SMC+NLO merging: POWHEG and MC@NLO

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Motivations for NLO+SMC

Comparisons of data with NLO results require now correcting for:

- Detector effects
- Underlying event
- Hadronization

All these corrections are estimated using a SMC generator.

- If NLO+SMC is implemented, an event sample can be generated, with hadronization effects and underlying event already included, that can be fed through the Detector simulator to be directly compared with data.
- Background modeling can become more accurate, benefitting from the available NLO results.

Two well proven methods: `MC@NLO` and `POWHEG`

Status of POWHEG

Most of it in `http://moby.mib.infn.it/~nason/POWHEG`,
Parts embedded in the HERWIG++ code
Up to now, the following processes have been implemented in POWHEG:

- $hh \to ZZ$ (Ridolfi, P.N., 2006)
- $hh \to Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \to Z/W$ (Alioli, Oleari, Re, P.N., 2008;)
  (Hamilton,Richardson,Tully, 2008;)
- $hh \to H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008)
- $hh \to H$, $hh \to HZ/W$ (Hamilton,Richardson,Tully, 2009;)
- $hh \to t + X$ (single top) (Alioli, Oleari, Re, P.N., 2009)
- VBF Higgs, (Oleari,P.N., 2009).
- The POWHEG BOX, (Alioli, Oleari, Re, P.N., 2010)
- $hh \to Z + \mathrm{jet}$, Preliminary (Alioli, Oleari, Re, P.N., 2010)

Status of MC@NLO

See http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/

- $hh \to H$ (gluon fusion)
- $hh \to Z/W$
- $hh \to (Z/W)(Z/W)$ (vector boson pairs)
- $hh \to Q\bar{Q}$
- $hh \to t + X\ (+W)$
- $hh \to h + W/Z$

Remarks on POWHEG

- POWHEG can be interfaced to any SMC program. Typical examples are provided with a PYTHIA and a HERWIG interface.
- In POWHEG, processes with more than two final state particles have been implemented, namely VBF Higgs production.
- A process with a singular Born term have been implemented ($Z + \mathrm{jet}$).
- A framework for the automated development of POWHEG implementations, the POWHEG BOX, has been published.

Remarks on MC@NLO

- MC@NLO implementation are tightly associated with a given shower Monte Carlo program. Most implementations are associated with HERWIG.
- Some MC@NLO implementation are also available within the HERWIG++ package.
- In a recent work (Torrielli, Frixione 2010), MC@NLO single vector boson production has been implemented within the PYTHIA virtuality ordered shower model.

NLO+SMC basics

Hardest emission in a Shower Monte Carlo

For illustration: assume there is only one radiating line.
SMC formula for hardest emission (P.N. 2004):

$$d\sigma = B(\Phi_B)d\Phi_B\left[\Delta_{t_0}^{\rm MC} + \underbrace{\Delta_t^{\rm MC}\frac{R^{\rm MC}(\Phi)}{B(\Phi_B)}d\Phi_r^{\rm MC}}_{d\,\Delta_t^{\rm MC}}\right]$$

- t is the radiation transverse momentum
- $B(\Phi_B)d\Phi_B$: Born differential cross section
- $\Delta_{t_0}^{\rm MC}$: No radiation probability down to the cutoff t_0
- $\Delta_t^{\rm MC}$: No radiation probability down to the scale t
- $R^{\rm MC}d\Phi_r^{\rm MC}$: SMC's real cross section, $\approx B\,\frac{1}{t}\,\frac{\alpha(t)}{2\pi}P(z)\,dz\,dt\,\frac{d\phi}{2\pi}$

$$\Delta_{t_l}^{\rm MC} = \exp\left[-\int_{t_l}^{t_h}\frac{dt'}{t'}\int dz\,\frac{\alpha_s(t')}{2\pi}\,P(z)\right] = \exp\left[-\int_{t_l}\frac{R^{\rm MC}}{B}d\Phi_r^{\rm MC}\right]$$

Hardest emission in a NLO+SMC: must be NLO accurate

It has the form:

$$d\sigma = \bar{B}^s(\Phi_B)d\Phi_B\left[\Delta^s_{t_0} + \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)}d\Phi_r\right] + [R(\Phi) - R^s(\Phi)]\,d\Phi$$

where $R \Rightarrow R^s$ in the soft and collinear limit,

$$\bar{B}^s(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^s(\Phi)\,d\Phi_r}_{\text{infinite}}\right]}_{\text{finite}}$$

Imagine that soft and collinear singularities in $R^{\rm MC}$ are regulated as in V.

and

$$\Delta^s_t = \exp\left[-\int_{t_l}\frac{R^s}{B}d\Phi_r\theta(t(\Phi) - t_l)\right]$$

Accuracy

Small t: $\frac{R_s(\Phi)}{B(\Phi_B)}\, d\Phi_{\mathrm{rad}} \approx \frac{\alpha_s(t)}{2\pi} P(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}$,

Also: $\bar{B} \approx B \times (1 + \mathcal{O}(\alpha_s))$

Thus: all features of SMC's are preserved at small t.

Large t: $\Delta \to 1, \quad d\sigma = \bar{B} \times \frac{R_s}{B}\, d\Phi + (R - R_s)\, d\Phi \approx R\, d\Phi$,
so: large t accuracy is preserved.

NLO accuracy: since $\Delta_{t_0} + \int \Delta_t \frac{R_s(\Phi)}{B(\Phi_B)} d\Phi_r = 1$, integrating in $d\Phi_r$ at fixed Φ_B

$$\int \delta(\Phi_B - \bar{\Phi}_B)\, d\sigma = \left[\bar{B} + \int (R - R_s)\, d\Phi_r\right]_{\Phi_B = \bar{\Phi}_B} = \left[B + V + \int R d\Phi_r\right]_{\Phi_B = \bar{\Phi}_B}$$

So: NLO accuracy is preserved for inclusive quantities.

In MC@NLO: $R^s d\Phi_r = R^{\rm MC} d\Phi_r^{\rm MC}$

Furthermore:
in MC@NLO the phase space parametrization Φ_B, $\Phi_r \Rightarrow \Phi$ is the one of the Shower Monte Carlo. We have:

$$\underbrace{\bar{B}^s(\Phi_B)d\Phi_B}_{\substack{\text{provided by }\texttt{MCatNLO}\\ \mathcal{S}\text{ event}}} \left[\underbrace{\Delta^s_{t_0} + \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)} d\Phi_r}_{\text{generated by }\texttt{HERWIG}}\right] + \underbrace{[R(\Phi) - R^s(\Phi)]\, d\Phi}_{\substack{\text{provided by }\texttt{MCatNLO}\\ \mathcal{H}\text{ event}}}$$

Recipe for MC@NLO

- Compute following cross section for $\mathcal{S}$ and $\mathcal{H}$ events:

$$\sigma_S = \int |\bar{B}^{\rm MC}(\Phi_B)|\, d\Phi_B, \;\; \sigma_H = \int |R - R^{\rm MC}|\, d\Phi$$

- Chose an $\mathcal{S}$ or $\mathcal{H}$ event with probability proportional to σ_S, σ_H
- For an $\mathcal{S}$ event:
  - generate Born kinematics with probability

$$|\bar{B}^{\rm MC}(\Phi_B)| = \left| B(\Phi_B) + \left[V(\Phi_B) + \int R^{\rm MC}(\Phi)\, d\Phi_r^{\rm MC} \right] \right|$$

  - Feed the Born kinematics to the MC for subsequent shower with weight ± 1, same sign as $\bar{B}^{\rm MC}(\Phi_B)$ (mostly $+1$).
- For an H event:
  - generate Radiation kinematics with probability $|R - R^{\rm MC}|$.
  - Feed to the MC (with weight ± 1, same sign as $R - R^{\rm MC}$)

Issues:

- Must use of the MC kinematic mapping $(\Phi_B, \Phi_r^{\mathrm{MC}}) \Rightarrow \Phi$.
- $R - R^{\mathrm{MC}}$ must be non singular: the MC must reproduce exactly the soft and collinear singularities of the radiation matrix element. (Many MC's are not fully accurate in the soft limit)
- $R - R^{\mathrm{MC}}$ can be negative: negative weights in the output.

In POWHEG: $R^s d\Phi_r = R F(\Phi)$

where $0 \leqslant F(\Phi) \leqslant 1$, and $F(\Phi) \Rightarrow 1$ in the soft or collinear limit.
$F(\Phi) = 1$ is also possible, and often adopted.
The parametrization $\Phi_B, \Phi_r \Rightarrow \Phi$ is within POWHEG, and there is complete freedom in its choice.

$$\underbrace{\bar{B}^s(\Phi_B) d\Phi_B}_{\texttt{POWHEG}} \left[\underbrace{\Delta^s_{t_0} + \Delta^s_t \frac{R^s(\Phi)}{B(\Phi_B)} d\Phi_r}_{\texttt{POWHEG}} \right] + \underbrace{\left[R(\Phi) - R^s(\Phi) \right] d\Phi}_{\texttt{POWHEG}}$$

All the elements of the hardest radiation are generated within POWHEG

Recipe

- POWHEG generates an event, with $t = t_{\texttt{powheg}}$
- The event is passed to a SMC, imposing no radiation with $t > t_{\texttt{powheg}}$.

Improvements over MC@NLO:

- Positive weighted events: $R - R_s = R(F-1) \geqslant 0$!
- Independence on the Shower MC: The hardest emission is generated by POWHEG; less hard emissions are generated by the shower.
- No issues with SMC inaccuracies

MC@NLO and POWHEG yield the exact total NLO cross section;

However, differential distributions are affected by induced higher order terms:

$$d\sigma = d\Phi_{B} \bar{B}\Big[\Delta_{t_0} + \Delta_t \tfrac{R_s}{B} d\Phi_r\Big] + (R - R_s)\, d\Phi, \qquad \bar{B} = B + \big[V + \textstyle\int R_s\, d\Phi_r\big]$$

