Measuring Grand Unification

Project : Sfitter team + Jean-Loic Kneur, Claire Adam.

Builds up on an earlier paper : « Measuring Supersymmetry »
Eur.Phys. J. C 54, 617-644 (2008), arXiv:0709.3985 [hep-ph]

Reminder (1) : SFitter ?

“If supersymmetry is discovered in the next generation of collider experiments, it will be crucial to determine its fundamental high-scale parameters from weak scale measurements.”

SFitter is a complex tool, used to determine the underlying fundamental parameters :

1. It uses as inputs sets of measurements (masses, mass differences, edges or thresholds) expected at LHC, ILC, or LHC+ILC.

2. For a given model (here MSSM), the spectrum at the electroweak scale is calculated by, in particular, Suspect (“A Fortran code for the Supersymmetric and Higgs Particle Spectrum in the MSSM”, hep-ph/0211331, *Abdelhak Djouadi, Jean-Loic Kneur and Gilbert Moultaka*)

SFitter uses both to fit the parameters, using combination of Markov chains and Minuit.

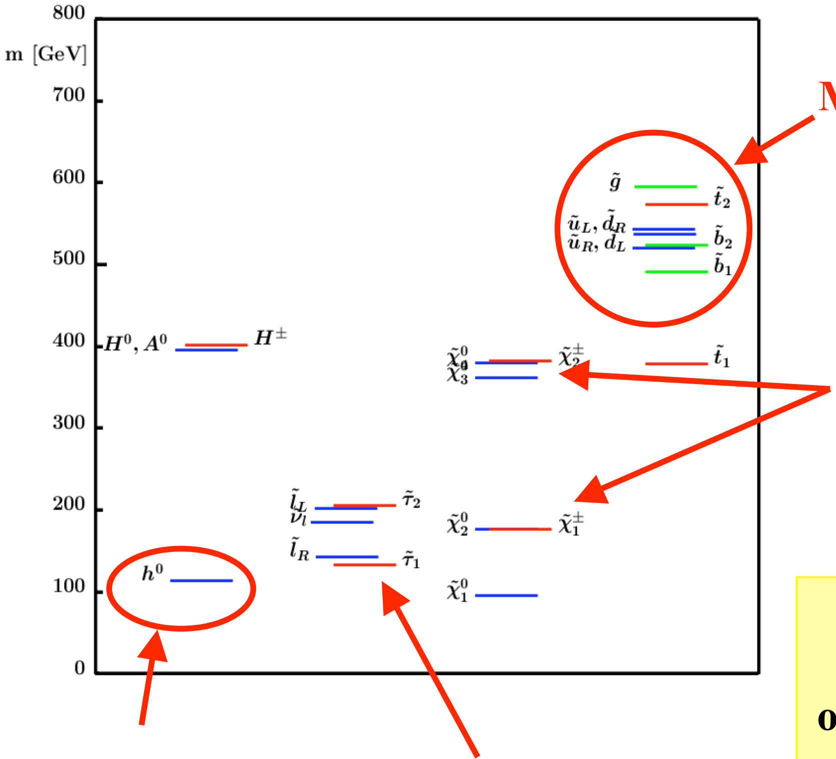
Previous SFitter publication (arXiv:0709.3985 [hep-ph].) :

“ For a “typical” point (SPS1a), and in two physics models (MSUGRA and MSSM), it was shown that a likelihood map could be built, maxima identified, and that the parameters could be extracted with some errors, properly including experimental and theory errors.”

Reminder (2) : SPS1a ?

