Gauge & Flavor Hierarchies from Weakly Nonlocal Braneworlds

Florian NORTIER

CEA Paris-Saclay, IPhT, Orme des Merisiers, Gif-sur-Yvette, France

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florian.nortier@ipht.fr

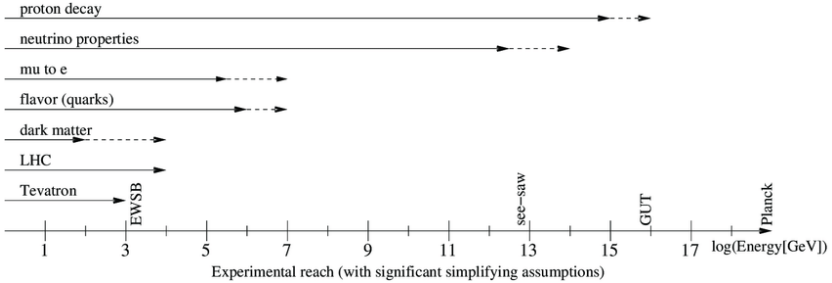
Several hierarchies in particle physics ⇒ BSM dynamical explanation?

- Gauge hierarchy (EW vs gravity scales): $\Lambda_{EW} \sim 100$ GeV $\ll \Lambda_P \sim 10^{18}$ GeV.
- Flavor hierarchy (neutrinos vs electron vs top quark): $M_\nu \ll M_e \ll M_t$.

BSM scales $\gg$ TeV-scale.
⇒ Stability of Higgs boson mass wrt BSM scales: $\delta M_H^2 \propto \Lambda_{BSM}^2$.
⇒ Models: weak scale SUSY; composite Higgses; extra dims. & branes; higher-deriv.; classicalization; etc.

BUT: No smoking guns of our favorite models at LHC ⇒ New (exotic) model building issues?

Talk: Extra dims. of space & branes + weak nonlocality.



Experimental reach (with significant simplifying assumptions)

Contents

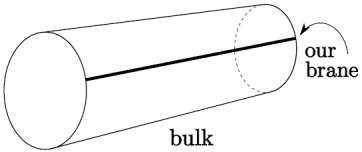
- Local EFT with compactified extra dims. of space & δ-like branes.
  $\Rightarrow$ Generates natural hierarchies thanks to geometry.
- Model with 1 flat extra dim. & N branes:

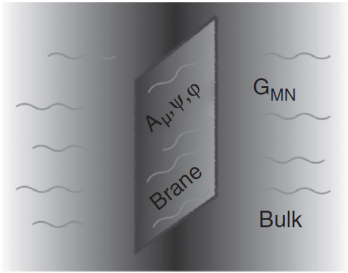
$$S_{5D} = \int d^4x\, dy \left[\mathcal{L}_{bulk} + \sum_{i=1}^{N} \delta(y - y_i)\, \mathcal{L}^{(i)}_{brane} \right] .$$

- Kaluza-Klein (KK) dim. reduction (5D $\rightarrow$ 4D) $\Rightarrow$ 4D KK-modes ϕ_n & bulk wave functions f_n:

$$\Phi(x, y) = \sum_{n}^{\infty} f_n(y)\, \phi_n(x) .$$

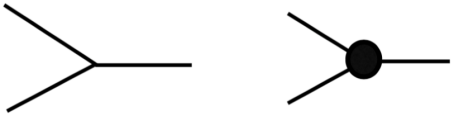
  $\Rightarrow$ 1 massless 0-mode + KK-tower with mass gap M_{KK}.

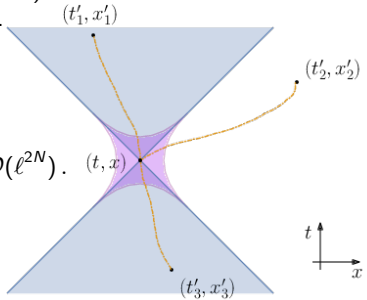




- Local QFT: Operators = Product of fields evaluated at same spacetime event.
- Weakly nonlocal theory: Interpolates btw IR local QFT & UV nonlocal theory above $\Lambda = 1/\ell$.
  $\Rightarrow$ Better UV behavior wrt local QFT $\Rightarrow$ Quantum gravity; Stability of the EW scale?
- Usually: Quantum gravity $\Rightarrow$ Min. length scale $\Rightarrow$ Weak nonlocality!
- Causality: Microcausality meaningless $\Rightarrow$ Macrocausality $\Rightarrow$ IR emergence of causality!
- Effect: Smearing of pointlike sources/vertices (∞-derivative operator).
  $\Rightarrow$ EFT: Pointlike source/vertex dressed by higher-dim. operators.
  ex: Heat kernel (Gaussian):

