Superfluid vortices:
from dense QCD to helium-4

Laurence Yaffe
University of Washington

based on work with Aleksey Cherman, Srimoyee Sen & Theodore Jacobson
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in strong and electromagnetic matter

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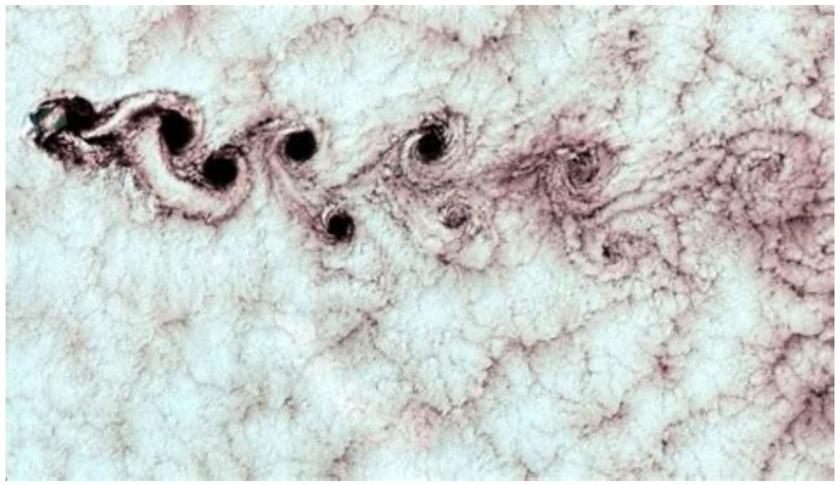

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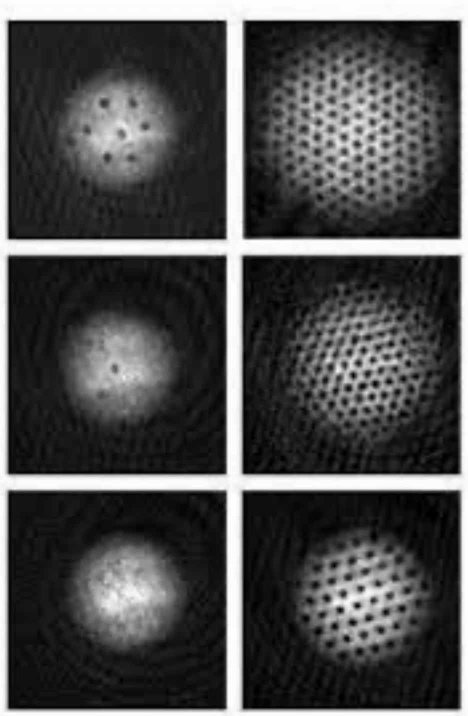

