Magnetic transitions in ultra-peripheral nuclear collisions

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[Danhoni and Navarra, Phys. Rev. C 103 (2021) 024902]

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Outline

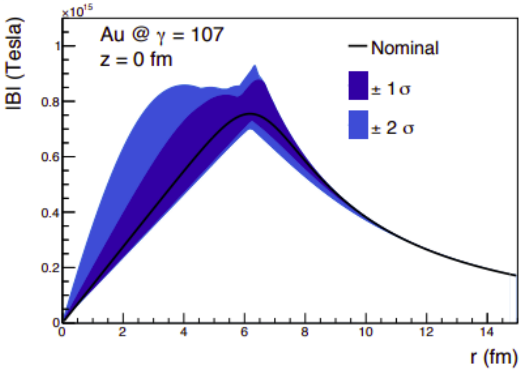
- Introduction
- The Semi-Classical Formalism
- The Quantum Formalism
- Conclusions

Magnetic Field

It is said that the magnetic field in heavy-ion collisions is the strongest in nature:

$$B \sim \gamma \alpha_{em} \frac{Ze}{R^2}$$

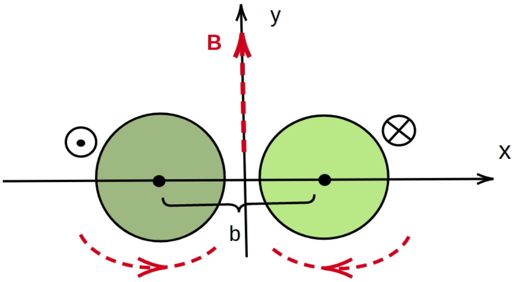
[Asakawa, Majumder and Muller (2010) Phys. Rev. C81 064912]



[Brandenburg, Zha and Xu (2021) Eur. Phys. J. A 57, no.10, 299]

Magnetic Field

A natural place to look for this field and its effects is in ultra-peripheral relativistic heavy-ion collisions (UPC's).



$N \rightarrow \Delta$

The strong magnetic field induces magnetic transitions like $N \rightarrow \Delta$ in the other nucleus. The deltas will keep moving with the nucleus and decay with 99% in $N + \pi$.

$$N \rightarrow \Delta \rightarrow N' \pi$$

- Use a classical field to study the forward pion production by the magnetic excitation of nucleons through the Δ.
- Replace this field with a flux of photons and use the photoproduction of pions as the analogous process of magnetic excitation.
- Compare the results obtained with the two formalisms.

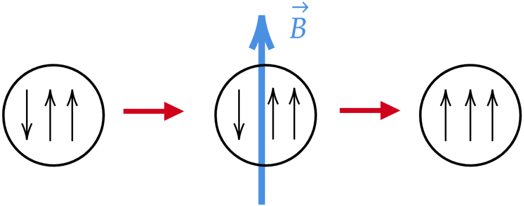
The Interaction Hamiltonian

The interaction Hamiltonian will be:

$$H_{int}(t) = -\vec{\mu}.\vec{B}(t)$$

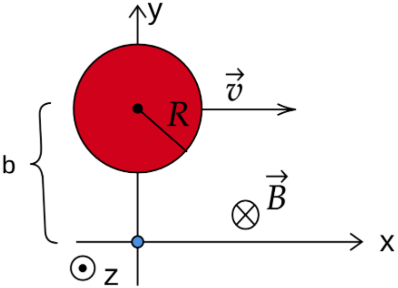
The magnetic dipole moment of the nucleon is given by:

$$\vec{\mu} = \sum_{i=u,d} \vec{\mu}_i = \sum_{i=u,d} \frac{q_i}{m_i} \vec{S}_i$$



Semi-Classical Calculation

We will consider a Pb-p ultra-peripheral collision and our reference frame to be the proton rest frame.



The Magnetic Field

The magnetic field is in the z direction. For simplicity, we will assume that the projectile-generated field is the same produced by a point charge:

$$B_z(t) = \frac{1}{4\pi} \frac{qv\gamma b}{((\gamma vt)^2 + b^2)^{3/2}}$$

[Asakawa, Majumder and Muller (2010) Phys. Rev. C81 064912]

Wave Function

The Hamiltonian acts on the spin states, so our wave functions will be:

$$\Psi = \phi(isospin) \otimes \chi(spin) = \frac{1}{\sqrt{2}}(\phi_S \otimes \chi_S + \phi_A \otimes \chi_A)$$

$$\begin{aligned} |p \ \uparrow\rangle &= \frac{1}{3\sqrt{2}}[udu(\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow - 2\uparrow\downarrow\uparrow) + duu(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow - 2\downarrow\uparrow\uparrow) \\ &+ uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)] \\ |\Delta^+ \uparrow\rangle &= \frac{1}{3}(uud + udu + duu)(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \end{aligned}$$

