

Study of an effective extension of the Standard Model and sensitivities to its most promising experimental signatures for the FCC-ee

Master 2 Subatomic Physic and Astroparticles - Internship

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Introduction

- Standard Model (of particle physics): theoretical framework which describes the interaction of elementary particles.
- Conceptual problems with the Standard Model: no gravity description, hierarchy problem, neutrinos mass, **strong CP violation problem**,...
- A solution to the latter problem: Peccei-Quinn mechanism. But, it leads to the existence of a new particle: the axion.
- The axion is searched at particle colliders, such as the current LHC or the future FCC-ee.

Our work in 2 parts:

- Theoretical study of a new model which extends the Standard Model by including an axion-like particle.
- Choice of a relevant experimental signature relative to the axion and study the sensibility of the detector IDEA @ FCC-ee to this signature.

Research group: PICSEL @ IPHC (R&D in CMOS sensors for FCC-ee and estimation of their impact on the physics performance).

Introduction

- 1 Theoretical context
- 2 Experimental context
- 3 Phenomenological study of an extension of the Standard Model
- 4 Simulation of the detector response with respect to a long-lived axion signature
- 5 Conclusion and perspective
- 6 References

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- 5 Conclusion and perspective
- 6 References

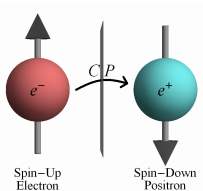
CP violation discovery in the weak interaction

Discrete transformations in Standard Model:

- C: charge conjugation (electrical charge inversion)
- P: parity (space inversion)
- T: time reversal
- and all possible combinations: CP, TC, TP, CPT

Initial assumption: the theory is invariant under these transformations but:

- C and P symmetries are violated in the weak sector (Lee and Yang's predictions, then Wu's measurement in 1956).
- CP symmetry (= particle/anti-particle symmetry) is also violated in the weak sector (kaons system in 1964).



Strong CP violation

Sources of CP violation in the Standard Model:

- In the weak sector: complex phase in the CKM matrix.
- There is another source in the strong sector: the θ -term.

What is the θ -term in QCD(Quantum ChromoDynamics)?

- Propagation of the gluons are traditionally described by the kinetic term:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu\nu}^A G^{\mu\nu}_A$$

$$\text{with: } G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + ig_s f_{BC}^A G_\mu^B G_\nu^C$$

f_{BC}^A are constant structure of SU(3) Lie group, A,B,C are colors indices.

- Lorentz and gauge invariance + renormalizability conditions authorize to include another term called θ -term that violates the CP symmetry:

$$\mathcal{L}_{\theta QCD} = -\frac{\theta_{QCD}}{4} G_{\mu\nu}^A \tilde{G}^{\mu\nu}_A \text{ with } \tilde{G}^{\mu\nu}_A = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma A}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the anti-symmetric tensor.

Strong CP violation

The term can be re-shaped as a surface term :

$$\mathcal{L}_{\theta_{QCD}} = -\frac{\theta_{QCD}}{2} \epsilon^{\mu\nu\rho\sigma} \partial_\mu (G_\nu^A \partial_\rho G_\sigma^A - \frac{2}{3} i G_\nu^A G_\rho^B G_\sigma^C)$$

But it does not vanish in QCD.

It generates non-perturbative effects such as a electric dipole moment for the neutron which is proportional to θ_{QCD} .

Experimental limits on the neutron electric dipole moment provide a limit on θ_{QCD} :

$$|\theta_{QCD}| < 5.6 \times 10^{-11}$$

Why so close to 0? Strong CP problem

Peccei-Quinn mechanism and introduction of the axion



Fig 2: Roberto Peccei (1942-2020) and Helen Quinn (1943-Present)

- Peccei-Quinn mechanism in 1977: imposing a new $U(1)$ symmetry which is spontaneously broken at the energy scale Λ .
- If this continuous symmetry were exact, then Goldstone theorem: existence of a massless scalar particle (Goldstone boson).
- As this symmetry is approximate (anomalous breaking due to the θ -term), existence of a scalar particle with a small mass (pseudo-Goldstone boson). It is named axion and noted a .