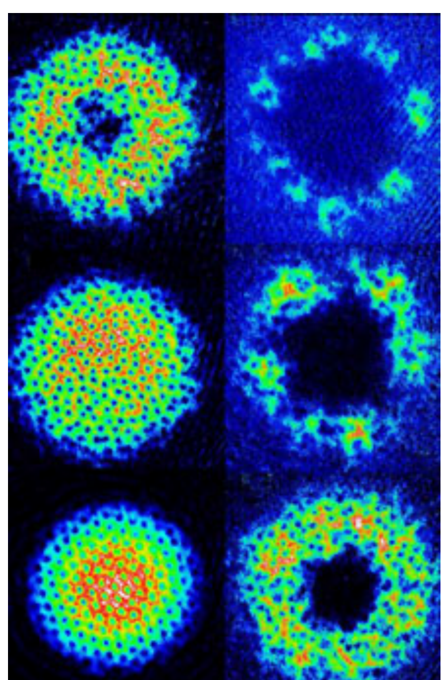

Peter Engels, JILA

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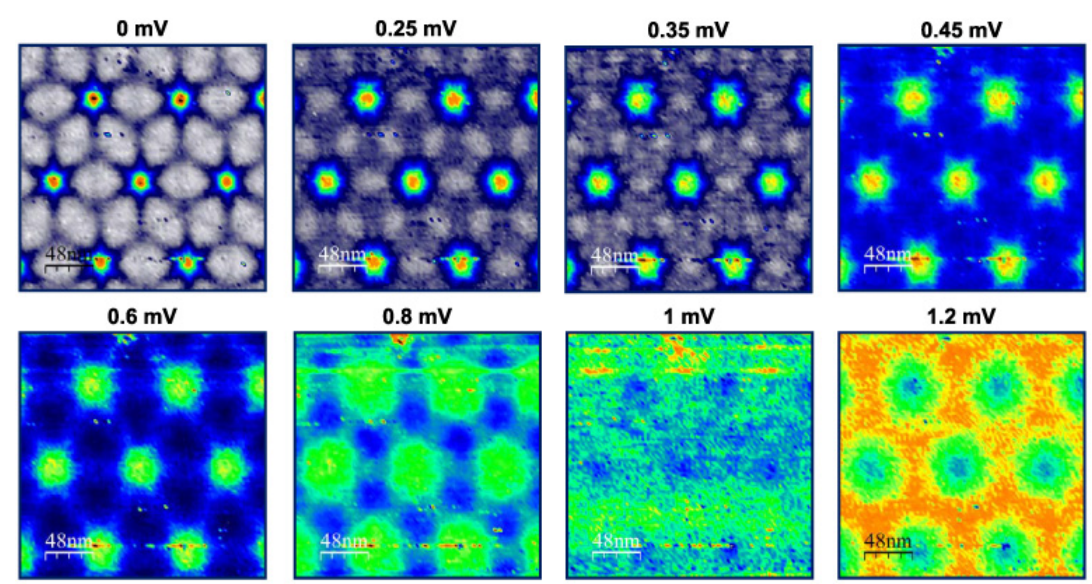

Suderow, Guillamón, Rodrigo, Vieira, 2014

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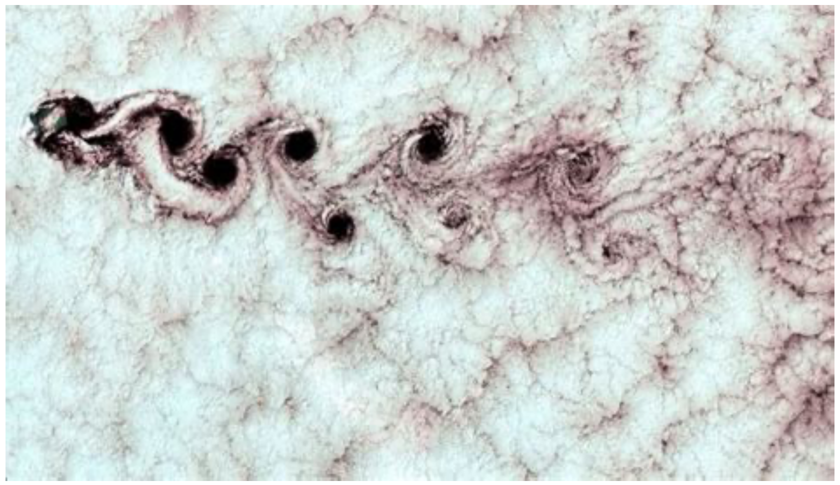