$m_0 = 100\text{GeV}$ $m_{1/2} = 250\text{GeV}$ $A_0 = -100\text{GeV}$ $\tan\beta = 10$ $\text{sign}(\mu) = +$

favorable for LHC and ILC (Complementarity)



Moderately heavy gluinos and squarks

“Physics Interplay of the LHC and ILC”
Editor G. Weiglein hep-ph/0410364

Heavy and light gauginos

Higgs at the limit of LEP reach

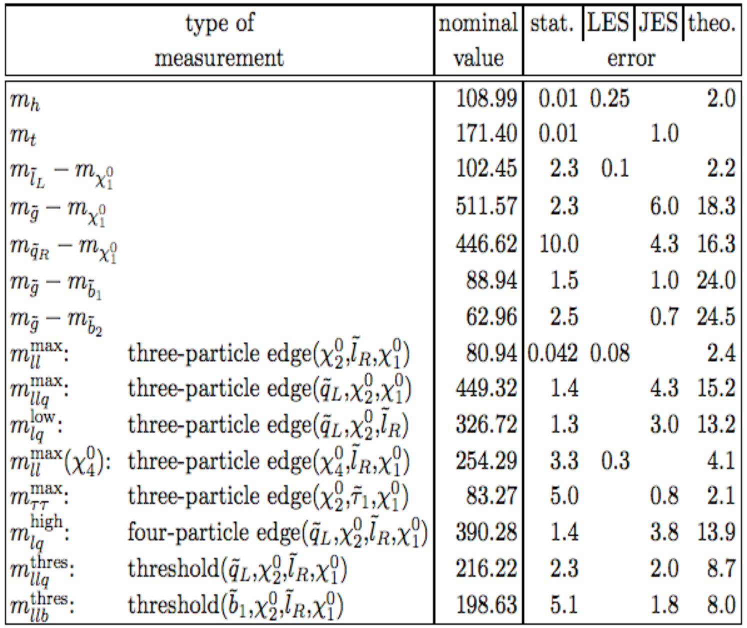
light sleptons

Further motivation :
The result of the EW fit (including b-physics observables, the anomalous moment of the muon and the relic density) yields a best-fit point… not too far from SPS1a !

Reminder (3) : experimental inputs

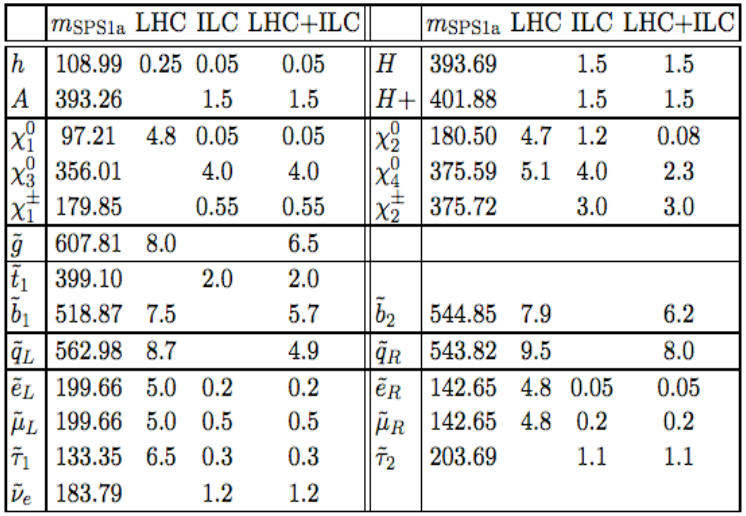
- LHC measures kinematical endpoints and mass difference, and covers better the strongly interacting sparticle sector,

- ILC has an impressive accuracy for particles which are light enough to be produced in pairs, and a somewhat better precision in the gaugino sector.



