Theoretical Study of the QCD Phase Diagram at High Densities

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Outline

1. Quantum Chromodynamics
2. Phase Structure of the Quark-Meson-Diquark model
3. Neutron Stars

Quantum Chromodynamics

QCD Particles

	Fermion			Boson
	Up	Charm	Top	Gluon
mass	$2.2\,\mathrm{MeV}$	$1.2\,\mathrm{GeV}$	$170\,\mathrm{GeV}$	
	Down	Strange	Bottom	
mass	$4.7\,\mathrm{MeV}$	$96\,\mathrm{MeV}$	$4.1\,\mathrm{GeV}$	

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Confinement: Can only observe color-neutral particle.

baryon
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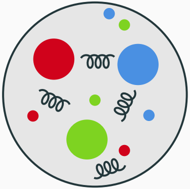

meson
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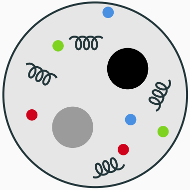

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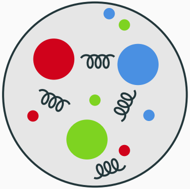

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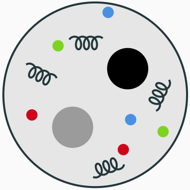
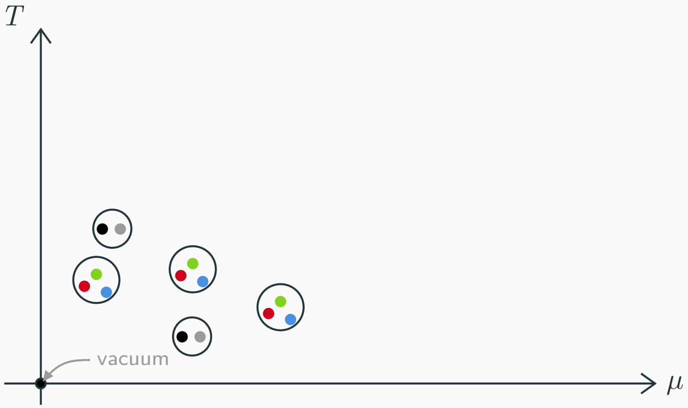


Chiral symmetry breaking: Quarks are much more massive in the vacuum.

QCD Phase Diagram



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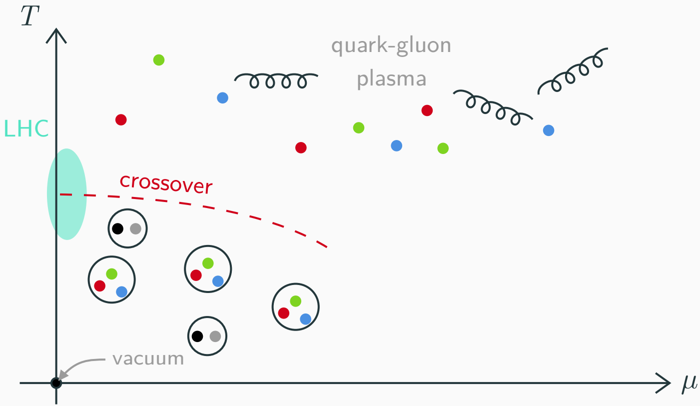

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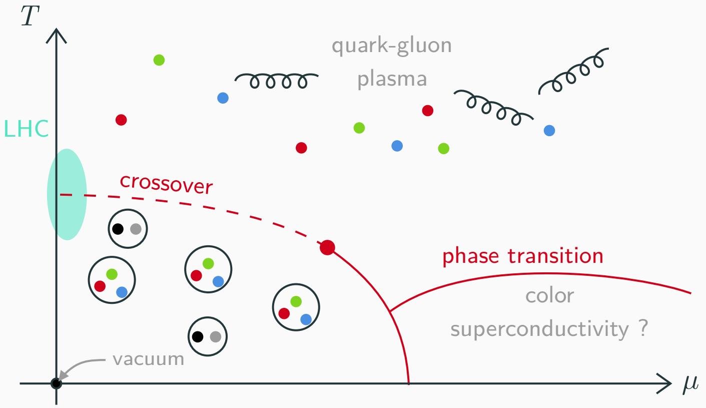

