Non-perturbative spectral properties of correlation functions at finite-temperature

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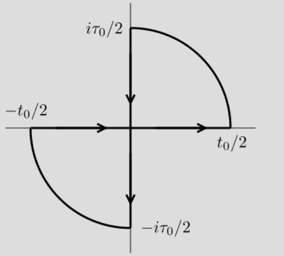
CRC-TR 211 Strong-interaction matter under extreme conditions

Local QFT beyond the vacuum

"Local QFT" → Define QFTs using a core set of physically-motivated assumptions, e.g. causality, Poincaré invariance, positive energy, ...

- This approach has led to many fundamental *non-perturbative* insights:

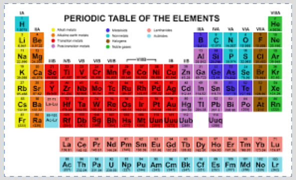
  → Relationship between Minkowski and Euclidean QFTs



  → *CPT* is a symmetry of *any* QFT

  → Connection between spin & particle statistics

  → Existence of dispersion relations, etc.



But... this framework only describes QFTs in the vacuum state

→ **Can one apply a similar approach to regimes where $T > 0$ or $\mu \neq 0$?**

Yes! *Important progress was made by J. Bros and D. Buchholz*

→ See: [*Z. Phys. C* 55 (1992), *Ann. Inst. H.Poincare Phys.Theor.* 64 (1996), *Nucl. Phys. B* 429 (1994), Nucl. Phys. B 627 (2002)]

Non-perturbative implications

- By demanding fields to be local ($[\Phi(x),\Phi(y)]=0$ for $(x\text{-}y)^2 < 0$) this imposes significant constraints on the structure of correlation functions

$\rightarrow$ For $T=1/\beta > 0$, the scalar spectral function has the representation:

$$\rho(p_0,\vec{p}) := \mathcal{F}\left[\langle\Omega_\beta|\,[\phi(x),\phi(y)]\,|\Omega_\beta\rangle\right] = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\;\epsilon(p_0)\,\delta\big(p_0^2-(\vec{p}-\vec{u})^2-s\big)\,\widetilde{D}_\beta(\vec{u},s)$$

Note: this is a ***non-perturbative*** representation!

"Thermal spectral density"

- In the limit of vanishing temperature one recovers the well-known *Källén-Lehmann* spectral representation:

$$\rho(p_0,\vec{p}) \xrightarrow{\beta\rightarrow\infty} 2\pi\,\epsilon(p_0)\int_0^\infty ds\;\delta\big(p^2-s\big)\,\rho(s)$$

e.g. $\rho(s)=\delta(s\text{-}m^2)$ for a massive free theory

Important question: *what does the thermal spectral density $\widetilde{D}_\beta(\boldsymbol{u},s)$ look like?*

Non-perturbative implications

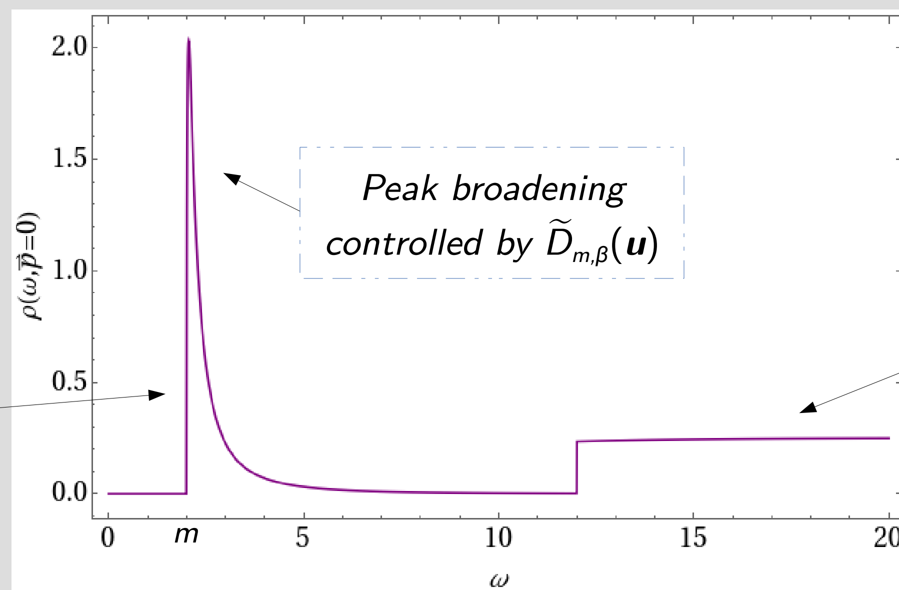
- A natural decomposition [Bros, Buchholz, *NPB* 627 (2002)] is:

$$\widetilde{D}_{\beta}(\vec{u},s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s-m^{2}) + \widetilde{D}_{c,\beta}(\vec{u},s)$$

"Damping factor"

Continuous component

Causes $T=0$ mass pole m to be screened by thermal effects



Fixes T-dependence of continuous spectral contributions

→ **Damping factors hold the key to understanding in-medium effects!**

Damping factors from Euclidean data

- The constraints imposed by locality offer new ways in which to understand, and compute, in-medium observables

- It turns out that these constraints also have significant implications in *Euclidean* spacetime

  → *Important to understand, since many non-perturbative techniques, e.g. lattice, functional methods (DSEs, FRG), are restricted to, or optimised for, calculations in imaginary time* τ

