Studying chiral imbalance using Chiral Perturbation Theory

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20–24 de junio de 2022
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Outline

- Chiral imbalance and axial chemical potential.

- Construction of the effective lagrangian for non zero chemical potential.

- Phenomenological consequences for $\mu_5 \neq 0$:

  Pion dispersion relation, the light quark condensate, the chiral and topological susceptibilities, the chiral charge density…

- Isospin chemical potential

  Pion and quark condensates and the isospin density.

Chiral Imbalance

The axial current is not conserved at the quantum level due to the $U(1)_A$ axial anomaly equation:

$$\partial^\mu J_{5,\mu} - 2i\widehat{m}_q J_5 = \frac{N_f}{2\pi^2}\partial^\mu K_\mu$$

$\langle N_L - N_R \rangle$

$$\frac{d}{dt}(Q_5^q - 2N_f T_5) \simeq 2i\int_{\mathrm{vol.}} d^3x\, \widehat{m}_q \overline{q}\gamma_5 q$$

In hot QCD a metastable topological charge $\langle T_5 \rangle$ may arise in a finite volumen.

$$\langle \Delta T_5 \rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\text{fireball}}$$

It may be associated with an axial chemical potential μ_5 .

Construction of the effective lagrangian

We consider the effective low-energy representation of Z(v, a, s, p, θ).

The construction of the most general, model-independent, effective lagrangian can be carried out within the framework of the external source method.

We introduce the so called ”spurion” fields:

$$\mathcal{L}_Q = A_\mu \bar{q} \gamma^\mu \left[Q_L(x) P_L + Q_R(x) P_R \right] q$$

Electromagnetic field: $Q_L = Q_R = Q$

Chiral imbalance: $Q_L = -Q_R = \frac{\mu_5}{F} \mathbb{1}, \quad A_\mu = F \delta_{\mu 0}.$

Isospin chemical potential: $Q_L = Q_R = Q \quad A_\mu = \Lambda \delta_{\mu 0}, \quad Q = \frac{\mu_I}{2\Lambda} \tau_3$

D. Espriu, A. Gómez Nicola and A. Vioque-Rodríguez, JHEP 06 (2020), 062.

arXiv:2205.14609v1

Construction of the effective lagrangian

The covariant derivative reads

$$d_\mu U \;=\; \partial_\mu U - iQ_R A_\mu U + iUQ_L A_\mu$$

the operator $\mathrm{tr}\left(U^\dagger d_\mu U\right)$ has to be considered as an additional operator for constructing the lagrangian

$$\mathrm{tr}\left(U^\dagger d_\mu U\right) = -\mathrm{tr}\left(U d_\mu U^\dagger\right) = 2i\delta_{\mu 0} N_f \mu_5$$

Chiral power counting:

$$U = \mathcal{O}(1) \qquad d_\mu U, Q_I = \mathcal{O}(p) \qquad \chi = \mathcal{O}(p^2)$$

The leading order lagrangian

At $\mathcal{O}(p)$, the only nontrivial operator is: $\mathrm{tr}\,(Q_L + Q_R)$

To $\mathcal{O}(p^2)$ the following operators are also allowed:

$$\mathrm{tr}\left[Q_R U Q_L U^\dagger\right] \qquad \mathrm{tr}(U^\dagger d_\mu U)\mathrm{tr}(U^\dagger d^\mu U) \qquad \mathrm{tr}[Q_L^2 + Q_R^2]$$

$$\mathrm{tr}[Q_L]^2 + \mathrm{tr}[Q_R]^2 \qquad \mathrm{tr}[Q_L]\mathrm{tr}[Q_R]$$

Therefore, at this order, the only modification to the chiral lagrangian is a constant term:

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{tr}\left[\partial_\mu U^\dagger \partial^\mu U + 2B_0\mathcal{M}\left(U + U^\dagger\right)\right] + 2\mu_5^2 F^2\left(1 - Z + \kappa_0\right)$$

Next to leading order lagrangian

All possible $\mathcal{O}(p^3)$ terms allowed by the symmetries vanish for the our choice of Q.

At $\mathcal{O}(p^4)$ we have new terms constructed out of the Q operators and $\mathrm{tr}\,(U^\dagger d_\mu U)$. The possible contributions (including Q operators) are of the form:

$$\boxed{ddQQ} \qquad \boxed{\chi QQ} \qquad \boxed{QQQQ}$$

Trace identities can be used to eliminate some of the operators.

