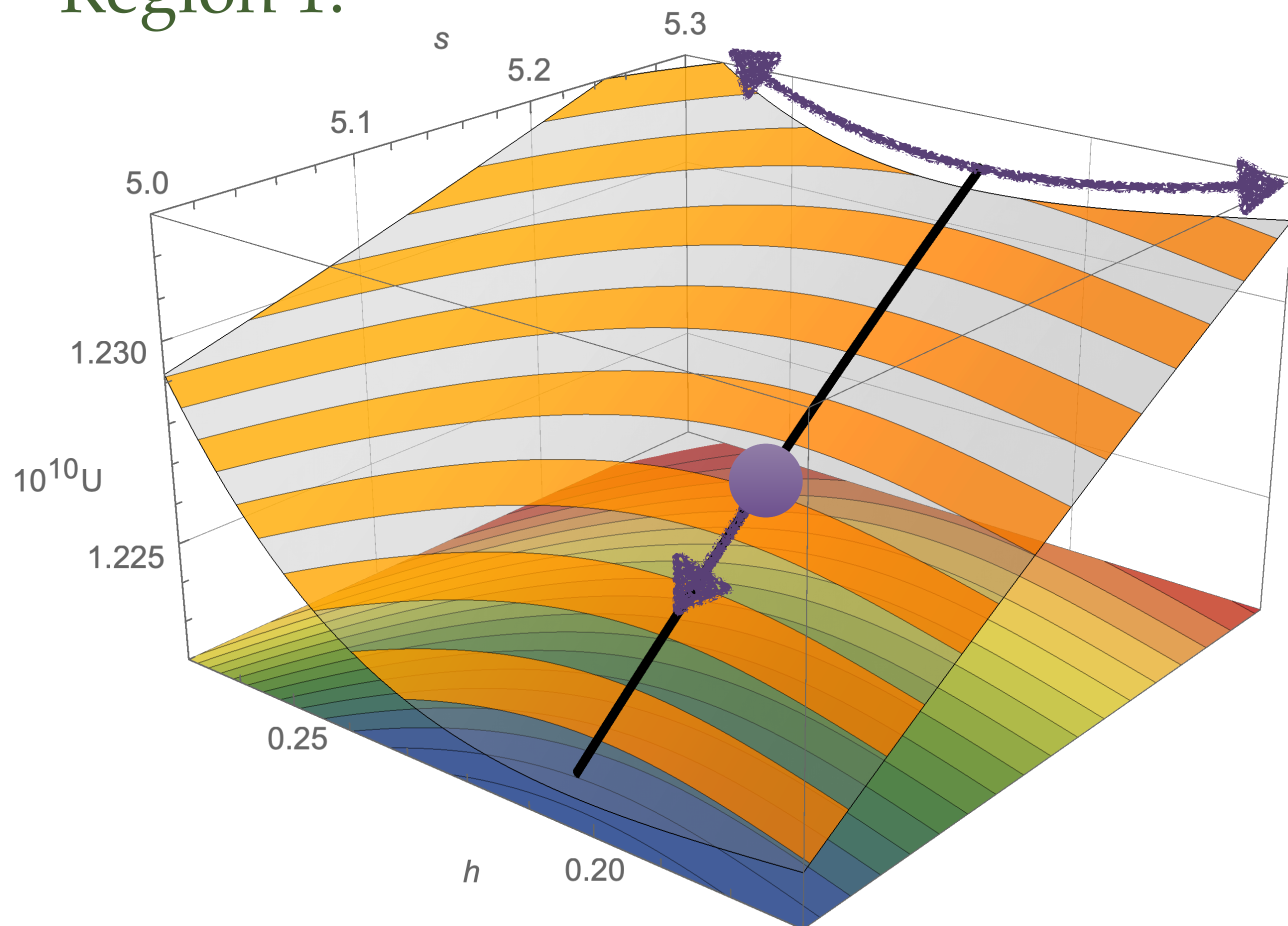
Higgs- R^2 Inflation, Trajectory



Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

Region 1.



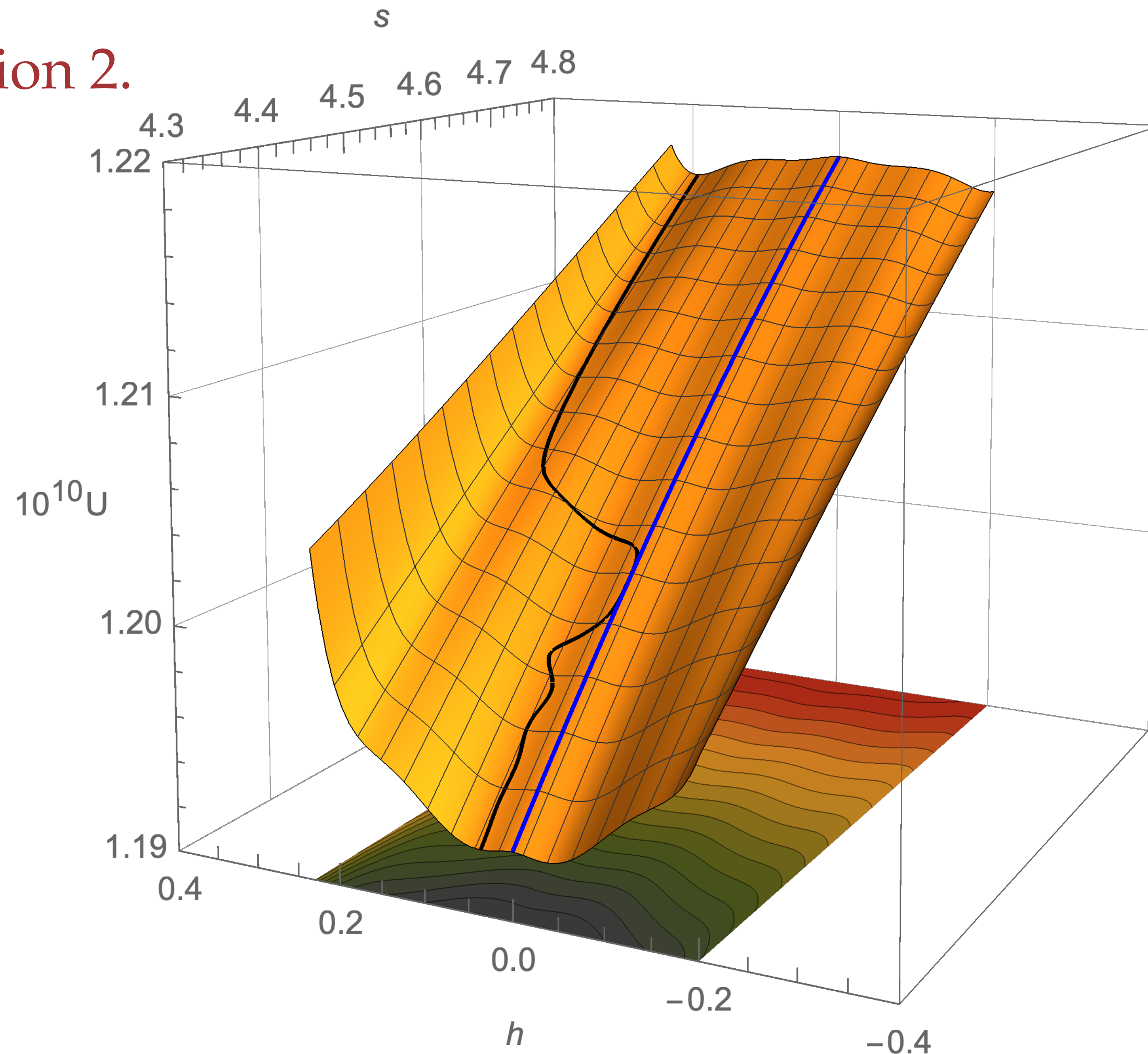
- “Valley” structure persists, exhibits an “attractor” behavior.
- *Isocurvature perturbations suppressed*
- CMB observables resemble the predictions of an “effective single-field” setup.
- “Slightly larger” n_s, r compared to constant λ .

$$n_s \approx 1 - \frac{2}{N_{\text{inf}}} - \frac{9}{2N_{\text{inf}}^2} + \frac{2M^2\xi^2 b}{\lambda_m(\lambda_m + 3M^2\xi^2)} + \dots \quad r \approx \frac{12}{N_{\text{inf}}^2} + \frac{24M^2\xi^2 b}{\lambda_m(\lambda_m + 3M^2\xi^2)N_{\text{inf}}} \ln \left(\frac{4M^2\xi N_{\text{inf}}}{(\lambda_m + 3M^2\xi^2)h_m^2} \right) + \dots$$

Higgs- R^2 Inflation, Trajectory

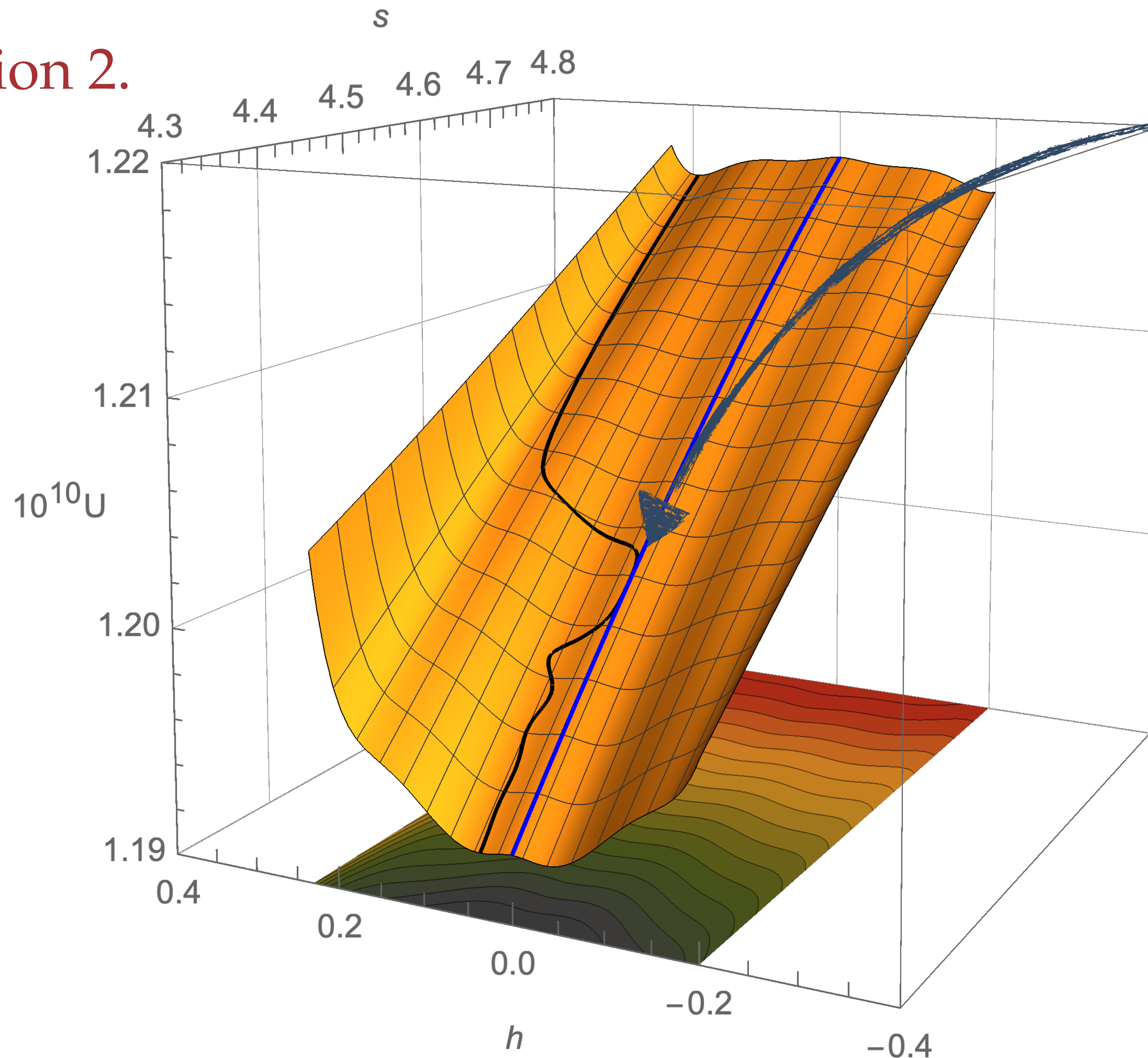
Region 2.

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]



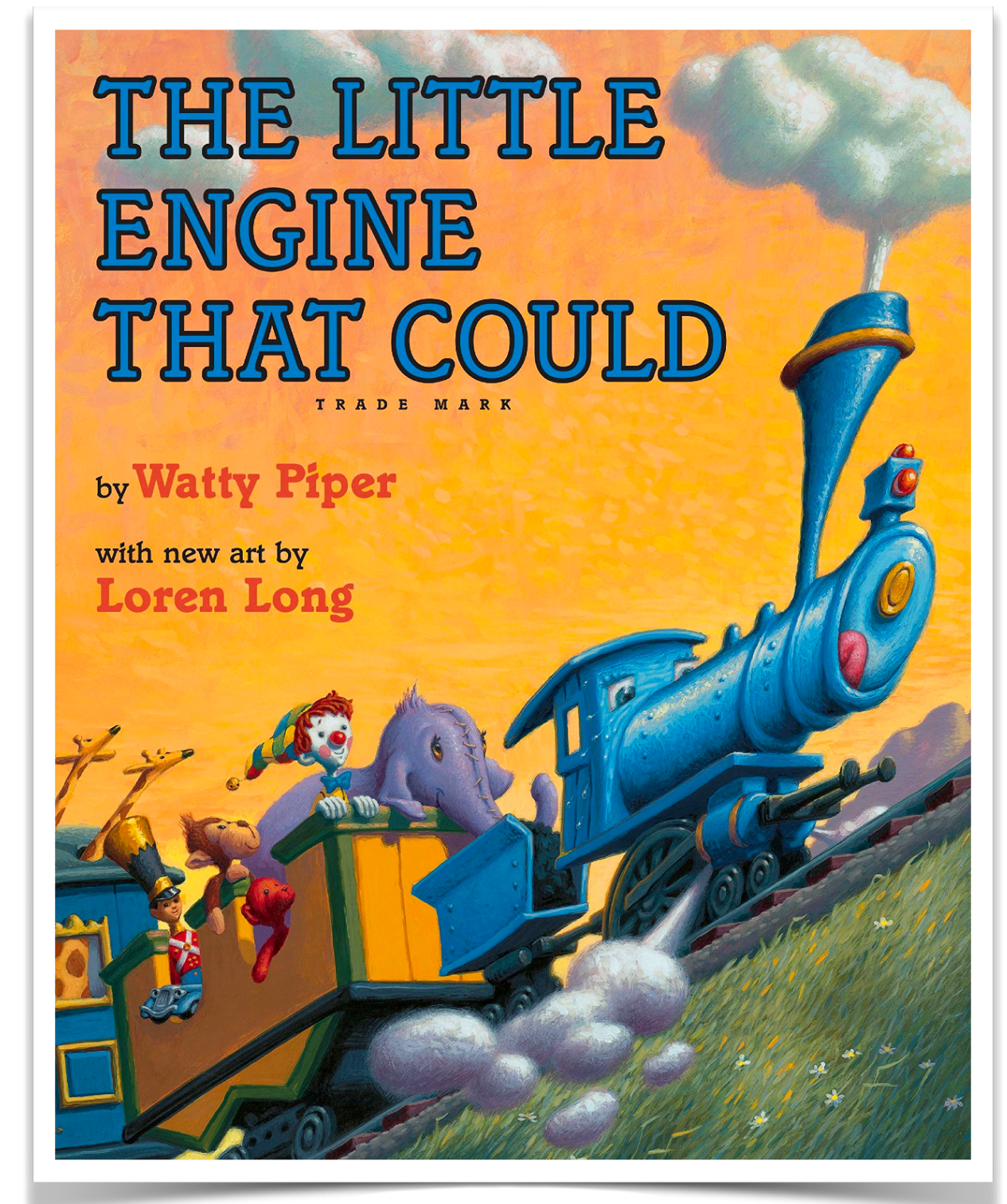
Higgs- R^2 Inflation, Trajectory

Region 2.



[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

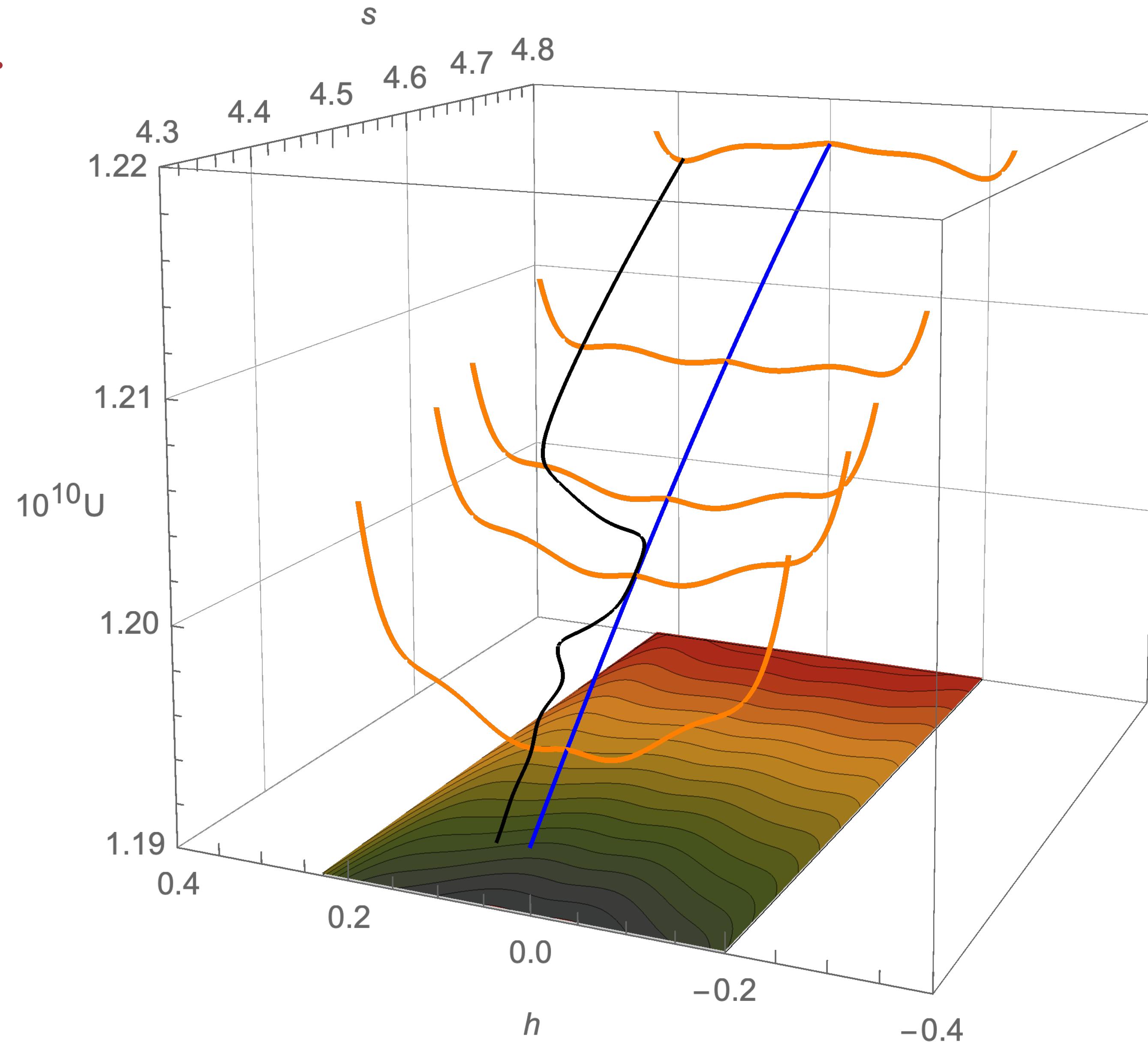
Inflaton “approaching” the $h = 0$ hill.



Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

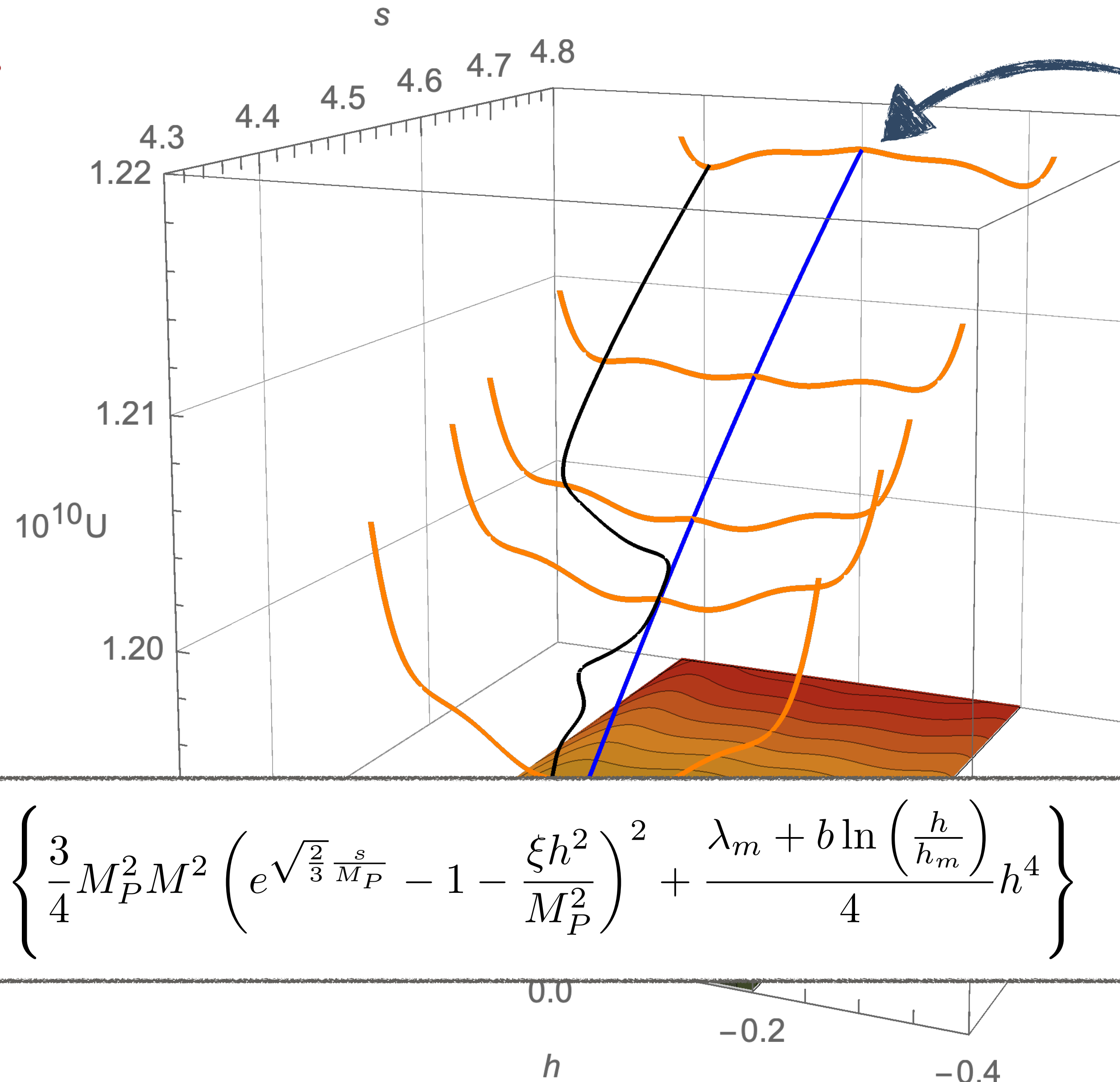
Region 2.



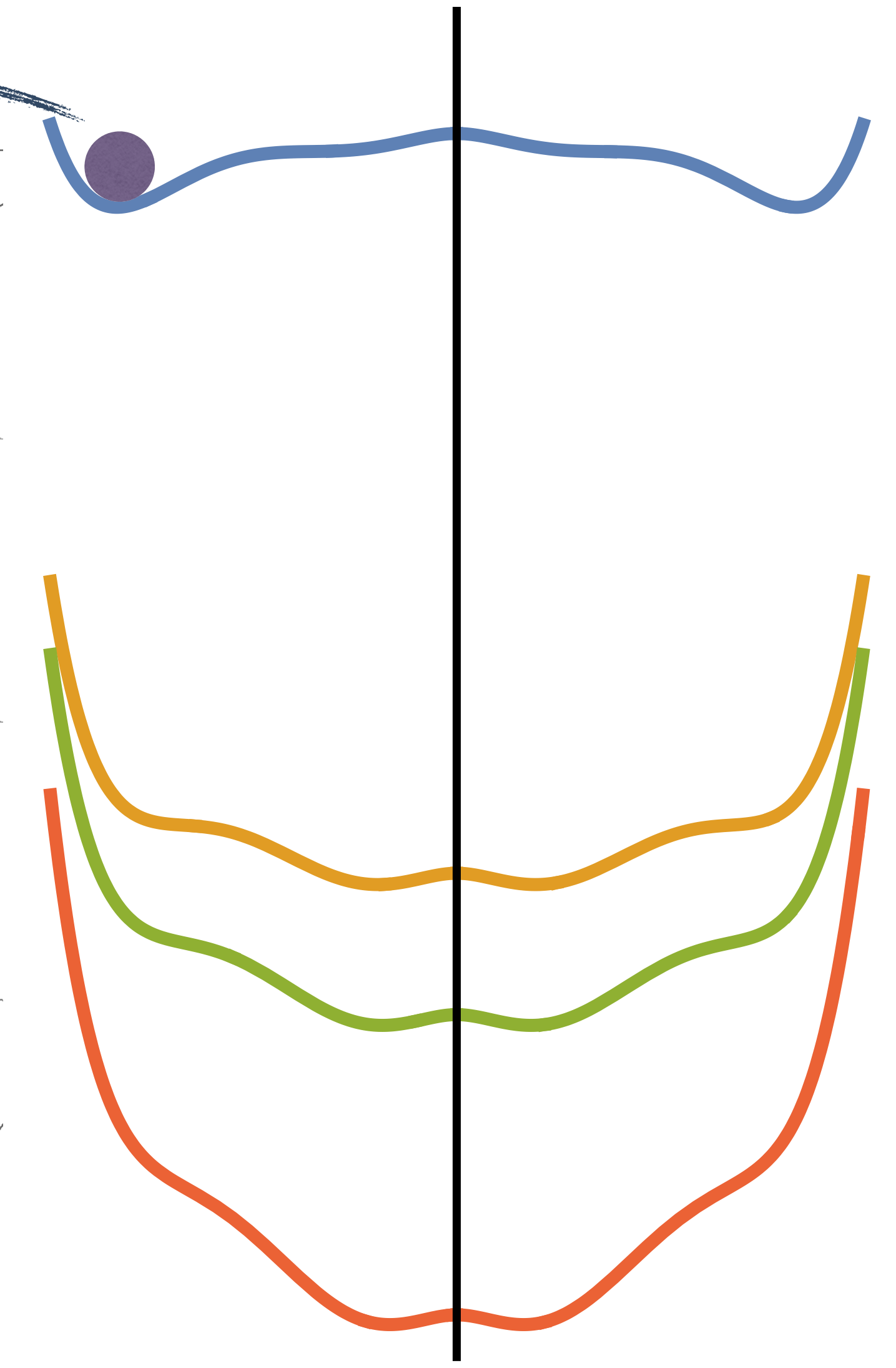
Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

Region 2.



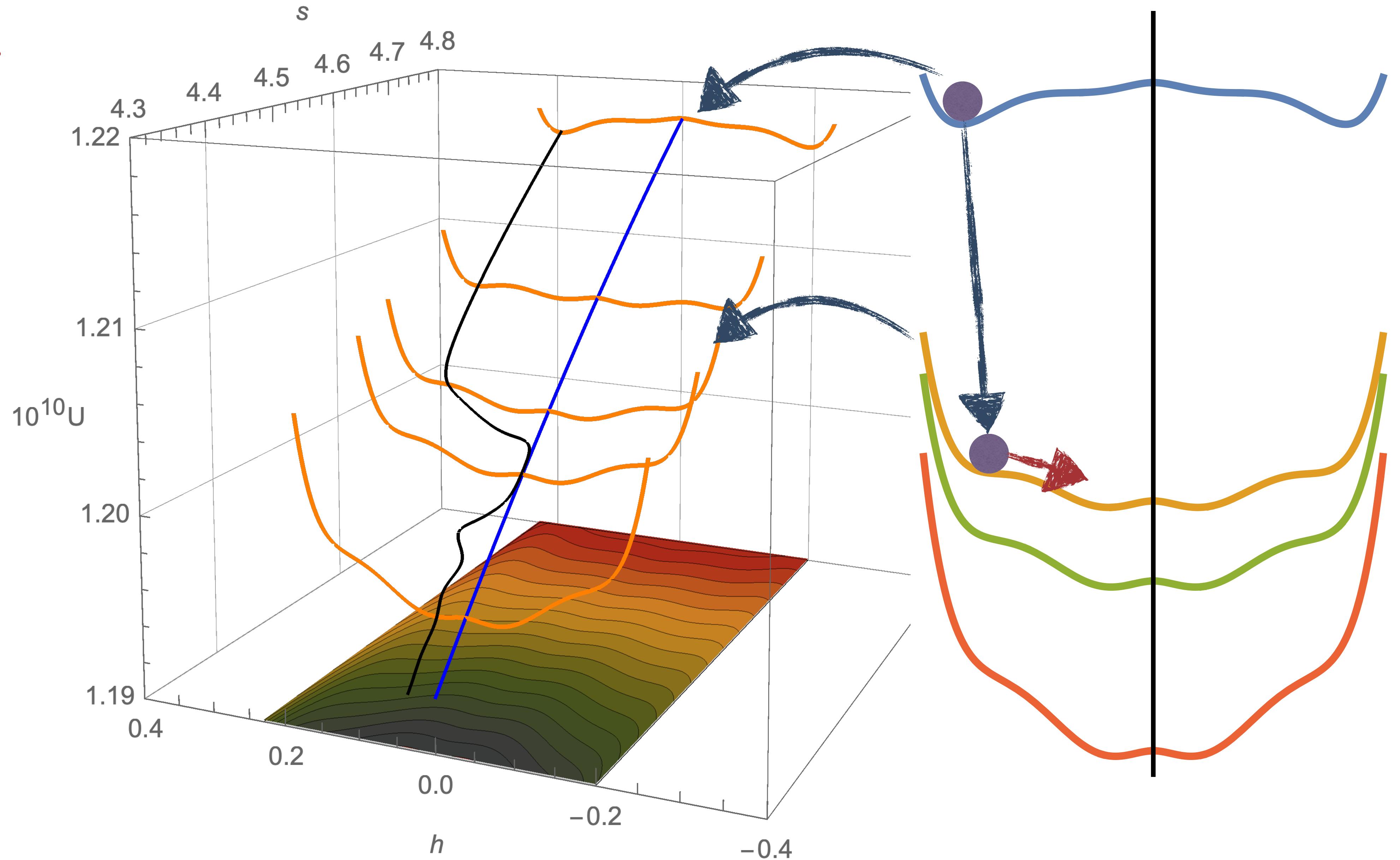
$$U = e^{-2\sqrt{\frac{2}{3}}\frac{s}{M_P}} \left\{ \frac{3}{4}M_P^2 M^2 \left(e^{\sqrt{\frac{2}{3}}\frac{s}{M_P}} - 1 - \frac{\xi h^2}{M_P^2} \right)^2 + \frac{\lambda_m + b \ln\left(\frac{h}{h_m}\right)}{4} h^4 \right\}$$



Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

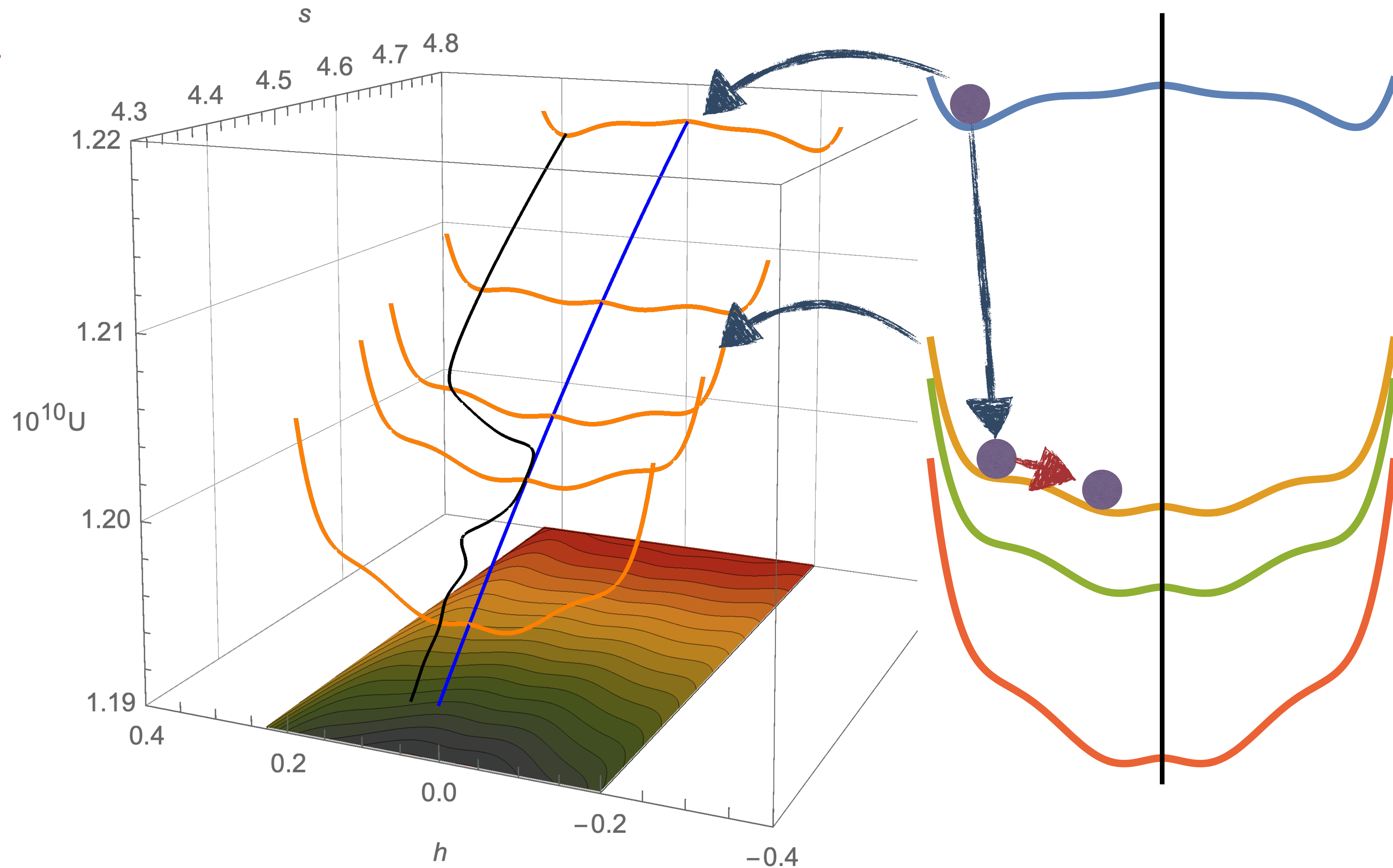
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Higgs- R^2 Inflation, Trajectory

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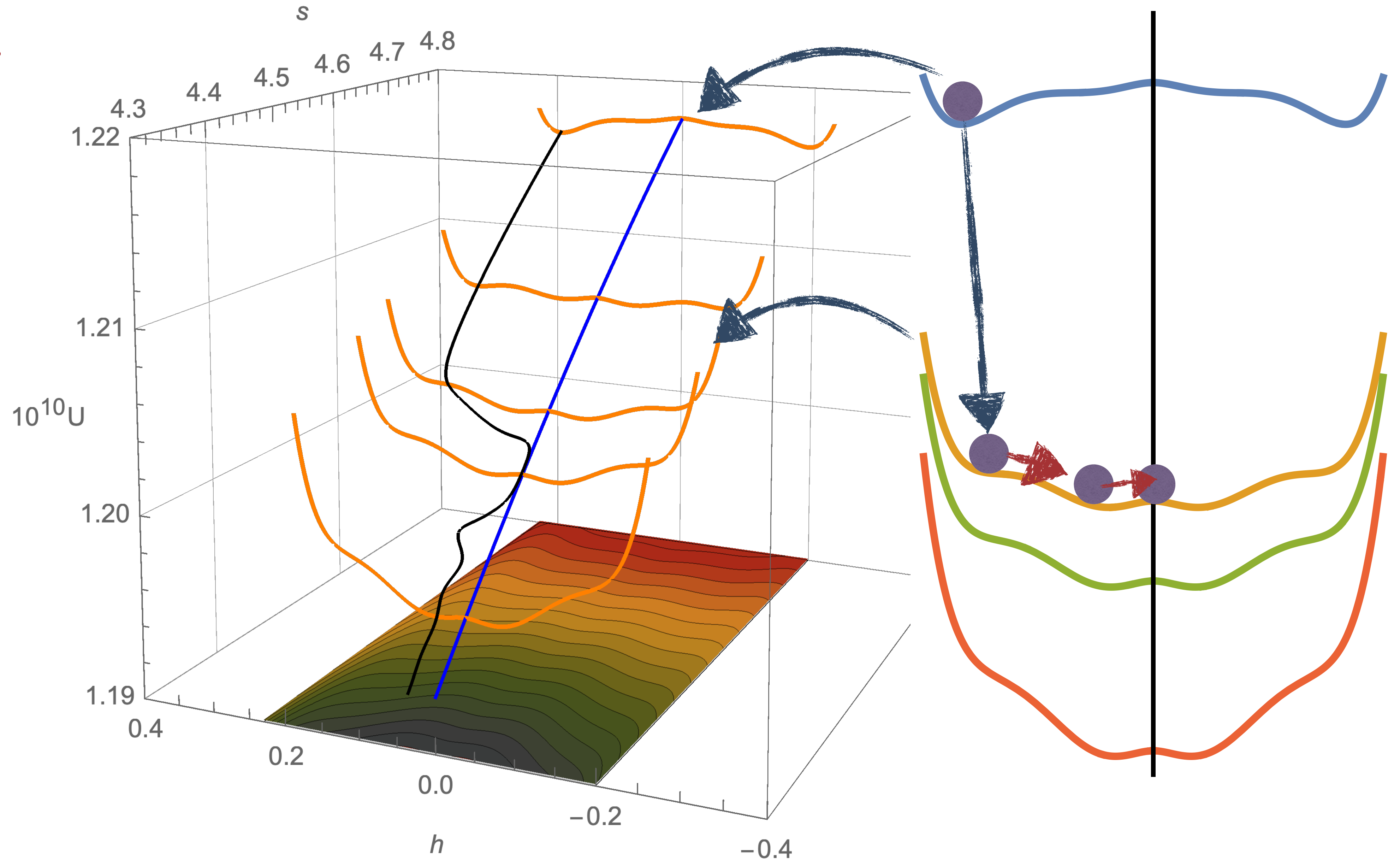
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Higgs- R^2 Inflation, Trajectory