- In many instances $T>0$ Euclidean data is used to extract observables, e.g. spectral functions from $\mathcal{W}_E(\tau) = \int d^3x\, \mathcal{W}_E(\tau, \vec{x})$

$$\mathcal{W}_E(\tau) = \int_0^{\infty} \frac{d\omega}{2\pi} \frac{\cosh\left[\left(\frac{\beta}{2} - |\tau|\right)\omega\right]}{\sinh\left(\frac{\beta}{2}\omega\right)} \rho(\omega)$$

Determine $\rho(\omega)$ given $W_E(\tau)$

→ *Problem is ill-conditioned, need additional information!*

Damping factors from Euclidean FRG data

- However, locality constraints imply that particle damping factors $D_{m,\beta}(\boldsymbol{x})$ can be directly calculated from Euclidean data, avoiding the inverse problem [P.L., 2201.12180]

$$D_{m,\beta}(\vec{x}) \sim e^{|\vec{x}|m} \int_0^\infty \frac{d|\vec{p}|}{2\pi}\, 4|\vec{p}| \sin(|\vec{p}||\vec{x}|)\, \widetilde{G}_\beta(0,|\vec{p}|).$$

p-space Euclidean propagator

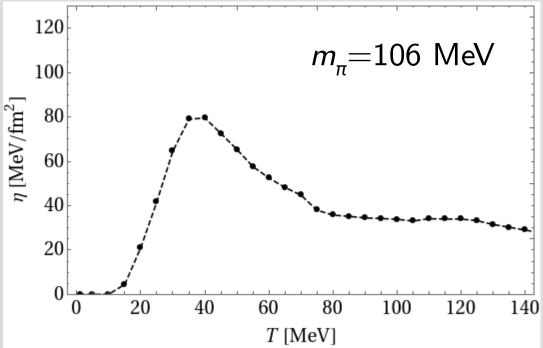
Holds for large separation $|\boldsymbol{x}|$

- In [P.L., R.-A. Tripolt, 2202.09142] pion propagator data from the quark-meson model (FRG calculation) was used to compute the damping factor at different values of T via the analytic relation above

- Fits to the resulting data were consistent with the form: $D_{m_\pi,\beta}(\vec{x}) = \alpha_\pi\, e^{-\gamma_\pi |\vec{x}|}$

- $D_{m,\beta}(\boldsymbol{x})$ can then be used as input for calculations, e.g. shear viscosity

Similar qualitative features to results from chiral perturbation theory



Damping factors from Euclidean FRG data

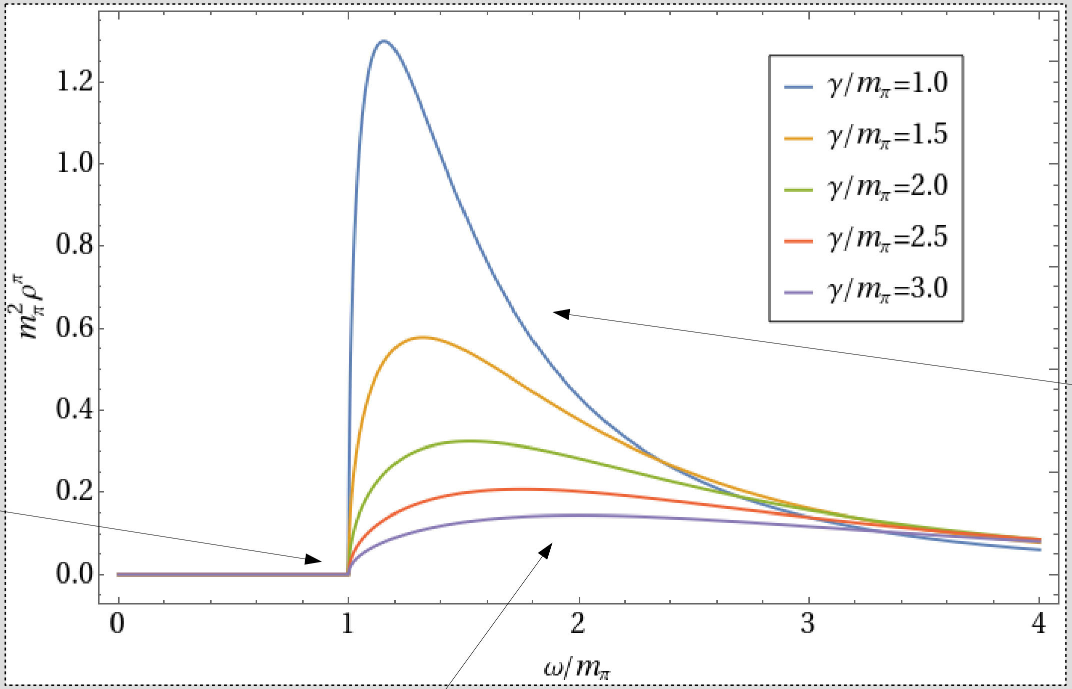
- Using the $T > 0$ spectral representation one finds:

$$D_{\pi,\beta}(\vec{x}) = \alpha\, e^{-|\vec{x}|\gamma}$$

Implies

$$\rho^{\pi}(\omega) = \epsilon(\omega)\theta(\omega^2 - m_\pi^2)\frac{4\alpha\,\gamma\sqrt{\omega^2 - m_\pi^2}}{(\omega^2 - m_\pi^2 + \gamma^2)^2}$$

Minimal energy needed to create particle with $\boldsymbol{p}=0$ $\rightarrow$ *cutoff at* $\omega = m_\pi$



Width $\gamma \rightarrow 0$ *as* $T \rightarrow 0$*, causing increasingly peaked behaviour*

As T *increases the width* γ *also increases, causing the thermal pion state to "melt"*

Damping factors from Euclidean lattice data

- In the FRG analysis *p*-space data was used to extract $D_{m,\beta}(\boldsymbol{x})$. Can one use *x*-space data instead? Yes!