The form of the explicit μ_5 corrections is:

$$\mathcal{L}_4(\mu_5) = \mathcal{L}_4^0(\mu_5 = 0) + \boxed{\kappa_1}\mu_5^2 \mathrm{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) + \boxed{\kappa_2}\mu_5^2 \mathrm{tr}\left(\partial_0 U^\dagger \partial^0 U\right) + \boxed{\kappa_3}\mu_5^2 \mathrm{tr}\left(\chi^\dagger U + U^\dagger \chi\right) + \boxed{\kappa_4}\mu_5^4$$

We use recent lattice analyses to estimate some of those constants.

Pion dispersion relation

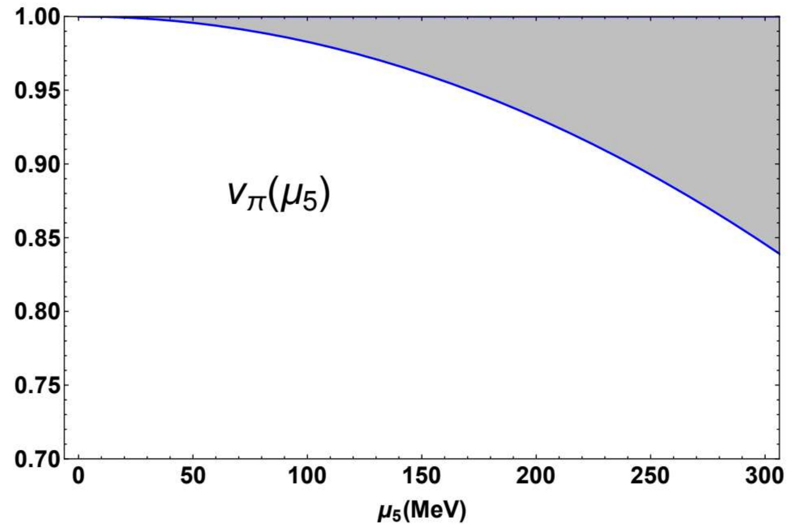
The spatial and time components of the pion decay constant are different:

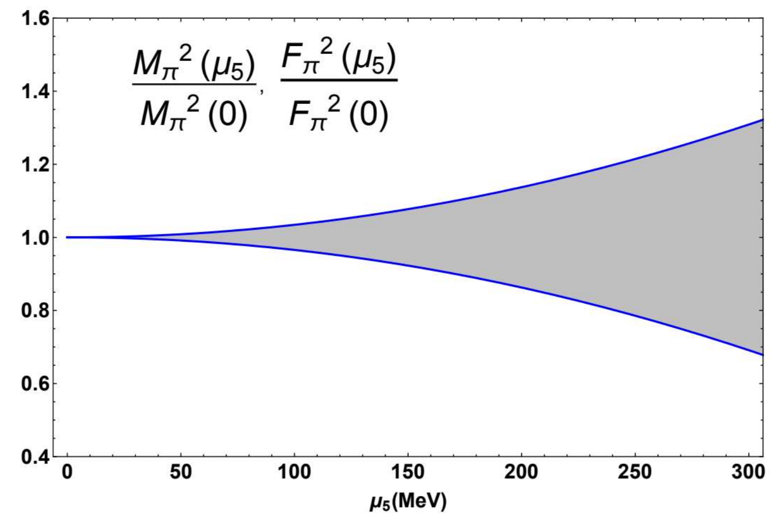
$$\left(F_\pi^t\right)^2(\mu_5) = F_\pi^2(0) + 4(\kappa_1+\kappa_2)\mu_5^2$$
$$\left(F_\pi^s\right)^2(\mu_5) = F_\pi^2(0) + 4\kappa_1\mu_5^2,$$

The two main physical consequences of that are the pion velocity and pion mass:

$$v_\pi(\mu_5) = 1 + 2\kappa_2\frac{\mu_5^2}{F^2} \Rightarrow \boxed{\kappa_2 < 0}$$

$$\left[M_\pi^2\right]^{\text{pole}}(\mu_5) = M_\pi^2(0) - 4\left(\kappa_1+\kappa_2-\kappa_3\right)\frac{\mu_5^2}{F^2}M^2$$