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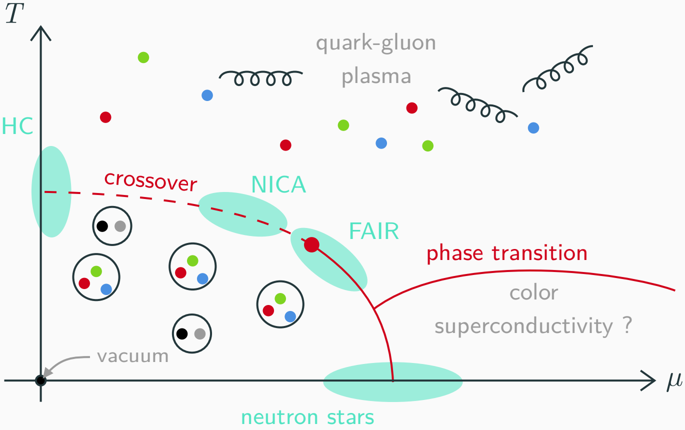
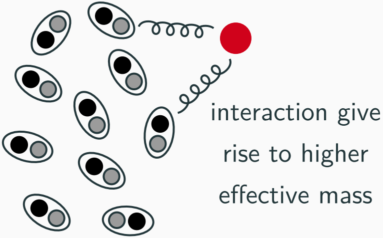


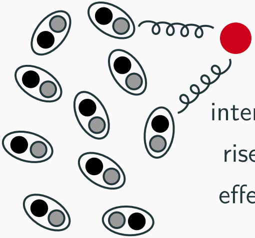
Chiral Condensate σ



$$\langle 0|\,\overline{q}q\,|0\rangle \neq 0$$

Order parameters of the phase diagram

Chiral Condensate σ



interaction give rise to higher effective mass

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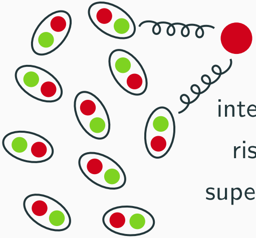
Diquark Condensate Δ



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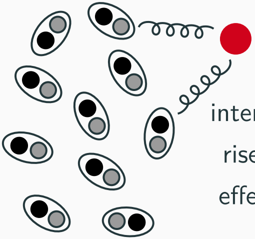

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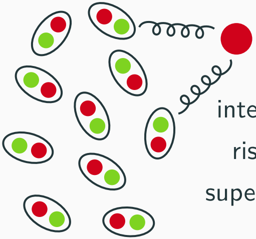

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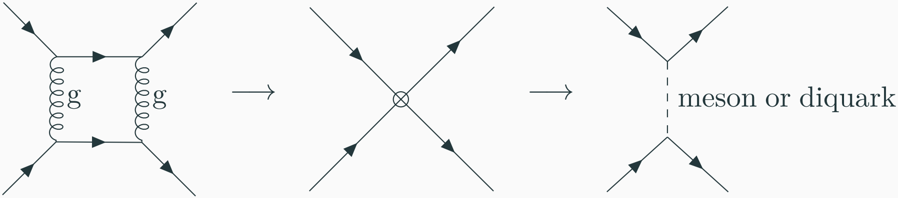
effective model of the order parameters

quark-meson-diquark model

The Quark-Meson-Diquark Model

Main idea: remove gluons from the theory and replace them by mesons and diquarks.



Meson and diquark can develop **vacuum expectation values** $\rightarrow$ order parameter for the phases.

Phase Structure of the Quark-Meson-Diquark model

Mean-field Approximation

Goal: Compute the partition function Z.

1st method: Mean-field Approximation (MFA)

$$Z = \int \mathcal{D}\sigma \mathcal{D}\pi \mathcal{D}\Delta \mathcal{D}\bar{q} \mathcal{D}q \; e^{-\beta(H-\mu N)}$$

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↓

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MFA

↓

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↑ thermodynamic potential

Functional Renormalization Group

2nd method: Functional Renormalization Group (FRG)

- Take into account bosonic fluctuations in a **non-perturbative** way.