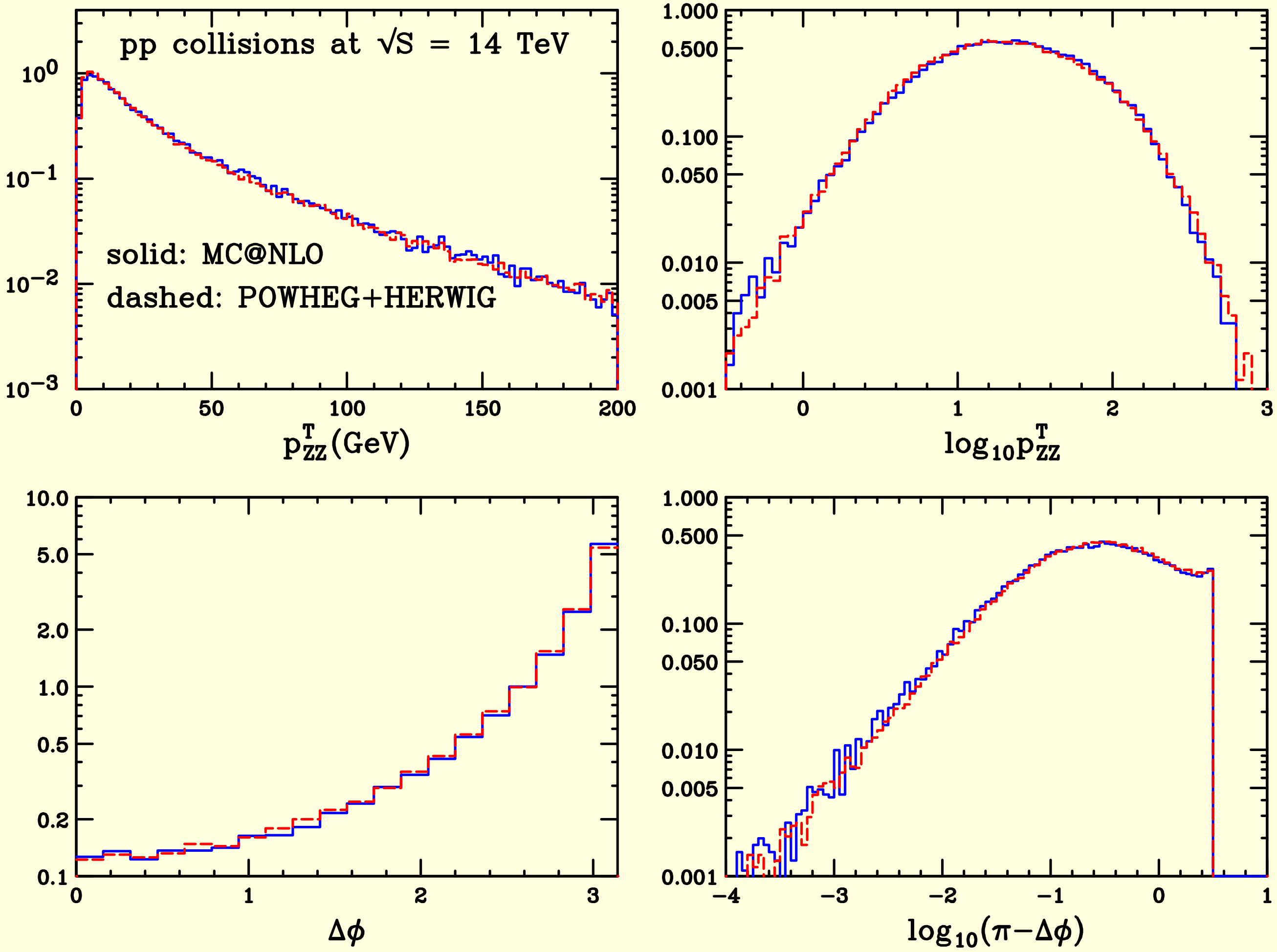
- The expression for $\Delta_{t_1,t} = \exp\Big[-\int \frac{R}{B} d\Phi_r \theta(k_T - t)\Big]$ generates terms of all orders, and suppresses the distributions at small p_T.
- Most important: $\bar{B}$, multiplied by R_s/B, generates NNLO terms.
  For large t:

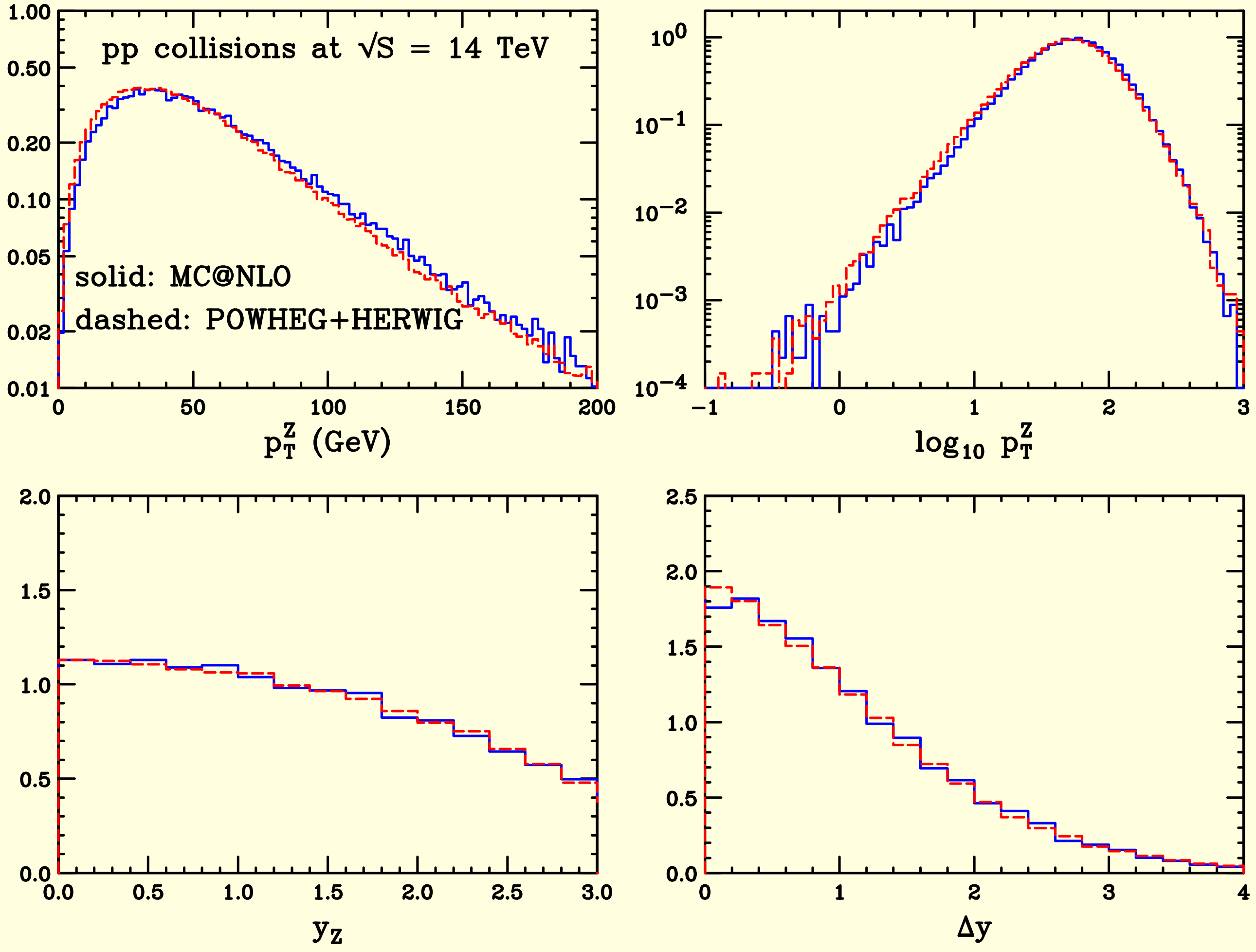
$$d\sigma = d\Phi_{B} \bar{B}\left[\Delta_{t_0} + \Delta_t \frac{R_s}{B} d\Phi_r\right] + (R - R_s)\, d\Phi \Rightarrow \left[\underbrace{\left(\frac{\bar{B}}{B} - 1\right) R_s}_{\text{NNLO}} + R\right] d\Phi$$

(if NLO corrections are positive, it typically enhances the distributions).