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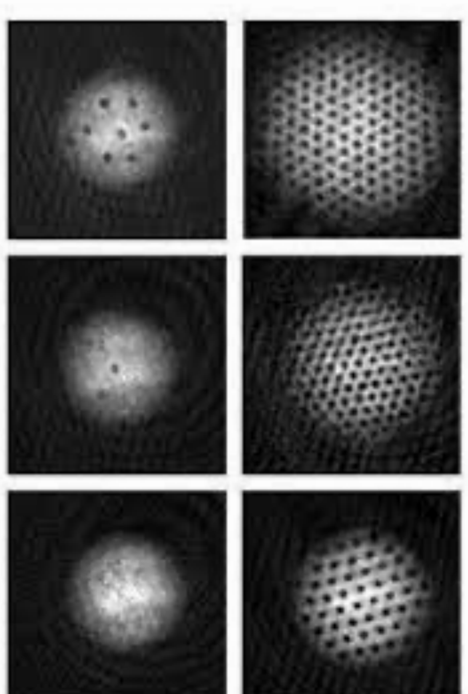

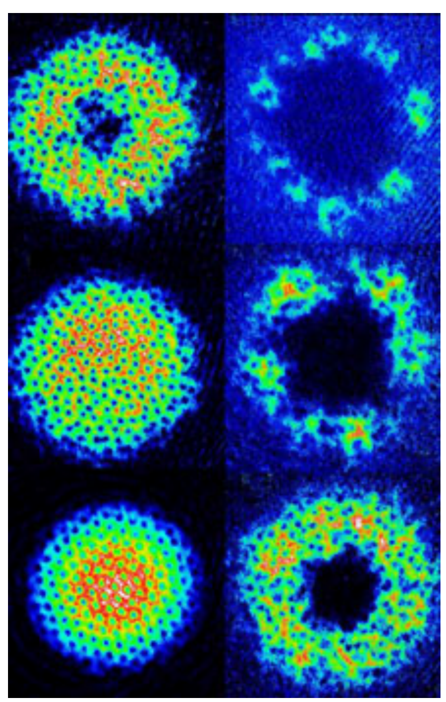

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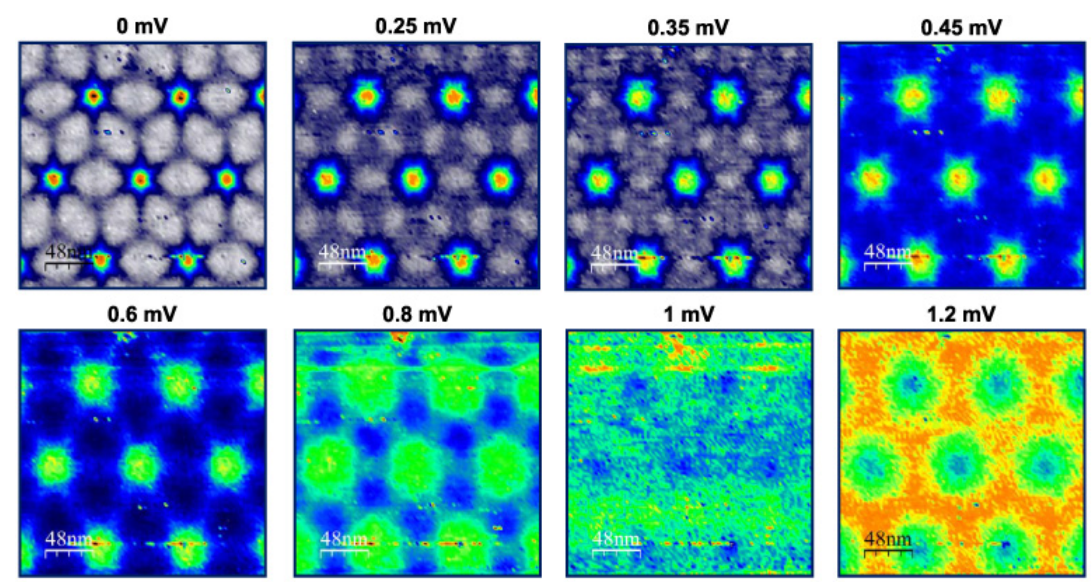