Axion properties

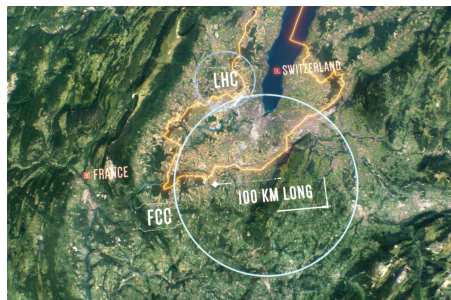
- **Mass:** $m_a \propto \frac{1}{\Lambda}$
The more the broken scale is high, the more the axion is light.
Traditional range scrutinized for QCD axions: few MeV at maximum.
- **Charge:** electrically neutral, no coloured.
- **Coupling with standard particles:** if Λ scale is high, the coupling will be low.
 - The axion can be long-lived and be "invisible" for particle detectors.
 - Good candidate for dark matter?

Axion-like particles can appear in other theoretical SM extensions.
→ Motivations for studying a unique effective model.

- 1 Theoretical context
- 2 Experimental context
- 3 Phenomenological study of an extension of the Standard Model
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FCC-ee

- Project of a future e^+e^- collider at CERN.
- 100km of circumference.
- Currently at feasibility study phase.
- Planned operation between 2040 and 2055.



Energies targeted:

type of events	Z peak	W^+W^-	hZ	$t\bar{t}$
\sqrt{s}	91 GeV	160 GeV	240 GeV	360 GeV

IDEA detector

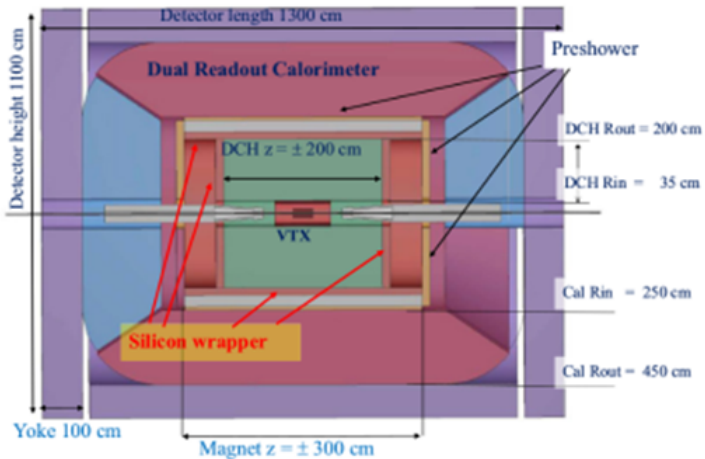


Fig 3: IDEA detector design

- 1 Theoretical context
- 2 Experimental context
- 3 Phenomenological study of an extension of the Standard Model**
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Model

- Effective theory : theory with a little number of parameters that characterized the studied system for a given energy scale Λ , non-renormalizable.
- Our model is composed by all the possible interactions terms with dimension 5 operators.
- Lagrangian density:

$$\begin{aligned}
 \mathcal{L}_{eff} &= \mathcal{L}_{SM} \\
 &+ \frac{1}{2}(\partial^\mu a)(\partial_\mu a) - \frac{m_a^2}{2}a^2 \\
 &+ g_s^2 \frac{C_{GG}}{\Lambda} a G_{\mu\nu}^A \tilde{G}_{\mu\nu}^A + g^2 \frac{C_{WW}}{\Lambda} a W_{\mu\nu}^i \tilde{W}_{\mu\nu}^i + g'^2 \frac{C_{BB}}{\Lambda} a B_{\mu\nu} \tilde{B}_{\mu\nu} \\
 &+ \frac{C_{a\Phi}}{\Lambda} [ia(i\bar{Q}_L Y_U \sigma^2 \Phi^* u_R - \bar{Q}_L Y_D \Phi d_R - \bar{L}_L Y_E \Phi e_R) + h.c.]
 \end{aligned}$$

Interactions

We would like to know the interactions between the axion and the other particles, and the corresponding Feynman rules.

Our work was to rewrite the Lagrangian density by:

- specifying the mass states instead of the gauge states for the bosons.
- breaking spontaneously the electroweak symmetry.

$$B_\mu = c_\omega A_\mu - s_\omega Z_\mu, \quad W_\mu^3 = s_\omega A_\mu + c_\omega Z_\mu,$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2),$$

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

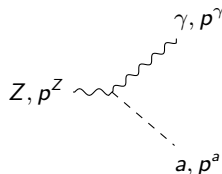
A^μ , Z^μ and W_μ^\pm are the vectorial fields for the photon, the Z boson W^+ and W^- .