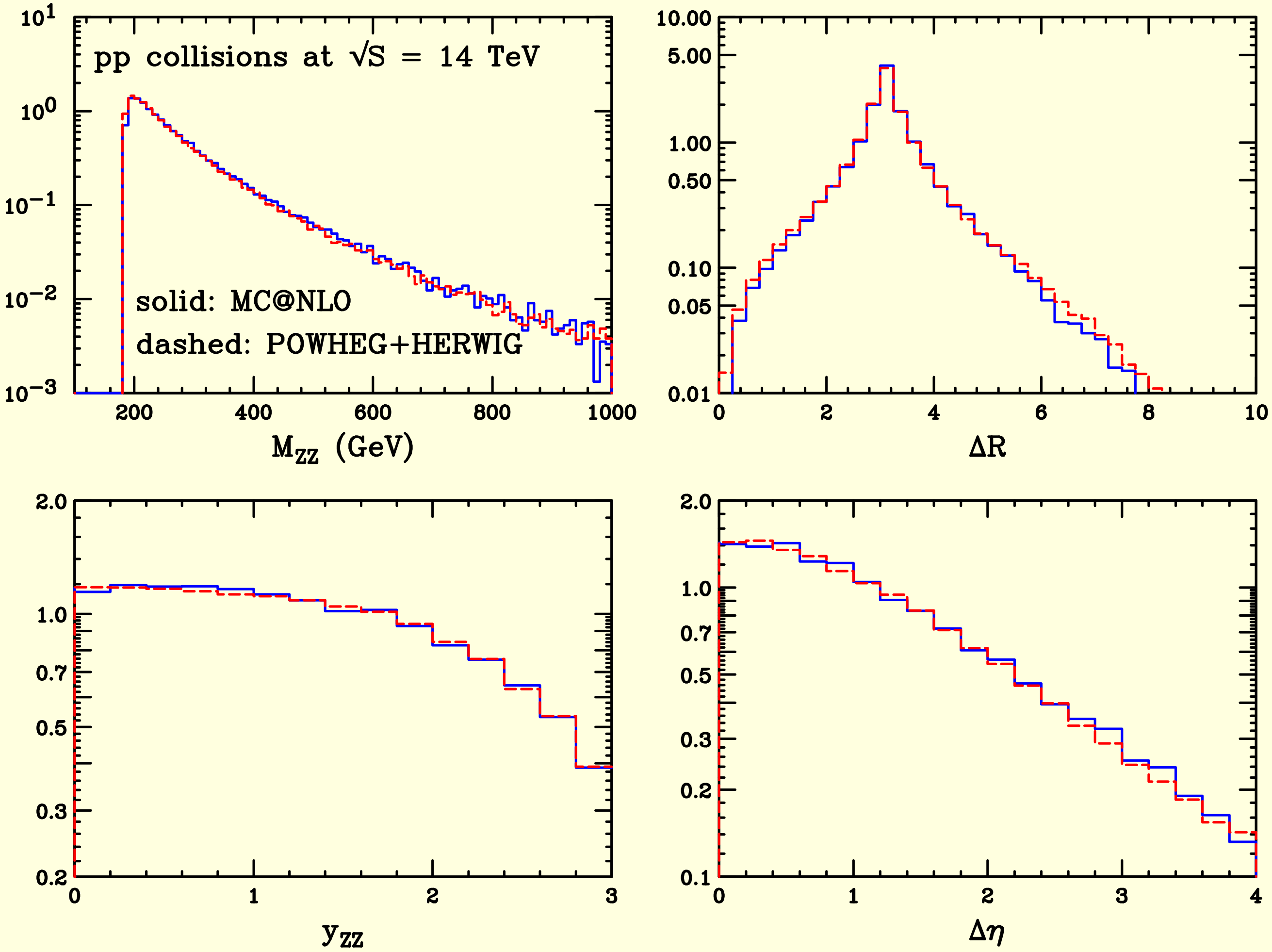
Comparisons of POWHEG+HERWIG vs. MC@NLO

Z pair production



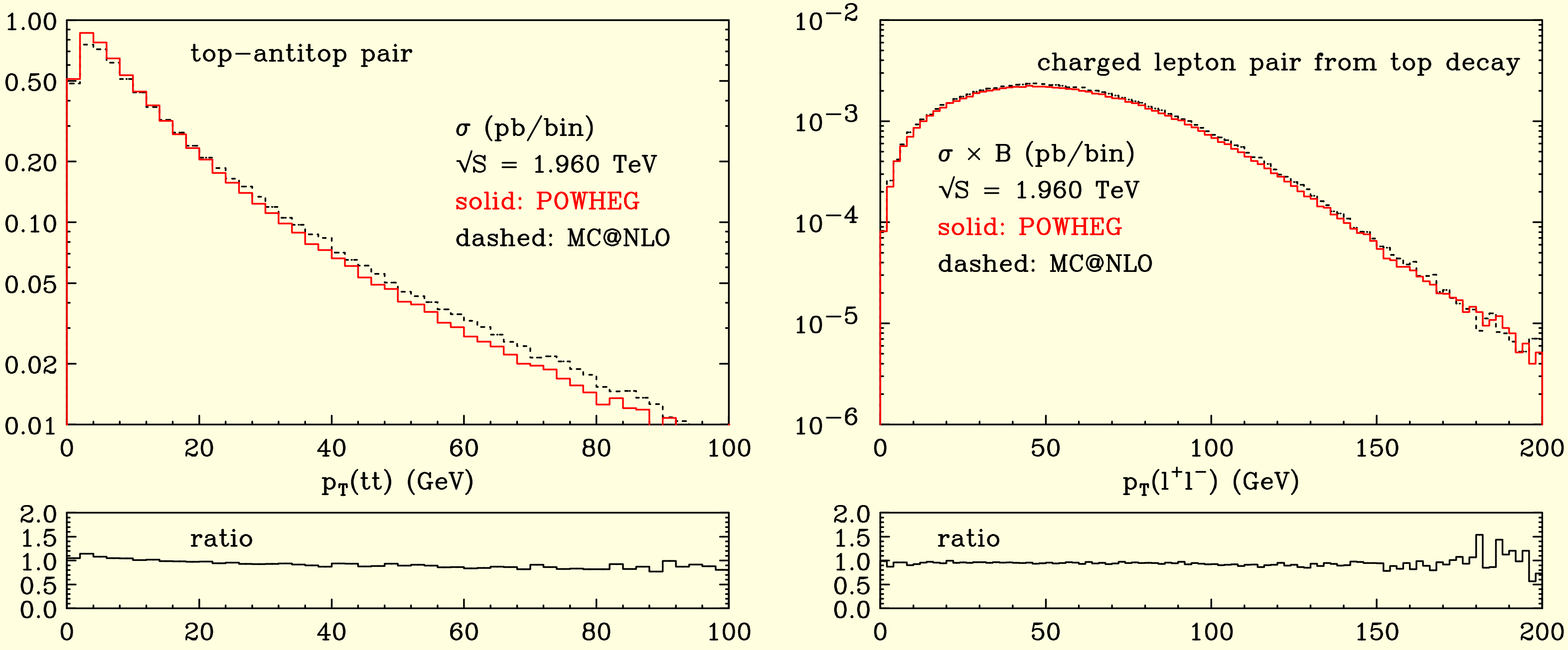


pp collisions at √S = 14 TeV
solid: MC@NLO
dashed: POWHEG+HERWIG
p_T^Z (GeV)
$\log_{10} p_T^Z$
y_Z
Δy



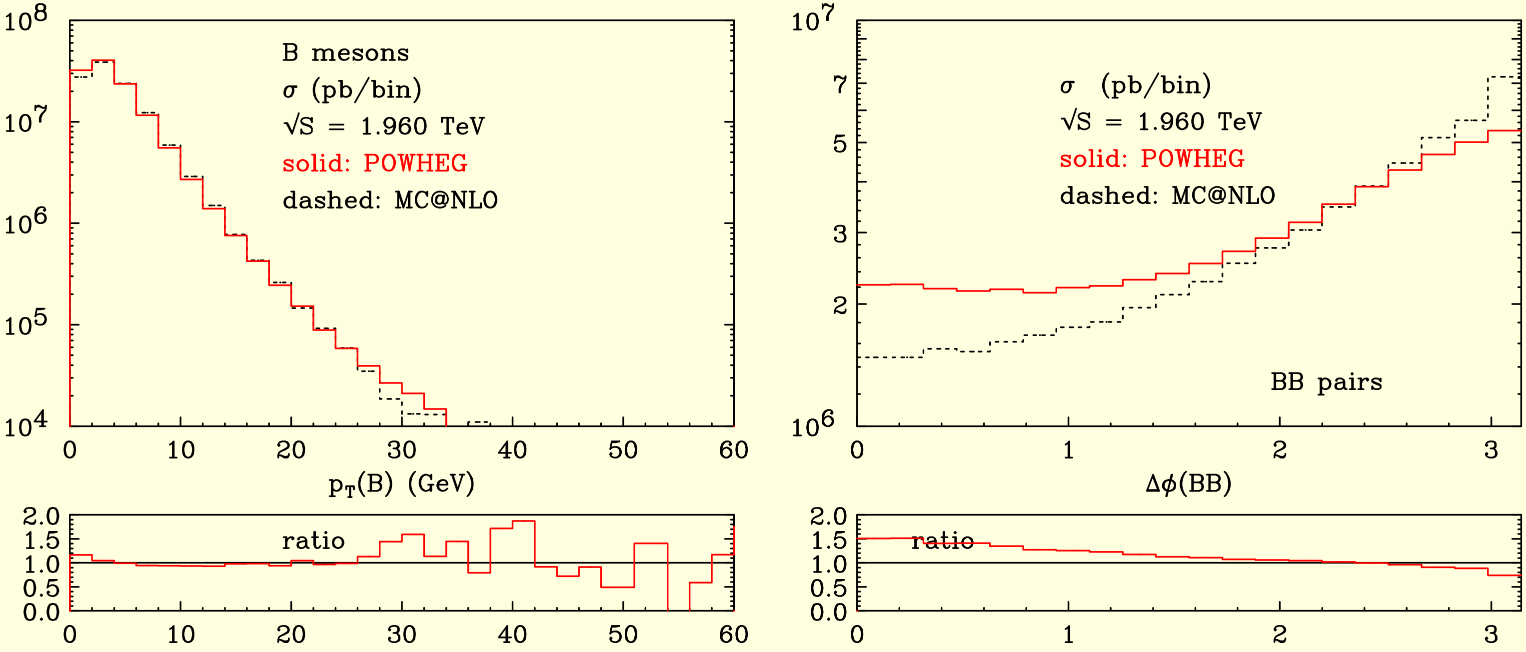
Remarkable agreement for most quantities;

POWHEG and MC@NLO comparison: Top pair production



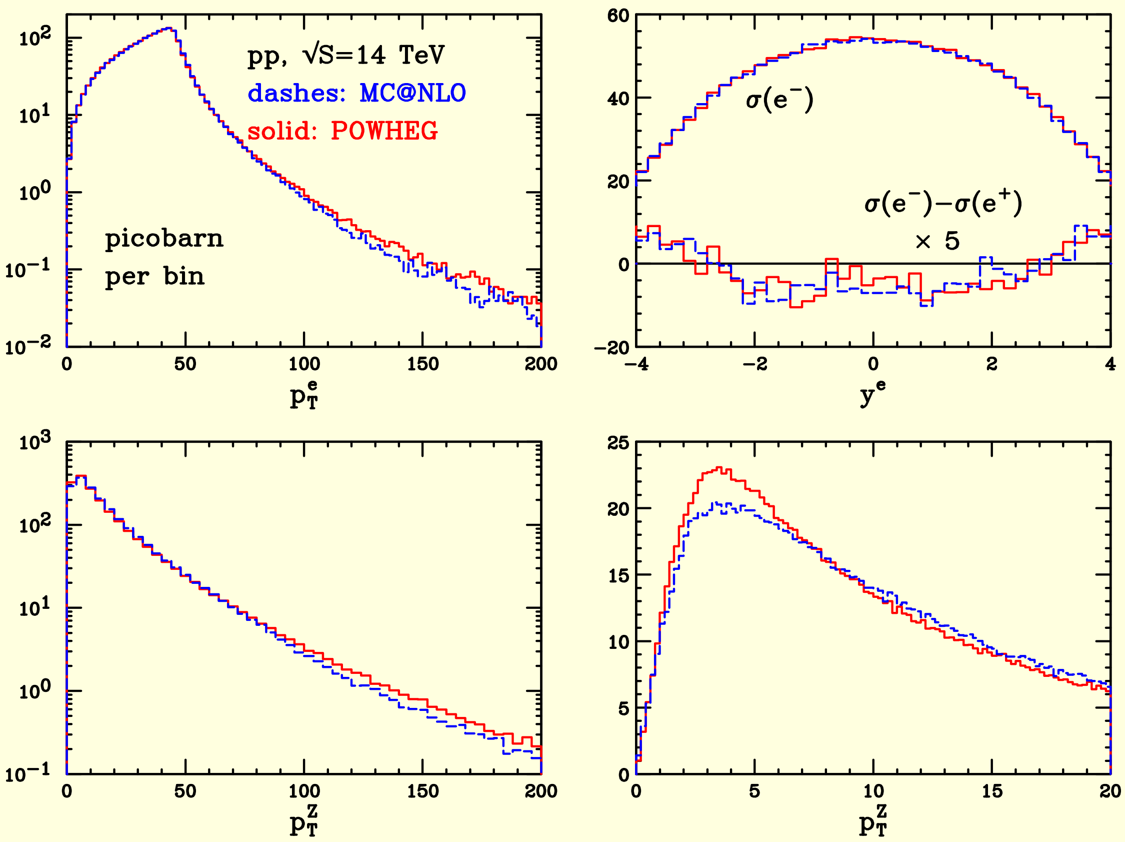
Good agreement for most observables considered
(differences can be ascribed to different treatment of higher order terms)