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superfluid vortices

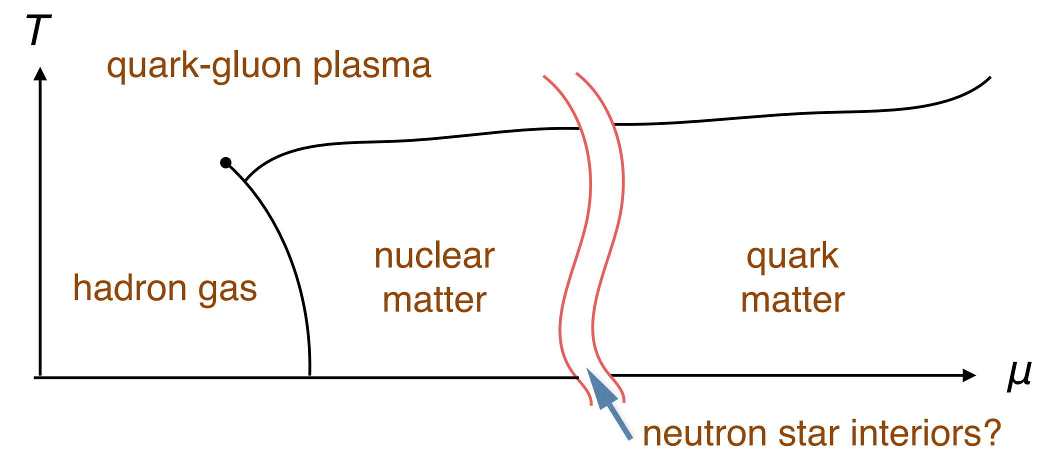
- signatures of spontaneously broken global $U(1)$ symmetry
- topologically stable collective excitations, $\pi_1(U(1)) = \mathbb{Z}$
- non-zero winding of order parameter, $\oint d\ell \cdot \nabla(\arg\langle\phi\rangle) = -2\pi\, n \in \mathbb{Z}$
- non-zero superfluid flow $\vec{v}_s = -\vec{\nabla}(\arg\langle\phi\rangle)/M$
- non-zero vorticity $\vec{\omega} \equiv \vec{\nabla} \times \vec{v}_s$
- quantized circulation, $\mathscr{C} = \oint d\ell \cdot \vec{v}_s = \int_{\mathcal{S}} d\Sigma \cdot \vec{\omega} = (2\pi/M)\, n$

nuclear matter

- dense “confined” hadronic phase

  ≈ Fermi liquid of neutrons

  - strongly coupled dynamics



- neutron pairing & condensation

  ➡ spontaneously broken $U(1)_B$ baryon number symmetry

  ➡ neutral superfluid

  ➡ vortex lattice in rotating neutron star interiors

- sensitively dependent on E&M, isospin breaking, …

idealized hadronic matter

- pure QCD, 3 flavor $SU(3)_f$ symmetric
  - ignore electromagnetism & weak interactions
  - degenerate, stable $n, p, \Lambda, \cdots$

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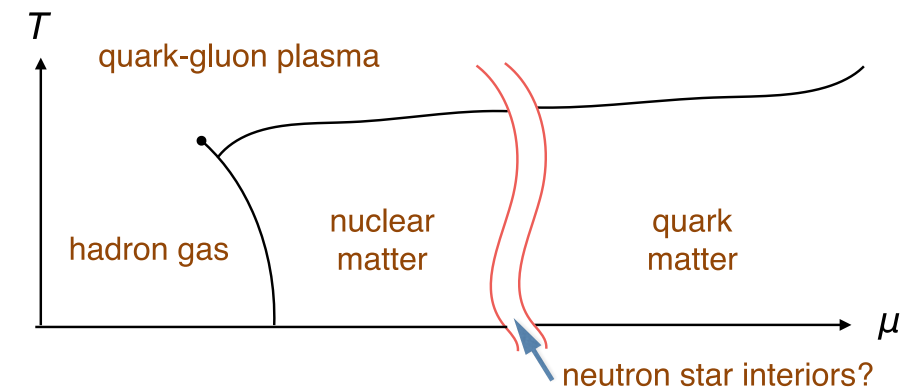
high density quark matter