  → A quantity of particular interest in lattice studies is the spatial correlator of particle-creating operators, defined:

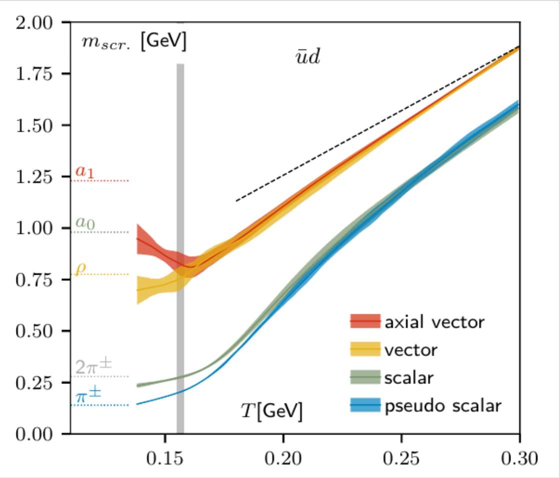
$$C(z) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} d\tau \, \mathcal{W}_E(\tau, \vec{x})$$

e.g. meson operators

$$\mathcal{W}_E(\tau, \vec{x}) = \langle \Omega_\beta | \overline{\psi} \Gamma \psi(x) \, \overline{\psi} \Gamma \psi(0) | \Omega_\beta \rangle$$

- Usually, the large-*z* behaviour of $C(z) \sim exp(-m_{scr}|z|)$ is used to extract particle *screening masses* m_{scr}

- This quantity is important for understanding phenomena such as quarkonium melting and (effective) chiral restoration in QCD



[HotQCD collaboration, *Phys. Rev. D* 100 (2019)]

Damping factors from Euclidean lattice data

- Spectral function constraints imply the following connection between the spatial correlator and thermal spectral density [P.L, O. Philipsen, *in preparation*]

$$C(z) = \frac{1}{2}\int_0^\infty ds \int_{|z|}^\infty dR\; e^{-R\sqrt{s}} D_\beta(R,s)$$

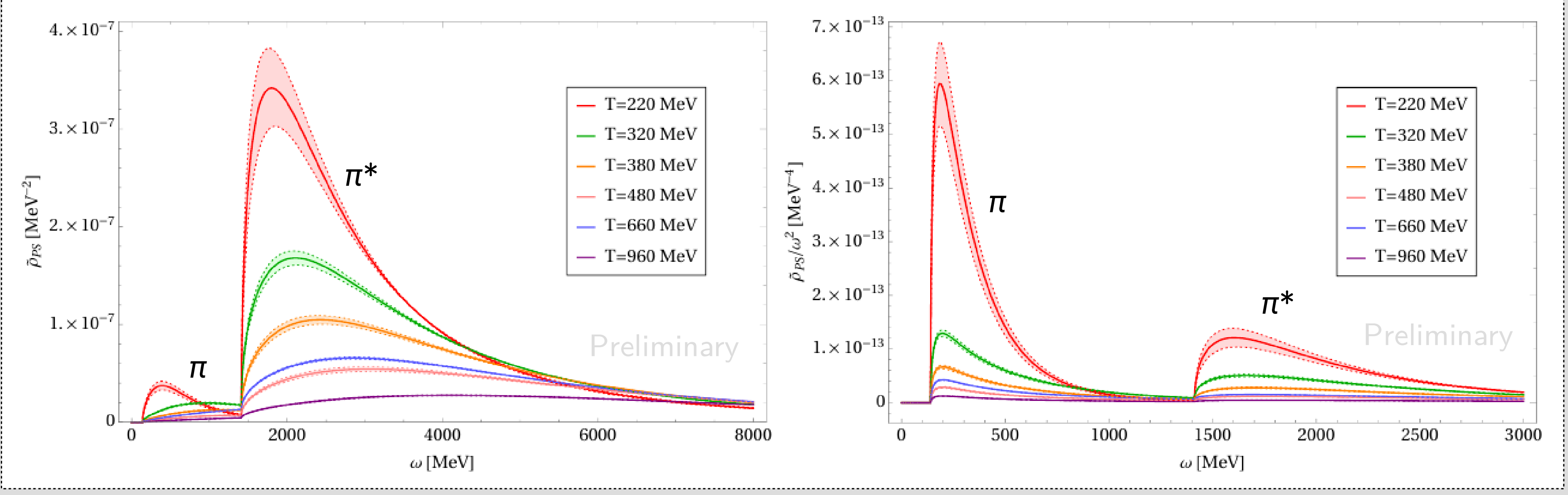
→ Damping factor of the lightest $T=0$ state: $D_{m,\beta}(|\vec{x}|=z) \sim -2e^{mz}\,\frac{dC(z)}{dz}, \quad z \to \infty$

- Once the damping factors of all contributing states are known, one can compute the corresponding spectral function, in particular $\rho(\omega, \boldsymbol{p}=0)$
- In QCD, perhaps the simplest spatial correlator example is that of the light quark pseudoscalar meson operator $\mathcal{O}^a_{\mathrm{PS}} = \overline{\psi}\gamma_5 \frac{\tau^a}{2}\psi$

<u>Goal</u>: Use lattice data from [Rohrhofer et al. *PRD* **100** (2019)] ($N_f=2$ with chiral fermions and physical masses) to compute the spectral function $\rho_{PS}(\omega)$