Bottom pair production



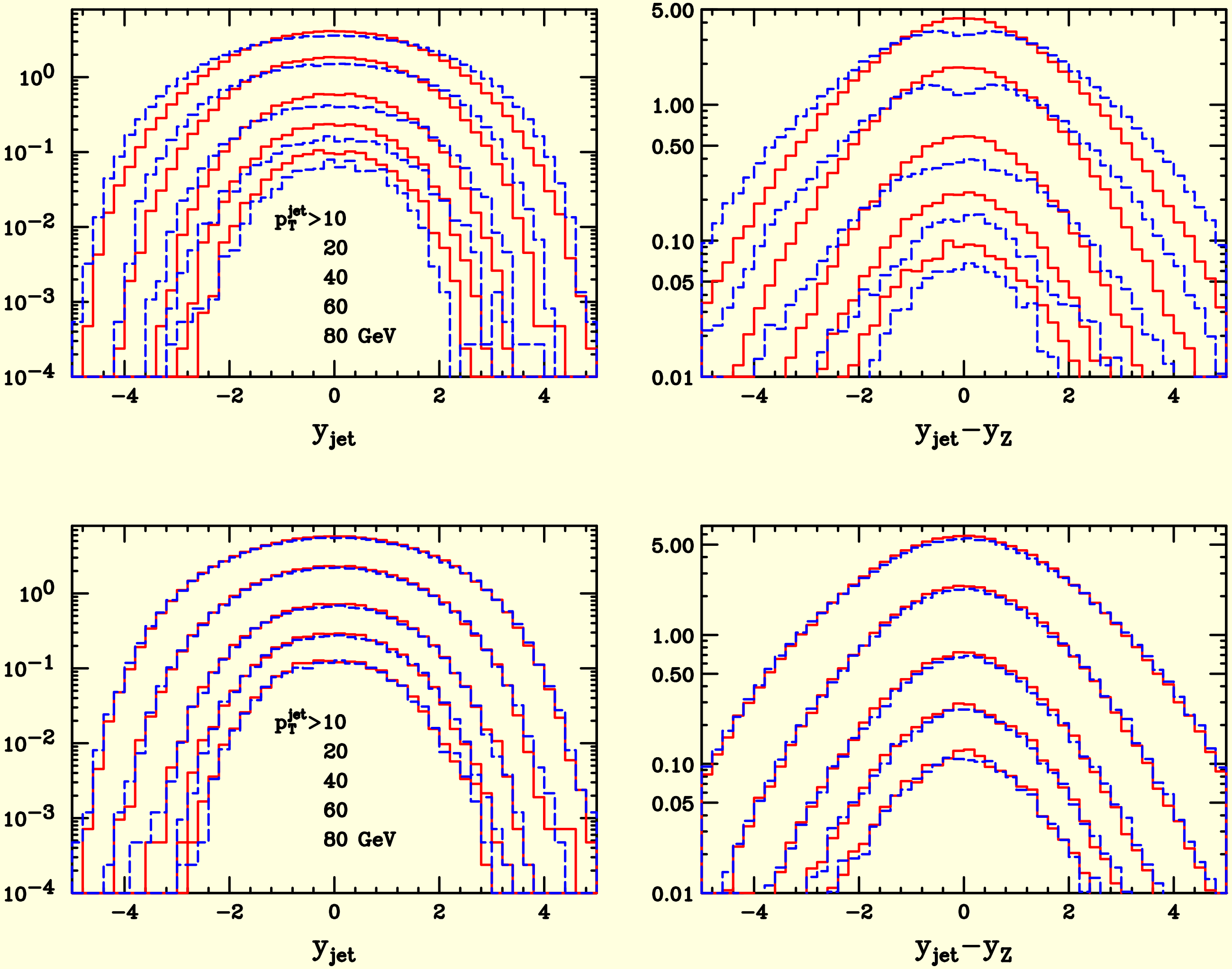
- Very good agreement for large scales (ZZ, $t\bar{t}$ production)
- Differences at small scales ($b\bar{b}$ at the Tevatron)
- POWHEG more reliable in extreme cases like $b\bar{b}, c\bar{c}$ at LHC (yields positive results, MC@NLO has problems with negative weights)

Z production: POWHEG+HERWIG vs. MC@NLO



Small differences in high and low p_T region

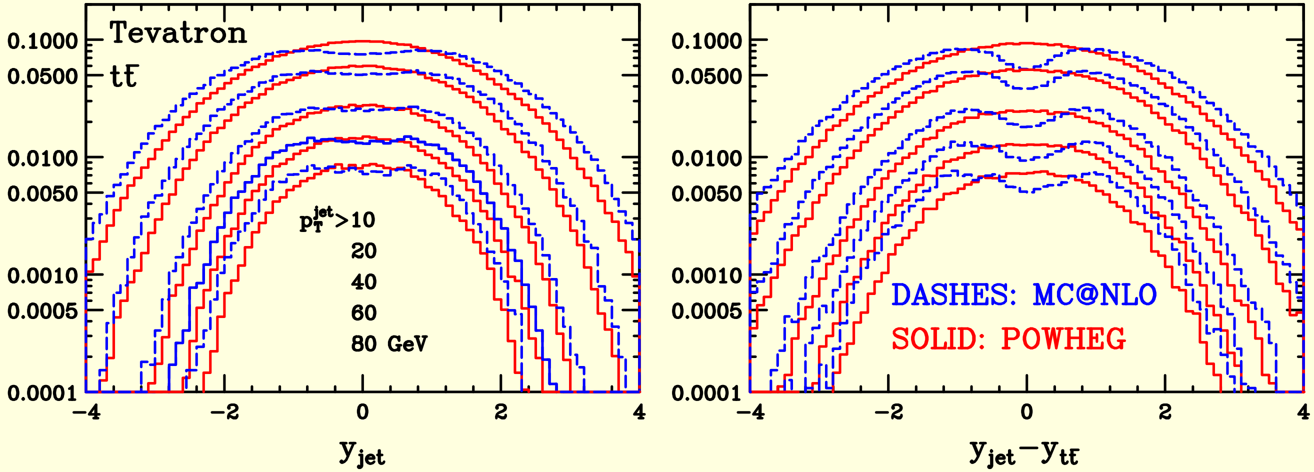
Z production: rapidity of hardest jet (TEVATRON)



