Beta Beams, EUROnu WP4

SEVENTH FRAMEWORK PROGRAMME

Beta Beam's Collective Effect Study

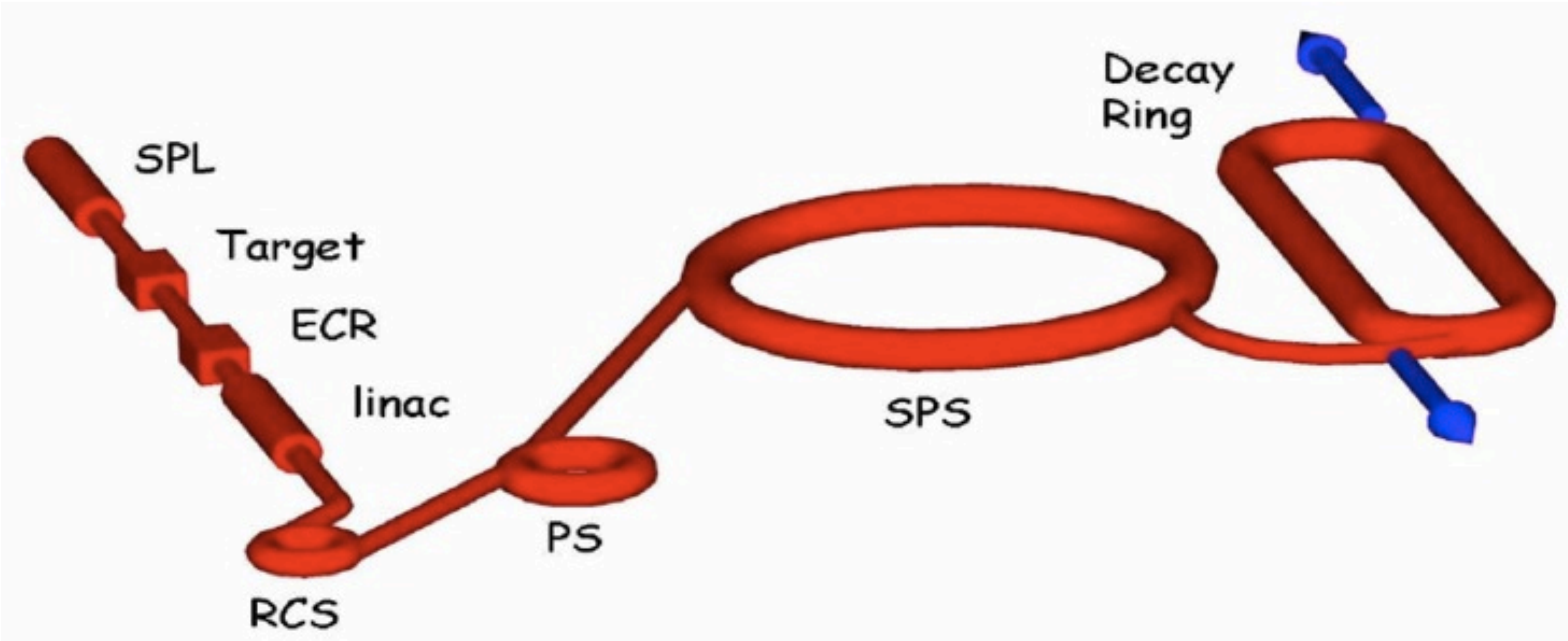
Christian Hansen

EUROν MEETING 2010/06/02

Many thanks to: E. Benedetto, A. Chancé, E. Metral, G. Rumolo & B. Salvant

Introduction

- Beta Beam's physics reach is optimized for high intensity ions beams with short bunch length
- Collective Effects will limit the final performance of accelerators
- Collective Effects has not yet been studied in detail for the CERN Beta Beam complex
- Plan to study all machines for all ions (FP6: ^{6}He & ^{18}Ne, FP7: ^{8}B & ^{8}Li)
  - So far focused on the Decay Ring for ^{6}He and ^{18}Ne
  - Results shown are based on FP6 design (FP6 database) with some edited values



Outline

- Direct Space Charge & Laslett's Tune Shift

- Transverse Broad Band Resonance:
  - Transverse Mode Coupling Instabilities (TMCI) Limit
  - HeadTail Results

- Longitudinal Broad Band Resonance:
  - Longitudinal Parameters
  - Microwave Instabilities Limits
  - HeadTail Results

Laslett's Tune Shifts

- Laslett's Tune Shifts take into account both DSC and Image Fields:
  - A particle in a bunch feels the collective Coulomb forces due to fields generated by the charge of other particles in the bunch → Direct Space Charge (DSC) → tune shift
  - Also Image Fields due to the surrounding vacuum pipe cause tune shift
- Grouped into Incoherent and Coherent (DSC only Incoherent) where the coherent tune shifts are due to either Penetrating or Non-Penetrating Fields

- ***Incoherent Tune Shift***

$$\Delta Q_{x,y}^{incoh} = -\frac{N r_0 R}{\pi\gamma\beta^2 Q_{x,y}}\left[\left(\frac{1-\beta^2}{B}+\beta^2\right)\frac{\varepsilon_{x,y}^{incoh}}{h^2}+\frac{1-\beta^2}{2B}\frac{\varepsilon_{x,y}^{dsc}}{a_y^2}\right]$$

Here added 1/2B myself, see backup slides

- ***Coherent Tune Shift using Penetrating Magnetic Fields***

$$\Delta Q_{x,y}^{coh} = -\frac{N r_0 R}{\pi\gamma\beta^2 Q_{x,y}}\left(\frac{1-\beta^2}{B}+\beta^2\right)\frac{\varepsilon_{x,y}^{coh}}{h^2}$$

- ***Coherent Tune Shift using Non-Penetrating Magnetic Fields***

$$\Delta Q_{x,y}^{coh} = -\frac{N r_0 R}{\pi\gamma\beta^2 Q_{x,y}}\left[\frac{1-\beta^2}{B}\frac{\varepsilon_{x,y}^{coh}}{h^2}+\beta^2\frac{\varepsilon_{x,y}^{incoh}}{h^2}\right]$$

Here, neglected χ_e and $\mathcal{F}$ (see Ng.)

K. Y. Ng
Physics of Intensity Dependent Instabilities
USPAS 2002

Laslett's Tune Shifts

- The absolute value of the tune shifts should be < 0.2

SC	DR ^{18}Ne	DR ^{6}He
ΔQ_{dsc_x}	-0.0409	-0.0083
ΔQ_{dsc_y}	-0.0946	-0.0192
ΔQ_x^{incoh}	-0.0409	-0.0083
ΔQ_y^{incoh}	-0.0946	-0.0192
$\Delta Q_x^{coh\ p}$	-1.7470e-04	-3.5564e-05
$\Delta Q_y^{coh\ p}$	-3.1937e-04	-6.5016e-05
$\Delta Q_x^{coh\ np}$	-6.2768e-05	-1.2765e-05
$\Delta Q_y^{coh\ np}$	-1.1475e-04	-2.3337e-05

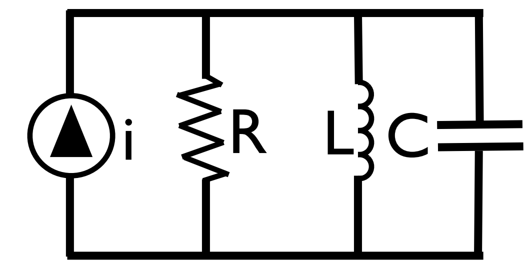
- We see that the effect of the image forces are negligible relatively to DSC
- DSC is more crucial for low energy so SPS and PS might have a big DSC problem in Beta Beams … to be studied in the future …

Impedances

Y.H. Chin, Impedance and Wake Fields

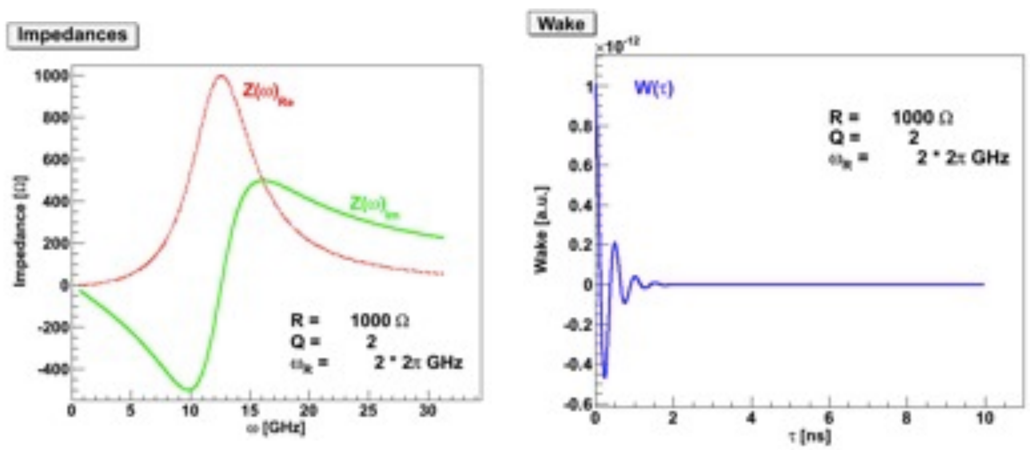
Resonance Impedance

- Wake fields can be trapped in discontinuities (e.g. cavities) in the vacuum chamber → resonance impedances → can be modeled with an RLC circuit:

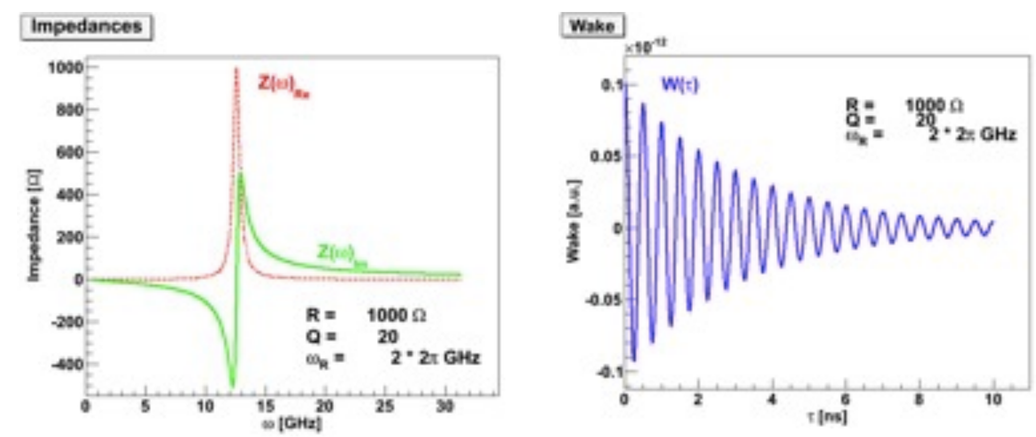


$$Z_{||}(\omega) = \frac{R_{||}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \qquad Z_{\perp}(\omega) = \frac{R_{\perp}\frac{\omega_r}{\omega}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$

Broad Band (low Q)



Narrow Band (high Q)

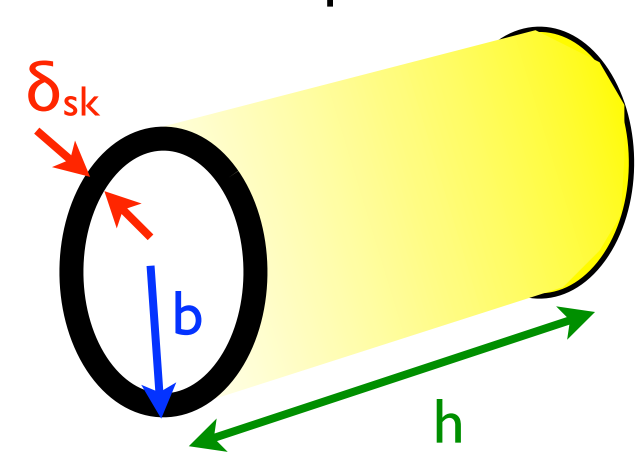


Resistive Wall Impedance

- Due to resistive beam pipe the image current is slowed down → wake field → impedance

$$Z_{||,rw}(\omega) = \frac{\omega}{2}(1-i)\frac{Z_0\delta_{sk}(\omega)h}{2\pi bc} \approx (1-i)\frac{\omega R}{2bc}\sqrt{\frac{2\rho}{\varepsilon_0|\omega|}}$$

$$Z_{\perp,rw}(\omega) = (sgn(\omega)-i)\frac{Z_0\delta_{sk}(\omega)h}{2\pi b^3} \approx (sgn(\omega)-i)\frac{R}{b^3}\sqrt{\frac{2\rho}{\varepsilon_0|\omega|}}$$



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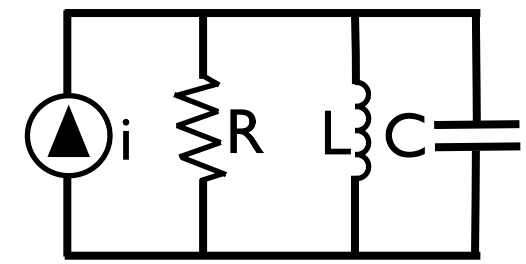
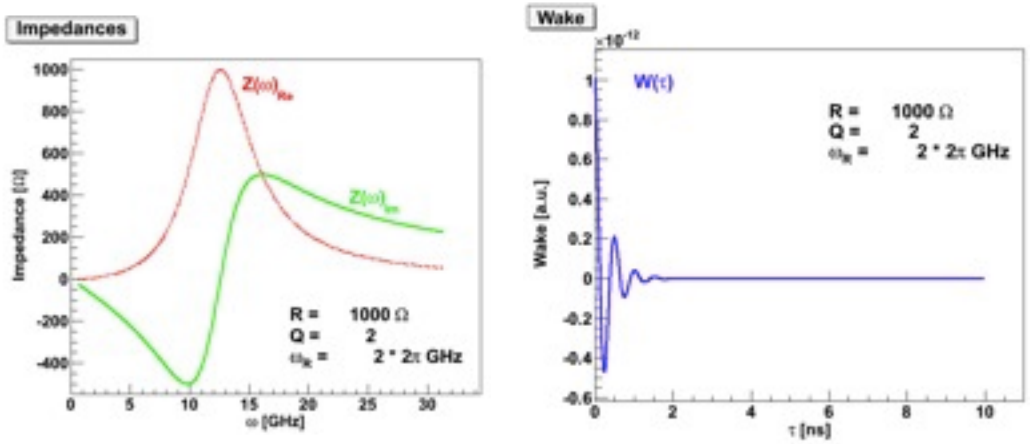
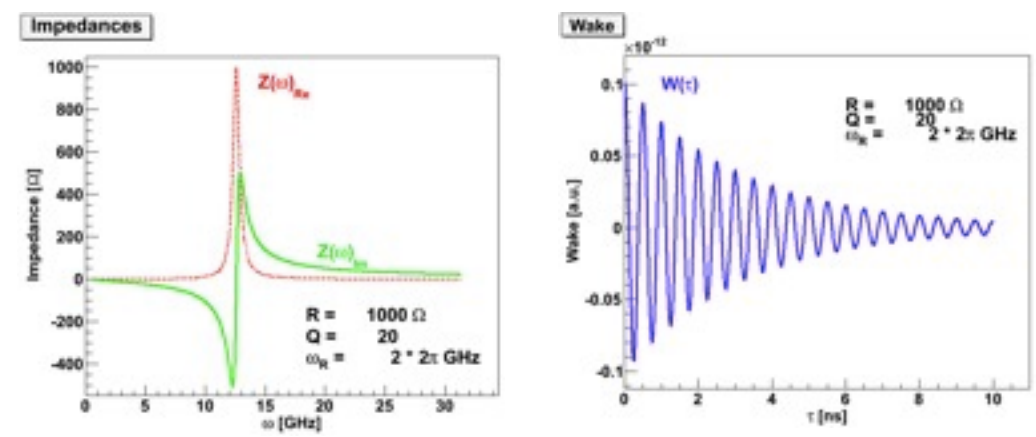


$$Z_{||}(\omega) = \frac{R_{||}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \qquad Z_{\perp}(\omega) = \frac{R_{\perp}\frac{\omega_r}{\omega}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$

