Study of the production of superheavy nuclei for S^3 on SPIRAL2

M2-PSA internship defense

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GANIL and SPIRAL2

Grand Accélérateur National d'Ions Lourds



- Research center in Caen established in 1983 by the CNRS and CEA.
- Nuclear physics, astrophysics, atomic physics, material sciences, radiobiology.
- Original complex: 5 cyclotrons to accelerate ${}^{12}C {}^{238}U$ beams.
- SPIRAL2 project: new linear accelerator to accelerate ¹H ²³⁸U beams at intensities 10 times larger than the original complex.

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Superheavy elements

- Definition may vary but generally $Z \ge 104$
- Existence only possible through shell effects



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Island of stability (next magic numbers after ²⁰⁸Pb)

M. Bender, W. Nazarewicz and P.-G. Reinhard, Phys. Lett. B, 515:42, 2001.

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The fusion-evaporation mechanism

Main mode of production of superheavy elements: fusion-evaporation reactions FUSION-QUASIFISSION FISSION Projectile Target CAPTURE FORMATION EVAPORATION GAMMAα **⊮**e-EMISSION

FUSION

. DEEXCITATION

- CN: compound nucleus
- ER: evaporation residue

Spectroscopy of superheavy elements

Objective: study the nuclear structure features of superheavy elements. Two types of spectroscopy:



The Super Separator Spectrometer (S^3)



F. Déchéry et al., Nucl. Instr. Meth. B, 376:125, 2016.

- Separator for fusion-evaporation with very low cross-sections
- Uses the high intensity beams from SPIRAL2

• Synthesis of nuclei in the superheavy and *N* = *Z* regions, plasma physics

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• Due in 2024

Motivation of the study

Excitation function

ER production cross-section as a function of E^* (excitation energy of the CN), E_{cm} (incident energy in the center-of-mass frame) or E_{lab} (incident energy in the laboratory frame).

- S³: we want to obtain the best possible ER production rates.
- It is necessary to select the beam energy so as to correspond to the maximum of the excitation function.
- We need to be able to simulate fusion-evaporation reactions → KEWPIE2 code (2015).



E. D. Donets and V. A. Shchegolev and V. A. Ermakov, Soviet Journal of Nuclear Physics, 2:723, 1966.

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Objective

Study the modeling of the fusion-evaporation mechanism with KEWPIE2 and draw information that will guide the future use of the code in preparing the experiments planned on S^3 , by comparing calculation results to experimental measurements.

General framework of KEWPIE2

- Code simulating the fusion-evaporation mechanism to obtain excitation functions for ER production.
- Cross-sections are calculated with :

$$\sigma_{ER}(E_{cm}) = \frac{\pi\hbar^2}{2\mu E_{cm}} \sum_{\ell \ge 0} (2\ell+1) P_{fus}(E_{cm},\ell) P_{surv}(E^*,\ell)$$

- The fusion and deexcitation phases can be considered independent of each other (Bohr independence hypothesis).
- Two fusion models are available in KEWPIE2 and it is also possible to directly specify fusion cross-sections.

General framework of KEWPIE2 — deexcitation

Dynamical cascade :



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Comparison of KEWPIE2 results with experimental data

We start by studying reactions using a ⁴⁰Ar projectile. A number of experimental excitation functions have been published in *D. Vermeulen et al., Z. Phys. A,* 318:157, 1984.



There is a large difference (up to several orders of magnitude) bewteen the KEWPIE2 results and the experimental measurements.

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Such a large discrepancy could result from a bad estimation of the fission barriers, which is a parameter the cross-sections are very sensitive to.

$$B_f = B_{LDM} - \Delta E_{sh}$$

$$\Delta E_{sh} = f \cdot \Delta E_{MN}$$

- The liquid drop fission barrier B_{LDM} can be calculated with 2 different modeles : Thomas-Fermi (TF) model (default) and Lublin-Strasbourg Drop (LSD) model.
- The shell-correction factor *f* can be adjusted (between 0 and 1).

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Other parameters of interest — fission

The fission decay width Γ_f^{BW} calculated with the Bohr-Wheeler model can be multiplied by a correction factor (Kramers-Strutinsky factor) to account for viscosity:

$$\Gamma_f = K \cdot S \cdot \Gamma_f^{BW}$$

$$K = \sqrt{1 + \left(\frac{\beta}{2\omega_{sd}}\right)^2 - \frac{\beta}{2\omega_{sd}}}$$
$$S = \frac{\hbar\omega_{gs}}{T_{gs}}$$