$$e^{\ell^2 \partial_x^2}\, \delta(x) = \sqrt{\frac{1}{4\pi\ell^2}}\, e^{-\frac{x^2}{4\ell^2}}\,,$$
$$= \delta(x) + \sum_{n=1}^{N-1} \frac{\ell^{2n}}{n!}\, \delta^{(n)}(x) + \mathcal{O}(\ell^{2N})\,.$$



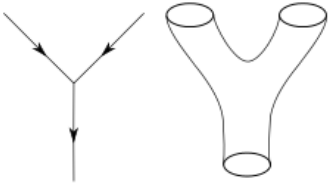


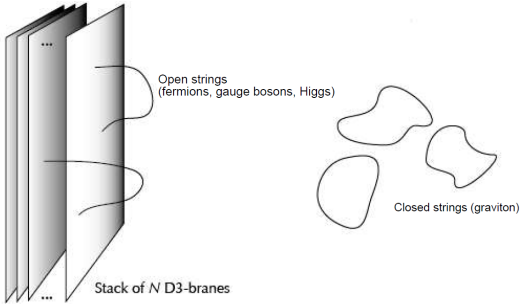
Witten, Nucl.Phys.B 268 (1986) 253-294 / Calcagni, Modesto, arXiv:1310.4957 [hep-th] / Calcagni, Modesto, arXiv:1404.2137 [hep-th]

- String theory $\Rightarrow$ 1D-extended objects (strings) $\Rightarrow$ Nonlocal scale = String scale $M_s = 1/\ell_s$.
- String field theory (SFT) = QFT formulation of string theory with ∞-deriv. operators.
- Truncation to 0-level sector (open bosonic SFT) $\Rightarrow$ Tachyon action $(-+\cdots+)$:

$$S = \int d^D x \left[\frac{1}{2}\phi\left(\Box+\mu^2\right)\phi - \frac{g}{3!}\left(e^{\ell_s^2\Box}\phi\right)^3\right] \iff S' = \int d^D x \left[\frac{1}{2}\phi\, e^{-2\ell_s^2\Box}\left(\Box+\mu^2\right)\phi - \frac{g}{3!}\phi^3\right].$$

- UV-completion of branewords: 6 extra dims. of space, D-brane stacks, UV-finiteness.





Efimov, Commun.Math.Phys. 5, 42–56 (1967) / Briscese, Modesto, arXiv:1803.08827 [hep-th] / Buoninfante, arXiv:2205.15348 [hep-th]

- Ghost-free ∞-deriv. QFT's $\Rightarrow$ Weakly nonlocal form factors = Transcendental entire functions.
- Prototype of UV-finite string-inspired scalar theory $(-+\cdots+)$; nonlocal scale $\Lambda = 1/\ell$:

$$S = \int d^D x \left[\frac{1}{2}\phi\, e^{\gamma\left(-\ell^2\Box\right)}\left(\Box - M^2\right)\phi - V(\phi)\right] \quad \Rightarrow \quad \Pi(k) = \frac{-ie^{-\gamma(\ell^2 k^2)}}{k^2 + M^2 - i\epsilon}\,.$$

- Unitarity (no new poles in $\Pi(k) \sim$ no ghost-like resonances) if:
  - Euclidean contour prescription for loop computations;
  - $e^{-\gamma(-\ell^2 E^2)} \searrow$ for $E \in \mathbb{I}\cup\mathbb{R} \Rightarrow$ SFT $e^{-\ell^2\Box}$ not valid for UV-complete theory!

