Axion effective field theory

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Outline of this talk

1. Let's understand axion fondamental interactions

   Typical Lagrangians and phenomenological implications

2. Let's build axion EFTs

   Integrating out chiral fermions from the path integral

3. Let's discus anomalies in EFTs

   A different approach from to so-called Fujikawa's method

A shift of paradigm

- To solve: the hierarchy problem

concretely: why the gravitational force is so much weaker than the other fundamental interactions?

Main candidate,

Supersymmetry : -enlarges Poincaré algebra (new energy scale)

-needs many new particles

-can preserve SM gauge group

- To solve: the strong CP puzzle

concretely: why matter and not anti-matter in our universe?

Main candidate,

'Peccei-Quinn' theory : -enforces CP-symmetry

-needs a new global 'no symmetry'
(anomalous+spontaneously broken)
(new energy scale)

-entangled with SM gauge group :
(careful!)

$$[SU(3)_c \otimes SU(2)_L \otimes U(1)_Y]_{local} \times [U(1)_{\mathcal{B},\mathcal{L},PQ}]_{global}$$

the **QCD axion**: « new » Goldstone bosons combination $\perp Z_L$

The key role of anomaly in QFT

The chiral anomaly of the SM

QFT Anomalies

Anomalies: classical symmetry broken at the quantum level

Example: « triangle anomalies » in massless QED

$$\mathscr{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

Two invariances: $\xrightarrow{\text{(Noether theorem)}}$ Two classicaly conserved currents:

- $\psi \to e^{i\theta_V}\psi$
- $\psi \to e^{-i\theta_A \gamma^5}\psi$

$$V^\mu = \bar{\psi}\gamma^\mu\psi\ , \quad \partial_\mu V^\mu = 0$$

$$A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi\ , \quad \partial_\mu A^\mu = 0$$

$$\Downarrow$$

At the quantum level:

$$V^\mu = \bar{\psi}\gamma^\mu\psi\ , \quad \partial_\mu V^\mu = 0 \quad \text{holds}$$

But axial symmetry is broken :

$$\partial_\mu A^\mu = \frac{1}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Fermionic path integral measure is not invariant: [Fujikawa]

$$Z = \int \mathscr{D}\psi \mathscr{D}\bar{\psi} e^{iS}$$

The Strong CP Puzzle in particle physics

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma^\mu D_\mu - m_q e^{\underset{\text{CPV}}{i\theta_{EW}}})q - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} - \theta_{QCD}\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\underset{\text{CPV}}{\tilde{G}^a_{\mu\nu}}$$

$\bar{q}$ → 4-component Dirac field

$U(1)_A$ chiral transformation: $q \rightarrow e^{i\gamma^5\theta_{EW}}q$ anomalous symmetry

the measure of the path integral is not invariant under this transformation

axial anomaly shifts quark mass phase to QCD vacuum

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma^\mu D_\mu - m_q)q - \frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} - (\overbrace{\theta_{QCD} - \underset{\neq 0}{\theta_{EW}}}^{\bar{\theta}})\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\tilde{G}^a_{\mu\nu}$$

Yukawa coupling to the Higgs are complex $\theta_{CKM} \neq 0$

Why is this strong CP-violation term so puzzling? $\mathcal{L}_{\cancel{CP}} = \bar{\theta}\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\tilde{G}^a_{\mu\nu}$

this induces a huge electric dipole moment for the neutron:

Theory: $|d_n| \sim |\bar{\theta}|10^{-16}e.cm$ vs Experiment: $|d_n| \lesssim 10^{-26}e.cm$

$\longrightarrow \bar{\theta} < 10^{-10}$

The strong CP problem
=Why is $\bar{\theta}$ so small?

The strong CP problem is really why the combination of QCD and EW parameters make up should be so small...

The Peccei-Quinn Axion Solution

axial anomaly: $\theta_{EW}^{\text{CPV}} \longleftrightarrow \theta_{QCD}^{\text{CPV}}$

Solution to the strong CP problem of QCD: add fields such that rotate $\bar{\theta}$ to the phase of a complex SM-singlet scalar who gets a VEV and dynamicaly drives $\bar{\theta} \to 0$ Peccei & Quinn

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma^\mu D_\mu - m_q e^{i\theta_{EW}})q - \frac{1}{4}G_a^{\mu\nu}G_{\mu\nu}^a - \theta_{QCD}\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\tilde{G}_{\mu\nu}^a$$

1. Introduce a new global axial $U(1)_{PQ}$ symmetry S.B. at high scale
   $\longrightarrow$ the low-energy theory has a **Goldstone boson** (the **axion** field)

2. Design $\mathcal{L}_{axion}$ such that $Q(q_L) \neq Q(q_R) \longrightarrow$ this makes the $U(1)_{PQ}$ **anomalous** : $\partial_\mu J^\mu \sim G_{\mu\nu}^a \tilde{G}_a^{\mu\nu}$

   net effect: $\mathcal{L}_{axion} = \mathcal{L}_{QCD} + \frac{a}{v}G_{\mu\nu}\tilde{G}^{\mu\nu} + \ldots$

3. Non-perturbative QCD effects induce:

$$\mathcal{L}_{axion} = \mathcal{L}_{ChPT}(\partial_\mu a, \pi, \eta, \eta', \ldots) + V_{eff}(\bar{\theta} + \frac{a}{v}, \pi, \eta, \ldots)$$

$$\sim -\Lambda_{QCD}^4 cos(\bar{\theta} + \frac{a}{v})$$

minimum of the potential: $\bar{\theta} + \frac{<a>}{v} = 0$ CP-violating term cancels!

CP symmetry is dynamically restored!

Axion or Axion-like

Some phenomenology

Axion Like Particles

typical coupling

- QCD axion has couplings correlated to its mass, $m_a \sim \Lambda^2_{QCD} \boxed{\frac{1}{f_a}}$

Non-trivial topology of the QCD vacuum

Current bounds push the mass well below the eV

-ALP: add an explicit mass term to get a new light pseudo scalar state

$$\mathscr{L}_{ALP} = \frac{1}{2}(\partial_\mu a \partial^\mu a - m_a^2 aa) + \text{couplings to SM particles}$$

No longer solve the strong CP problem

May be a DM candidate

Few might arise from string theory

Mass window spans over sub-eV to few GeV

If the mass is greater than a few GeV: LHC could say something!

How to tackle ALP-SM couplings?

BSM Higgs strategy

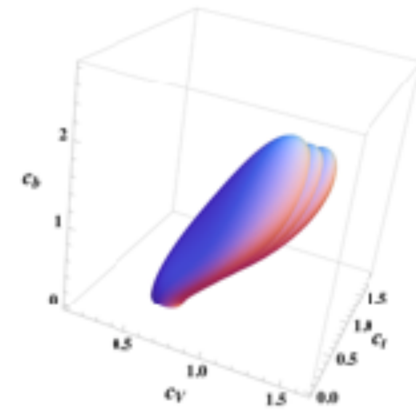
Toy model

(simple, intuitive, model independant, etc.)

Ex: Higgs kappa-framework

$$\mathscr{L}^{BSM}_{Higgs} \supset \kappa_W g^{SM}_{hWW} hW^+W^- + \kappa_Z g^{SM}_{hZZ} hZZ + \kappa_t g^{SM}_{htt} h\bar{t}t + \ldots$$

experimental data ⟶

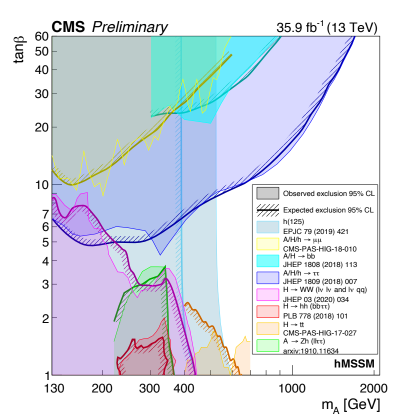


Ultra-Violet model

(solve problems, complicated, many parameters, etc.)

Ex: MSSM

experimental data ⟶



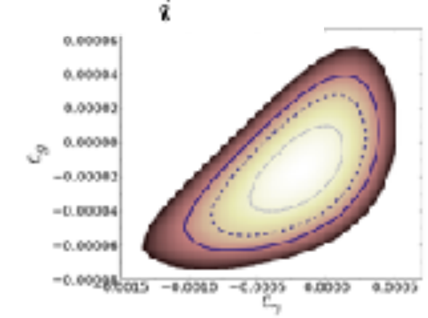
Effective Field Theory

(phenomenological QFT, model independant, etc.)

Ex: SMEFT

$$\mathcal{L}_{\text{SM-EFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

experimental data ⟶



BSM Axion strategy

Ultra-Violet model

Ex: PQWW axion

KSVZ invisible axion

DFSZ invisible axion

} QCD axion

ALP models

etc.

On going theoretical effort

↔

Effective Field Theory

$$\mathscr{L}_{SM-ALP-EFT} = \mathscr{L}_{SM} + \mathscr{L}_a + \mathscr{L}_{a-SM}$$

Ex:

$$\mathscr{L}^{D=5}_{a-SM} \supset \sum_f C_{ff}\frac{\partial^\mu a}{\Lambda}\bar{f}\gamma_\mu\gamma_5 f + C_{GG}\frac{a}{\Lambda}G_{\mu\nu}\tilde{G}^{\mu\nu} + C_{\gamma\gamma}\frac{a}{\Lambda}F_{\mu\nu}\tilde{F}^{\mu\nu} + C_{\gamma Z}\frac{a}{\Lambda}F_{\mu\nu}\tilde{Z}^{\mu\nu} + C_{ZZ}\frac{a}{\Lambda}Z_{\mu\nu}\tilde{Z}^{\mu\nu} + C_{WW}\frac{a}{\Lambda}W_{\mu\nu}\tilde{W}^{\mu\nu}$$

$$\mathscr{L}^{D\geq 6}_{a-SM} \supset \frac{C_{ah}}{\Lambda^2}(\partial_\mu a)(\partial^\mu a)H^\dagger H + \ldots$$

Which basis for ALP-SM couplings?

On going theoretical effort

Axion couplings

Energy

At energies below f_a (SSB):

$$\mathcal{L}_{axion} \supset \frac{\partial_\mu a}{2 f_a} j_a^\mu + \# \frac{a}{f_a} G\tilde{G} + \# \frac{a}{f_a} F\tilde{F} + \# \frac{a}{f_a} Z\tilde{F} + \# \frac{a}{f_a} Z\tilde{Z} + \# \frac{a}{f_a} W\tilde{W}$$

LHC regime

free from (complex) low energy QCD effects
probe different couplings than low energy experiments

electroweak couplings recently computed
do not follow the expected pattern
J.Q. and C. Smith, arXiv:1903.12559, 2006.06778, 2010.13683;
J.Q., C. Smith and P.N.H. Vuong , arXiv:2112.00553

At energies below Λ_{QCD} : $a - \eta' - \pi^0 - \eta - \ldots$ mixing

axion mass: $m_a = m_\pi \frac{f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} \sim \frac{\Lambda^2_{QCD}}{f_a}$

axion couplings to electrons, nucleons, mesons, photons, ...

(EDMs)

mostly explored:

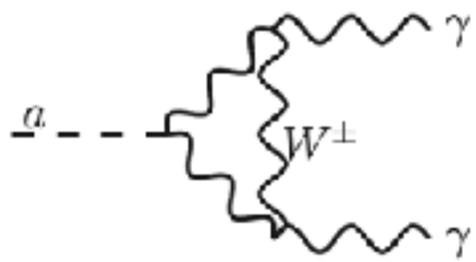
$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - \boxed{1.92}\right)$$

model dep.

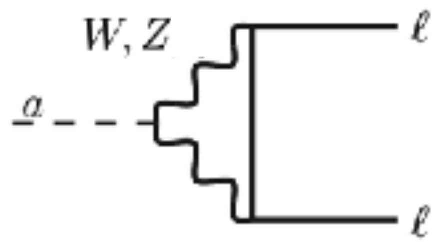
model indep.
below confinement

Axion electroweak couplings

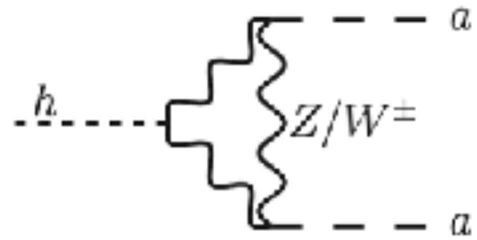
- $a \to \gamma\gamma$:



- $a \to ll$:



- $h \to aa$:

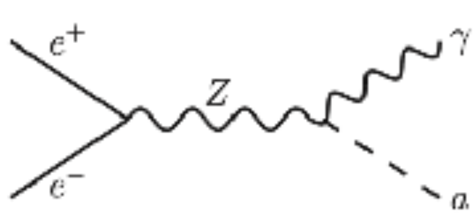


ALP electroweak couplings matters

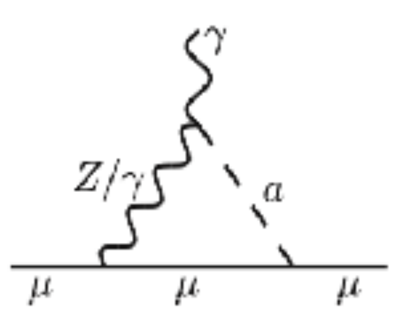
They need to be crucially explored at the LHC!

...