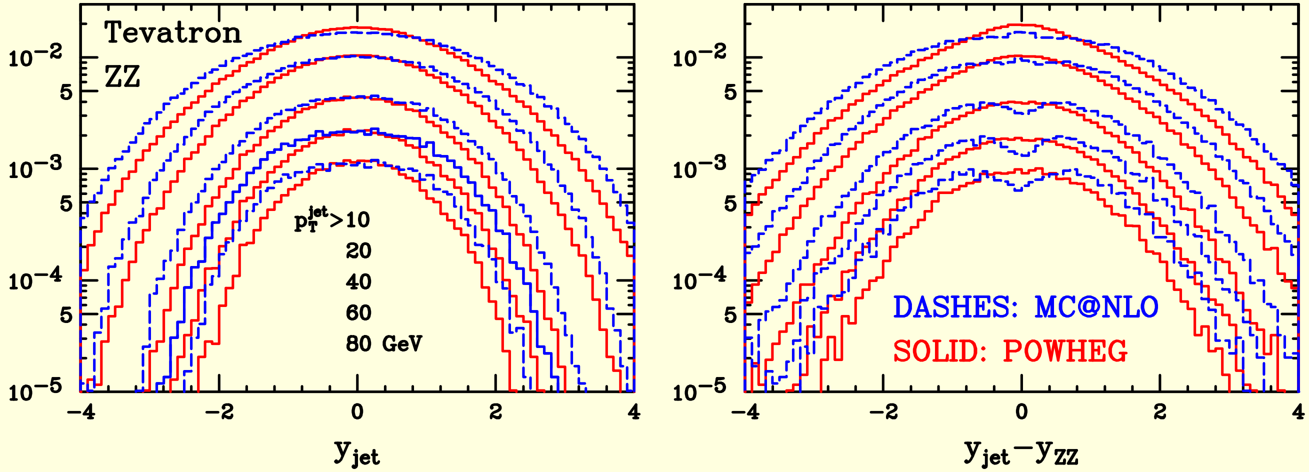
POWHEG+HERWIG
MC@NLO

POWHEG+PYTHIA
PYTHIA

Dip in central region in MC@NLO also in $t\bar{t}$ and ZZ

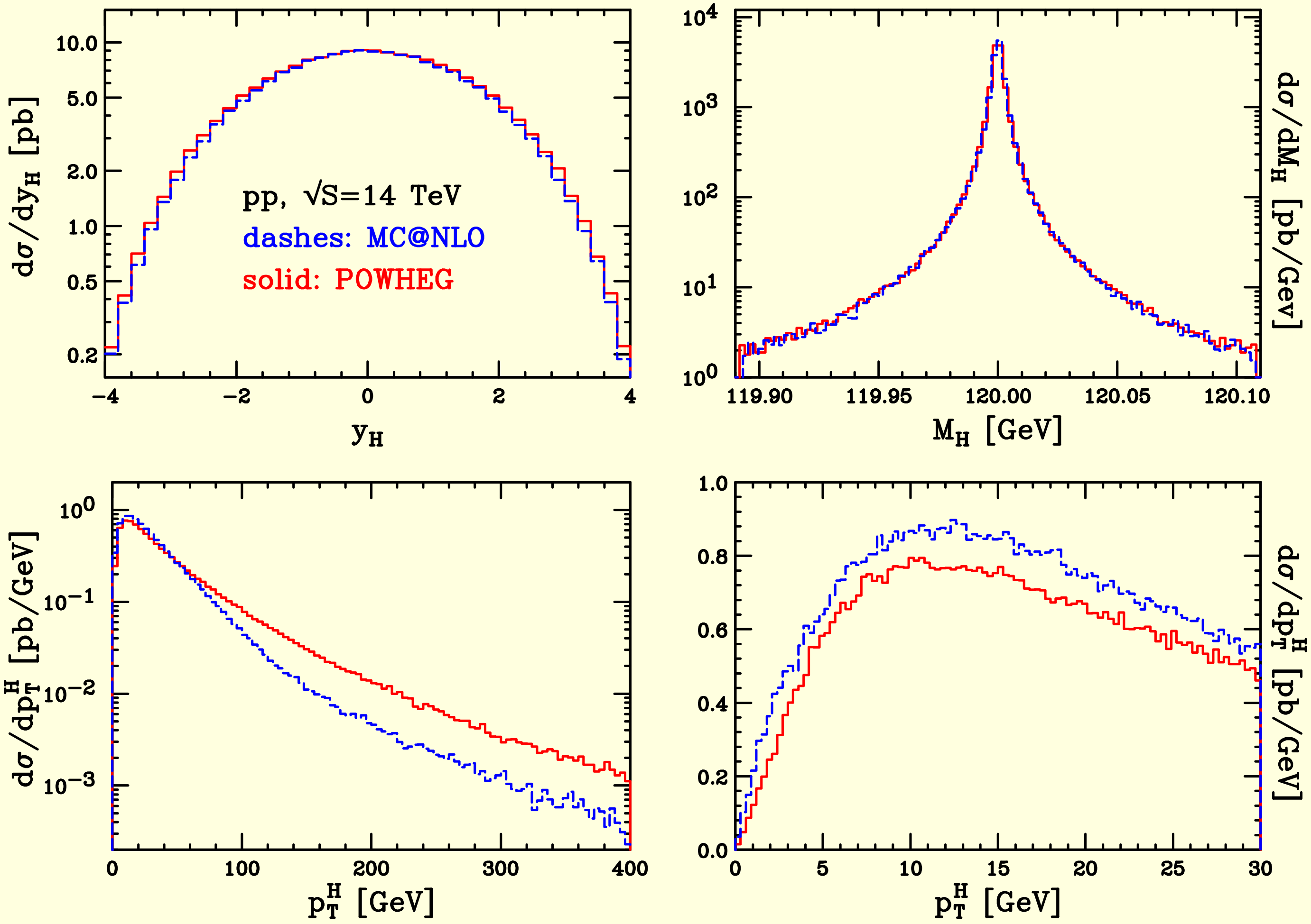


POWHEG+HERWIG
MC@NLO

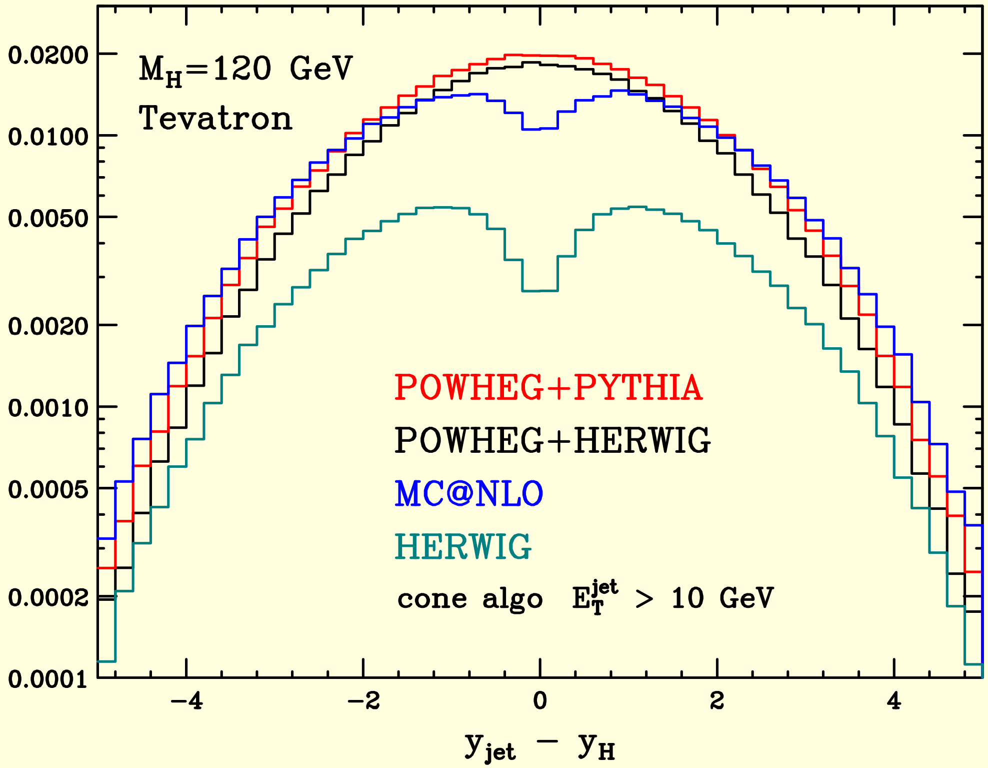


POWHEG+HERWIG
MC@NLO

Higgs boson via gluon fusion at LHC

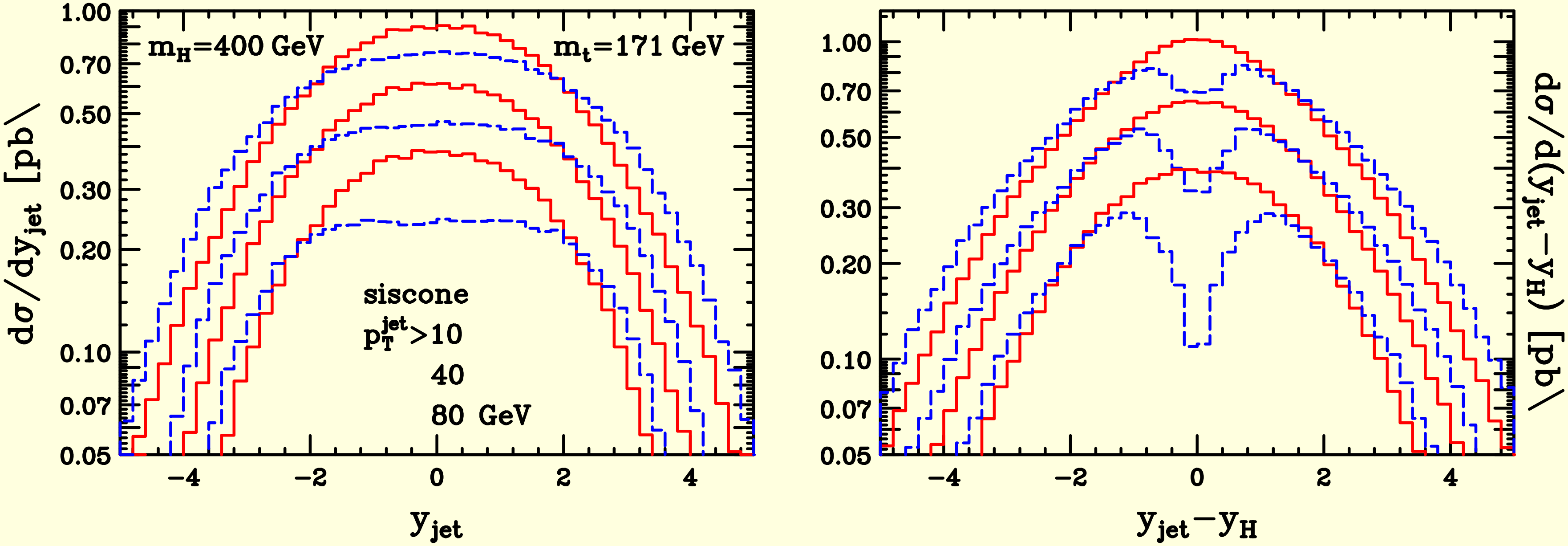


Jet rapidity in h production



Dip in MC@NLO inerithed from even deeper dip in HERWIG (MC@NLO tries to fill dead regions in HERWIG, a mismatch remains).

Gets worse for larger E_T cuts:



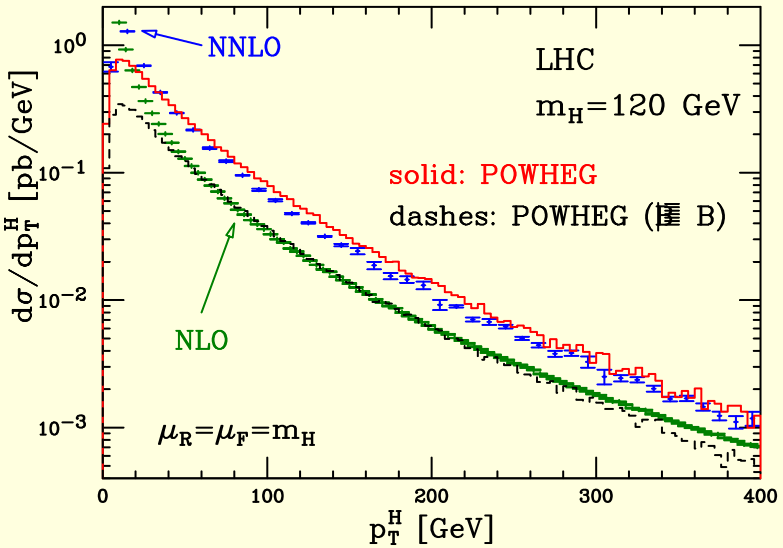
Questions:

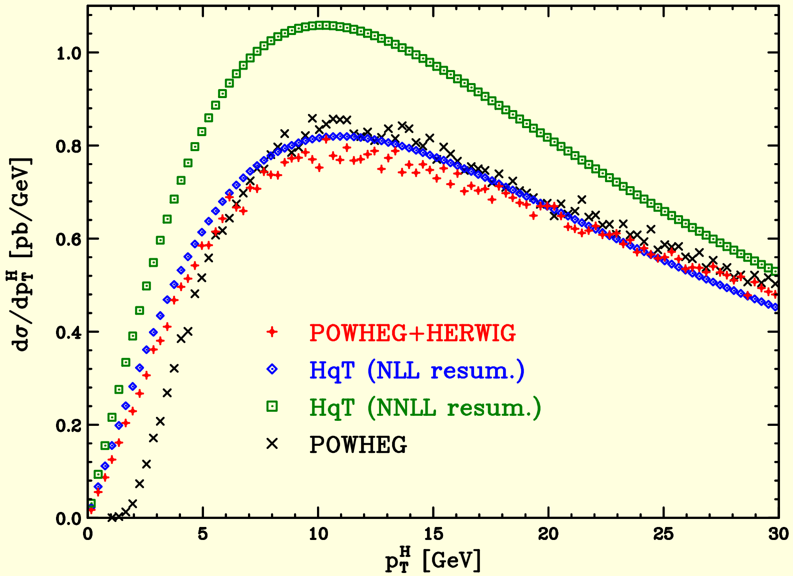
Why MC@NLO has a dip in the hardest jet rapidity?

Why POWHEG has no dip?

(Some have (wrongly) argued that the dip is filled by the harder p_T spectrum)

Hard p_T spectrum: POWHEG vs. NNLO vs. NNLL





$$d\sigma = \bar{B}\, d\Phi_B \left\{ \Delta_{t^0} + \Delta_t \frac{R}{B} d\Phi_r \right\} \approx \frac{\bar{B}}{B} R\, d\Phi_B\, d\Phi_r = \underbrace{\{1 + \mathcal{O}(\alpha_s)\}}_{\approx 2 \text{ for here!}} R\, d\Phi$$

Large enhancement because of the large K factor in Higgs production.

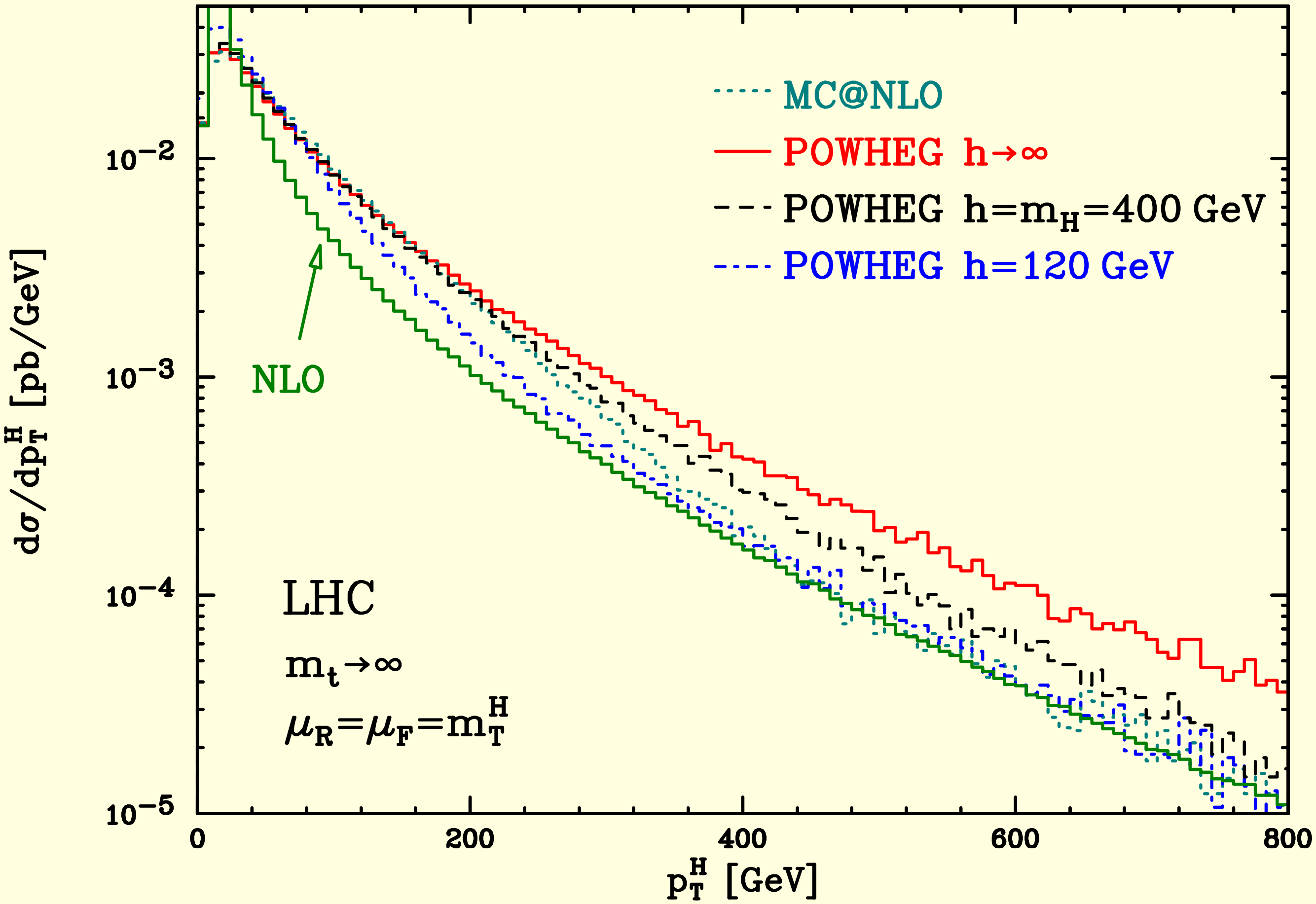
Higher p_T spectrum because of the choice $R_s = R$.
(Better agreement with NNLO this way)