Broad Band (low Q)



Narrow Band (high Q)



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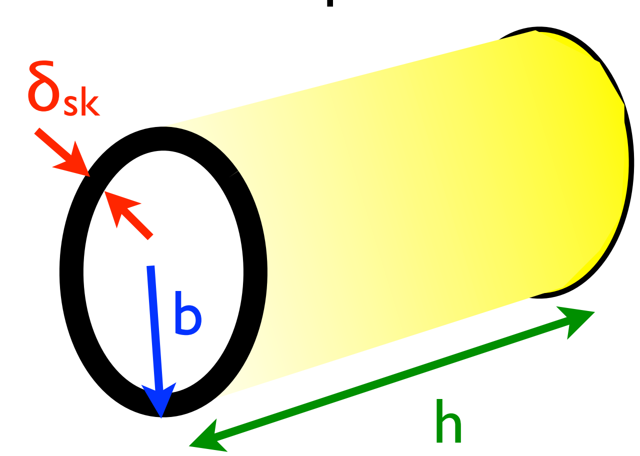


Inputs for Broad Band Resonance Impedance

- Have assumed same values for the DR as for SPS to know how much better the DR need to be

Parameters	DR ^{18}Ne	DR ^{6}He
$Q_{\parallel}$	1.00	1.00
$\omega_{r,\parallel}$ [GHz]	6.28	6.28
$\lvert Z_{\parallel}/n \rvert$ [Ω]	10.00	10.00
$R_{s,\parallel}$ [MΩ]	0.221	0.221
$Q_{\perp}$	1.00	1.00
$\omega_{r,\perp}$ [GHz]	6.28	6.28
$R_{s,\perp}$ [MΩ/m]	20.00	20.00

Inputs for Chromaticity

- Used

ξ_x = **0.05 and** ξ_y = **0.1** for DR where $\eta > 0$

$\perp$

Transverse

TMCI Limit

- With high bunch intensity the wake fields couple the modes together so the different head-tail modes can **not** be treated separately as is done in Sacherer's Formula

- Instead a Transverse Mode Coupling Instability (TMCI) appears above a threshold for number of particles per bunch:

E. Métral, CERN, Overview of Single-Beam Coherent Instabilities in Circular Accelerators

$$N^{th}_{b_{x,y}} = \frac{32}{3\sqrt{2}\pi} \frac{Q_{x,y}|\eta|\varepsilon_l^{2\sigma}\,\omega_r}{Z^2\beta^2 c} \left(\Re\left[Z^{BB}_{\perp_{x,y}}\right]_{max}\right)^{-1} \left(1 + \frac{\omega_{\xi_{x,y}}}{\omega_r}\right)$$

- Where $\varepsilon_l^{2\sigma} = \frac{\pi}{2}\beta^2 E_{tot}\tau_b\delta_{max}$ in eVs (for dimension analysis: Js/C)

	DR ^{18}Ne	DR ^{6}He
ε_l (2σ) [eVs]	43.200	14.464
$\rho[Z^{BB}_{\perp_y}]_{max}$ $\left[\frac{M\Omega}{m}\right]$	21.327	21.327
$N_B/N^{th}_{b_x}$	22.859	4.635
$N_B/N^{th}_{b_y}$	41.646	8.445

- Worst for ^{18}Ne in DR: N_B needs to be reduced by a factor 42 OR $R_\perp^{DR} = R_\perp^{SPS}/42$

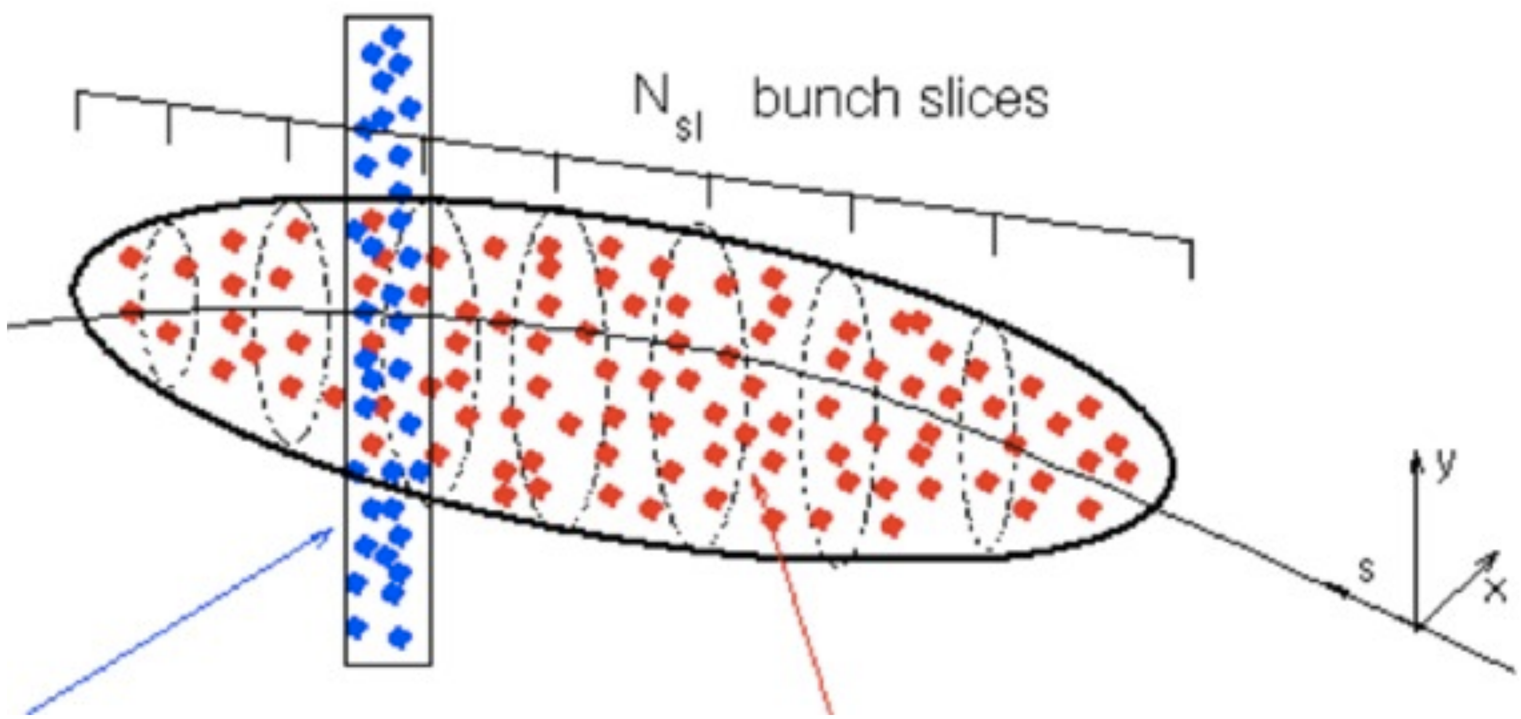
- Tried to improve N^{th}/N_B by tuning chromaticity, but didn't help (Here $|\xi_x| = 0.05$ & $|\xi_y| = 0.1$)

HEADTAIL

By Giovanni Rumolo

E. Benedetto, CERN,
Beam stability in the SPL-Proton Driver accumulator for a Neutrino Factory at CERN

- HEADTAIL is a multiparticle tracking code
- The bunch is sliced longitudinally
- The impedance is assumed to be localized at a few positions around the ring
- At each impedance location, each slice leaves a wake-field behind and gets a kick by the field generated by the preceding slices
- The bunch is then transferred to the next impedance location via a transport matrix



- For the Beta Beam Studies the possibility of bunches with ^{18}Ne and ^{6}He was added to the code

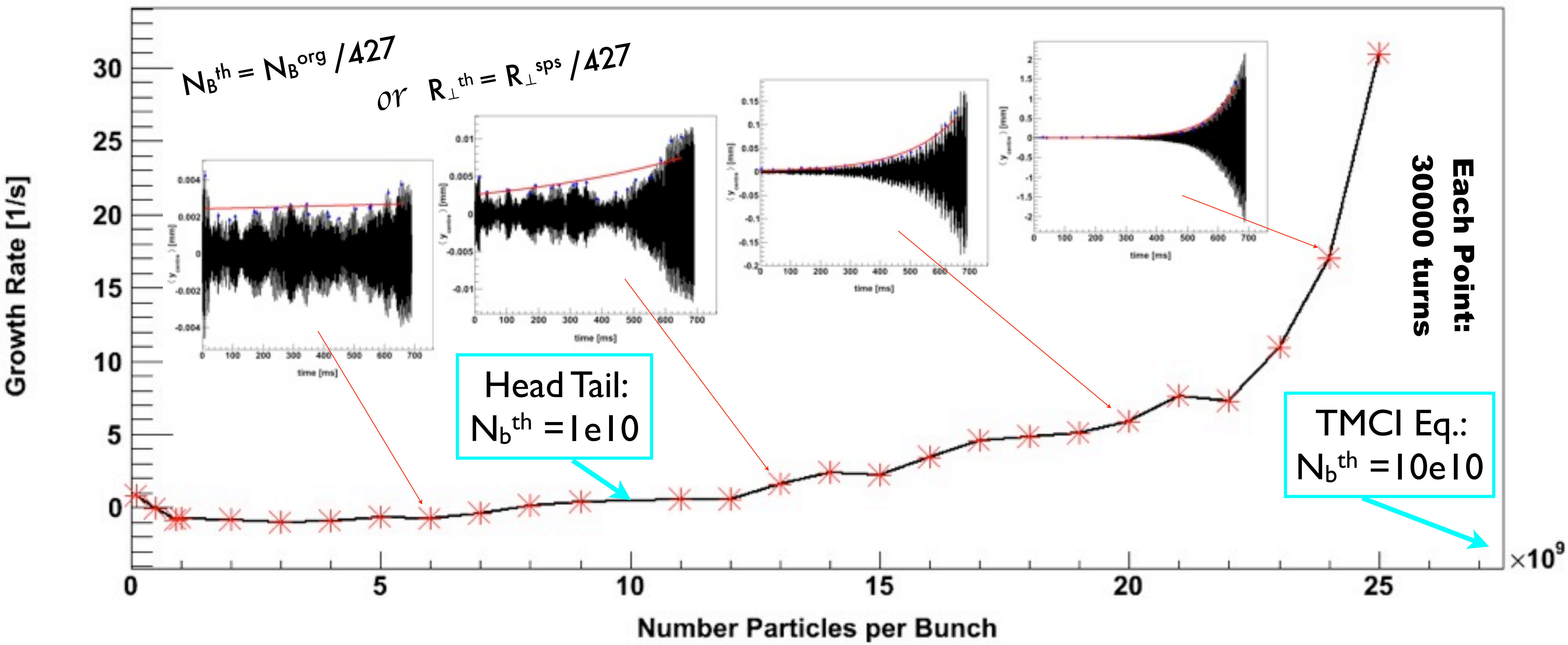
DR ^{18}Ne - Transversal Broad Band

- A Least Square Fit to the exponential gives $\langle y_c \rangle_0$ and the Growth Rate, $1/\tau$

$$\langle y_c \rangle = \langle y_c \rangle_0 \, e^{t/\tau}$$

DR ^{18}Ne
Transv. Broad Band Res.
N_B^{org} = 4.27e12
$R_\perp^{org}$ = 20 MΩ/m
ξ_x^{org} = 0.05, ξ_y^{org} = 0.1
(η > 0 for DR)

- Growth Rate as a function of ion bunch intensity in the Decay Ring:



- HeadTail indicates that for the current anticipated bunch intensity a 427 times smaller shunt impedance than SPS is needed for the DR

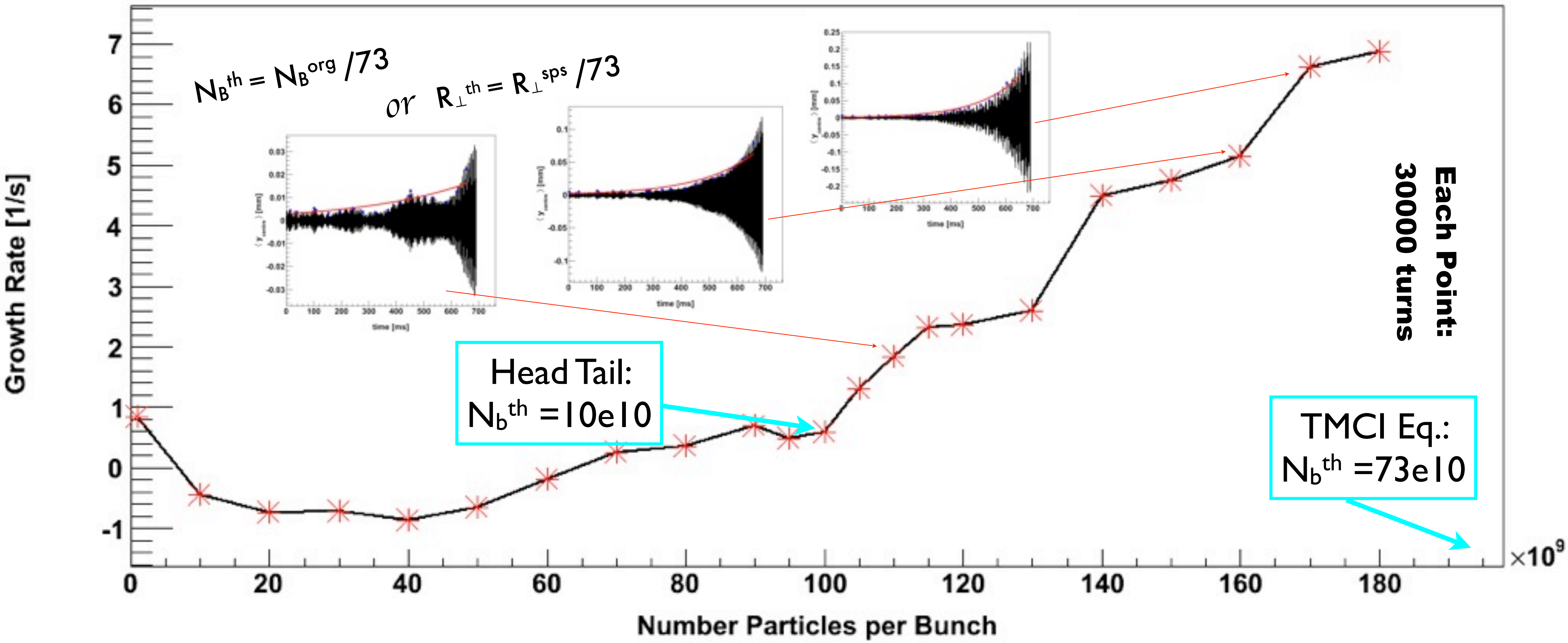
DR ^{6}He - Transversal Broad Band

- A Least Square Fit to the exponential gives $\langle y_c \rangle_0$ and the Growth Rate, 1/ τ

$$\langle y_c \rangle = \langle y_c \rangle_0 \, e^{t/\tau}$$

DR ^{6}He
Transv. Broad Band Res.
N_B^{org} = 7.24e12
$R_\perp^{org}$ = 20 MΩ/m
ξ_x^{org} = 0.05, ξ_y^{org} = 0.1
(η > 0 for DR)