Interactions

Interactions between the axion and the other bosons:

3 bosons interactions	4 bosons interactions	5 bosons interactions
agg	$aggg$	$aW^+W^-\gamma\gamma$
$a\gamma\gamma$	$aW^+W^-\gamma$	$aW^+W^-\gamma Z$
aZZ	aW^+W^-Z	aW^+W^-ZZ
$aZ\gamma$		
aW^+W^-		

Example of a Feynman rule for the $aZ\gamma$ interaction vertex:



$$\text{Factor : } -i8 \frac{e^2}{s_w c_w} \frac{C_{\gamma Z}}{\Lambda} p_\mu^a p_\rho^Z \epsilon^{\mu\nu\rho\sigma}$$

$$\text{with } C_{\gamma Z} = c_w^2 C_{WW} - s_w^2 C_{BB}$$

Production at FCC-ee

Production at FCC-ee with $\sqrt{s} = 91$ GeV (Z peak):
3 possible diagrams at LO (Leading-Order) of the theory.



Involving 3 coupling constants: $C_{\gamma\gamma}/\Lambda$, $C_{\gamma Z}/\Lambda$ and C_{ZZ}/Λ .

Considering a same value for the C/Λ couplings, the first diagram is dominant because the Z is on-shell.

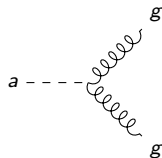
Axion decay modes

The decay modes of the axion (at LO) depend on its mass:

Decay mode	gg	ggg	$\gamma\gamma$	$Z\gamma$	ZZ	W^+W^-
m_a (GeV)	any	any	any	> 91	> 182	> 160

Decay mode	$W^+W^-\gamma$	$W^+W^-\gamma\gamma$	W^+W^-Z	$W^+W^-Z\gamma$	W^+W^-ZZ
m_a (GeV)	> 160	> 160	> 251	> 251	> 342

Partial decay widths (at LO) have been derived. For example: $a \rightarrow gg$

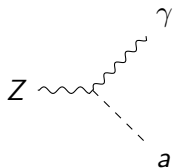


$$\Gamma(a \rightarrow gg) = 32\pi\alpha_s^2 m_a^3 \frac{|C_{gg}|^2}{\Lambda^2}$$

Axion in decay mode of MS particles

Axions can appear also in exotic decays mode of MS particles.

For instance, decay mode $Z \rightarrow a\gamma$

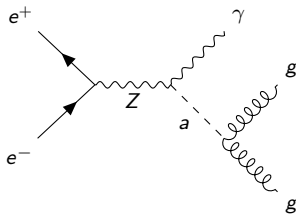


$$\Gamma(Z \rightarrow a\gamma) = \frac{e^4(m_Z^2 - m_a^2)^3}{6\pi s_\omega^2 c_\omega^2 m_Z^3} \left| \frac{C_{Z\gamma}}{\Lambda} \right|^2$$

- 1 Theoretical context
- 2 Experimental context
- 3 Phenomenological study of an extension of the Standard Model
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- 6 References

Choice of an experimental signature for the FCC-ee

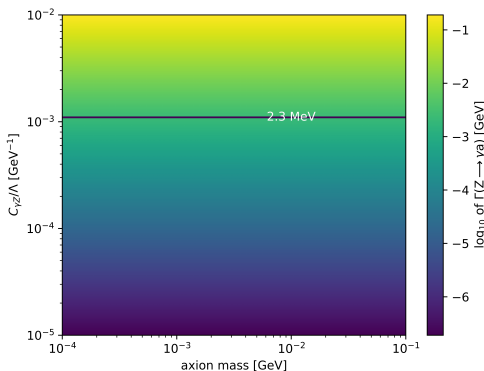
- FCC-ee energy: Z production on mass shell.
- The Z boson can decay in axion-photon according to the coupling $C_{\gamma Z}/\Lambda$.
- We choose the decay mode of the axion in two gluons according to the coupling C_{gg}/Λ .
 - The decay mode of the axion in two photons has been already investigated in the FCC-ee community.
 - The 2 gluons will give two jets in the final state. The tracks of the charged particle contained in the jets can allow us to reconstruct the axion decay vertex.
- We would like a long-lived axion decaying in the detector volume.



Study of the signature

A first constraint on this process is the uncertainty on the experimentally-measured Z total-width which is known at 2.3 MeV.