Perturbation Theory

The intensity of the magnetic field is of order $eB \simeq m_\pi^2$. The energy that can be transferred to a nucleon is:

$$\sqrt{eB} \simeq m_\pi$$

And we know that:

$$M_\Delta - M_N \simeq 2m_\pi$$

Initial state: $|\psi^0\rangle = |p\rangle$ $\Rightarrow P = |c_\Delta|^2$

Final state: $|\psi^1\rangle = c_N |p\rangle + c_\Delta |\Delta\rangle$

Transition Amplitude

Under magnetic field excitation, the amplitude for the transition $|p\uparrow\rangle \rightarrow |\Delta^+\uparrow\rangle$ using time-dependent perturbation theory will be given by:

$$c_\Delta = -i\int_{-\infty}^{t} e^{iE_{fi}t'}\,\langle\Delta^+\uparrow|H_{int}|p\uparrow\rangle\,dt'$$

Giving the transition element:

$$\langle\Delta^+\uparrow|H_{int}|p\uparrow\rangle = \frac{\sqrt{2}Be}{3m}$$

Cross Section

The cross section for $p \to \Delta$ is given by:

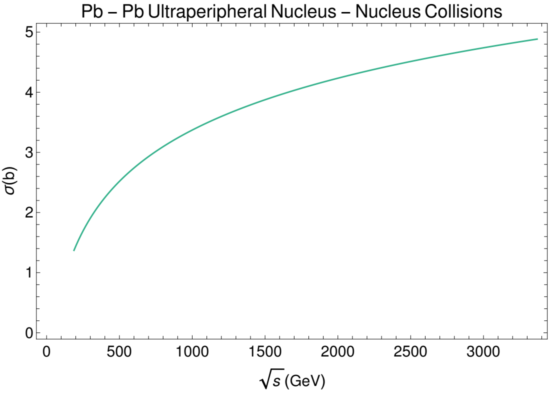
$$\sigma = \int |c_\Delta|^2 d^2 b \qquad (1)$$

Substituting c_Δ we get:

$$\sigma = \frac{Z^2 e^4}{9\pi m^2} \left(\frac{E_{fi}}{v\gamma}\right)^2 \int_R^\infty \left[K_1\left(\frac{E_{fi}\, b}{v\gamma}\right)\right]^2 b\, db$$

Cross Section

$$\sigma_{tot} = A\sigma$$



Cross section as a function of the center of mass frame.

Pion Production

If we suppose every event will produce a pion, the number of pions detected can be obtained by:

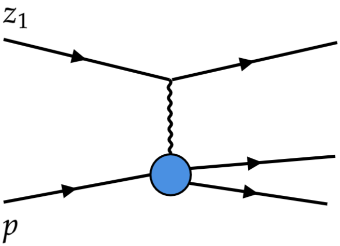
$$\frac{dR}{dt} = \mathcal{L} \times \sigma$$

Energy	Cross Section (b)	Luminosity ($cm^{-2}s^{-1}$)	Number of Pions (s^{-1})
5.00 TeV	5.39	6.2×10^{27}	3.34×10^{4}
2.76 TeV	4.63	1×10^{26}	4.63×10^{2}
5.20 TeV	5.43	6×10^{27}	3.28×10^{4}

The Photon Flux Approach

In this formalism the magnetic field is replaced by a flux of photons. The projectile becomes a source of photons.

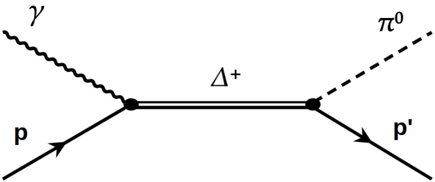
$$\sigma_{tot} = (\text{flux of photons}) \otimes \sigma_{\gamma}$$



Photoproduction of the Neutron Pion

In order to compute the total cross section, first, we need to compute the process:

$$\gamma + p \rightarrow p + \pi^0$$



For that, we need an expression for this reaction that can be used in high energy.

