SOFT-WALL STABILIZATION

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GDR Terascale, Saclay, March 29th 2010

Collaboration with J.A.Cabrer and M.Quirós

OUTLINE

- Introduction
- Soft Wall models (models with 1 brane)
- Soft Wall stabilization and spectra
- A class of models
- Conclusions

OPEN QUESTIONS IN THE SM (AND BEYOND)

- What is the origin of Electroweak Symmetry Breaking?
- Why is the scale of the Z and W bosons 10^{17} times smaller than the Planck mass? (Hierarchy Problem)
- Why is there such a huge hierarchy in the masses of the Standard Model fermions?
- What is the origin of neutrino masses?
- If there is Supersymmetry, how is it broken?
- If there is a Grand Unified Theory, how is it broken to the SM, and why are there no colored Higgses?

All these issues can be addressed in models with Extra Dimensions

RS MODELS

Randall & Sundrum '99

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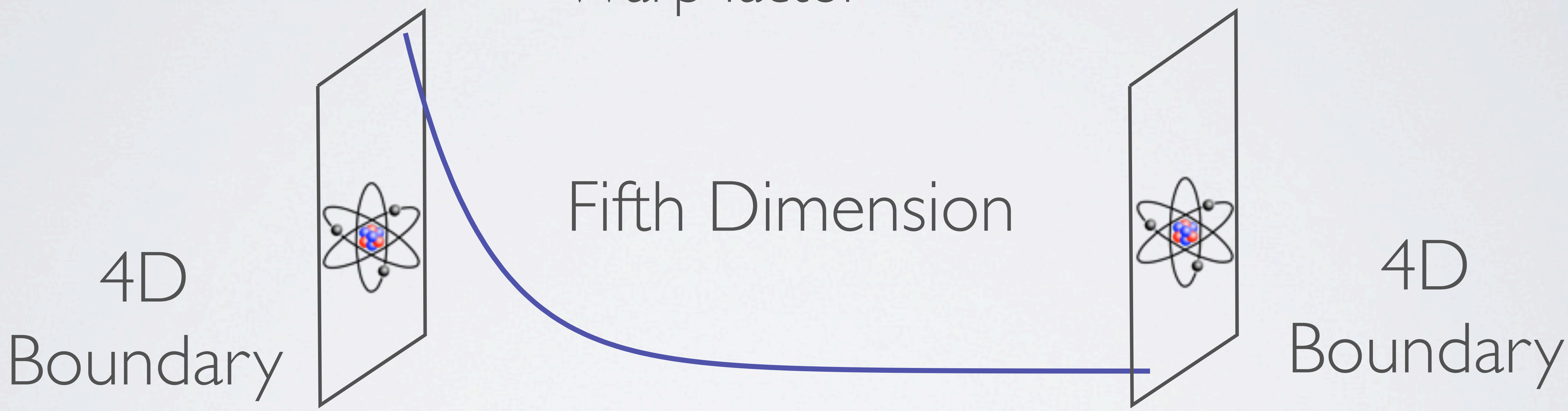
$$ds^2 = e^{-2\,k\,y} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2$$

Warp factor



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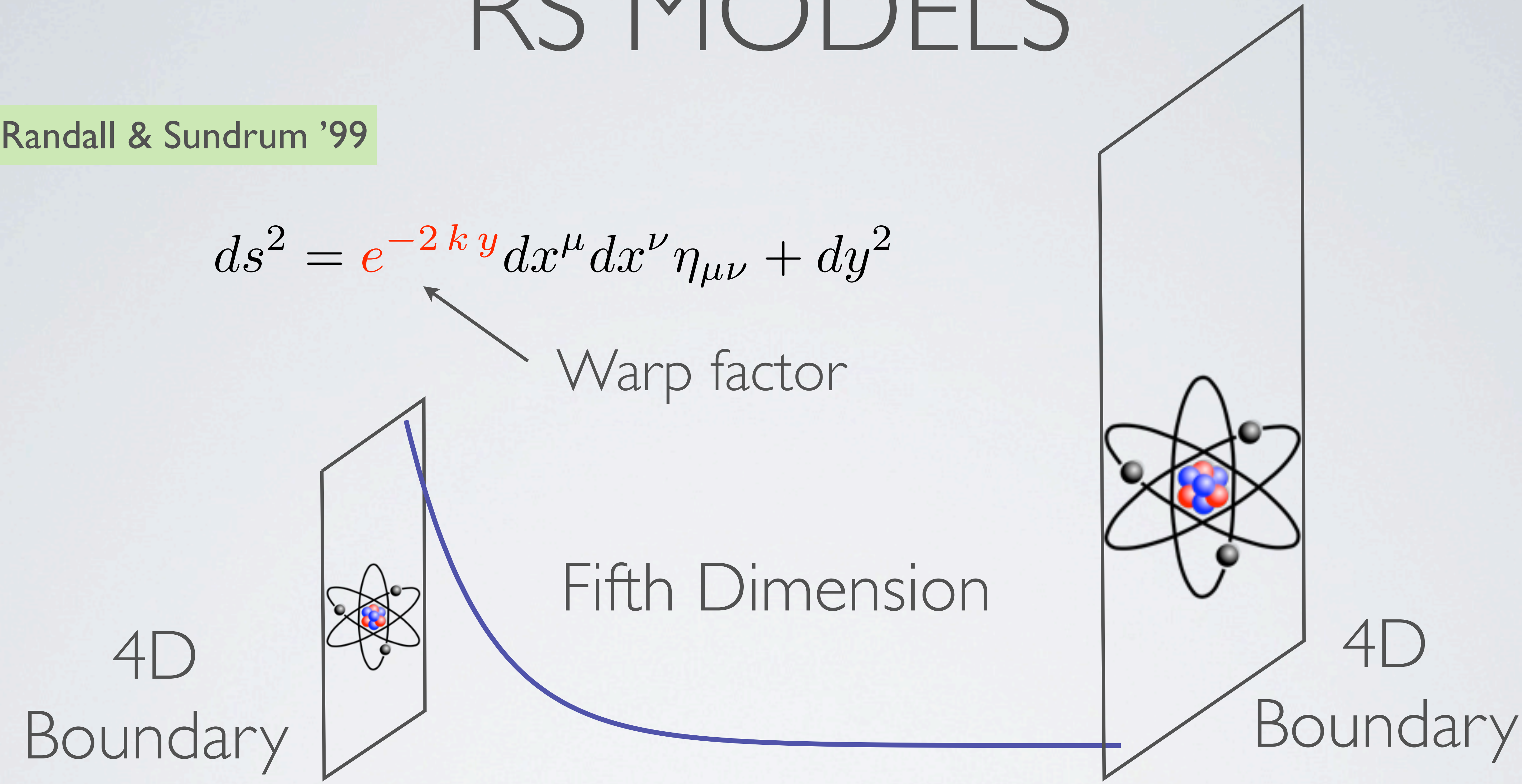

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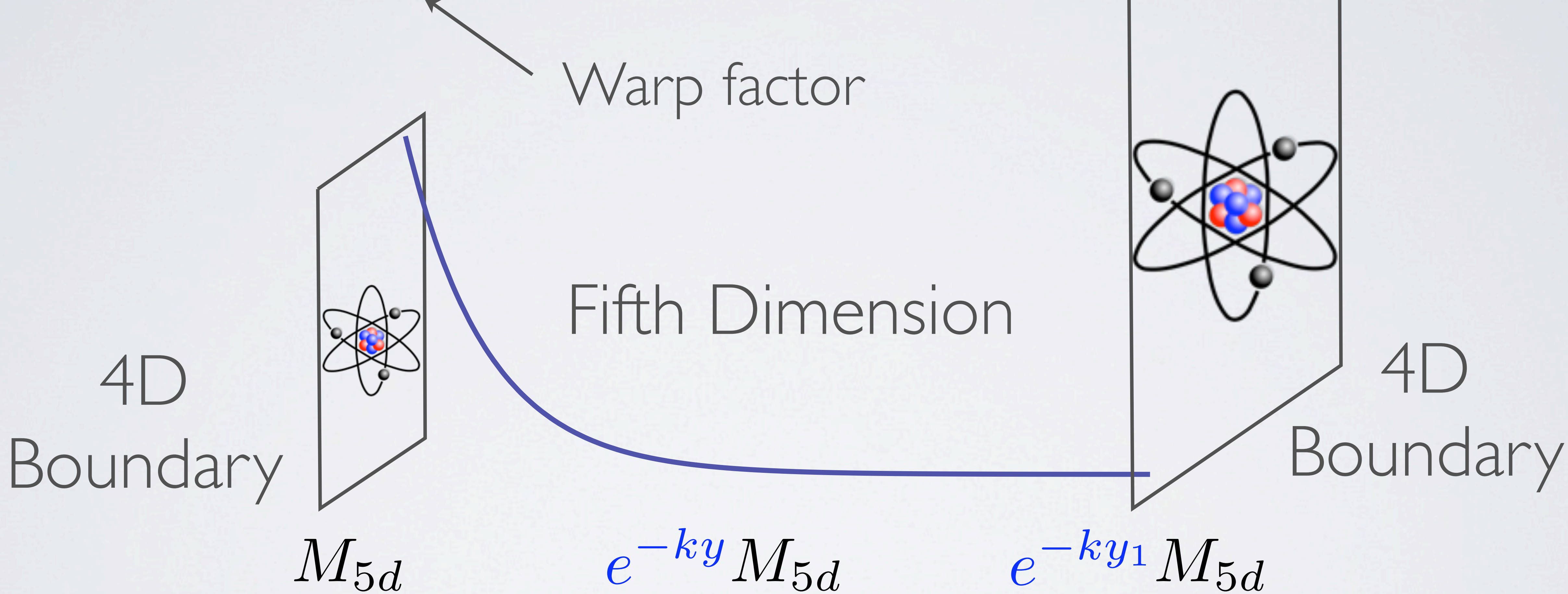