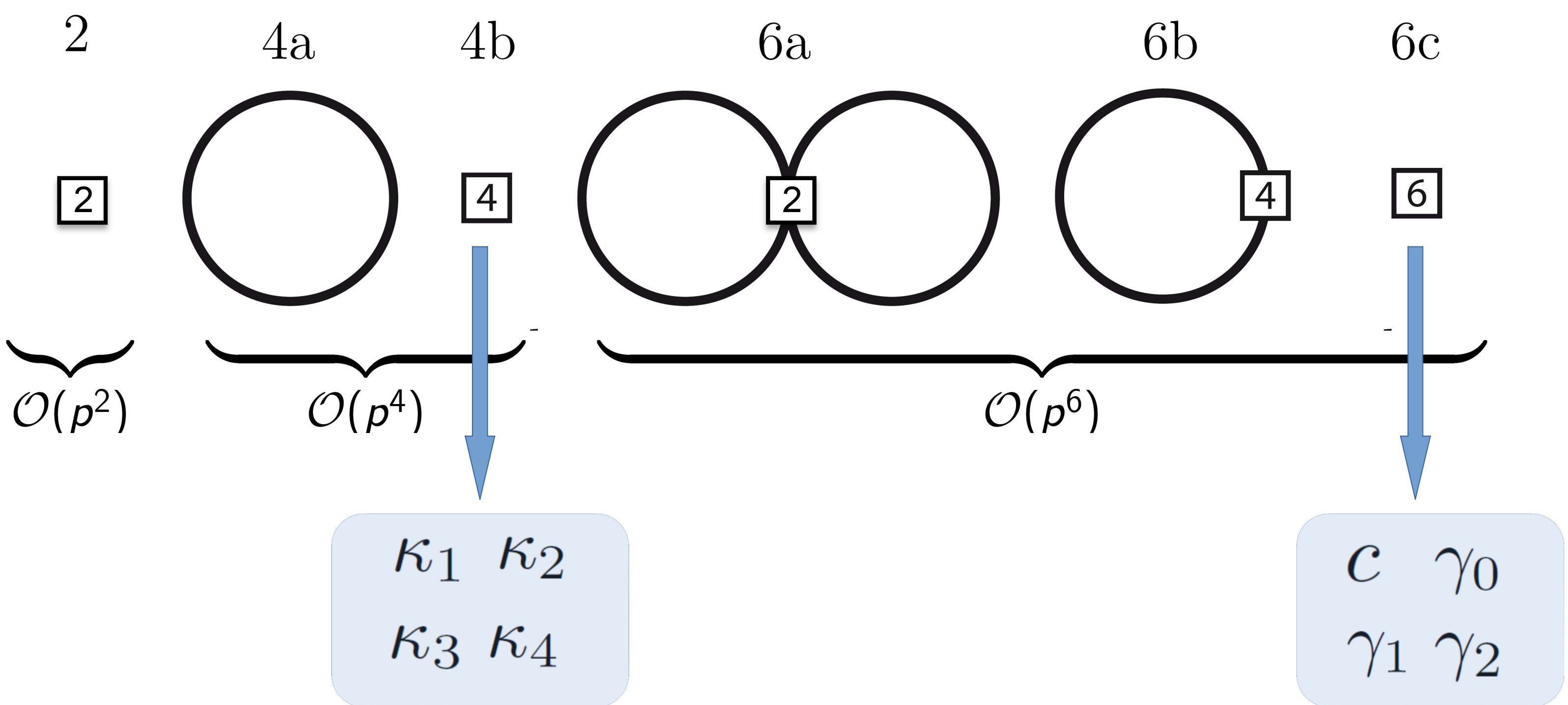
Vacuum energy density

$$\epsilon(T,\mu_5) = -(\beta V)^{-1}\log Z(T,\mu_5)$$

At $\mathcal{O}(p^2)$, it only contributes the constant part of the LO lagrangian

$$\epsilon_2(\mu_5) = -F^2M^2 - 2\mu_5^2F^2\left(1 - Z + \kappa_0\right)$$

The $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ include:



Quark condensate and scalar susceptibility

The first correction to the quark condensate is temperature independent:

$$\langle \bar{q}q \rangle_l^{LO} = -2B_0F^2$$

$$\langle \bar{q}q \rangle_l^{\rm NLO}(T,\mu_5) = \langle \bar{q}q \rangle_l^{\rm NLO}(T,0) + 4\kappa_3 \frac{\mu_5^2}{F^2} \langle \bar{q}q \rangle_l^{\rm LO}$$

For the ratio $\langle \bar{q}q \rangle_l^{\rm NNLO}(T,\mu_5)/\langle \bar{q}q \rangle_l^{\rm NNLO}(0,\mu_5)$ the κ_i dependence reduces to the combinations:

$$\boxed{\kappa_a = 2\kappa_1 - \kappa_2} \qquad \boxed{\kappa_b = \kappa_1 + \kappa_2 - \kappa_3}$$