- Growth Rate as a function of ion bunch intensity in the Decay Ring:



- HeadTail indicates that for the current anticipated bunch intensity a 73 times smaller shunt impedance than SPS is needed for the DR

II

Longitudinal

Longitudinal Parameters

- The longitudinal parameters are not clear and/or incorrect in our "FP6 database"
- Sorting things out together with Antoine Chancé
- We have succeeded quit well for the DR
- Still working on SPS; Antoine has recently done an RF simulation (with the ESME 2D program) to achieve the longitudinal parameters from SPS

Longitudinal Parameters - DR

- In the DR the reference values are the maximum momentum spread, δ_m, (due to a collimator) and the voltage, V, so we want to solve for bunch length, L_b, and emittance, ε_l

S.Y.Lee, Accelerator Physics

- In the phase-space with coordinates (Φ, δ) the synchrotron Hamiltonian is

$$H = \frac{1}{2} h \omega_{rev} \eta \delta^2 + \frac{\omega_{rev} ZeV}{2\pi\beta^2 E_{tot}} \left[\cos\phi - \cos\phi_s + (\phi - \phi_s)\sin\phi_s\right]$$

A. Chancé

- The DR is a Storage Ring so $\Phi_s = 0$ ☺

- If θ_b is the maximum phase advance for a particle then that particle will pass two points: $(0, \delta_m)$ and (θ_b, δ), and since Hamiltonian is a constant of motion $H(\Phi=0, \delta=\delta_m) = H(\Phi=\theta_b, \delta=0)$

$$-\frac{1}{2} h \omega_{rev} \eta \delta_m^2 = \frac{\omega_{rev} ZeV}{2\pi\beta^2 E_{tot}} \left[\cos\theta_b - 1\right] \quad \Rightarrow \quad \theta_b = \arccos\left[1 - \frac{\pi h \eta E_{tot} \beta^2}{ZeV} \delta_m^2\right]$$

- Since $L_b = (2\theta_b / 2\pi)(2\pi\rho / h) = 2\rho\theta_b/h$

$$L_b = \frac{2\rho}{h} \arccos\left[1 - \frac{\pi h \eta E_{tot} \beta^2}{ZeV} \delta_m^2\right]$$

Longitudinal Parameters - DR

- The phase space trajectory of the separatrix, that separates the phase space into inside and outside the bunch, we get by using the point ($\Phi=0$, $\delta=\delta_m$) and the fact that the hamiltonian is a constant of motion, so $H(\Phi, \delta) = H(\Phi=\theta_b, \delta=0)$

$$\frac{1}{2}h\omega_{rev}\eta\delta^2 + \frac{\omega_{rev}ZeV}{2\pi\beta^2 E_{tot}}[\cos\phi - 1] = \frac{1}{2}h\omega_{rev}\eta\delta_m^2 \quad \Rightarrow \quad \delta(\phi) = \sqrt{\delta_m^2 - \frac{\omega_{rev}ZeV}{2\pi\beta^2 E_{tot}}[\cos\phi - 1]}$$

A. Chancé

- The phase-space area of this bunch we get by

$$A = \int_0^{\theta_b} \delta(\phi)d\phi = \ldots = \delta_m G\left(\frac{\theta_b}{2}\right) \quad \text{where} \quad G(\phi) = \frac{8}{\sin\phi}\left[E(\sin\phi) - \cos^2\phi K(\sin\phi)\right]$$

- To get the area in (Δt, ΔE) phase space, ε_l, from the area in (Φ, δ) phase space, A, we convert: $\varepsilon_l = \rho/(\beta hc) \cdot \beta^2 E_{tot} A = \beta\rho E_{tot}/(hc)\, A$

$$\varepsilon_l = \frac{\beta\rho E_{tot}}{hc}\delta_m G\left(\frac{\theta_b}{2}\right)$$

Longitudinal Parameters - DR

- For small amplitude oscillations the phase space ellipse (in the phase space (Φ, δ)) of a particle is defined by it's maximum values (θ_b, δ_m) that follows the relation

$$\frac{\delta_m}{\theta_b} = \frac{Q_s}{h|\eta|}$$

S.Y.Lee,
Accelerator Physics

- Using $\theta_b = hL_b/(2\rho)$ we get the test relation that *should* be fulfilled for a matched bunch

$$\frac{\rho|\eta|\delta_m}{Q_s L_b/2} = 1$$

Longitudinal Parameters - DR

	DR ^{18}Ne	DR ^{6}He
δ_{max}	2.500e-03	2.500e-03
eV [MeV]	1.196e+01	2.000e+01
L_b^{*} [m] $= \frac{2\rho}{h}\arccos\left(1 - \frac{\pi h \eta E_{tot}(\beta\delta_{max})^2}{ZeV}\right)$	1.970	1.970
ε_l^{*} [eVs] $= \frac{\beta\rho E_{tot}\delta_{max}}{hc} G\{\theta_b/2\}$	42.947	14.358
$Q_s = \sqrt{\frac{hZeV\lvert\eta\cos\phi_s\rvert}{2\pi\beta^2 E_{tot}}}$	3.653e-03	3.653e-03
$\theta_b = \frac{hL_b}{2\rho}$ [rad]	0.827	0.827
$\frac{\rho\lvert\eta\rvert\delta_{max}^{*}}{Q_s L_b/2}$	0.972	0.972

Microwave Instability

- Longitudinal Broad Band Impedance, $Z_{||bb}(\omega)$, can cause internal bunch oscillations which can cause bunch lengthening and increase in energy spread

- The "Keil-Schnell Criterion" gives an approximate upper allowed limit on number bunch particles

$$N_b^{th} = \frac{2\pi\beta^2|\eta|E_{tot}F}{Z^2e^2\left|\frac{Z_{||}}{n}\right|}\left(\frac{\delta_{max}}{2}\right)^2\frac{\tau_b}{4}$$

S.Y.Lee, Accelerator Physics

	DR ^{18}Ne	DR ^{6}He
σ_δ	1.250e-03	1.250e-03
τ_b [ns]	6.572	6.572
$\left\|\frac{Z_{\|\|}}{n}\right\|$ [Ω]	10.000	10.000
N_b^{th}	2.146e+11	1.794e+12
N_B/N_b^{th}	19.881	4.038

- For ^{18}Ne in DR: N_B needs to be reduced by a factor 20 OR $R_{||}^{DR} = R_{||}^{SPS}/20$

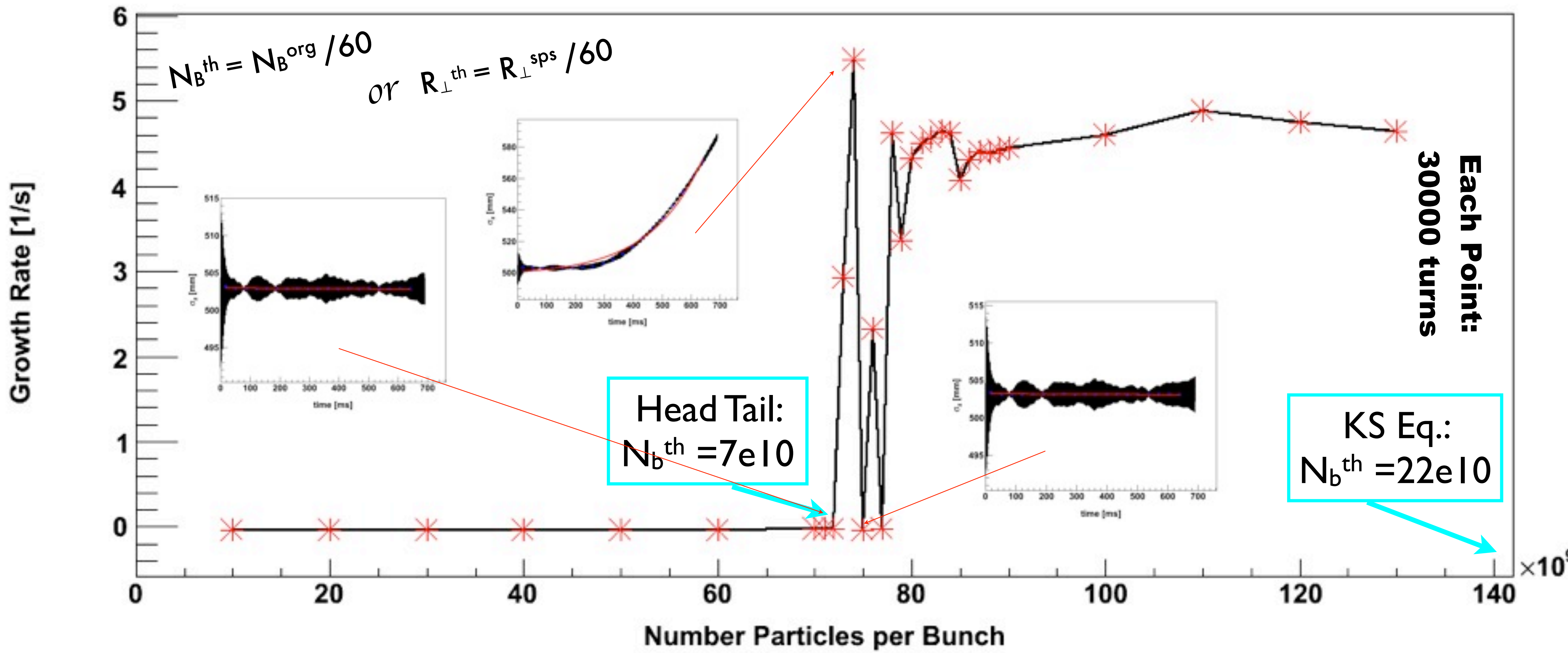
DR ^{18}Ne - Longitudinal Broad Band

DR ^{18}Ne
Long. Broad Band Res.
N_B^{org} = 4.27e12
$R_{||}^{org}$ = 0.2 MΩ
ξ_x^{org} = 0.05, ξ_y^{org} = 0.1
(η > 0 for DR)

- A Least Square Fit to the exponential gives σ_0 and the Growth Rate, $1/\tau$

$$\sigma_z = \sigma_0 e^{t/\tau}$$

- Growth Rate as a function of ion bunch intensity in the Decay Ring:



- HeadTail indicates that for the current anticipated bunch intensity a 60 times smaller longitudinal shunt impedance than SPS is needed for the DR

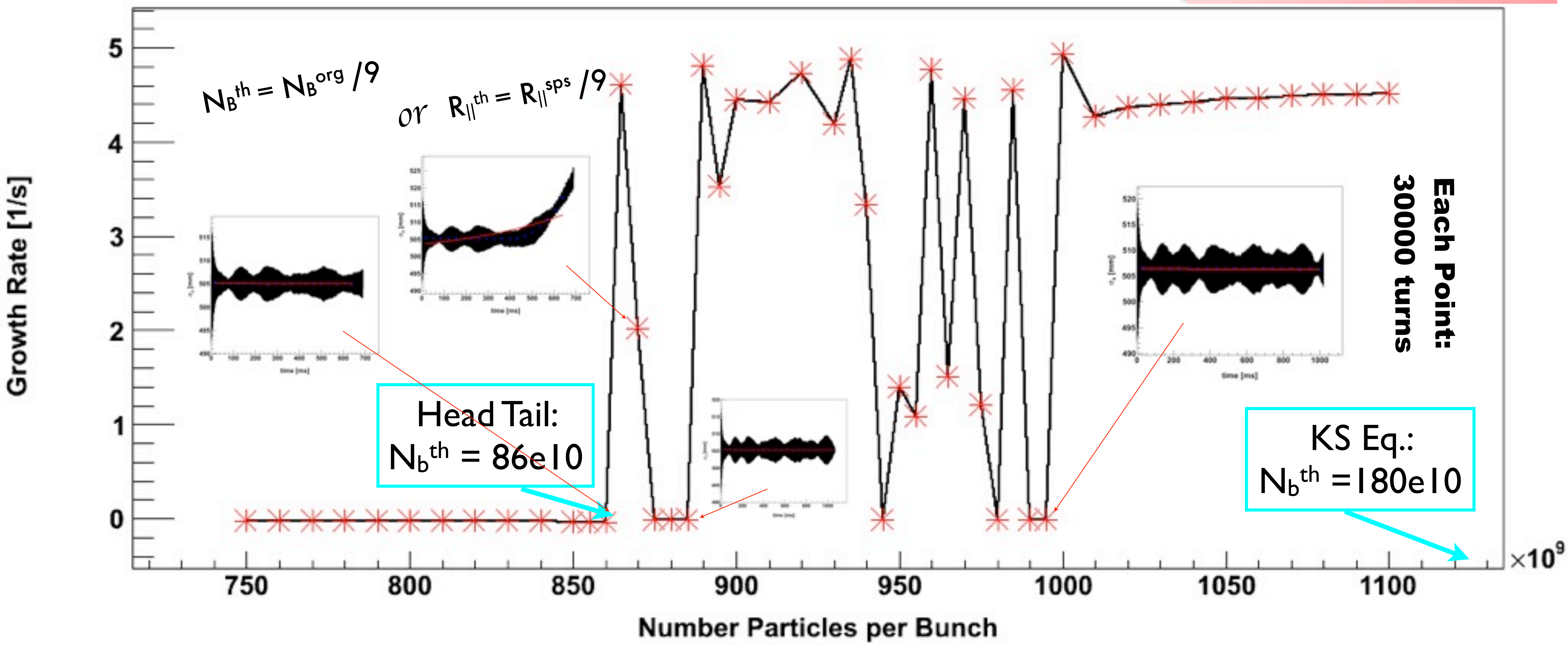
DR ^{6}He - Longitudinal Broad Band

- A Least Square Fit to the exponential gives σ_0 and the Growth Rate, $1/\tau$

$$\sigma_z = \sigma_0 e^{t/\tau}$$

DR ^{6}He
Long. Broad Band Res.
N_B^{org} = 7.24e12
$R_{||}^{org}$ = 0.2 MΩ
ξ_x^{org} = 0.05, ξ_y^{org} = 0.1
($\eta > 0$ for DR)

- Growth Rate as a function of ion bunch intensity in the Decay Ring:

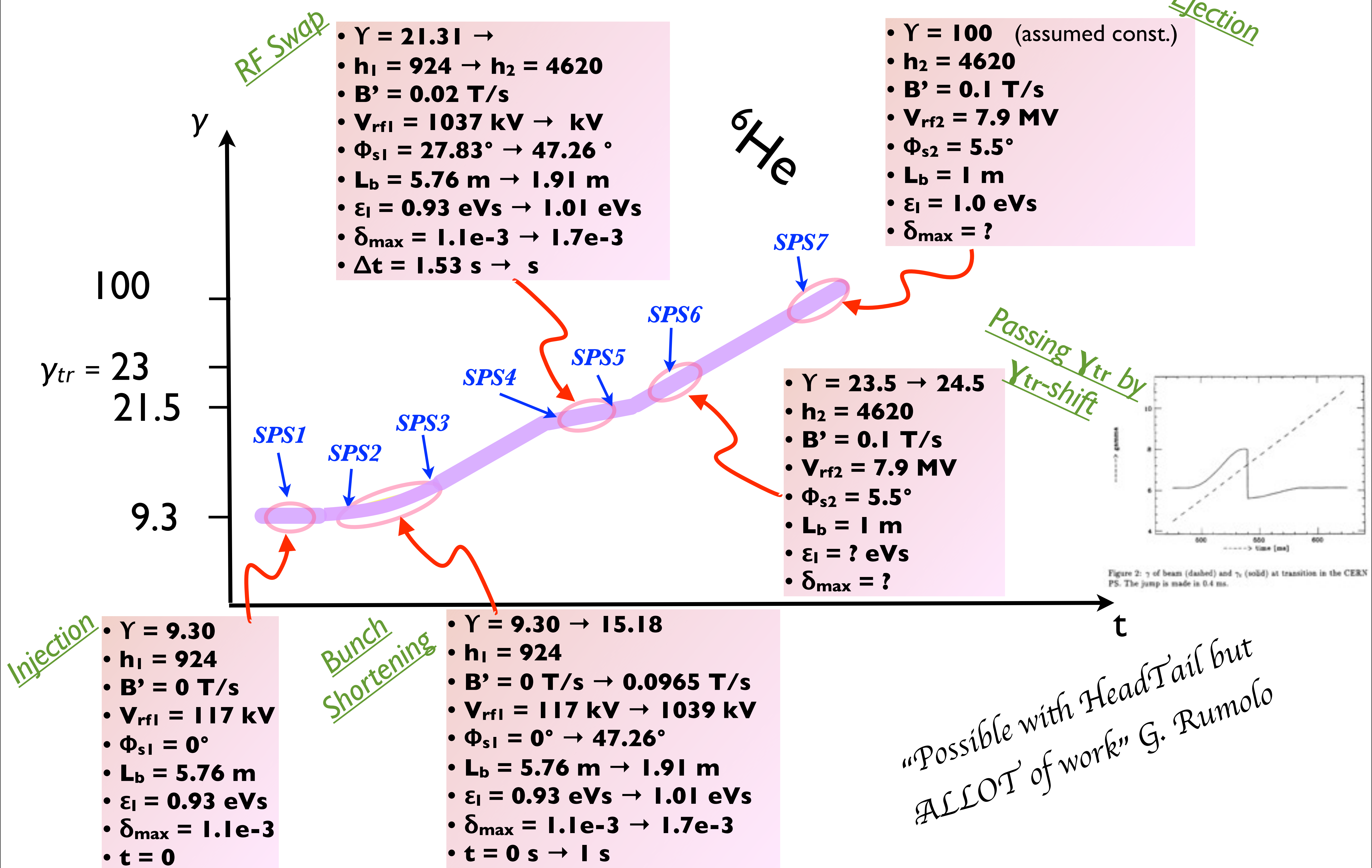


- HeadTail indicates that for the current anticipated bunch intensity a 9 times smaller longitudinal shunt impedance than SPS is needed for the DR

- Will do head-tail mode coupling and decoupling analysis to explain this behavior

Longitudinal Parameters - SPS

- SPS RF Program is being developed with the use of ESME (A. Chancé)



Conclusions

- According to HEADTAIL simulations the DR have to have
  - 430 times better transversal shunt impedance than SPS (^{18}Ne) and
  - 60 times better longitudinal shunt impedance than SPS (^{18}Ne)

To Do

- Finish the SPS RF program with ESME simulations
- Study the instabilities at some crucial parts in the SPS RF cycle
- Include other instabilities like those due to Resistive Wall Impedance
- Try to improve Beta Beam's result by tuning the chromaticity

- If result does not improve allot:
  - Redesign the Beta Beam - N_B, γ_{tr}, …
  - Study impact on physics reach

Backup Slides

Input Values (I)

Parameters	SPS Inj. ^{18}Ne	SPS Inj. ^{6}He	SPS3 ^{18}Ne	SPS3 ^{6}He	SPS4 ^{18}Ne	SPS4 ^{6}He
Z	10	2	10	2	10	2
A	18	6	18	6	18	6
h	924	924	924	924	924	924
C [m]	6911.6	6911.6	6911.6	6911.6	6911.6	6911.6
γ_{tr}	24.0	24.0	24.0	24.0	24.0	24.0
V_{RF} [MV]	5.646e-03	1.166e-01	1.000e+00	1.000e+00	1.000e+00	1.000e+00
dB/dt [T/s]	0.00	0.00	0.10	0.10	0.02	0.02
γ	15.5	9.3	13.0	13.0	21.5	21.5
δ_{max}	2.37e-04	5.37e-04	1.67e-03	1.67e-03	1.67e-03	1.67e-03
E_{rest} [MeV]	16767.10	5605.54	16767.10	5605.54	16767.10	5605.54
M	20	20	20	20	20	20
L_b [m]	5.984	5.984	1.197	1.197	1.197	1.197
N_b	2.48e+11	7.15e+11	2.45e+11	6.75e+11	2.45e+11	6.75e+11
N_m	1	1	1	1	1	1
$t_{1/2}$ [s]	1.67	0.81	1.67	0.81	1.67	0.81
T_c [s]	3.60	6.00	3.60	6.00	3.60	6.00
Q_x	26.13	26.13	26.13	26.13	26.13	26.13
Q_y	26.18	26.18	26.18	26.18	26.18	26.18
$\langle \beta \rangle_x$ [m]	54.55	54.55	54.55	54.55	54.55	54.55
$\langle \beta \rangle_y$ [m]	54.59	54.59	54.59	54.59	54.59	54.59
$\langle D \rangle_x$ [m]	1.83	1.83	1.83	1.83	1.83	1.83
ξ_x	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
ξ_y	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10
ε_{N_x} (1σ) [πm·rad]	1.23e-05	1.23e-05	1.23e-05	1.23e-05	1.23e-05	1.23e-05
ε_{N_y} (1σ) [πm·rad]	6.60e-06	6.60e-06	6.60e-06	6.60e-06	6.60e-06	6.60e-06
ε_l (full) [eVs]	1.76	0.80	0.90	0.90	0.90	0.90
b_x [cm]	28.4	28.4	28.4	28.4	28.4	28.4
b_y [cm]	6.9	6.9	6.9	6.9	6.9	6.9
ρ [Ωm]	1.0e-07	1.0e-07	1.0e-07	1.0e-07	1.0e-07	1.0e-07

Input Values (2)

Parameters	SPS5 ^{18}Ne	SPS5 ^{6}He	SPS6 ^{18}Ne	SPS6 ^{6}He
Z	10	2	10	2
A	18	6	18	6
h	4620	4620	4620	4620
C [m]	6911.6	6911.6	6911.6	6911.6
γ_{tr}	24.0	24.0	24.0	24.0
V_{RF} [MV]	7.900e+00	7.900e+00	7.900e+00	7.900e+00
dB/dt [T/s]	0.02	0.02	0.10	0.10
γ	21.5	21.5	23.5	23.5
δ_{max}	1.67e-03	1.67e-03	1.67e-03	1.67e-03
E_{rest} [MeV]	16767.10	5605.54	16767.10	5605.54
M	20	20	20	20
L_b [m]	1.197	1.197	1.197	1.197
N_b	2.45e+11	6.75e+11	2.45e+11	6.75e+11
N_m	1	1	1	1
$t_{1/2}$ [s]	1.67	0.81	1.67	0.81
T_c [s]	3.60	6.00	3.60	6.00
Q_x	26.13	26.13	26.13	26.13
Q_y	26.18	26.18	26.18	26.18
$\langle \beta \rangle_x$ [m]	54.55	54.55	54.55	54.55
$\langle \beta \rangle_y$ [m]	54.59	54.59	54.59	54.59
$\langle D \rangle_x$ [m]	1.83	1.83	1.83	1.83
ξ_x	-0.05	-0.05	-0.05	-0.05
ξ_y	-0.10	-0.10	-0.10	-0.10
ε_{N_x} (1σ) [πm·rad]	1.23e-05	1.23e-05	1.23e-05	1.23e-05
ε_{N_y} (1σ) [πm·rad]	6.60e-06	6.60e-06	6.60e-06	6.60e-06
ε_l (full) [eVs]	0.90	0.90	0.90	0.90
b_x [cm]	28.4	28.4	28.4	28.4
b_y [cm]	6.9	6.9	6.9	6.9
ρ [Ωm]	1.0e-07	1.0e-07	1.0e-07	1.0e-07

Input Values (3)

Parameters	SPS Ej. ^{18}Ne	SPS Ej. ^{6}He	DR ^{18}Ne	DR ^{6}He
Z	10	2	10	2
A	18	6	18	6
h	4620	4620	924	924
C [m]	6911.6	6911.6	6911.6	6911.6
γ_{tr}	24.0	24.0	27.0	27.0
V_{RF} [MV]	7.900e+00	7.900e+00	1.196e+01	2.000e+01
dB/dt [T/s]	0.10	0.10	0.00	0.00
γ	100.0	100.0	100.0	100.0
δ_{max}	4.73e-04	1.07e-03	2.50e-03	2.50e-03
E_{rest} [MeV]	16767.10	5605.54	16767.10	5605.54
M	20	20	20	20
L_b [m]	1.197	1.197	1.967	1.970
N_b	2.45e+11	6.75e+11	2.45e+11	6.75e+11
N_m	1	1	20	15
$t_{1/2}$ [s]	1.67	0.81	1.67	0.81
T_c [s]	3.60	6.00	3.60	6.00
Q_x	26.13	26.13	22.23	22.23
Q_y	26.18	26.18	12.16	12.16
$\langle \beta \rangle_x$ [m]	54.55	54.55	148.25	148.25
$\langle \beta \rangle_y$ [m]	54.59	54.59	173.64	173.64
$\langle D \rangle_x$ [m]	1.83	1.83	-0.60	-0.60
ξ_x	1.00	0.05	0.05	0.05
ξ_y	1.00	0.10	0.10	0.10
ε_{N_x} (1σ) [πm·rad]	1.23e-05	1.23e-05	1.48e-05	1.48e-05
ε_{N_y} (1σ) [πm·rad]	6.60e-06	6.60e-06	7.90e-06	7.90e-06
ε_l (full) [eVs]	2.20	1.00	42.89	14.36
b_x [cm]	28.4	28.4	16.0	16.0
b_y [cm]	6.9	6.9	16.0	16.0
ρ [Ωm]	1.0e-07	1.0e-07	1.0e-07	1.0e-07

Calculated Values (I)

	SPS Inj. ^{18}Ne	SPS Inj. ^{6}He	SPS3 ^{18}Ne	SPS3 ^{6}He	SPS4 ^{18}Ne	SPS4 ^{6}He
$N_B = N_b \frac{1-2^{-N_m T_c/(\gamma t_{1/2})}}{1-2^{-T_c/(\gamma t_{1/2})}}$	2.48e+11	7.15e+11	2.45e+11	6.75e+11	2.45e+11	6.75e+11
r_0 [m] = $r_p Z^2/A$	8.53e-18	1.02e-18	8.53e-18	1.02e-18	8.53e-18	1.02e-18
E_{tot} [GeV] = $\gamma \cdot E_{rest}$	260.39	52.30	217.97	72.87	360.49	120.52
$\beta = \sqrt{1-1/\gamma^2}$	1.00	0.99	1.00	1.00	1.00	1.00
$\eta = \{1/\gamma_{tr}\}^2 - \{1/\gamma\}^2$	-2.41e-03	-9.75e-03	-4.18e-03	-4.18e-03	-4.27e-04	-4.27e-04
T_{rev} [ms] = $C/(\beta c)$	23.1026	23.1882	23.1231	23.1231	23.0796	23.0796
ω_{rev} [MHz] = $2\pi/T_{rev}$	0.27	0.27	0.27	0.27	0.27	0.27
$\sigma_\delta = \delta_{max}/2$	1.19e-04	2.69e-04	8.34e-04	8.34e-04	8.34e-04	8.34e-04
τ_b [ns] = $L_b / (\beta c)$	20.00	20.08	4.00	4.00	4.00	4.00
I_b [A] = ZeN_B / τ_b	19.87	11.41	98.02	54.01	98.20	54.11
$\varepsilon_l^{2\sigma}$[eVs] = $\frac{\pi}{2}\beta^2 E_{tot}\tau_b\delta_{max}$	1.93	0.88	2.27	0.76	3.77	1.26
ω_s [kHz] = $Q_s \cdot \omega_{rev}$	0.08	0.69	1.17	0.90	0.36	0.28
ω_x [MHz] = $Q_x \cdot \omega_{rev}$	7.11	7.08	7.10	7.10	7.11	7.11
ω_y [MHz] = $Q_y \cdot \omega_{rev}$	7.12	7.10	7.12	7.12	7.13	7.13
ω_c [GHz] = $\beta c / b_{min(x,y)}$	4.34	4.32	4.33	4.33	4.34	4.34
$\Delta Q_{\xi_x} = \xi_x \delta_{max} Q_x$	-3.10e-04	-7.02e-04	-2.18e-03	-2.18e-03	-2.18e-03	-2.18e-03
$\Delta Q_{\xi_y} = \xi_y \delta_{max} Q_y$	-6.21e-04	-1.41e-03	-4.37e-03	-4.37e-03	-4.37e-03	-4.37e-03

Calculated Values (2)