- $e^+e^- \to a\gamma$:



- Muon anomalous magnetic moment:



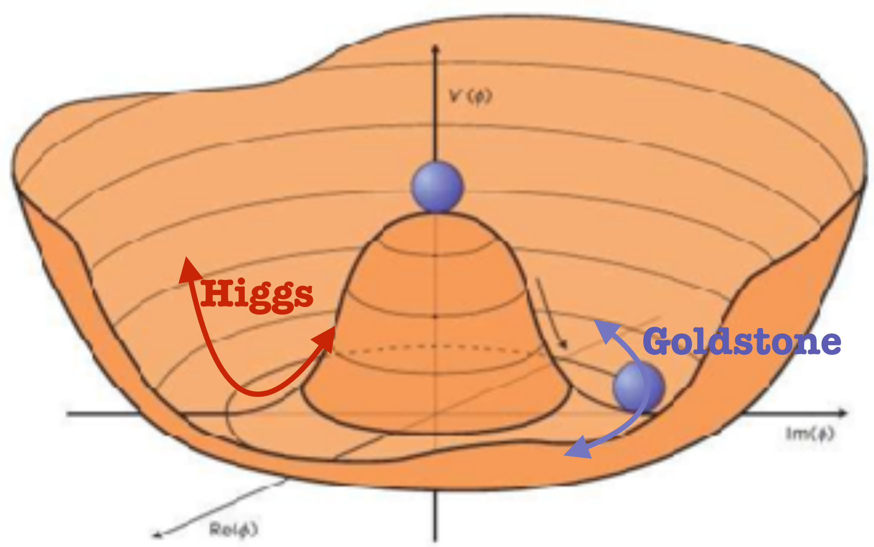
Why axions « have » derivative couplings?

An axionic toy model: simple QED extension

- local $U(1)_{em}$, new scalar field ϕ:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R + (y\phi\bar{\psi}_L\psi_R + h.c.) + \partial_\mu\phi^\dagger\partial^\mu\phi - V(\phi)$$

→ Goldstone boson (**axion**) remnant of $U(1)_{PQ}$ S.S.B.



Linear representation: $\phi(x) = v + \sigma(x) + ia(x)$

$x + iy$

Polar representation: $\phi(x) = \frac{1}{\sqrt{2}}(v + \sigma(x))e^{-ia(x)/v}$

$\rho^{i\theta}$

Linear representation

$$\phi(x) = v + \sigma(x) + i\boxed{a(x)}$$

$$\mathcal{L}_{\mathrm{Linear}} \supset \frac{1}{2}\partial_\mu a^0 \partial^\mu a^0 + \frac{m}{v}\boxed{a\ \bar{\psi} i \gamma_5 \psi}$$

(no tree-level couplings
to gauge fields)

→The axion is a usual pseudo-scalar with no derivative couplings to fermions

Polar representation

$$\phi(x) = \rho e^{-ia(x)/v}$$

To remove « a » from the Yukawa terms $(y\phi\bar{\psi}_L\psi_R + h.c.)$

One **reparametrizes** fermion fields:

$$\psi_L(x) \to \exp(i\alpha a^0(x)/v)\psi_L(x)\ ,\ \ \psi_R(x) \to \exp(i(\alpha+1)a^0(x)/v)\psi_R(x)$$

→Fermion kinetic term induce **derivative interactions**

$$\bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R$$

$$\boxed{\delta\mathcal{L}_{\mathrm{Der}}} = -\frac{\partial_\mu a^0}{v}(\alpha\bar{\psi}_L\gamma^\mu\psi_L + (\alpha+1)\bar{\psi}_R\gamma^\mu\psi_R) = -\frac{\partial_\mu a^0}{2v}((2\alpha+1)\bar{\psi}\gamma^\mu\psi + \bar{\psi}\gamma^\mu\gamma_5\psi)$$

$$\longrightarrow \mathcal{L}_{\mathrm{Polar}} \supset \frac{1}{2}\partial_\mu a^0\partial^\mu a^0 + \boxed{\delta\mathcal{L}_{\mathrm{Der}}} + \boxed{\ ?\ }$$

Polar representation

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \sigma^0(x))e^{-ia^0(x)/v}$$

- Fermionic path integral measure is not invariant under the **fermion reparametrisation**: [Fujikawa]

new local interaction (**anomaly** - Jacobian of the transformation)

$$\boxed{\delta\mathcal{L}_{\rm Jac}} = \frac{e^2}{16\pi^2 v} a^0 \underbrace{(\alpha - (\alpha+1))}_{Q(q_L) \;-\; Q(q_R)} F_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{e^2}{16\pi^2 v} a^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\longrightarrow \mathcal{L}_{\rm Polar} \supset \frac{1}{2}\partial_\mu a^0 \partial^\mu a^0 + \boxed{\delta\mathcal{L}_{\rm Der}} + \boxed{\delta\mathcal{L}_{\rm Jac}}$$

The 2HDM

$$V_{2\text{HDM}} = m_1^2\Phi_1^\dagger + m_2^2\Phi_2^\dagger\Phi_2 + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_2^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)$$

After **Spontaneous Symmetry Breaking** :

- two neutral scalar Higgs bosons: **h** and **H**
- a pair of charged Higgs boson $H^\pm$
- a pseudo-scalar **A**

$$\mathscr{L}_{\text{Yukawa}} = -\bar{u}_R\mathbf{Y}_u q_L\Phi_1 - \bar{d}_R\mathbf{Y}_d q_L\Phi_2^\dagger - \bar{e}_R\mathbf{Y}_e\ell_L\Phi_2^\dagger + h.c.$$

linear or polar rep. !

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 + H_1 + iP_1 \\ H_1^- \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} H_2^+ \\ v_2 + H_2 + iP_2 \end{pmatrix}$$

with $v_1^2 + v_2^2 = v^2 \sim (246\ \text{GeV})^2$

-neutral CP-even Higges: $\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \# & \# \\ \# & \# \end{pmatrix}\begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$

-charged Higges: $\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \# & \# \\ \# & \# \end{pmatrix}\begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix}$

-neutral CP-odd Higges: $\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} P_2 \\ P_1 \end{pmatrix}$

Two standard axion models

PQWW axion :

Peccei, Quinn '77
Weinberg '78
Wilczek '78

axion identified with a phase in a 2HDM ($f_a \sim v_{ew}$) : **ruled out**

phenomenology calls for $f_a \gg v_{ew}$ (« invisible axion »)

method: mix it with a complex SM singlet with « big » VEV

KSVZ axion :

Kim '79
Shifman, Vainshtein, Zakharov '80

New « heavy » electrically neutral quark, charged under $U(1)_{PQ}$
+ a new complex scalar singlet

$$\mathscr{L}_{KSVZ} = \mathscr{L}_{SM} + \bar{\Psi}_{L,R} \not{D} \Psi_{L,R} + y \bar{\Psi}_L \Psi_R \phi + V(\phi)$$

DFSZ axion :

Zhitnitskii '80
Dine, Fischler, Srednicki '81

2HDM, SM quarks and leptons are charged under $U(1)_{PQ}$
+ a new complex scalar singlet

DFSZ axion couplings

1. in the linear representation

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 + H_1 + i\boxed{P_1} \\ H_1^- \end{pmatrix} \qquad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} H_2^+ \\ v_2 + H_2 + i\boxed{P_2} \end{pmatrix}$$

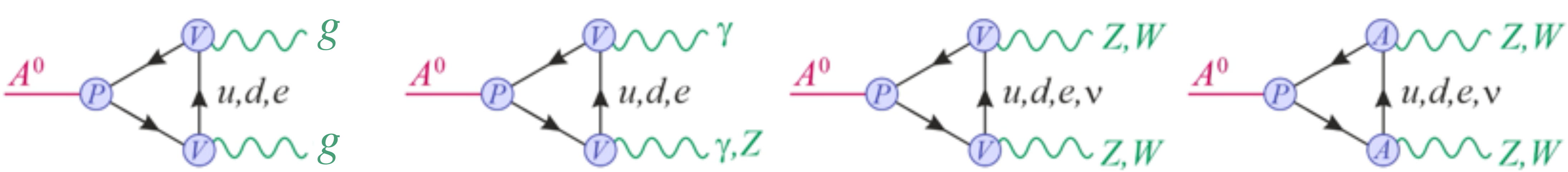
Standard 2HDM phenomenology with $\boxed{A}$

- <u>Axion couplings to fermions :</u> mass-dependent **pseudoscalar couplings**

$$\mathscr{L}_{Aff} = -i \sum_{f=u,d,e} \frac{m_f}{f_a} \chi^f A(\bar{\psi}_f \gamma_5 \Psi_f) \qquad \text{with} \qquad \chi^d = \chi^e = \frac{1}{\chi^u} = \tan\beta$$

- <u>Axion couplings to gauge bosons:</u> **No** $A \rightarrow VV$ **at tree level**

couplings to SM gauge bosons **at one loop**:



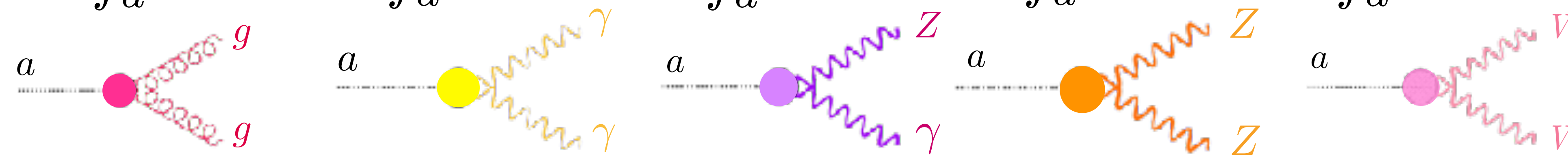
Amplitudes know for a long time J.Gunion et al., PRD 46 (1992) 2907

Finite and **non anomalous** (no ambiguity of any kind)

DFSZ axion couplings to SM gauge fields

1. Axion is a pseudo-scalar with no derivative couplings

$$\mathcal{L}_{\rm axion} \supset \# \frac{a}{f_a} G\tilde{G} + \# \frac{a}{f_a} F\tilde{F} + \# \frac{a}{f_a} Z\tilde{F} + \# \frac{a}{f_a} Z\tilde{Z} + \# \frac{a}{f_a} W\tilde{W}$$



J.Gunion et al., PRD 46 (1992) 2907

in the limit $m_{u,d,e} \to \infty$

$$\begin{aligned}\mathcal{L}^{\rm eff}_{\rm axion} = \frac{a^0}{16\pi^2 f_a} \Big(& g_s^2 \mathcal{N}_C G_{\mu\nu}\tilde{G}^{\mu\nu} \\ & + e^2 \mathcal{N}_{em} F_{\mu\nu}\tilde{F}^{\mu\nu} \\ & + \frac{2e^2}{c_W s_W}(\mathcal{N}_0 - s_W^2 \mathcal{N}_{em}) Z_{\mu\nu}\tilde{F}^{\mu\nu} \\ & + \frac{e^2}{c_W^2 s_W^2}(\mathcal{N}_1 - 2 s_W^2 \mathcal{N}_0 + s_W^4 \mathcal{N}_{em}) Z_{\mu\nu}\tilde{Z}^{\mu\nu} \\ & + 2g^2 \mathcal{N}_2 W_{\mu\nu}\tilde{W}^{\mu\nu} \Big)\end{aligned}$$

$$\mathcal{N}_C = \frac{1}{2}\left(x + \frac{1}{x}\right)$$

$$\mathcal{N}_{em} = N_C\left(\frac{4}{9}x + \frac{1}{9x}\right) + \frac{1}{x}$$

$$\mathcal{N}_0 = \frac{1}{4}\left(N_C\left(\frac{2}{3}x + \frac{1}{3x}\right) + \frac{1}{x}\right)$$

$$\mathcal{N}_1 = \frac{1}{12}\left(N_C\left(x + \frac{1}{x}\right) + \frac{1}{x}\right)$$

$$\mathcal{N}_2 = \frac{1}{12}\left(N_C\left(x + \frac{1}{x}\right) + \frac{3}{2x}\right)$$

$(x \equiv \cot\beta)$

Breaks EW symmetry: $\mathcal{N}_0 \neq \mathcal{N}_1 \neq \mathcal{N}_2$

DFSZ axion couplings

2. in the polar representation

$$\Phi_1 = \frac{1}{\sqrt{2}} \exp\left\{ i \frac{a}{v} x \right\} \begin{pmatrix} \sqrt{2} H_1^+ \\ v_1 + H_1^0 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \exp\left\{ -i \frac{a}{v} \frac{1}{x} \right\} \begin{pmatrix} \sqrt{2} H_2^+ \\ v_2 + H_2^0 \end{pmatrix}$$

Fermion reparametrization: $\psi \to \exp\left\{ i \frac{PQ(\psi)}{v} a \right\} \psi$

Consequence 1 : non-invariance of the kinetic terms

- Axion **derivative** couplings to fermions :

$$\mathcal{L}_{Der} = -\frac{1}{2f_a} \partial_\mu a \sum_{u,d,e,\nu} \chi_V^f (\bar{\psi}_f \gamma^\mu \psi_f) + \chi_A^f (\bar{\psi}_f \gamma^\mu \gamma^5 \psi_f)$$

Freedom/ambiguity in the PQ charge

	u	d	e	ν
χ_V	$2\alpha + x$	$2\alpha + \frac{1}{x}$	$2\beta + \frac{1}{x}$	β
χ_A	x	$\frac{1}{x}$	$\frac{1}{x}$	$-\beta$

Consequence 2 : non-invariance of the fermionic measure

- Anomalous axion couplings to SM gauge fields at **tree-level** :

(Jacobian of the transformation)