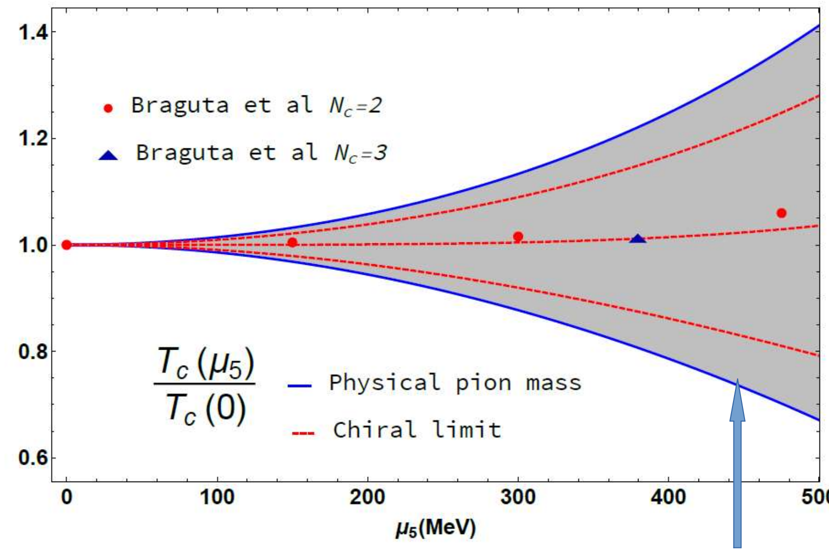
Although in the chiral limit it depends only on κ_a :

$$\left. \frac{\langle \bar{q}q \rangle_l (T,\mu_5)}{\langle \bar{q}q \rangle_l (0,\mu_5)} \right|_{M=0} = 1 - \frac{T^2}{8F^2}\left[1 - 2\kappa_a \frac{\mu_5^2}{F^2}\right] - \frac{T^4}{384F^4} + \mathcal{O}\left(\frac{1}{F^6}\right)$$

The coefficient that regulates the dependence of the scalar susceptibility with μ_5 near the chiral limit is $\kappa_1 - \kappa_3$.

Critical temperature

The ChPT curve for the physical pion mass lies very close to the chiral limit one.

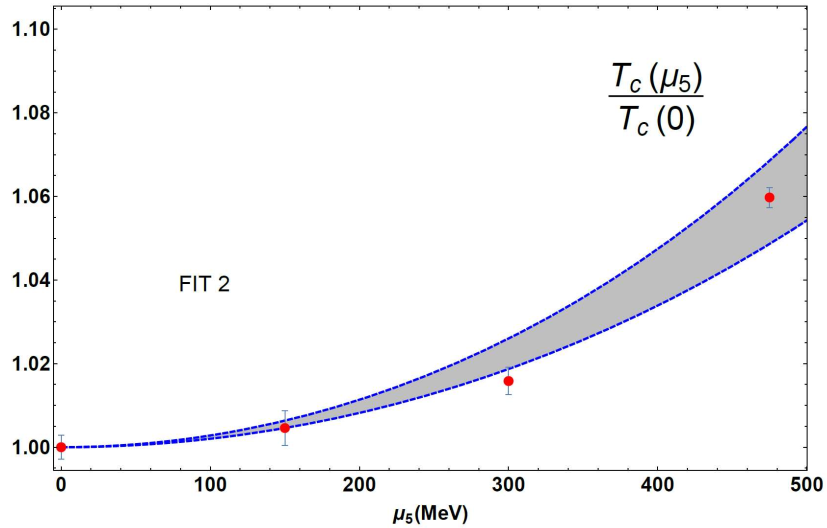


Lattice points clearly fall into the uncertainty given by the natural values range.

Natural values

The chiral limit approach is a robust approximation.

FIT	$\kappa_a \times 10^3$	$\kappa_b \times 10^3$	χ^2/dof	R^2	# points $\mu_5 \neq 0$
Fit 1 $(M = 0)$	1.7 ± 0.6	—	0.01	1.00	2
Fit 2 $(M = 0)$	2.3 ± 0.4	—	1.41	0.99	3
Fit 3	2.3 (fixed)	0 ± 1	1.36	0.99	3
Fit 4 $(M = 0\ \mathcal{O}(\mu_5^2))$	2.5 ± 0.4	—	1.85	0.99	3



Lattice data Braguta et all (2015)

