Acoustic analogue of a quantum effect : Casimir forces

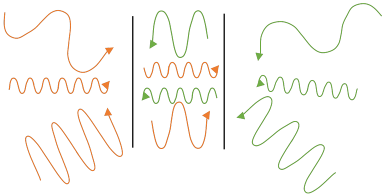
Cathy Gellenoncourt

Neel Institute, Grenoble

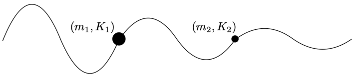
Strasbourg University, Master PSA defense 2022, directed by Cédric Poulain and Jérôme Duplat

Context and Goals

- Team at Neel Institut: particle-wave duality and Casimir forces;
- Particle wave duality system: non fixed mass string on a rope;
- Casimir effect: 2 plates attracting each other;
- Our system: 2 fixed beads equivalent to the Casimir cavity.



Original and theoretical Casimir set up.

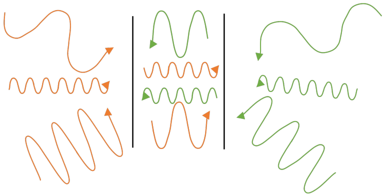


Two bead-strings on rope.

Table of Contents

Original quantum approach

- Each mode of the vacuum fluctuations is associated to a harmonic oscillator (cf quantum field theory);
- Energy of each mode: $\frac{1}{2}\hbar\omega$;
- Zero point energy: sum of the energies of each modes.



Original and theoritical Casimir set up.

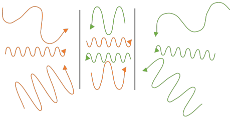
- Potential energy:

$$U(d) = -(\frac{\pi^2\hbar c}{720d^3})L^2 \quad (1)$$

- Resulting attractive force:

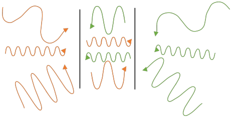
$$F(d) = -\frac{\pi^2\hbar c}{240d^4} \quad (2)$$

Extention of this quantum effect and analogies

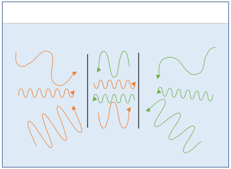


Original and theoritical Casimir set up.

Extention of this quantum effect and analogies

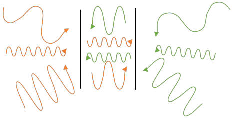


Original and theoritical Casimir set up.

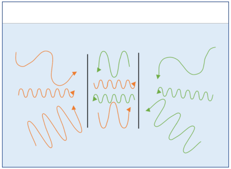


3D hydro-acoustic analogue for attractive forces.

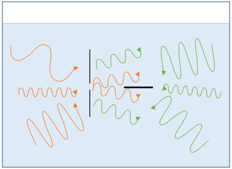
Extention of this quantum effect and analogies



Original and theoritical Casimir set up.

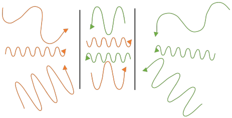


3D hydro-acoustic analogue for attractive forces.

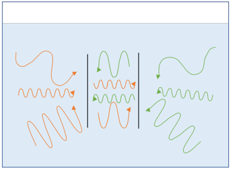


3D hydro-acoustic analogue for repulsive forces.

Extention of this quantum effect and analogies

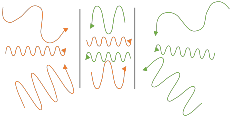


Original and theoretical Casimir set up.

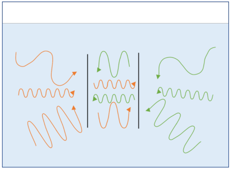


3D hydro-acoustic analogue for attractive forces.

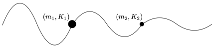
Extention of this quantum effect and analogies



Original and theoretical Casimir set up.