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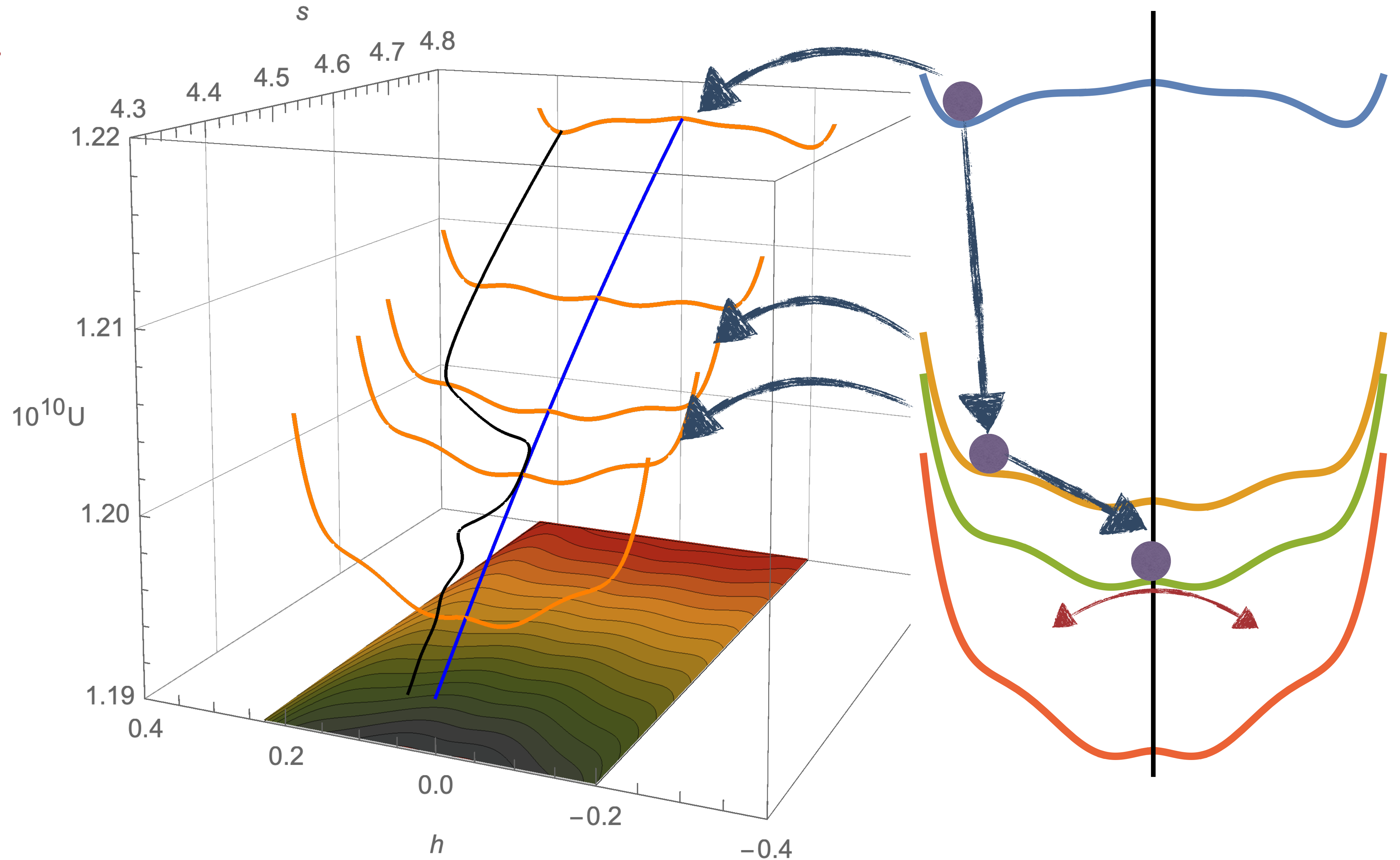
Region 2.



Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

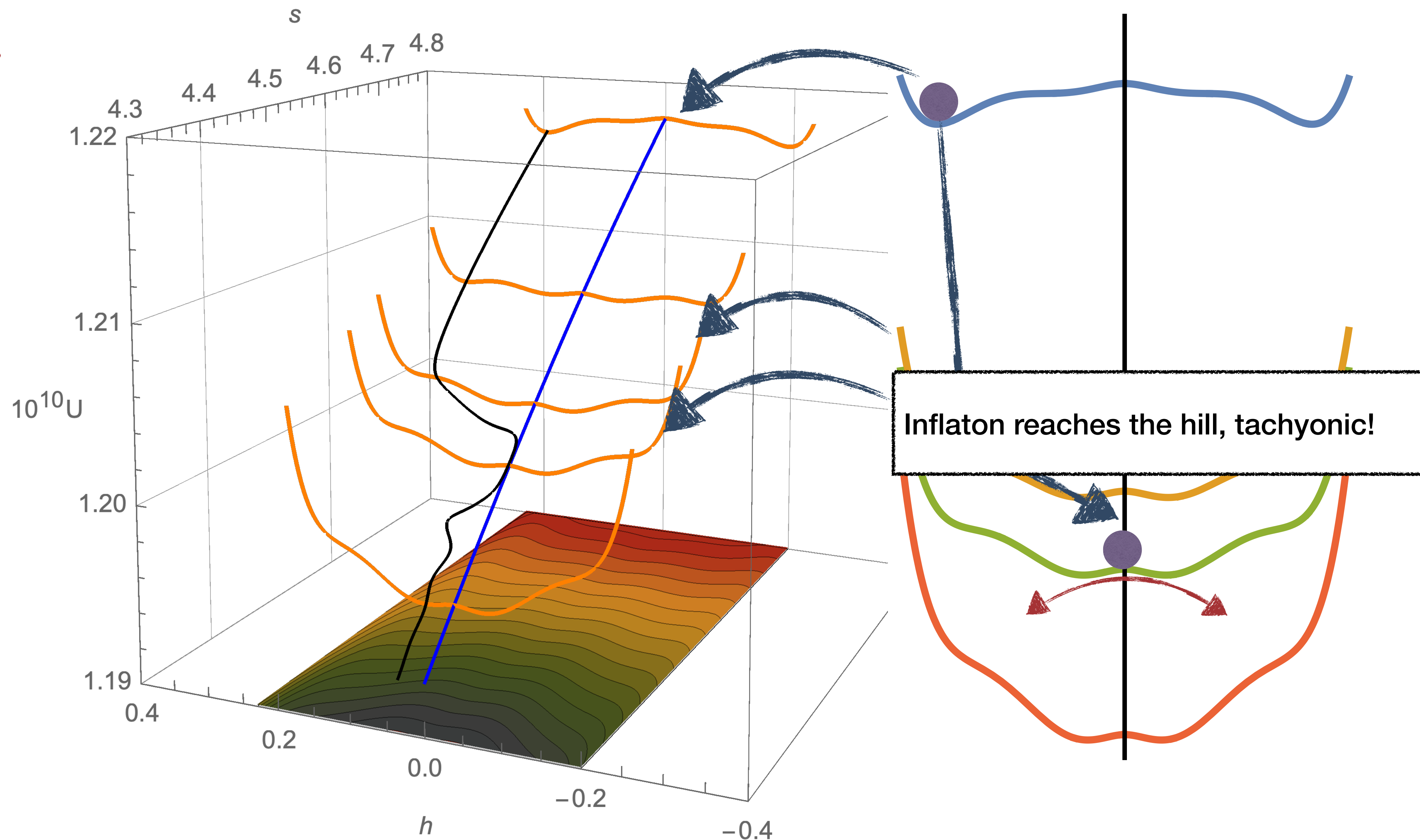
Region 2.



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[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

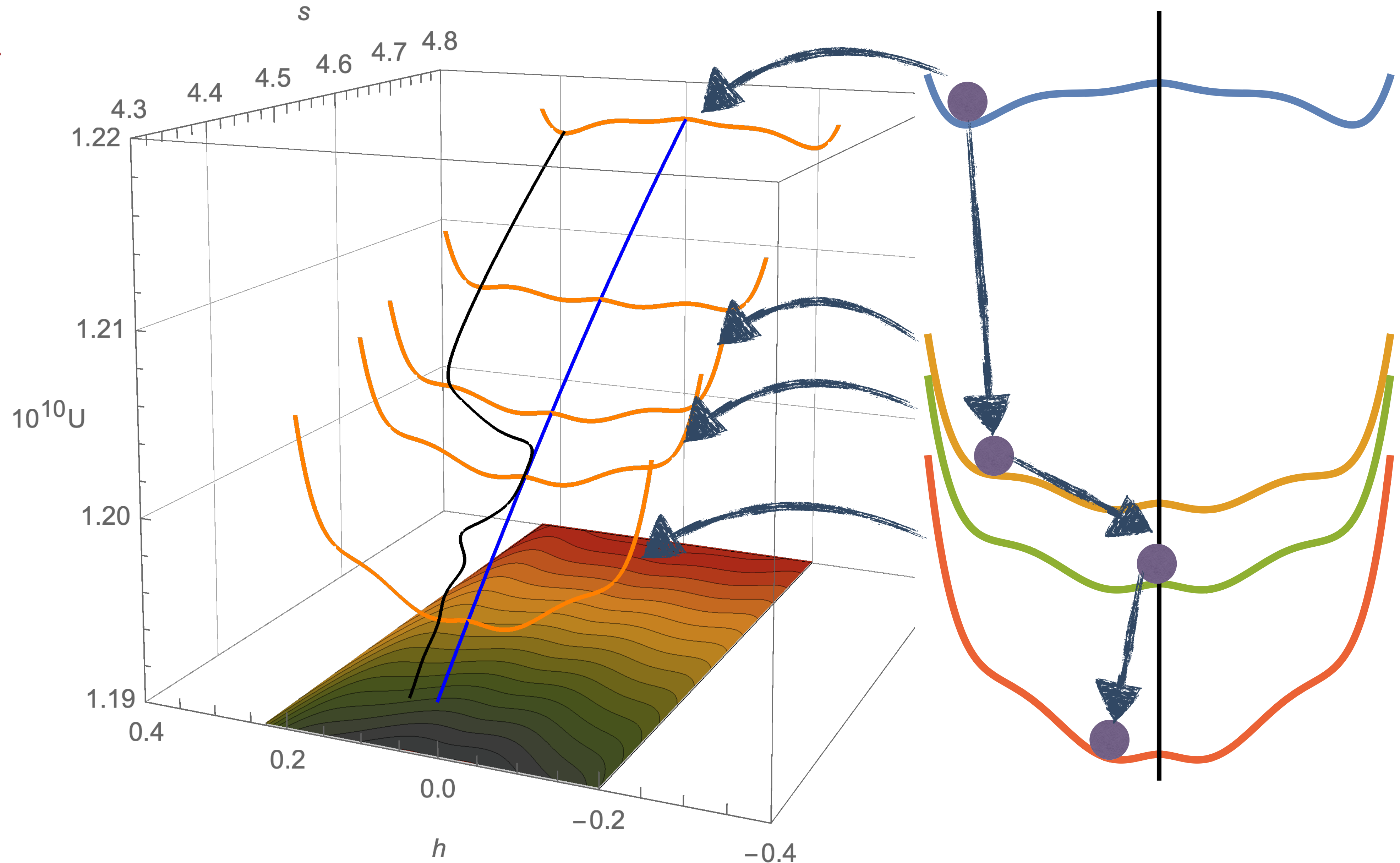
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Higgs- R^2 Inflation, Trajectory

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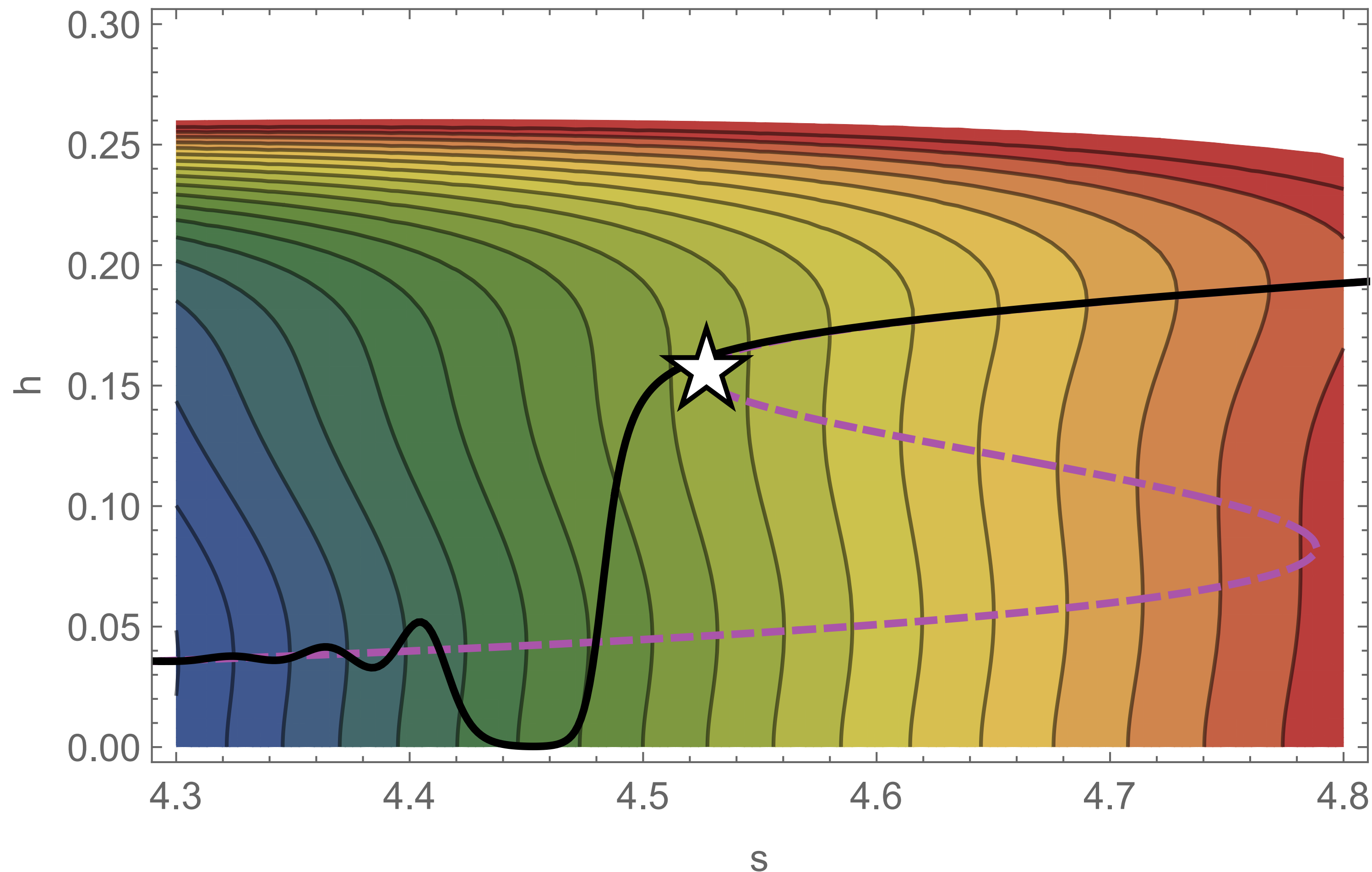
Region 2.



Higgs- R^2 Inflation, Trajectory

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

Region 2.



Dashed, Purple : $\frac{\partial U}{\partial h} = 0$

Valley structure departure, second valley emerges at smaller h .

$$\frac{h_{local\ min}}{h_m} = e^{-\frac{3}{4}} + \frac{\sqrt{5b^2 - 16b\lambda_m - 48bM^2\xi^2}}{4b}$$

$$0 \leq \xi < \frac{1}{4\sqrt{3}M} \sqrt{5b - 16\lambda_m}, \quad \lambda_m < \frac{5b}{16}.$$

Departure induced by the nonzero b , differs from constant λ !

Nonminimal coupling ξ determines position of the hill-reaching.

Higgs- R^2 Inflation, Perturbations

Second order perturbed action with $\phi^a(t, \vec{x}) = \phi_0^a(t) + \delta\phi^a(t, \vec{x})$, $ds^2 = -(1 + 2\psi)dt^2 + a(t)^2(1 - 2\psi)\delta_{ij}dx^i dx^j$.
with the perturbation equations

$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{\parallel}) H\dot{\mathcal{R}} + \frac{k^2}{a^2}\mathcal{R} = -2\frac{H^2}{\dot{\phi}_0}\eta_{\perp} \left[\dot{Q}_N + \left(3 - \eta_{\parallel} + \frac{\dot{\eta}_{\perp}}{H\eta_{\perp}} \right) HQ_N \right]$$

$$\ddot{Q}_N + 3H\dot{Q}_N + \left(\frac{k^2}{a^2} + M_{\text{eff}}^2 \right) Q_N = 2\dot{\phi}_0\eta_{\perp}\dot{\mathcal{R}}.$$

$$\eta_{\parallel} \equiv -\frac{\ddot{\phi}_0}{\dot{\phi}_0 H}$$

$$\eta_{\perp} \equiv \frac{U_N}{\dot{\phi}_0 H}$$

$$Q^a \equiv \delta\phi^a + \frac{\dot{\phi}^a}{H}\psi$$

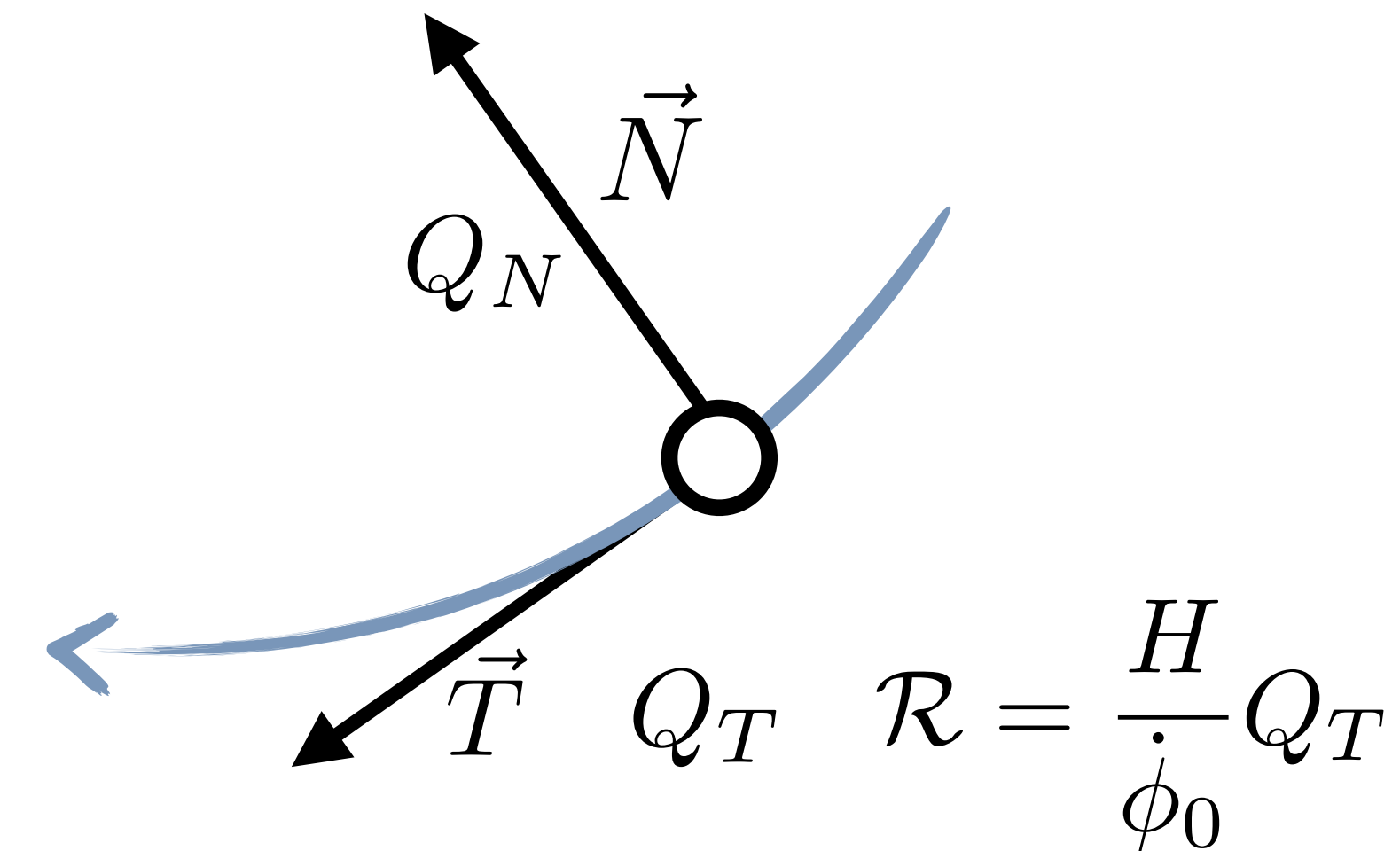
$$\dot{\theta} \equiv H\eta_{\perp}$$

$$M_{\text{eff}}^2 = U_{NN} + H^2\epsilon_{\mathcal{R}} - \dot{\theta}^2.$$

[S. Cespedes et. al, JCAP 05 (2012) 008]

[A. Achucarro et. al, Phys. Rev. D86 (2012) 121301]

[S. Groot Nibbelink and B.J.W. van Tent, Class. Quant. Grav. 19 (2002) 613] and many more...