The topological susceptibility

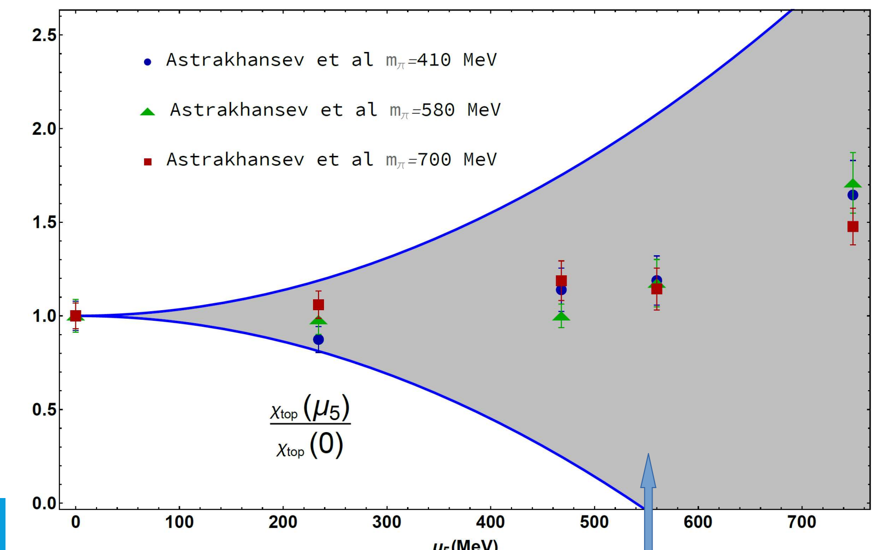
The dependence of χ_{top} with low and moderate μ_5 is controlled by the κ_3 constant:

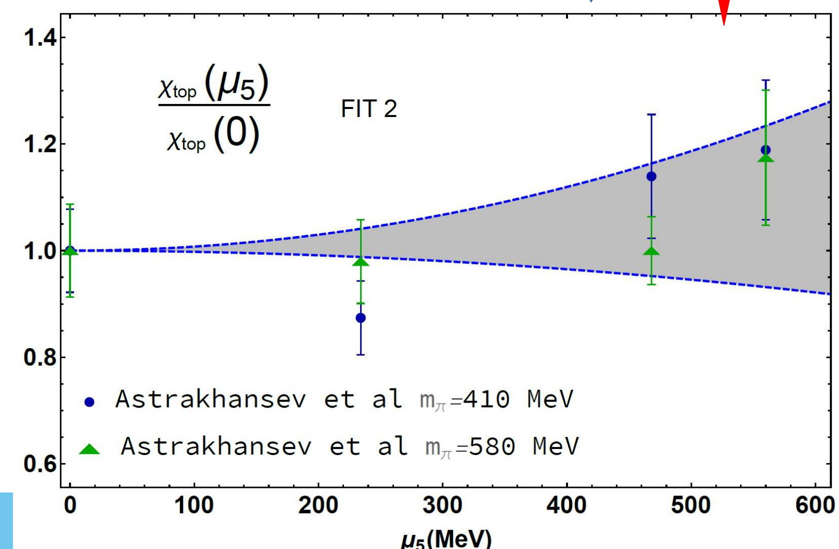
$$\frac{\chi_{top}(\mu_5)}{\chi_{top}(0)} = 1 + 4\frac{\kappa_3 \mu_5^2}{F^2} + \mathcal{O}(1/F^4)$$

The fit shows:

- The results for κ_3 are compatible with zero.
- The error bands are much narrower than the natural values.

FIT	$\kappa_3 \times 10^3$	R^2	χ^2/dof	# points $\mu_5 \neq 0$
Fit 1	0.1 ± 1.4	0.99	1.20	2 ($m_\pi = 410$ MeV)+2 ($m_\pi = 580$ MeV)
Fit 2	0.5 ± 0.9	0.99	1.13	3 ($m_\pi = 410$ MeV)+3 ($m_\pi = 580$ MeV)