Fundamental cutoff scale is redshifted

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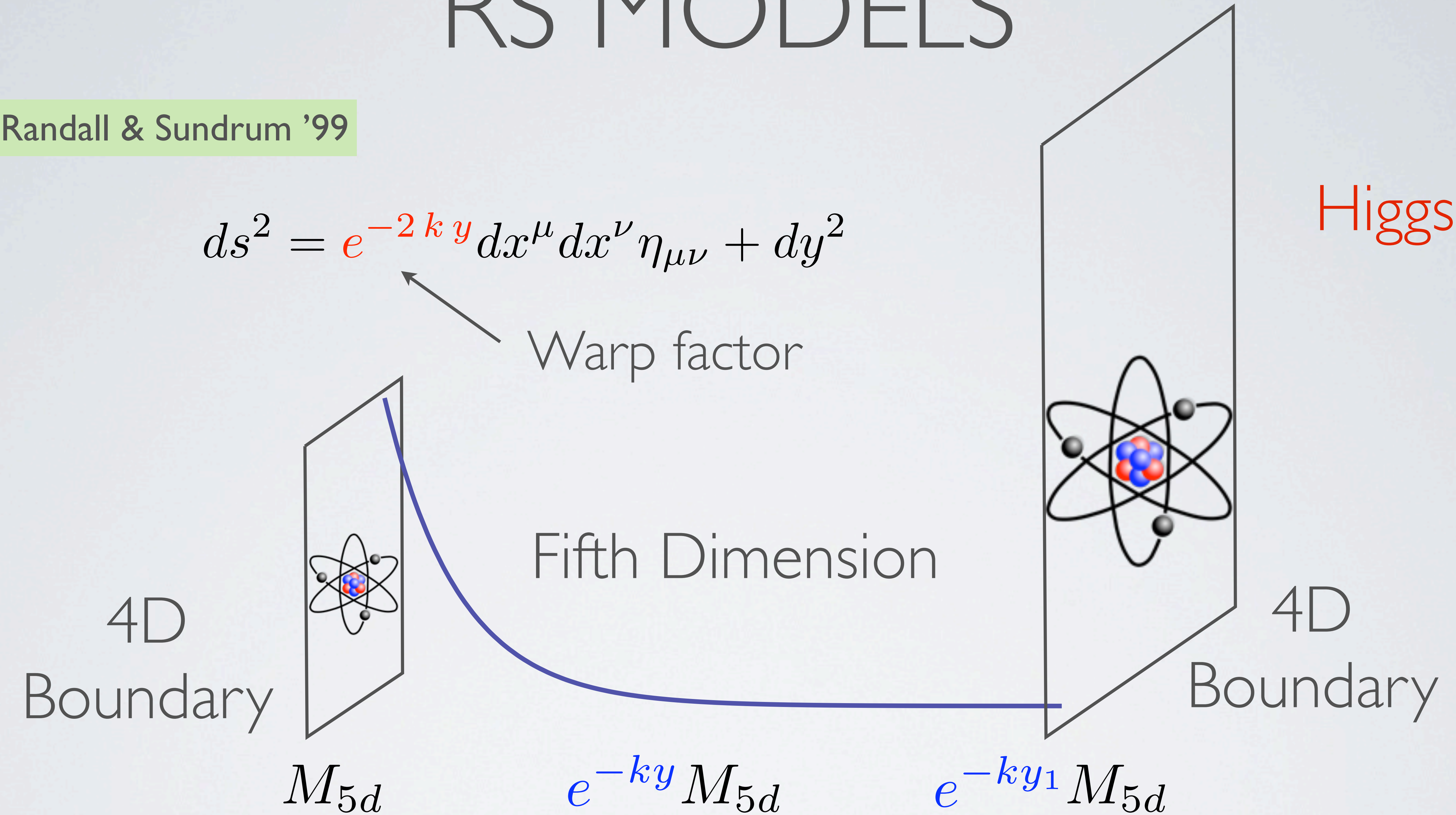

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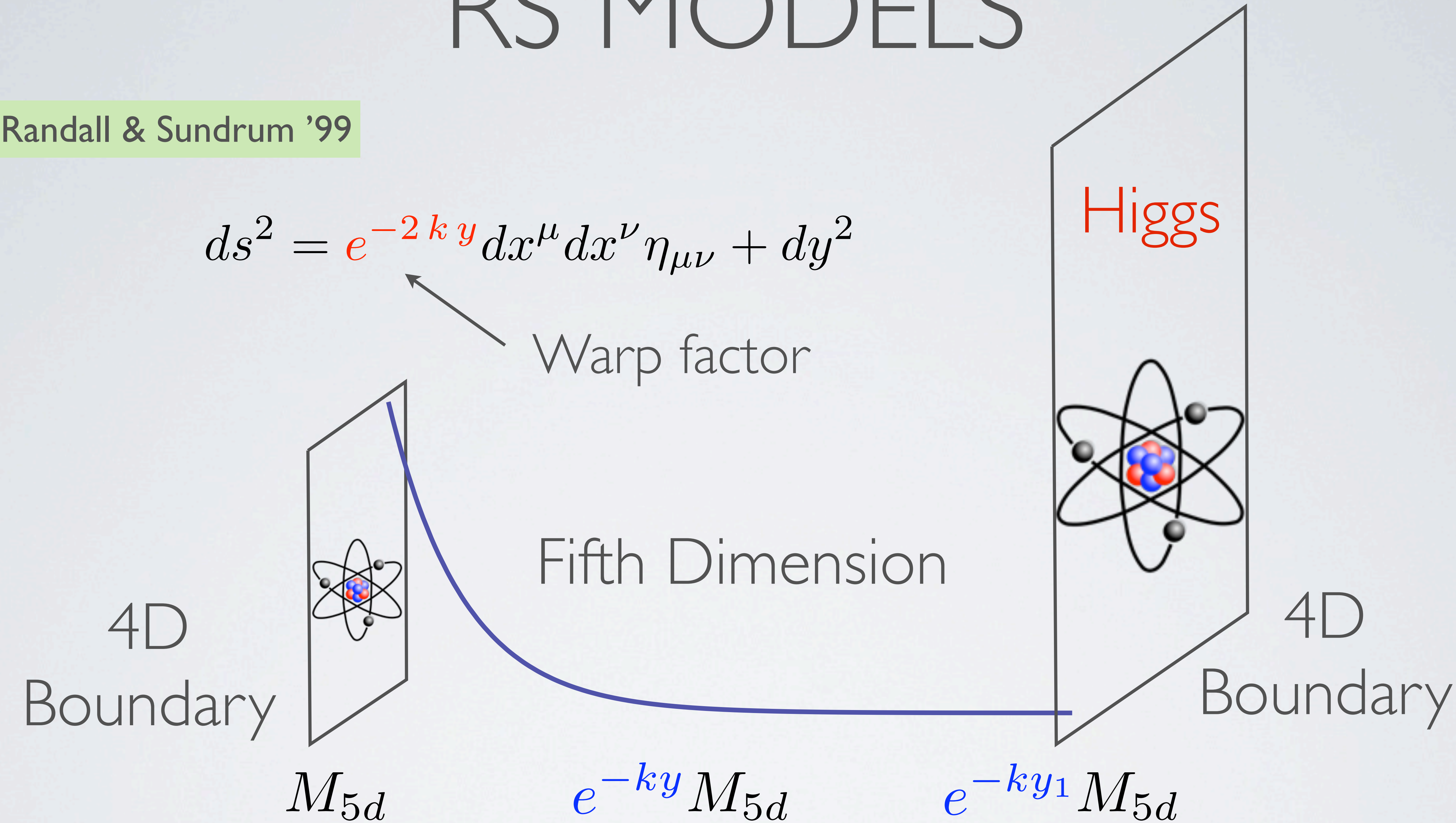


Fundamental cutoff scale is redshifted

STABILIZATION PROBLEM

- Pure 5D Gravity with negative Cosmological Constant (and appropriate brane tensions) has RS as a solution.

- BUT: Interbrane distance is UNDETERMINED

- There is an extra massless mode (RADION)

$$g_{MN} = g_{MN}^{RS} + \begin{pmatrix} h_{\mu\nu} & \\ & h_{55} \end{pmatrix}$$

- How to fix the length of the extra dimension?
- How to generate potential and mass for the radion

- Can be solved by adding a scalar field

Goldberger & Wise '99

Do we need two branes?