Damping factors from Euclidean lattice data

- **Step 1**: Perform fits to the spatial correlator data $C_{PS}(z)$ to obtain the functional dependence at different temperatures ($Ae^{-Bz}+Ce^{-Dz}$ ansatz describes the data very well)
- **Step 2**: Calculate the corresponding damping factors from $C_{PS}(z)$ (for π and π^*)
- **Step 3**: Use $D_{m,\beta}$ to compute $\rho_{PS}(\omega)$ analytically using the spectral representation



- The π and π^* dominate the spectral function at these T, and the π has a pronounced peak in some range $T > T_{pc}$ [P.L, O. Philipsen, *in preparation*]

  $\rightarrow$ ***Non-perturbative effects still important above T_{pc}***

- Screening masses are defined: $\boldsymbol{m_{scr} = m_{T=0} + \gamma}$, *where* $\gamma \rightarrow 0$ *for* $T \rightarrow 0$. This approach can also be used to extract the *T-dependence of* f_π / f_{π^*}

Summary & outlook

- Local QFT is an analytic framework that attempts to address the fundamental question "*what is a QFT?*"

- The framework can be extended to $T > 0$, and this has important implications, including:

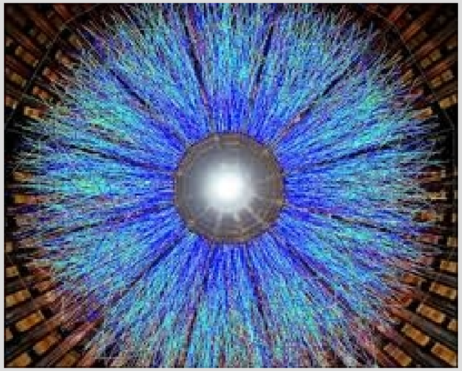
  → Spectral representations for thermal correlators

  → Ability to extract real-time observables from Euclidean data

  → Interpretation of screening masses

- So far only real scalar fields $\Phi(x)$ with $T > 0$ considered, but this approach can be extended (higher spin, $\mu \neq 0$). *Work in progress!*

→ This framework provides a way of obtaining **non-perturbative** insights into the phase structure of QFTs, and the resulting in-medium phenomena



[Brookhaven National Lab]

Backup: *Local QFT*

- In the 1960s, A. Wightman and R. Haag pioneered an approach which set out to answer the fundamental question "*what is a QFT?*"
- The resulting approach, *Local QFT*, defines a QFT using a core set of physically motivated axioms

Axiom 1 (Hilbert space structure). *The states of the theory are rays in a Hilbert space $\mathcal{H}$ which possesses a continuous unitary representation $U(a, \alpha)$ of the Poincaré spinor group $\overline{\mathscr{P}_+^\uparrow}$.*

Axiom 2 (Spectral condition). *The spectrum of the energy-momentum operator P^μ is confined to the closed forward light cone $\overline{V}^+ = \{p^\mu \mid p^2 \geq 0,\, p^0 \geq 0\}$, where $U(a, 1) = e^{iP^\mu a_\mu}$.*

Axiom 3 (Uniqueness of the vacuum). *There exists a unit state vector $|0\rangle$ (the vacuum state) which is a unique translationally invariant state in $\mathcal{H}$.*

Axiom 4 (Field operators). *The theory consists of fields $\varphi^{(\kappa)}(x)$ (of type (κ)) which have components $\varphi_l^{(\kappa)}(x)$ that are operator-valued tempered distributions in $\mathcal{H}$, and the vacuum state $|0\rangle$ is a cyclic vector for the fields.*

Axiom 5 (Relativistic covariance). *The fields $\varphi_l^{(\kappa)}(x)$ transform covariantly under the action of $\overline{\mathscr{P}_+^\uparrow}$:*

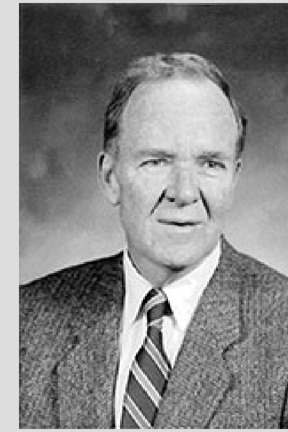
$$U(a, \alpha)\varphi_i^{(\kappa)}(x)U(a, \alpha)^{-1} = S_{ij}^{(\kappa)}(\alpha^{-1})\varphi_j^{(\kappa)}(\Lambda(\alpha)x + a)$$

where $S(\alpha)$ is a finite dimensional matrix representation of the Lorentz spinor group $\overline{\mathscr{L}_+^\uparrow}$, and $\Lambda(\alpha)$ is the Lorentz transformation corresponding to $\alpha \in \overline{\mathscr{L}_+^\uparrow}$.

Axiom 6 (Local (anti-)commutativity). *If the support of the test functions f, g of the fields $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$ are space-like separated, then:*

$$[\varphi_l^{(\kappa)}(f), \varphi_m^{(\kappa')}(g)]_\pm = \varphi_l^{(\kappa)}(f)\varphi_m^{(\kappa')}(g) \pm \varphi_m^{(\kappa')}(g)\varphi_l^{(\kappa)}(f) = 0$$

when applied to any state in $\mathcal{H}$, for any fields $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$.



A. Wightman

[R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and all that* (1964).]

R. Haag

[R. Haag, *Local Quantum Physics*, Springer-Verlag (1992).]