- asymptotic freedom ⇒ weakly coupled
- dense deconfined "CFL"phase

  ≈ Fermi liquid of quarks

- quark pairing & di-quark condensation

  ➡ "color superconductor"



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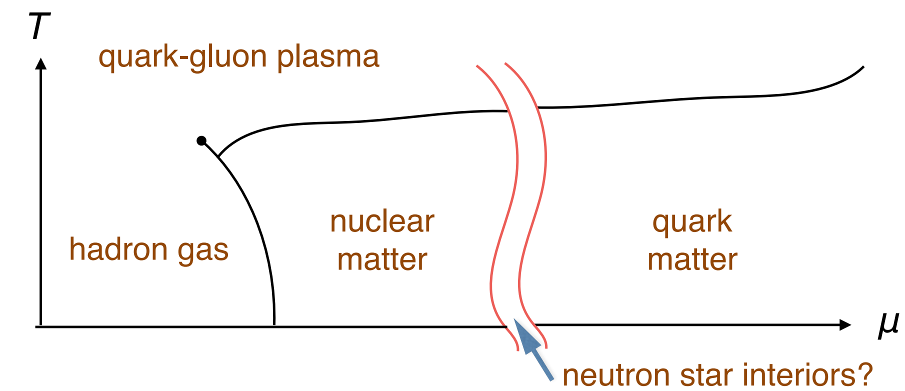

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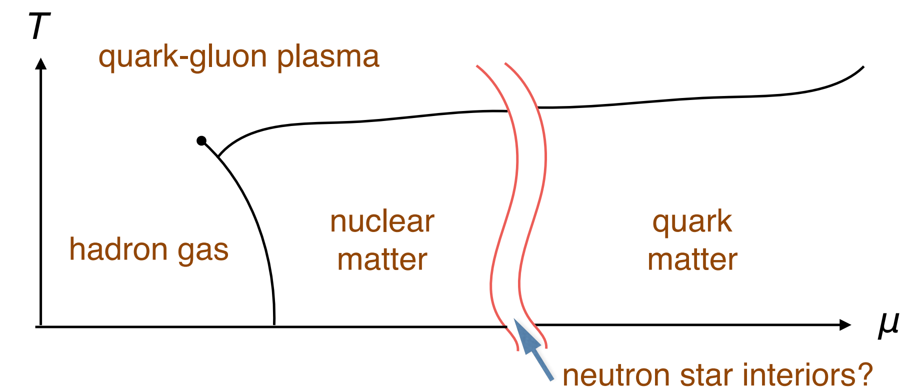

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  ➡ ~~“color superconductor”~~

  - spontaneously broken $U(1)_B$ baryon number symmetry
  - unbroken $SU(3)$ flavor symmetry, fully Higged $SU(3)_{\text{color}}$
  - gauge symmetries cannot truly break

  ➡ neutral superfluid

phase continuity?

- Schäfer-Wilczek conjecture, 1998:
  - identical symmetry realizations, corresponding low-lying excitations
  - no distinguishing local order parameters
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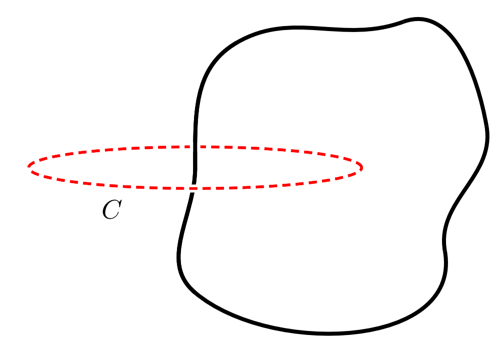
- compatible vortex properties? No!
    - phase of Wilson loop linking vortex = "topological" order parameter

$$\mathcal{O} \equiv \lim_{r\to\infty} \arg \langle e^{\oint A} \rangle_{\text{vortex}} = \begin{cases} \pm 2\pi/3 & \text{CFL phase} \\ 0 & \text{hadronic phase} \end{cases}$$