 β : reduced friction coefficient (5 zs⁻¹ by default) — free parameter

Other parameters of interest — level density

- The nuclear level density plays an important role in the calculation of the various decay widths.
- Level-density parameter a.
- Four different expressions for a: Fermi gas model (a₀), Tōke and Świątecki (a₁), Reisdorf (a₂), Nerlo-Pomorska et al. (a₃).
- Ignatyuk's prescription:

$$a_{gs}(E^*) = a \left[1 + (1 - e^{-E^*/E_d}) rac{\Delta E_{sh}}{E^*}
ight]$$

• E_d : shell-damping energy (19 MeV by default) — free parameter

χ^2 analysis

Procedure

To adjust the model we proceed to do a χ^2 analysis. The χ^2 value is to be compared to the number of degrees of freedom, equal here to the number of experimental values *N*. We can define a reduced χ^2 that is to be compared to 1 : $\chi_r^2 = \frac{\chi^2}{N}$.

KEWPIE2 configuration	χ^2_r	RMSE (mb)
Default	8089877.74	63.960
f = 0, 5	697661.67	25.130
f = 0, 1	35251.76	4.024
LSD	1525.57	2.411
$LSD + a_2$	1393.55	2.401
$LSD + a_2 + \beta = 4 \ zs^{-1}$	619.75	1.713
$LSD + a_2 + \beta = 3 \ zs^{-1}$	1353.48	2.310
$LSD + a_2 + E_d = 16 \text{ MeV}$	598.32	1.634
$LSD + a_2 + E_d = 15 \text{ MeV}$	517.25	1.576
$LSD + a_2 + E_d = 14 \; MeV$	520.48	1.642

 $^{40}Ar + {}^{165}Ho$

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Results for 40 Ar + 165 Ho and 40 Ar + 169 Tm



"LSD + a_2 + E_d = 15 MeV" configuration

The amplitudes and positions of the maxima of the excitation functions are indeed well reproduced. However, the 3n / 2n + 3n channels appear to still be underestimated.

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Results for ${}^{40}Ar + {}^{171}Yb$ and ${}^{40}Ar + {}^{174}Yb$



"LSD + a_2 + E_d = 15 MeV" configuration

The positions of the maxima are well reproduced, and so are the orders of magnitude except for the 2n+3n channel as before, as well as for the 6n + 7n channels.

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Taking γ emission into account

The 2n / 2n+3n channels may be underestimated because until now γ emission, which would compete with fission, has not been taken into account. We can do so in KEWPIE2 with the SMLO model.

Configuration KEWPIE2		χ_r^2	RMSE (mb)
LSD	3n	1263.79	2.579
	4n	877.88	1.648
	5n	1873.70	3.676
	6n	2171.63	0.638
	Total	1525.56	2.411
$LSD + a_2 + E_d = 15 \; MeV$	3n	1170.87	2.482
	4n	687.23	1.347
	5n	293.10	1.861
	6n	156.10	0.196
	Total	517.25	1.576
LSD + SMLO	3n	360.21	1.38
	4n	259.21	1.628
	5n	682.72	2.441
	6n	1221.88	0.520
	Total	623.85	1.709
$LSD + SMLO + E_d = 20 \text{ MeV}$	3n	293.70	1.248
	4n	486.70	2.227
	5n	534.47	2.045
	6n	981.15	0.476
	Total	590.50	1.768

 $^{40}Ar + {}^{165}Ho$

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Results for $^{40}\mathrm{Ar}$ + $^{165}\mathrm{Ho}$ and $^{40}\mathrm{Ar}$ + $^{169}\mathrm{Tm}$

"LSD + SMLO" configuration



The amplitudes for the 2n / 2n+3n channels are no longer underestimated in a substantial manner.

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Results for ${}^{40}Ar + {}^{208}Pb$

We now consider a significantly heavier target (²⁰⁸Pb) and compare the KEWPIE2 results with experimental values from *D. Ackermann, Diplomarbeit, Technische Hochschule Darmstadt, 1989*, as well as results from the HIVAP code.



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Results for ${}^{18}O + {}^{238}U$

We now change both projectile and target and compare the KEWPIE2 results with experimental values from *E. D. Donets, V. A. Shchegolev, V. A. Ermakov, Soviet Journal of Nuclear Physics, 2:723, 1966*, as well as results from the HIVAP code.



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Conclusion

- We could successfully reproduce the positions of the maxima and the orders of magnitude of excitation functions for reactions with a ⁴⁰Ar projectile and a range of targets. However the 6n channel was sometimes underestimated.
- The results were not as good when changing target and projectile ($^{18}O + ^{238}U$ reaction).
- Taking γ emission into account and using the LSD fission barriers generally lead to better results.
- There is still much to do, the next step is to simulate reactions with various mass numbers to get a better global picture of KEWPIE2's ability to model fusion-evaporation reactions.
- Globally the simulation works relatively well, but the extrapolation power is low since we need to adjust for different reactions.