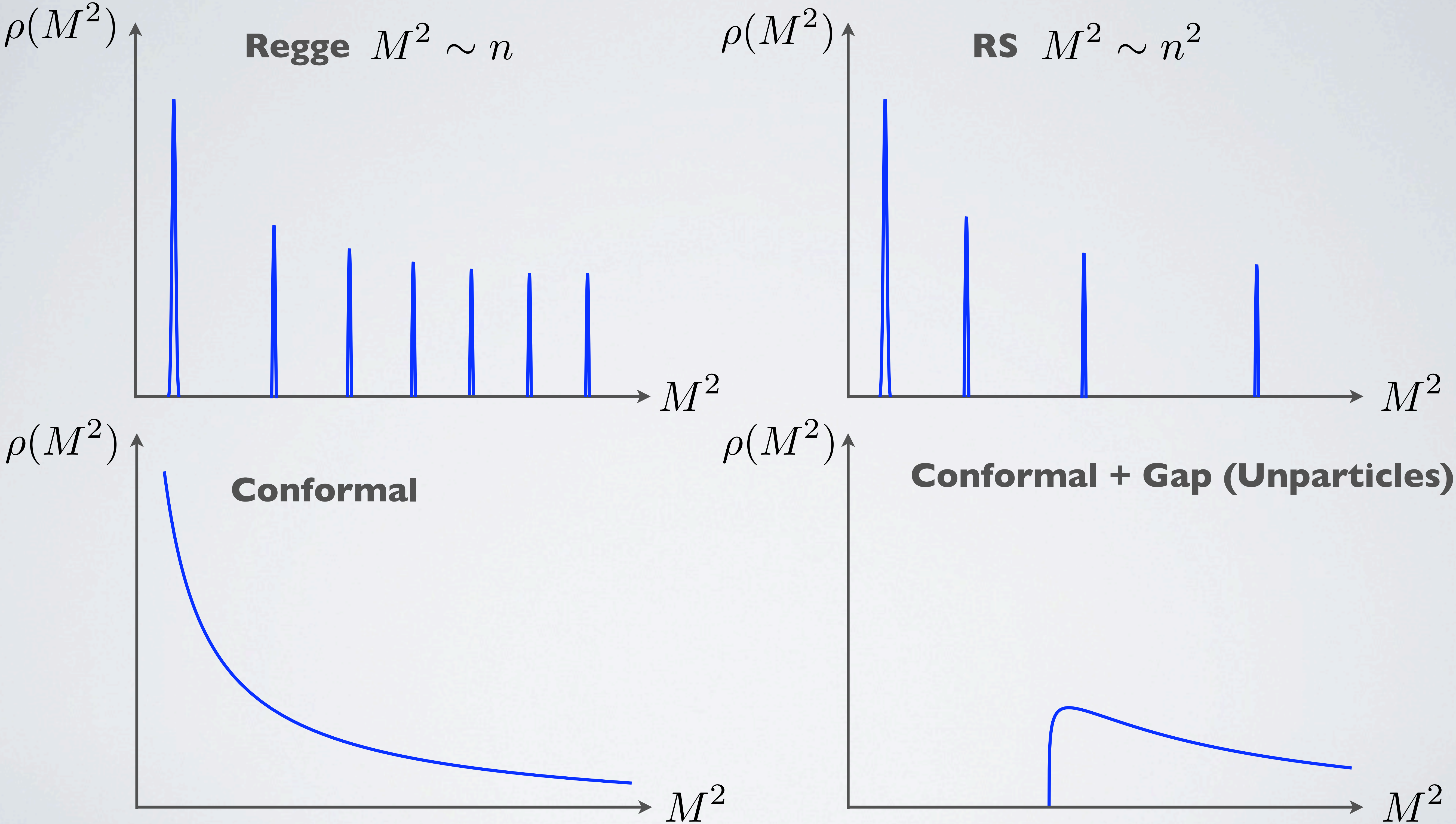
GAUGE/GRAVITY DUALITY

Gravity/Gauge theory correspondence asserts that the 5D theory is dual to a strongly coupled 4D gauge theory

KK modes on 5d side ⟷ Resonances ("mesons") on 4d side

- RS with two branes: KK spectrum is roughly $m_n^2 \sim n^2$
- In 4D strongly coupled gauge theories many more possibilities.

POSSIBLE SPECTRA



IR brane can be replaced by SOFT WALL

SUPERPOTENTIAL METHOD

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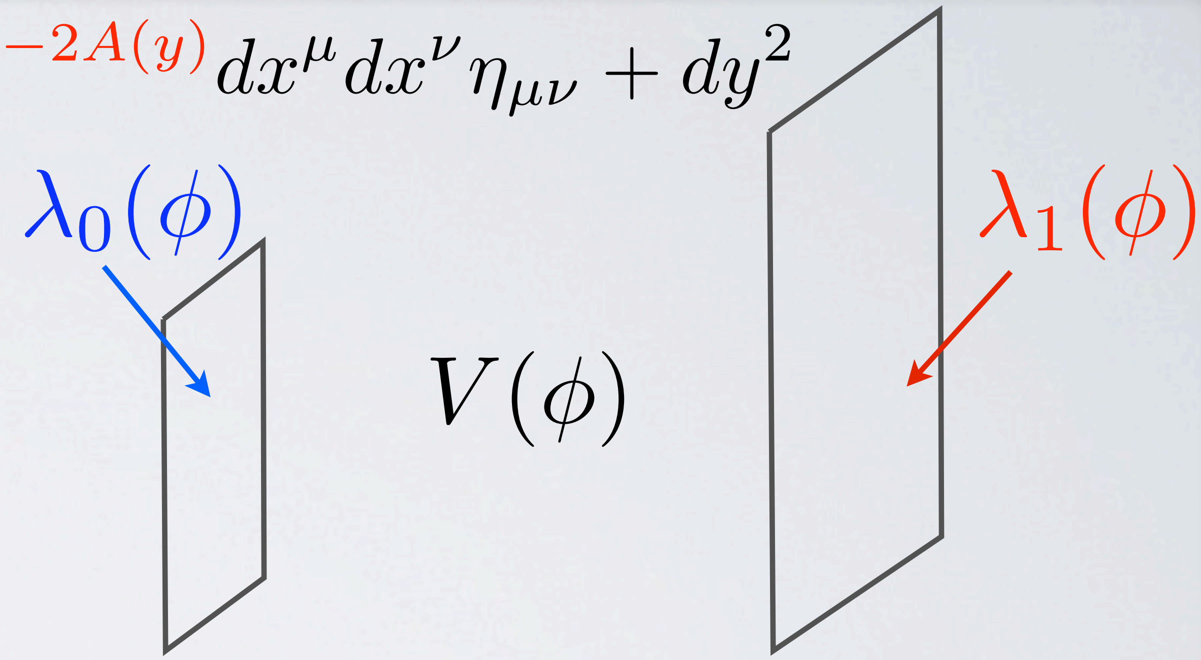
- Gravity + scalar field with bulk and brane potential

- Solve Einstein equations coupled to scalar

$$ds^2 = e^{-2A(y)} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2$$



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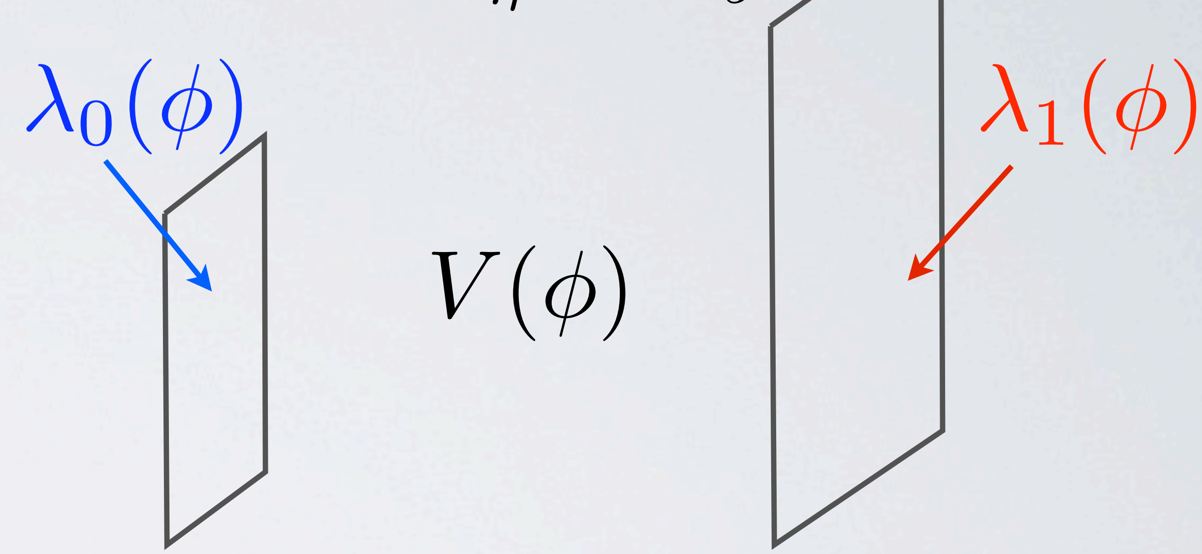

- Solve Einstein equations
  coupled to scalar

- Define a "Superpotential" $V(\phi) = 3W'(\phi)^2 - 12W^2(\phi)$

NO SUSY INVOLVED

- Einstein equations become $\phi'(y) = W'(\phi)$ $\quad A'(y) = W(\phi)$

- Boundary values from extremizing the 4D potentials

$$V_i(\phi) = \lambda_i(\phi) - 6\,W(\phi)$$

DeWolfe et al '99, Brandhuber & Sfetsos '99

SOFT WALLS

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Soft Walls models only possess a single (UV) brane, but nevertheless exhibit a finite length in the 5th dimension. The IR brane is replaced by a curvature singularity at which the metric vanishes.