Partial decay width of the Z boson in axion and photon :



Study of the signature

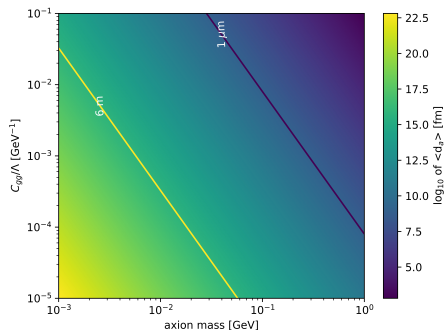
We would like to reconstruct the axion decay vertex from the tracks coming the 2 jets.
 → Instrumental constraint on the flight distance of the axion: IDEA radius ≈ 6 m,
 vertex-detector resolution $\approx \mu\text{m}$.

The mean flight distance of the axion $\langle d_a \rangle$ in the detector frame can be computed from the decay width of the axion in gluon pair (considering BR=100%):

$$\langle d_a \rangle = \gamma \beta c \tau$$

$$\text{with } \tau = \frac{\hbar}{\Gamma_{a \rightarrow gg}}$$

$$\text{and } \beta \gamma = \frac{1}{2} \left(\frac{m_Z}{m_a} + \frac{m_a}{m_Z} \right)$$



Generation and simulation

We generate events corresponding to the chosen signature:

- The Lagrangian density was already implemented in the program `FEYNRULES`.
- Using the programs `MADGRAPH_AMC@NLO 5` and `PYTHIA 8` to generate the Monte-Carlo events.
- Using the program `DELPHES` to simulate the response of the detector.
- No simulation of the beam noise (program `GUINEAPIG`).

Benchmarks definition:

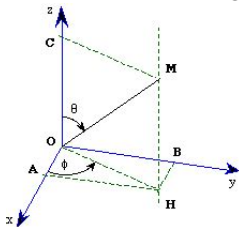
- axion mass of 1 MeV, 3 MeV, 20 MeV.
- axion flight distance of 1 μm , 1 cm, 1 m, 5 m.

→ Producing 10,000 events sample for each benchmark.

First result on the detector response: photon reconstruction

Matching between generated photons coming from the Z decay and the reconstructed photons:

- Matching is achieved by choosing the closest reconstructed photon in the transverse plane.
- In other terms, minimizing the observable $\Delta R = \sqrt{\Delta\varphi^2 + \Delta\eta^2}$.

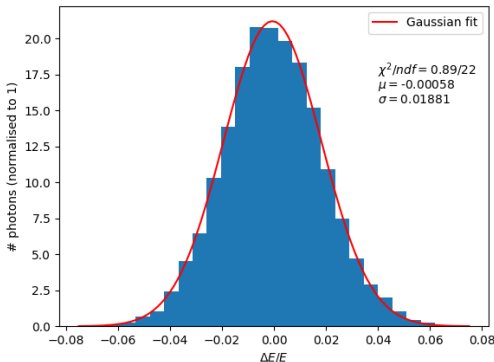


with the pseudo-rapidity $\eta = -\ln \left[\tan \left(\frac{\theta}{2} \right) \right]$

- Cut on ΔR : $\Delta R < 0.05$
- $\frac{\text{Successful associations}}{\text{generated events}} = \text{photon reconstruction efficiency} \times \text{matching efficiency} = 0.99\%$.

First result on the detector response: photon reconstruction

Resolution on the photon energy: $\frac{\Delta E}{E\gamma}$



Estimated resolution on the photon energy: 1.8%,
which is consistent with the calorimeter resolution $\frac{11\%}{\sqrt{E}}$ for the photon.

First result on the detector response: influence of the axion flight distance

Impact of the mean flight distance $\langle d_a \rangle$ on reconstructed observables:

mean flight distance $\langle d_a \rangle$	1 μm	1 m	5 m
ratio of axions decaying in the detector volume	100.0%	99.8%	69.9%
jet multiplicity ($p_T > 20$ GeV)	1.8	1.8	1.8
missing transverse energy [GeV]	1.1	1.1	1.1

Results: the (fast-)simulation does not take care of the flight distance of the axion.