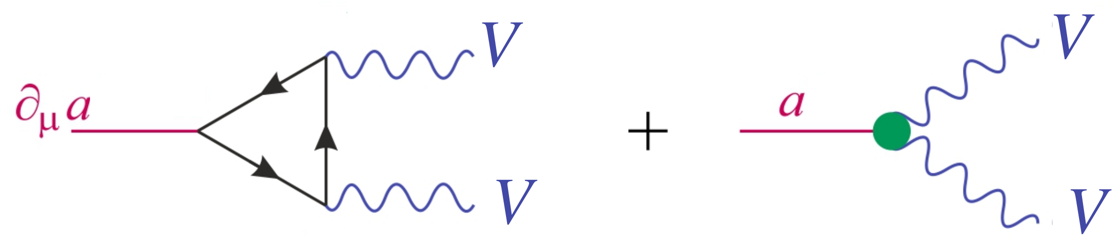
$$\begin{aligned} \delta\mathcal{L}_{Jac} &= \frac{a}{16\pi^2 v} g_s^2 \mathcal{N}_C G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} \\ &+ \frac{a}{16\pi^2 v} g^2 \mathcal{N}_L W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} \\ &+ \frac{a}{16\pi^2 v} g'^2 \mathcal{N}_Y B_{\mu\nu} \tilde{B}^{\mu\nu} \end{aligned} \qquad \begin{aligned} \mathcal{N}_C &= \frac{1}{2}\left(x + \frac{1}{x} \right) \\ \mathcal{N}_L &= -\frac{1}{2}(3\alpha + \beta) \\ \mathcal{N}_Y &= \frac{1}{2}(3\alpha + \beta) + \frac{4}{3}x + \frac{1}{3x} + \frac{1}{x} \end{aligned}$$

DFSZ axion couplings to SM gauge fields

2. Axion has derivative couplings to fermions

couplings to SM gauge bosons at one loop:

Generically :



Amplitudes need to be carefully regularized

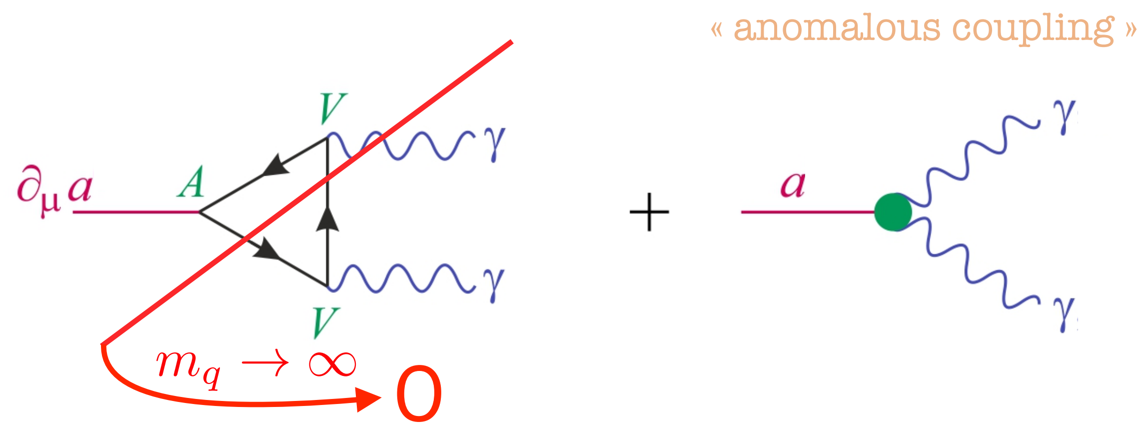
Divergences and ambiguities due to QFT anomalies

DFSZ axion couplings to SM gauge fields

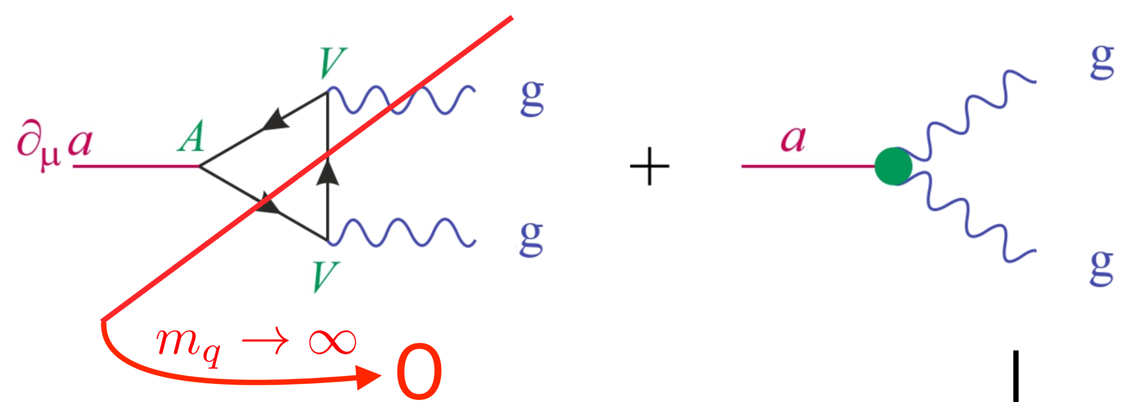
2. Axion has derivative couplings to fermions

Effective couplings to SM gauge bosons at one loop:

Axion-photons :



Axion-gluons:



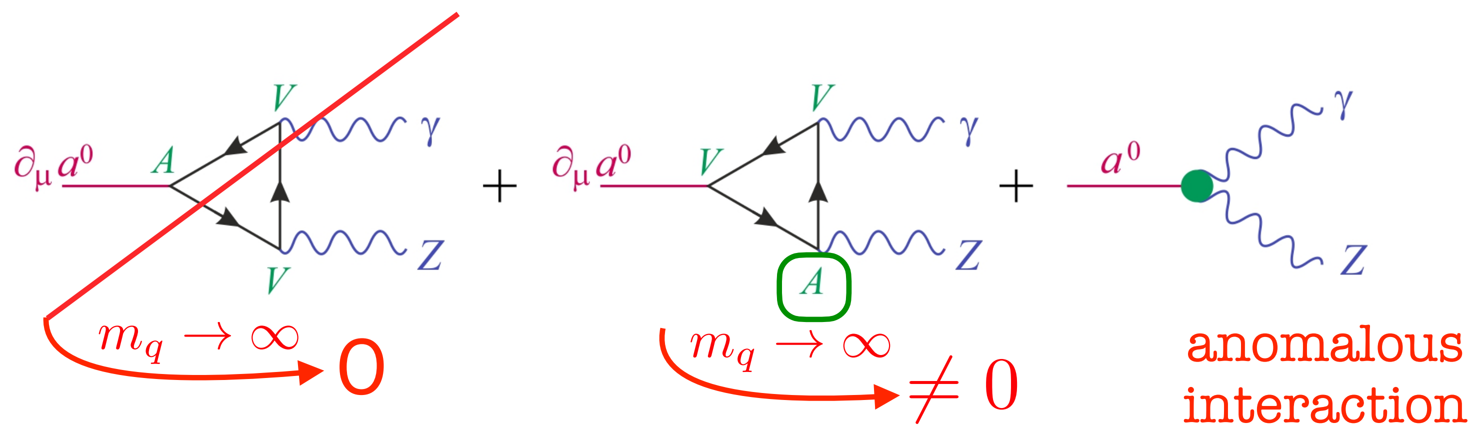
Convenient book-keeping of the effect of heavy fermions

DFSZ axion couplings to SM gauge fields

2. Axion has derivative couplings to fermions

Effective couplings at one loop:

$\underline{a \to \gamma Z:}$



1. **Vector current is not conserved** (anomalous as Baryon-number current)

One has to consider both couplings:

$(\partial_\mu a)\bar{\psi}\gamma^\mu\gamma^5\psi$ **and** $(\partial_\mu a)\bar{\psi}\gamma^\mu\psi$

↓

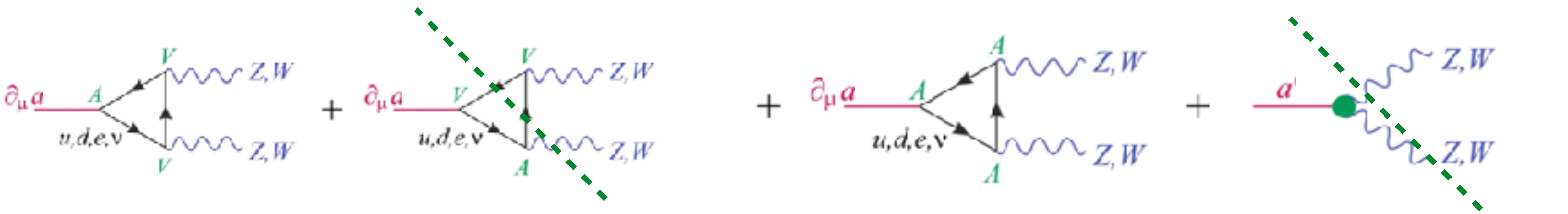
not a reliable **book-keeping** of the effect of heavy fermions

DFSZ axion couplings to SM gauge fields

2. Axion has derivative couplings to fermions

Effective couplings at one loop:

$\underline{a \to ZZ, W^+W^- :}$



contribute | partially contribute | contribute | **does not** contribute

Freedom/ambiguity in the PQ charge cancel exactly

2. The anomalous contact int. does cancel out systematically with the anomalous part to the triangle graphs

$$\mathcal{L}_{\text{axion-gauge}} = \boxed{\delta\mathcal{L}_{\text{Der}}} + \boxed{\delta\mathcal{L}_{\text{Jac}}}$$

~~finite+divergence~~ ~~anomaly~~

« Polar = Linear »

Polar representation:

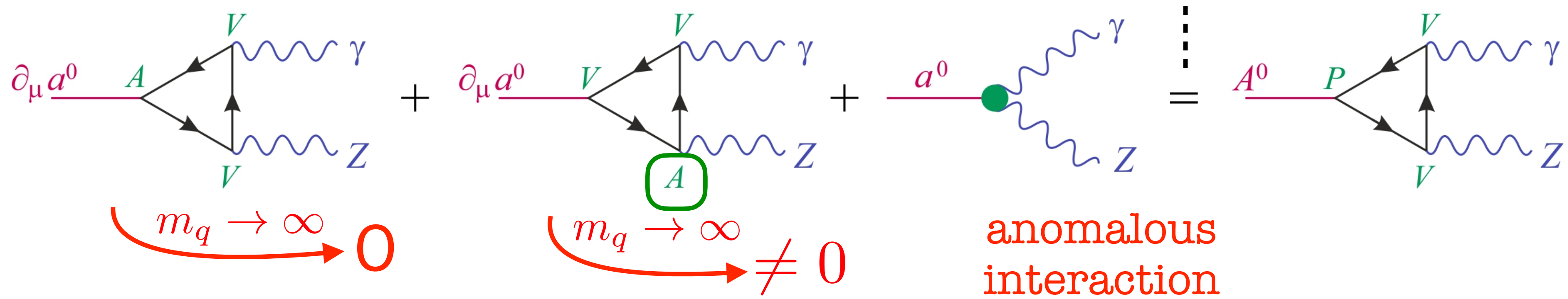
Axial current $A = \bar{\psi}\gamma^\mu\gamma_5\psi$

Vector current $V = \bar{\psi}\gamma^\mu\psi$

Linear representation:

Pseudo-scalar current $P = \bar{\psi}\gamma_5\psi$

- $a \to \gamma Z$:



anomalous interaction

Vector current is not conserved

One has to consider both couplings:

$(\partial_\mu a)\bar{\psi}\gamma^\mu\gamma^5\psi$ **and** $(\partial_\mu a)\bar{\psi}\gamma^\mu\psi$

not a reliable **book-keeping** of the effect of heavy fermions

- idem for ZZ and WW

DFSZ axion summary

$$\mathcal{L}^{\rm eff} = \frac{a^0}{16\pi^2 v}\left(g_s^2\mathcal{N}^{gg}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + e^2\mathcal{N}^{\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{2e^2}{c_W s_W}\left(\mathcal{N}_1^{\gamma Z} - s_W^2\mathcal{N}_2^{\gamma Z}\right)Z_{\mu\nu}\tilde{F}^{\mu\nu}\right.$$
$$\left. + \frac{e^2}{c_W^2 s_W^2}\left(\mathcal{N}_1^{ZZ} - 2s_W^2\mathcal{N}_2^{ZZ} + s_W^4\mathcal{N}_3^{ZZ}\right)Z_{\mu\nu}\tilde{Z}^{\mu\nu} + 2\mathcal{N}^{WW}g^2 W^+_{\mu\nu}\tilde{W}^{-,\mu\nu}\right)$$

in the limit $m_{u,d,e} \to \infty$

Linear $a^0\bar{\psi}\gamma_5\psi$	Anomalous interactions	Polar $\partial_\mu a^0\bar{\psi}\gamma^\mu\gamma_5\psi$ AVV	AAA	$\partial_\mu a^0\bar{\psi}\gamma^\mu\psi$ VAV
$\mathcal{N}^{gg} = \frac{1}{2}\left(x + \frac{1}{x}\right)$	$\mathcal{N}^{gg}$	0	$-$	$-$
$\mathcal{N}^{\gamma\gamma} = \frac{4}{3}\left(x + \frac{1}{x}\right)$	$\mathcal{N}^{\gamma\gamma}$	0	$-$	$-$
$\mathcal{N}_1^{\gamma Z} = \frac{1}{2}\left(x + \frac{1}{x}\right)$	$\mathcal{N}_L$	0	$-$	$\mathcal{N}_1^{\gamma Z} - \mathcal{N}_L$
$\mathcal{N}_2^{\gamma Z} = \mathcal{N}^{\gamma\gamma}$	$\mathcal{N}^{\gamma\gamma}$	0	$-$	0
$\mathcal{N}_1^{ZZ} = \frac{1}{4}x + \frac{1}{3x}$	$\mathcal{N}_L$	$\frac{\beta}{16}$	$-\frac{1}{2}\mathcal{N}_1^{ZZ} + \frac{\beta}{16}$	$\frac{3}{2}\mathcal{N}_1^{ZZ} - \mathcal{N}_L - \frac{\beta}{8}$
$\mathcal{N}_2^{ZZ} = \mathcal{N}_1^{\gamma Z}$	$\mathcal{N}_L$	0	0	$\mathcal{N}_2^{ZZ} - \mathcal{N}_L$
$\mathcal{N}_3^{ZZ} = \mathcal{N}^{\gamma\gamma}$	$\mathcal{N}^{\gamma\gamma}$	0	0	0
$\mathcal{N}^{WW} = \frac{x}{4} + \frac{3}{8x}$	$\mathcal{N}_L$	$\frac{3}{2}\mathcal{N}^{WW} - \frac{3}{2}\mathcal{N}_1^{\gamma Z} + \frac{\beta}{16}$	$-\frac{1}{2}\mathcal{N}^{WW} + \frac{\beta}{16}$	$\frac{3}{2}\mathcal{N}_1^{\gamma Z} - \mathcal{N}_L - \frac{\beta}{8}$

J.Gunion et al., PRD 46 (1992) 2907

$x = v_2/v_1 = 1/\tan\beta$

J.Q. and C. Smith, arXiv:1903.12559

Effective interactions are not always equal to anomalous interactions!