Backup: *Local QFT beyond the vacuum*

- **Idea**: Look for a generalisation of the standard axioms that is compatible with $T > 0$, and approaches the vacuum case for $T \to 0$

Axiom 1 (**Hilbert space structure**). *The states of the theory are rays in a Hilbert space $\mathcal{H}$ which possesses a continuous unitary representation $U(a, \alpha)$ of the Poincaré spinor group $\overline{\mathscr{P}_+^\uparrow}$.*

→ H_β is defined for fixed $\beta = 1/T$

Axiom 2 (**Spectral condition**). *The spectrum of the energy-momentum operator P^μ is confined to the closed forward light cone $\overline{V}^+ = \{p^\mu \mid p^2 \geq 0,\, p^0 \geq 0\}$, where $U(a, 1) = e^{iP^\mu a_\mu}$.*

→ Replaced by the KMS condition

$$\langle \Omega_\beta | \phi(x_1) \cdots \phi(x_k) \phi(x_{k+1}) \cdots \phi(x_n) | \Omega_\beta \rangle$$
$$= \langle \Omega_\beta | \phi(x_{k+1}) \cdots \phi(x_n) \phi(x_1 + i(\beta, \vec{0})) \cdots \phi(x_k + i(\beta, \vec{0})) | \Omega_\beta \rangle$$

Axiom 3 (**Uniqueness of the vacuum**). *There exists a unit state vector $|0\rangle$ (the vacuum state) which is a unique translationally invariant state in $\mathcal{H}$.*

→ Instead, thermal background state $|\Omega_\beta\rangle$

Axiom 4 (**Field operators**). *The theory consists of fields $\varphi^{(\kappa)}(x)$ (of type (κ)) which have components $\varphi_l^{(\kappa)}(x)$ that are operator-valued tempered distributions in $\mathcal{H}$, and the vacuum state $|0\rangle$ is a cyclic vector for the fields.*

✓ Fields are still distributions

Axiom 5 (**Relativistic covariance**). *The fields $\varphi_l^{(\kappa)}(x)$ transform covariantly under the action of $\overline{\mathscr{P}_+^\uparrow}$:*

$$U(a, \alpha) \varphi_i^{(\kappa)}(x) U(a, \alpha)^{-1} = S_{ij}^{(\kappa)}(\alpha^{-1}) \varphi_j^{(\kappa)}(\Lambda(\alpha)x + a)$$

where $S(\alpha)$ is a finite dimensional matrix representation of the Lorentz spinor group $\overline{\mathscr{L}_+^\uparrow}$, and $\Lambda(\alpha)$ is the Lorentz transformation corresponding to $\alpha \in \overline{\mathscr{L}_+^\uparrow}$.

→ The fields no longer transform under general unitary Lorentz transformations

Axiom 6 (**Local (anti-)commutativity**). *If the support of the test functions f, g of the fields $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$ are space-like separated, then:*

$$[\varphi_l^{(\kappa)}(f), \varphi_m^{(\kappa')}(g)]_\pm = \varphi_l^{(\kappa)}(f) \varphi_m^{(\kappa')}(g) \pm \varphi_m^{(\kappa')}(g) \varphi_l^{(\kappa)}(f) = 0$$

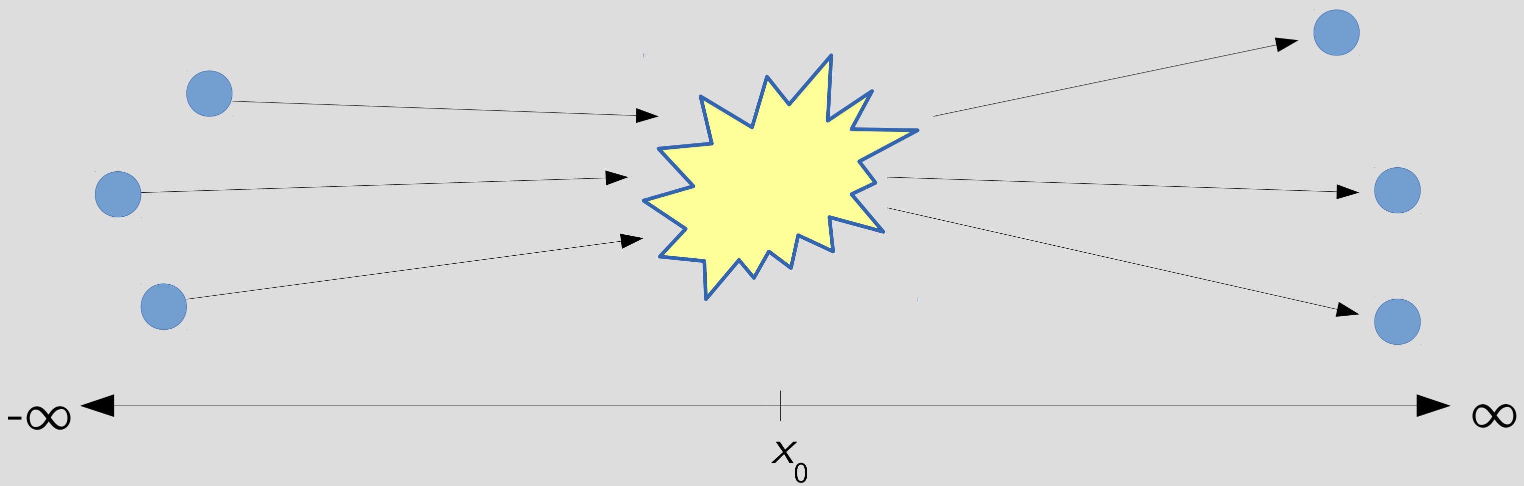
when applied to any state in $\mathcal{H}$, for any fields $\varphi_l^{(\kappa)}, \varphi_m^{(\kappa')}$.