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- 2 Experimental context
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- 6 References

Conclusion and perspective

Exploring the axion physics at the FCC-ee collider

- Understanding the theoretical motivations of the QCD axion: consequence of the solution (Peccei-Quinn mechanism) to the strong CP violation problem.
- Performing a phenomenological study of an effective model extending the Standard Model by including an axion and its interactions:
 - Deriving the practical Feynman rules from the Lagrangian density (expressed with gauge states and before the electroweak symmetry breaking).
 - Investigating how the axion is produced at the FCC-ee in terms of cross-sections.
 - Calculating partial decay width corresponding to the two-body decays of the axion, and to the exotic decays of standard particles.
 - Checking that the results are consistent with the numerical predictions of the program `FEYNRULES/MADGRAPH_AMC@NLO`.

Conclusion and perspective

- Choosing an experimental signature to study: $e^+e^- \rightarrow Z \rightarrow \gamma a (\rightarrow gg)$ where the axion a can have a long flight distance before its decay.
- Estimating the detector IDEA sensibility to this signature:
 - Determining the interesting parameter region.
 - Generating Monte-Carlo events corresponding to the signal by using high-energy physics software.
 - Applying the fast-simulation of the detector IDEA.
 - First results: consistent simulation for the photon reconstruction/identification where as long distance flight are not considered.

Perspective:

- Tuning the detector simulation for handling the flight distance of the axion.
- Finalizing the sensitivity study by generating background events and extracting a signal/background ratio.
- Improving the simulation by including the beam noise (program GUINEAPIG).
- Finally, other signatures can be also studied. Example: $a \rightarrow \gamma\gamma$.

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- 5 Conclusion and perspective
- 6 References

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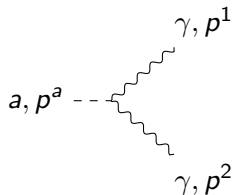
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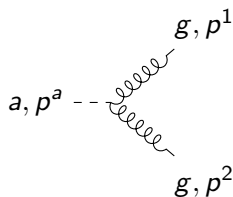
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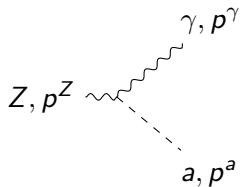
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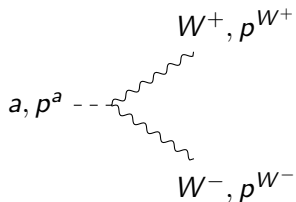
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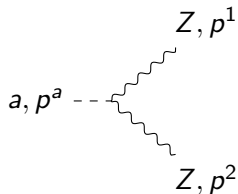
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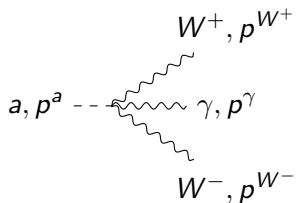
$$\text{Facteur : } -i8 \frac{e^2}{s_\omega c_\omega} \frac{C_{\gamma Z}}{\Lambda} p_\mu^a p_\rho^Z \epsilon^{\mu\nu\rho\sigma}$$



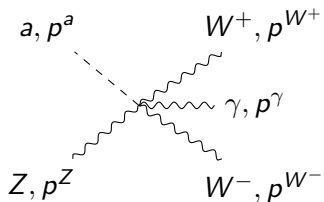
$$\text{Facteur : } -i8 g_W^2 \frac{C_{WW}}{\Lambda} \epsilon^{\mu\nu\rho\sigma}$$



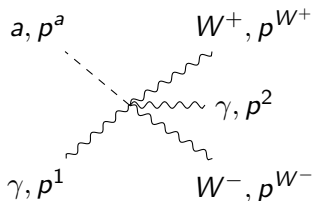
$$\text{Facteur : } -i4 \frac{e^2}{s_w^2 c_w^2} \frac{C_{ZZ}}{\Lambda} \epsilon^{\mu\nu\rho\sigma}$$



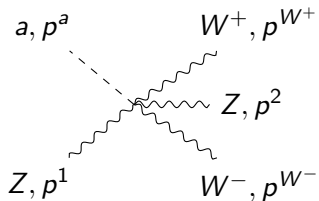
$$\text{Facteur : } -i8 \frac{g_W^3 s_w C_{WW}}{\Lambda} (p_\mu^{W^+} - p_\mu^{W^-} + p_\mu^\gamma) \epsilon^{\mu\nu\rho\sigma}$$