Thank you for your attention!

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The SIRIUS detection system

Spectroscopy and Identification or Rare Isotopes Using S³ (detection system for decay spectroscopy after separation)

- Implantation detector: 10 × 10 cm² DSSD (*Double-sided Silicon Strip Detector*)
- 4 10×10 cm² Si tunnel detectors
- 5 Ge clover detectors (EXOGAM)



Currently: only the DSSD and 1 tunnel detector

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Thomas-Fermi (TF) model (fission barriers)

$$B_{LDM} = P \cdot F(X)$$

$$F(X) = \begin{cases} 0.595553 - 0.124136(X - X_1) & \text{for } 30 \le X \le X_1 \\ 1.99749 \cdot 10^{-4}(X_0 - X)^3 & \text{for } X_1 \le X \le X_0 \end{cases}$$

$$X = \frac{Z_C^2}{A_C(1 - k_s I_c^2)}$$

$$X_0 = 48.5428$$

$$X_1 = 34.15$$

$$P = A_C^{2/3} (1 - k_s I_c^2)$$

$$k_s = 1.9 + (Z_c - 80)/75$$

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Lublin-Strasbourg Drop (LSD) model (fission barriers)

$$B_{LDM} = B_{max} \exp\left[-\left(\frac{I_C - I_0}{\Delta_I}\right)^2\right]$$

$$B_{max} = a_0 + a_1 Z_C + a_2 Z_C^2 10^{-2} + a_3 Z_C^3 10^{-4}$$

$$I_C = (A_C - 2Z_C)/A_C$$

$$I_0 = a_4 + a_5 Z_C 10^{-4}$$

$$\Delta_I = a_6 + a_7 Z_C 10^{-2} + a_8 Z_C^2 10^{-4}$$

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Level-density parameter (1)

• Tōke and Świątecki:

$$a_{1} = \frac{A}{14.61} \left(1 + 3.114 \frac{B_{s}}{A^{1/3}} + 5.626 \frac{B_{k}}{A^{2/3}} \right) \left(1 - \frac{I^{2}}{9} \right)$$

• Reisdorf:

$$a_{2} = A\left(0.04543r_{0}^{3} + 0.1355r_{0}^{2}\frac{\mathcal{B}_{s}}{\mathcal{A}^{1/3}} + 0.1426r_{0}\frac{\mathcal{B}_{k}}{\mathcal{A}^{2/3}}\right)$$

• Nerlo-Pomorska et al.:

$$a_3 = 0.092A + 0.036A^{2/3}\mathcal{B}_s + 0.275A^{1/3}\mathcal{B}_k - 0.00146\frac{Z^2}{A^{1/3}}\mathcal{B}_c$$

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Level-density parameter (2)

• Surface term:

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$$\mathcal{B}_{s} = 1 + \frac{2}{5}\alpha_{2}^{2} - \frac{4}{105}\alpha_{2}^{3} - \frac{66}{175}\alpha_{2}^{4}$$

• Curvature term:
$$\mathcal{B}_k = 1 + \frac{2}{5}\alpha_2^2 + \frac{16}{105}\alpha_2^3 - \frac{82}{175}\alpha_2^4$$

• Coulomb term:
$$\mathcal{B}_c = 1 - \frac{1}{5}\alpha_2^2 - \frac{4}{105}\alpha_2^3 + \frac{51}{245}\alpha_2^4$$

Ground state:

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$$\alpha_2 = \sqrt{\frac{5}{4\pi}}\beta_2$$

Saddle point:

$$\alpha_2 = \frac{7}{3}y - \frac{938}{765}y^2 + 9.499768y^3 - 8.050944y^4$$

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Empirical Barrier Distribution (EBD) model (fusion)

$$\sigma_{cap} = \int_0^{E_{cm}} \sigma_{cap}(E_{cm}, B) D(B) \, \mathrm{d}B$$
$$\sigma_{cap}(E_{cm}, B) = \pi R^2 \left(1 - \frac{B}{E_{cm}}\right)$$
$$D(B) = \frac{1}{\sqrt{2\pi}w} \exp\left[-\frac{(B - B_0)^2}{2w^2}\right]$$

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Wentzel-Kramers-Brillouin (WKB) approximation (fusion)

$$P_{cap}(E_{cm}, J_C) = \frac{1}{1 + \exp(2\Omega)}$$
$$\Omega = \int_{r_{int}}^{r_{out}} dr \sqrt{\frac{2\mu}{\hbar^2} [V(r) - E_{cm}]}$$
$$V(r) = V_N(r) + V_{coul}(r) + V_{cent}(r)$$

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