Remember that $\mathcal{N}_L$ is ambiguous

Implication for ALPs searches

How to construct a truly **axion-like** basis?

$$\mathcal{L}^{\rm eff}_{ALP} = \frac{1}{2}(\partial_\mu a^0 \partial^\mu a^0 - m_a^2 a^0 a^0) + \mathcal{L}_{\rm KSVZ-like} + \mathcal{L}_{\rm DFSZ-like}$$

KSVZ like: New, heavy, electrically neutral quark, charged under $U(1)_{\rm PQ}$

$$\mathcal{L}^{\rm eff}_{\text{KSVZ-like}} = \frac{a^0}{16\pi^2 f_a}\left(g_s^2 \mathcal{N}_C G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + g^2 \mathcal{N}_L W_{\mu\nu}\tilde{W}^{\mu\nu} + g'^2 \mathcal{N}_Y B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$$

- Typically assuming some heavy **vector-like** fermions
- No direct coupling to SM fermions
- Manifestly symmetric under $SU(3)_C \otimes SU(2)_L \otimes U(1)_L$

Implication for ALPs searches

How to construct a truly **axion-like** basis?

$$\mathcal{L}_{ALP}^{\text{eff}} = \frac{1}{2}(\partial_\mu a^0 \partial^\mu a^0 - m_a^2 a^0 a^0) + \mathcal{L}_{\text{KSVZ}-\text{like}} + \mathcal{L}_{\text{DFSZ}-\text{like}}$$

DFSZ like: 2HDM, SM quarks and leptons are charged under $U(1)_{\text{PQ}}$

$$\mathcal{L}_{\text{DFSZ-like}}^{\text{eff}} = -\frac{1}{2f_a}\partial_\mu a \sum_{f\,=\,\text{chiral fermions}} \chi_V^f \bar{\psi}_f \gamma^\mu \psi_f + \chi_A^f \bar{\psi}_f \gamma^\mu \gamma^5 \psi_f$$

$$+ \frac{a}{16\pi^2 f_a}\left(g_s^2 \mathcal{N}_C G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + g^2 \mathcal{N}_L W_{\mu\nu} \tilde{W}^{\mu\nu} + g'^2 \mathcal{N}_Y B_{\mu\nu} \tilde{B}^{\mu\nu}\right)$$

- **Vector currents do contribute** to physical observables
- Spurious $\mathscr{B}$ and $\mathscr{L}$ violation included
- Axion-like $\Rightarrow$ **need to impose anomaly cancellation**!

Implication for ALPs searches

How to construct a truly **axion-like** basis?

$$\mathcal{L}_{ALP}^{\text{eff}} = \frac{1}{2}(\partial_\mu a^0 \partial^\mu a^0 - m_a^2 a^0 a^0) + \mathcal{L}_{\text{KSVZ}-\text{like}} + \mathcal{L}_{\text{DFSZ}-\text{like}}$$

KSVZ like: New, heavy, electrically neutral quark, charged under $U(1)_{\text{PQ}}$

$$\mathcal{L}_{\text{KSVZ-like}}^{\text{eff}} = \frac{a^0}{16\pi^2 f_a}\left(g_s^2 \mathcal{N}_C G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + g^2 \mathcal{N}_L W_{\mu\nu} \tilde{W}^{\mu\nu} + g'^2 \mathcal{N}_Y B_{\mu\nu} \tilde{B}^{\mu\nu}\right)$$

- Typically assuming some heavy **vector-like** fermions
- No direct coupling to SM fermions
- Manifestly symmetric under $SU(3)_C \otimes SU(2)_L \otimes U(1)_L$

DFSZ like: 2HDM, SM quarks and leptons are charged under $U(1)_{\text{PQ}}$

$$\mathcal{L}_{\text{DFSZ-like}}^{\text{eff}} = -\frac{i}{f_a} a^0 \sum_{f\,=\,\text{chiral fermions}} m_f \chi_A^f (\bar{\psi}_f \gamma_5 \psi_f)$$

Anomaly cancellation taken into account!

Simple pseudo-scalar couplings

- One should not build EFTs with both **anomalous couplings** and **vectorial-axial fermion couplings** : because of **anomaly cancellations**!
- Effective interactions are not always equal to anomalous interactions!

Several interesting phenomenological aspects

Baryon & Lepton number, Seesaw, GUTs

Axion and Baryon & Lepton number

2HDM of type II: $\mathcal{L}_{\text{Yukawa}} = -\bar{u}_R \mathbf{Y}_u q_L \Phi_1 - \bar{d}_R \mathbf{Y}_d q_L \Phi_2^\dagger - \bar{e}_R \mathbf{Y}_e \ell_L \Phi_2^\dagger + h.c.$

$$\boxed{U(1)_{\mathcal{B}} \otimes U(1)_{\mathcal{L}}} \otimes \boxed{U(1)_1 \otimes U(1)_2} \otimes SU(2)_L \otimes SU(3)_C \xrightarrow[x = v_2/v_1]{\text{EWSB}} U(1)_{\mathcal{B}} \otimes U(1)_{\mathcal{L}} \otimes U(1)_{em} \otimes SU(3)_C$$

2 neutral Goldstone bosons: a, Z_L

$a \perp Z_L$

$$PQ(\Phi_1, \Phi_2, \phi) = \left(x \; , \; -\frac{1}{x} \; , \; \frac{1}{2}\left(x + \frac{1}{x}\right)\right)$$

DFSZ

$$\Rightarrow PQ(q_L, u_R, d_R, \ell_L, e_R) = (\alpha, \alpha + x, \alpha + \frac{1}{x}, \beta, \beta + \frac{1}{x})$$

2 parameters ambiguity

At this stage no way to fix α & β

Ambiguity due to the invariance of the Yukawa couplings under $\mathscr{B}$ & $\mathscr{L}$

$\Rightarrow$ to be used to accommodate $\mathscr{B}, \mathscr{L}$ violation

Axion and the seesaw mechanism

Majorana mass term: $\mathcal{L}_{\nu_R} = -\frac{1}{2}\bar{\nu}_R^C \mathbf{M}_R \nu_R + \bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_i + h.c.\ .$

$$\Rightarrow \begin{cases} \bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_1 : PQ(\nu_R) = \beta + x = 0\ , \\ \bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_2 : PQ(\nu_R) = \beta - \frac{1}{x} = 0\ . \end{cases}$$

still: $PQ(q_L, u_R, d_R, \ell_L, e_R) = (\alpha, \alpha + x, \alpha + \frac{1}{x}, \beta, \beta + \frac{1}{x})$

- No ambiguity on β since $U(1)_{\mathscr{L}}$ has never been a symmetry: β is fixed
- Introduce operator and then set β, **not the contrary!**

νDFSZ: $\mathcal{L}_{\nu_R} = -\frac{1}{2}\bar{\nu}_R^C \mathbf{Y}_R \nu_R \phi + \bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_i + h.c.\ .$

$$\Rightarrow \quad PQ(\nu_R) = -PQ(\phi)/2 \ \neq 0$$

$$\Rightarrow \begin{cases} \bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_1 \Rightarrow \beta = -\frac{1}{4}\left(5x + \frac{1}{x}\right) \\ \bar{\nu}_R \mathbf{Y}_\nu \ell_L \Phi_2 \Rightarrow \beta = -\frac{1}{4}\left(x - \frac{3}{x}\right) \end{cases}$$

still: ....

- $U(1)_{\mathscr{L}} \subset U(1)_1 \times U(1)_2$ does not correspond to the usual Lepton number
- $U(1)_{\mathscr{L}}$: never occurs at low energy
- **axion = majoron** and still solve the strong CP-problem

Axion and GUT

- Let's embed the axion into $SU(5)$ $\begin{cases} \mathscr{B}-\mathscr{L} \text{ conserving} \\ \mathscr{B}+\mathscr{L} \text{ violating} \end{cases}$

$\longrightarrow$ one of the ambiguity immediately disappears:

$$3\alpha+\beta=-\left(x+\frac{1}{x}\right)\equiv 2\underbrace{\mathcal{N}_{SU(5)}}_{\text{anomaly coefficients}}$$

Rq: constraint not compatible with instanton requirement: $3\alpha+\beta=0$

$\mathscr{L}^{\text{eff}}_{\text{inst}} \propto l_L^3 q_L^9$

- In axion models, PQ charges of the 2 Higgs doublets and the fermions are the same up to the value of α and β

$\rightarrow$ this comes from the orthogonality condition among Goldstone bosons (Yukawa couplings)

$\Rightarrow$the low energy phenomenology of the axion is the same in all these models since axions couplings are independent of α and β!

often obscured by the normalisation of the PQ charges

Conclusion

- Axion couplings to gauge bosons are not anomaly-driven
- Essential to keep the axion vector currents in ALP basis
- Axion-electroweak couplings do not always follow the expected pattern
  $\rightarrow$ must be kept in mind for ALP searches
- Switching to the linear representation is safer (no ambiguity)
- The ambiguity in PQ charges to be used to accommodate $\mathscr{B}$, $\mathscr{L}$ violation
- The low energy phenomenology of the DFSZ axion is the same in all models since axion couplings are independent of those ambiguities

often obscured by the normalisation of the PQ charges

1. Let’s understand axion fondamental interactions

Typical Lagrangians and phenomenological implications

→ 2. Let’s build axion EFTs

Integrating out chiral fermions from the path integral

Generalities with fermions in gauge theory

We start from a generic UV Lagrangian exhibiting some set of local symmetries and involving fermionic degrees of freedom :

$$\mathscr{L}_{\text{UV}}^{\text{fermion}} = \bar{\Psi}\left(i\partial_\mu\gamma^\mu + g_{_V} V_\mu\gamma^\mu - g_{_A} A_\mu\gamma^\mu\gamma^5\right)\Psi$$

This theory is invariant under a set of gauge transformation :

$$V_\mu \to V_\mu + \partial_\mu\theta(x)$$
$$A_\mu \to A_\mu - \partial_\mu\theta(x)$$
$$\Psi \to e^{ig_{_V}\theta(x) + ig_{_A}\theta(x)\gamma^5}\Psi$$

Our goal is to integrate out the fermion to get the EFT
(i.e get the tower of effective interactions by performing an inverse mass expansion)

This obviously means that the fermion to be integrated out should be massive, which forces the axial gauge symmetry to be spontaneously broken

Spontaneous symmetry breaking & fermion mass

Let's include the **complex scalar field**, ϕ_A which by acquiring a VEV, will spontaneously break the axial gauge symmetry :

$$\mathscr{L}_{\mathrm{UV}}^{\mathrm{fermion}} = \bar{\Psi}\left(i\partial_\mu\gamma^\mu + g_{_V}V_\mu\gamma^\mu - g_{_A}A_\mu\gamma^\mu\gamma^5 - y_\Psi\phi_A\right)\Psi$$

To focus on **manifest gauge invariance**, it is convenient to include the **Goldstone boson**, π_A, which explicitly enters the **exponential representation** of ϕ_A :

$$\phi_A = (v + \sigma_A)\, e^{i2g_A\frac{\pi_A}{v}\gamma_5}$$

is gauge invariant, so it plays no rôle!