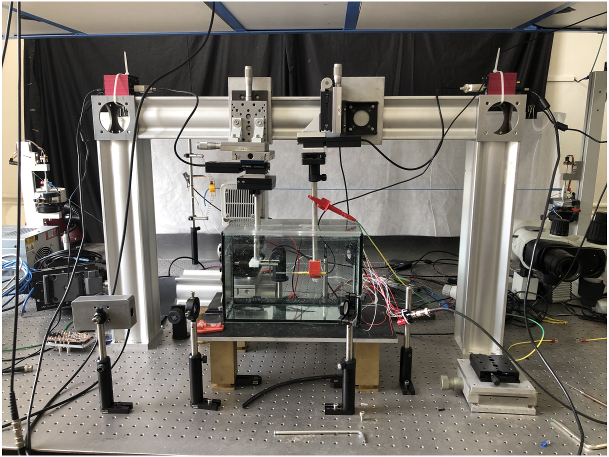


3D hydro-acoustic analogue for attractive forces.



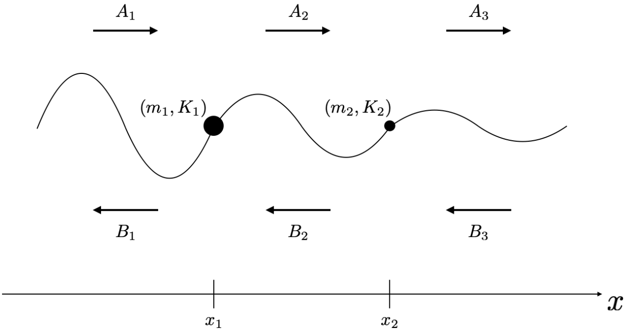
1D acoustic anologue.

3D acoustic analogue



Experiment set up for the 3D analogue study

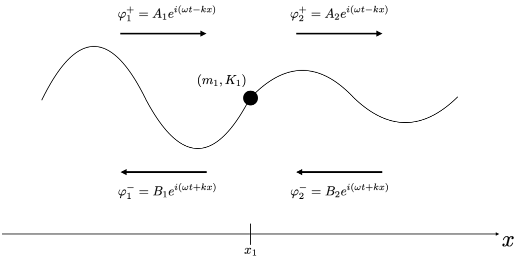
1D acoustic analogue



Bead-strings on a rope.

The one bead-string system

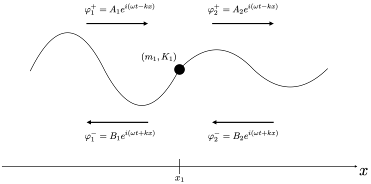
$$\frac{\partial^2 \varphi}{\partial t^2} - c^2 \frac{\partial^2 \varphi}{\partial x^2} = \Box \varphi = -\frac{m}{\mathcal{T}} \left[\frac{\partial^2 \varphi}{\partial t^2} + \omega_p^2 \varphi(x,t) \right] \delta(x - x_1) \quad (3)$$



$$c = \sqrt{\mathcal{T}/\rho_0}$$
$$\omega_p = \sqrt{K/m}$$

Bead-string on a rope.

The one bead-string system

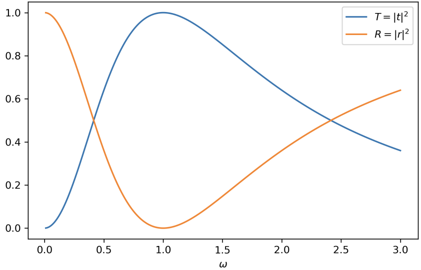


Bead-string on a rope.

$$\beta = \frac{\Omega(\omega)}{\omega} = \frac{mc}{2\mathcal{T}} \frac{(\omega_p^2 - \omega^2)}{\omega}$$
$$r(\omega) = -\frac{i\beta}{1+i\beta}$$
$$t(\omega) = \frac{1}{1+i\beta}$$
$$T = |t(\omega)|^2 = 1 - R$$

$$\begin{pmatrix} A_2 \\ B_1 \end{pmatrix} = \begin{pmatrix} t(\omega) & r(\omega) \\ r(\omega) & t(\omega) \end{pmatrix} \begin{pmatrix} A_1 \\ B_2 \end{pmatrix}$$

Transparency for one bead-string

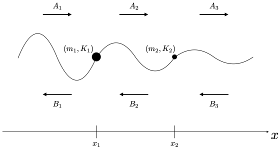


$$T = |t(\omega)|^2 = \frac{1}{1+\beta^2}$$
$$\beta = \frac{\Omega}{\omega}$$

Transmission and reflection coefficients (in energy) for the one bead-string system.

Transparency regime: bead and field are decoupled when $\omega = \omega_p$.

