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Strasbourg University, Master PSA defense 2022, directed by Cédric Poulain and Jérôme Duplat

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Context and Goals

- Team at Neel Institut: particle-wave duality and Casimir forces;
- **Particle wave duality system: non fixed mass string on a rope;**
- Casimir effect: 2 plates attracting each other;
- Our system: 2 fixed beads equivalent to the Casimir cavity.

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Original and theoretical Casimir set up.

Two bead-strings on rope.

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[Casimir forces](#page-3-0)

[Original quantum approach](#page-3-0)

Original quantum approach

- \blacksquare Each mode of the vacuum fluctuations is associated to a harmonic oscillator (cf quantum field theory);
- Energy of each mode: $\frac{1}{2}\hbar\omega$;
- Zero point energy: sum of the energies of each modes.

min Lime $\frac{1}{2}$

Original and theoritical Casimir set up.

Potential energy:

$$
U(d) = -(\frac{\pi^2 \hbar c}{720 d^3}) L^2 \quad (1)
$$

Resulting attractive force:

$$
F(d)=-\frac{\pi^2\hbar c}{240d^4}\qquad (2)
$$

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Extention of this quantum effect and analogies

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Original and theoritical Casimir set up.

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Original and theoritical Casimir set up.

3D hydro-acoustic analogue for attractive forces.

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Extention of this quantum effect and analogies

Original and theoritical Casimir set up.

3D hydro-acoustic analogue for attractive forces.

3D hydro-acoustic analogue for repulsive forces.

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Extention of this quantum effect and analogies

Original and theoretical Casimir set up.

3D hydro-acoustic analogue for attractive forces.

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3D hydro-acoustic analogue for attractive forces.

1D acoustic anologue.

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3D acoustic analogue

Experiment set up for the 3D analogue study

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 L_{1D} acoustic analogue

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1D acoustic analogue

Bead-strings on a rope.

 $-1D$ acoustic analogue

[Analogies](#page-10-0)

The one bead-string system

$$
\frac{\partial^2 \varphi}{\partial t^2} - c^2 \frac{\partial^2 \varphi}{\partial x^2} = \Box \varphi = -\frac{m}{\mathcal{T}} \left[\frac{\partial^2 \varphi}{\partial t^2} + \omega_p^2 \varphi(x, t) \right] \delta(x - x_1) \quad (3)
$$

Bead-string on a rope.

 $-1D$ acoustic analogue

 L [The one bead system](#page-12-0)

The one bead-string system

$$
\beta = \frac{\Omega(\omega)}{\omega} = \frac{mc}{27} \frac{(\omega_p^2 - \omega^2)}{\omega}
$$

$$
r(\omega) = -\frac{i\beta}{1 + i\beta}
$$

$$
t(\omega) = \frac{1}{1 + i\beta}
$$

$$
T = |t(\omega)|^2 = 1 - R
$$

Bead-string on a rope.

$$
\begin{pmatrix} A_2 \ B_1 \end{pmatrix} = \begin{pmatrix} t(\omega) & r(\omega) \\ r(\omega) & t(\omega) \end{pmatrix} \begin{pmatrix} A_1 \\ B_2 \end{pmatrix}
$$

 $-1D$ acoustic analogue

[The one bead system](#page-12-0)

Transparency for one bead-string

Transmission and reflection coefficients (in energy) for the one bead-string system.

Transparency regime: bead and field are decoupled when $\omega = \omega_p$.

 $-1D$ acoustic analogue

 L [Extention to the two beads system](#page-14-0)

Extention to the two beads system

$$
\begin{pmatrix} A_3 \\ B_1 \end{pmatrix} = \frac{1}{d} \begin{pmatrix} 1 & \alpha \\ \alpha & 1 \end{pmatrix} \begin{pmatrix} A_1 \\ B_3 \end{pmatrix} = C \begin{pmatrix} A_1 \\ B_3 \end{pmatrix}
$$

Bead-strings on a rope.

- \blacksquare d and α are complexes;
- C can be diagonal with eigenvalues λ_+ and λ_-

$$
\begin{pmatrix} A_3 \ B_1 \end{pmatrix} = \begin{pmatrix} \frac{1+\alpha}{d} & 0 \\ 0 & \frac{1-\alpha}{d} \end{pmatrix} \begin{pmatrix} A_1 \\ B_3 \end{pmatrix}
$$

$$
\begin{aligned}\n\text{Transparency} &\Longrightarrow \lambda_+ = 1 \text{ and} \\
\lambda_- = 1 &\Longrightarrow \beta \tan(2kx_0) = 1.\n\end{aligned}
$$

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 $-1D$ acoustic analogue

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Transparency in the two bead-strings system

 β tan $(2kx_0)=1$, at high frequencies $\omega=(n+\frac{1}{2})$ $\frac{1}{2}$) $\frac{\pi c}{2x_0}$ $2x_0$

Transparency conditions plot for two bead-strings.