Use the flexibility in POWHEG to choose $R_s \neq R$

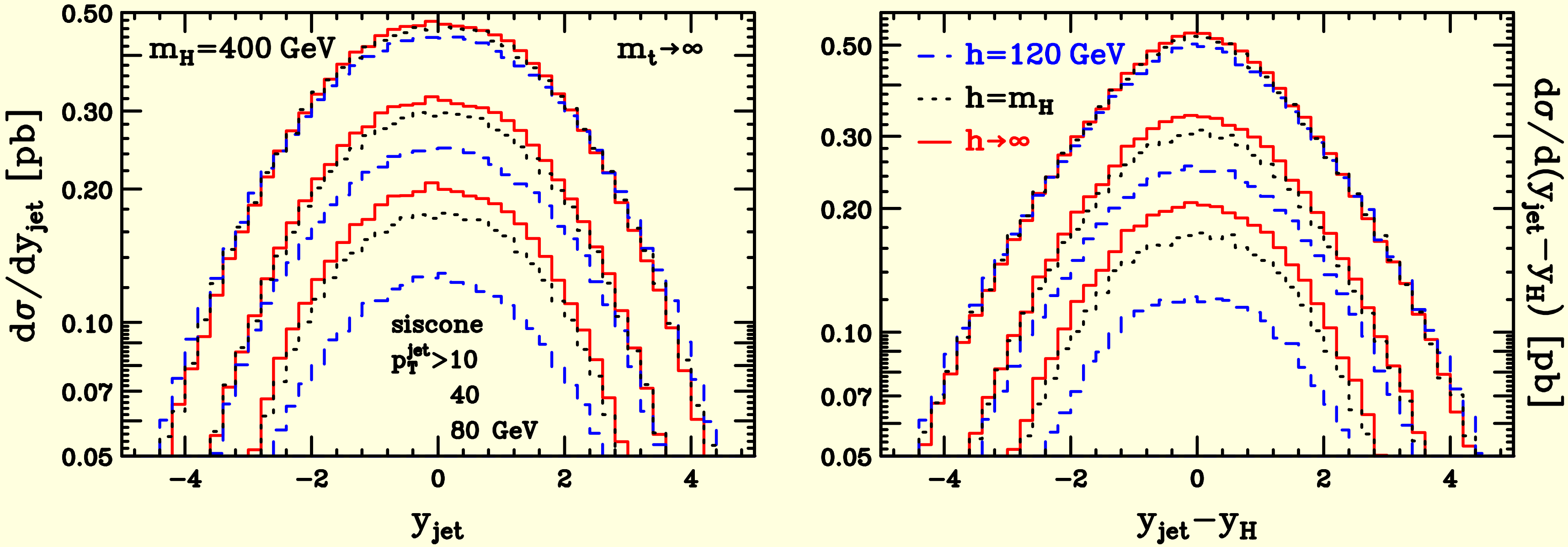
$$R_s = R \frac{h^2}{k_T^2 + h^2}$$

Agrees with NLO at high p_T.

However ...



No dips arise in the jet rapidity distributions:



So: extra radiation at high k_T and dips are unrelated issues in POWHEG.

Why is there a dip in MC@NLO?

For large k_T:

$$d\sigma = \frac{\bar{B}^{\mathrm{MC}}}{B} R^{\mathrm{MC}}\, d\Phi_B d\Phi_r^{\mathrm{MC}} + [R - R^{\mathrm{MC}}] d\Phi$$

$$= \underbrace{R d\Phi}_{\text{no dip}} + \underbrace{\left(\frac{\bar{B}^{\mathrm{MC}}}{B} - 1\right)}_{\substack{\mathcal{O}(\alpha_s),\\ \text{large for Higgs!}}} \times \underbrace{R^{\mathrm{MC}}}_{\text{Herwig dip}} d\Phi$$

So: a contribution with a dip is added to the exact NLO result;

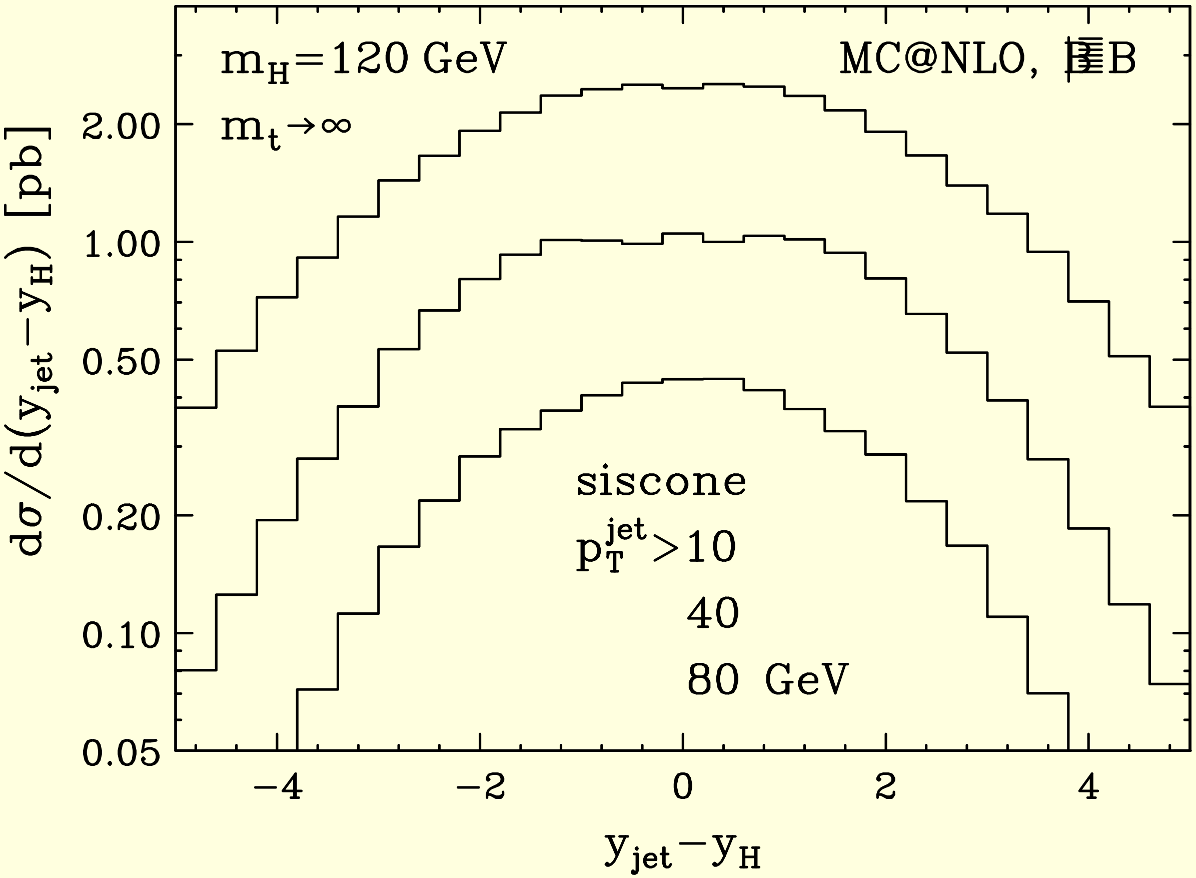
The contribution is $\mathcal{O}(\alpha_s R)$, i.e. NNLO!

but is large in processes with large K-factors.

Can we test this hypothesis? Replace $\bar{B}^{\mathrm{MC}}(\Phi_n) \Rightarrow B(\Phi_n)$ in MC@NLO!

the dip should disappear ...

MC@NLO with $\bar{B}^{\mathrm{MC}}$ replaced by B



No visible dip is present! (see also Hamilton,Richardson,Tully, 2009)

Summary of MC@NLO and POWHEG comparisons

- Fairly good agreement on most distributions
- Areas of disagreement can be tracked back to NNLO terms, arising mostly because of the use of an NLO inclusive cross section (the $\bar{B}$ function) to shower out the hardest radiation.
- In POWEG, since the hardest radiation is generated by POWHEG itself, one has the flexibility of tuning the magnitude of these NNLO terms.
- For MC@NLO, these NNLO terms can generate unphysical behaviour in physical distributions, reflecting the structure of the underlying shower Monte Carlo (dead zones, depletion away from jets, etc.).