	SPS5 ^{18}Ne	SPS5 ^{6}He	SPS6 ^{18}Ne	SPS6 ^{6}He
$N_B = N_b \frac{1-2^{-N_m T_c/(\gamma t_{1/2})}}{1-2^{-T_c/(\gamma t_{1/2})}}$	2.45e+11	6.75e+11	2.45e+11	6.75e+11
r_0 [m] $= r_p Z^2/A$	8.53e-18	1.02e-18	8.53e-18	1.02e-18
E_{tot} [GeV] $= \gamma \cdot E_{rest}$	360.49	120.52	394.03	131.73
$\beta = \sqrt{1-1/\gamma^2}$	1.00	1.00	1.00	1.00
$\eta = \{1/\gamma_{tr}\}^2 - \{1/\gamma\}^2$	-4.27e-04	-4.27e-04	-7.47e-05	-7.47e-05
T_{rev} [ms] $= C/(\beta c)$	23.0796	23.0796	23.0755	23.0755
ω_{rev} [MHz] $= 2\pi/T_{rev}$	0.27	0.27	0.27	0.27
$\sigma_\delta = \delta_{max}/2$	8.34e-04	8.34e-04	8.34e-04	8.34e-04
τ_b [ns] $= L_b / (\beta c)$	4.00	4.00	4.00	4.00
I_b [A] $= ZeN_B / \tau_b$	98.20	54.11	98.22	54.12
$\varepsilon_l^{2\sigma}$ [eVs] $= \frac{\pi}{2}\beta^2 E_{tot} \tau_b \delta_{max}$	3.77	1.26	4.12	1.38
ω_s [kHz] $= Q_s \cdot \omega_{rev}$	2.26	1.75	0.90	0.70
ω_x [MHz] $= Q_x \cdot \omega_{rev}$	7.11	7.11	7.11	7.11
ω_y [MHz] $= Q_y \cdot \omega_{rev}$	7.13	7.13	7.13	7.13
ω_c [GHz] $= \beta c / b_{min(x,y)}$	4.34	4.34	4.34	4.34
$\Delta Q_{\xi_x} = \xi_x \delta_{max} Q_x$	-2.18e-03	-2.18e-03	-2.18e-03	-2.18e-03
$\Delta Q_{\xi_y} = \xi_y \delta_{max} Q_y$	-4.37e-03	-4.37e-03	-4.37e-03	-4.37e-03

Calculated Values (3)

	SPS Ej. ^{18}Ne	SPS Ej. ^{6}He	DR ^{18}Ne	DR ^{6}He
$N_B = N_b \frac{1-2^{-N_m T_c/(\gamma t_{1/2})}}{1-2^{-T_c/(\gamma t_{1/2})}}$	2.45e+11	6.75e+11	4.27e+12	7.24e+12
r_0 [m] $= r_p Z^2/A$	8.53e-18	1.02e-18	8.53e-18	1.02e-18
E_{tot} [GeV] $= \gamma \cdot E_{rest}$	1676.71	560.55	1676.71	560.55
$\beta = \sqrt{1-1/\gamma^2}$	1.00	1.00	1.00	1.00
$\eta = (1/\gamma_{tr})^2 - (1/\gamma)^2$	1.64e-03	1.64e-03	1.27e-03	1.27e-03
T_{rev} [ms] $= C/(\beta c)$	23.0558	23.0558	23.0558	23.0558
ω_{rev} [MHz] $= 2\pi/T_{rev}$	0.27	0.27	0.27	0.27
$\sigma_\delta = \delta_{max}/2$	2.37e-04	5.34e-04	1.25e-03	1.25e-03
τ_b [ns] $= L_b / (\beta c)$	3.99	3.99	6.56	6.57
I_b [A] $= ZeN_B / \tau_b$	98.31	54.17	1041.99	353.19
$\varepsilon_l^{2\sigma}$[eVs]$=\frac{\pi}{2}\beta^2 E_{tot}\tau_b\delta_{max}$	4.98	3.75	43.20	14.46
ω_s [kHz] $= Q_s \cdot \omega_{rev}$	2.05	1.58	1.00	1.00
ω_x [MHz] $= Q_x \cdot \omega_{rev}$	7.12	7.12	6.06	6.06
ω_y [MHz] $= Q_y \cdot \omega_{rev}$	7.14	7.14	3.31	3.31
ω_c [GHz] $= \beta c / b_{min(x,y)}$	4.34	4.34	1.87	1.87
$\Delta Q_{\xi_x} = \xi_x \delta_{max} Q_x$	1.24e-02	1.40e-03	2.78e-03	2.78e-03
$\Delta Q_{\xi_y} = \xi_y \delta_{max} Q_y$	1.24e-02	2.80e-03	3.04e-03	3.04e-03

RF Values - No Acc.

	SPS Inj. ^{18}Ne	SPS Inj. ^{6}He	DR ^{18}Ne	DR ^{6}He
$(\delta_{max})^* = \frac{h\varepsilon_l c}{\rho E_{tot} G\{\theta_b/2\}}$	2.373e-04	5.370e-04	-	-
δ_{max}	2.373e-04	5.370e-04	2.500e-03	2.500e-03
eV^* [MeV] $= \frac{\pi h\|\eta\| E_{tot}\{\beta\delta_{max}\}^2}{Z(1-\cos\theta_b)}$	5.646e-03	1.166e-01	-	-
eV [MeV]	5.646e-03	1.166e-01	1.196e+01	2.000e+01
L_b^* [m] $= \frac{2\rho}{h}\arccos\left(1 - \frac{\pi h\eta E_{tot}(\beta\delta_{max})^2}{ZeV}\right)$	-	-	1.970	1.970
L_b [m]	5.984	5.984	1.967	1.970
ε_l^* [eVs] $= \frac{\beta\rho E_{tot}\delta_{max}}{hc} G\{\theta_b/2\}$	-	-	42.883	14.358
ε_l [eVs]	1.760	0.800	42.890	14.360
$Q_s = \sqrt{\frac{hZeV\|\eta\cos\phi_s\|}{2\pi\beta^2 E_{tot}}}$	2.778e-04	2.543e-03	3.653e-03	3.653e-03
$\theta_b = \frac{hL_b}{2\rho}$ [rad]	2.513	2.513	0.826	0.827
$\frac{\rho\|\eta\|\delta^*_{max}}{Q_s L_b/2}$	0.757	0.757	0.973	0.972

RF Values - Acc. (I)

	SPS3 ^{18}Ne	SPS3 ^{6}He	SPS4 ^{18}Ne	SPS4 ^{6}He
$(\delta_{max})^* = ?$	??	??	??	??
δ_{max}	1.668e-03	1.668e-03	1.668e-03	1.668e-03
eV^* [MeV] = ?	??	??	??	??
eV [MeV]	1.000e+00	1.000e+00	1.000e+00	1.000e+00
L_b^* [m] = ?	??	??	??	??
L_b [m]	1.197	1.197	1.197	1.197
ε_l^* [eVs] = ?	??	??	??	??
ε_l [eVs]	0.900	0.900	0.900	0.900
$\phi_s\ [^\circ] = \mathrm{asin}\left(\frac{2\pi\rho^2 B'(t)}{V_{rf}}\right)$	49.49	49.49	8.75	8.75
$Q_s = \sqrt{\frac{hZeV\lvert\eta\cos\phi_s\rvert}{2\pi\beta^2 E_{tot}}}$	4.293e-03	3.321e-03	1.314e-03	1.016e-03
$\theta_b = \frac{hL_b}{2\rho}$ [rad]	0.503	0.503	0.503	0.503
$\frac{\rho\lvert\eta\rvert\delta_{max}^*}{Q_s L_b/2}$	2.986	3.860	0.997	1.289

RF Values - Acc. (2)

	SPS5 ^{18}Ne	SPS5 ^{6}He	SPS6 ^{18}Ne	SPS6 ^{6}He	SPS Ej. ^{18}Ne	SPS Ej. ^{6}He
$(\delta_{max})^* = ?$	??	??	??	??	??	??
δ_{max}	1.668e-03	1.668e-03	1.668e-03	1.668e-03	4.734e-04	1.068e-03
eV^* [MeV] = ?	??	??	??	??	??	??
eV [MeV]	7.900e+00	7.900e+00	7.900e+00	7.900e+00	7.900e+00	7.900e+00
L_b^* [m] = ?	??	??	??	??	??	??
L_b [m]	1.197	1.197	1.197	1.197	1.197	1.197
ε_l^* [eVs] = ?	??	??	??	??	??	??
ε_l [eVs]	0.900	0.900	0.900	0.900	2.200	1.000
$\phi_s\,[^\circ]=\mathrm{asin}\left(\frac{2\pi\rho^2 B'(t)}{V_{rf}}\right)$	1.10	1.10	5.52	5.52	5.52	5.52
$Q_s=\sqrt{\frac{hZeV\lvert\eta\cos\phi_s\rvert}{2\pi\beta^2 E_{tot}}}$	8.305e-03	6.424e-03	3.313e-03	2.562e-03	7.512e-03	5.810e-03
$\theta_b=\frac{hL_b}{2\rho}$ [rad]	2.514	2.514	2.514	2.514	2.514	2.514
$\frac{\rho\lvert\eta\rvert\delta^*_{max}}{Q_s L_b/2}$	0.158	0.204	0.069	0.089	0.190	0.553

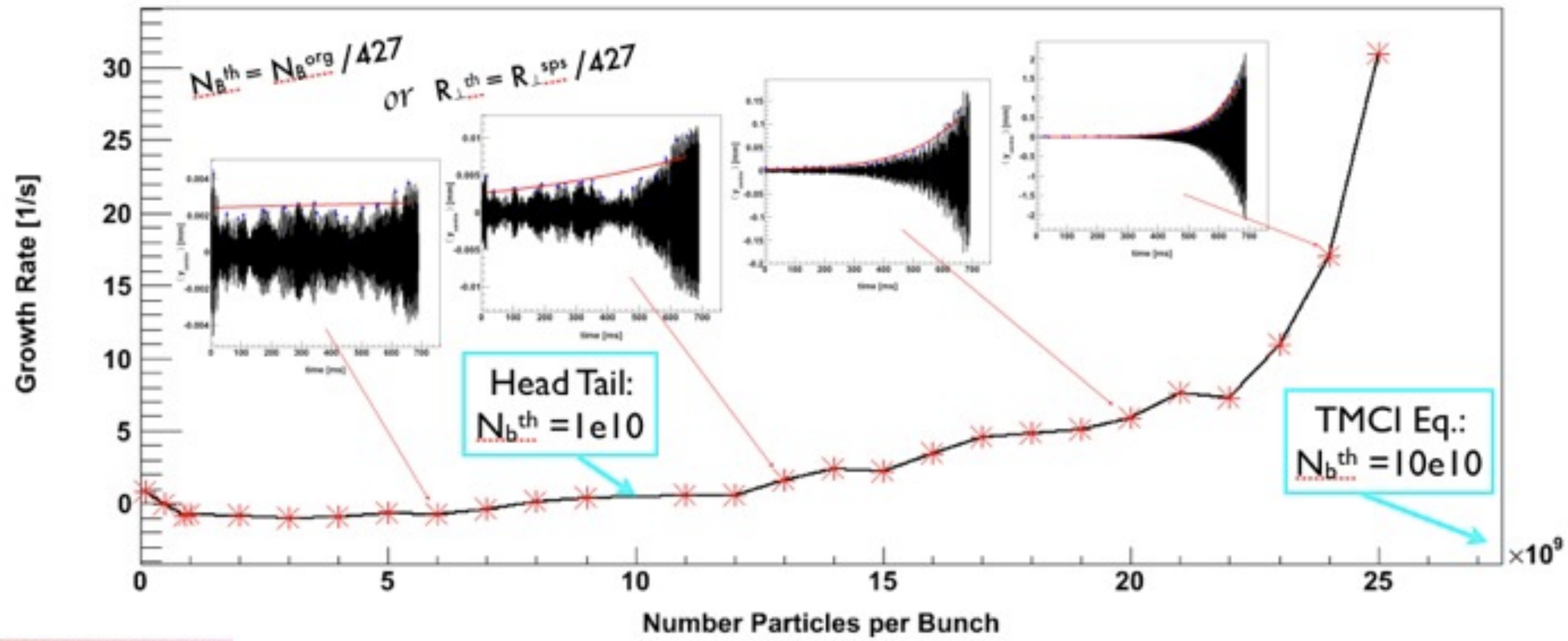
Beta Beam Instability Studies

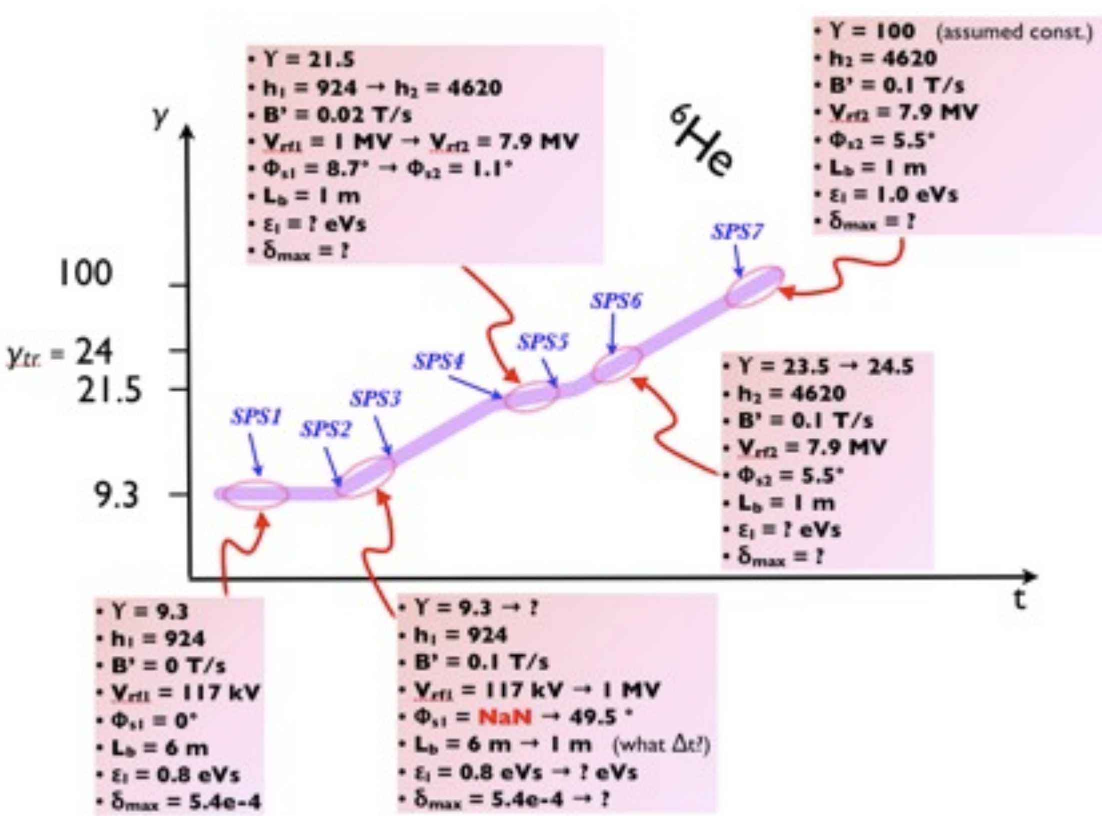
C. Hansen

- Collective Effect studies with the "Head Tail" simulation program will be made to study instabilities for all beams in the Beta Beam complex

- Instability dependencies of bunch intensities are being investigated for the Decay Ring

  (To the right: Instability growth rate (1/τ) due to transversal broad band impedance for ^{6}He in DR)