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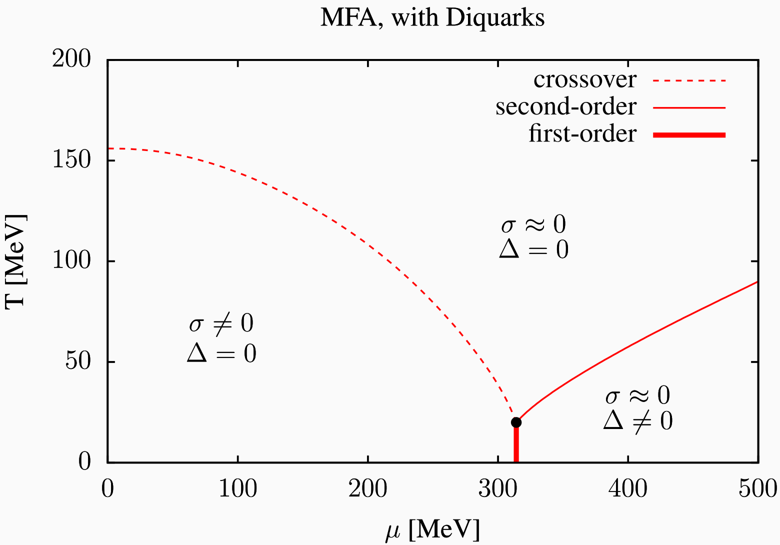
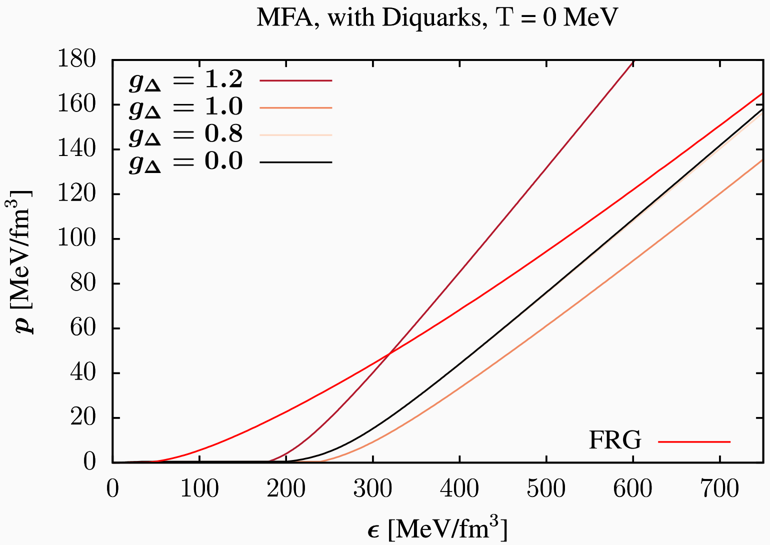
- Take into account bosonic fluctuations in a **non-perturbative** way.
- Integrate fluctuations momentum-shell by momentum-shell.
- Diquark not included yet but give insight into the importance of fluctuation.

Phase Diagram

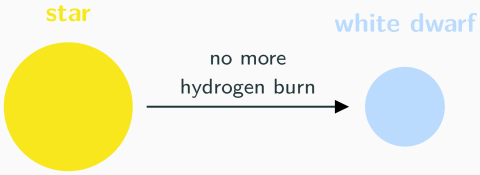


Equation of State



Neutron Stars

Origin of a Neutron Star



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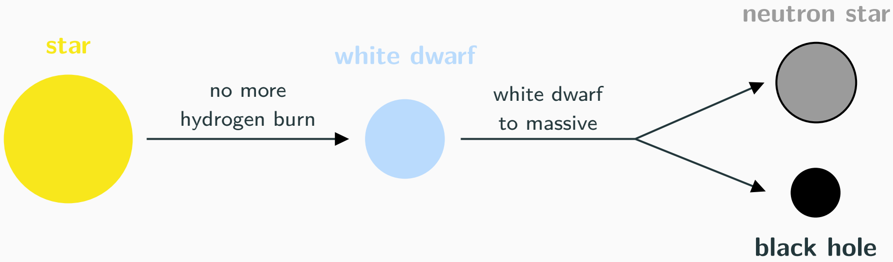