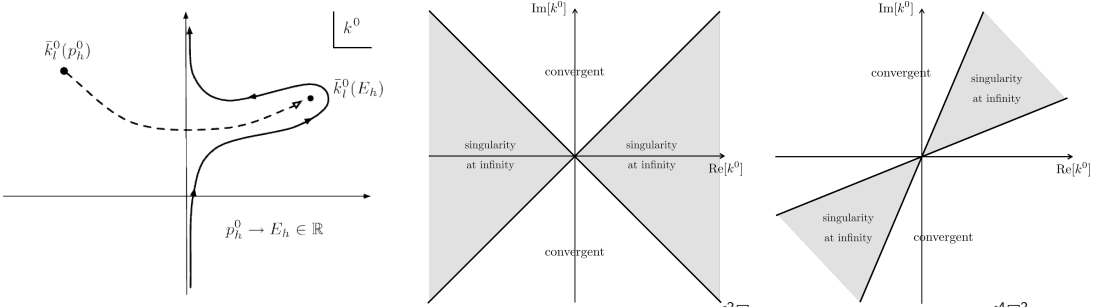


Figure: Left: Euclidean contour prescription. Middle: Analiticity of $e^{-\ell^2\Box}$. Right: Analiticity of $e^{-\ell^4\Box^2}$.

Tomboulis, arXiv:hep-th/9702146 / Modesto, Rachwal, arXiv:1503.00261 [hep-th] / Modesto, Piva, Rachwal, arXiv:1506.06227 [hep-th] / Modesto, Rachwal, Int.J.Mod.Phys.D 26 (2017) 11, 1730020 / Modesto, arXiv:2103.04936 [gr-qc] / Modesto, arXiv:2103.05536 [hep-th]

- ∞-deriv. gauge/gravity QFT's $\Rightarrow$ Competition kinetic vs interaction terms:
  - Prototype of pure gauge Yang-Mills theory $(-+\cdots+)$; nonlocal scale $\Lambda = 1/\ell$:

$$\mathcal{L}_{YM} = -\frac{1}{4g_{YM}}\left[\text{tr}\, F\, e^{\gamma\left(-\ell^2\mathcal{D}^2\right)}\, F + \mathcal{V}_{YM}\right] .$$

  - Prototype of pure gravity theory:

$$\mathcal{L}_{gr} = -\frac{2}{\kappa_D^2}\sqrt{|g|}\left[R - \frac{1}{2}\,R\left(\frac{e^{\gamma\left(-\ell^2\Box\right)}-1}{\ell^2\Box}\right)R + R_{\mu\nu}\left(\frac{e^{\gamma\left(-\ell^2\Box\right)}-1}{\ell^2\Box}\right)R^{\mu\nu} + \mathcal{V}_{gr}\right] .$$

    $\Rightarrow$ No ghost-like resonances!

- Asympt. polynomial form factors; $p_N(z)$ = polynomial of degree $N \in \mathbb{N}^*$:

$$\left|e^{\gamma\left(\ell^2 z\right)}\right| \underset{|z|\to\infty}{\longrightarrow} p_N\left(|z|\right) .$$

  $\Rightarrow$ Deep-UV $\sim$ Lee-Wick QFT's $\Rightarrow$ Perturbative renormalization program!
  $\Rightarrow$ Superrenorm. & asympt. free / UV-finite & asympt. safety (any spacetime dim. D).

- Recent extension to include matter & Higgs fields.
  $\Rightarrow$ Same tree-level amplitudes as local theories $\Rightarrow$ Nonlocality at loop-level!

Contents

Toy model:

- SFT-inspired Euclidean model: local kinetic term + nonlocal interactions ⇒ Tree-level nonlocality.
- 1 flat extra dim. compactified on the ordifold $S^1/\mathbb{Z}_2 \sim$ interval $[0, \pi\rho]$.
- 1 bulk scalar field $\Phi(x, y)$ + associated smeared field ($\Lambda = 1/\ell$):

$$\widetilde{\Phi}(x, y) = e^{\ell^2\left(\partial_\mu^2 + \partial_y^2\right)} \Phi(x) \,.$$

- KK-decomposition & normalization of bulk wave functions:

$$\Phi(x, y) = \sum_n \phi_n(x)\, f_n(y) \,, \quad M_n = \frac{n}{\rho} \,, \quad \oint dy\ f_n(y)\, f_m(y) = \delta_{nm} \,.$$

- Smeared KK-fields & bulk wave functions

$$\widetilde{\phi}_n(x) = e^{\ell^2 \partial_\mu^2}\, \phi_n(x) \,, \quad \widetilde{f}_n(x) = e^{-\left(\frac{n\ell}{\rho}\right)^2} f_n(y) \,.$$

⇒ KK-modes with $M_n \gg \Lambda$ have suppressed 4D effective coupings ($\propto f_n$'s overlaps) & 4D propagators.
⇒ Shadow extra dim. = Suppressed KK-mode effects wrt local theory! ⇒ Way to hide an extra dim.!