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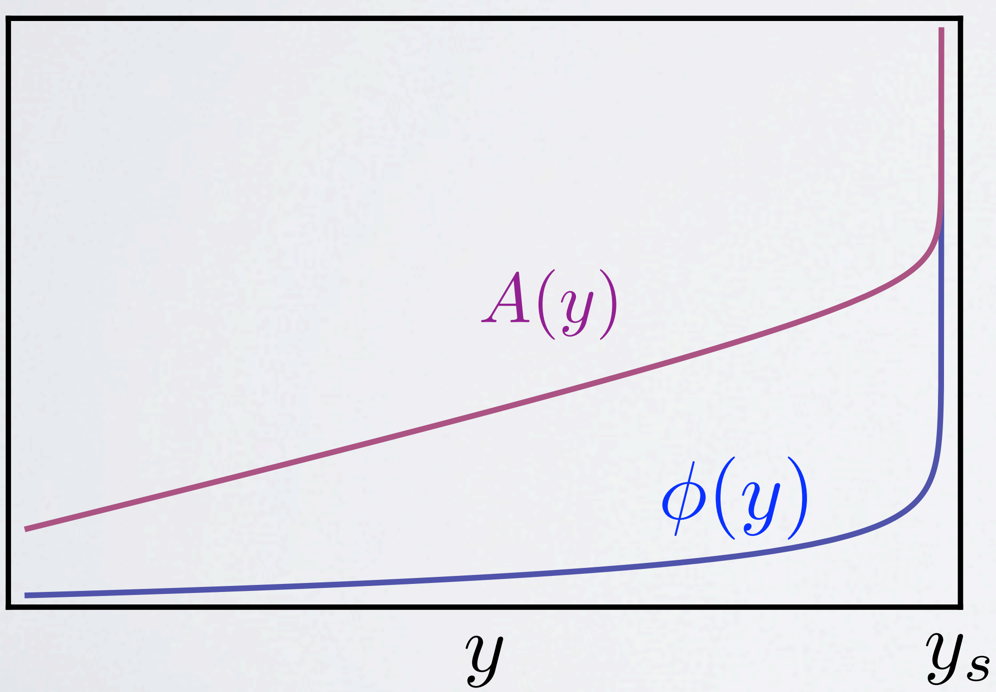

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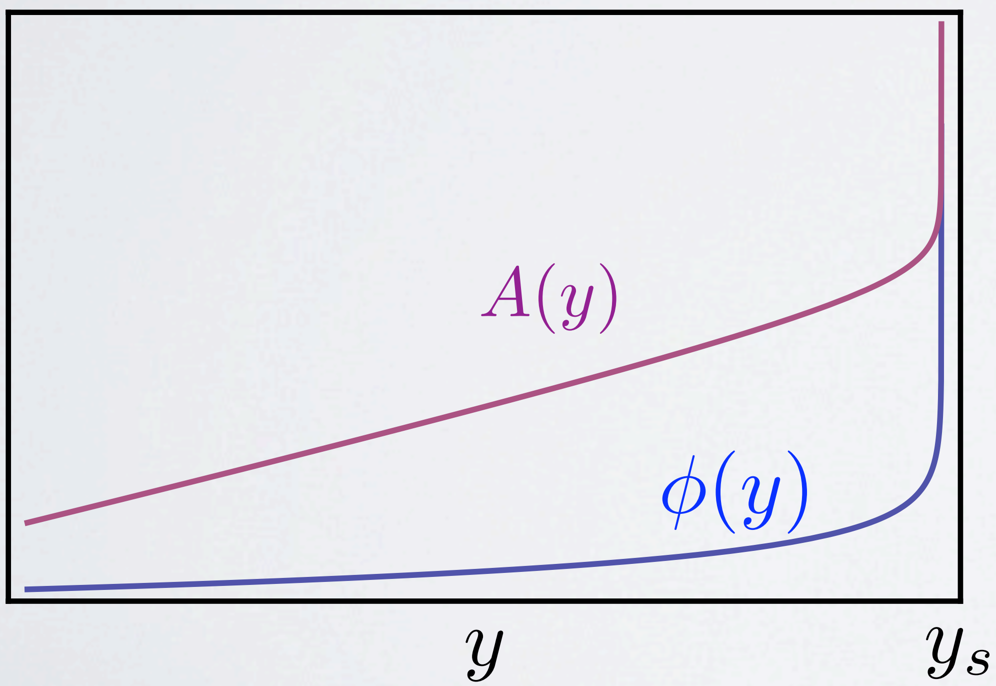

$$ds^2 = e^{-2A(y)} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2$$

Profiles diverge at finite y if $W(\phi) \sim \phi^2$ or faster!

SOFT WALL STABILIZATION

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The Warping $exp(-k\ y_s)$ affects the Mass scale:

- The Unparticle mass gap
- The level spacing in the discrete case

Warped down

SOFT WALL STABILIZATION

The Warping ***exp(-k y_s)*** affects the Mass scale:
- The Unparticle mass gap
- The level spacing in the discrete case

Warped down

- Notice that $e^{k\,y_1} = 10^{16} \Longrightarrow k\,y_1 \approx 37$
- Choose some suitable *W* such that

$$ky_s = \int_{\phi_0}^{\infty} \frac{1}{W'(\phi)} \approx 37$$

- Now shift superpotential $W \to W + k$

$$A(y) \to A(y) + k\,y$$

- Shift does not change position of singularity

Remember

$$\phi'(y) = W'(\phi)$$

$$A'(y) = W(\phi)$$

Cabrer, GG & Quirós '09

SPECTRA WITH SOFT WALLS

SPECTRA WITH SOFT WALLS

- In the conformally flat frame, the KK spectrum of any bulk field follows a Schrödinger Equation

$$-\psi''(z) + \hat{V}(z)\psi(z) = m^2\psi(z)$$

Depends on the background

Proper Length coordinates

$$ds^2 = e^{-2A(y)} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2$$

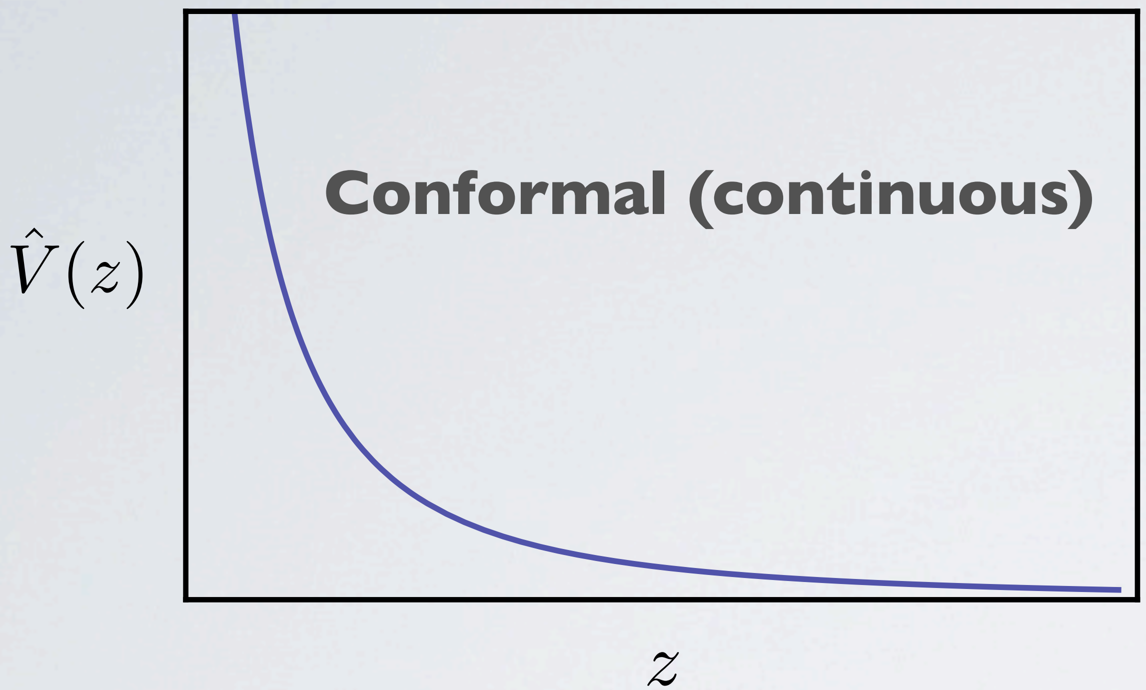
Conformally flat coordinates

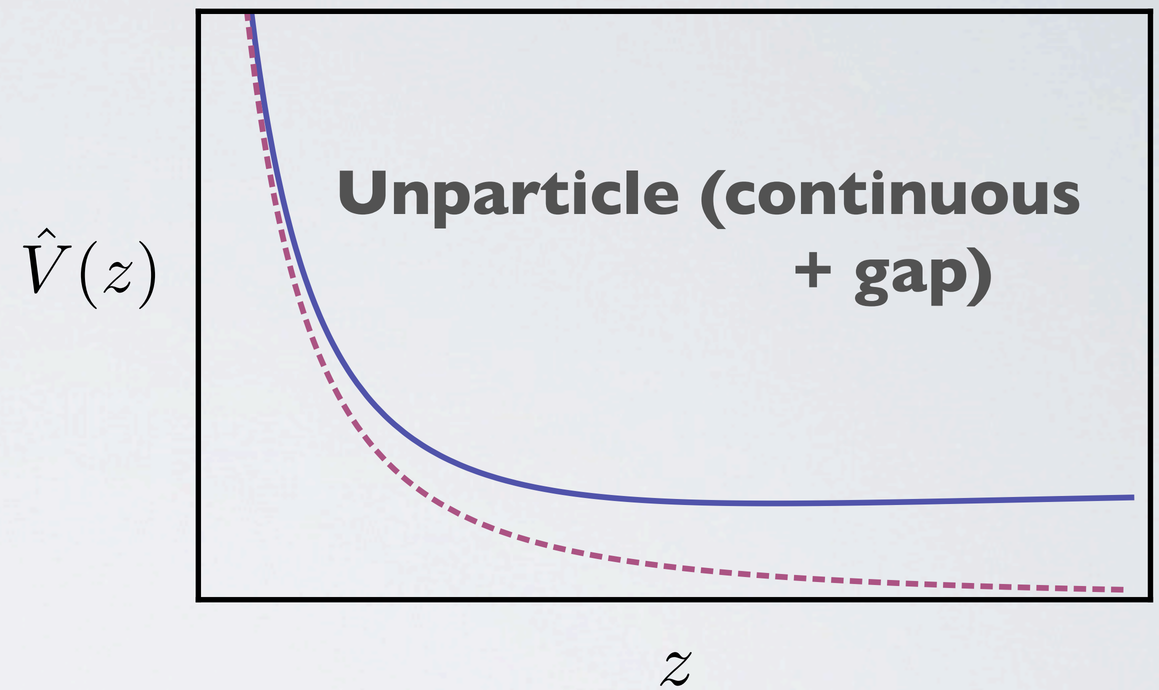
$$ds^2 = e^{-2A(z)} (dx^\mu dx^\nu \eta_{\mu\nu} + dz^2)$$