type of		nominal	stat.	LES	JES	theo.
measurement		value	error			
m_h		108.99	0.01	0.25		2.0
m_t		171.40	0.01		1.0	
$m_{\tilde{l}_L} - m_{\chi_1^0}$		102.45	2.3	0.1		2.2
$m_{\tilde{g}} - m_{\chi_1^0}$		511.57	2.3		6.0	18.3
$m_{\tilde{q}_R} - m_{\chi_1^0}$		446.62	10.0		4.3	16.3
$m_{\tilde{g}} - m_{\tilde{b}_1}$		88.94	1.5		1.0	24.0
$m_{\tilde{g}} - m_{\tilde{b}_2}$		62.96	2.5		0.7	24.5
$m_{ll}^{\rm max}$:	three-particle edge(χ_2^0,$\tilde{l}_R$,χ_1^0)	80.94	0.042	0.08		2.4
$m_{llq}^{\rm max}$:	three-particle edge($\tilde{q}_L$,χ_2^0,χ_1^0)	449.32	1.4		4.3	15.2
$m_{lq}^{\rm low}$:	three-particle edge($\tilde{q}_L$,χ_2^0,$\tilde{l}_R$)	326.72	1.3		3.0	13.2
$m_{ll}^{\rm max}(\chi_4^0)$:	three-particle edge(χ_4^0,$\tilde{l}_R$,χ_1^0)	254.29	3.3	0.3		4.1
$m_{\tau\tau}^{\rm max}$:	three-particle edge(χ_2^0,$\tilde{\tau}_1$,χ_1^0)	83.27	5.0		0.8	2.1
$m_{lq}^{\rm high}$:	four-particle edge($\tilde{q}_L$,χ_2^0,$\tilde{l}_R$,χ_1^0)	390.28	1.4		3.8	13.9
$m_{llq}^{\rm thres}$:	threshold($\tilde{q}_L$,χ_2^0,$\tilde{l}_R$,χ_1^0)	216.22	2.3		2.0	8.7
$m_{llb}^{\rm thres}$:	threshold($\tilde{b}_1$,χ_2^0,$\tilde{l}_R$,χ_1^0)	198.63	5.1		1.8	8.0

TABLE II: LHC measurements in SPS1a, taken from [19]. Shown are the nominal values (from SuSpect) and statistical errors, systematic errors from the lepton (LES) and jet energy scale (JES) and theoretical errors. All values are given in GeV.



	$m_{\rm SPS1a}$	LHC	ILC	LHC+ILC		$m_{\rm SPS1a}$	LHC	ILC	LHC+ILC
h	108.99	0.25	0.05	0.05	H	393.69		1.5	1.5
A	393.26		1.5	1.5	$H+$	401.88		1.5	1.5
χ_1^0	97.21	4.8	0.05	0.05	χ_2^0	180.50	4.7	1.2	0.08
χ_3^0	356.01		4.0	4.0	χ_4^0	375.59	5.1	4.0	2.3
$\chi_1^\pm$	179.85		0.55	0.55	$\chi_2^\pm$	375.72		3.0	3.0
$\tilde{g}$	607.81	8.0		6.5					
$\tilde{t}_1$	399.10		2.0	2.0					
$\tilde{b}_1$	518.87	7.5		5.7	$\tilde{b}_2$	544.85	7.9		6.2
$\tilde{q}_L$	562.98	8.7		4.9	$\tilde{q}_R$	543.82	9.5		8.0
$\tilde{e}_L$	199.66	5.0	0.2	0.2	$\tilde{e}_R$	142.65	4.8	0.05	0.05
$\tilde{\mu}_L$	199.66	5.0	0.5	0.5	$\tilde{\mu}_R$	142.65	4.8	0.2	0.2
$\tilde{\tau}_1$	133.35	6.5	0.3	0.3	$\tilde{\tau}_2$	203.69		1.1	1.1
$\tilde{\nu}_e$	183.79		1.2	1.2					

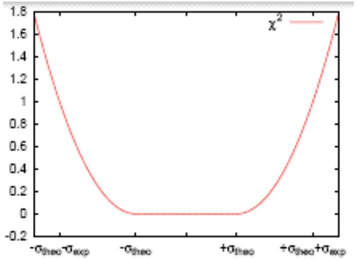
TABLE I: Errors for the mass determination in SPS1a, taken from [19]. Shown are the nominal parameter values (from SuSpect), the error for the LHC alone, from the LC alone, and from a combined LHC+LC analysis. Empty boxes indicate that the particle cannot, to current knowledge, be observed or is too heavy to be produced. All values are given in GeV.