Randall, Sundrum, arXiv:hep-ph/9905221 / Gherghetta, Pomarol, arXiv:hep-ph/0003129 / Hosotani, Mabe, arXiv:hep-ph/0503020

Local Randall–Sundrum model (RS1):

- Spacetime = Slice of AdS_5 $(-++++)$ of proper length L:

$$ds^2 = g_{MN} dx^M dx^N = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 .$$

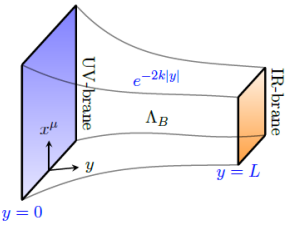
- 4D EW Higgs field localized on IR-brane $\Rightarrow$ Redshifted Higgs VEV on IR-brane:

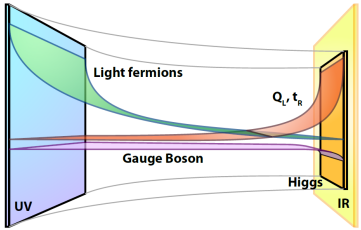
$$v_4 = e^{-kL} v_5 \sim 100 \text{ GeV} \Rightarrow \text{Warp transmutation of } \Lambda_{EW}!$$

  + Redshifted nonpert. IR-brane cutoff $\Rightarrow \Lambda_{IR} = e^{-kL} \Lambda_{UV} \Rightarrow$ Stability wrt radiative corrections!

- Bulk fermions (quasilocalized 0-modes): 4D Yukawa $\propto$ overlap with Higgs field $\Rightarrow$ Flavor hierarchy!

- LHC bounds $\Rightarrow \Lambda_{IR} \gg M_{KK} = e^{-kL} k \geq \mathcal{O}(1)$ TeV $\Rightarrow$ Strong little hiearchy problem.





Weak nonlocality at LHC: Biswas, Okada, arXiv:1407.3331 [hep-ph] / Su, Li, Nicolaidou, Chen, Wu, Paganis, arXiv:2108.10524 [hep-ph]

Weakly nonlocal RS1 model: Nortier, arXiv:2112.15592 [hep-th]

Stringy warped throats: Klebanov, Strassler, arXiv:hep-th/0007191 / Reece, Wang, arXiv:1003.5669 [hep-ph]

- 4D weakly nonlocal toy models:
  - Nonlocal scale $\Lambda = 1/\ell$ stabilizes Higgs-like scalar masses $\Rightarrow \delta M_H^2 \sim \Lambda^2$.
  - Nonlocal scale bounds from toy models: $\Lambda \geq \mathcal{O}(1)$ TeV.
- String-inspired weak nonlocality in RS1 $\Rightarrow$ 4D Higgs field $H(x)$ on IR-brane is smeared:

$$\widetilde{H}(x) = e^{\gamma\left(\ell_5^2\Box\right)} H(x)\,, \quad \gamma\left(\ell_5^2\Box\right) = \gamma\left(\ell_5^2\, g^{\mu\nu}\partial_\mu\partial_\nu\right)\,.$$

  $\Rightarrow \ell_4 = e^{kL}\ell_5 \quad \Rightarrow$ Warp transmutation of nonlocal scale!
  $\Rightarrow$ Stabilizes brane-localized Higgs boson mass at TeV-scale .
- If $\ell_5 \sim \ell_s$ (string length) $\Rightarrow$ Redshift of $M_s = 1/\ell_s$ along warped throats (Klebanov-Strassler).