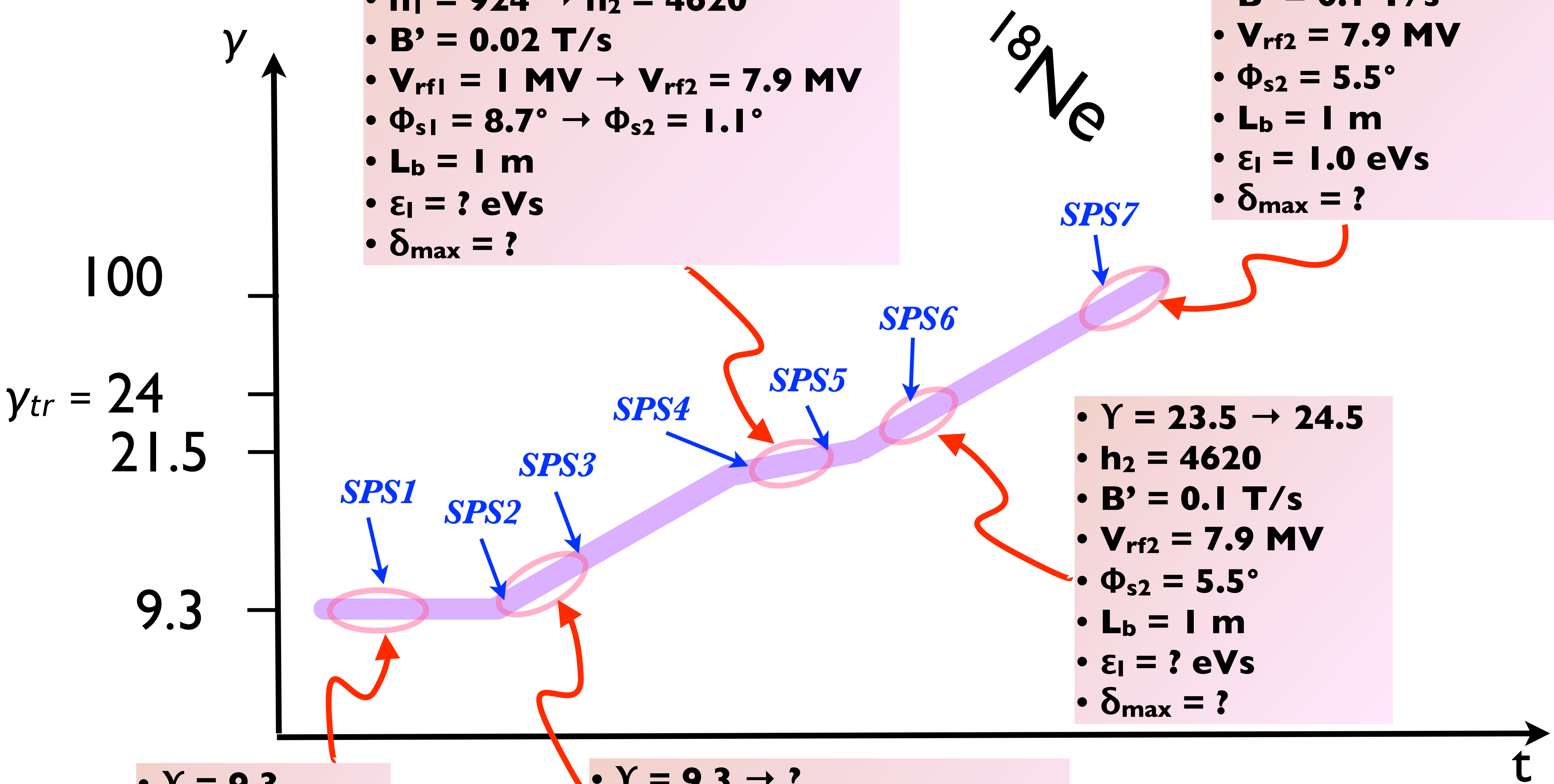


- The extra impedance due to beam loading at the special RF cavity in the Decay Ring will have to be taken into account

- The SPS' RF programs for the Beta Beams (left) are currently being developed in detail (A. Chancé) for the Instability Studies

Longitudinal Parameters - SPS

- Areas in the SPS cycle where to investigate instabilities:



- Υ = 9.3
- h_1 = 924
- B' = 0 T/s
- V_{rf1} = 5.6 kV
- Φ_{s1} = 0°
- L_b = 6 m
- ε_l = 1.76 eVs
- δ_{max} = 2.4e-4

- Υ = 9.3 → ?
- h_1 = 924
- B' = 0.1 T/s
- V_{rf1} = 120 kV → 1 MV
- Φ_{s1} = NaN° → ? °
- L_b = 6 m → 1 m (what Δt?)
- ε_l = 0.8 eVs → ? eVs
- δ_{max} = 2.4e-4 → ?

- Υ = 21.5
- h_1 = 924 → h_2 = 4620
- B' = 0.02 T/s
- V_{rf1} = 1 MV → V_{rf2} = 7.9 MV
- Φ_{s1} = 8.7° → Φ_{s2} = 1.1°
- L_b = 1 m
- ε_l = ? eVs
- δ_{max} = ?

- Υ = 23.5 → 24.5
- h_2 = 4620
- B' = 0.1 T/s
- V_{rf2} = 7.9 MV
- Φ_{s2} = 5.5°
- L_b = 1 m
- ε_l = ? eVs
- δ_{max} = ?

- Υ = 100 (assumed const.)
- h_2 = 4620
- B' = 0.1 T/s
- V_{rf2} = 7.9 MV
- Φ_{s2} = 5.5°
- L_b = 1 m
- ε_l = 1.0 eVs
- δ_{max} = ?

"Possible with HeadTail but ALLOT of work" G. Rumolo

Transversal Instability Limits

HeadTail and Formulas

$\xi = \xi^{org}$	HeadTail	Transverse Mode Coupling (TMCI Eq.)
DR ^{18}Ne; BB$\perp$	$R_\perp = R_\perp^{sps}/427$	$R_\perp = R_\perp^{sps}/42$
DR ^{6}He; BB$\perp$	$R_\perp = R_\perp^{sps}/73$	$R_\perp = R_\perp^{sps}/9$
SPS Ej. ^{18}Ne; BB$\perp$	$N_B = N_B^{org}/??$	$N_B = N_B^{org}/10$
SPS Ej. ^{6}He; BB$\perp$		$N_B = N_B^{org}/4$
SPS Inj. ^{18}Ne; BB$\perp$	$N_B = N_B^{org}/??$	$N_B = N_B^{org}/14$
SPS Inj. ^{6}He; BB$\perp$		$N_B = N_B^{org}$

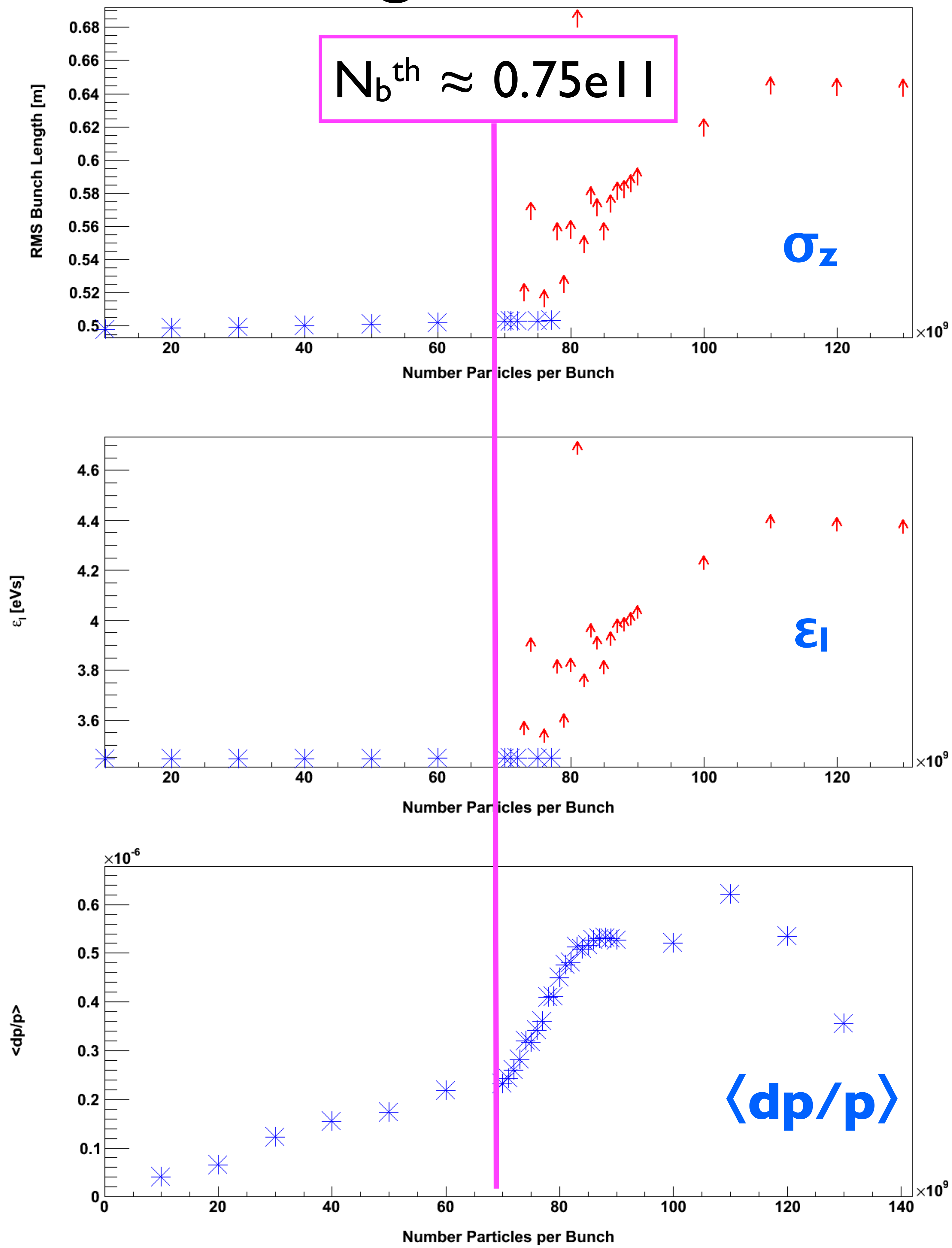
Longitudinal Instability Limits

HeadTail and Formulas

$\xi = \xi^{org}$	HeadTail	Micro Wave Instabilities (Keil Schnell)
DR ^{18}Ne; BB||	$R_{\|\|} = R_{\|\|}^{sps}/60$	$R_{\|\|} = R_{\|\|}^{sps}/20$
DR ^{6}He; BB||	$R_{\|\|} = R_{\|\|}^{sps}/9$	$R_{\|\|} = R_{\|\|}^{sps}/4$
SPS Ej. ^{18}Ne; BB||		$N_B = N_B^{org}/44$
SPS Ej. ^{6}He; BB||		$N_B = N_B^{org}/8$
SPS Inj. ^{18}Ne; BB||		$N_B = N_B^{org}/36$
SPS Inj. ^{6}He; BB||		$N_B = N_B^{org}$

DR ^{18}Ne - Longitudinal Broad Band

Each point:
30000 turns



Stability Limit:

Head Tail:
$N_B = N_B^{org}/57$

Keil Schnell:
$N_B = N_B^{org}/20$

DR ^{18}Ne
Longitudinal Broad Band Resonance

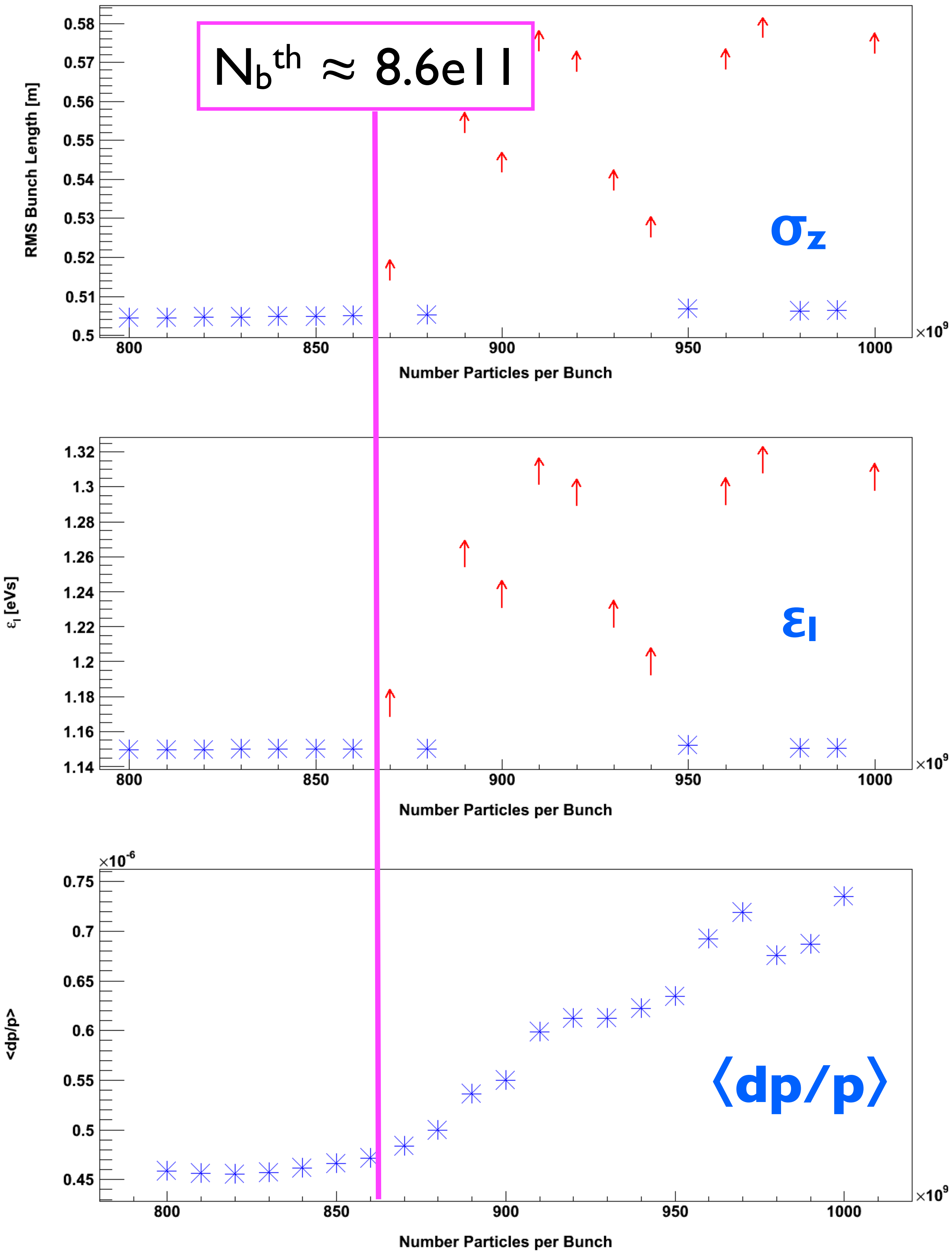
$N_B^{org} = 4.27e12$
$R_{||}^{org} = 0.2\ M\Omega$

$\xi_x^{org} = 0.05$, $\xi_y^{org} = 0.1$ for DR ($\eta > 0$)

KS: $N_B = N_B^{org}/20$
BI: $N_B = N_B^{org}/10$

DR ^{6}He - Longitudinal Broad Band

Each point: 30000 turns



Stability Limit:

Head Tail: $N_B = N_B^{org}/8$

Keil Schnell: $N_B = N_B^{org}/4$

DR ^{6}He Longitudinal Broad Band Resonance

$N_B^{org} = 7.24e12$

$R_{||}^{org} = 0.2\ M\Omega$

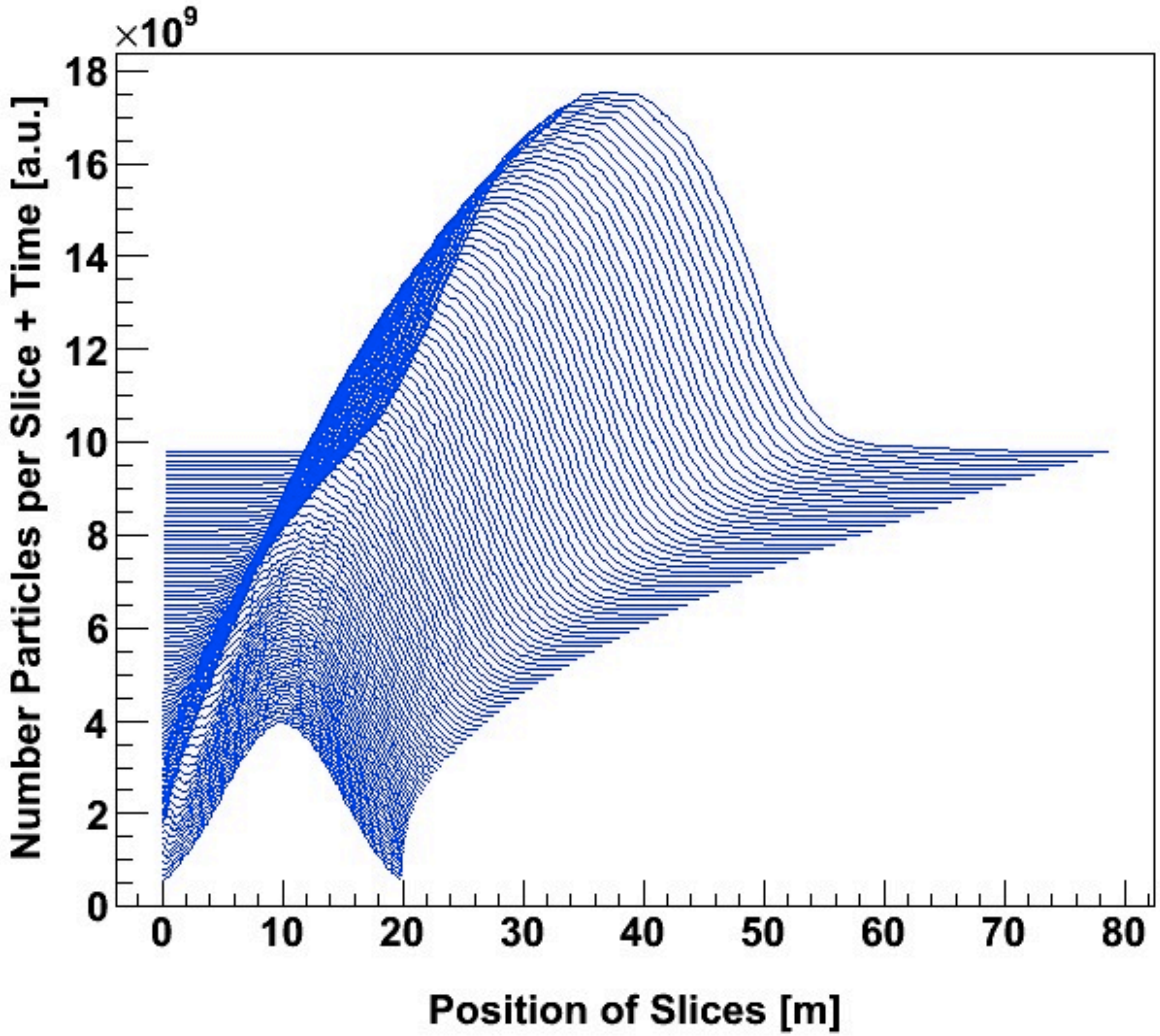
$\xi_x^{org} = 0.05$, $\xi_y^{org} = 0.1$ **for DR** ($\eta > 0$)

KS: $N_B = N_B^{org}/4$

BI: $N_B = N_B^{org}/2$

SPS ^{18}Ne Injection - Long. Broad Band

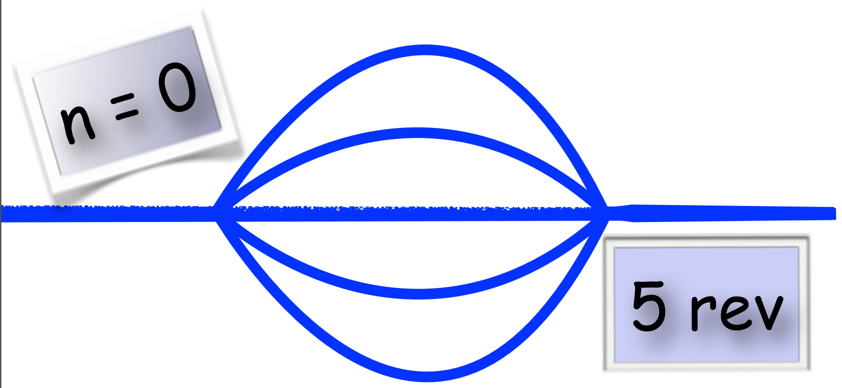
$N_B = N_B^{org}$ and $\xi = \xi^{org}$

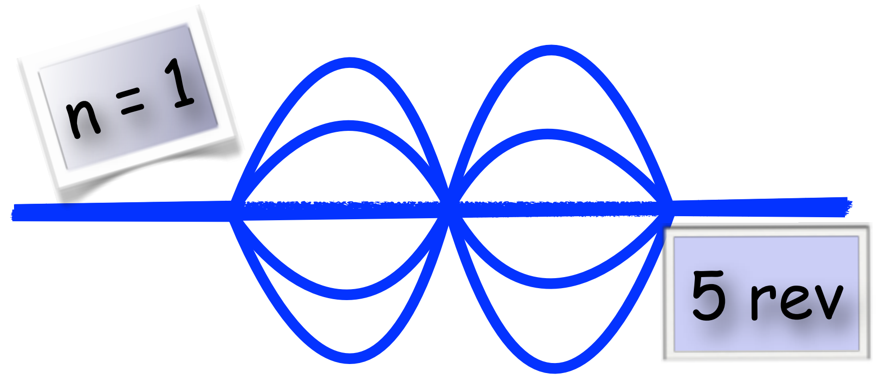


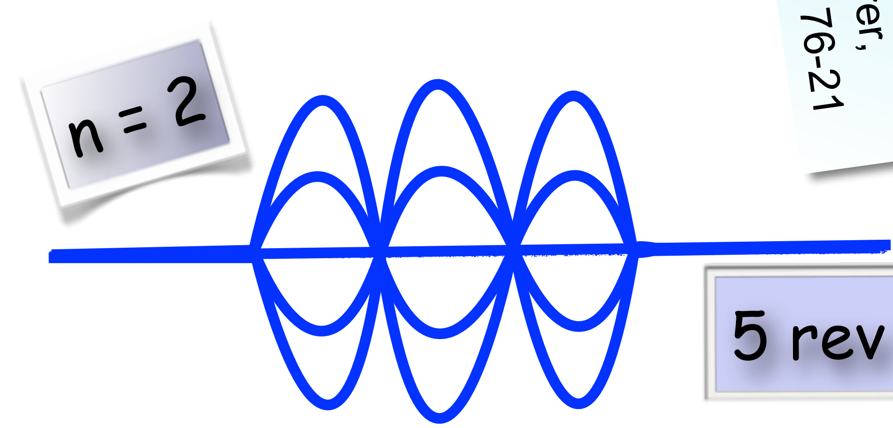
Head Tail Modes (*n*)

- The different ways particles in the front (head) of the bunch are positioned compared to particles in the back (tail) of the bunch are grouped in different "modes"
- The "head-tail mode number", *n*, defines how the head and tail couples in that mode
- The signal of a bunch in a position monitor shows *n* nodes for mode *n*:

F.J. Sacherer, CERN/PS/BR 76-21

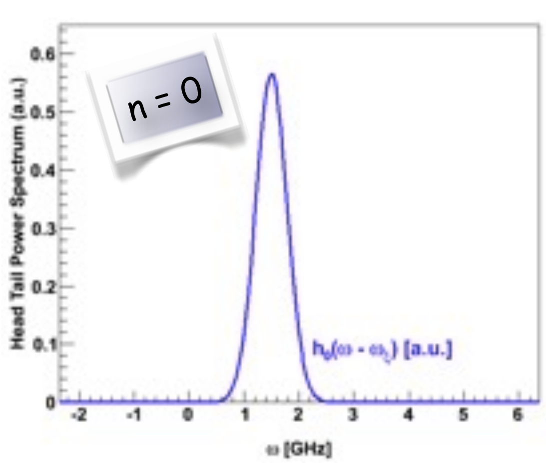


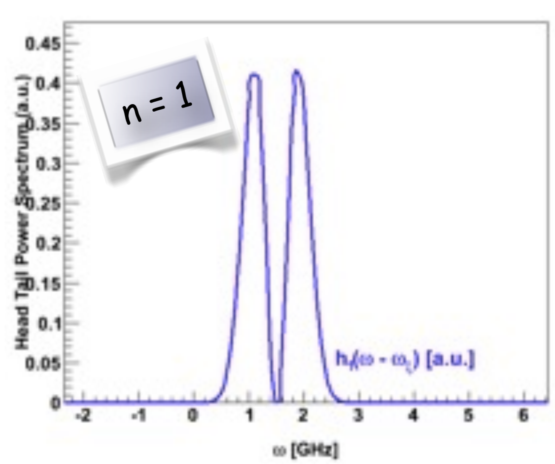


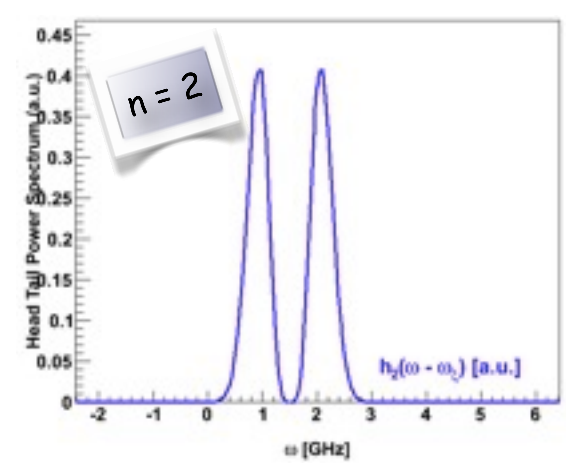


- These *head-tail modes in time domain* can be Fourier transformed and squared to get the "head-tail power spectrum", $h_n(\omega)$:

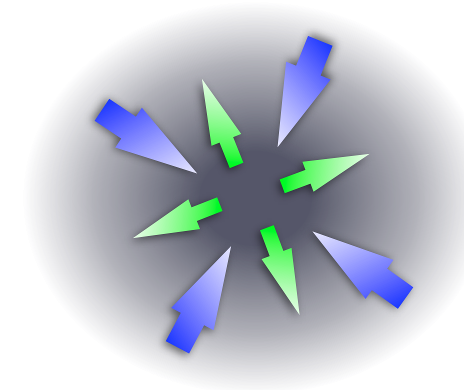
$$p_n(t) = \begin{cases} \cos\left[(n+1)\pi\frac{t}{\tau_b}\right] & , n = 0, 2, 4, \ldots \\ \sin\left[(n+1)\pi\frac{t}{\tau_b}\right] & , n = 1, 3, 5, \ldots \end{cases} \quad \Rightarrow \quad \left|\mathcal{F}\left(p_n(t)\right)\right|^2 = h_n(\omega)$$







Direct Space Charge



- A particle in a bunch feels the collective Coulomb forces due to fields generated by the charge of the other particles in the bunch

- For relativistic beams the repulsive E forces are cancelled

  by the contracting B forces → tune shift due to space charge ∝ γ^{-2}

$$\Delta Q_{dsc_{x,y}} = -\frac{\lambda r_0 R}{2\beta\gamma^2\epsilon_{N_{x,y}}}$$

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Physics of Collective Beam Instabilities

- Assuming Gaussian bunches the peak line charge density near the bunch center is

$\lambda = N/\left(\sqrt{2\pi}\sigma_z\right)$ and the full bunch length $L_b = 4\sigma_z$

- For ions $r_0 = r_p Z^2/A$ so we get the tune shift

$$\Delta Q_{dsc_{x,y}} = -\frac{2N_B r_p Z^2 R}{\sqrt{2\pi} A L_b \beta\gamma^2\epsilon_{N_{x,y}}}$$

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Physics of Collective Beam Instabilities

- If absolute value is more than 0.2 it could cause the tune to cross over the resonance lines

Direct Space Charge

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Physics of Intensity Dependent Instabilities
USPAS 2002

$$\Delta Q_{dsc_{x,y}} = -\frac{1}{B}\frac{N_B r_0 R}{\pi\gamma^3\beta^2 Q_{x,y}}\frac{\varepsilon^{dsc}_{x,y}}{2\sigma_y^2}$$

Exchanged $a_{x,y} \rightarrow \sqrt{2}\sigma_{x,y}$ and $\varepsilon \rightarrow \varepsilon^{1\sigma}$ to assume Gaussian instead of Uniform.

where

$$\varepsilon^{dsc}_x = \frac{\sigma_y^2}{\sigma_x(\sigma_y+\sigma_x)} \qquad \sigma_x = \sqrt{\frac{\langle\beta_x\rangle\varepsilon^{1\sigma}_{N_x}}{\gamma\beta} + \langle D_x\rangle^2\left(\frac{dp}{p}\right)^2_{max}}$$

$$\varepsilon^{dsc}_y = \frac{\sigma_y}{\sigma_y+\sigma_x} \qquad \sigma_y = \sqrt{\frac{\langle\beta_y\rangle\varepsilon^{1\sigma}_{N_y}}{\gamma\beta}}$$

DSC	DR ^{18}Ne	DR ^{6}He	SPS Ej. ^{18}Ne	SPS Ej. ^{6}He	SPS Inj. ^{18}Ne	SPS Inj. ^{6}He
ΔQ_{dsc_x}	-0.0402	-0.0082	-0.0109	-0.0036	-0.0916	-0.0881
ΔQ_{dsc_y}	-0.0930	-0.0189	-0.0149	-0.0049	-0.1252	-0.1204

(added the factor 1/B myself; B is the bunching factor)

Direct Space Charge

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- For elliptical beam according to Ng

$$\Delta Q_{dsc_{x,y}} = -\frac{8N_B r_0 R}{\sqrt{2\pi} L_b \beta\gamma^2 \sqrt{\epsilon_{N_{x,y}}} \left[\sqrt{\epsilon_{N_{x,y}}} + \sqrt{\epsilon_{N_{y,x}} \langle\beta_{y,x}\rangle / \langle\beta_{x,y}\rangle}\ \right]}$$