Prospects in POWHEG

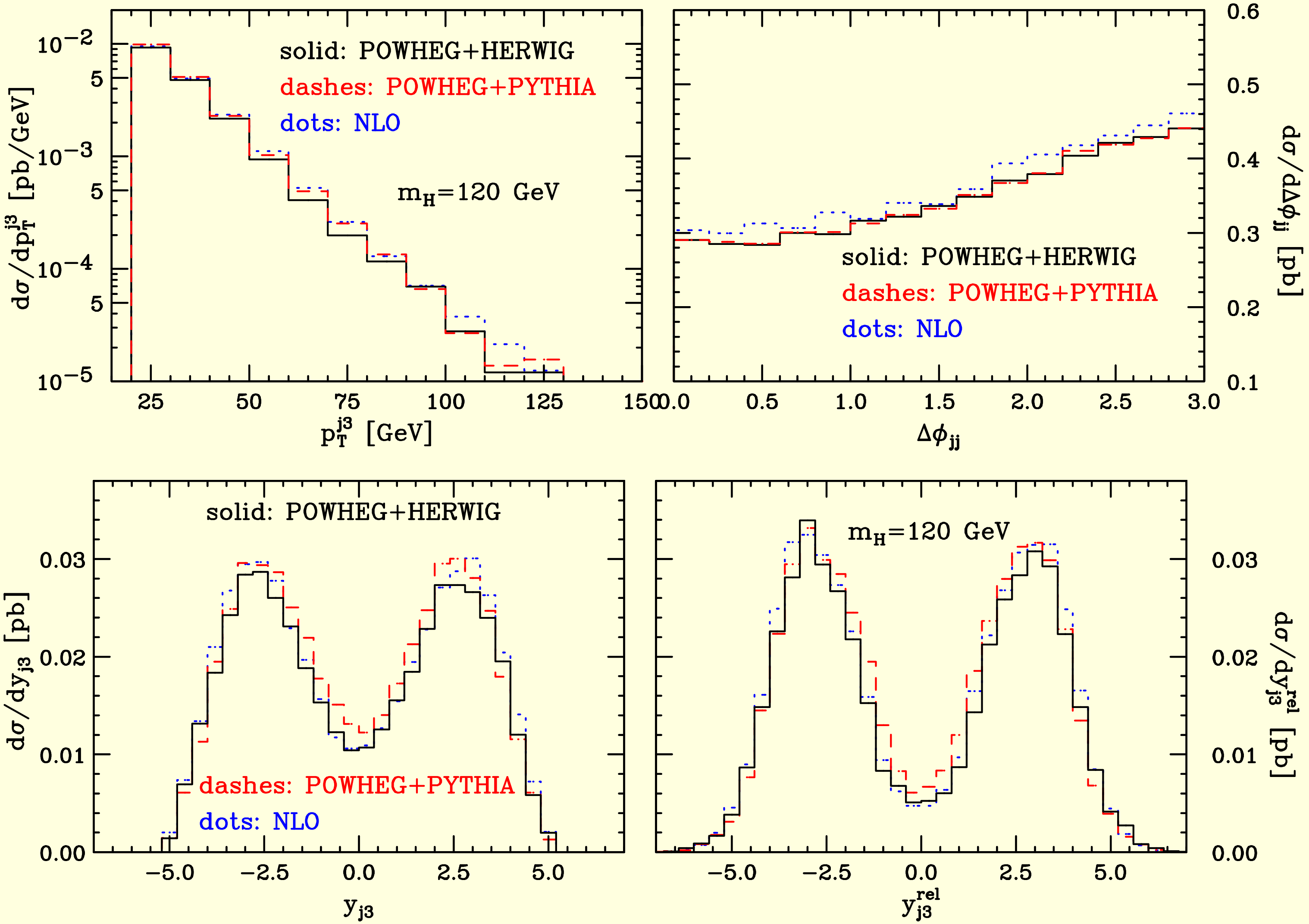
Towards automation: the POWHEG BOX

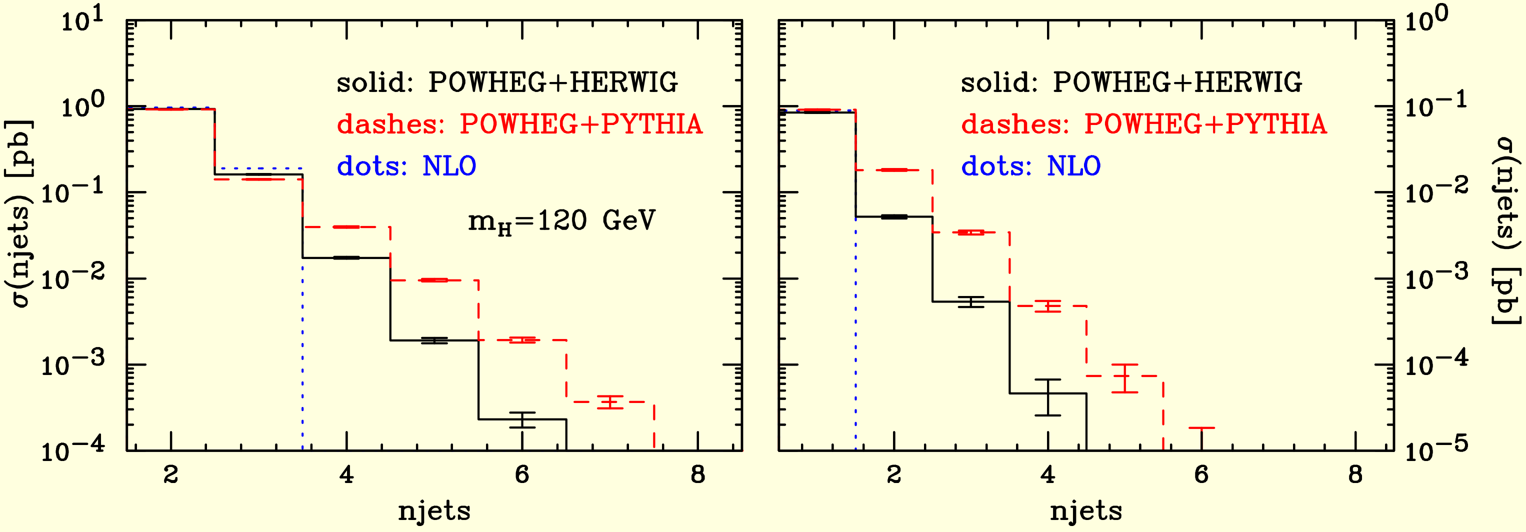
The MIB (Milano-Bicocca) group (Alioli, Oleari, Re, P.N.) is working on an automatic implementation of POWHEG for generic NLO processes.

The framework has been tested in processes already implemented, like single vector boson production and single top production

The new processes $hh \to Z + 1\mathrm{jet}$, and the VBF higgs production, have been implemented in this framework.

Higgs bosons in VBF



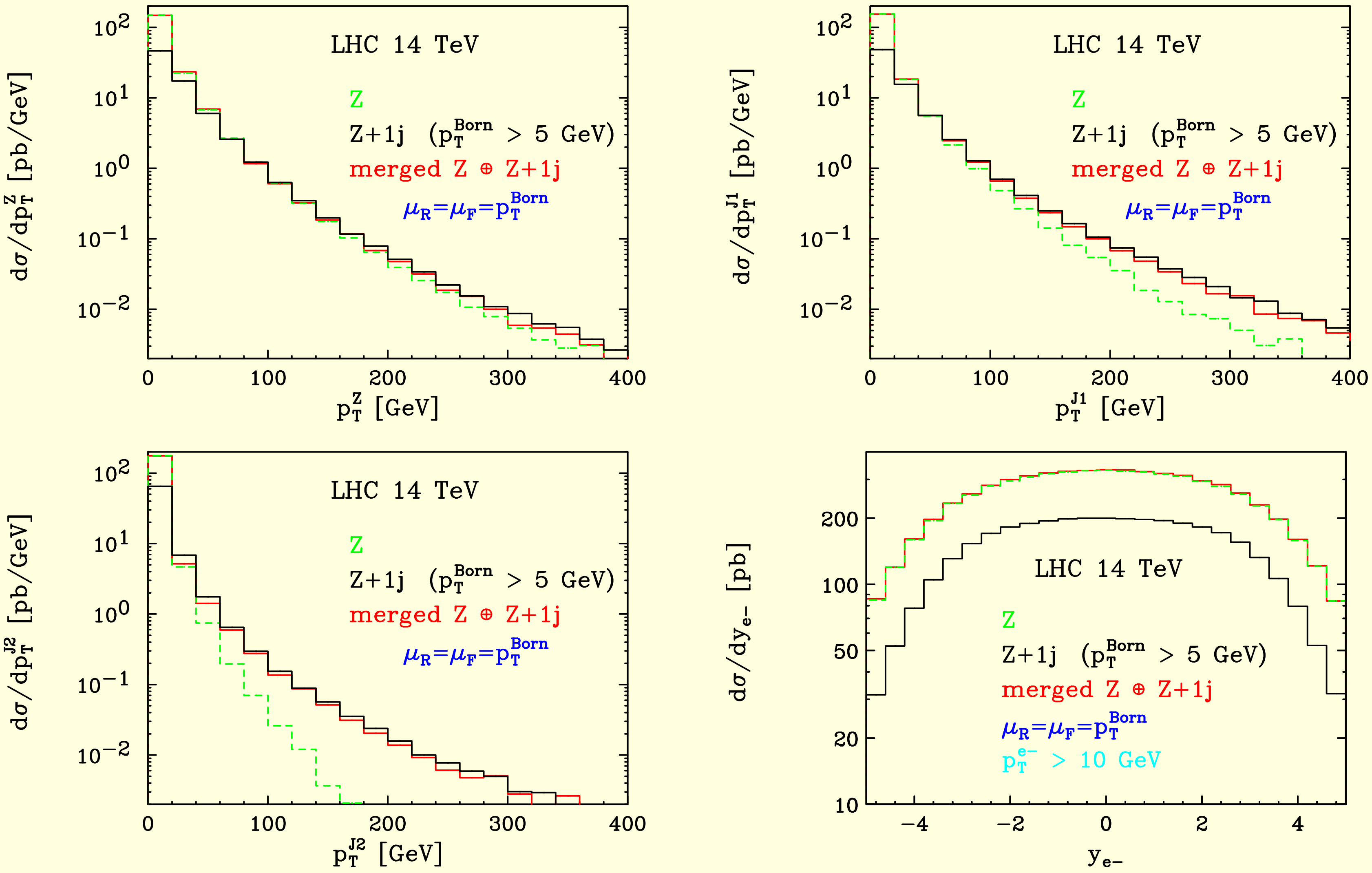


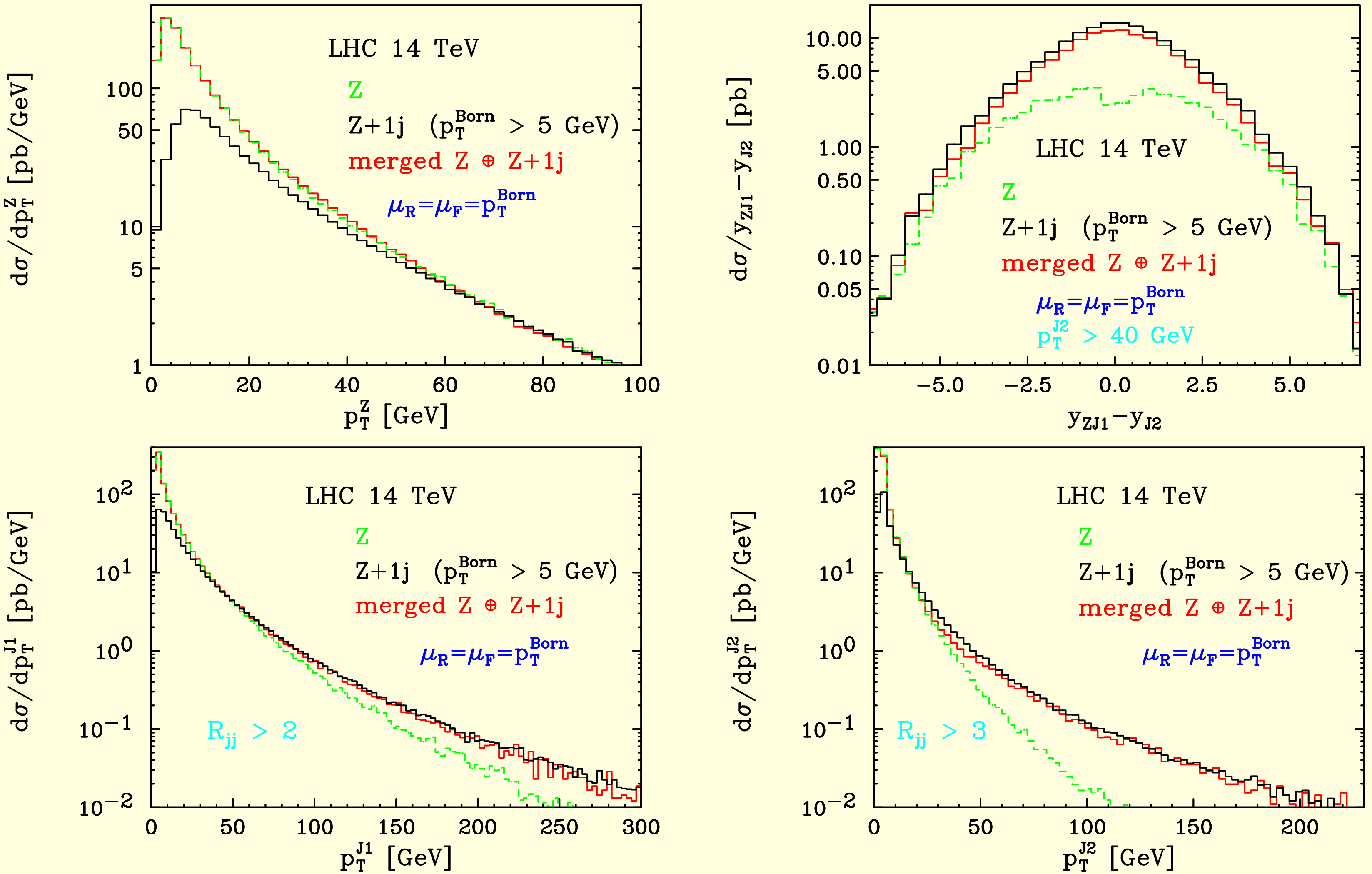
$$p_T^j > 20\,\mathrm{GeV}, \qquad |y_j| < 5$$