Extention to the two beads system



Bead-strings on a rope.

$$\begin{pmatrix} A_3 \\ B_1 \end{pmatrix} = \frac{1}{d} \begin{pmatrix} 1 & \alpha \\ \alpha & 1 \end{pmatrix} \begin{pmatrix} A_1 \\ B_3 \end{pmatrix} = C \begin{pmatrix} A_1 \\ B_3 \end{pmatrix}$$

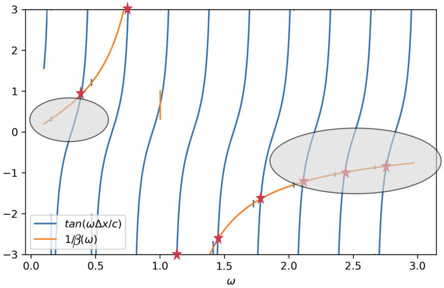
- d and α are complexes;
- C can be diagonal with eigenvalues λ_+ and λ_-

$$\begin{pmatrix} A_3 \\ B_1 \end{pmatrix} = \begin{pmatrix} \frac{1+\alpha}{d} & 0 \\ 0 & \frac{1-\alpha}{d} \end{pmatrix} \begin{pmatrix} A_1 \\ B_3 \end{pmatrix}$$

- Transparency $\Longrightarrow \lambda_+ = 1$ and $\lambda_- = 1 \Longrightarrow \beta \tan(2kx_0) = 1$.

Transparency in the two bead-strings system

$$\beta \tan(2kx_0) = 1, \text{ at high frequencies } \omega = (n + \tfrac{1}{2})\frac{\pi c}{2x_0}$$



Transparency conditions plot for two bead-strings.

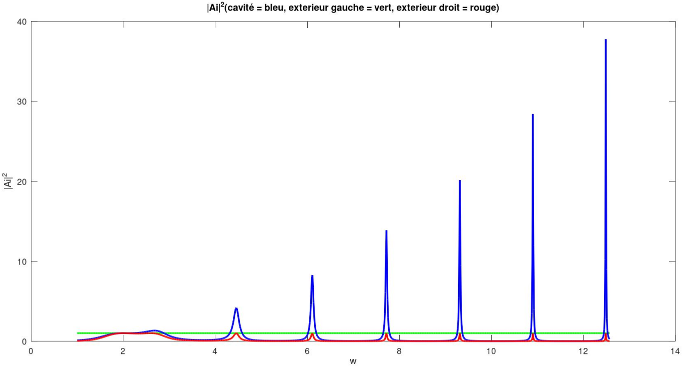
Pseudo-transparency: transparency in amplitude but there is a phase shift between outgoing fields.

Resonance in the two bead-strings system

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = \frac{1}{d} \begin{pmatrix} 1 + i\beta & -i\beta e^{-2ikx_0} \\ -i\beta e^{-2ikx_0} & 1 + i\beta \end{pmatrix} \begin{pmatrix} A_1 \\ B_3 \end{pmatrix} = D \begin{pmatrix} A_1 \\ B_3 \end{pmatrix}$$

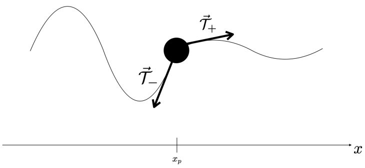
- D can be diagonal
- eigenvalues λ_S and λ_A with eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- Resonance condition:
  - field is amplified or depleted $\Longrightarrow$ extremum of eigenvalues
  - $\beta \tan(2kx_0) = 1$ or -1

Fields and resonance regime



$|A_i|^2$ with respect to ω for one incident wave in the 2 bead-strings system.

Forces calculation for one bead-string



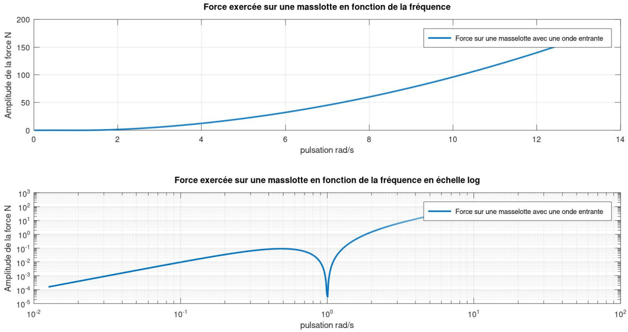
Force on one bead-string.