DSC	DR ^{18}Ne	DR ^{6}He	SPS Ej. ^{18}Ne	SPS Ej. ^{6}He	SPS Inj. ^{18}Ne	SPS Inj. ^{6}He
ΔQ_{dsc_x}	-0.2410	-0.0491	-0.0287	-0.0095	-0.2418	-0.2327
ΔQ_{dsc_y}	-0.3570	-0.0727	-0.0393	-0.0130	-0.3303	-0.3177

- But if we assume round beam this becomes a factor 2 bigger than Chao's equation (prev. slide)

$$\Delta Q_{dsc_{x,y}} = -\frac{4N_B r_0 R}{\sqrt{2\pi} L_b \beta\gamma^2 \epsilon_{N_{x,y}}}$$

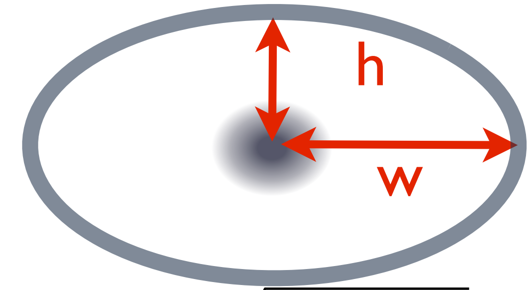
- So let's divide Ng's equation by 2

$$\Delta Q_{dsc_{x,y}} = -\frac{4N_B r_0 R}{\sqrt{2\pi} L_b \beta\gamma^2 \sqrt{\epsilon_{N_{x,y}}} \left[\sqrt{\epsilon_{N_{x,y}}} + \sqrt{\epsilon_{N_{y,x}} \langle\beta_{y,x}\rangle / \langle\beta_{x,y}\rangle}\ \right]}$$

DSC	DR ^{18}Ne	DR ^{6}He	SPS Ej. ^{18}Ne	SPS Ej. ^{6}He	SPS Inj. ^{18}Ne	SPS Inj. ^{6}He
ΔQ_{dsc_x}	-0.1205	-0.0245	-0.0144	-0.0048	-0.1209	-0.1163
ΔQ_{dsc_y}	-0.1785	-0.0364	-0.0196	-0.0065	-0.1651	-0.1589

Image Coefficients for Elliptical Vacuum Chambers

- Assume the beam is centered, then



$$\epsilon = \sqrt{w^2 - h^2}$$

$$\varepsilon_y^{incoh} = -\varepsilon_x^{incoh} = \frac{h^2}{12\epsilon^2}\left[(1+k'^2)\left(\frac{2K(k)}{\pi}\right)^2 - 2\right]$$

$$\varepsilon_y^{coh} = \frac{h^2}{4\epsilon^2}\left[\left(\frac{2K(k)}{\pi}\right)^2 - 1\right]$$

$$\varepsilon_x^{coh} = \frac{h^2}{4\epsilon^2}\left[1 - \left(\frac{2K(k)k'}{\pi}\right)^2\right]$$

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where

$$k' = \left(\frac{1 + 2\sum_{s=1}^{\infty}(-1)^s q^{s^2}}{1 + 2\sum_{s=1}^{\infty} q^{s^2}}\right)^2$$

$$q = \frac{w-h}{w+h}$$

$$k = \sqrt{1 - k'^2}$$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}}$$

When w = h (e.g. for the DR) then

$$\varepsilon_y^{incoh} = \varepsilon_x^{incoh} = 0$$

$$\varepsilon_y^{coh} = \varepsilon_x^{coh} = 1/2$$

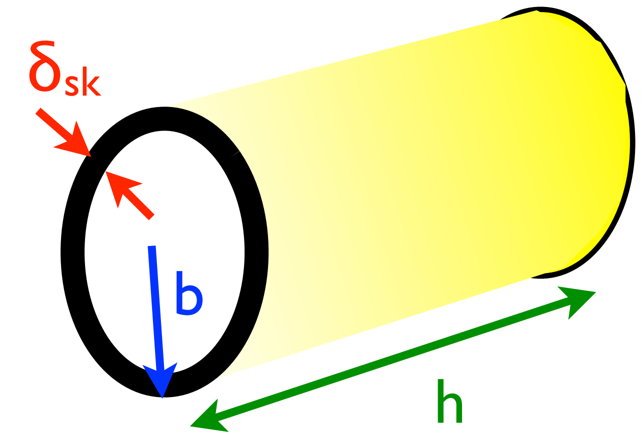
Resistive Wall Impedance

- Since the conductivity of the beam pipe is not perfect the image current is slowed down, radiates a wake field which gives an impedance

- To get the Resistive Wall Impedance one takes into account that the EM fields penetrate the pipe material to a thickness called "Skin Depth", that equals

Stainless Steel:
Magnetic Permeability = $\mu = \mu_{ss}\,\mu_0$
$= 1.05 \cdot 4\pi \cdot 10^{-7}$ Vs/Am
Resistivity = $\rho = 1 \cdot 10^{-7}\ \Omega$m

$$\delta_{sk}(\omega) = \sqrt{\frac{2\rho}{|\omega|\mu}}$$



where ρ is the materials "bulk resistance"
and then gets the "resistant" (real) and "reactive" (imaginary) parts for the longitudinal and transverse impedances of a cylindrical model with length *h* (circumference of the ring is used for *h*), radius *b* and thickness δ_{sk}

Y.H. Chin, Impedance and Wake Fields

$$Z_{||,rw}(\omega) = \frac{\omega}{2}(1-i)\frac{Z_0\delta_{sk}(\omega)h}{2\pi bc} \approx (1-i)\frac{\omega R}{2bc}\sqrt{\frac{2\rho}{\varepsilon_0|\omega|}}$$

$$Z_{\perp,rw}(\omega) = (sgn(\omega)-i)\frac{Z_0\delta_{sk}(\omega)h}{2\pi b^3} \approx (sgn(\omega)-i)\frac{R}{b^3}\sqrt{\frac{2\rho}{\varepsilon_0|\omega|}}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$h = C = 2\pi R$$

$$\mu \approx \mu_0$$

- … To be "plugged in" in Sacherer's formulas … (see coming slides)

Resonance Impedances

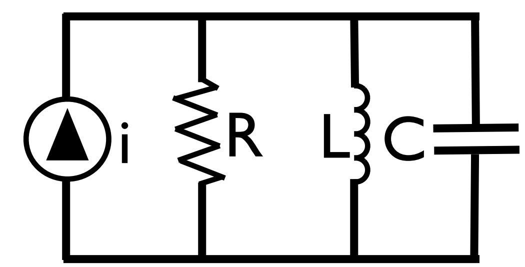
$\omega < \omega_c$

- Wake Fields trapped in cavities or discontinuities in the vacuum chamber cause Resonance Impedances

- Resonance Impedances consist of a real (resistive) part and a imaginary (reactive) part:

$$Z = Z_{Re} + iZ_{Im}$$

→ We see an analogy between Resonance Wake Fields and Electronic Circuits



→ The Impedance of "high order modes" Wakes can be modeled with the RLC circuit

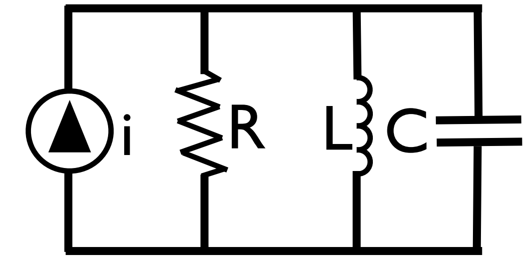
Y.H. Chin, Impedance and Wake Fields

$$\rightarrow \quad Z_{||}(\omega) = \frac{R_{||}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \quad , \quad Z_{\perp}(\omega) = \frac{R_{\perp}\frac{\omega_r}{\omega}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$

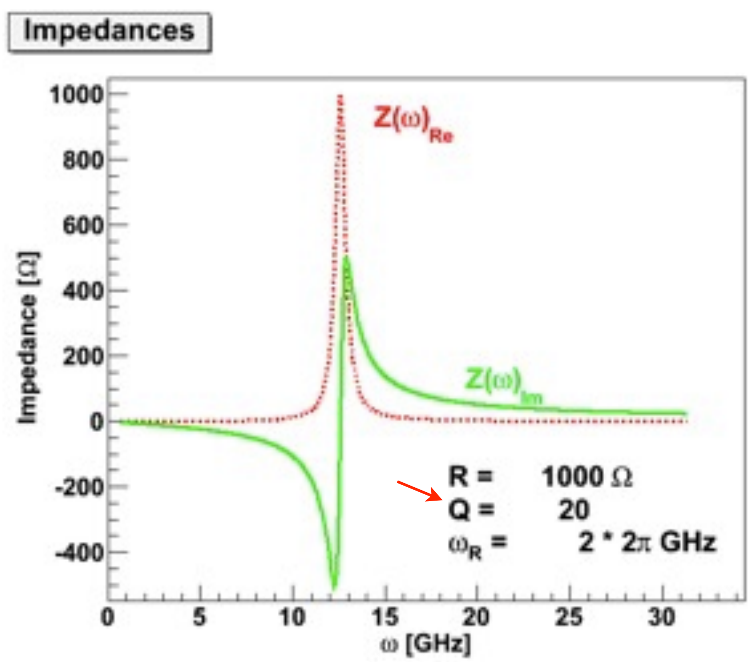
where $Q = R\sqrt{C/L}$ is the "Quality Factor" and $\omega_r = 1/\sqrt{LC}$ is the characteristic frequency for the RLC circuit, or for the pipe it is the "Characteristic Frequency" for the structure causing the Wake Field and $R_{||}$ and $R_{\perp}$ are the "Shunt Impedances"
Take the Inverse FT to get the Wake Fields →

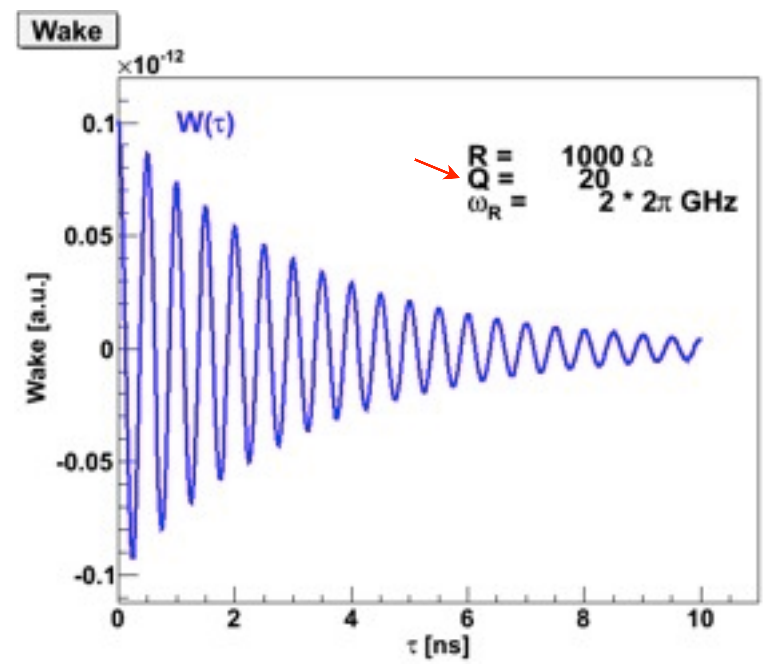
$$W_{||}(\tau) = \frac{e^{-\omega_r \tau/2Q}}{C}\left[\cos\left(\omega_r \tau \sqrt{1 - 1/(4Q^2)}\right) - \frac{1}{\sqrt{4Q^2}}\sin\left(\omega_r \tau \sqrt{1 - 1/(4Q^2)}\right)\right], \quad \tau > 0, \quad = 0 \quad \tau < 0$$

Narrow & Broad Band



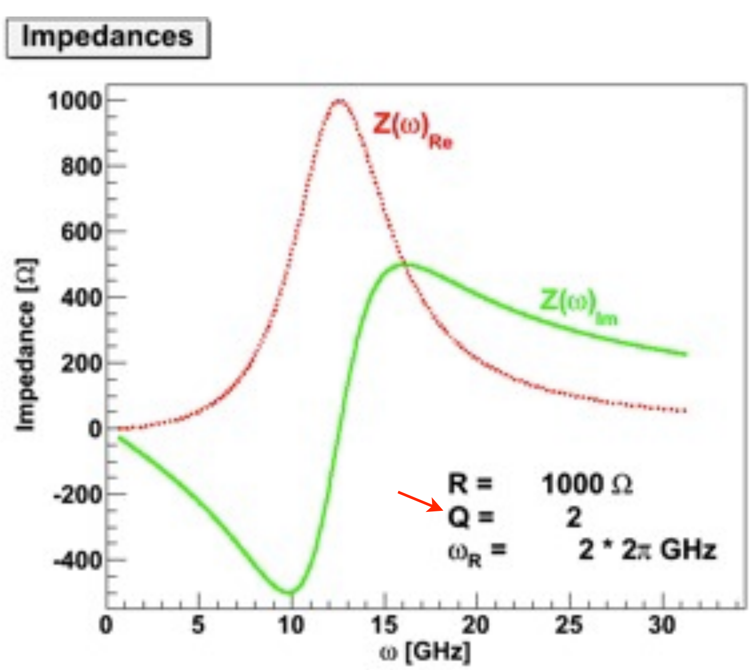
- From the RLC circuit model we see the behavior of the resonant wake fields and the real and imaginary part of the impedance in the case of high quality factor; Narrow Band

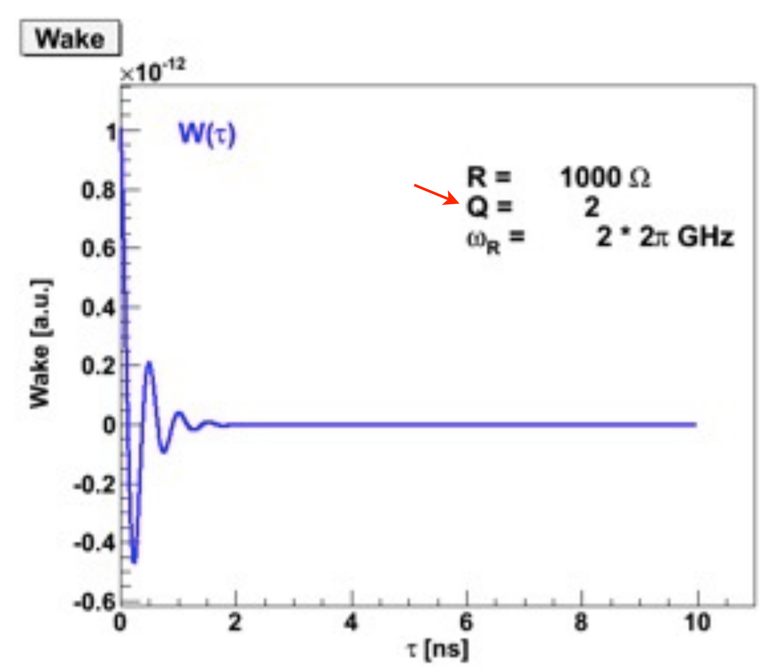




High Q → Narrow Band →
Long Lasting Wake Field →
Multi Bunch Instabilities

and in the case of low quality factor; Broad Band





Low Q → Broad Band →
Short Lasting Wake Field →
Single Bunch Instabilities