Jones and Scadron Formula

A simple parametrization for this process can be found in an article written by Jones and Scadron:

$$\frac{d\sigma_\gamma}{d\Omega} = \frac{\alpha}{12}\frac{\omega}{mW}\frac{\sin^2(\delta)}{\Gamma}[|F_+^*|^2 f(\theta) + |G_+^*|^2 g(\theta)]$$

Where :

$$W = \sqrt{m^2 + 2m\omega}$$

$$\begin{aligned} F_+^* &= (G_M^* - 3G_E^*) \\ G_+^* &= (G_M^* + G_E^*) \end{aligned}$$

$$\sin^2\delta = |m_\Delta\Gamma/(s - m_\Delta^2 + im_\Delta\Gamma)|^2$$

[H. F Jones, M. D. Scadron, Ann. Phys. 81, 1 (1973)]

Adjusting to the Data

Adjusting Jones and Scadron parametrizaton to the data:

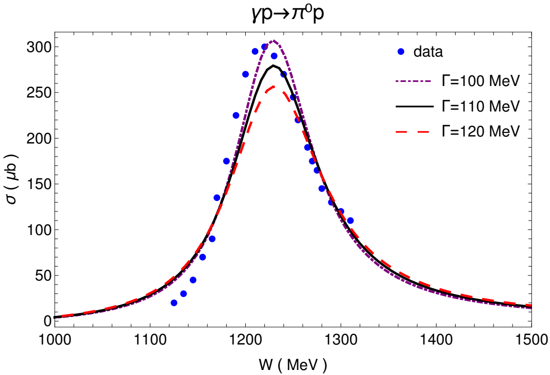
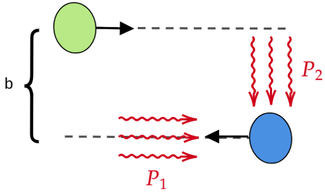


Figure: Jones and Scadron parametrization compared to the data for several Γ

Pascalutsa and Phillips, Phys. Rev. C 67, 055202 (2003)

Nucleus-Nucleus interaction



The number of equivalent photons incident on the target per unit area:

[Bertulani, Baur (1988) Phys. Reports 163, 5–6]

$$N(\omega, b) = \frac{Z_1^2 \alpha}{\pi} \left(\frac{xc}{b^2 v^2}\right)^2 [K_1^2(x) + 1/\gamma^2 K_0^2(x)]$$

where

$$x = \frac{\omega b}{\gamma v}$$

Equivalent Photon Approximation

The photon spectra is given by: [Bertulani, Baur (1988) Phys. Reports 163, 5–6]

$$n(\omega) = \frac{Z_1^2\alpha}{\pi}\left[2\xi K_0(\xi)K_1(\xi) - \xi^2(K_1^2(\xi) - K_0^2(\xi))\right]$$

and

$$\xi = \frac{\omega(R_A + R_B)}{\gamma}$$

The average energy of an emitted photon can be estimated as:

$$\bar{\omega} = \frac{\int_0^{\gamma m_N} n(\omega)\omega d\omega}{\int_0^{\gamma m_N} n(\omega) d\omega}$$

For the LHC, $\gamma \approx 1000$ which means $\omega = 10 GeV$ [arXiv:2011.00726 [hep-ph]]

The Total Cross Section

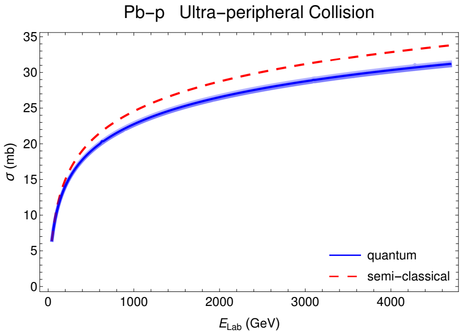
The cross section of this process can be written as a function of the nucleon-photon cross section: [Bertulani, Baur (1988) Phys. Reports 163, 5–6]

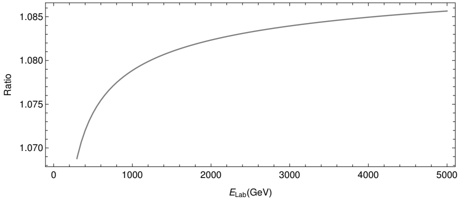
$$\sigma = \int \frac{d\omega}{\omega} n(\omega) \sigma_{\gamma N \to N\pi}$$

Where $n(\omega)$ is the photon spectrum.

Pb-p Ultra-peripheral Collision

Plotting the results for both formalisms:





Comparing the curves we can observe that the difference between the curves approaches 9% for high energies.

[arXiv:2011.00726 [hep-ph]]

Conclusions

- These results suggest that the classical approximation of the magnetic field reproduces most of the photon interaction in photoproduction in high energies.
- In heavy-ion collisions, one can treat classically the magnetic field.
- The photoproduction of the neutral pion can be used to test this idea.

Thank You!