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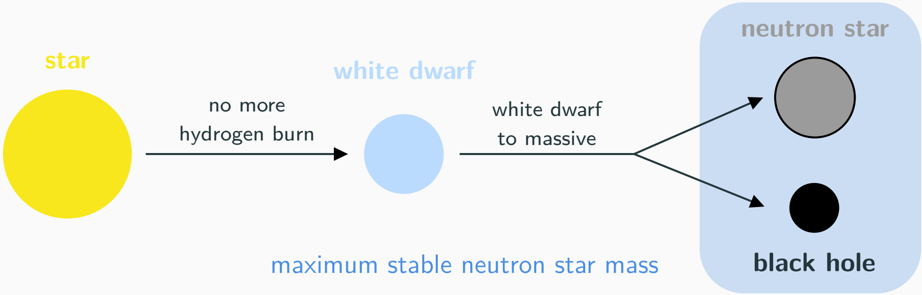
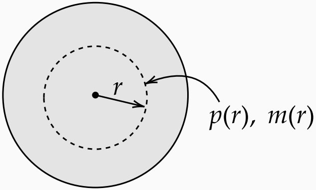


maximum stable neutron star mass
depend on microscopic structure

Tolman-Oppenheimer-Volkoff Equation

neutron star



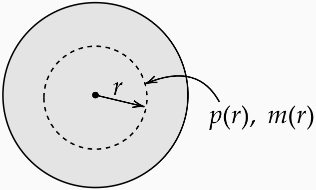
Tolman-Oppenheimer-Volkoff equation

$$\frac{dp}{dr} = -\frac{(\epsilon(r) + p(r))\left[m(r) + 4\pi r^3 p(r)\right]}{r(r - 2Gm(r))}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon(r)$$

Tolman-Oppenheimer-Volkoff Equation

neutron star



need microscopique model

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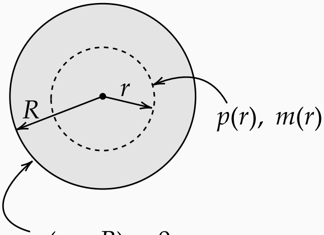

$p(r = R) = 0$
$m(r = R) = M$

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Tolman-Oppenheimer-Volkoff equation

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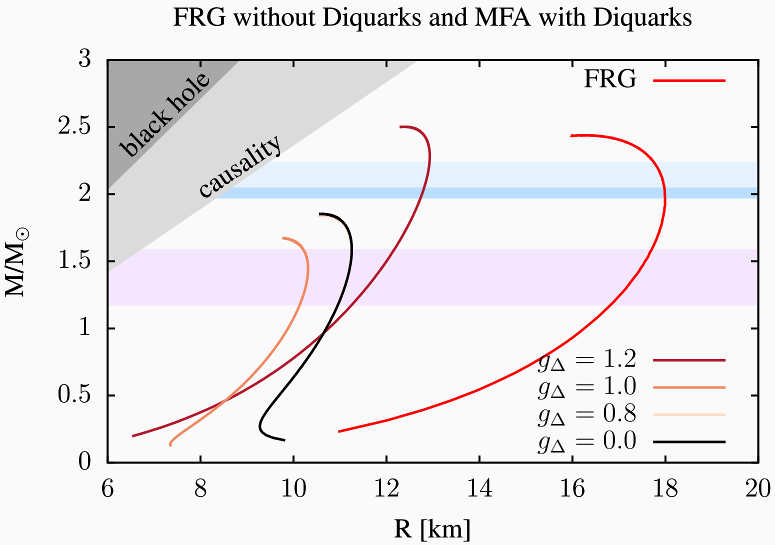
Solve for a variety of central pressure $p(r = 0) = p_0$ to obtain relation between mass and radius.

Mass-radius Relationship



Tidal Deformability

Tidal deformability λ: Ability of a star to deform under an external gravitational field.