$y_s < \infty\,,\qquad z_s = z(y_s)$ can be finite or infinite

NON CONFINING POTENTIALS

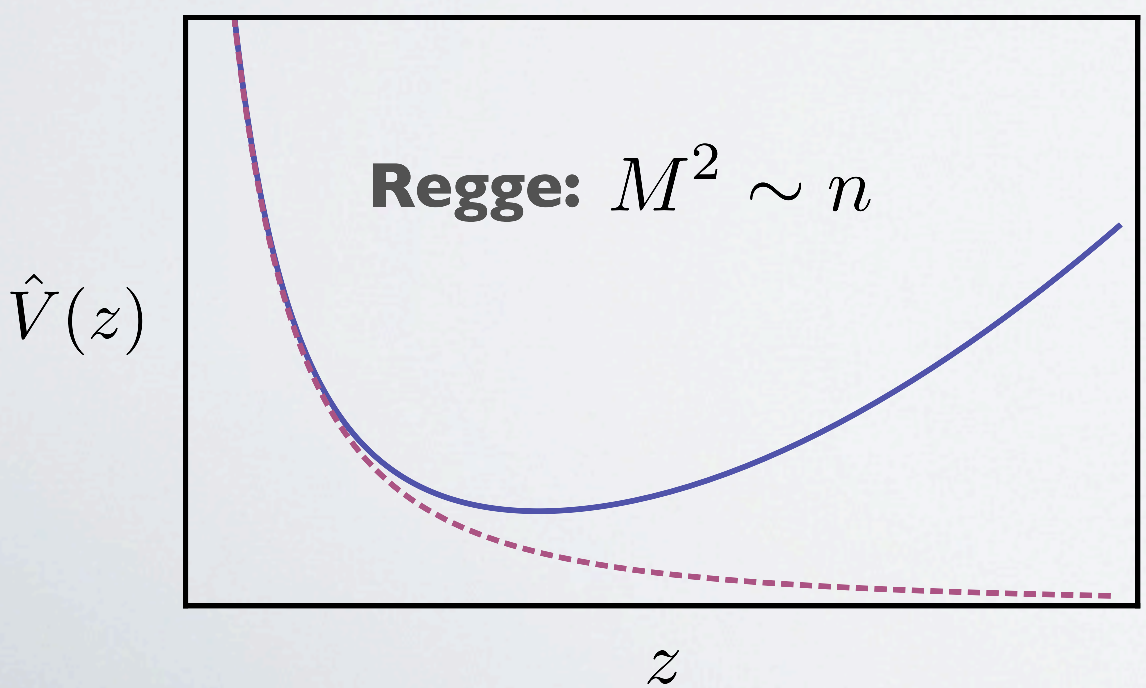
Conformal length infinite

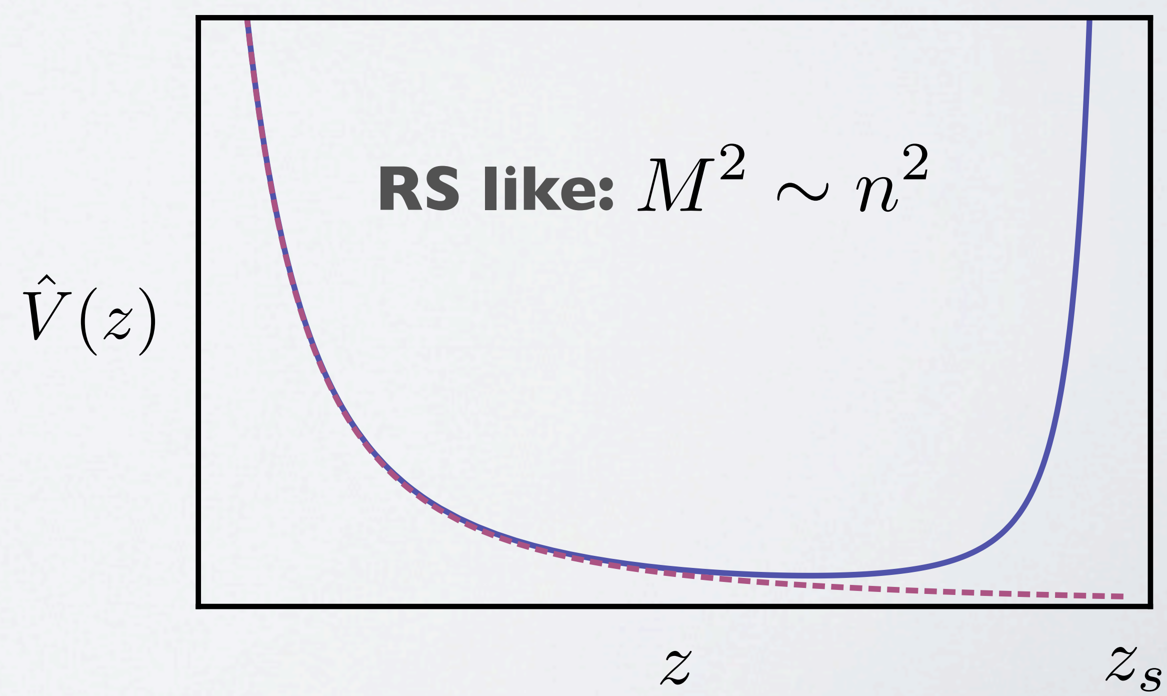




CONFINING POTENTIALS

Conformal length finite or infinite





SOFT WALL SPECTRA

<table>
<tr><td>$W(\phi)$</td><td>$\leq \phi^2$</td><td>$> \phi^2$
$< e^{\phi}$</td><td>e^{ϕ}</td><td>$e^{\phi}\phi^{\beta}$
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<tr><td>y_s</td><td>∞</td><td colspan="5">finite</td></tr>
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<tr><td rowspan="2">mass spectrum</td><td colspan="2" rowspan="2">continuous</td><td rowspan="2">continuous w/ mass gap</td><td colspan="3">discrete</td></tr>
<tr><td>$m_n \sim n^{2\beta}$</td><td colspan="2">$m_n \sim n$</td></tr>
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Gursoy et al '07,
Cabrer, GG & Quirós '09

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Asymptotic behaviour of W

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Singularity in "proper distance"

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Asymptotic form of the spectrum

Gursoy et al '07,
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Asymptotic behaviour of W

Finite Length

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Singularity in "conformal distance"

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Asymptotic behaviour of W

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Singularity in "conformal distance"

Asymptotic form of the spectrum

Finite Length

Mass gap appears

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Asymptotic form of the spectrum