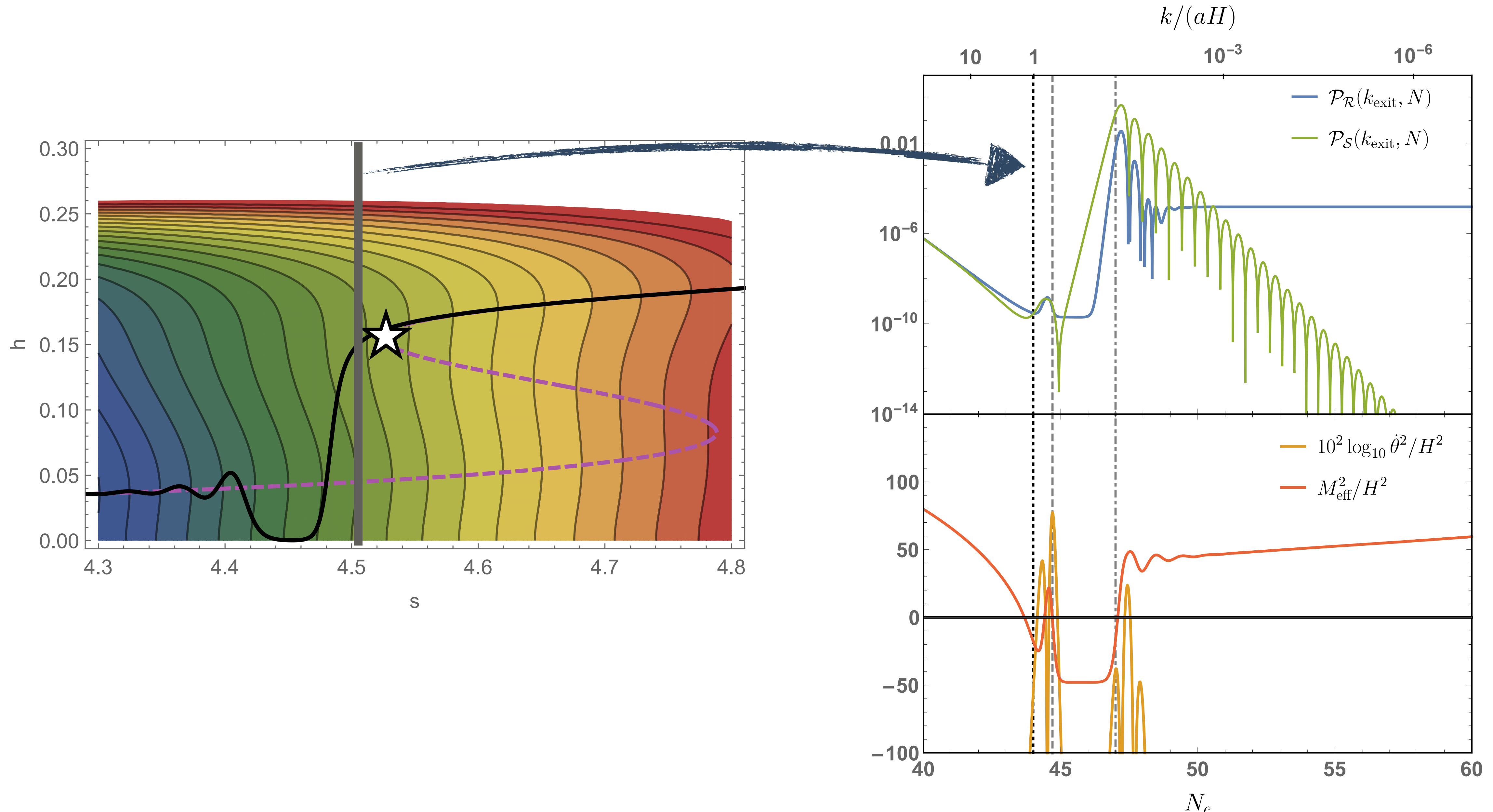


- $M_{\text{eff}}^2 < 0$ leads to *tachyonic growth of Q_N* , then gets *sourced to \mathcal{R}* through *turns in the trajectory*.

- Tachyonic mass at the “hill” of the potential at $h = 0$, $M_{\text{eff}}^2 \simeq -3M^2\xi \left(1 - e^{-\sqrt{\frac{2}{3}}s} \right)$

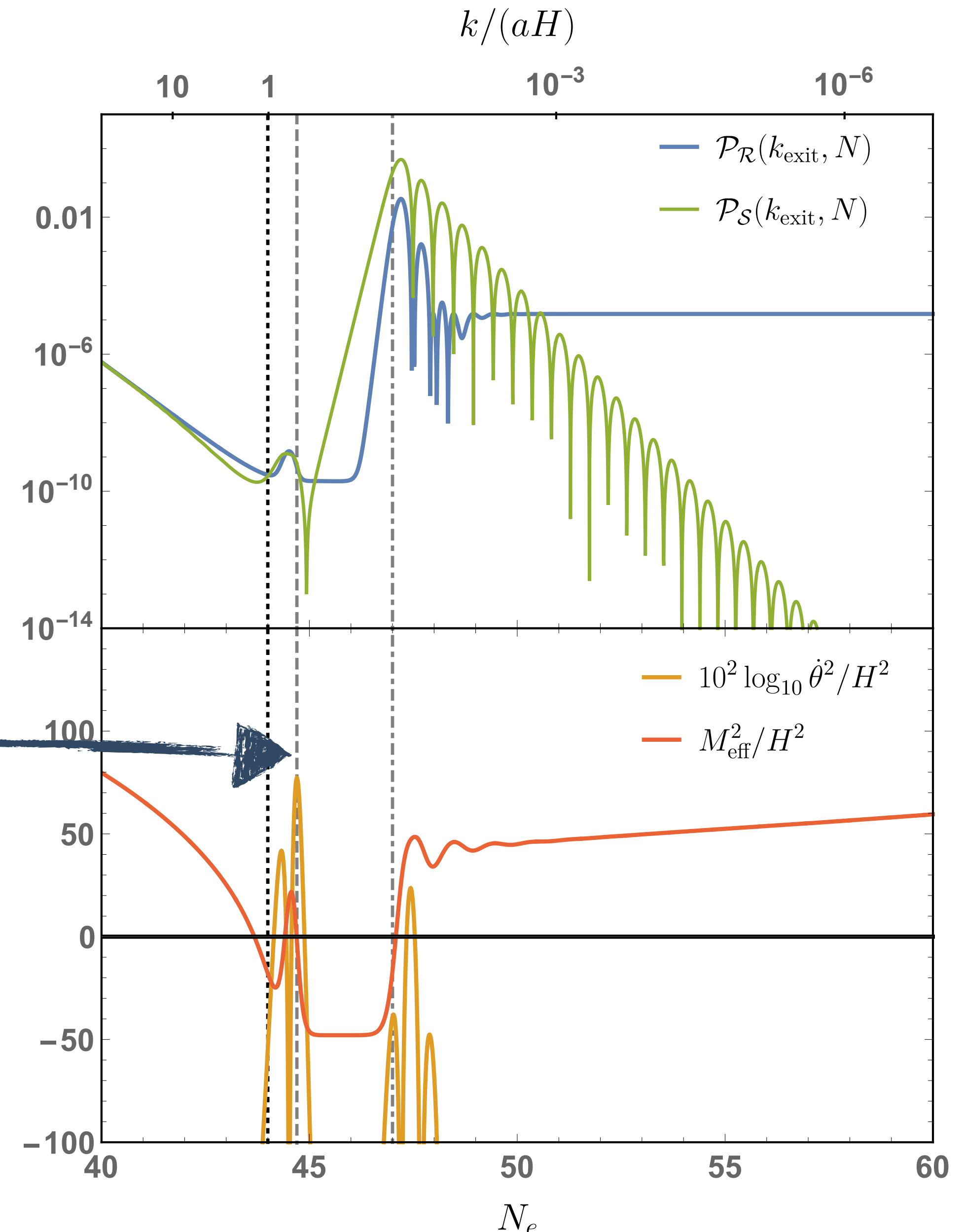
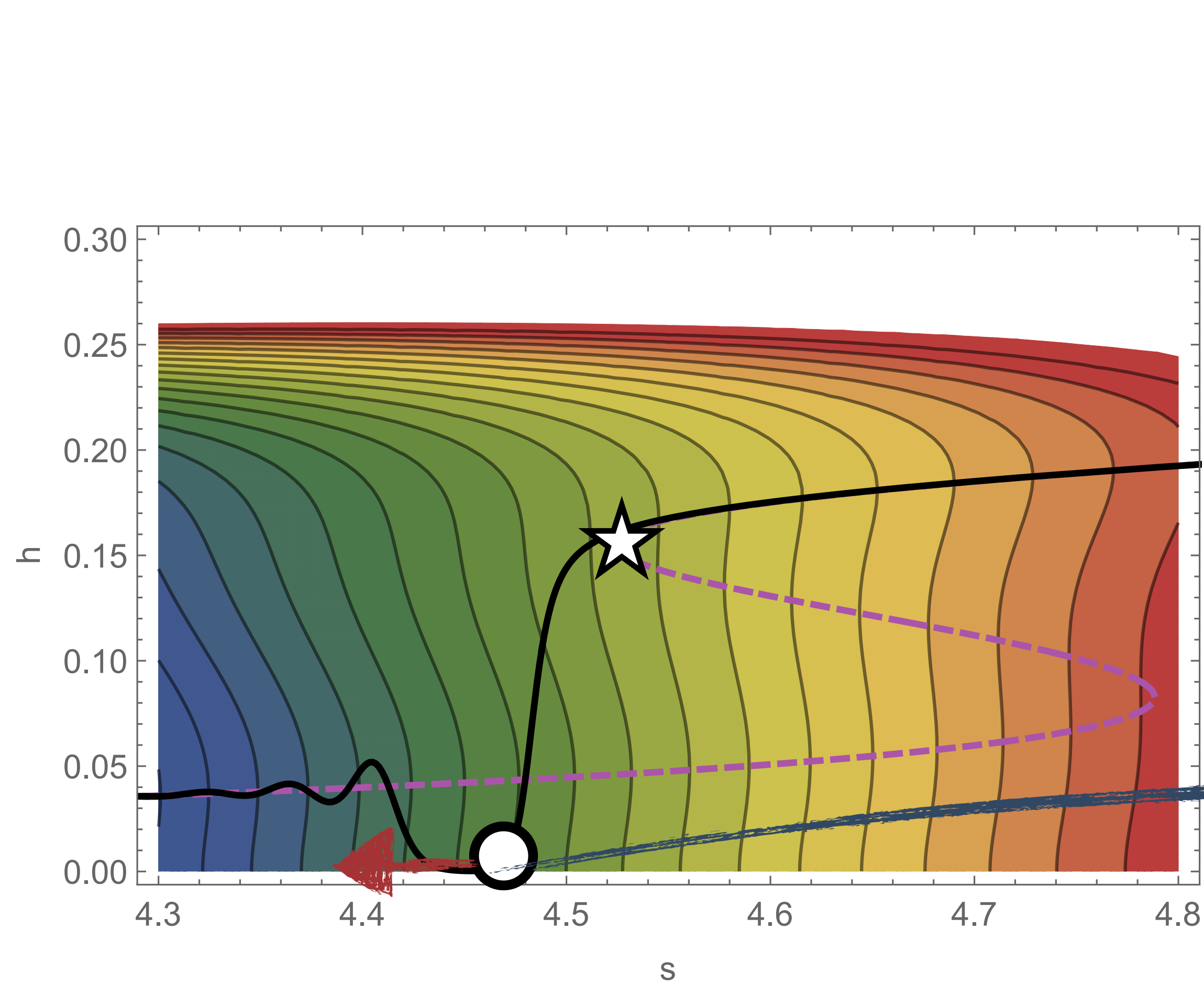
Higgs- R^2 Inflation, Perturbations

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]