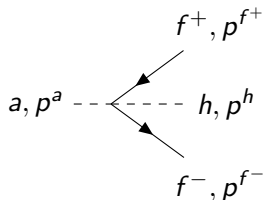
$$\text{Facteur : } -i32g_W^4 s_\omega c_\omega \frac{C_{WW}}{\Lambda} \epsilon^{\mu\nu\rho\sigma}$$



$$\text{Facteur : } -i32g_W^4 s_\omega^2 \frac{C_{WW}}{\Lambda} \epsilon^{\mu\nu\rho\sigma}$$



$$\text{Facteur : } -i32g_W^4 c_\omega^2 \frac{C_{WW}}{\Lambda} \epsilon^{\mu\nu\rho\sigma}$$



$$\text{Facteur : } -i \frac{y_{f^+ f^-} C_{a\Phi}}{f_a}$$

$$\mathcal{L}_{a\gamma\gamma} = 2e^2 \frac{C_{\gamma\gamma}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu \partial_\rho A_\sigma$$

$$\mathcal{L}_{aZZ} = 2 \frac{e^2}{s_w^2 c_w^2} \frac{C_{ZZ}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} \partial_\mu Z_\nu \partial_\rho Z_\sigma$$

$$\mathcal{L}_{aZ\gamma} = 4 \frac{e^2}{s_w c_w} \frac{C_{\gamma Z}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu \partial_\rho Z_\sigma$$

$$\mathcal{L}_{agg} = 2g_s^2 \frac{C_{GG}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} \partial_\mu G_\nu^A \partial_\rho G_\sigma^A$$

$$\mathcal{L}_{aggg} = 2g_s^2 \frac{C_{GG}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} f_{BC}^A [-ig((\partial_\mu G_\nu^A)G_\rho^B G_\sigma^C + G_\mu^B G_\nu^C (\partial_\rho G_\sigma^A))]$$

$$\mathcal{L}_{aW^+W^-} = 2g_W^2 \frac{C_{WW}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} (\partial_\mu W_\nu^+ \partial_\rho W_\sigma^-)$$

$$\mathcal{L}_{aW^+W^-\gamma} = i8g_W^3 s_w \frac{C_{WW}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} [(\partial_\mu W_\nu^+) A_\rho W_\sigma^- - (\partial_\mu W_\nu^-) A_\rho W_\sigma^+ + W_\mu^+ W_\nu^- \partial_\rho A_\sigma]$$

$$\mathcal{L}_{aW^+W^-Z} = i8g_W^3 c_w \frac{C_{WW}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} [\partial_\mu W_\nu^+ Z_\rho W_\sigma^- - (\partial_\mu W_\nu^-) Z_\rho W_\sigma^+ + W_\mu^+ W_\nu^- \partial_\rho Z_\sigma]$$

$$\mathcal{L}_{aW^+W^-Z\gamma} = 32g_W^4 s_w c_w \frac{C_{WW}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} [A_\mu W_\nu^+ Z_\rho W_\sigma^-]$$

$$\mathcal{L}_{aW^+W^-\gamma\gamma} = 32g_W^4 s_w^2 \frac{C_{WW}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} [A_\mu W_\nu^+ A_\rho W_\sigma^-]$$

$$\mathcal{L}_{W^+W^-ZZ} = 32g_W^4 c_w^2 \frac{C_{WW}}{\Lambda} a\epsilon^{\mu\nu\rho\sigma} [Z_\mu W_\nu^+ Z_\rho W_\sigma^-]$$

$a \rightarrow \gamma\gamma$

$$iM = P_\rho^r P_\sigma^s (-4e^2 \frac{C_{\gamma\gamma}}{\Lambda} \epsilon^{\mu\nu\rho\sigma} p_\mu^1 p_\nu^2) \quad (1)$$

Avec P_ρ^r et P_σ^s les vecteurs polarisation où r et s sont les différentes polarisations possible. On nommera par la suite $C = -4e^2 \frac{C_{\gamma\gamma}}{\Lambda}$. On peut donc exprimer $|M|^2$ en multipliant par le complexe conjugué :

$$|M|^2 = |C|^2 \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} P_\rho^r P_{\rho'}^{r*} P_\sigma^s P_{\sigma'}^{s*} p_\mu^1 p_\nu^2 p_{\mu'}^1 p_{\nu'}^2, \quad (2)$$

$$|\bar{M}|^2 = \sum_{r,s} |M|^2. \quad (3)$$

$$\begin{aligned} |\bar{M}|^2 &= \frac{1}{2} |C|^2 \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} (-\eta_{\rho\rho'}) (-\eta_{\sigma\sigma'}) p_\mu^1 p_\nu^2 p_{\mu'}^1 p_{\nu'}^2 \\ &= |C|^2 [\delta^{\mu\nu'} \delta^{\nu\mu'} - \delta^{\mu\mu'} \delta^{\nu\nu'}] p_\mu^1 p_\nu^2 p_{\mu'}^1 p_{\nu'}^2 \\ &= |C|^2 [(p_\mu^1 p^{2\mu})(p_\nu^1 p^{2\nu}) - (p_\mu^1 p^{1\mu})(p_\nu^2 p^{2\nu})]. \end{aligned} \quad (4)$$