Finite Length

Mass gap appears

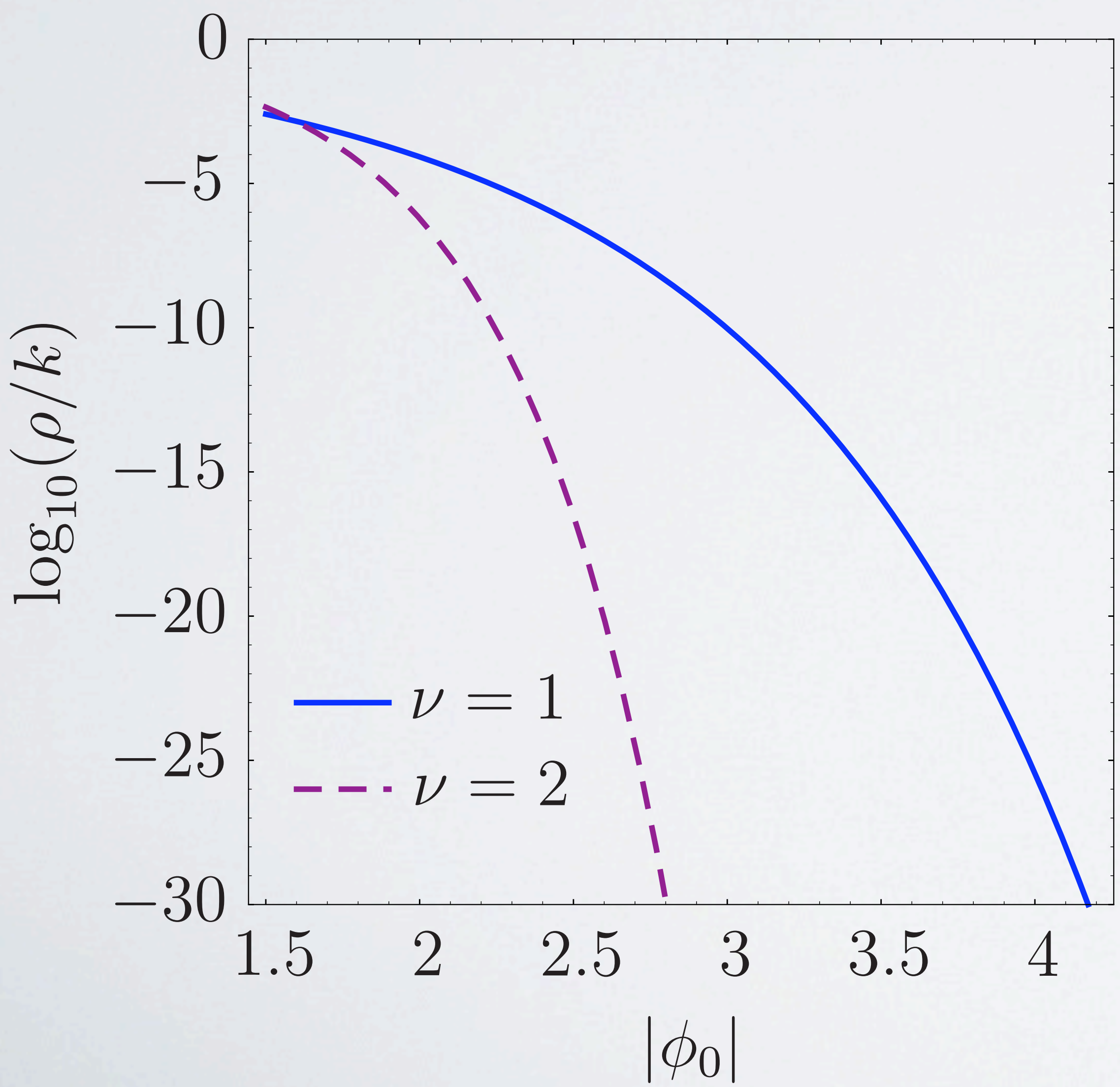
Spectrum discrete

Gursoy et al '07,
Cabrer, GG & Quirós '09

PARTICULAR MODELS

Consider the class of models $W(\phi) = k(1 + e^{\nu\phi})$

$$ky_s = \frac{1}{\nu^2} e^{-\nu\phi_0} \approx 37$$ for O(1) negative values for ϕ_0



Spectrum can be

- **Continuous**
- **Continuous+gap**
- **Discrete**

THE (GRAVITON) SPECTRUM

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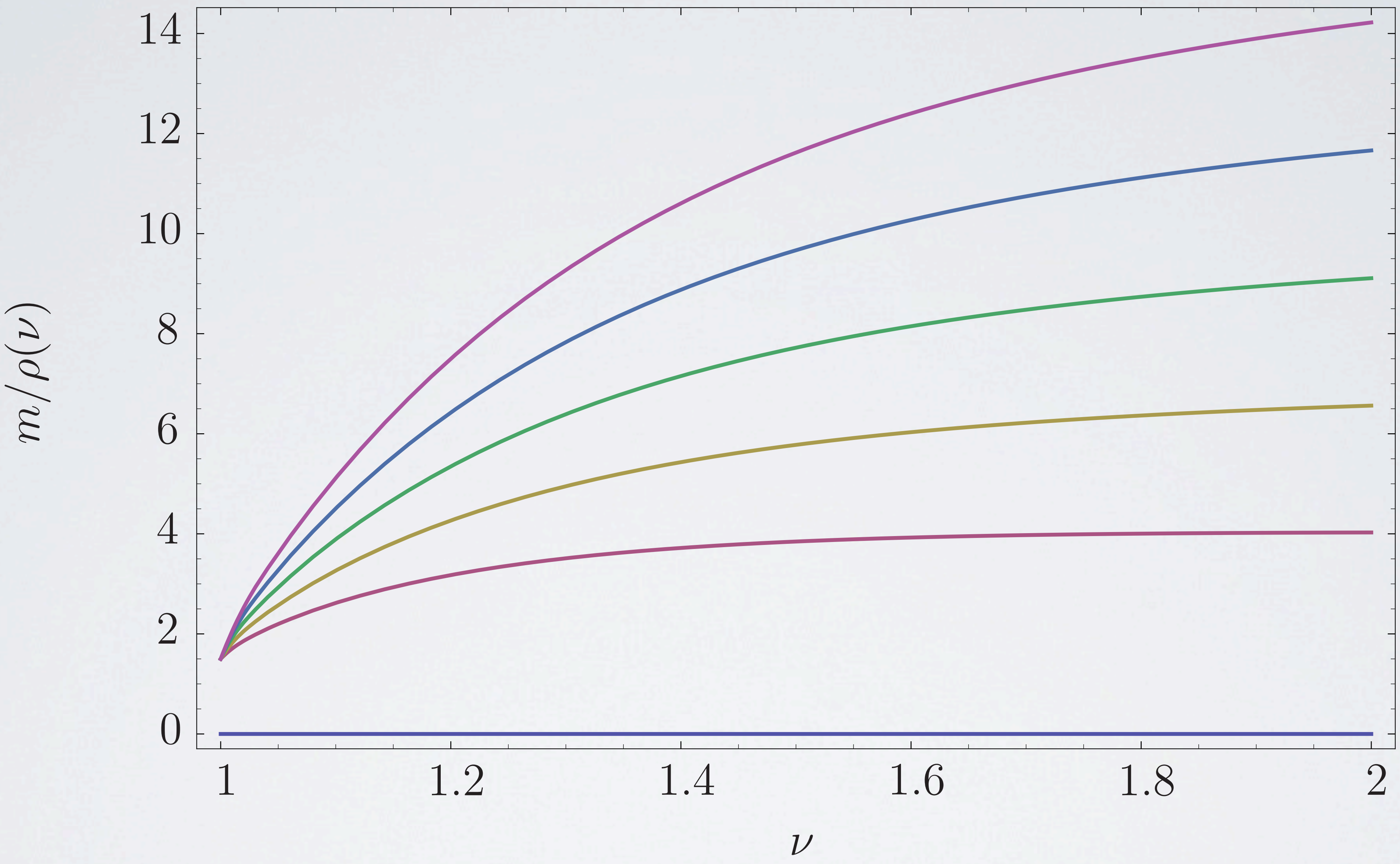

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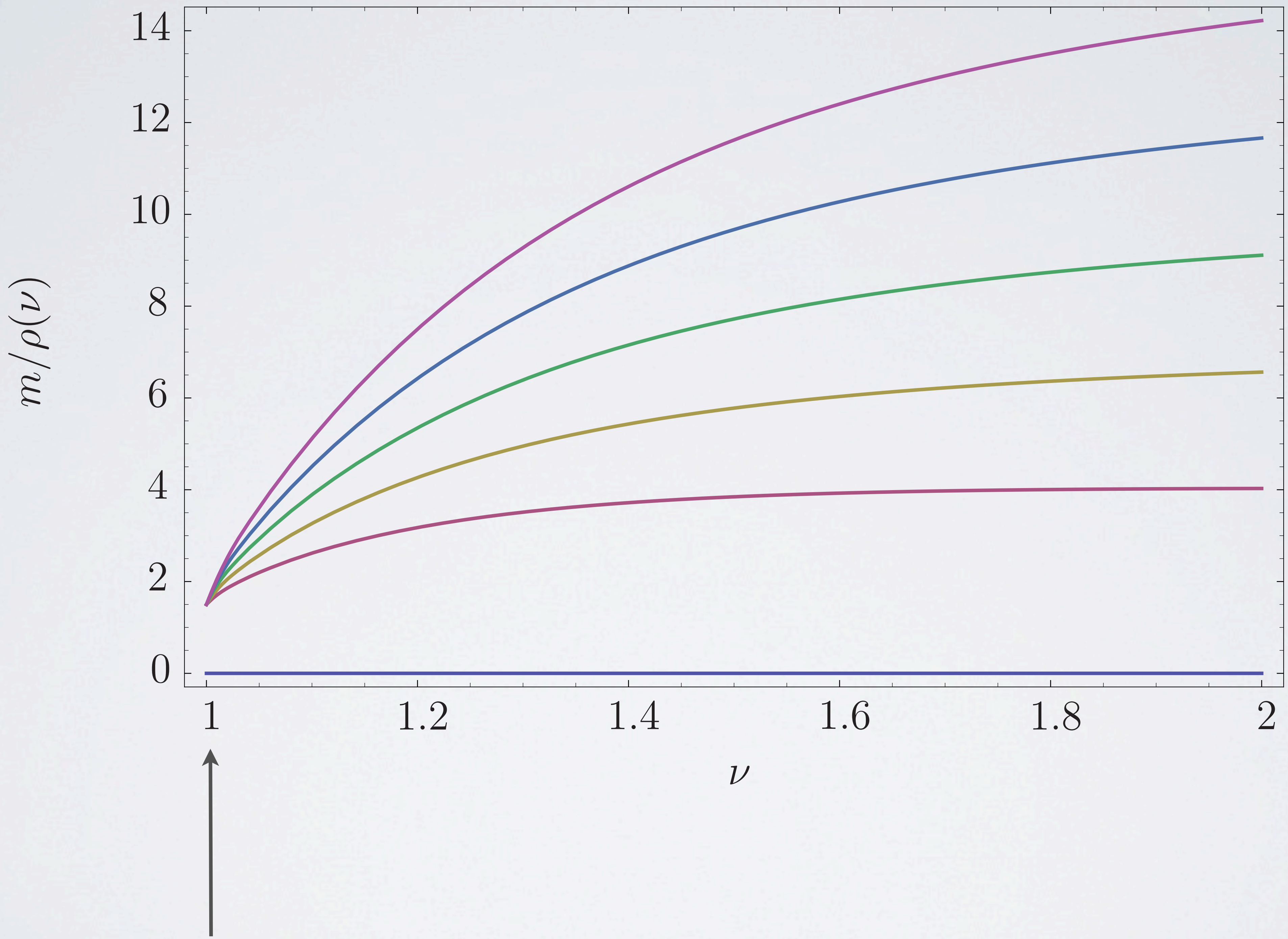
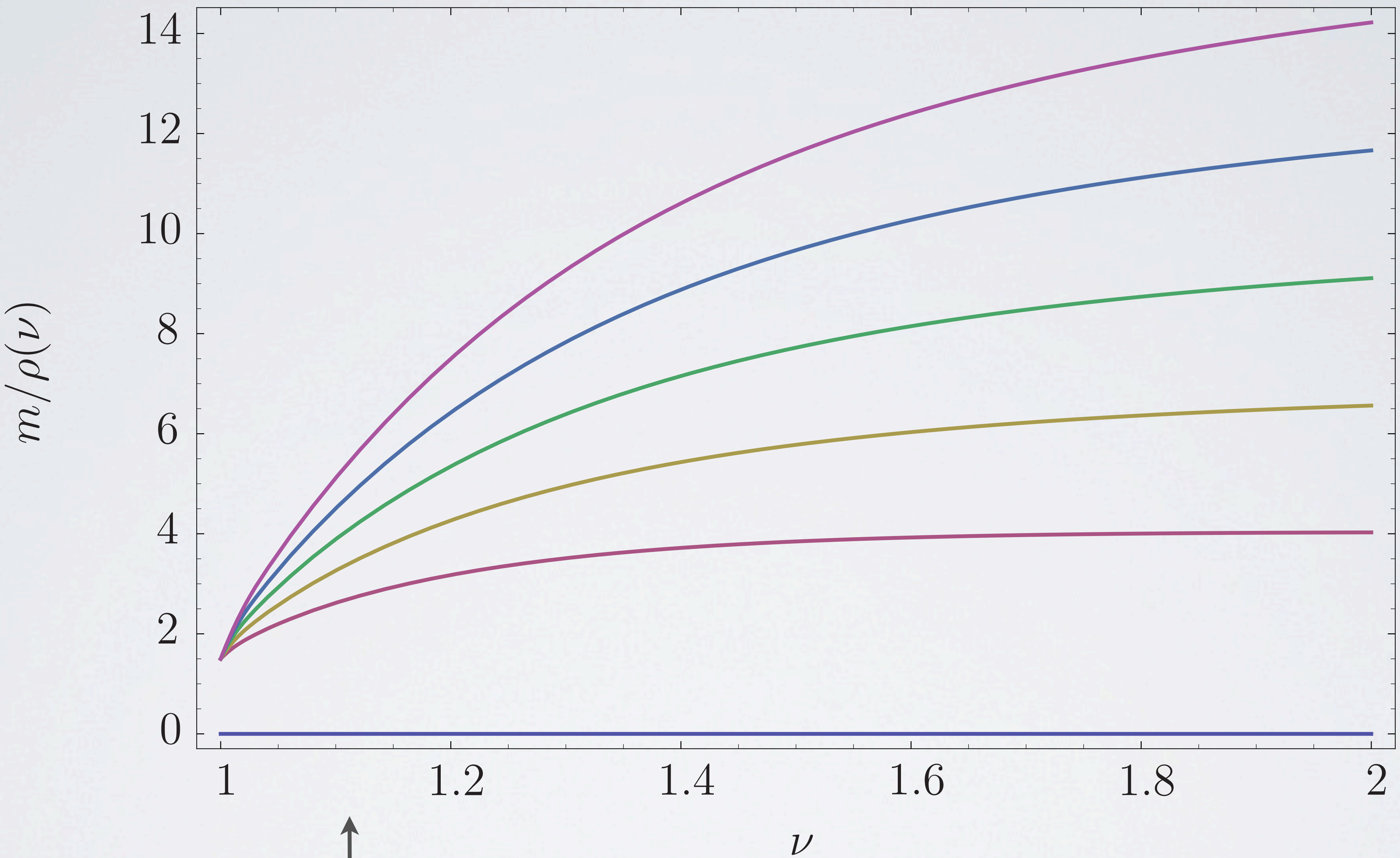