✓ Locality is unaffected by the properties of the background state. *This is important!*

Backup: *Damping factors from asymptotic dynamics*

- Since all observable quantities are computed using correlation functions, which are characterised by *damping factors*, one can use these to gain new insights into the properties of QFTs when $T>0$

- It has been proposed [Bros, Buchholz, *NPB* 627 (2002)] that these quantities are controlled by the large-time x_0 dynamics of the theory



Important: Interactions with the thermal background persist, even for large x_0

$\rightarrow$ ***Need to take this into account in definition of scattering states!***

Backup: *Damping factors from asymptotic dynamics*

- Idea: thermal scattering states are defined by imposing an asymptotic field condition [*NPB* 627 (2002)]:

Asymptotic fields Φ_0 are assumed to satisfy dynamical equations, but only at large x_0

In Φ^4 theory

"Asymptotic coupling"

$$(\partial^2 + m^2)\phi_0(x) + \frac{\lambda}{3!}\phi_0^3(x) \xrightarrow{|x_0|\to\infty} 0$$

"Asymptotic mass"

- Given that the thermal spectral density has the decomposition

$$\widetilde{D}_\beta(\vec{u}, s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s - m^2) + \widetilde{D}_{c,\beta}(\vec{u}, s)$$

it follows that:

1. The continuous contribution to $\langle\Omega_\beta|\phi(x)\phi(y)|\Omega_\beta\rangle$ is **suppressed** for large x_0

2. The particle damping factor $\widetilde{D}_{m,\beta}(\boldsymbol{u})$ is **uniquely fixed** by the asymptotic field equation

- This means that the non-perturbative thermal effects experienced by particle states are entirely controlled by the asymptotic dynamics!

Backup: Φ^4 theory for $T > 0$

- Applying the asymptotic field condition for Φ^4 theory, the resulting damping factors have the form [*NPB* 627 (2002)]:

$\rightarrow$ **For $\boldsymbol{\lambda} < 0$:**

$$D_{m,\beta}(\vec{x}) = \frac{\sin(\kappa|\vec{x}|)}{\kappa|\vec{x}|}$$

$\rightarrow$ **For $\boldsymbol{\lambda} > 0$:**

$$D_{m,\beta}(\vec{x}) = \frac{e^{-\kappa|\vec{x}|}}{\kappa_0|\vec{x}|}$$

where κ is defined with $r = m/T$:

$$\kappa = T\sqrt{|\lambda|}K(r), \qquad K(r) = \sqrt{\int \frac{d^3\hat{\vec{q}}}{(2\pi)^3 2\sqrt{|\hat{\vec{q}}|^2 + r^2}} \frac{1}{e^{\sqrt{|\hat{\vec{q}}|^2 + r^2}} - 1}}$$

$\rightarrow$ *The parameter κ has the interpretation of a thermal width: $\kappa \rightarrow 0$ for $T \rightarrow 0$*, or equivalently κ^{-1} *is mean-free path*

- Now that one has the exact dependence of $D_{m,\beta}(\boldsymbol{x})$ on the external physical parameters, in this case T, m and λ, one can use this to calculate observables ***analytically***

Backup: Φ^4 theory for $T > 0$

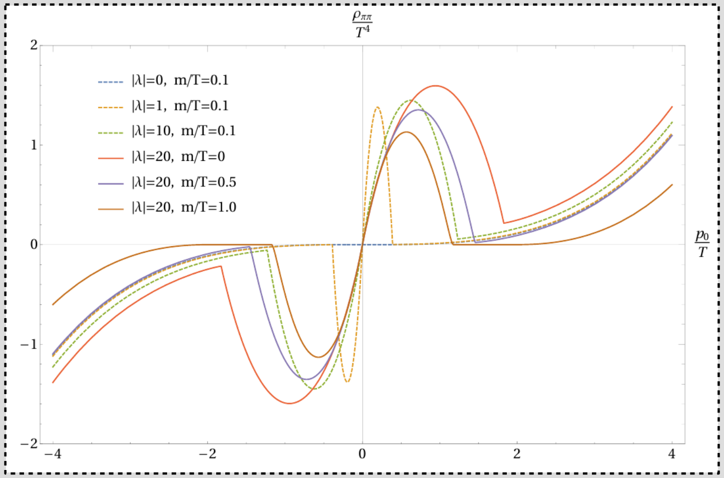
- Of particular interest is the *shear viscosity* η, which measures the resistance of a medium to sheared flow

  $\rightarrow$ This quantity can be determined from the spectral function of the spatial traceless energy-momentum tensor

$$\rho_{\pi\pi}(p_0) = \lim_{\vec{p}\to 0} \mathcal{F}\left[\langle\Omega_\beta|\left[\pi^{ij}(x), \pi_{ij}(y)\right]|\Omega_\beta\rangle\right](p)$$

  ... and η is recovered via the *Kubo relation* $\eta = \frac{1}{20} \lim_{p_0\to 0} \frac{d\rho_{\pi\pi}}{dp_0}$

- Using $D_{m,\beta}(\boldsymbol{x})$ for $\lambda < 0$, the EMT spectral function $\rho_{\pi\pi}$ has the form:

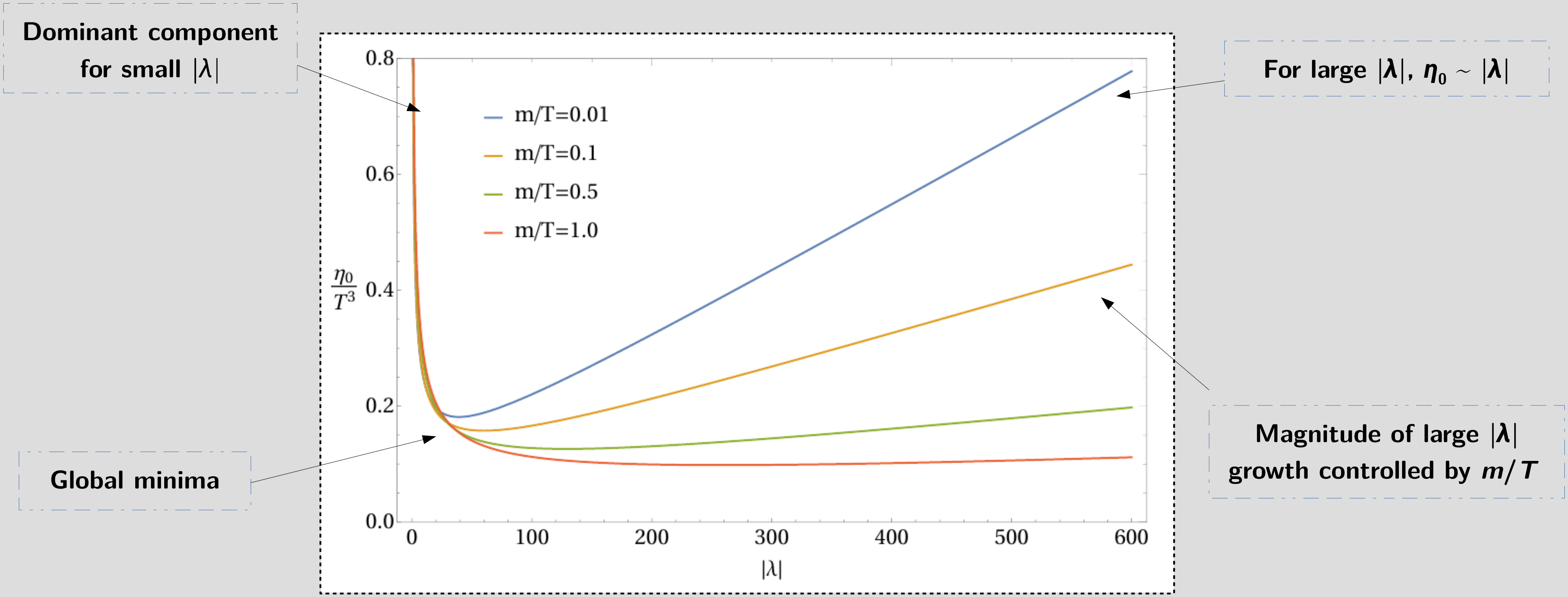


- The presence of interactions causes resonant peaks to appear $\rightarrow$ peaked when $p_0 \sim \kappa = 1/\ell$
- For $\lambda \to 0$ the free-field result is recovered, as expected
- The dimensionless ratio m/T controls the magnitude of the peaks

Backup: Φ^4 theory for $T > 0$

- Applying Kubo's relation, the shear viscosity η_0 arising from the asymptotic states can be written [P.L., R.-A. Tripolt, J. M. Pawlowski, D. H. Rischke, PRD 104, 065010 (2021)]

$$\eta_0 = \frac{T^3}{15\pi}\left[\frac{\mathcal{K}_3\left(\frac{m}{T},0,\infty\right)}{\sqrt{|\lambda|}} + \sqrt{|\lambda|}\,\mathcal{K}_1\left(\frac{m}{T},0,\infty\right) + \frac{\mathcal{K}_4\left(\frac{m}{T},\sqrt{|\lambda|}K\left(\frac{m}{T}\right),\sqrt{|\lambda|}K\left(\frac{m}{T}\right)\right)}{4|\lambda|}\right]$$



$\rightarrow$ For fixed coupling, η_0/T^3 is entirely controlled by functions of m/T

Backup: Φ^4 theory for $T > 0$

- What about the case $\lambda > 0$? $\rightarrow$ η_0 *diverges!*

  Why? – *The particle damping factor* $D_{m,\beta}(\boldsymbol{u})$ *does not decay rapidly enough at large momenta*

- This characteristic is related to the "bad" UV behaviour of the quartic interaction, i.e. the triviality of Φ^4 appears to have an impact beyond $T=0$!

- In [PRD 104, 065010 (2021)] it was shown more generally that the finiteness of η_0 is related to the existence of thermal equilibrium

If the KMS condition holds $\Longrightarrow$ η_0 is finite

- This procedure demonstrates that asymptotic dynamics can be used to explore the non-perturbative properties of QFTs when T>0

  $\rightarrow$ *Can also calculate other observables, e.g. transport coefficients, entropy density, pressure, etc.*

Backup: *spectral representations*

- For thermal asymptotic states, the spectral function $\rho_{\pi\pi}$ has the form

$$\rho_{\pi\pi}(p_0) = \sinh\left(\frac{\beta}{2}p_0\right)\int \frac{d^3\vec{q}}{(2\pi)^4}\frac{2}{3}|\vec{q}|^4 \int_{-\infty}^{\infty} dq_0\, \frac{\widetilde{C}_\beta(q_0,\vec{q})\,\widetilde{C}_\beta(p_0-q_0,\vec{q})}{\sinh\left(\frac{\beta}{2}q_0\right)\sinh\left(\frac{\beta}{2}(p_0-q_0)\right)}$$