Sachant que les particules 1 et 2 sont des photons, $p_\mu^1 p^{1\mu} = 0$ et $p_\nu^2 p^{2\nu} = 0$. De plus, en se plaçant dans le référentiel de l'axion, c'est à dire du centre de masse, et en utilisant la conservation de la quantité de mouvement ainsi que de l'énergie, nous démontrons que $p_\mu^1 p^{2\mu} = \frac{m_a^2}{2}$, avec m_a la masse de l'axion. Ainsi :

$$|\bar{M}|^2 = |C|^2 \frac{m_a^4}{4} . \quad (5)$$

$$\Gamma_{i \rightarrow f} = \frac{p^*}{32\pi^2 m_a^2} \int |\bar{M}|^2 d\Omega \quad (6)$$

$$\text{avec } p^* = \frac{1}{2m_a} \sqrt{[(m_a^2 - (m_1 + m_2)^2)][m_a^2 - (m_1 - m_2)^2]} . \quad (7)$$

$$\Gamma(a \rightarrow \gamma\gamma) = \frac{4\pi\alpha^2 m_a^3}{\Lambda^2} |C_{\gamma\gamma}|^2 \quad \text{avec } \alpha = \frac{e^2}{4\pi} \quad (8)$$

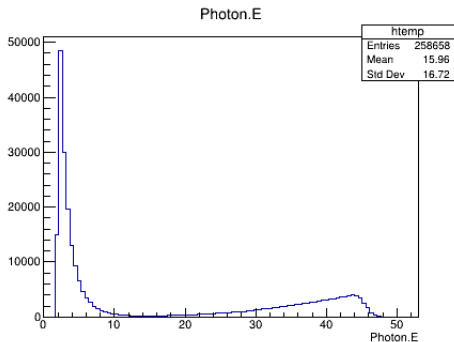


Fig 4: Energy of all the reconstructed photon per Delphes for a 10000 events sample (GeV).

If we consider the Lagrangian density of the QED :

$$\mathcal{L}_{QED} = \bar{\Psi} i D^\mu \gamma_\mu \Psi - m \bar{\Psi} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \text{avec} \quad D^\mu = \partial^\mu - ieQA^\mu \quad (9)$$

We can add terms that are Lorentz en gauge invariant. It is possible to check that the following term own these properties.

$$\mathcal{L}_{\theta_{QED}} = -\frac{\theta_{QED}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad \text{avec} \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} . \quad (10)$$

Where $\epsilon^{\mu\nu\rho\sigma}$ is the anti-symmetric tensor.

We can bring out the CP violation by re-shape the kinetics terms in term of electric and magnetic Field

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(E^2 - B^2), \quad -\frac{\theta_{QED}}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \theta_{QED} \vec{E} \cdot \vec{B} \quad (11)$$

	ρ	\vec{j}	\vec{E}	\vec{B}	$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$	$-\frac{\theta_{QED}}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$
T	+	-	+	-	+	-
C	-	-	-	-	+	+
P	+	-	-	+	+	-
CP	-	+	+	-	+	-
TC	-	+	-	+	+	-
TP	+	+	-	-	+	+
CPT	-	-	+	+	+	+

Table 1: Transformation of several observables of electromagnetism by the discrete transformations C, P and T, as well as by their combinations.

The term θ can be written as a surface term. Indeed, by using the properties of the anti-symmetric tensor, the coupling can be put in the form of a total derivative :

$$\mathcal{L}_\theta = -\frac{\theta}{2}\partial_\rho[\epsilon^{\mu\nu\rho\sigma}A_\sigma\partial_\mu A_\nu] \quad (12)$$

This term does not contribute to the equations of motion of the free field by imposing that the field A_μ be suppressed at infinity. It does not contribute either to the perturbative interactions because these are expressed from the solutions of the free field.