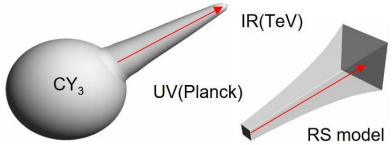
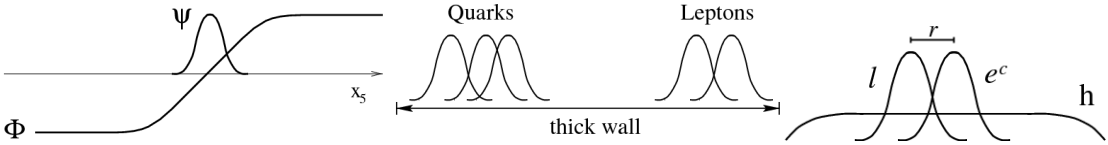


Figure: Klebanov-Strassler throat versus Randall-Sundrum model

Local Arkani-Hamed–Schmaltz (AS) model:

- Goal: Generate small interaction couplings (naturalness).
  $\Rightarrow$ Suppressed proton decay; Flavor hierarchy, Neutrino masses, etc.
- Split fermions: Bulk fermion wave functions peaked at $\neq$ points along a flat exta dim. $[0, \pi\rho]$
- Domain wall from bulk scalar Φ coupled to bulk fermions Ψ:
  - Trapped chiral fermions (0-modes) inside domain wall.
  - Position of Gaussian wave functions controlled by bulk masses.

  $\Rightarrow$ Suppressed 4D fermion operators from suppressed overlaps!
- 5D fields: Gauge bosons + Fermions Ψ + Higgs boson h.
  $\Rightarrow$ Main constraints: FCNC's (KK-gauge bosons) $\Rightarrow M_{KK} = 1/\rho \geq 100$ TeV.



Goal: $\neq$ UV-origin of split fermions: domain wall $\rightarrow$ multiple fuzzy branes
$\Rightarrow$ Same low-energy pheno. & constraints as AS model BUT $\neq$ UV-origin.

Model (5D EFT):

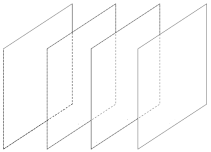
- Yukawa interactions: 1 local bulk Higgs field $H(x,y)$ + 4D weakly nonlocal Weyl fermions $\psi_L^{(1)}(x)$ & $\psi_R^{(2)}(x)$ localized on 2 $\neq$ δ-like branes (nonlocal length scale $\ell \ll \rho$):

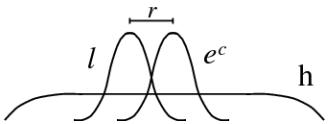
$$\mathcal{L} = \mathcal{L}_H - \sum_{i=1}^{2} \delta(y-y_i)\, \psi_{L/R}^{(i)\dagger} \left(i\bar{\sigma}\right) \psi_{L/R}^{(i)} + Y\, \widetilde{\Psi}_L^{(1)\dagger} H\, \widetilde{\Psi}_R^{(2)} + \text{H.c.}$$

Smeared fermions:

$$\widetilde{\Psi}_{L/R}^{(1/2)}(x,y) = e^{\ell^2\left(\partial_\mu^2+\partial_y^2\right)} \psi_{L/R}^{(1/2)}(x)\, \delta\left(y-y_{1/2}\right) \quad \Rightarrow \quad y_4 \propto \sqrt{\frac{\ell}{\rho}}\, e^{\frac{-r^2}{8\ell^2}} \ll 1$$

- Natural 5D Yukawa couplings $Y \sim \ell^{3/2}$ + large interbrane distance $r = |y_2 - y_1| \gg \ell$
$\Rightarrow$ Suppressed effective 4D Yukawa couplings $y_4 \ll 1$!





IV. Conclusion & Outlook

Motivations for weak nonlocality:

- Include gravity in UV complete quantum theory of Nature (string theory, ∞-deriv. QFT's).
- Soft UV-behavior + smearing effect on interaction vertices & pointlike sources.
- Potential path towards asympt. freedom/safety?
- Potential new path to stabilize EW scale?

Applications to braneworlds:

- Shadow extra dim.: KK-excitations ($M_n \gg \Lambda$) have suppressed couplings.
- Fuzzy branes: Suppressed couplings btw fields localized on 2 $\neq$ branes.
- Warp transmutation of scales: Nonlocal scale redshifted along a warped extra dim.

$\Rightarrow$ New model building issues for energy frontier!

Outlook:

- Towards a weakly nonlocal SM $\Rightarrow$ Hierarchy problem? Quantum gravity?
- Towards a weakly nonlocal RS-like model $\Rightarrow$ New features of fuzzy branes? UV-complete?
- Study weakly nonlocal pheno. $\Rightarrow$ Collider signatures?
- ...