... which after applying the generalised KL representation, together with the Kubo relation, implies

$$\begin{aligned}\eta_0 = \frac{T^5}{240\pi^5}\int_0^\infty ds\int_0^\infty dt\int_0^\infty d|\vec{u}|\int_0^\infty d|\vec{v}|\,|\vec{u}||\vec{v}|\,\widetilde{D}_\beta(\vec{u},s)\,\widetilde{D}_\beta(\vec{v},t) \\ \times\Bigg[4\left[1+\epsilon(|\vec{u}|-|\vec{v}|)\right]\left\{\frac{|\vec{v}|}{T}\mathcal{I}_3\left(\frac{\sqrt{t}}{T},0,\infty\right)+\frac{|\vec{v}|^3}{T^3}\mathcal{I}_1\left(\frac{\sqrt{t}}{T},0,\infty\right)\right\} \\ +\left\{\mathcal{I}_4\left(\frac{\sqrt{t}}{T},\frac{|\vec{v}|}{T},\frac{s-t+(|\vec{u}|+|\vec{v}|)^2}{2(|\vec{u}|+|\vec{v}|)T}\right)+\epsilon(|\vec{u}|-|\vec{v}|)\,\mathcal{I}_4\left(\frac{\sqrt{t}}{T},\frac{|\vec{v}|}{T},\frac{s-t+(|\vec{v}|-|\vec{u}|)^2}{2(|\vec{v}|-|\vec{u}|)T}\right)\right\}\Bigg]\end{aligned}$$

- The model dependence of η_0 factorises, and is controlled by the thermal spectral density $D_\beta(\mathbf{u},s)$

Backup: *Euclidean spectral relations*

- One can use the assumptions of local QFT at finite T to put constraints on the the structure of Euclidean correlation functions

→ From the KMS condition and locality:

$$\mathcal{W}_E(\tau,\vec{x}) = \frac{1}{\beta}\sum_{N=-\infty}^{\infty} w_N(\vec{x})e^{\frac{2\pi i N}{\beta}\tau}$$

- The Fourier coefficients of the Euclidean two-point function are then related to the thermal damping factors as follows [P.L., 2201.12180]:

$$w_N(\vec{x}) = \frac{1}{4\pi|\vec{x}|}\left[D_m(\vec{x})e^{-|\vec{x}|\sqrt{m^2+\omega_N^2}} + \int_0^\infty ds\, e^{-|\vec{x}|\sqrt{s+\omega_N^2}}D_c(\vec{x},s)\right]$$

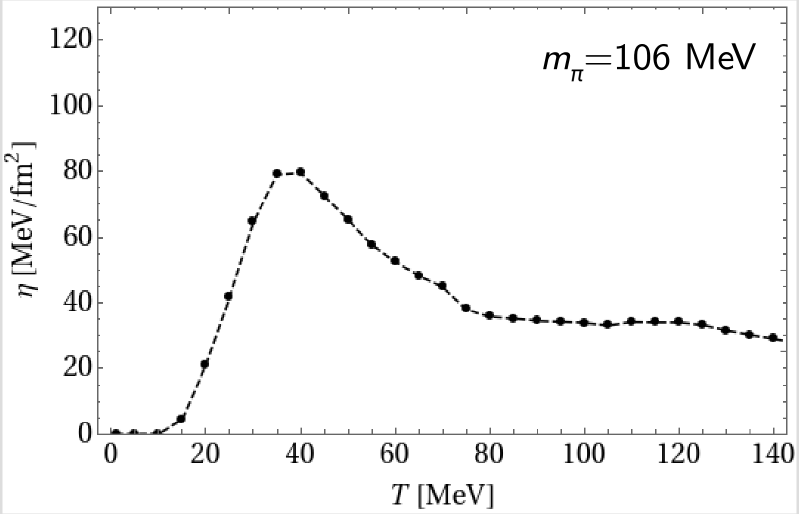
→ *The continuous component $D_c(\mathbf{x},s)$ is exponentially suppressed!*

- $\omega_N = 2\pi NT$ are the Matsubara frequencies. For $N=0$ this leads to:

$$\int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} d\tau\, \mathcal{W}_E(\tau,\vec{x}) \sim \frac{1}{4\pi|\vec{x}|}D_{m,\beta}(\vec{x})\, e^{-|\vec{x}|m}$$

Backup: *damping factors from Euclidean data*

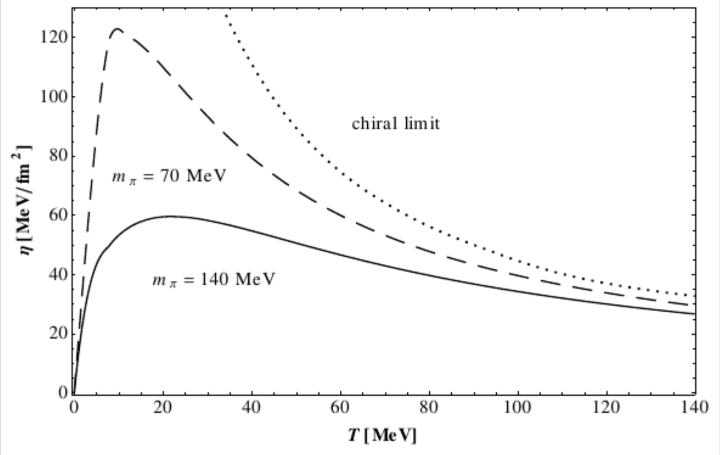
- Using the analytic relations derived in [PRD 104, 065010 (2021)] for the shear viscosity as a function of the damping factor, the numerically extracted values for $D_{\pi,\beta}(\boldsymbol{x})$ can be used to compute the shear viscosity



η vanishes for $T\to 0$, and appears to level out at large T

- Can compare these results with those obtained using chiral perturbation theory

$\to$ *Very similar qualitative features!*



[R. Lang, N. Kaiser, W. Weise, EPJ A 48 (2012)]