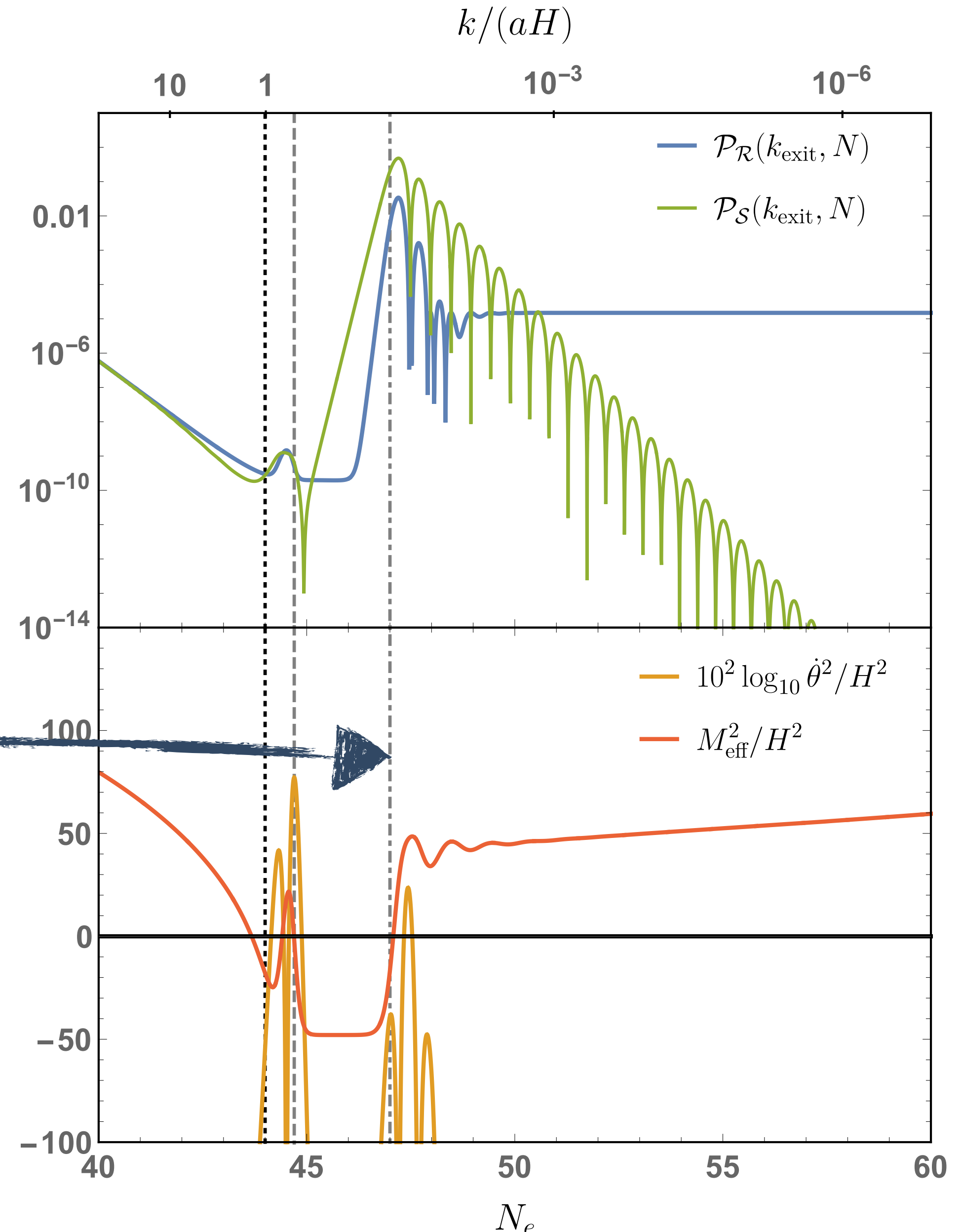
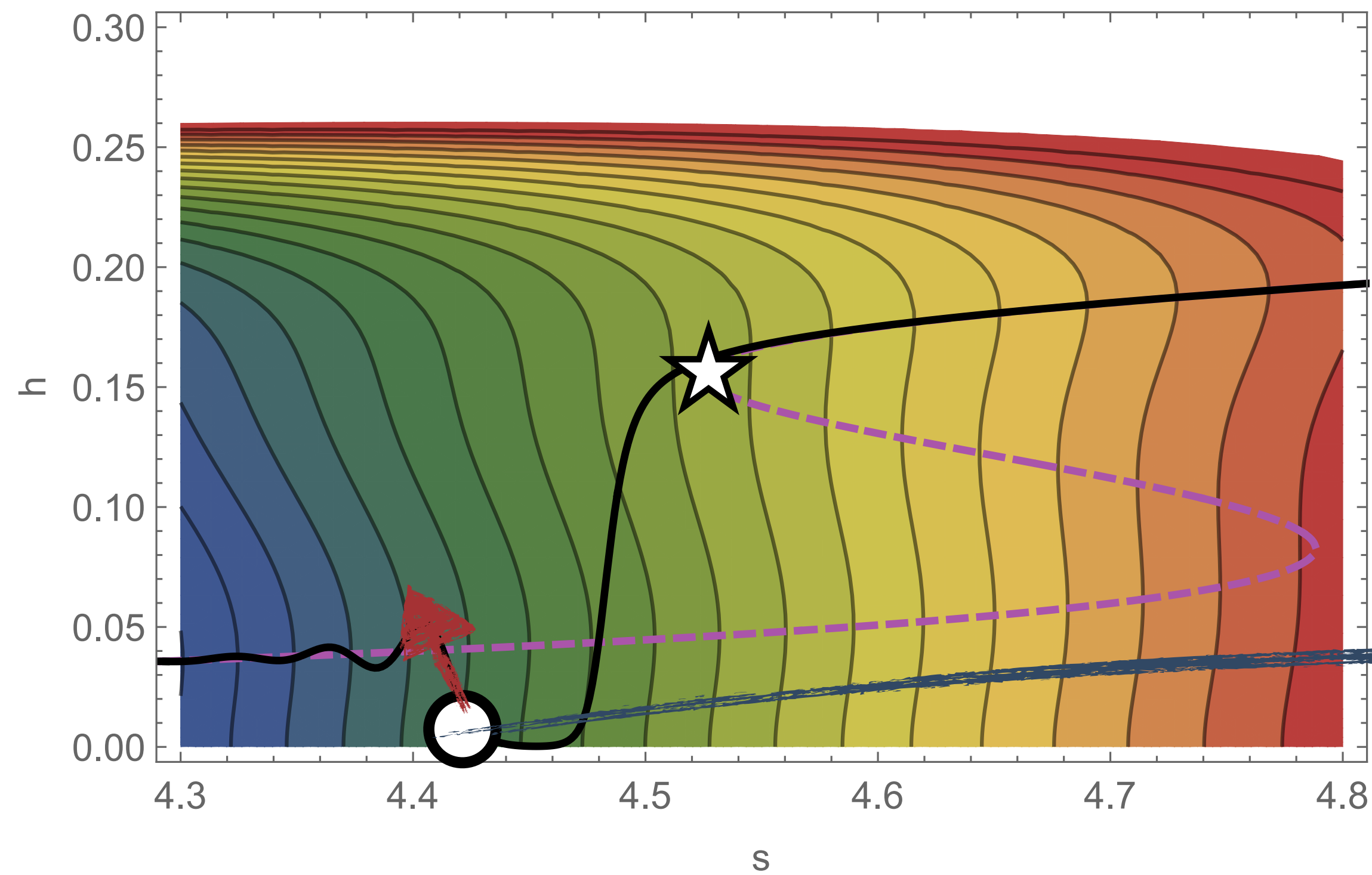
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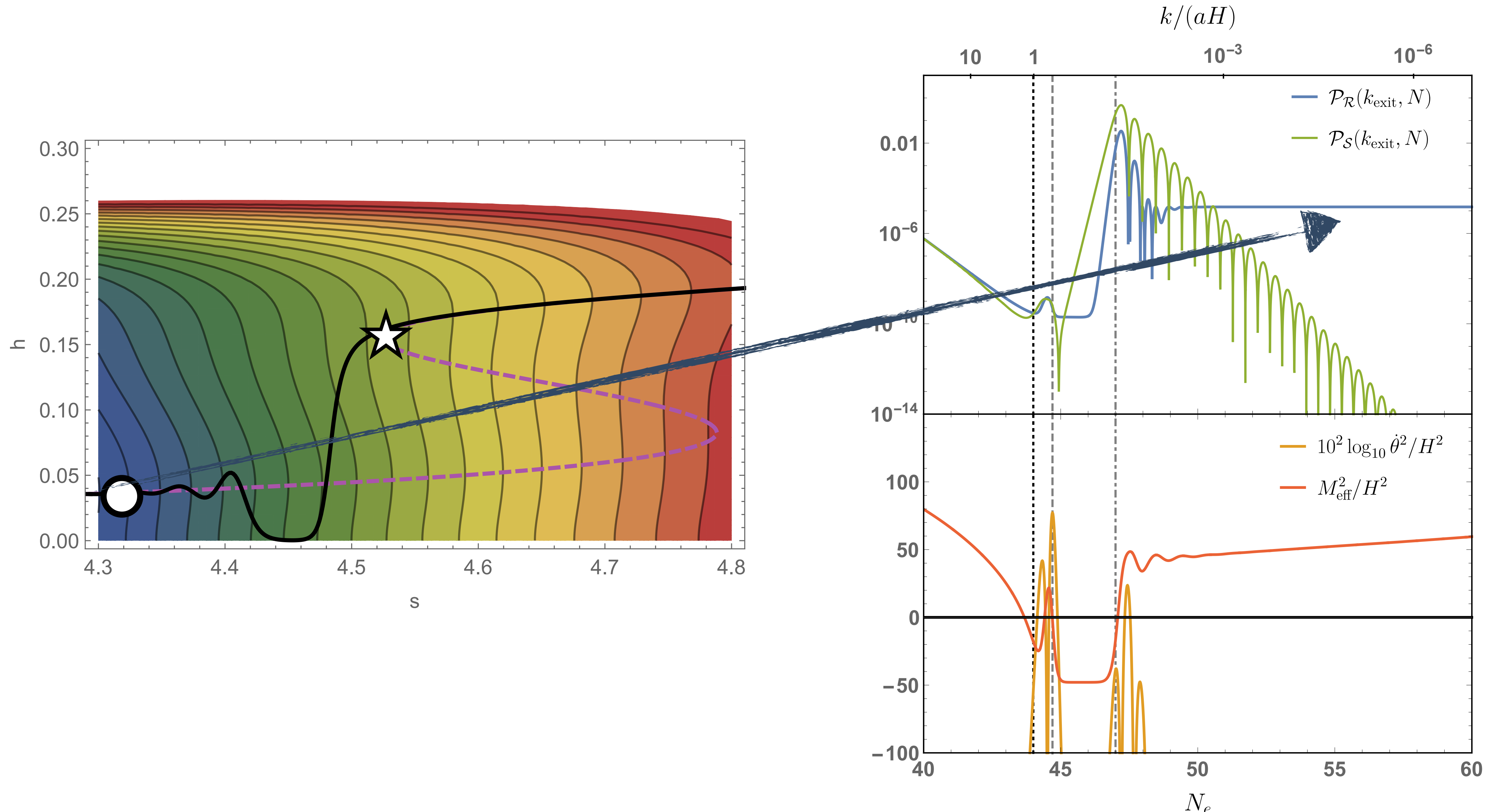
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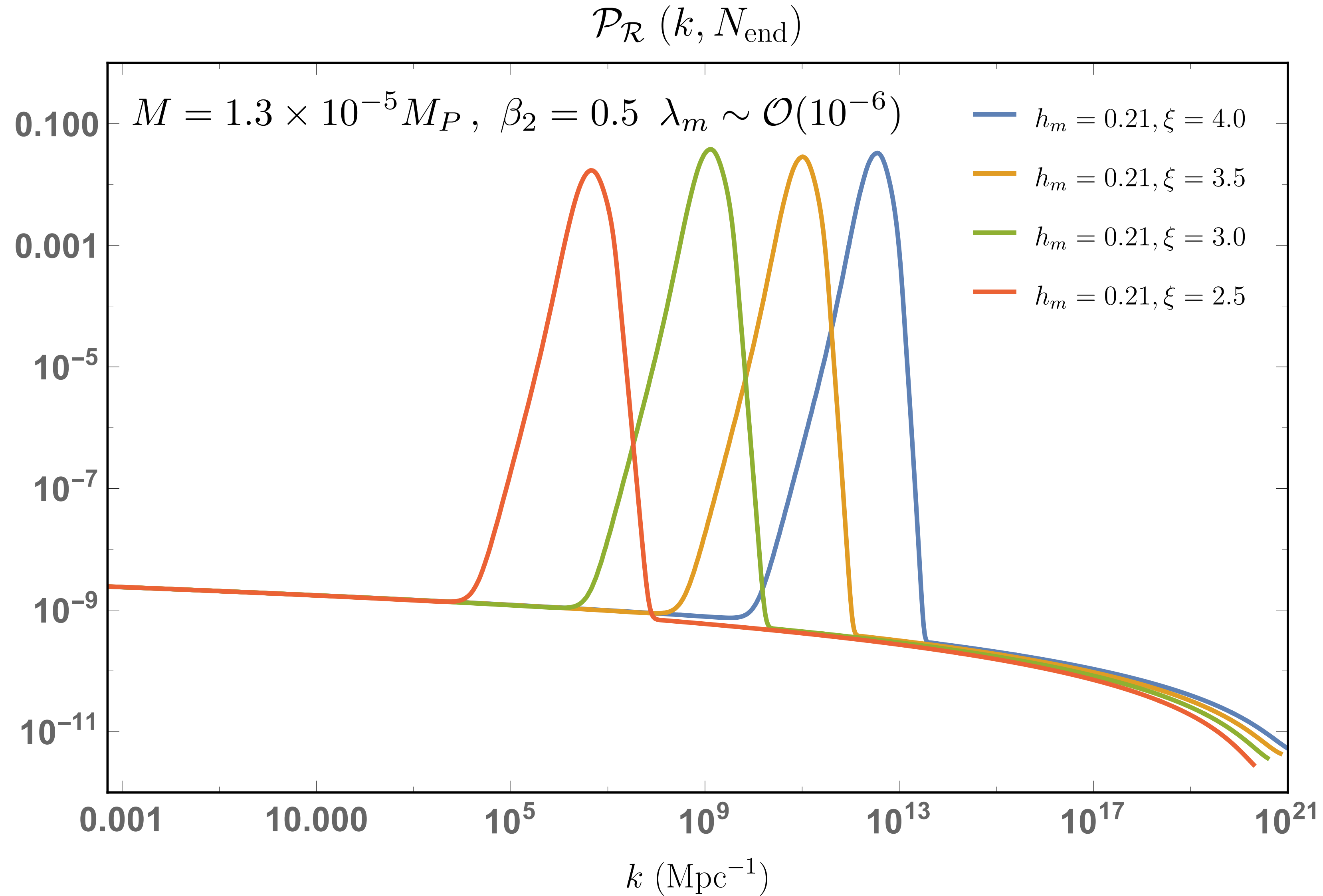
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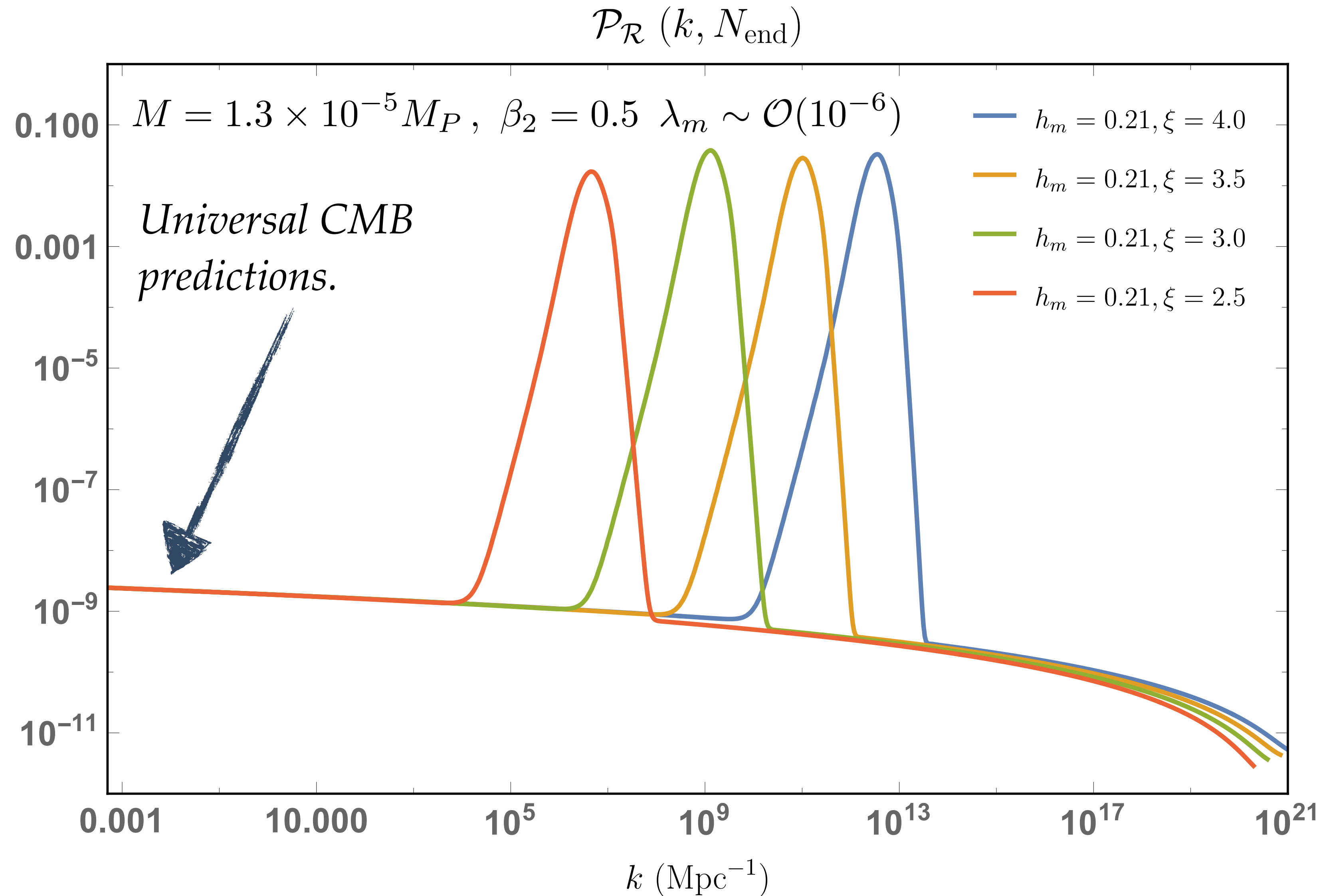
Higgs- R^2 Inflation, Power Spectrum

[**DYC**, K. Kohri, S.C. Park, arXiv:2205.14813]