One does not need to fully define the UV theory « EFT-oriented »

Thanks to the exponential parametrization of the Goldstone boson this theory is still manifestly gauge invariant, with the **shift transformation** :

$$\pi_A \to \pi_A - v\theta$$

Linear representation

By contrast, if one insists on **manifest renormalizability**, it is convenient to insert the **Goldstone boson linearly** :

$$\phi_A = \underbrace{v + \sigma_A} + \underbrace{i\pi_A}$$

A $U(1)_A$ gauge transformation mixes these 2 components

$\longrightarrow$ It is only by fully specifying the **UV theory** that one could maintain **both manifest renormalizability** & **manifest gauge invariance**

« Polar $\sim$ linear »

It seems legitimate to consistently account for spontaneously breaking of the axial gauge symmetry, to consider the **exponential parametrization** of the Goldstone boson,

$$\mathscr{L}_{\mathrm{UV}}^{\mathrm{fermion}} = \bar{\Psi}\Big(i\partial_\mu\gamma^\mu + g_{_V} V_\mu \gamma^\mu - g_{_A} A_\mu \gamma^\mu \gamma^5 - M \underbrace{e^{i\,2g_{_A}\frac{\pi_A}{v}\gamma^5}}_{1+\frac{2g_A}{v}\boxed{i\pi_A\gamma_5}+\dots}\Big)\Psi$$

Taylor expansion

The $\pi_A \to \bar{\Psi}\Psi$ coupling is of pseudoscalar type

and is the same as it would be in the **linear rep.** of ϕ_A

(whatever the breaking chain)

- To evaluate the **one-loop effective action**, we will **truncate** the expansion to that leading term, since we are interested only in operators at most linear in a given Goldstone boson.
- Issues related to the apparent **non-renormalizability** of the exponential parametrization will not affect our developments

Fermion reparametrization

Instead of truncating the exponential parametrization, there is an **exact** procedure to recover a linearized Lagrangian which allows to transfer the Goldstone dependence from the Yukawa sector to the gauge sector, perform a field-dependent reparametrization of the fermion fields:

$$\Psi \to \Psi = e^{-ig_A \frac{\pi_A(x)}{v}\gamma^5}\Psi$$

is now gauge invariant

so the mass term does not cause any trouble even for a chiral gauge symmetry
and could easily be factorized out for an EFT mass expansion

$$\longrightarrow \quad \mathscr{L}_{\mathrm{UV}}^{\mathrm{fermion}} = \bar{\Psi}\left(i\partial_\mu\gamma^\mu - M + g_V V_\mu \gamma^\mu - g_A\left[A_\mu - \frac{\partial_\mu \pi_A(x)}{v}\right]\gamma^\mu\gamma^5\right)\Psi$$

This quadratic operator has the virtue of being **manifestly gauge invariant**
(but not renormalizable)

- The Goldstone boson ensures the theory stays gauge invariant :
  so one should **not** get rid of them by moving to the **unitary gauge**

- This form looks particularly well suited for an inverse mass expansion since $M \sim v$
  but one should not be tempted to neglect the dim 5 operator at this stage!

Anomalies

One crucially important caveat:

the **fermion field reparametrization** does not leave the **path integral measure** invariant

(the field being chiral)

The Jacobian of the transformation sums up to additional terms in the Lagrangian of the form:

$$\mathscr{L}_{\mathrm{UV}} \supset \mathscr{L}_{\mathrm{UV}}^{\mathrm{Jac}} = \frac{1}{16\pi^2} g_A g_V^2 \left[\frac{\pi_A(x)}{v} F_V^{\mu\nu} \tilde{F}_V^{\mu\nu} \right] + \frac{1}{48\pi^2} g_A^3 \left[\frac{\pi_A(x)}{v} F_A^{\mu\nu} \tilde{F}_A^{\mu\nu} \right]$$

These terms **explicitly break the gauge invariance**

(they get shifted under $\pi_A \to \pi_A - v\theta$)

- For **abelian** gauge theories, this is not a serious problem since the change in the Lagrangian sum up to an innocuous **surface term**
- But since **our goal** is to consider **SM gauge interactions**, this issue must be addressed

Two main ways to deal with Anomalies

1) If one want to hold the interactions to be gauged, a first possibility consists in tunning the chiral fermionic content such that the **total contribution to the anomaly vanishes** (as in SM)

Goldstones are allowed to be moved to and from the mass terms without generating a Jacobian

i.e strict equivalence between the $\bar{\Psi}\left(\partial_\mu \pi_A \gamma^\mu \gamma^5\right)\Psi$ and $\bar{\Psi}\left(M\gamma_5 \pi_A / v\right)\Psi$ couplings can be viewed as the transcription of the non-anomalous Ward identity $\partial_\mu A^\mu = 2iMP$ $\begin{cases} A^\mu = \bar{\Psi}\gamma^\mu\gamma^5\Psi \\ P = \bar{\Psi}\gamma^5\Psi \end{cases}$

2) give up gauge invariance to a global symmetry

Scenario with combined situations

Generically, the theory corresponds to :

$$\mathscr{L}_{\mathrm{UV}} \supset \mathscr{L}_{\mathrm{UV}}^{\mathrm{fermion}} + \mathscr{L}_{\mathrm{UV}}^{\mathrm{Jac}}$$

Would be Goldstones

$$\supset \bar{\Psi}\left[i\partial_\mu \gamma^\mu - M + \left(\mathrm{V}_\mu - \frac{\boxed{\partial_\mu}\pi_V}{v_V} \right)\gamma^\mu - \left(\mathrm{A}_\mu - \frac{\boxed{\partial_\mu}\pi_A}{v_A} \right)\gamma^\mu \gamma^5 \right.$$

Goldstones

$$\left\{ \begin{aligned} & \left. - \left(0 - \frac{\boxed{\partial_\mu}\pi_S}{v_S} \right)\gamma^\mu - \left(0 - \frac{\boxed{\partial_\mu}\pi_P}{v_P} \right)\gamma^\mu \gamma^5 \right] \Psi \\ & + \frac{1}{16\pi^2}\frac{\pi_P}{v_P}\left(F_V^{\mu\nu}\tilde{F}_V^{\mu\nu} + \frac{1}{3}F_A^{\mu\nu}\tilde{F}_A^{\mu\nu} \right) + \frac{1}{16\pi^2}\frac{\pi_S}{v_S}F_A^{\mu\nu}\tilde{F}_V^{\mu\nu} \end{aligned} \right.$$

So, let us proceed and integrate out the fermion field
involving **local partial derivative** in its quadratic operator

Evaluation: Feynman diagrams vs path integral

- If one decides to use Feynman diagrams to integrate out fermions, one will have to deal with common divergent triangle amplitudes that one will have to carefully regularise

  Even if this is a standard manipulation in QFT, the potential **spread of the anomaly** due to **group mixing** have to be considered with high care

  J.Q. and C. Smith, arXiv:1903.12559

- In the functional approach, the fact that the axial couplings are anomalous manifests itself by the presence of ambiguities in the functional trace

  (the ambiguity is localised in the Dirac matrix traces if one choose to use dimensional regularisation)

  J.Q., C. Smith, Pham Ngoc Hoa Vuong, arXiv: 2112.00553

Functional matching at one loop

$$\int d^d x \mathscr{L}_{\text{EFT}}^{\text{1-loop}}[\phi] = \frac{i}{2} \ln\det\left(- \frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta\varphi^2}\Big|_{\Phi=\Phi_c[\phi]} \right)\Bigg|_{\text{hard}}$$

The functional approach is powerful because it exists an elegant method to compute this determinant based on:

- **method of regions** : one just needs hard region no IR details
- **CDE** : one works with $D_\mu, \phi \Rightarrow$ directly gauge invariant operators, no momentum-space Feynman rules

Integrating-out heavy fermions

S.A.R. Ellis, JQ, P. N. H. Vuong, T. You, Z. Zhang, arXiv:2006.16260

Generic UV Lagrangian involving fermions:

$$\mathcal{L}_{\mathrm{UV}}[\phi, \Psi] = \mathcal{L}_0[\phi] + \bar{\Psi}\left(\not{P} - M - X[\phi]\right)\Psi$$

Generic couplings with background fields:

$$X[\phi] = \underbrace{W_0[\phi]}_{\text{scalar}} + \underbrace{iW_1[\phi]\gamma^5}_{\text{pseudo-scalar}} + \overbrace{V_\mu[\phi]\gamma^\mu}^{\text{vector}} + \overbrace{A_\mu[\phi]\gamma^\mu\gamma^5}^{\text{axial-vector}}$$

$$S_{\mathrm{eff}}^{\text{1-loop}} = -i\,\mathrm{Tr}\ln\left(\not{P} - M - X[\phi]\right)$$

After integrating out heavy fermions:

$$\mathcal{L}_{\mathrm{eff}}^{\text{1-loop}} = i\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\left[\frac{-1}{\not{q}+M}\left(-\not{P} + W_0[\phi] + iW_1[\phi]\gamma^5 + V_\mu[\phi]\gamma^\mu + A_\mu[\phi]\gamma^\mu\gamma^5\right)\right]^n$$

-Expand order by order (ex: up to n=6)
-Integrate over momentum q (careful to γ^5 in D-dimension)
-Evaluate the Dirac traces

Covariant Derivative Expansion and Dim. Reg.

$$\mathcal{L}_{\text{eff}}^{\text{1-loop}} = i\,\text{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\left[\frac{-1}{\not{q}+M}\left(-\not{P}+W_0[\phi]+iW_1[\phi]\boxed{\gamma^5}+V_\mu[\phi]\gamma^\mu+A_\mu[\phi]\gamma^\mu\boxed{\gamma^5}\right)\right]^n$$

"subtlety": How to have enough freedom in **dim. reg.** to choose which currents are conserved or not?

- In d>4 dimension: $\{\gamma^\mu,\gamma^5\}=0$ & trace cyclicity can **not** hold simultaneously

- The usual ambiguity (choice of integration variables) $\longrightarrow$ ambiguity on the location of γ^5
  for divergent integrals
  t' Hooft & Veltman

- One can uses this ambiguity → free parameters → decide if a symmetry is broken **or not**
  (in Slavnov–Taylor identity)

$$\text{tr}\left(\gamma_a\not{V}^i\gamma_b\not{V}^j\gamma_c\not{P}\gamma_d\not{A}^k\gamma^5\right)\Big|_{d=4-\epsilon}\longrightarrow\boxed{\alpha_1}\text{tr}\left(\gamma_a\not{V}^i\gamma^5\gamma_b\not{V}^j\gamma_c\not{P}\gamma_d\not{A}^k\right)\Big|_{d=4-\epsilon}+\boxed{\beta_1}\text{tr}\left(\gamma_a\not{V}^i\gamma_b\not{V}^j\gamma^5\gamma_c\not{P}\gamma_d\not{A}^k\right)\Big|_{d=4-\epsilon}$$
$$+\boxed{\theta_1}\text{tr}\left(\gamma_a\not{V}^i\gamma_b\not{V}^j\gamma_c\not{P}\gamma^5\gamma_d\not{A}^k\right)\Big|_{d=4-\epsilon}+\boxed{\eta_1}\text{tr}\left(\gamma_a\not{V}^i\gamma_b\not{V}^j\gamma_c\not{P}\gamma_d\not{A}^k\gamma^5\right)\Big|_{d=4-\epsilon}$$

Scenario with combined situations

Generically, the theory corresponds to :

$$\mathscr{L}_{\text{UV}} \supset \mathscr{L}_{\text{UV}}^{\text{fermion}} + \mathscr{L}_{\text{UV}}^{\text{Jac}}$$

Would be Goldstones

$$\supset \bar{\Psi}\left[i\partial_\mu\gamma^\mu - M + \left(\mathrm{V}_\mu - \frac{\boxed{\partial_\mu}\pi_V}{v_V}\right)\gamma^\mu - \left(\mathrm{A}_\mu - \frac{\boxed{\partial_\mu}\pi_A}{v_A}\right)\gamma^\mu\gamma^5\right.$$

Goldstones

$$\left. - \left(0 - \frac{\boxed{\partial_\mu}\pi_S}{v_S}\right)\gamma^\mu - \left(0 - \frac{\boxed{\partial_\mu}\pi_P}{v_P}\right)\gamma^\mu\gamma^5\right]\Psi$$

$$+\frac{1}{16\pi^2}\frac{\pi_P}{v_P}\left(F_V^{\mu\nu}\tilde{F}_V^{\mu\nu} + \frac{1}{3}F_A^{\mu\nu}\tilde{F}_A^{\mu\nu}\right) + \frac{1}{16\pi^2}\frac{\pi_S}{v_S}F_A^{\mu\nu}\tilde{F}_V^{\mu\nu}$$

$$\longrightarrow \quad \mathscr{L}_{\text{EFT}}^{\text{1loop}} \supset \underline{\omega_{AVV}}\frac{\partial_\mu\pi_P}{v_P}\mathrm{V}_\nu\tilde{F}_V^{\mu\nu} + \underline{\omega_{AAA}}\frac{\partial_\mu\pi_P}{v_P}\left(\mathrm{A}_\nu - \frac{\partial_\nu\pi_A}{v_A}\right)\tilde{F}_A^{\mu\nu}$$

Ambiguous
(free parameters)

$$+\underline{\omega_{VVA}}\frac{\partial_\mu\pi_S}{v_S}\mathrm{V}_\nu\tilde{F}_A^{\mu\nu} + \underline{\omega_{VAV}}\frac{\partial_\mu\pi_S}{v_S}\left(\mathrm{A}_\nu - \frac{\partial_\nu\pi_A}{v_A}\right)\tilde{F}_V^{\mu\nu}$$

- To fix these ambiguities, we would like to **impose** the vector and axial **gauge invariance**
  (since this should be equivalent to impose the **Ward identities**)

However, all these operators are gauge invariant!