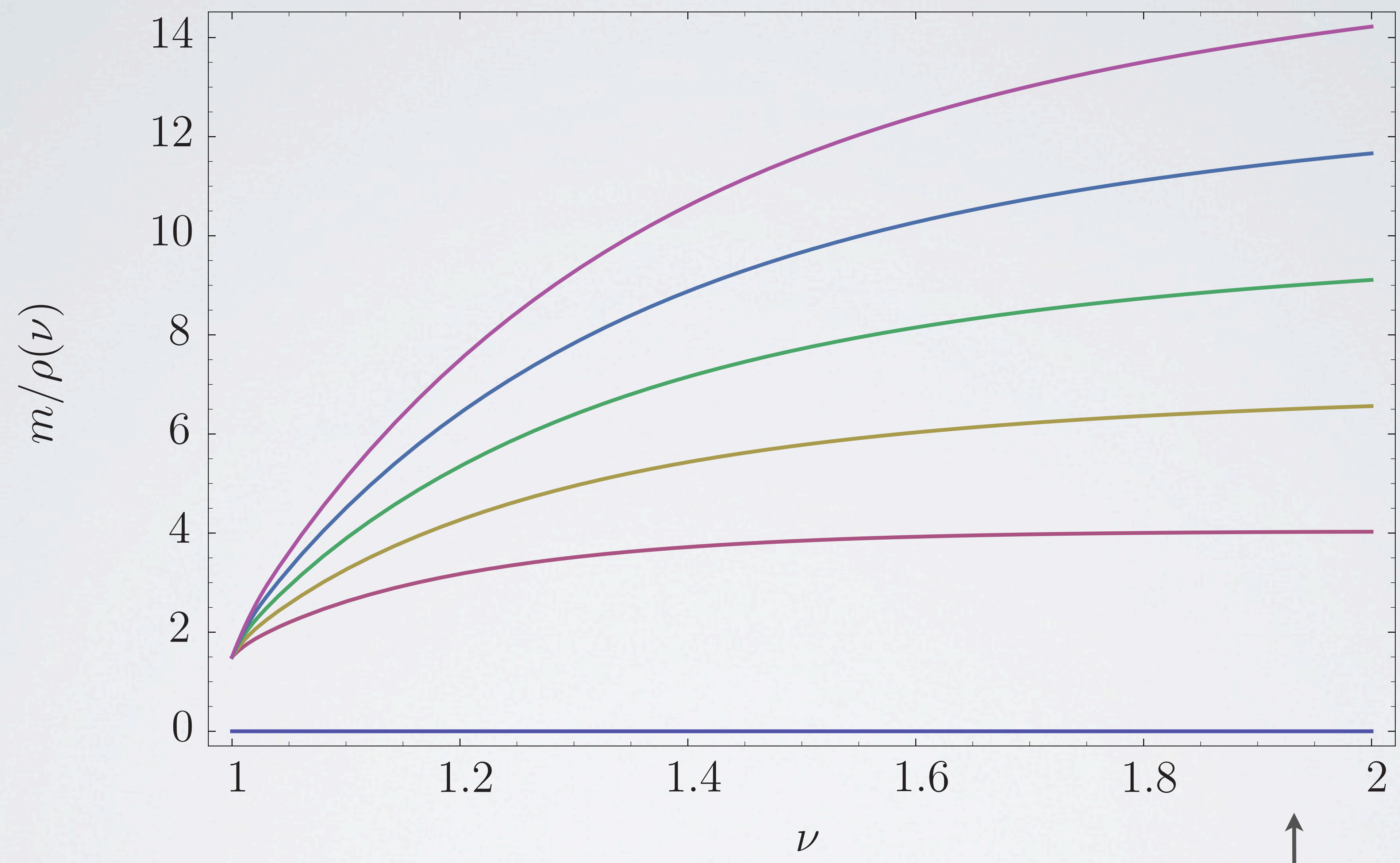
gap + continuous

THE (GRAVITON) SPECTRUM



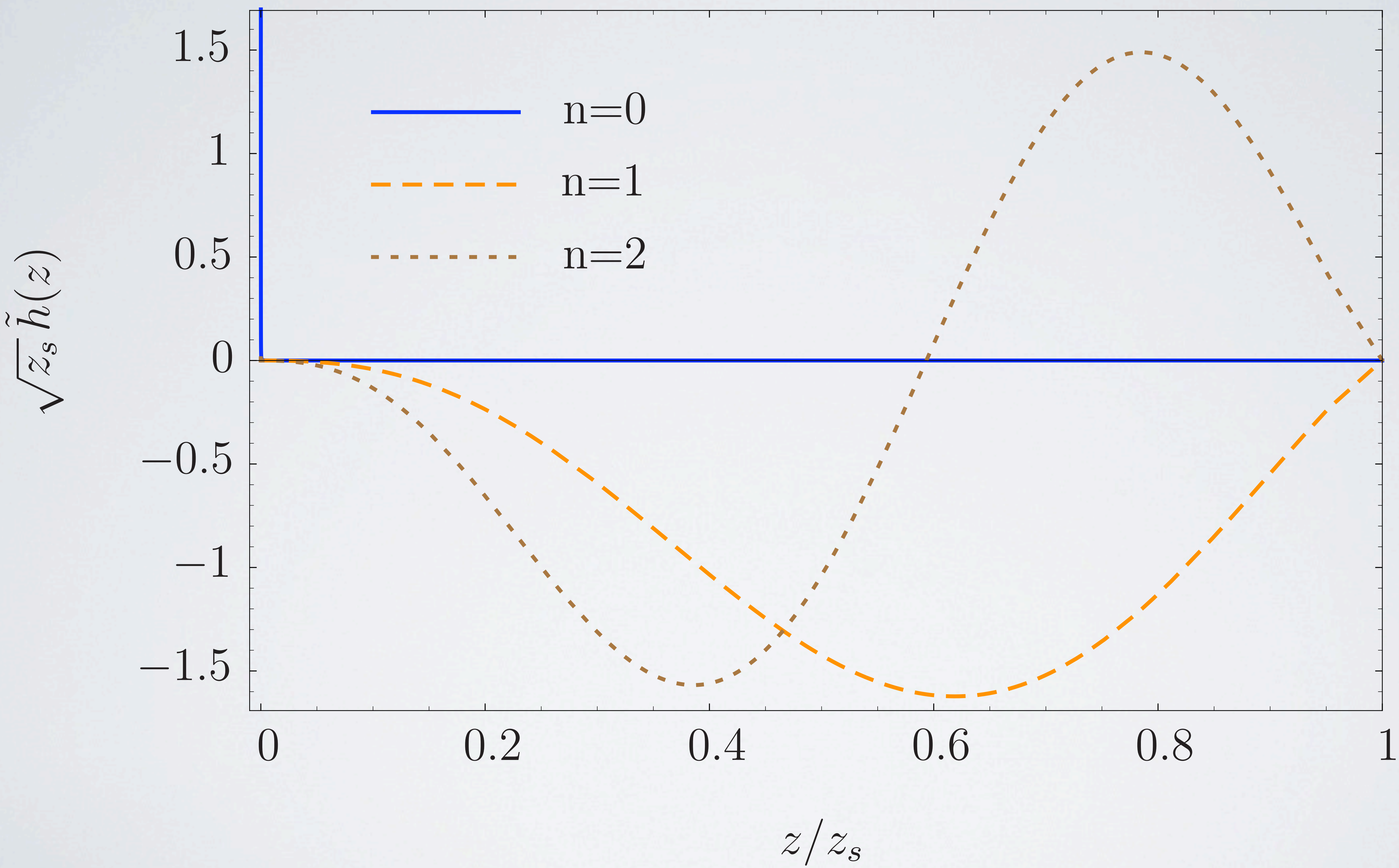
gap+very densely spaced discretuum

THE (GRAVITON) SPECTRUM



Discrete, hard-wall like

WAVE FUNCTIONS



THE RADION SPECTRUM

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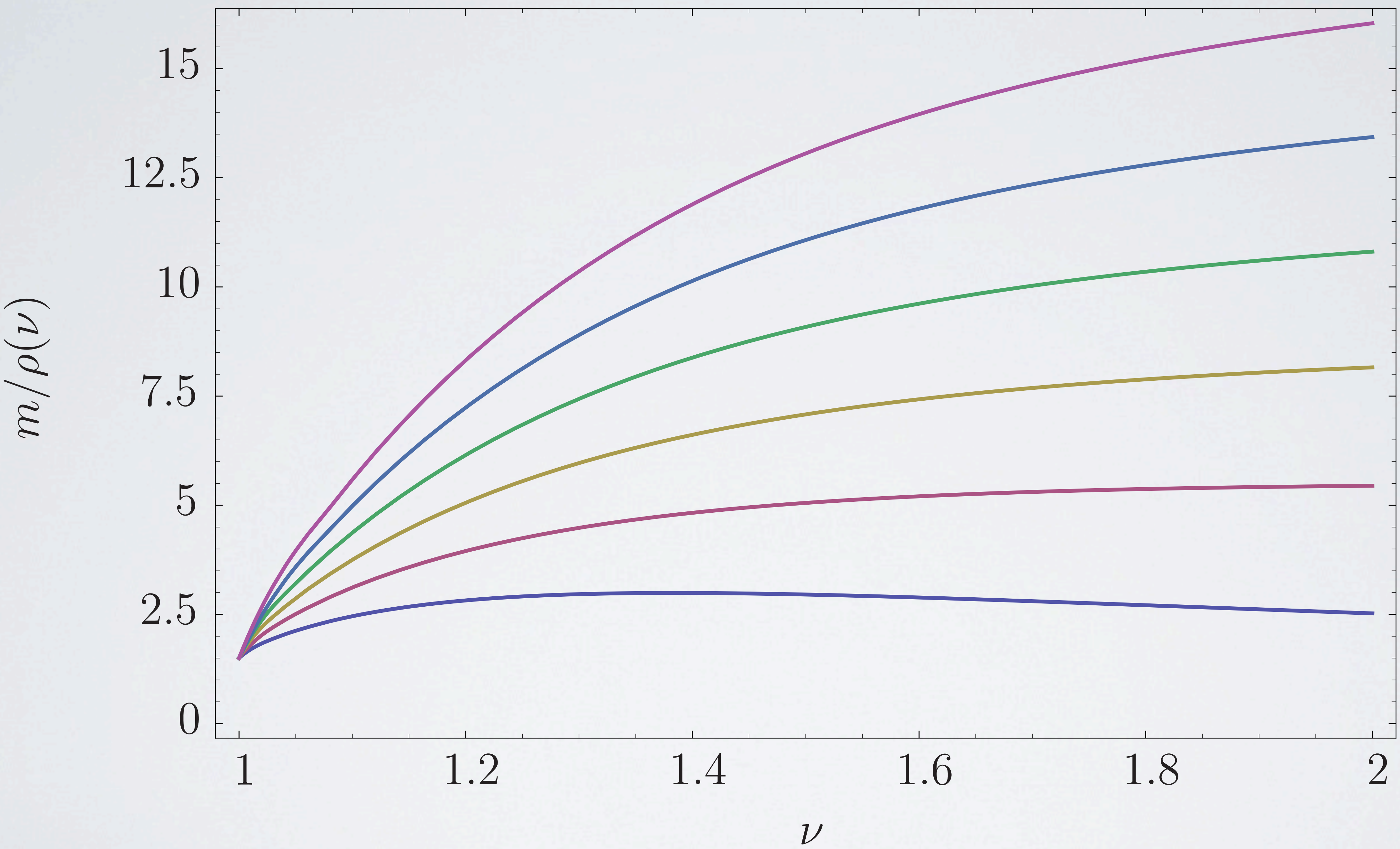

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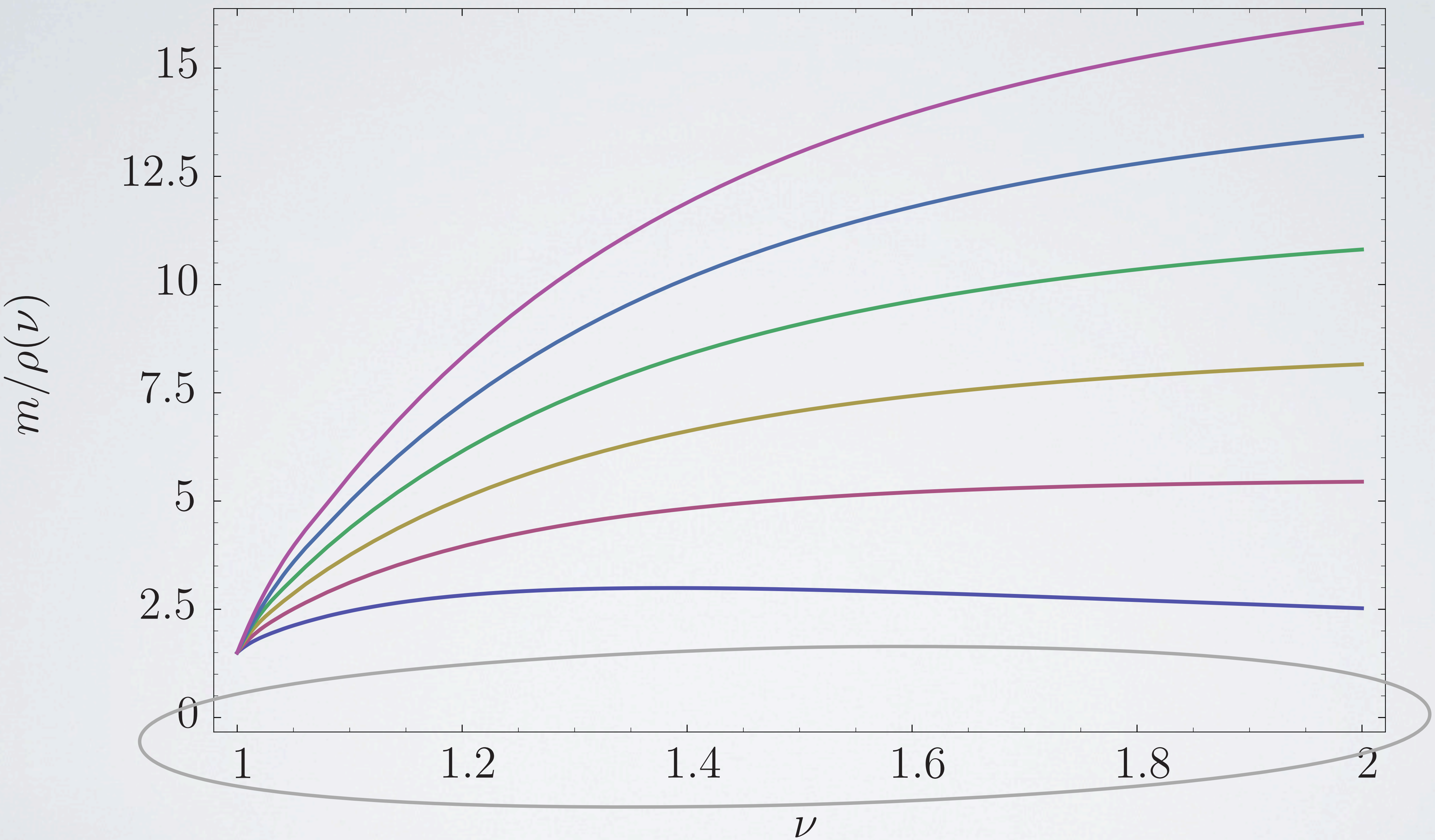

NO ZERO MODE

CONCLUSIONS

- RS models provide way of obtaining electroweak and fermion mass hierarchy
- Stabilization can be achieved by adding extra scalar field
- IR brane can be consistently replaced by Soft Walls
- Spectra of Soft Wall models richer than in usual RS (gapped continuum, gapped, discretuum, Regge-like, etc.)
- Stabilization can be achieved without ANY fine tuning