Errors are split between :

- Statistical ⇒ Gaussian or Poisson, uncorrelated
- Experimental systematics (e.g luminosity, efficiency) ⇒ Gaussian, correlated
- Theoretical ⇒ follow the “Rfit Scheme”

No information within theory errors: flat distribution

$$\chi^2 = \sum_{\text{measurements}} \begin{cases} 0 & \text{for } |x_{\text{data}} - x_{\text{pred}}| < \sigma_{\text{theo}} \\ \left(\frac{|x_{\text{data}} - x_{\text{pred}}| - \sigma_{\text{theo}}}{\sigma_{\text{exp}}} \right)^2 & \text{for } |x_{\text{data}} - x_{\text{pred}}| \geq \sigma_{\text{theo}} \end{cases}$$



Theoretical errors used for the MSSM fit :

0.5% for the masses of colorless particles (neutralinos, charginos, sleptons)

1% for the masses of gluinos and squarks

In the previous study :

- Full likelihood map fit, identify and classify primary/secondary minima (Markov chains)
- Minuit is used to refine the identified minima

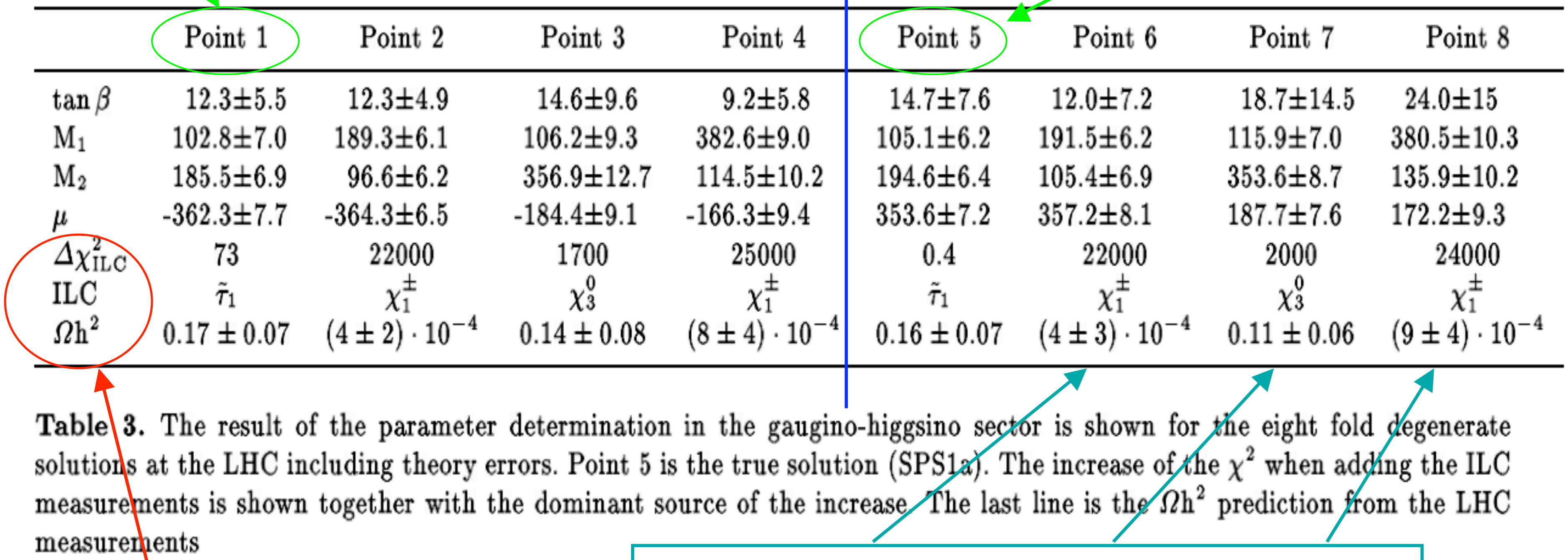
For us : Start from the identified minima
Toys are used to obtain a reliable error estimate
(data smearing + Minuit ⇒ distributions)