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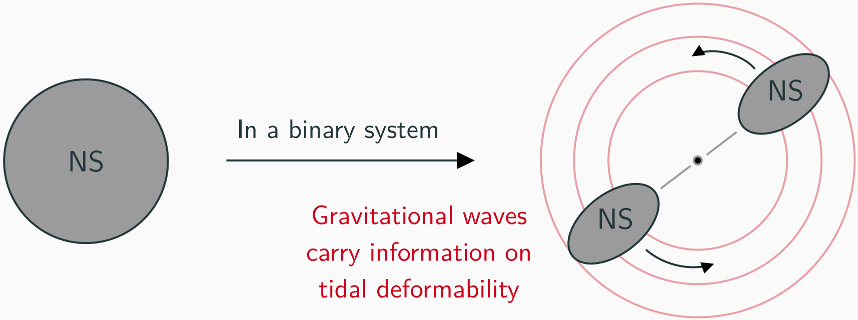
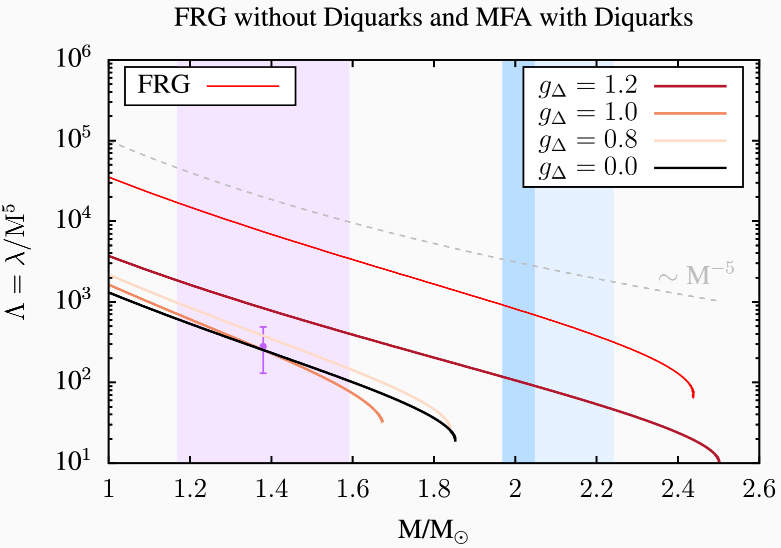
How to measure it:



Tidal Deformability



Summary and Outlook

Summary:

- Phase strucutre of the quark-meson-diquark model and results on neutron star observables.
- Maybe visible effect of diquarks with high coupling?
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Outlook:

- Implement the FRG with diquarks.
- Build hybrid model of neutron star including neutrons.

Thank you!

The Quark-Meson-Diquark Model

Lagrangian of the model:

$$\mathcal{L}_{\mathrm{QMD}} = \overline{q}\Big(-i\partial\!\!\!/ - \mu\gamma^0\Big)q \;+\; g_m\overline{q}(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})q$$

$$+\; ig_{\Delta}\Big(\Delta^*\overline{q}\gamma_5\tau_2\lambda_2 C\overline{q}^T + \Delta q^T C\gamma_5\tau_2\lambda_2 q\Big)$$

$$+\; \frac{1}{2}(\partial_\mu\sigma)\Big(\partial^\mu\sigma\Big) + \frac{1}{2}(\partial_\mu\vec{\pi})\Big(\partial^\mu\vec{\pi}\Big) + (\partial_\mu\Delta)\Big(\partial^\mu\Delta^*\Big) + U(\sigma,\vec{\pi},\Delta)$$

The Quark-Meson-Diquark Model

Lagrangian of the model:

Quark dynamics

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The Quark-Meson-Diquark Model

Lagrangian of the model:

Quark dynamics — Interaction quark-meson

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Interaction quark-diquark

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meson dynamics — diquark dynamics

The Quark-Meson-Diquark Model

Lagrangian of the model:

$$\mathcal{L}_{QMD} = \underbrace{\bar{q}\left(-i\not{\partial} - \mu\gamma^0\right)q}_{\text{Quark dynamics}} + \underbrace{g_m \bar{q}(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau})q}_{\text{Interaction quark-meson}}$$

$$+ \underbrace{ig_\Delta\left(\Delta^*\bar{q}\gamma_5\tau_2\lambda_2 C\bar{q}^T + \Delta q^T C\gamma_5\tau_2\lambda_2 q\right)}_{\text{Interaction quark-diquark}}$$

$$+ \underbrace{\frac{1}{2}(\partial_\mu\sigma)\left(\partial^\mu\sigma\right) + \frac{1}{2}(\partial_\mu\vec{\pi})\left(\partial^\mu\vec{\pi}\right)}_{\text{meson dynamics}} + \underbrace{(\partial_\mu\Delta)\left(\partial^\mu\Delta^*\right)}_{\text{diquark dynamics}} + \underbrace{U(\sigma,\vec{\pi},\Delta)}_{\text{meson and diquark self interaction}}$$