Scenario with combined situations

Generically, the theory corresponds to :

$$\mathscr{L}_{\rm UV} \supset \mathscr{L}_{\rm UV}^{\rm fermion} + \mathscr{L}_{\rm UV}^{\rm Jac}$$

Would be Goldstones

$$\supset \bar{\Psi}\Big[i\partial_\mu\gamma^\mu - M + \left(\mathrm{V}_\mu - \frac{\boxed{\partial_\mu}\pi_V}{v_V}\right)\gamma^\mu - \left(\mathrm{A}_\mu - \frac{\boxed{\partial_\mu}\pi_A}{v_A}\right)\gamma^\mu\gamma^5$$

Goldstones

$$\left\{\begin{aligned} &- \left(0 - \frac{\boxed{\partial_\mu}\pi_S}{v_S}\right)\gamma^\mu - \left(0 - \frac{\boxed{\partial_\mu}\pi_P}{v_P}\right)\gamma^\mu\gamma^5\Big]\Psi \\ &+ \frac{1}{16\pi^2}\frac{\pi_P}{v_P}\left(F_V^{\mu\nu}\tilde{F}_V^{\mu\nu} + \frac{1}{3}F_A^{\mu\nu}\tilde{F}_A^{\mu\nu}\right) + \frac{1}{16\pi^2}\frac{\pi_S}{v_S}F_A^{\mu\nu}\tilde{F}_V^{\mu\nu}\end{aligned}\right.$$

- The would be Golstones (π_V and π_A) and the Goldstones (π_S and π_P) are **both** writing with local derivative acting on them

However, to minimise the number of integrals to regulazise, one might move back to the mass term the would be Golstones

- But more importantly, this would allow to obtain gauge-variant operators from which we hope to leverage ambiguities on several Wilson coefficients…

Gauge variation & surface term

$$\mathscr{L}_{\text{UV}} \supset \mathscr{L}_{\text{UV}}^{\text{Jac}} + \bar{\Psi}\left[i\partial_\mu\gamma^\mu - M\left(1 + \frac{\pi_V}{v_V} + \frac{\pi_A}{v_A}\gamma^5\right) + \text{V}_\mu - \text{A}_\mu + \frac{\partial_\mu\pi_S}{v_S}\gamma^\mu + \frac{\partial_\mu\pi_P}{v_P}\gamma^\mu\gamma^5\right]\Psi$$

$$\longrightarrow \mathscr{L}_{\text{EFT}} \supset \mathscr{L}_{\text{UV}}^{\text{Jac}} + \omega_{AVV}\frac{\partial_\mu\pi_P}{v_P}\text{V}_\nu\tilde{F}_V^{\mu\nu} + \omega_{VVA}\frac{\partial_\mu\pi_S}{v_S}\text{V}_\nu\tilde{F}_A^{\mu\nu}$$

gauge invariant

$$+\boxed{\omega_{VAV}\frac{\partial_\mu\pi_S}{v_S}\text{A}_\nu\tilde{F}_V^{\mu\nu}} + \boxed{\omega_{AAA}\frac{\partial_\mu\pi_P}{v_P}\text{A}_\nu\tilde{F}_A^{\mu\nu}}$$

gauge **variant**

only π_S dependent axial gauge variant operator of the EFT

only π_P dependent axial gauge variant operator of the EFT

Since the theory is axial gauge invariant by construction:

$$\delta_A(\omega_{VAV}\dots) = \partial_\mu(\dots) \qquad \delta_A(\omega_{AAA}\dots) = \partial_\mu(\dots)$$

surface terms do not contribute to the theory

- So gauge invariance constraint does not remove any ambiguity
- One should move the gauge variation of π_S and π_P being originally surface term to the bulk of the theory

One needs at least two operators which violate each axial gauge invariance so one can form a non trivial equation with ω_{VAV}, ω_{AAA}.

The trick

One way to realise this is to **gauge the global symmetry** See Bonnefoy et al. , arXiv:2011.10025

i.e introduce fictious gauge fields associated to the π_P and π_S Goldstone bosons :

$$\mathscr{L}_{\text{UV}}^{\text{fermion}} \supset \bar{\Psi}\left[i\partial_\mu\gamma^\mu - M\left(1+\frac{\pi_V}{v_V}+\frac{\pi_A}{v_A}\gamma^5\right) + \mathrm{V}_\mu - \mathrm{A}_\mu + \left(\boxed{S_\mu} - \frac{\partial_\mu \pi_S}{v_S}\right)\gamma^\mu + \left(\boxed{P_\mu} - \frac{\partial_\mu \pi_P}{v_P}\right)\gamma^\mu\gamma^5\right]\Psi$$

$$\longrightarrow \quad \mathscr{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{VVA}\left(\mathrm{S}_\mu - \frac{\partial_\mu \pi_S}{v_C}\right)\mathrm{V}_\nu \tilde{F}_A^{\mu\nu} + \omega_{AVV}\left(\mathrm{P}_\mu - \frac{\partial_\mu \pi_P}{v_C}\right)\mathrm{V}_\nu \tilde{F}_V^{\mu\nu}$$
$$+\omega_{VAV}\left(\mathrm{S}_\mu - \frac{\partial_\mu \pi_S}{v_C}\right)\underline{\mathrm{A}_\nu}\tilde{F}_V^{\mu\nu} + \eta_{ASV}\underline{\pi_A}F_S\tilde{F}_V$$
$$+\omega_{AAA}\left(\mathrm{P}_\mu - \frac{\partial_\mu \pi_P}{v_C}\right)\underline{\mathrm{A}_\nu}\tilde{F}_A^{\mu\nu} + \eta_{APA}\underline{\pi_A}F_P\tilde{F}_A$$

Now it exists several axial gauge variant operators

Then gauge invariance imposes a non trivial constrain on Wilson coefficients

Gauge invariance to remove ambiguities

Now, gauge invariance is no longer automatic:

$$\mathscr{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{VVA}\left(\mathrm{S}_\mu - \frac{\partial_\mu \pi_S}{v_C}\right)\mathrm{V}_\nu \tilde{F}_A^{\mu\nu} + \omega_{AVV}\left(\mathrm{P}_\mu - \frac{\partial_\mu \pi_P}{v_C}\right)\mathrm{V}_\nu \tilde{F}_V^{\mu\nu}$$

$$+\omega_{VAV}\left(\mathrm{S}_\mu - \frac{\partial_\mu \pi_S}{v_C}\right)\underline{\mathrm{A}}_\nu \tilde{F}_V^{\mu\nu} + \eta_{ASV}\underline{\pi_A} F_S \tilde{F}_V$$

$$+\omega_{AAA}\left(\mathrm{P}_\mu - \frac{\partial_\mu \pi_P}{v_C}\right)\underline{\mathrm{A}}_\nu \tilde{F}_A^{\mu\nu} + \eta_{APA}\underline{\pi_A} F_P \tilde{F}_A$$

Vector gauge invariance:
$\Rightarrow \omega_{VVA} = \omega_{AVV} = 0$

Axial gauge invariance:

$\Rightarrow \quad \omega_{VAV} = \eta_{AVS}$

$\Rightarrow \quad \omega_{AAA} = \eta_{APA}$

- η_{AVS} and η_{APA} are fully calculable, unambiguous coefficients

Though in **the physical case**, none of these interactions exist
since they require the presence of the fictious P_μ and S_μ gauge fields

- The determination of ω_{VAV} and ω_{AAA} is now transparent to concretely derive EFTs

$$\boxed{\mathscr{L}_{\text{EFT}}^{\text{1loop}} \supset \frac{1}{4\pi^2}\frac{\partial^\mu \pi_S}{v_S} A^\nu \tilde{F}_V^{\mu\nu} + \frac{1}{12\pi^2}\frac{\partial^\mu \pi_P}{v_P} A^\nu \tilde{F}_A^{\mu\nu}}$$

Application to axion phenomenology

axion couplings to massive SM gauge fields

$$
\begin{aligned}
\mathscr{L}_{\rm EFT}^{\rm 1loop} \supset \sum_f \frac{1}{4\pi^2}\Big[& \left(g_V^{PQ} g_A^Z g_V^Z\right)^f \left(a\, F_{\mu\nu}^{A^Z} \tilde{F}_{\mu\nu}^{V^Z}\right) + \frac{1}{3}\left(g_A^{PQ} g_A^Z g_A^Z\right)^f \left(a\, F_{\mu\nu}^{A^Z} \tilde{F}_{\mu\nu}^{A^Z}\right)\Big] \\
+\frac{1}{4\pi^2}\Big[& \left(g_V^{PQ} g_A^W g_V^W\right)^f \left(a\, F_{\mu\nu}^{A^W} \tilde{F}_{\mu\nu}^{V^W}\right) + \frac{1}{3}\left(g_A^{PQ} g_A^W g_A^W\right)^f \left(a\, F_{\mu\nu}^{A^W} \tilde{F}_{\mu\nu}^{A^W}\right)\Big] \\
+\frac{1}{4\pi^2}\Big[& \left(g_V^{PQ} g_A^Z g_V^\gamma\right)^f \left(a\, F_{\mu\nu}^{A^Z} \tilde{F}_{\mu\nu}^{V^\gamma}\right)\Big]
\end{aligned}
$$

fermion SM gauge charge

fermion PQ charge

Summary

$$\mathcal{L}^{\text{eff}} = \frac{a^0}{16\pi^2 v}\left(g_s^2 \mathcal{N}^{gg} G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + e^2\mathcal{N}^{\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{2e^2}{c_W s_W}\left(\mathcal{N}_1^{\gamma Z} - s_W^2\mathcal{N}_2^{\gamma Z}\right) Z_{\mu\nu}\tilde{F}^{\mu\nu} \right.$$
$$\left. + \frac{e^2}{c_W^2 s_W^2}\left(\mathcal{N}_1^{ZZ} - 2s_W^2\mathcal{N}_2^{ZZ} + s_W^4\mathcal{N}_3^{ZZ}\right) Z_{\mu\nu}\tilde{Z}^{\mu\nu} + 2\mathcal{N}^{WW} g^2 W^+_{\mu\nu}\tilde{W}^{-,\mu\nu}\right)$$

in the limit $m_{u,d,e} \to \infty$

Linear $a^0\bar{\psi}\gamma_5\psi$	Polar			
	Anomalous interactions	$\partial_\mu a^0\bar{\psi}\gamma^\mu\gamma_5\psi$ AVV	AAA	$\partial_\mu a^0\bar{\psi}\gamma^\mu\psi$ VAV
$\mathcal{N}^{gg} = \frac{1}{2}\left(x + \frac{1}{x}\right)$	$\mathcal{N}^{gg}$	0	$-$	$-$
$\mathcal{N}^{\gamma\gamma} = \frac{4}{3}\left(x + \frac{1}{x}\right)$	$\mathcal{N}^{\gamma\gamma}$	0	$-$	$-$
$\mathcal{N}_1^{\gamma Z} = \frac{1}{2}\left(x + \frac{1}{x}\right)$	$\mathcal{N}_L$	0	$-$	$\mathcal{N}_1^{\gamma Z} - \mathcal{N}_L$
$\mathcal{N}_2^{\gamma Z} = \mathcal{N}^{\gamma\gamma}$	$\mathcal{N}^{\gamma\gamma}$	0	$-$	0
$\mathcal{N}_1^{ZZ} = \frac{1}{4}x + \frac{1}{3x}$	$\mathcal{N}_L$	neutrino taken massless $\frac{\beta}{16}$	$-\frac{1}{2}\mathcal{N}_1^{ZZ} + \frac{\beta}{16}$	$\frac{3}{2}\mathcal{N}_1^{ZZ} - \mathcal{N}_L - \frac{\beta}{8}$
$\mathcal{N}_2^{ZZ} = \mathcal{N}_1^{\gamma Z}$	$\mathcal{N}_L$	0	0	$\mathcal{N}_2^{ZZ} - \mathcal{N}_L$
$\mathcal{N}_3^{ZZ} = \mathcal{N}^{\gamma\gamma}$	$\mathcal{N}^{\gamma\gamma}$	0	0	0
$\mathcal{N}^{WW} = \frac{x}{4} + \frac{3}{8x}$	$\mathcal{N}_L$	$\frac{3}{2}\mathcal{N}^{WW} - \frac{3}{2}\mathcal{N}_1^{\gamma Z} + \frac{\beta}{16}$	$-\frac{1}{2}\mathcal{N}^{WW} + \frac{\beta}{16}$	$\frac{3}{2}\mathcal{N}_1^{\gamma Z} - \mathcal{N}_L - \frac{\beta}{8}$

$x = v_2/v_1 = 1/\tan\beta$

J.Gunion et al., PRD 46 (1992) 2907

J.Q. and C. Smith, arXiv:1903.12559

Conclusion

- We have build EFTs dealing with gauge and anomalous symmetries
- The functional method is a natural framework to use
- Careful treatment of Dirac traces involving γ_5 in Dimensional Regularisation

1. Let’s understand axion fondamental interactions

Typical Lagrangians and phenomenological implications

2. Let’s build axion EFTs

Integrating out chiral fermions from the path integral

3. Let’s discus anomalies in EFTs

A different approach from to so-called Fujikawa’s method

Anomalies from the path integral

- path-integral measure for gauge theories with fermions is not invariant under the chiral transformation

« Path integral for gauge theories with fermions ». K. Fujikawa, Phys.Rev. D21 (1980) 2848

gives rise the the Adler-Bell-Jackiw anomaly and the anomalous Ward-Takahashi identities

- Anomaly of ungauged axial U(1) current : $j^5_\mu = \bar\Psi\gamma^\mu\gamma_5\Psi$

understood as a non-invariance of the path integral measure under local transformations on fermion fields :

$$\Psi' = e^{i\gamma_5\alpha(x)}\Psi$$
$$\bar\Psi' = \bar\Psi e^{i\gamma_5\alpha(x)}$$

- local matrix transformation $\Psi(x) \to U(x)\Psi(x) \longrightarrow [d\Psi][d\bar\Psi] \to (\text{Det}\ \mathcal{U}\text{Det}\ \bar{\mathcal{U}})^{-1}[d\Psi][d\bar\Psi]$

$$U(x) = e^{i\alpha(x)} \qquad \bar{\mathcal{U}}\mathcal{U} = 1 \qquad [d\Psi][d\bar\Psi] \to [d\Psi][d\bar\Psi]$$

$$U(x) = e^{i\gamma_5\alpha(x)} \qquad \bar{\mathcal{U}} = \mathcal{U} \qquad [d\Psi][d\bar\Psi] \to \boxed{(\text{Det}\ \mathcal{U})^{-2}}[d\Psi][d\bar\Psi]$$

$$[d\Psi][d\bar\Psi] \to \exp\Big[-2i\int d^4x\ \alpha(x)\mathcal{A}(x)\Big][d\Psi][d\bar\Psi]$$