Natural values

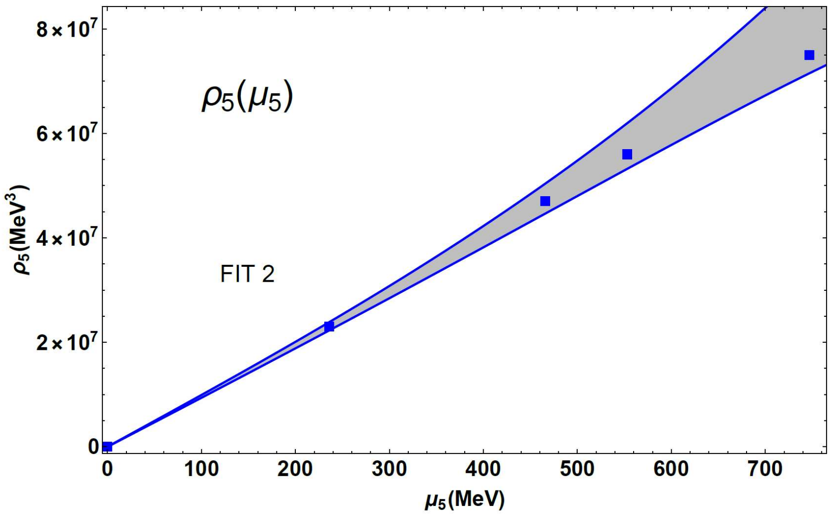
The chiral charge density

We perform a fit of

$$\rho_5(T,\mu_5)|_{M=0} = 4F^2\mu_5\left(1 - Z + \kappa_0 + \kappa_2\frac{\pi^2T^4}{10F^4}\right) + 4\kappa_4\mu_5^3 - 6\frac{\gamma_0}{F^2}\mu_5^5 + \mathcal{O}(1/F^4)$$

The linear dependence fits very well the lowest μ_5 lattice points.

Fit 1 is consistent with Fit 2.



Lattice data Astrakhantsev et all (2019)

FIT	κ_0	$\kappa_4 \times 10^3$	$\gamma_0 \times 10^5$	R^2	# points $\mu_5 \neq 0$
Fit 1	3.2 ± 0.1	0 (fixed)	0 (fixed)	0.99	3
Fit 2	3.1 ± 0.1	7.1 ± 3.6	4.6 ± 2.4	0.99	4

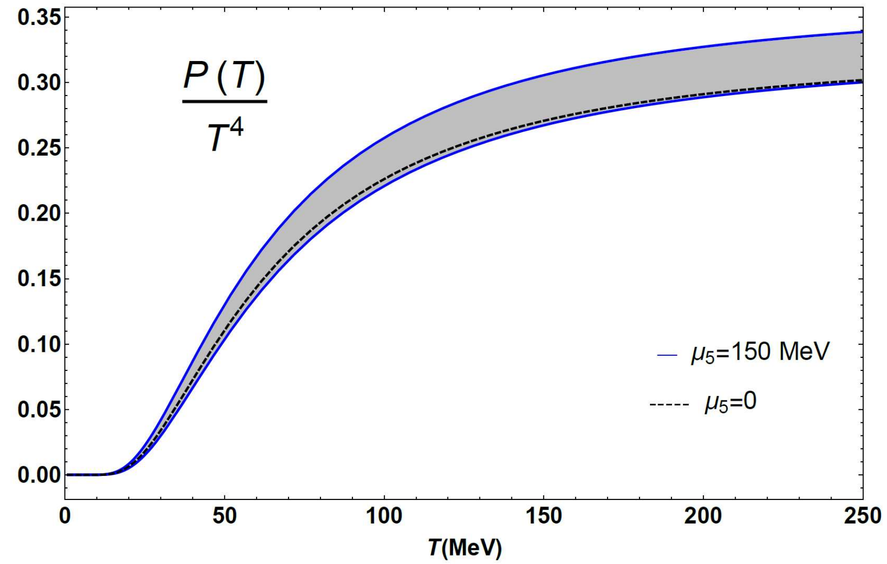
Presure and speed of sound

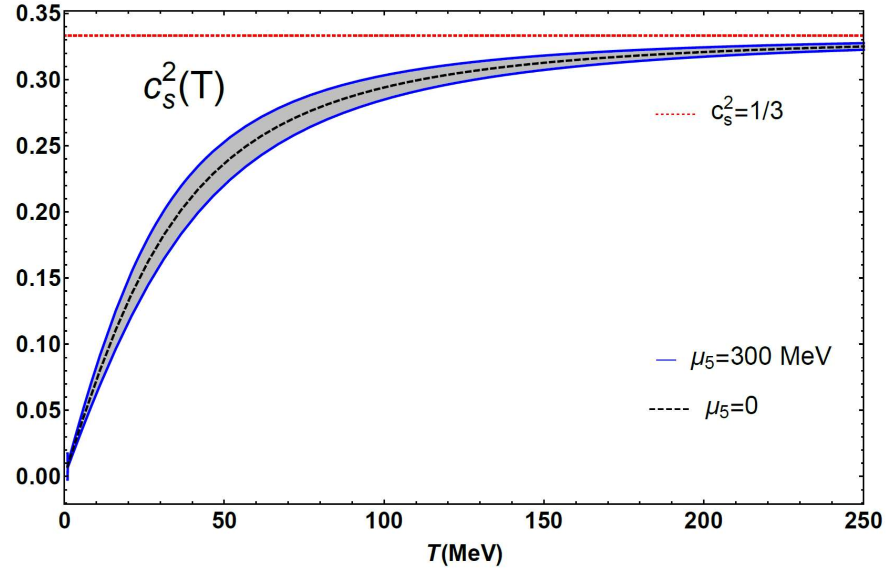
The μ_5 corrections to the pressure are parametrized by κ_2 and κ_b.