With LHC only : “4+4” solutions

“almost True” solution

mirrors :
$\mu<0$ | $\mu>0$

“ true solution ”
$M1< M2 < \mu$



	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Point 7	Point 8
$\tan\beta$	12.3±5.5	12.3±4.9	14.6±9.6	9.2±5.8	14.7±7.6	12.0±7.2	18.7±14.5	24.0±15
M_1	102.8±7.0	189.3±6.1	106.2±9.3	382.6±9.0	105.1±6.2	191.5±6.2	115.9±7.0	380.5±10.3
M_2	185.5±6.9	96.6±6.2	356.9±12.7	114.5±10.2	194.6±6.4	105.4±6.9	353.6±8.7	135.9±10.2
μ	-362.3±7.7	-364.3±6.5	-184.4±9.1	-166.3±9.4	353.6±7.2	357.2±8.1	187.7±7.6	172.2±9.3
$\Delta\chi^2_{\rm ILC}$	73	22000	1700	25000	0.4	22000	2000	24000
ILC	$\tilde{\tau}_1$	$\chi_1^\pm$	χ_3^0	$\chi_1^\pm$	$\tilde{\tau}_1$	$\chi_1^\pm$	χ_3^0	$\chi_1^\pm$
Ωh^2	0.17 ± 0.07	$(4 \pm 2) \cdot 10^{-4}$	0.14 ± 0.08	$(8 \pm 4) \cdot 10^{-4}$	0.16 ± 0.07	$(4 \pm 3) \cdot 10^{-4}$	0.11 ± 0.06	$(9 \pm 4) \cdot 10^{-4}$

Table 3. The result of the parameter determination in the gaugino-higgsino sector is shown for the eight fold degenerate solutions at the LHC including theory errors. Point 5 is the true solution (SPS1a). The increase of the χ^2 when adding the ILC measurements is shown together with the dominant source of the increase. The last line is the Ωh^2 prediction from the LHC measurements

Swaps : $M2<M1< \mu$, $M1<\mu<M2$, $M2<\mu<M1$

- Adding ILC : allows to lift the degeneracy. M-Stau1 very important to distinguish point 1 / 5
- Relic density (calculated using Micromegas) : is not sensitive to a swap between M2 and μ, but allows to see if M1 is correct.

Model definition ?   MSSM, but …

Some parameters are fixed and harmless (“standard stuff”) :

- Trilinear couplings are set to zero for the 1rst generation
- Use an average mass for Left and Right light squarks (u,d,s,c)
- α_S and M_{top} are included in the fit

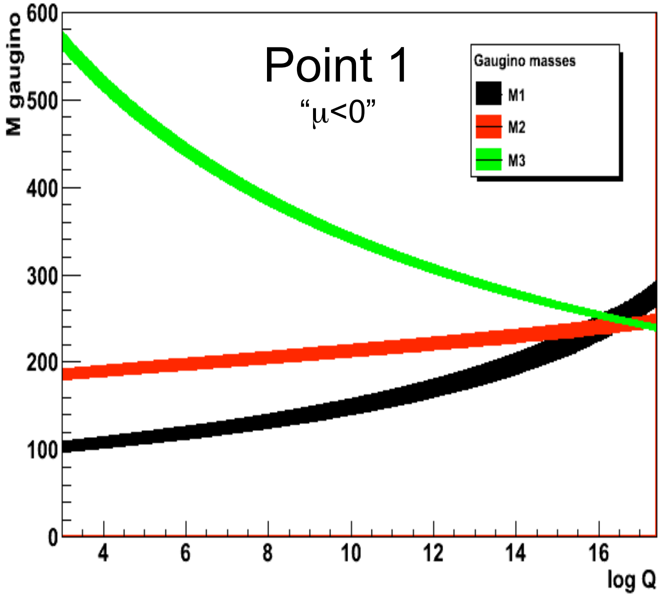
A close look at the fit result for the “true point 5” shows that the values are “off” compared to SPS1a values (by 1/6th to 1/3rd of the RMS)