$\text{Det}\ U = e^{\text{Tr}\ \ln\ U}$

$\lim_{x\to 0}(\ln\ (1+x)) \sim x$

anomaly function : $\mathcal{A}(x) = \boxed{\text{Tr}[\gamma_5]}\,\boxed{\delta^4(x-x)}$

at first sight no definite result : 0 $\qquad\infty$ need to introduce a regulator

...

$$\mathcal{A}(x) = -\frac{1}{16\pi^2}F_{\mu\nu}\tilde F^{\mu\nu}$$

Anomalies from the path integral à la Fujikawa

Path integral in Euclidean space: $Z = \int \mathcal{D}\Psi\, \mathcal{D}\bar{\Psi}\, \exp\left(-\int \mathrm{d}^4x\, \bar{\Psi} i \not{D} \Psi\right)$ (Dirac fermion in a vector gauge theory)

$i\not{D}$: hermitian

$$\mathrm{i}\not{D}\Psi_n = \underbrace{\lambda_n}_{\text{real}}\Psi_n$$

$[\gamma_5, \not{D}] \neq 0$: both operators can not be simultaneously diagonalized

origin of the anomalous behavior

In quantum theory, the primary importance is attached to the **Lorentz-covariant « energy »** operator $\not{D}$, over γ_5 (« chirality asymmetry » operator)

So one starts with the Dirac basis: $\Psi(x) = \sum_n a_n \varphi_n(x)$, $\bar{\Psi}(x) = \sum_n \varphi_n^\dagger(x)\bar{b}_n$ with $\int \mathrm{d}^4x\, \varphi_n^\dagger(x)\varphi_m(x) = \delta_{mn}$

$$A(x) \equiv \sum_n \varphi_n(x)^\dagger \gamma_5 \varphi_n(x)$$

primary def. of the anomaly

ill-defined because conditionally convergent quantity

(what is your prescription to sum: +1-1+1-1+...?)

$$A(x) = \lim_{M\to\infty}\left(\sum_n \varphi_n(x)^\dagger \gamma_5 e^{-(\lambda_n/M)^2}\varphi_n(x)\right)$$

regularize the large eigenvalues (with a specific basis dependent regulator)

only the zero-modes matter

$$= \lim_{M\to\infty}\left(\sum_n \varphi_n(x)^\dagger \gamma_5 e^{-(\not{D}/M)^2}\varphi_n(x)\right)$$

$$= \lim_{M\to\infty} \mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\gamma_5 e^{-ikx} e^{-(\not{D}/M)^2} e^{ikx}$$

changing the basis vectors to « **plane waves** »

« extraction of the gauge field dependence of A(x) by using the plane waves which have no gauge field dependence by themselves. »

$$= \lim_{M\to\infty} \mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\gamma_5 \exp\left(\frac{-1}{2M^2}\{2[ik_\mu + D_\mu(x)]^2 + [\gamma^\mu,\gamma^\nu]F_{\mu\nu}(x)\}\right)$$

$$= \lim_{M\to\infty} \mathrm{Tr}\gamma_5\{[\gamma^\mu,\gamma^\nu]F_{\mu\nu}\}^2\left(\frac{1}{2M^2}\right)^2\frac{1}{2!}\int\frac{d^4k}{(2\pi)^4}e^{-k^\mu k^\mu/M^2}$$

$$= \frac{1}{2}\left(\frac{-1}{8\pi^2}\right)\mathrm{Tr}\,{}^*F^{\mu\nu}F_{\mu\nu}(x).$$

K. Fujikawa, Phys.Rev. D21 (1980) 2848

$$\boxed{\begin{aligned} j_\mu^5 &= \bar{\Psi}\gamma^\mu\gamma_5\Psi \\ \partial^\mu j_\mu^5(x) &= \frac{i}{8\pi^2}\mathrm{Tr}\,[F_{\mu\nu}\tilde{F}^{\mu\nu}] \end{aligned}}$$

Anomalies from the path integral revisited

- Fujikawa does the « direct » evaluation of the measure :

$$\mathscr{D}\psi\mathscr{D}\bar{\psi} \to \text{"A"}\mathscr{D}\psi\mathscr{D}\bar{\psi}$$

- What about an « indirect » (as a ratio) evaluation :

≠ less straightforward

invariance under the labeling of the path integral

$$Z = \int \mathscr{D}\psi\mathscr{D}\bar{\psi}e^{iS[\psi,\bar{\psi}]} \quad = \quad Z' = \int \mathscr{D}\psi'\mathscr{D}\bar{\psi}'e^{iS[\psi',\bar{\psi}']}$$

with $\psi' = e^{i\gamma_5\theta(x)}\psi$

$$\text{"A"} = \frac{\#_1}{\#_2} = \frac{\int \mathscr{D}\psi\mathscr{D}\bar{\psi}e^{iS[\psi,\bar{\psi}]}}{\int \mathscr{D}\psi\mathscr{D}\bar{\psi}e^{iS[\psi,\bar{\psi}]-\int dx\bar{\psi}[2im\theta\gamma_5+(\gamma^\mu\partial_\mu\theta)\gamma_5]\psi}}$$

$$\text{"A"} = \frac{\det(i\not{D} - m)}{\det(i\not{D} - m - 2im\theta\gamma_5 - (\not{\partial}\theta)\gamma_5)}$$

It exists very efficient **EFT** methods to evaluate those « two » **functional determinants**

« indirect » evaluation of the anomaly through Covariant Derivative Expansion (CDE) of functional determinants

B. Filoche, R. Larue, J.Q., P.N.H. Vuong, arXiv:2205.02248

Covariant Derivative Expansion in short

Usual context in EFTs : integrating out a fermion of mass m,

$$\mathrm{Tr}\log(i\not{D} - m)$$ (one-loop piece)

$$= \int d^4x d^4q \; e^{iq.x}\mathrm{tr}\log(i\not{D} - m)e^{-iq.x}$$

1) Fourier transforms ($x \to q$) : use plane wave basis (rq: from the start unlike Fujikawa)

(BCH)

$$= \int d^4x d^4q \; \mathrm{tr}\log(i\not{D} - \not{q} - m)$$

2) Taylor expansion of the log :

$$= -\int d^4x d^4q \; \sum_{n=1}^{\infty} \frac{1}{n}\mathrm{tr}[\frac{1}{q+m} i\not{D}]^n$$

Interesting factorisation between Wilson coefficients and operators : $(\int dq \; C(q)) \times Op$.

Concrete example with the ABJ anomaly

path integral: $$Z \equiv \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left(i \int \mathrm{d}^4x \bar{\psi}(i\partial\!\!\!/ - V\!\!\!/ - m)\psi \right) = \int \mathcal{D}\psi\mathcal{D}\bar{\psi} e^{iS}$$

Axial tranformation: $$\psi \to e^{i\theta(x)\gamma_5}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\theta(x)\gamma_5}.$$

Jacobian : $$\mathcal{D}\psi\mathcal{D}\bar{\psi} \to J[\theta]\mathcal{D}\psi\mathcal{D}\bar{\psi}, \quad J[\theta] = \frac{\det(iD\!\!\!\!/ - m)}{\det(iD\!\!\!\!/ - m - 2im\theta\gamma_5 - (D\!\!\!\!/\theta)\gamma_5)} \equiv \exp\left[\int \mathrm{d}^4x\, \mathcal{A}(x)\right]$$

$$\mathcal{A}_{n=5}^{m\gamma_5} = i\left(2m^6\mathcal{I}_i^5 - 16m^4\mathcal{I}[q^2]_i^5 + 48m^2\mathcal{I}[q^4]_i^5\right)\mathrm{tr}(D\!\!\!\!/D\!\!\!\!/D\!\!\!\!/D\!\!\!\!/\,\theta\gamma_5)$$
$$= -\frac{i}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\theta\,\mathrm{tr}\left(F_{\mu\nu}F_{\rho\sigma}\right)$$

$$\mathcal{A} = \int \frac{\mathrm{d}^4q}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{tr}\left[\frac{-1}{q\!\!\!/ + m}\left(-iD\!\!\!\!/ + 2im\theta\gamma_5 + (\partial\!\!\!/\theta)\gamma_5\right)\right]^n\Bigg|_{\text{carrying }\theta\text{ dependence}}$$

$$\mathcal{A}_{n=4,\mathrm{fin}}^{\partial\!\!\!/\gamma_5} = i(m^4\mathcal{I}_i^4 - 4m^2\mathcal{I}[q^2]_i^4)\mathrm{tr}(D\!\!\!\!/D\!\!\!\!/D\!\!\!\!/(\partial\!\!\!/\theta)\gamma_5)$$
$$= \frac{i}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}(D_\mu D_\nu D_\rho(\partial_\sigma\theta))$$

Using BMHV scheme of dimensional regularization:

$$\mathcal{A}_{n=4,\mathrm{div}}^{\partial\!\!\!/\gamma_5} = \frac{-i}{16\pi^2}\left(\varepsilon\frac{2}{\varepsilon} + \mathcal{O}(\varepsilon)\right)\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}(D_\mu D_\nu D_\rho(\partial_\sigma\theta))$$
$$\xrightarrow[\varepsilon\to 0]{} -\frac{i}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}(D_\mu D_\nu D_\rho(\partial_\sigma\theta)).$$

$$\mathcal{A} = \mathcal{A}^{m\gamma_5} + \mathcal{A}^{\partial\!\!\!/\gamma_5} = \mathcal{A}_{n=5}^{m\gamma_5} = -\frac{i}{8\pi^2}\theta\,\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$$

Covariant Anomaly in chiral gauge field theory

path integral: $$Z \equiv \int \mathcal{D}\psi\mathcal{D}\bar\psi \exp\left(i\int d^4x\bar\psi(i\partial\!\!\!/ - V\!\!\!/ - A\!\!\!/\gamma_5 - m)\psi\right) = \int \mathcal{D}\psi\mathcal{D}\bar\psi e^{iS}$$

Axial tranformation: $$\psi \to e^{i\theta(x)\gamma_5}\psi, \quad \bar\psi \to \bar\psi e^{i\theta(x)\gamma_5}.$$

In which current do we want « to put » the anomaly?

Or we could do :

NEW

Vector tranformation: $$\psi \to e^{i\theta(x)}\psi, \quad \bar\psi \to \bar\psi\, e^{-i\theta(x)}$$

Jacobian : $$\mathcal{D}\psi\mathcal{D}\bar\psi \to J[\theta]\mathcal{D}\psi\mathcal{D}\bar\psi, \qquad J[\theta] = \frac{\det(iD\!\!\!/ - m)}{\det(iD\!\!\!/ - m - 2im\theta\gamma_5 - (D\!\!\!/\theta)\gamma_5)} \equiv \exp\left[\int d^4x\,\mathcal{A}(x)\right]$$

$$\mathcal{A} = \int \frac{d^4q}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{tr}\left[\frac{-1}{\not q + m}(-iD\!\!\!/ + 2im\theta\gamma_5 + (\partial\!\!\!/\theta)\gamma_5)\right]^n\Bigg|_{\text{carrying } \theta \text{ dependence}}$$

$$\mathcal{A}^{m\gamma_5} = \frac{-i}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\,\theta\left(F^V_{\mu\nu}F^V_{\rho\sigma} + \frac{1}{3}F^A_{\mu\nu}F^A_{\rho\sigma}\right)$$

$$\mathcal{A}^{\partial\!\!\!/\gamma_5} = \alpha\,\mathrm{tr}\left[\theta\,\epsilon^{\mu\nu\rho\sigma}\frac{1}{2}F^V_{\mu\nu}F^V_{\rho\sigma}\right] + \beta\,\mathrm{tr}\left[\theta\,\epsilon^{\mu\nu\rho\sigma}\frac{1}{2}F^A_{\mu\nu}F^A_{\rho\sigma}\right]$$

Imposing axial and vector gauge invariance :

Fix the free parameters: $\alpha = 0, \quad \beta = -i/12\pi^2$.

$$\mathcal{A}^{\partial\!\!\!/\gamma_5} = \frac{-i}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\,\theta\left(\frac{2}{3}F^A_{\mu\nu}F^A_{\rho\sigma}\right)$$

$$\boxed{\mathcal{A} \equiv \bar{\mathcal{A}}^{\partial\!\!\!/\gamma_5} + \mathcal{A}^{m\gamma_5} = \frac{-i}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\,\theta\left(F^V_{\mu\nu}F^V_{\rho\sigma} + F^A_{\mu\nu}F^A_{\rho\sigma}\right)}$$

This is the so-called covariant anomaly
i.e the breaking of an axial global symmetry in a vector and axial gauge field theory

More anomalies

See B. Filoche, R. Larue, J.Q., P.N.H. Vuong, arXiv:2205.02248

- Straightforward to recover Bardeen's result (1969) regarding the consistent anomaly (fermion reparametrisation associated to a gauge transformation)

- Also straightforward to extend in curved space-time, to recover the so-called axial gravitational anomaly :

$$J[\theta]^2 = \frac{\det\left(\sqrt{-g}^2(\not{D}^2 + m^2)\right)}{\det\left(\sqrt{-g}^2(\not{D}^2 + m^2 + 4im^2\theta\gamma_5)\right)}$$

$$\mathcal{A}^{\mathrm{grav}} = \frac{-i}{384\pi^2}\epsilon^{\mu\nu\rho\sigma}R^{\alpha\beta}{}_{\mu\nu}R_{\alpha\beta\rho\sigma}$$

-Alternative way to derive the scale anomaly (but without the need to introduce space-time curvature)

The scale transformation $x_\mu \to x'_\mu = e^\sigma x_\mu$ induces:

$$\frac{\partial}{\partial x^\mu} \to \frac{\partial}{\partial x'^\mu} = e^{-\sigma}\frac{\partial}{\partial x^\mu}, \qquad \mathrm{d}^d x \to \mathrm{d}^d x' = e^{d\sigma}\mathrm{d}^d x,$$

$$\psi(x) \to \psi'(x') = e^{-(d-1)\sigma/2}\psi(x),$$
$$\bar\psi(x) \to \bar\psi'(x') = e^{-(d-1)\sigma/2}\bar\psi(x),$$
$$A_\mu(x) \to A'_\mu(x') = A_\mu(x') = e^{-\sigma}A_\mu(x)$$

$$J[\sigma] = \frac{\det(i\not{D} - m)}{\det(i\not{D} - m + \sigma m - i\frac{d-1}{2}(\not\partial\sigma))}$$

$$\mathcal{A}_{\mathrm{scale}} = \frac{\sigma}{24\pi^2}\mathrm{tr}\,(F_{\mu\nu})^2$$

Peculiarities and interest of the method

-One first regularizes an ill-defined quantity (Jacobian) inserting as much freedom as needed and secondly call for coherence (covariance, integrability) of the obtained theory to fixe those ambiguities

« Usually » it works the opposite way, as one firstly calls for a well defined theory (free of any ambiguity) and secondly perform the regularization. Side effect: each theory calls for a specific regularization.