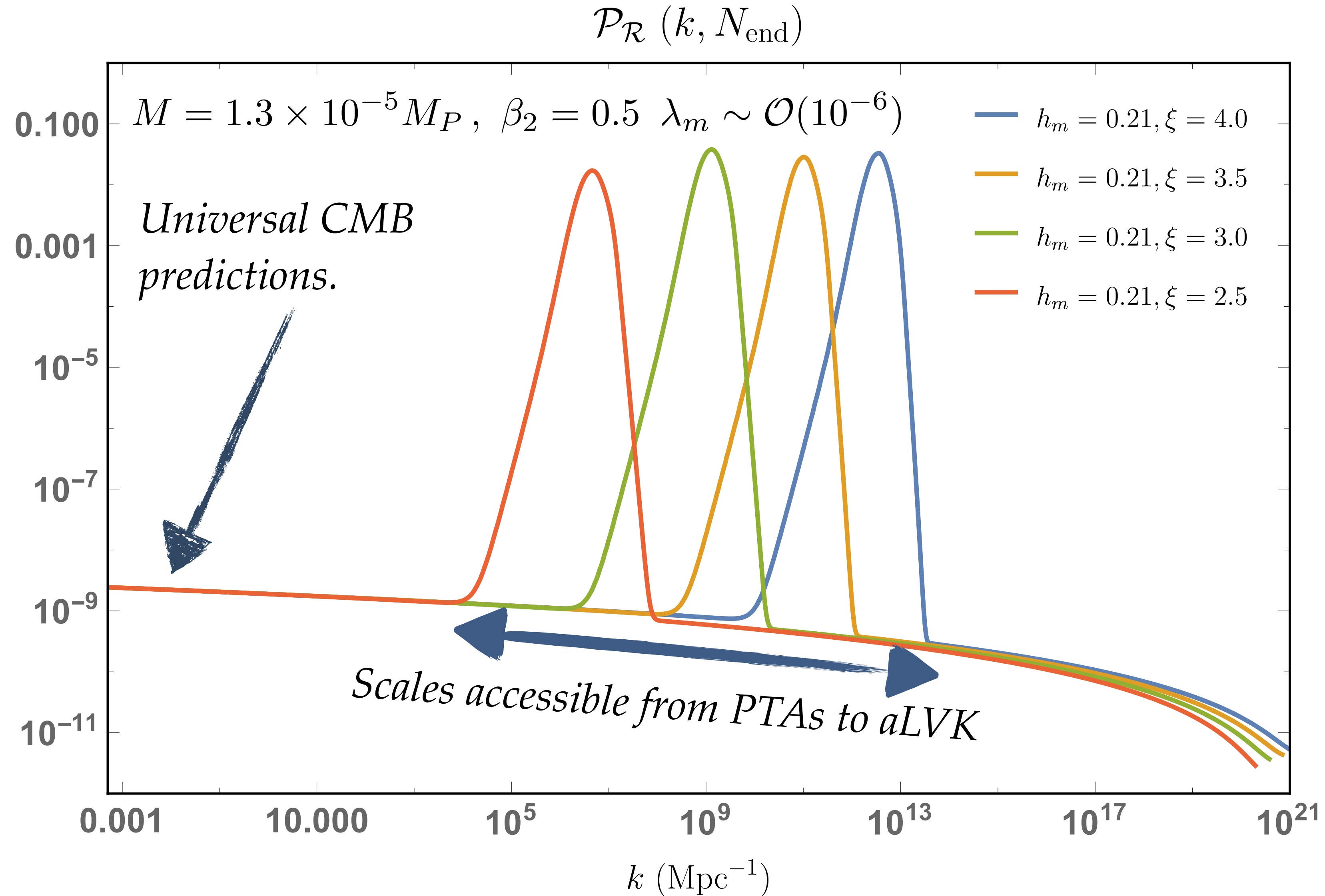
Higgs- R^2 Inflation, Power Spectrum

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

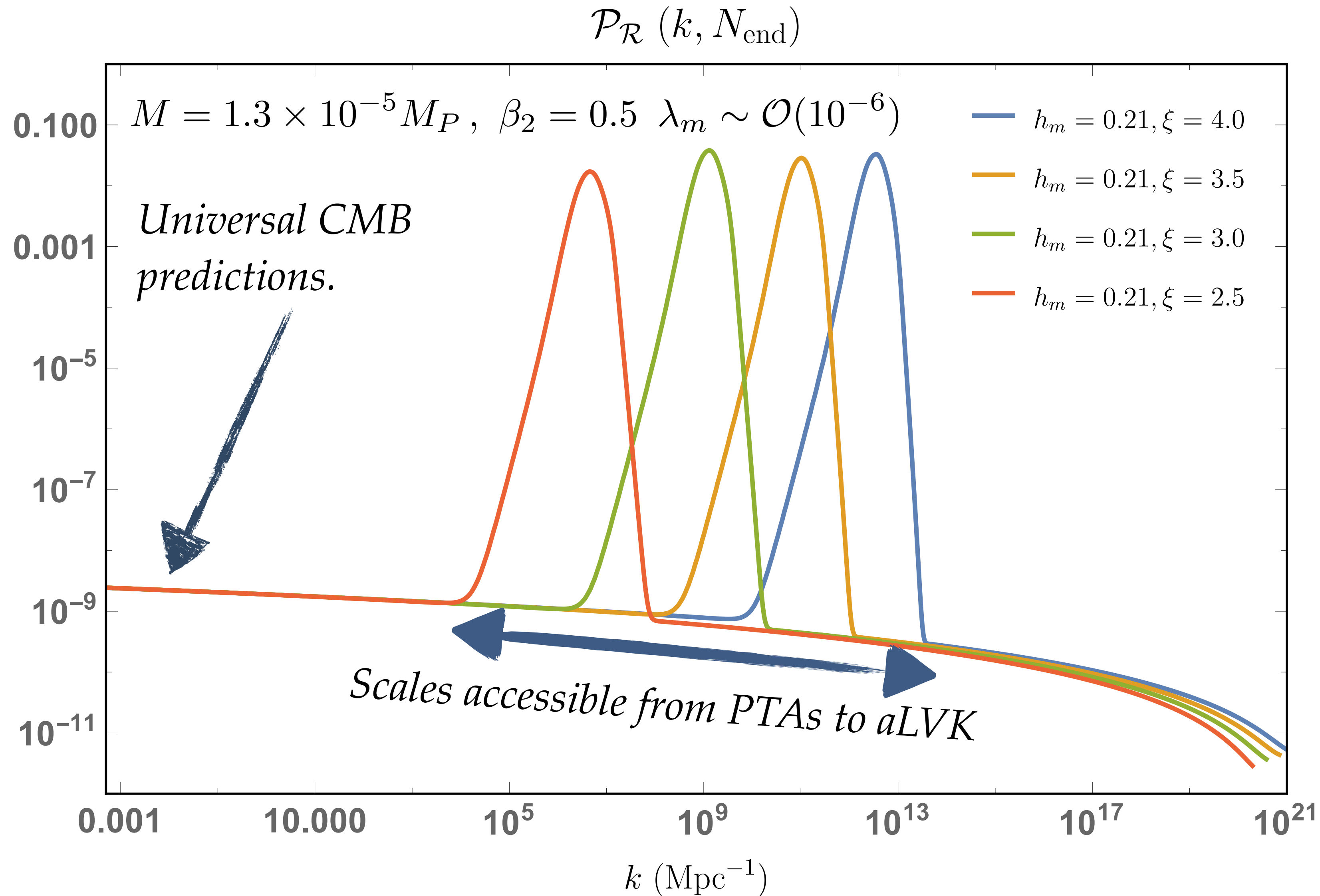


Higgs- R^2 Inflation, Power Spectrum

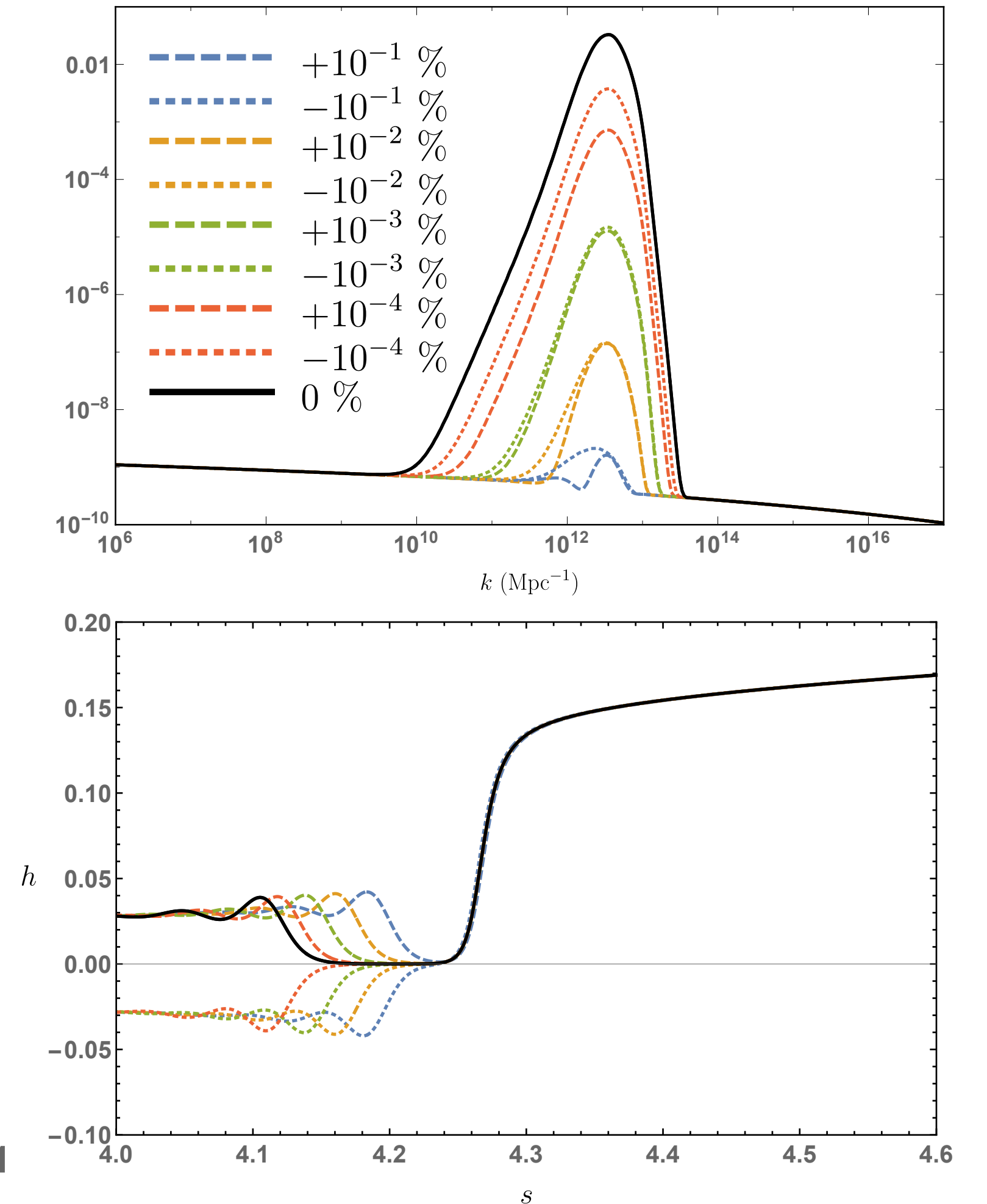
[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]



Higgs- R^2 Inflation, Power Spectrum



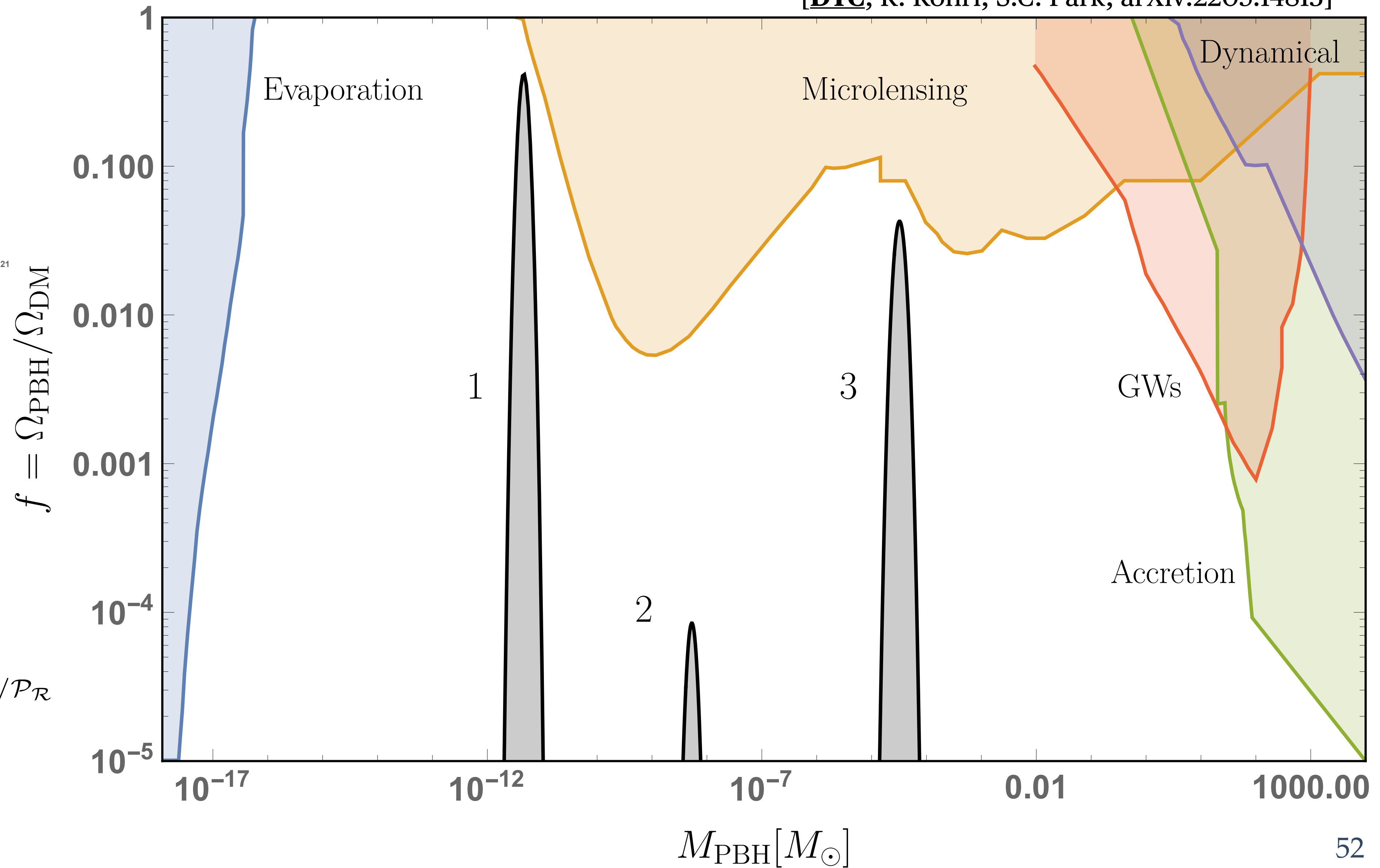
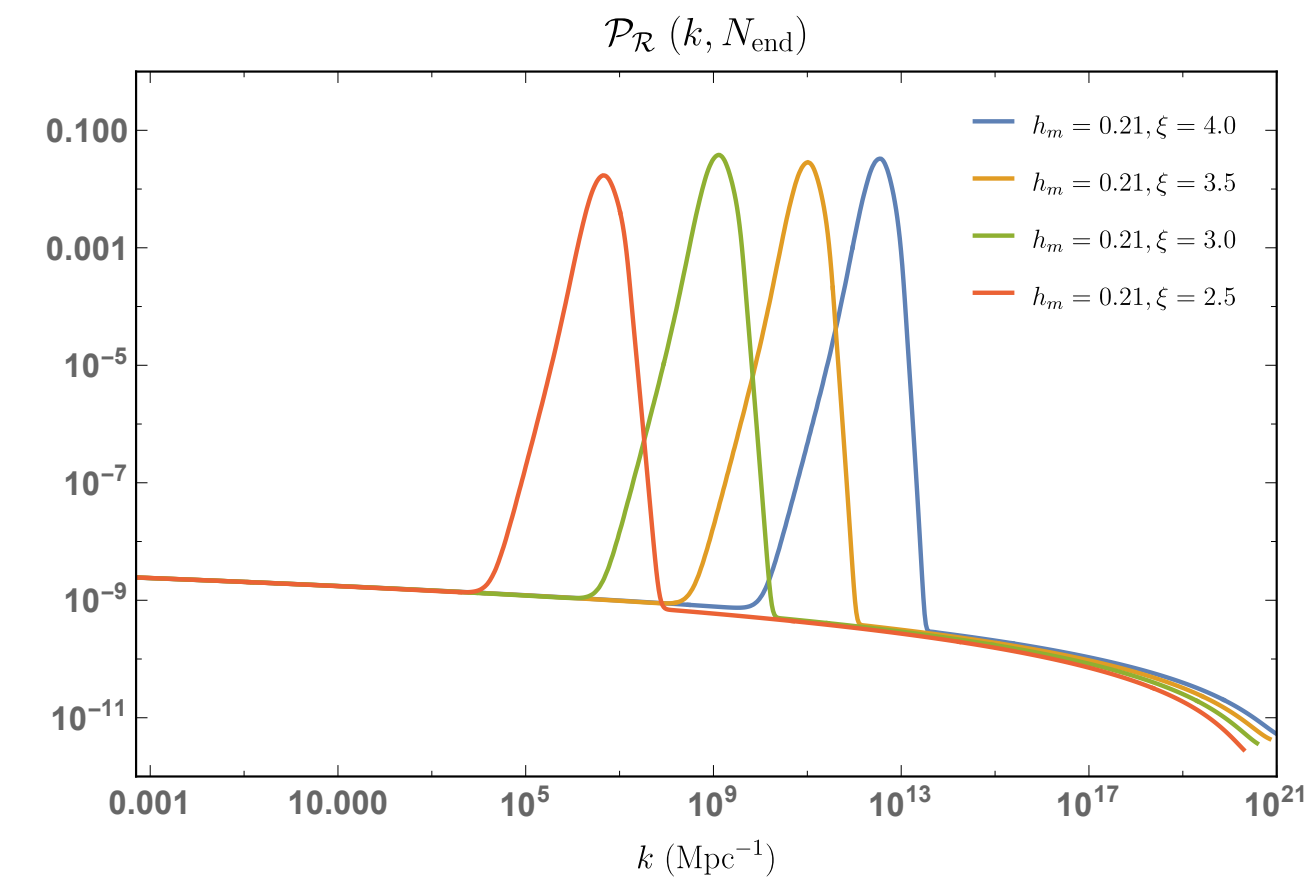
[**DYC**, K. Kohri, S.C. Park, arXiv:2205.14813]



$$\frac{\delta \lambda_m}{\lambda_m} \equiv \frac{\lambda_m^{\text{dev}} - \lambda_m}{\lambda_m} \sim 10^{-4} \%$$

Phenomena — Primordial Black Holes

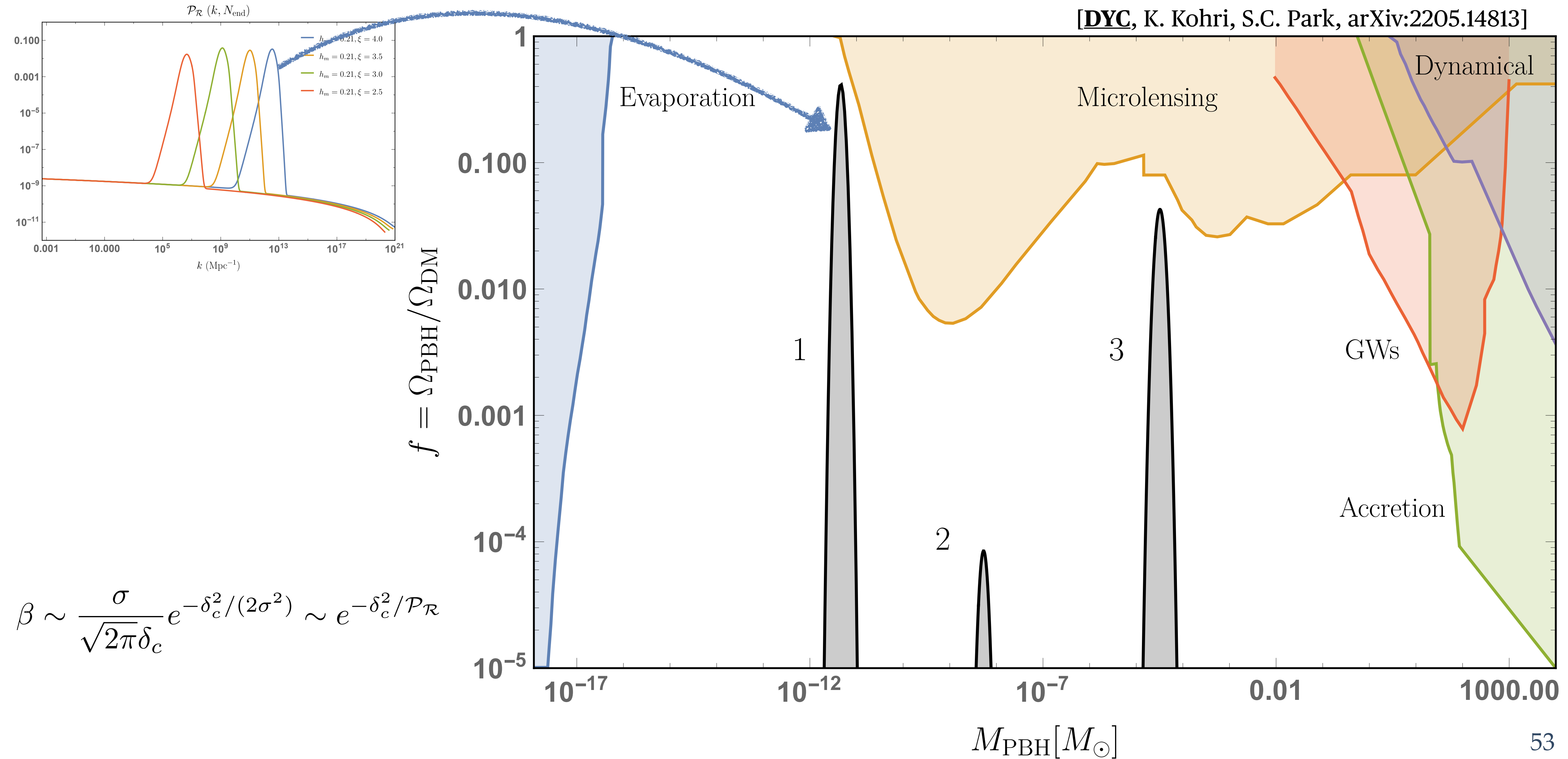
[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]



$$\beta \sim \frac{\sigma}{\sqrt{2\pi}\delta_c} e^{-\delta_c^2/(2\sigma^2)} \sim e^{-\delta_c^2/\mathcal{P}_{\mathcal{R}}}$$