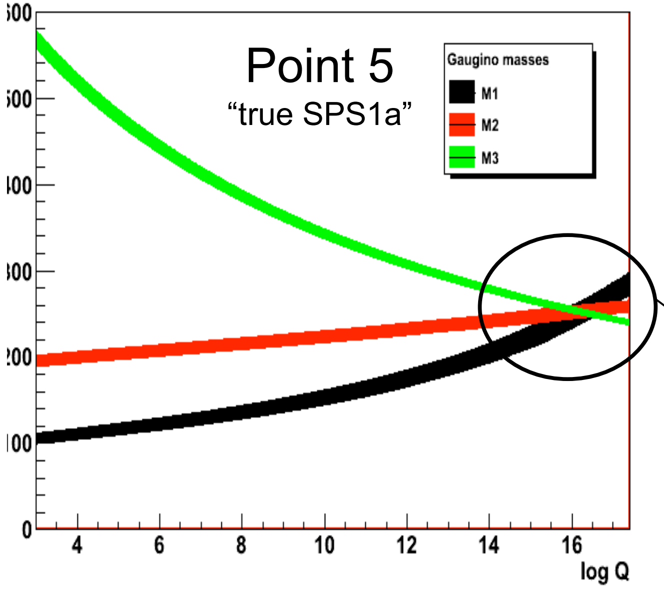
⇒ This is understood, and due to several sources :

- Atau and Ab are unknown, we chose to fix then at zero : effect non negligible

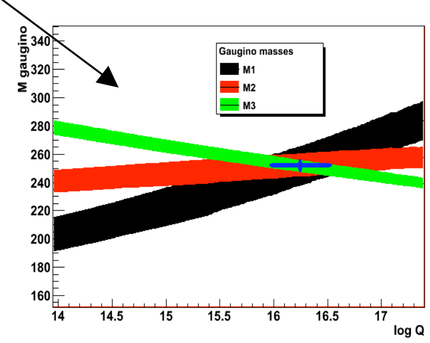
- the stau and stop sector are not well measured @LHC : we let them free in the fit, and this introduces a shift.

Next step : extrapolation using suspect

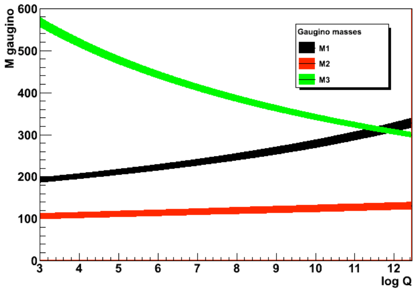




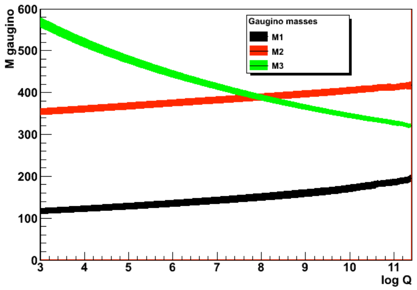
width = RMS
few 1000 toys



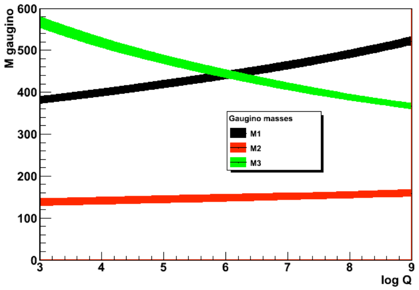
Points 2&6



Points 3&7



Points 4&8



Measuring unification ?