Pseudo-transparency: transparency in amplitude but there is a phase shift between outgoin[g fi](#page-14-0)[eld](#page-16-0)[s](#page-14-0)[.](#page-15-0)

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Resonance in the two bead-strings system

$$
\begin{pmatrix} A_2 \ B_2 \end{pmatrix} = \frac{1}{d} \begin{pmatrix} 1 + i\beta & -i\beta e^{-2ikx_0} \\ -i\beta e^{-2ikx_0} & 1 + i\beta \end{pmatrix} \begin{pmatrix} A_1 \\ B_3 \end{pmatrix} = D \begin{pmatrix} A_1 \\ B_3 \end{pmatrix}
$$

D can be diagonal

eigenvalues λ_S and λ_A with eigenvectors $\begin{pmatrix} 1 \ 1 \end{pmatrix}$ 1

 $\Big)$ and $\Big($ 1 −1 \setminus

Resonance condition:

- **■** field is amplified or depleted \implies extremum of eigenvalues
- β tan(2kx₀) = 1 or −1

 $-1D$ acoustic analogue

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Fields and resonance regime

 $|A_i|^2$ with respect to ω for one incident wave in the 2 bead-strings system.

 $-1D$ acoustic analogue

[Forces calculation](#page-18-0)

Forces calculation for one bead-string

Force on one bead-string.

Reflection and transmission \implies momentum transfert;

Forces:
\n
$$
F_{x_0}(t) = \frac{\tau}{2} \left[\left(\frac{\partial Re(\varphi_1)}{\partial x} \right)^2 \Big|_{x_0^-} - \left(\frac{\partial Re(\varphi_2)}{\partial x} \right)^2 \Big|_{x_0^+} \right];
$$

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Forces calculation for one bead-string

For one incident wave $(A_1 = 1, B_2 = 0, x_0 = 0)$: $\langle F(t) \rangle = \frac{TA^2}{c^2}$ $\frac{\epsilon^{-1/2}}{c^2} \frac{(\Omega \omega)^2}{\omega^2 + \Omega^2}$ $ω^2 + Ω^2$

Force on one bead-string with one incident wave.

 $-1D$ acoustic analogue

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Forces calculation for one bead-string

For two incident waves ($A_1 = B_2 = 1$, $x_0 = 0$, dephasing $\Delta \Phi$): $\langle F(t) \rangle = \frac{2\mathcal{T}(Ak)^2}{\beta}$ $\frac{(\mathcal{A}\mathsf{R})}{\beta}(1-\mathcal{T})\sin(\Delta\Phi)$

Force on one bead-string with two incident waves and a $\frac{\pi}{2}$ dephasing.

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Forces calculation for two bead-strings

For two bead-strings system:

■ Forces: $F_{x_0}(t) = \frac{7}{2}$ " *∂*Re(*φ*2) $\left.\frac{\partial e(\varphi_2)}{\partial x}\right)^2\Bigg|_{x=0} - \left(\frac{\partial Re(\varphi_3)}{\partial x}\right)$ $|x_0^{-}$ \cdot \cd $\left.\frac{\partial e(\varphi_3)}{\partial x}\right)^2\Bigg|_{x^+}$ 1 $F_{-x_0}(t) = \frac{T}{2}$ " *∂*Re(*φ*1) $\left.\frac{\partial e(\varphi_1)}{\partial x}\right)^2\Big|_{x=0}$ − *∂*Re(*φ*2) $\left.\frac{\partial e(\varphi_2)}{\partial x}\right)^2\Bigg|_{-x_0^+}$ 1 ; Casimir force: *<*F−x0*>*−*<*Fx0*>* 2 ; Mean force: $\frac{\langle F_{-\mathsf{x}_0} \rangle + \langle F_{\mathsf{x}_0} \rangle}{2}$;

 L_{1D} acoustic analogue

[Forces calculation](#page-18-0)

Forces and fields

Casimir and mean forces amplitude between two bead-strings.

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[Conclusion and further perspectives](#page-23-0)

To sum up

Study of a quantum effect: Casimir forces

- Acoustic analogue of a quantum effect:
	- 3D acoustic analogue;
	- 1D acoustic analogue: bead-strings on rope;
		- **Forces**:
		- Two important regimes: transparency and resonance.
- **Further perspectives**
	- \blacksquare Finish the resonances and transparencies study;
	- **Link these phenomenons to forces:**
	- Summation over all frequencies and phases to describe the system under an quantum white noise (fields and forces).
- \blacksquare Benefits of this study:
	- particle-wave duality study;
	- nanometric technologies.

[Conclusion and further perspectives](#page-23-0)

Thank you for your attention. Any questions?