- Reflection and transmission $\Longrightarrow$ momentum transfert;
- Forces:
  $F_{x_0}(t) =$
  $\frac{\mathcal{T}}{2}\left[\left(\frac{\partial Re(\varphi_1)}{\partial x}\right)^2\Big|_{x_0^-} - \left(\frac{\partial Re(\varphi_2)}{\partial x}\right)^2\Big|_{x_0^+}\right];$

Forces calculation for one bead-string

For one incident wave ($A_1 = 1$, $B_2 = 0$, $x_0 = 0$):

$$< F(t) >= \frac{\mathcal{T} A^2}{c^2} \frac{(\Omega\omega)^2}{\omega^2+\Omega^2}$$

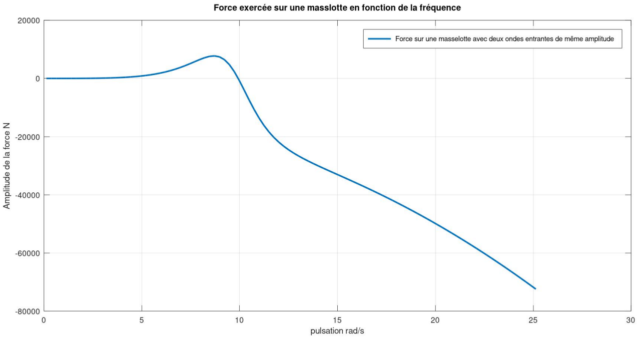


Force on one bead-string with one incident wave.

Forces calculation for one bead-string

For two incident waves ($A_1 = B_2 = 1$, $x_0 = 0$, dephasing $\Delta\Phi$):

$$< F(t) >= \frac{2\mathcal{T}(Ak)^2}{\beta}(1 - T)\sin(\Delta\Phi)$$



Force on one bead-string with two incident waves and a $\frac{\pi}{2}$ dephasing.

Forces calculation for two bead-strings

For two bead-strings system:

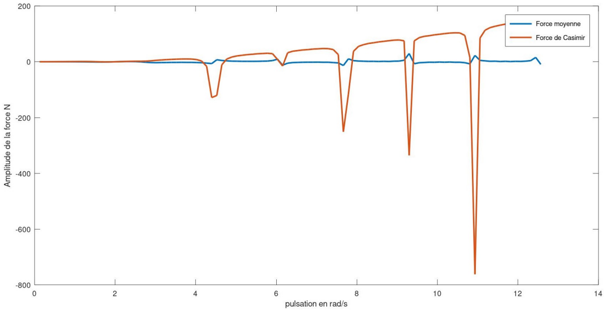
- Forces:

$$F_{x_0}(t) = \frac{\mathcal{T}}{2}\left[\left(\frac{\partial Re(\varphi_2)}{\partial x}\right)^2\Big|_{x_0^-} - \left(\frac{\partial Re(\varphi_3)}{\partial x}\right)^2\Big|_{x_0^+}\right]$$

$$F_{-x_0}(t) = \frac{\mathcal{T}}{2}\left[\left(\frac{\partial Re(\varphi_1)}{\partial x}\right)^2\Big|_{-x_0^-} - \left(\frac{\partial Re(\varphi_2)}{\partial x}\right)^2\Big|_{-x_0^+}\right];$$

- Casimir force: $\frac{<F_{-x_0}>-<F_{x_0}>}{2}$;
- Mean force: $\frac{<F_{-x_0}>+<F_{x_0}>}{2}$;

Forces and fields



Casimir and mean forces amplitude between two bead-strings.

To sum up

Study of a quantum effect: Casimir forces

- Acoustic analogue of a quantum effect:
  - 3D acoustic analogue;
  - 1D acoustic analogue: bead-strings on rope;
    - Forces;
    - Two important regimes: transparency and resonance.
- Further perspectives
  - Finish the resonances and transparencies study;
  - Link these phenomenons to forces;
  - Summation over all frequencies and phases to describe the system under an quantum white noise (fields and forces).
- Benefits of this study:
  - particle-wave duality study;
  - nanometric technologies.

Thank you for your attention.
Any questions?