1. For any set of parameters (e.g 3 gaugino masses Mi), and each Q2 step, build :

$$\chi^2(Q^2) = \sum_{i,j}^{N} (M_i - < M_i >)(C_p^{-1})_{ij}(M_j - < M_j >)$$

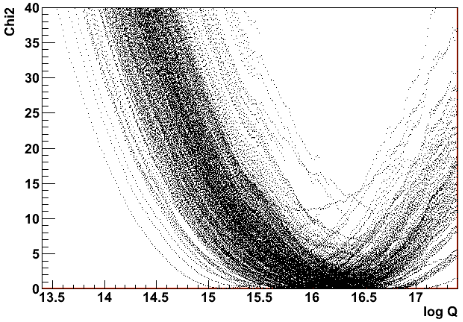
$\Rightarrow$ can build a χ^2_{95}

2. If we assume unification, with a mass m_U : we can build a χ^2_{ave}

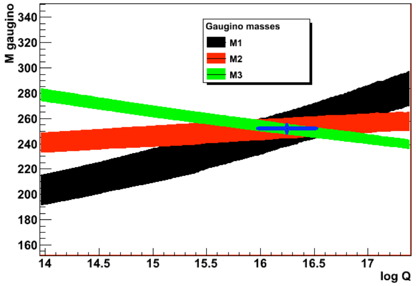
$$\chi^2_{ave}(Q^2) = \sum_{i,j}^{N} (M_i - m_U)(C_p^{-1})_{ij}(M_j - m_U)$$

$$m_U(Q^2) = \left(\sum_{i,j} (C_p^{-1})_{ij}\right)^{-1} \left(\sum_{i,j} (C_p^{-1})_{ij} M_j\right)$$

3. The Q^2 for which the χ^2_{ave} is minimal is the “unification scale candidate” and the corresponding m_U is the unified mass candidate :
(for gauginos it will be $m_{1/2}$).



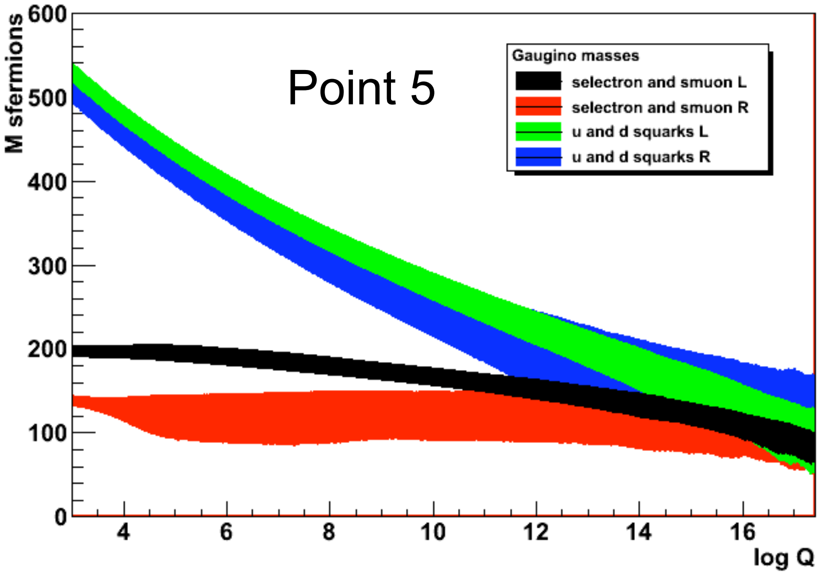
4. We “declare unification” if $\chi^2_{ave} < \chi^2_{95}$



$M_{1/2}$ = 251.5 ± 5.8
Q = 16.2 ± 0.27

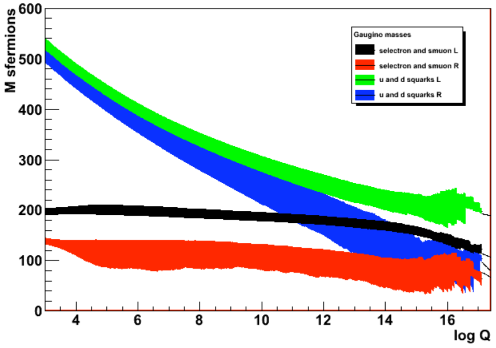
And unification is “declared” for :
95.5 % of the “point 5” toys
83% of the “point 1” toys

Same procedure with the Sfermions (1rst generation)

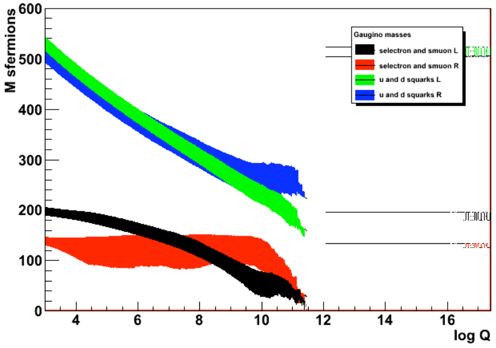


Conclusion : given the “flatness” in Q2, adding scalars to gauginos does not improve the precision