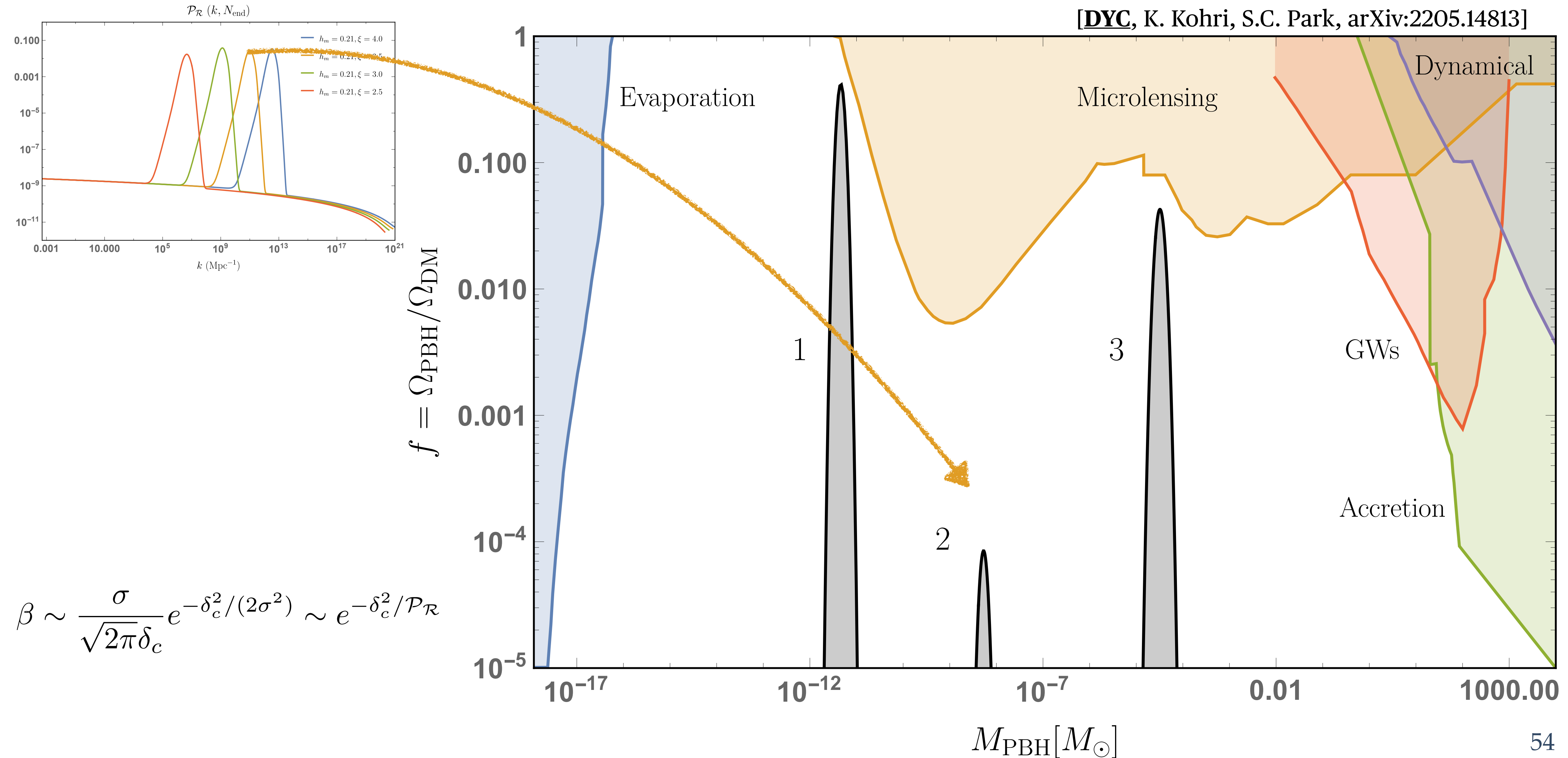
Phenomena — Primordial Black Holes

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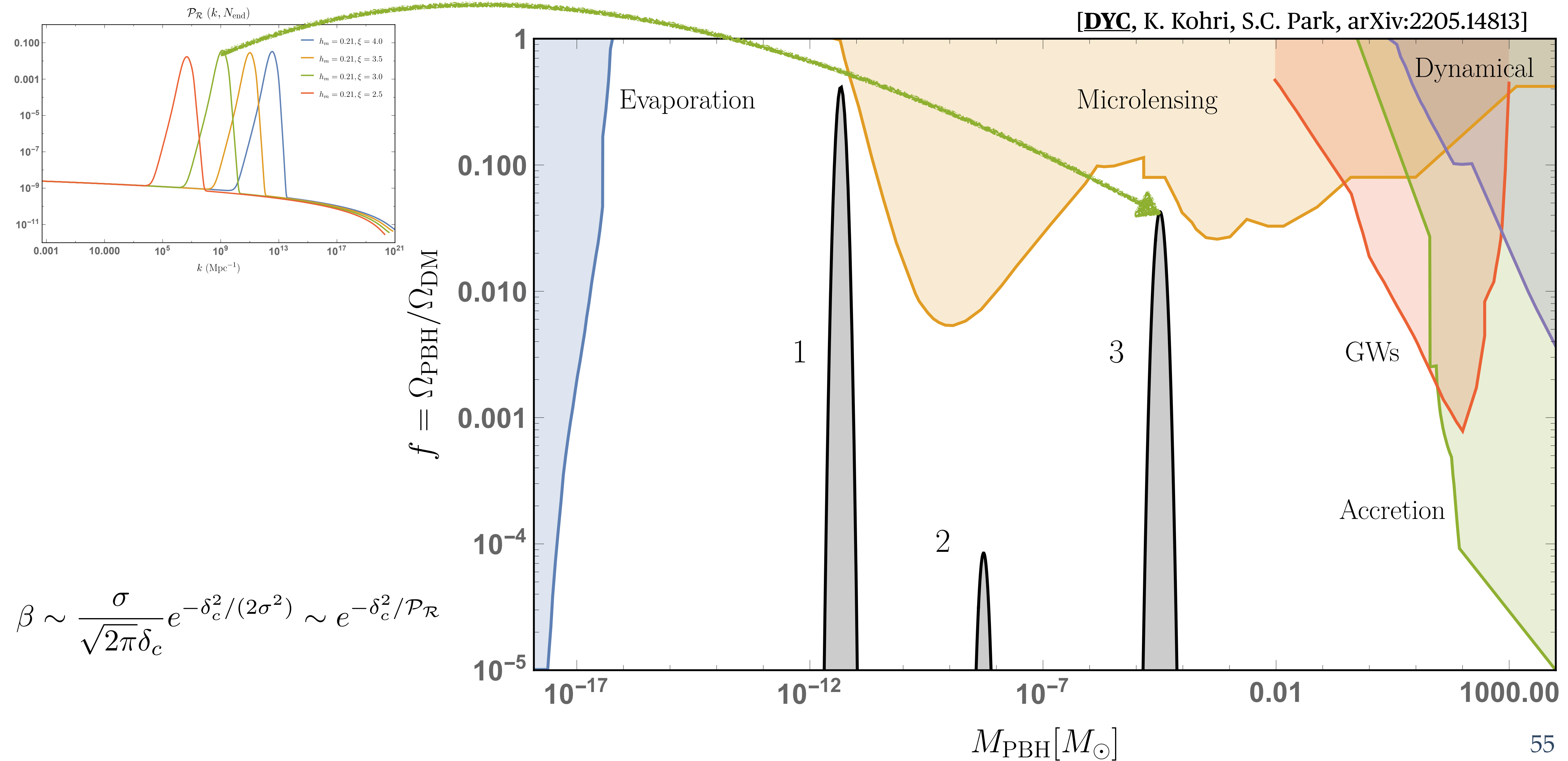
Phenomena — Primordial Black Holes

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]



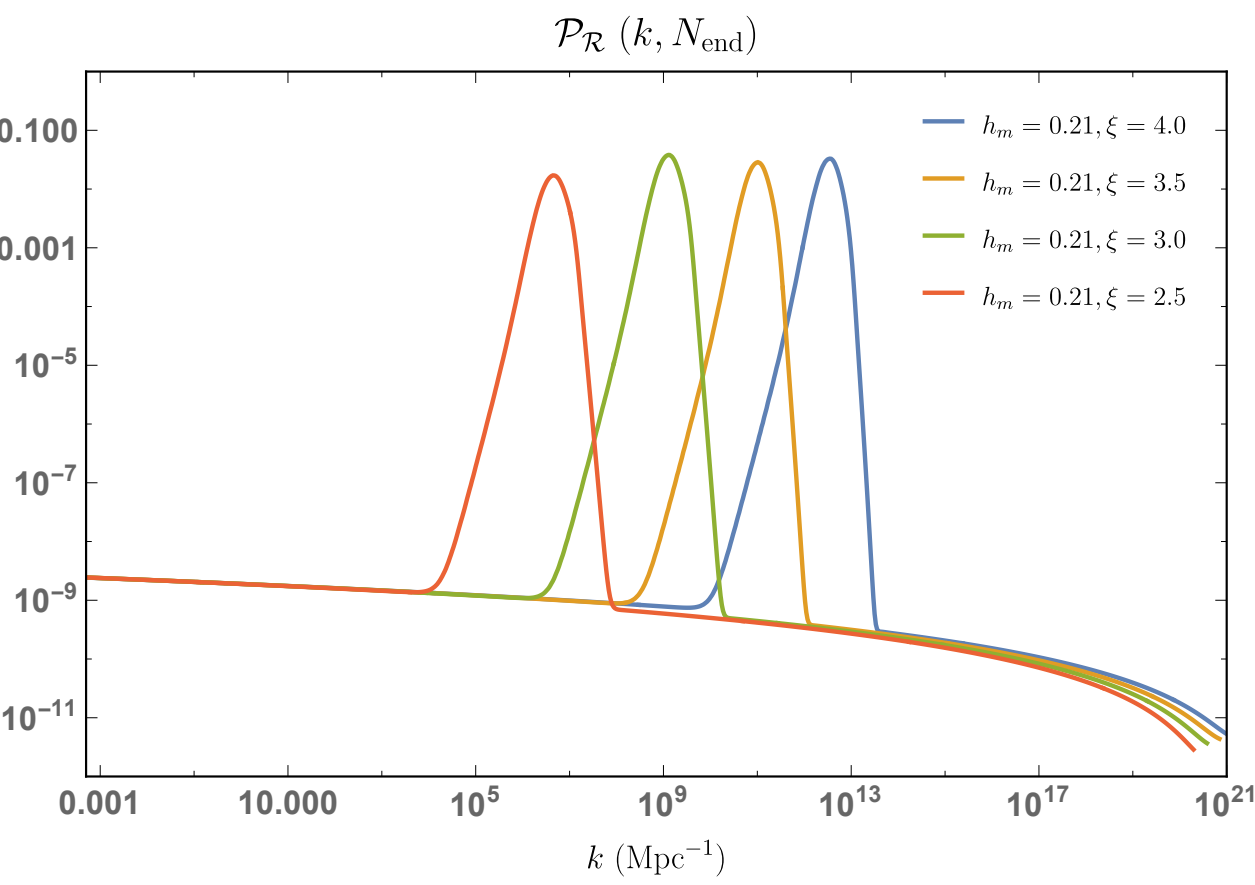
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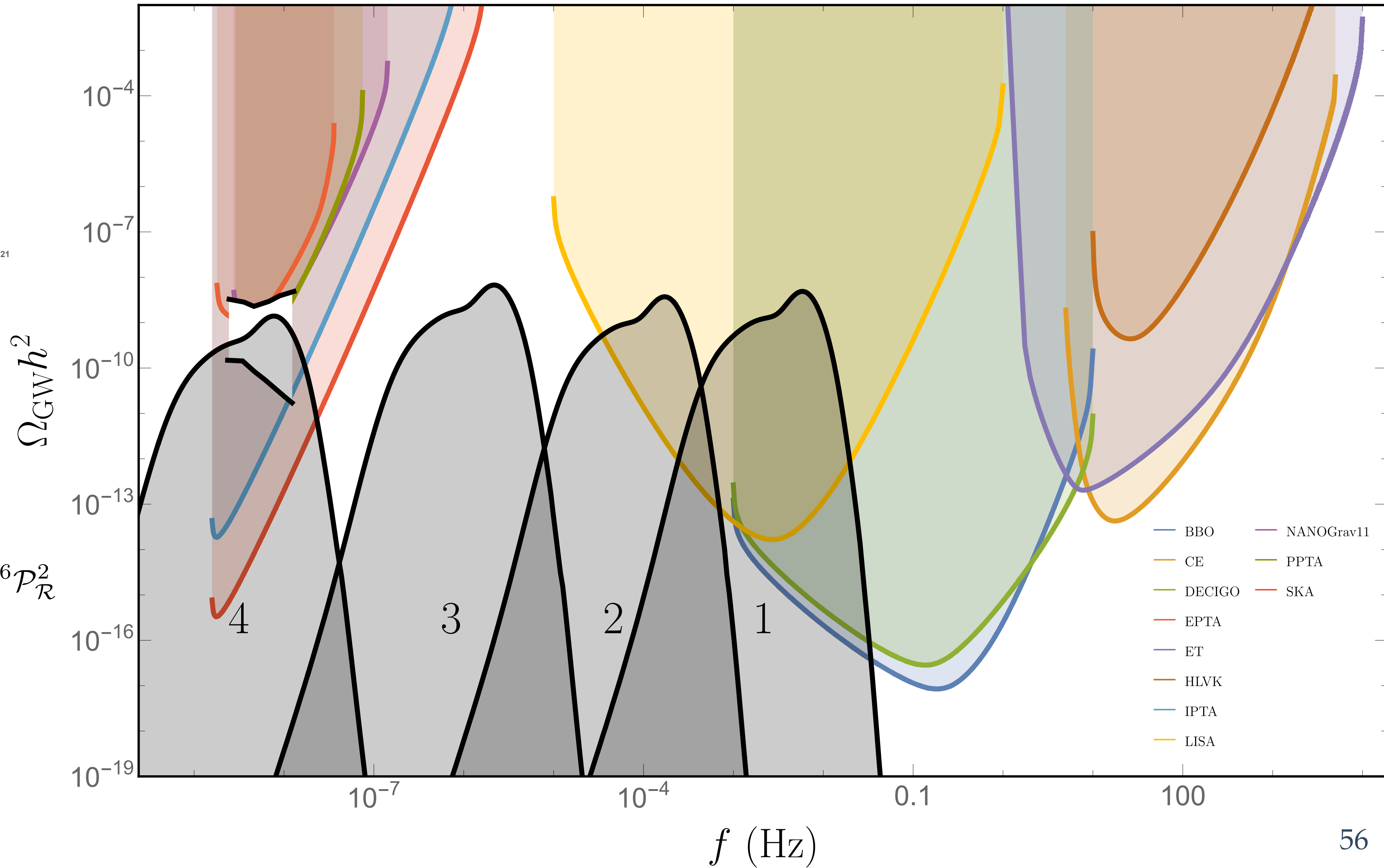


Phenomena — Second order GWs

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

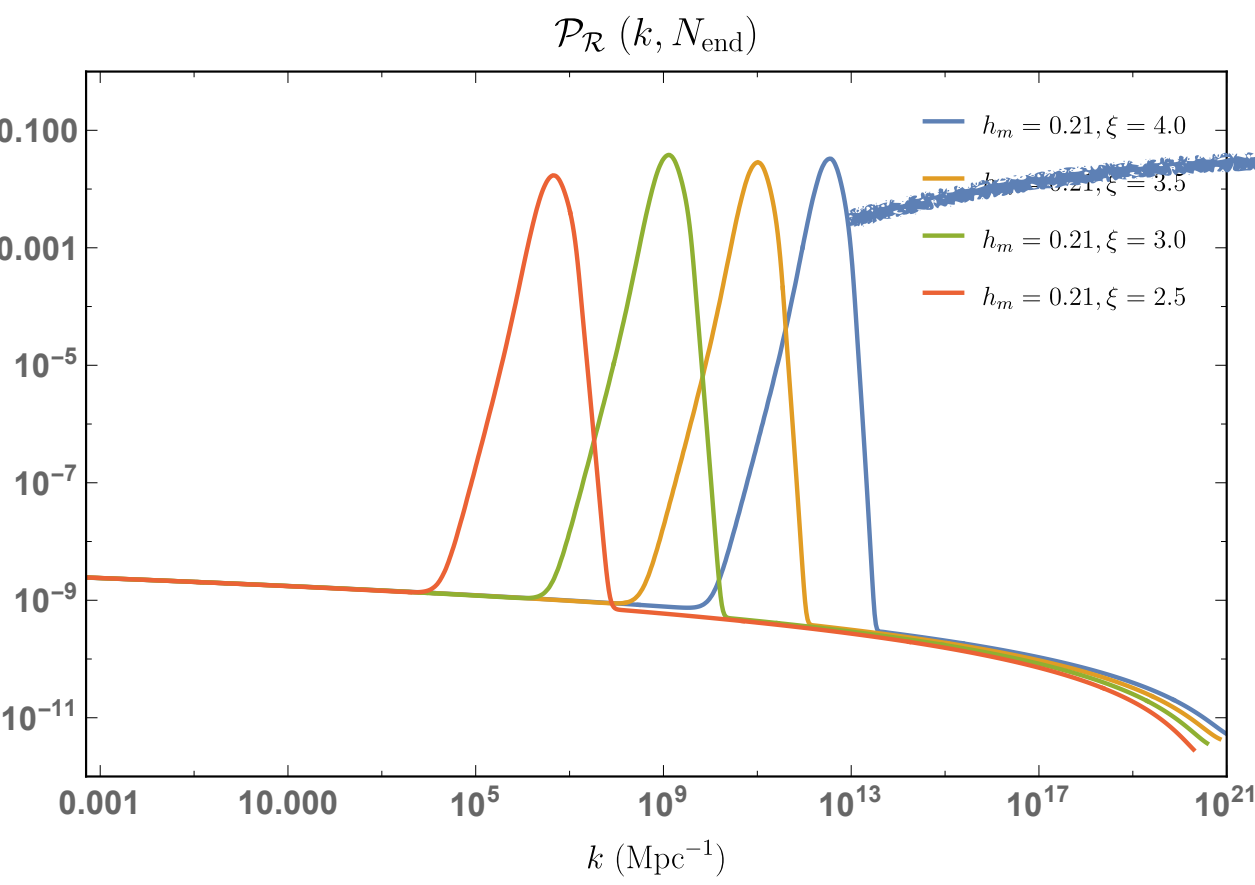


$$\Omega_{\text{GW}} h^2 \sim \frac{1}{12} \Omega_{r,0} h^2 \times \mathcal{P}_{\mathcal{R}}^2 \sim 10^{-6} \mathcal{P}_{\mathcal{R}}^2$$