$$p_T^{\mathrm{tag}} > 30\,\mathrm{GeV}, \quad |y_{j_1} - y_{j_2}| > 4.2, \qquad y_{j_1} \times y_{j_2} < 0, \quad m_{jj} > 600\,\mathrm{GeV}$$

veto jet: $\min(y_1, y_2) < y_j < \max(y_1, y_2)$

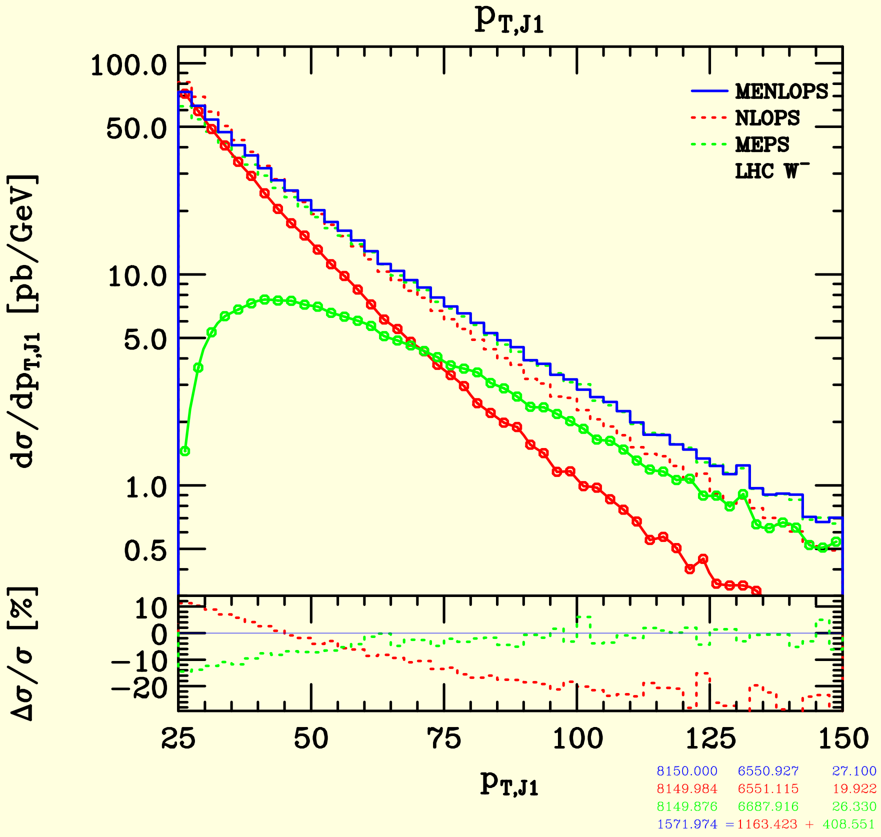
Preliminary: merge Z and $Z+1$ jet samples

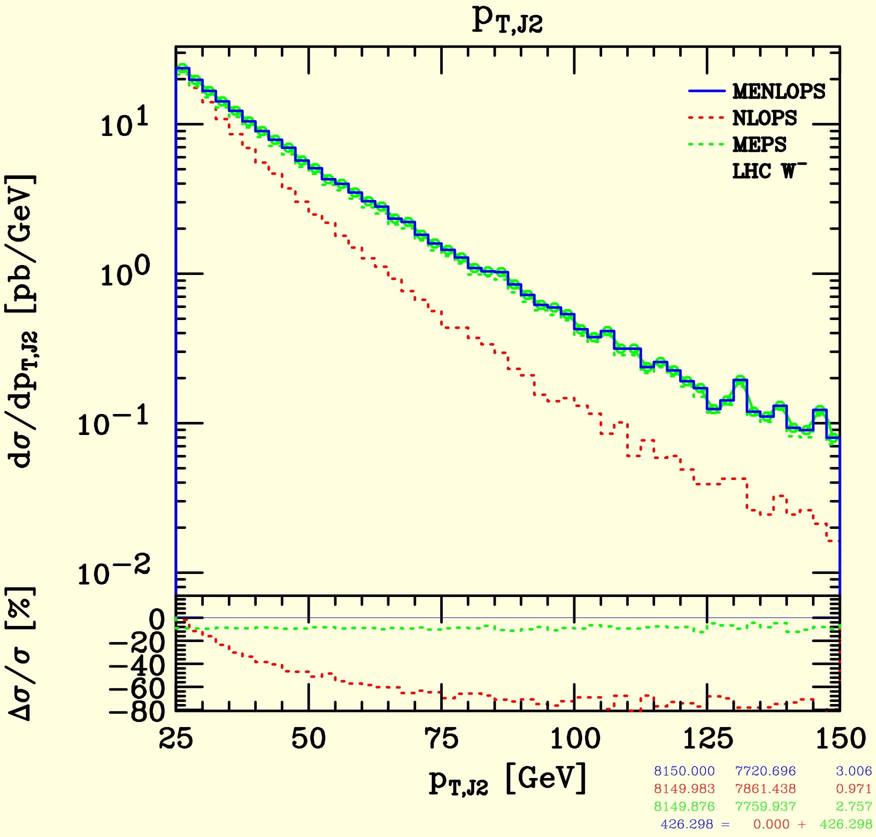


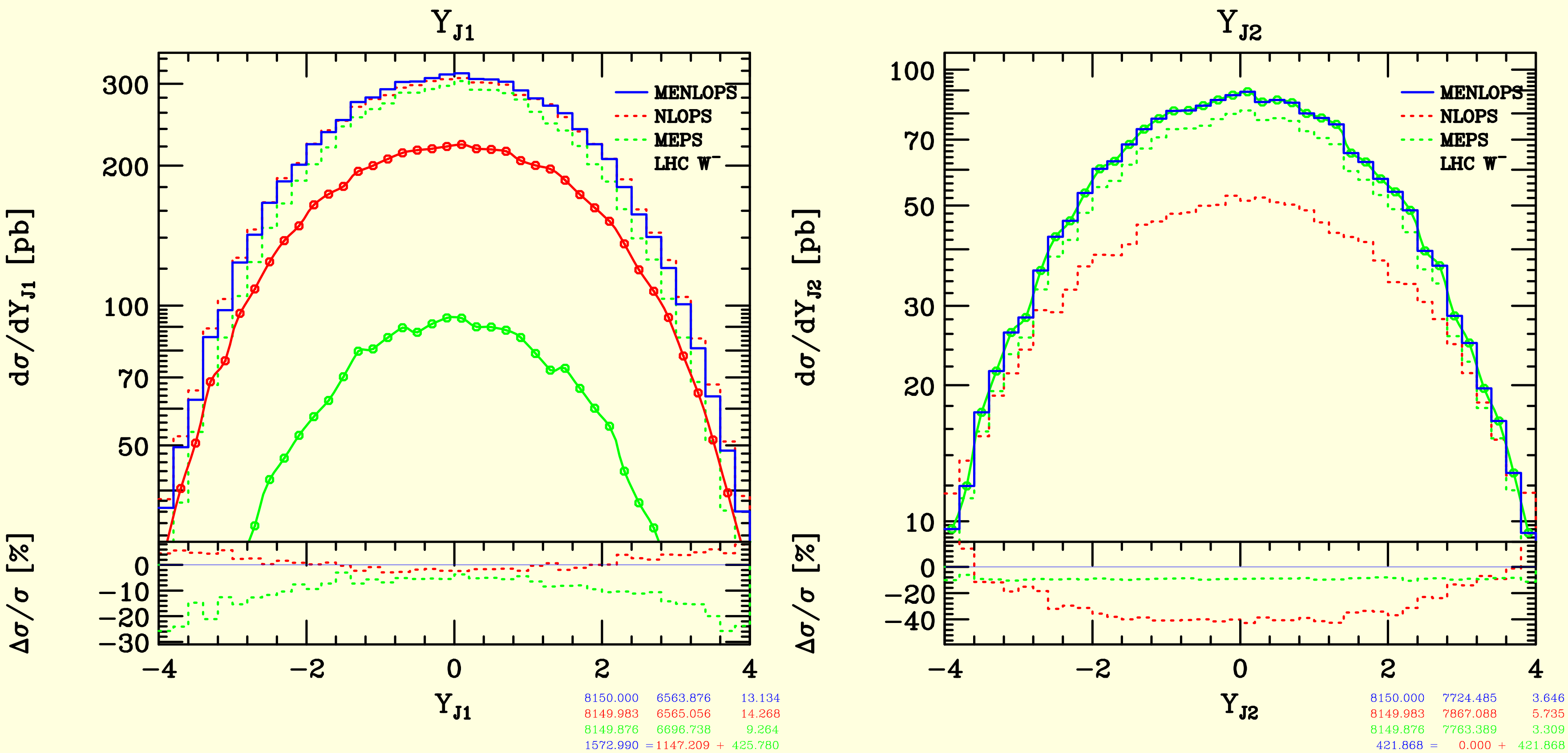


(with S.Alioli, E. Re, C. Oleari, P.N.)

Preliminary: merge POWHEG and MEPS samples







(K. Hamilton, P.N)

Conclusions

- NLO accuracy with Shower MC has become a reality in recent years.
- The `POWHEG` method is progressing, with new processes being included
- Progress in understanding agreement and differences between `MC@NLO` and `POWHEG`
- A path to full automation of `POWHEG` implementations of arbitrary NLO calculations is open
- Many interesting problems are just being addressed: interfacing `POWHEG` to CKKW style showers; CKKW at NLO, etc.