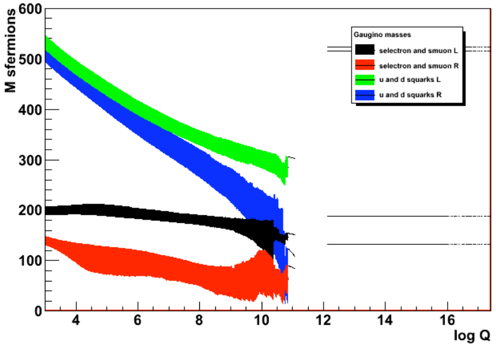
Points 2&6



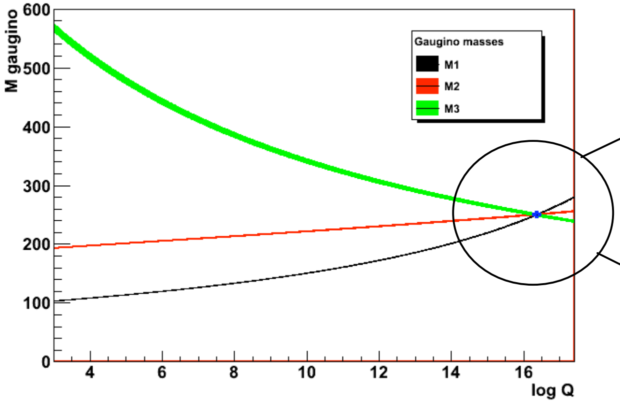
Points 3&7

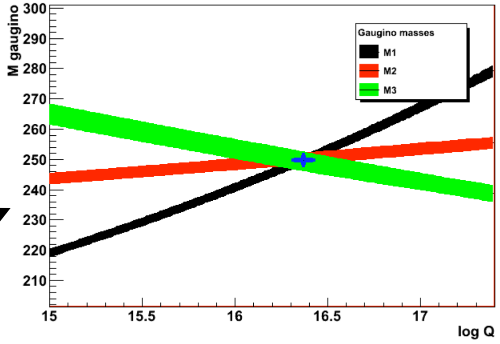


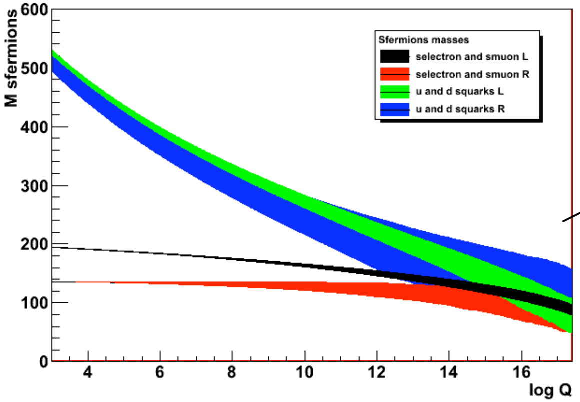
Points 4&8



Adding ILC to the LHC :







Name	Unified Parameter	Unification Scale
$m_{1/2}$	249.7±1.7	16.37±0.05
$m_0^{1/2Gen}$	96.3±10	16.7±0.6
m_0^{3Gen}	119.4±23	15.3±1.2
m_0	104±9.3	16.0±0.6
A_0	-176±173	15.3±4.4

Measured with :
Atau, Ab, Atop

Can even play with fermions of the 3rd generation, but it does not really improve : generations 1 and 2 are leading the game

Conclusion

At the LHC :

- the sign of μ is not measurable
- the 4 degenerated solutions correspond to swaps of M1,M2,μ

- out of the 2x4 combinations, 2x1 "unify" and they are hardly distinguishable

Thus, we will not be able to "prove" unification @LHC,
but asking for unification will lift the ambiguity.

Adding ILC to the LHC :

- no more ambiguity, unification can be "proven"
- m0, m1/2, Q can me measured.