-Systematically use dimensional regularization (no hand-made regulator)

Freedom/ambiguity is then in the definition of γ_5 in d-dimensions

The anomalous interactions for various setup originate from a single general computation involving parameters fixed by physical arguments

-Evaluate the covariant and consistent anomaly from the path integral having then the possibility to choose in which current the anomaly has to stand

Convenient for model building (think about axion models)

-Several public codes available to build EFTs. BSM models involving QFT anomalies are so far out of reach, **but not for long!**

Indeed, this method allows to compute anomalous interactions in a self-consistent manner in the path integral formalism.

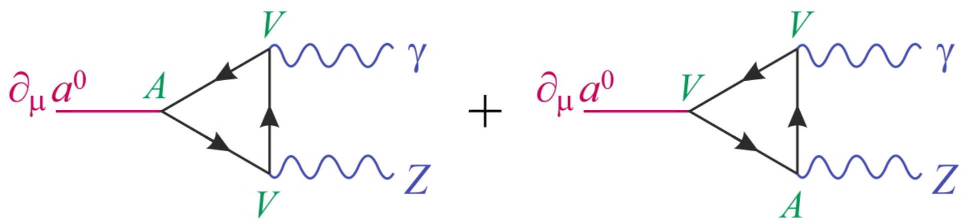
Conclusion

- Axion-electroweak couplings do not always follow the 'expected pattern' $\rightarrow$ must be kept in mind for ALP searches
- We have build EFTs dealing with gauge and anomalous symmetries
- The functional method is a natural framework to use
- We have discussed how to properly deal with several kind of ambiguities in these EFT computations
- We have presented a new way to evaluate anomalies from the path integral

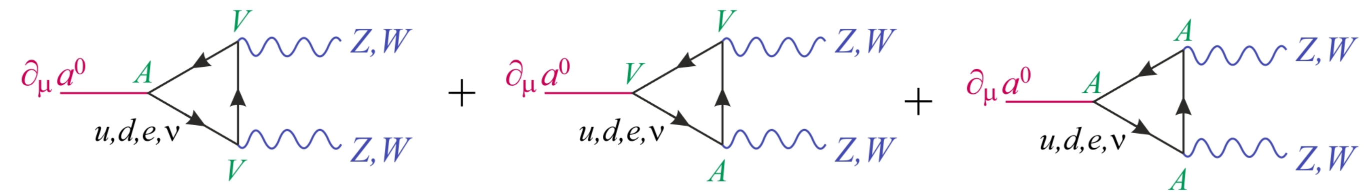
Spare slides

Consistent regularisation of divergent triangle diagrams

$$a^0 \to \gamma Z$$



$$a^0 \to ZZ, W^+W^-$$



Symmetry group point of view:

Gauge current must be conserved

PQ current is not conserved (anomalous)

Chiral Point of view:

Vector current are conserved **or not**

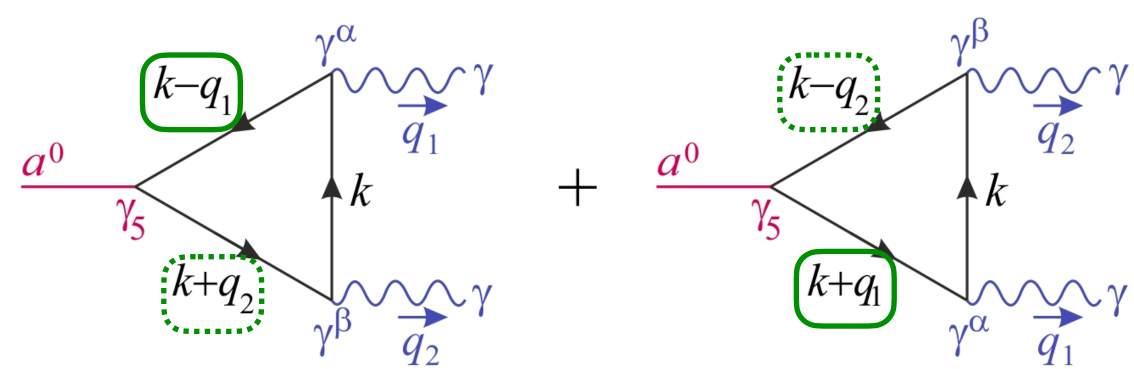
Axial current are conserved **or not**

⇒one needs to have enough freedom in the regularisation to choose which currents are conserved or not

Weinberg:

keep track of the ambiguities in the loop momentum routine

⟶ lead to 2 free parameters to tune Ward identities



Careful: Pauli-Villars or dim. reg. enforces automatically this parametric choice and there is no reason for this to hold in general for EW gauge bosons.

What are the axion couplings to all SM gauge bosons?

- It is generally believed that: $\mathcal{L}_{\rm PQ}^{\rm eff} = \frac{a^0}{16\pi^2 f_a}\left(g_s^2 \mathcal{N}_C G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + g^2 \mathcal{N}_L W_{\mu\nu}\tilde{W}^{\mu\nu} + g'^2 \mathcal{N}_Y B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$
  Georgi, Kaplan, Randall (1986)

  function only of PQ charges

Rotation from the weak eigenstate basis to the mass eigenstate basis:

$$
\begin{aligned}
\mathcal{L}_{\rm PQ}^{\rm eff,P} = \frac{a^0}{16\pi^2 f_a}\Big(& g_s^2 \mathcal{N}_C G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} \\
& + e^2 \mathcal{N}_{em} F_{\mu\nu}\tilde{F}^{\mu\nu} \\
& + \frac{2e^2}{c_W s_W}(\mathcal{N}_L - s_W^2 \mathcal{N}_{em}) Z_{\mu\nu}\tilde{F}^{\mu\nu} \\
& + \frac{e^2}{c_W^2 s_W^2}((1-2s_W^2)\mathcal{N}_L + s_W^4 \mathcal{N}_{em}) Z_{\mu\nu}\tilde{Z}^{\mu\nu} \\
& + 2g^2 \mathcal{N}_L W^+_{\mu\nu}\tilde{W}^{\mu\nu}_- \Big)
\end{aligned}
$$

$$
\begin{aligned}
W^3_\mu &= c_W Z_\mu + s_W A_\mu \\
B_\mu &= -s_W Z_\mu + s_W A_\mu \\
\mathcal{N}_{em} &= \mathcal{N}_L + \mathcal{N}_Y
\end{aligned}
$$

Manifestly $SU(2)_L \otimes U(1)_Y$ invariant

Why no electroweak symmetry breaking effect? (massive chiral fermions)

Ambiguity in the fermion PQ charges

- It is generally believed that: $\mathcal{L}^{\rm eff}_{\rm PQ} = \frac{a^0}{16\pi^2 f_a}\left(g_s^2 \mathcal{N}_C G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + g^2 \mathcal{N}_L W_{\mu\nu}\tilde{W}^{\mu\nu} + g'^2 \mathcal{N}_Y B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$

  Georgi, Kaplan, Randall (1986)

  function only of PQ charges

- When SM fermions are charge under $U(1)_{\rm PQ}$:

  There are actually four entangled $U(1)_{\rm Y}\otimes U(1)_{\rm PQ}\otimes U(1)_{\mathcal{B}}\otimes U(1)_{\mathcal{L}}$: Charges are ambiguous!

$U(1)_{PQ}$ charges when $a^0 \to a^0 + v\theta$:

$$\boxed{\begin{array}{c}\Phi_1 \to \exp(i\theta x)\Phi_1\ ,\quad \Phi_2 \to \exp(-i\theta/x)\Phi_2\ ,\quad \psi \to \exp(i\chi_\psi\theta)\psi\ ,\\[2ex] \chi_{q_L} = \alpha\ ,\ \chi_{u_R} = \alpha + x\ ,\ \chi_{d_R} = \alpha + \frac{1}{x}\ ,\ \chi_{\ell_L} = \beta\ ,\ \chi_{e_R} = \beta + \frac{1}{x}\ ,\end{array}}$$

$$x = v_2/v_1 = 1/\tan\beta$$

free parameters: conservation B&L

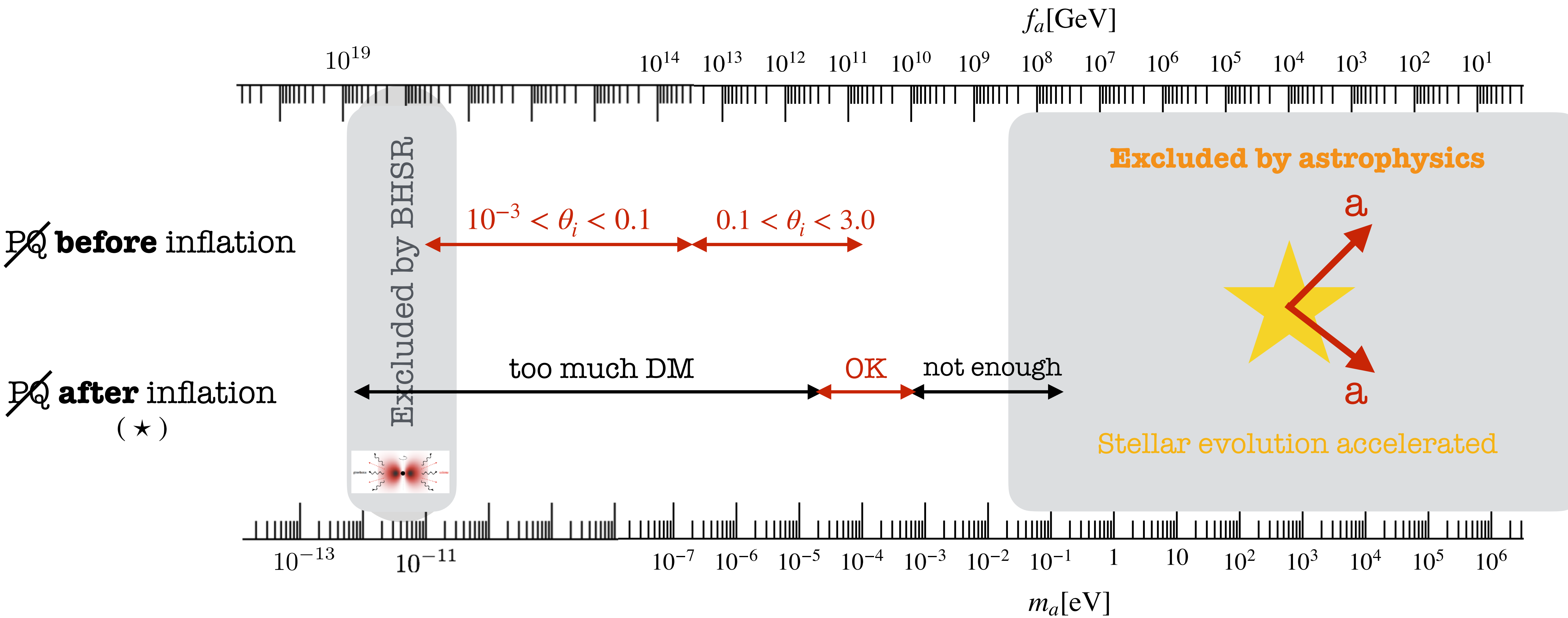
$$\longrightarrow \quad \mathcal{N}_C = \frac{1}{2}\left(x + \frac{1}{x}\right) \qquad \mathcal{N}_L = -\frac{1}{2}\left(3\alpha + \beta\right) \qquad \mathcal{N}_{em} = \frac{4}{3}\left(x + \frac{1}{x}\right)$$

$$\boxed{\delta\mathcal{L}_{\rm Jac}} = \frac{a^0}{16\pi^2 v}\left(g_s^2\mathcal{N}_C G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + e^2\mathcal{N}_{em}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{2e^2}{c_W s_W}\left(\boxed{\mathcal{N}_L} - s_W^2\mathcal{N}_{em}\right)Z_{\mu\nu}\tilde{F}^{\mu\nu}\right.$$
$$\left. + \frac{e^2}{c_W^2 s_W^2}\left((1-2s_W^2)\boxed{\mathcal{N}_L} + s_W^4\mathcal{N}_{em}\right)Z_{\mu\nu}\tilde{Z}^{\mu\nu} + 2\boxed{\mathcal{N}_L}g^2 W^+_{\mu\nu}\tilde{W}^{-,\mu\nu}\right)$$

$3\alpha + \beta$ reflects anomaly in the $\mathscr{B} + \mathscr{L}$ current $\Rightarrow$spurious interactions

Landscape

Axions should be very light and feebly interacting



(★) for $N_{DW} > 1$, predictions spoiled by topological defects

Axion DM constraints from **laboratory** experiments, from **stars** and **cosmos** observations