Chiral limit

κ_2

Natural values





The speed of sound depends only on κ_b.

Isospin chemical potential

The QCD lagrangian including a nonzero isospin chemical potential

$$\mathcal{L}_{QCD} = \bar{q}\left(i \not{D} - \mathcal{M}\right) q + \frac{\mu_I}{2}\bar{q}\gamma_0\tau_3 q$$

In addition to the standard terms $\mathrm{Tr}\left[U\tau_3 U^\dagger \tau_3\right]$ is also allowed since it breaks chiral symmetry but preserves $U(1)_{I_3}$.

The most general $\mathcal{O}(p^2)$ lagrangian at nonzero μ_I is given by

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}\left[d_\mu U d^\mu U^\dagger + \chi^\dagger U + \chi U^\dagger + \frac{1}{2}a_1\mu_I^2 U\tau_3 U^\dagger \tau_3\right] + \frac{1}{4}a_2 F^2 \mu_I^2$$

$\frac{1}{4}a_2 F^2 \mu_I^2 \leftarrow \mathrm{Tr}(Q^2)$

$\chi = M^2 + 2iB_0 j\tau_1$

We allow for a nontrivial vacuum configuration

$$\begin{cases} U(x) = A\exp\left[i\frac{\tau^a\pi^a(x)}{F}\right]A \\ A = \cos(\alpha/2)\mathbb{1} + i\sin(\alpha/2)\tau_1 \end{cases}$$

LO critical value : $\mu_c = \frac{M}{\sqrt{1-a_1}}$ → The constant a_1 displaces the critical value.

Next to Leading Order

- Constant terms coming from $\mathcal{L}_4$:

$$\epsilon_4^{4Q} = -\hat{q}_1\mu_I^4\sin^4\alpha - \frac{1}{2}\hat{q}_2M^2\mu_I^2\cos\alpha - \frac{1}{2}\hat{q}_3M^2\mu_I^2\cos^3\alpha - \hat{q}_4B_0\mu_I^2 j\sin\alpha - \hat{q}_5B_0\mu_I^2 j\sin\alpha\cos^2\alpha - \hat{q}_6\mu_I^4\cos^2\alpha - \hat{q}_7\mu_I^4$$

$\hat{q}_i$ will be renormalized to absorb the loop divergences.

- One-loop of $\mathcal{L}_2$.
- Linear term proportional to π_1 coming from $\mathcal{L}_2$.

Observables of interest:

$$\langle\bar{q}q\rangle(\mu_I) = \langle\bar{u}u + \bar{d}d\rangle = \frac{\partial\epsilon(\mu_I, j)}{\partial m} \qquad \langle i\bar{q}\gamma_5\tau_1 q\rangle(\mu_I) = \frac{\partial\epsilon(\mu_I, j)}{\partial j} \qquad n_I(\mu_I) = -\frac{\partial\epsilon(\mu_I, j)}{\partial\mu_I}$$