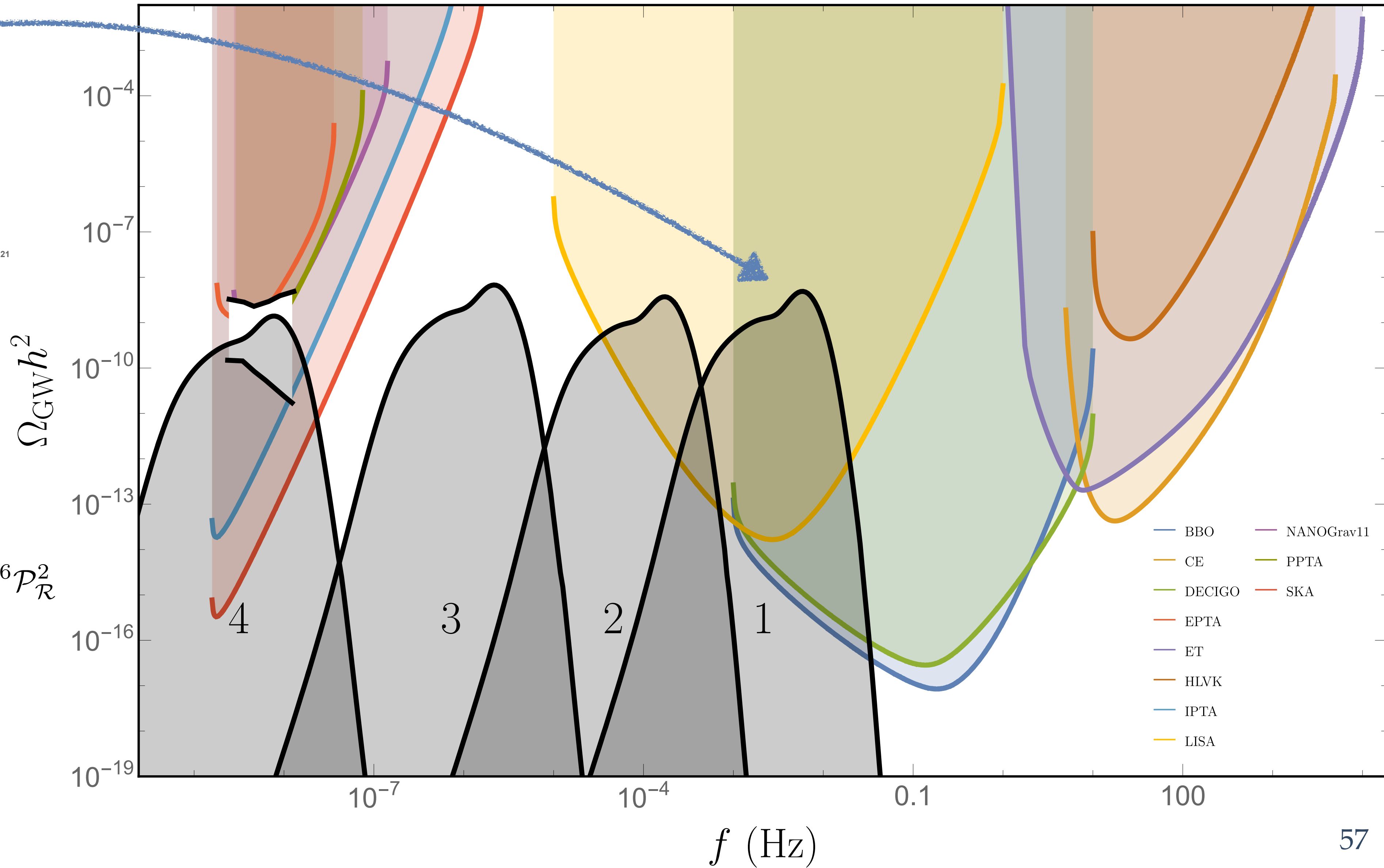


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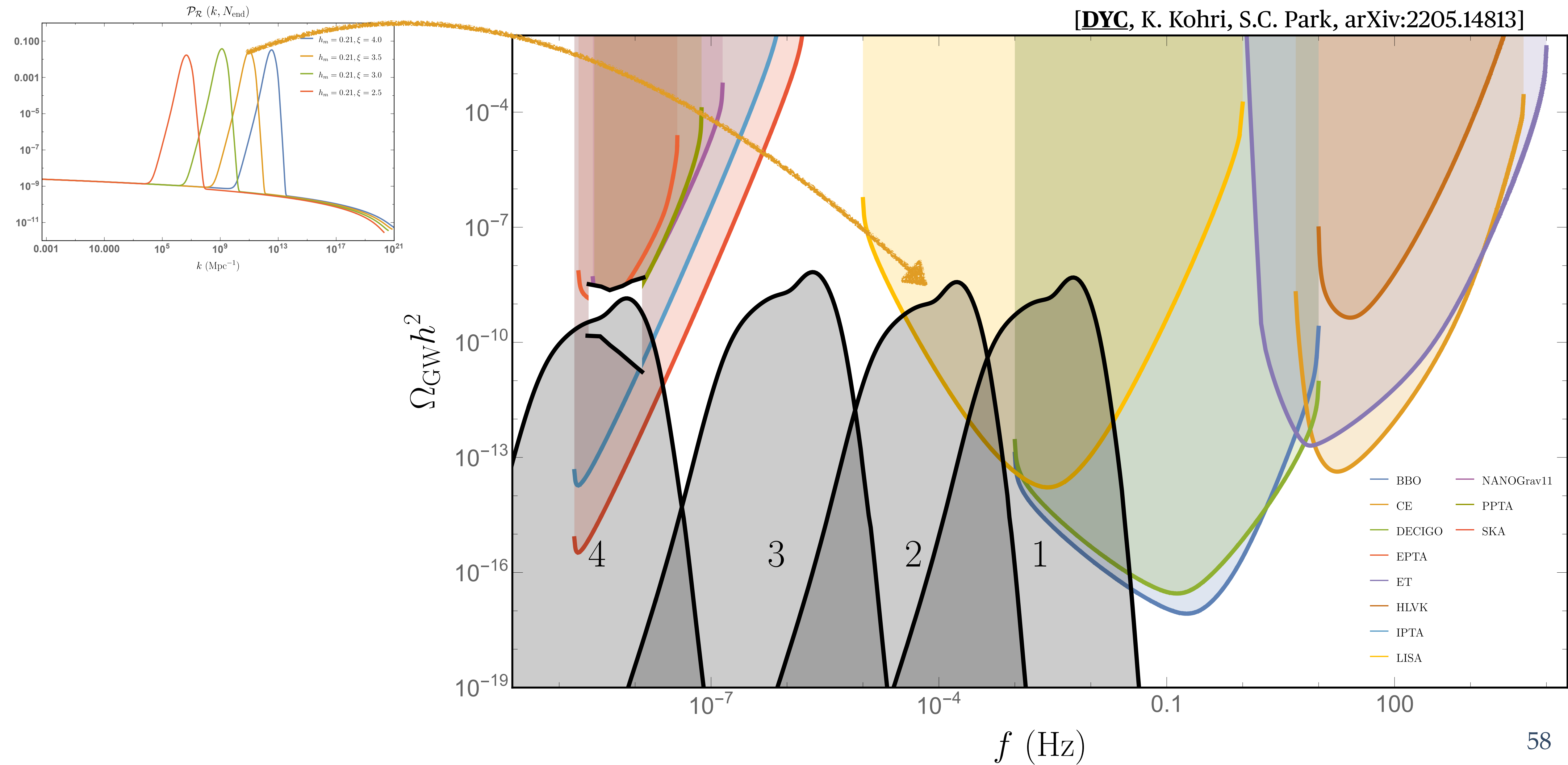


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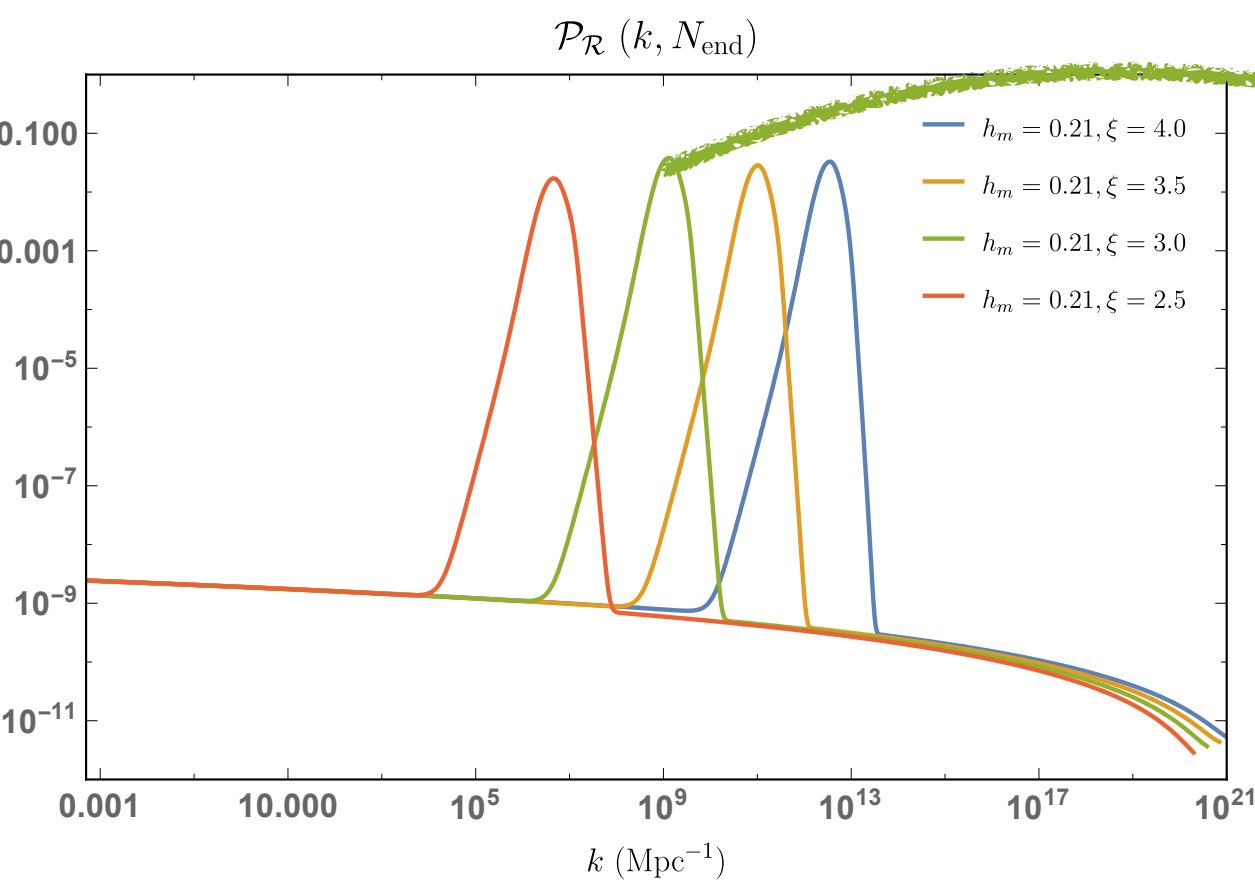
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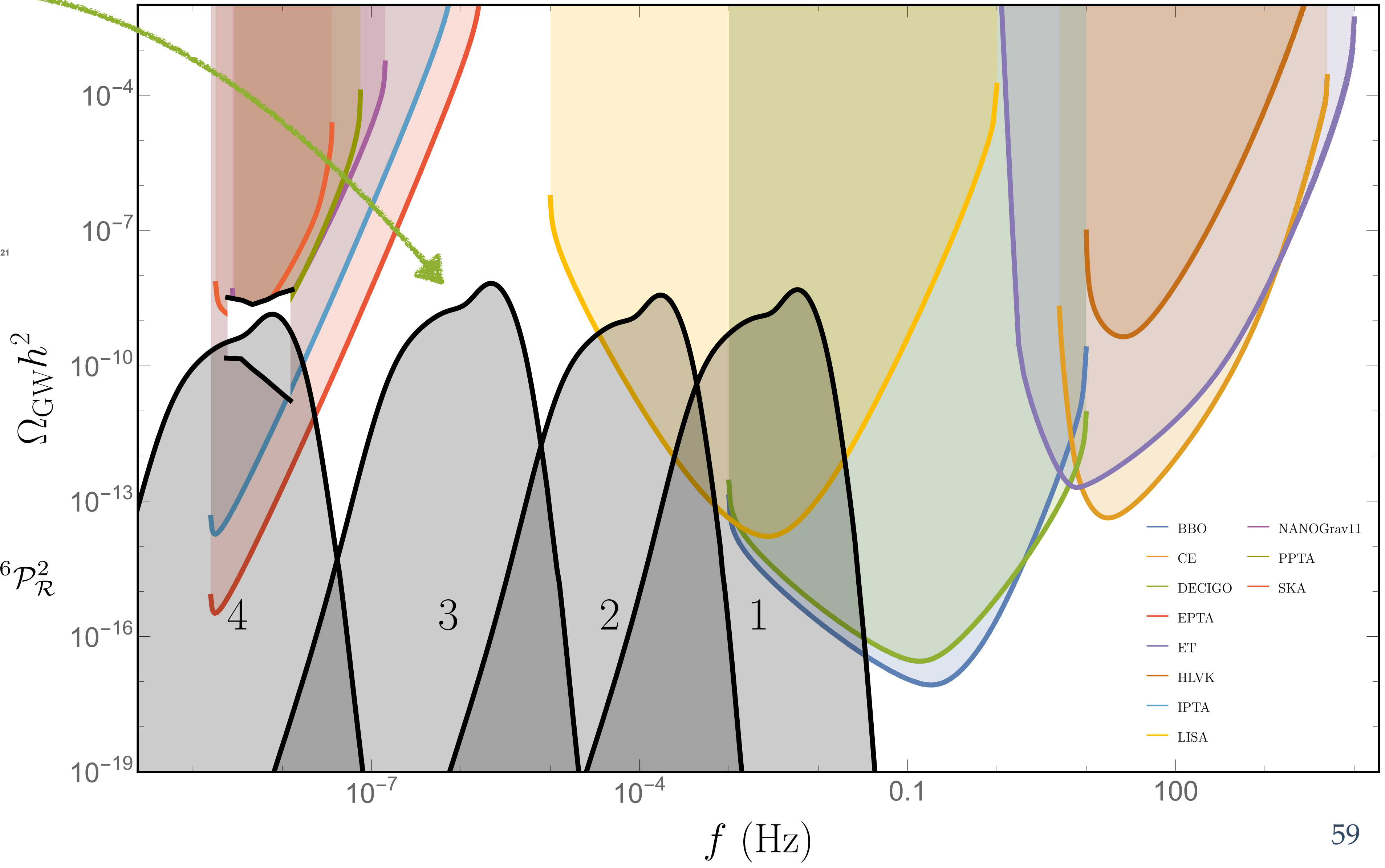


Phenomena — Second order GWs

[DYC, K. Kohri, S.C. Park, arXiv:2205.14813]

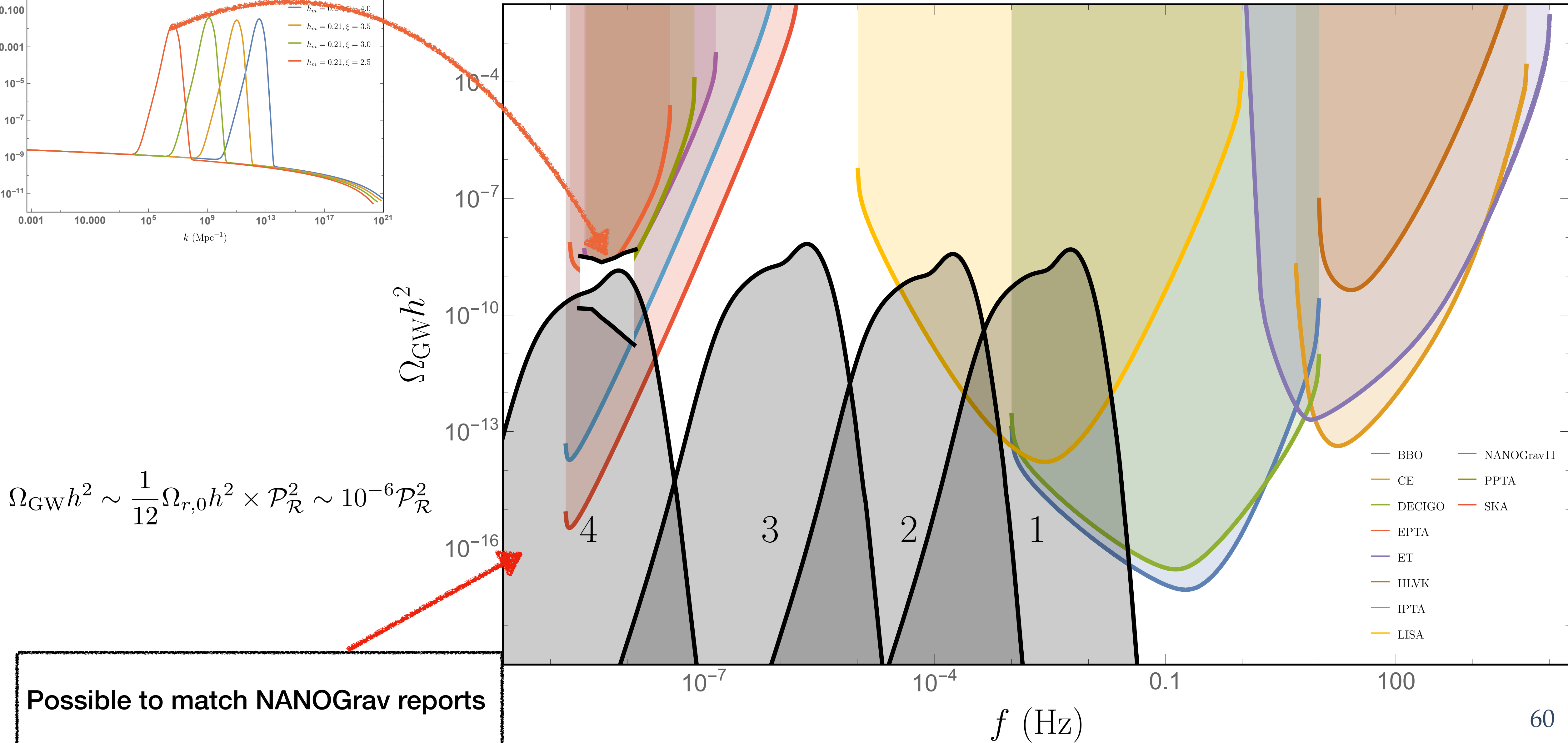
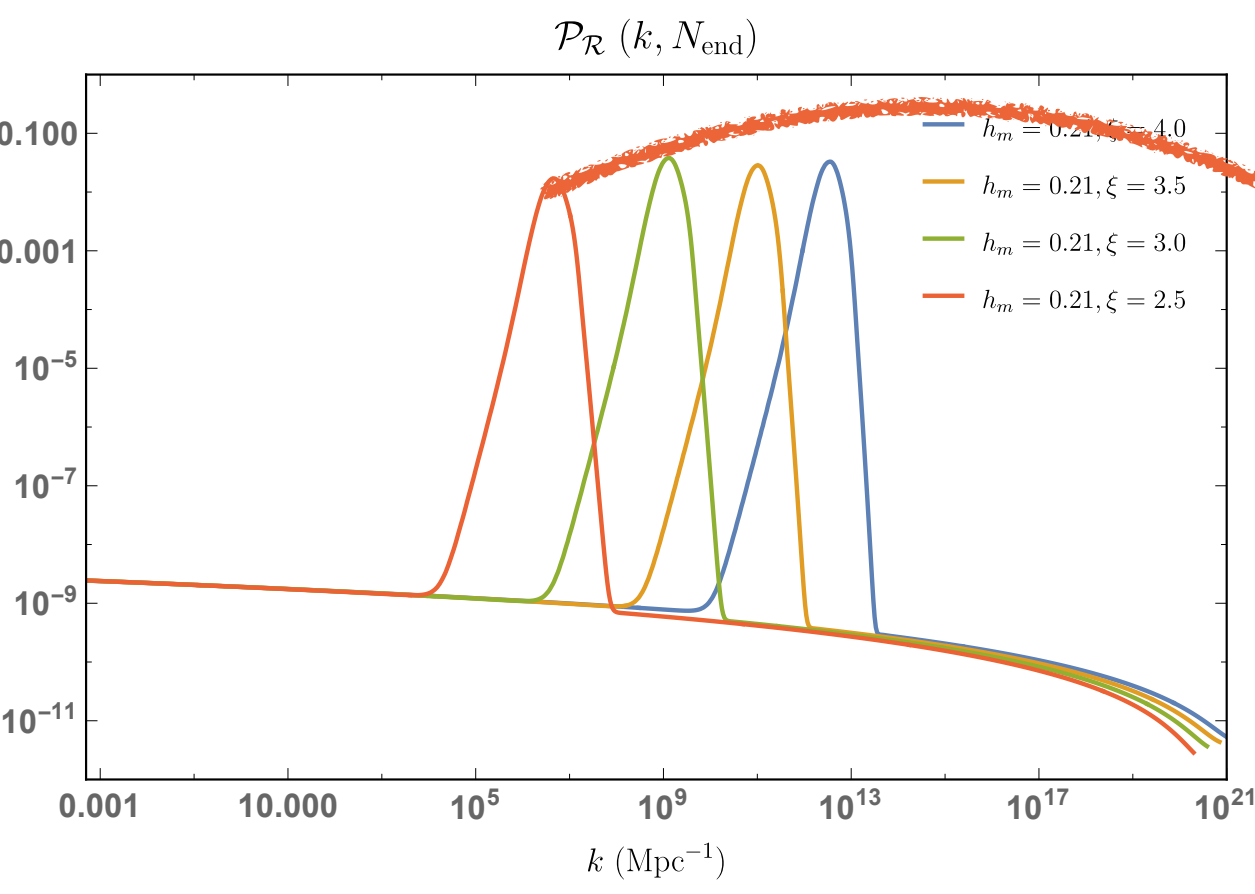


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Phenomena — Second order GWs

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$$\Omega_{\text{GW}} h^2 \sim \frac{1}{12} \Omega_{r,0} h^2 \times \mathcal{P}_{\mathcal{R}}^2 \sim 10^{-6} \mathcal{P}_{\mathcal{R}}^2$$

Possible to match NANOGrav reports

Summary & Outlooks

- ❖ Primordial Black Holes, an appealing DM candidate, can be produced through large curvature perturbations, that can be associated with second order gravitational waves.
- ❖ The Higgs- R^2 model, featuring both a *distinctive valley* and a *noticeable hill*, can exhibit a near inflection point USR phase and / or a tachyonic instability induced by the running λ .
- ❖ Apart from USR induced PBHs / GWs, that limit a significant PBH abundance within the mass range $M_{\text{PBH}} \in (10^{-16}, 10^{-15})M_{\odot}$, the *tachyonic instability* can produce a wide range of PBHs / GWs from *LIGO/Virgo to PTA frequencies*.
- ❖ Further distinguishable features? Direct correlation to collider observables? (m_{top})

“Thank you!”