To ensure that n_I vanishes below μ_c^{NLO} →

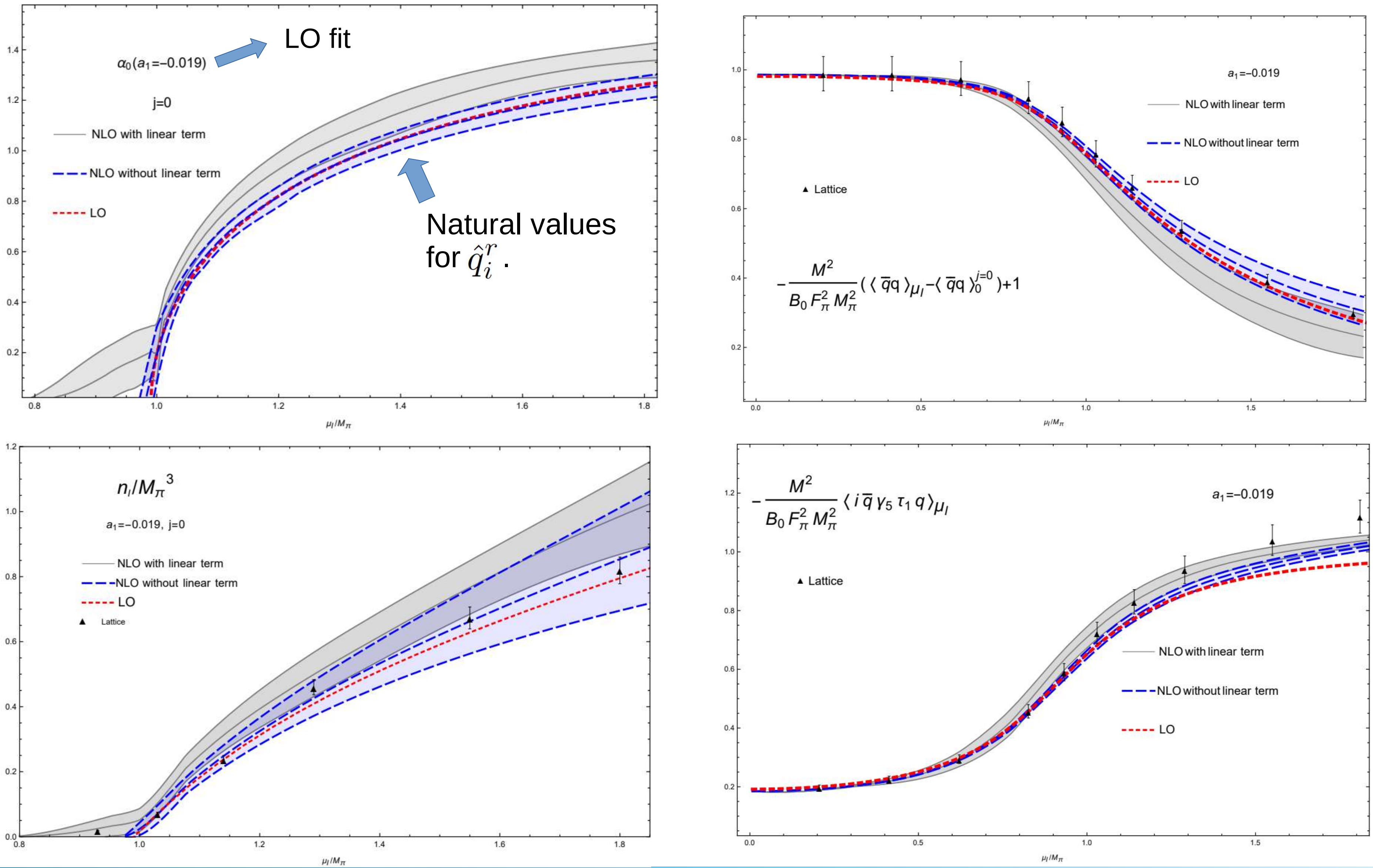
$$a_1, a_2 = \mathcal{O}\left(\frac{1}{F^2}\right)$$

$$\hat{q}_6^r(\mu) + \hat{q}_7^r(\mu) = 0 + \mathcal{O}\left(\frac{1}{F^2}\right)$$

Critical value $\beta_2(\mu_I)\big|_{\mu_I=\mu_c} = 0$:

$$\epsilon = \beta_0(\mu_I) + \beta_2(\mu_I)\alpha^2 + \mathcal{O}(\alpha^4) \Rightarrow \beta_2(\mu_I) = \frac{1}{2}F_\pi^2\left[M_\pi^2 - (1-a_1)\mu_I^2\right] + \frac{1}{4}\mu_I^2\left[M^2(\hat{q}_2^r + 3\hat{q}_3^r) + 4\hat{q}_6^r\mu_I^2\right] - \underbrace{\frac{F^2(\mu_I^4 - M^4)}{2\mu_I^2}}_{\text{linear term}} + \mathcal{O}\left(\frac{1}{F^2}\right)$$

Numerical results



Conclusions

- We have analyzed the effective chiral lagrangian for nonzero chiral imbalance for two light flavours, through its dependence with the axial chemical potential μ_5 .
- We have analyzed several observables and the dependence of their μ_5 corrections with the κ_i constants.
- A consistent picture with lattice data emerges, allowing to determine some LEC combinations.

average values of κ_a and κ_3 ⟹ $\kappa_1 \simeq 0.8 \times 10^{-3}$, $\kappa_2 \simeq -0.5 \times 10^{-3}$, $\kappa_3 \simeq 0.3 \times 10^{-3}$

- We discuss new μ_I-corrections to the quark and pion condensates and the isospin density including the effect of new LECs.