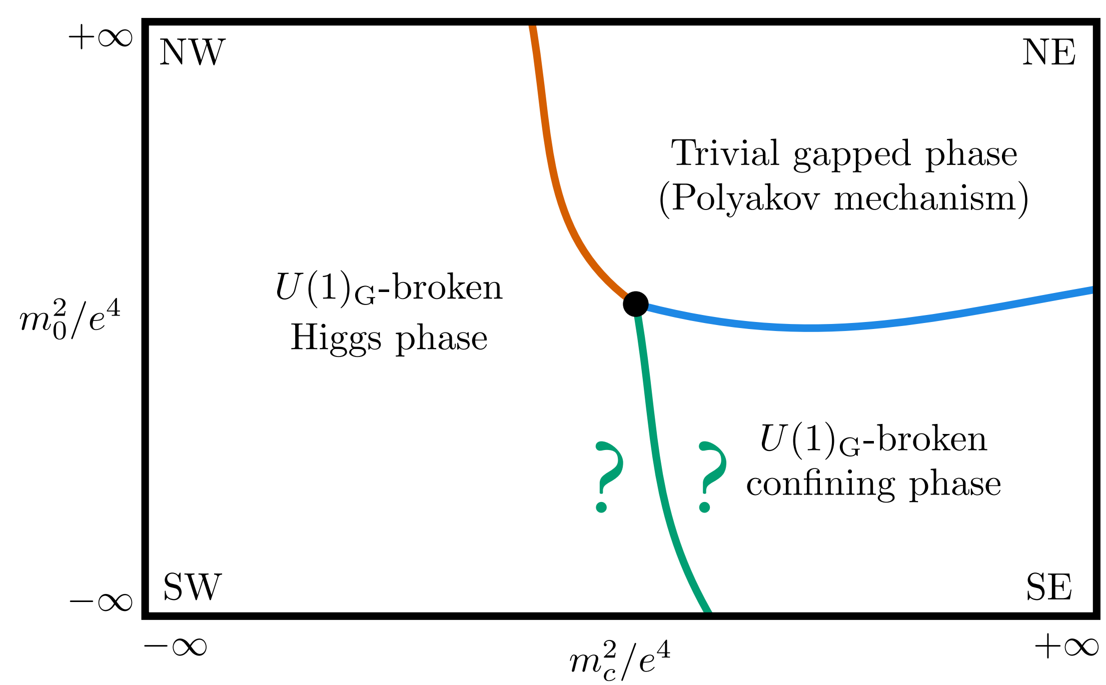
➡ non-trivial particle-vortex braiding statistics in CFL phase

- inconsistent with smooth phase continuity

3D Abelian-Higgs model

$$S = \int d^3x \left[\frac{1}{4e^2} F_{\mu\nu}^2 + |D_\mu \phi_+|^2 + |D_\mu \phi_-|^2 + m_c^2 \left(|\phi_+|^2 + |\phi_-|^2 \right) + |\partial_\mu \phi_0|^2 + m_0^2 \, |\phi_0|^2 \right.$$
$$- \epsilon \left(\phi_+ \phi_- \phi_0 + \text{h.c.} \right) + \lambda_c \left(|\phi_+|^4 + |\phi_-|^4 \right) + \lambda_0 |\phi_0|^4$$
$$\left. + g_c \left(|\phi_+|^6 + |\phi_-|^6 \right) + g_0 |\phi_0|^6 + \cdots + V_{\rm m}(\sigma) \right] .$$

- 3D $U(1)$ gauge theory ⇒ monopole-driven confinement
- $\phi_\pm \approx$ diquark condensates, $\phi_0 \approx$ dibaryon interpolating field
- single $U(1)_{\rm G}$ global symmetry ≈ baryon number symmetry



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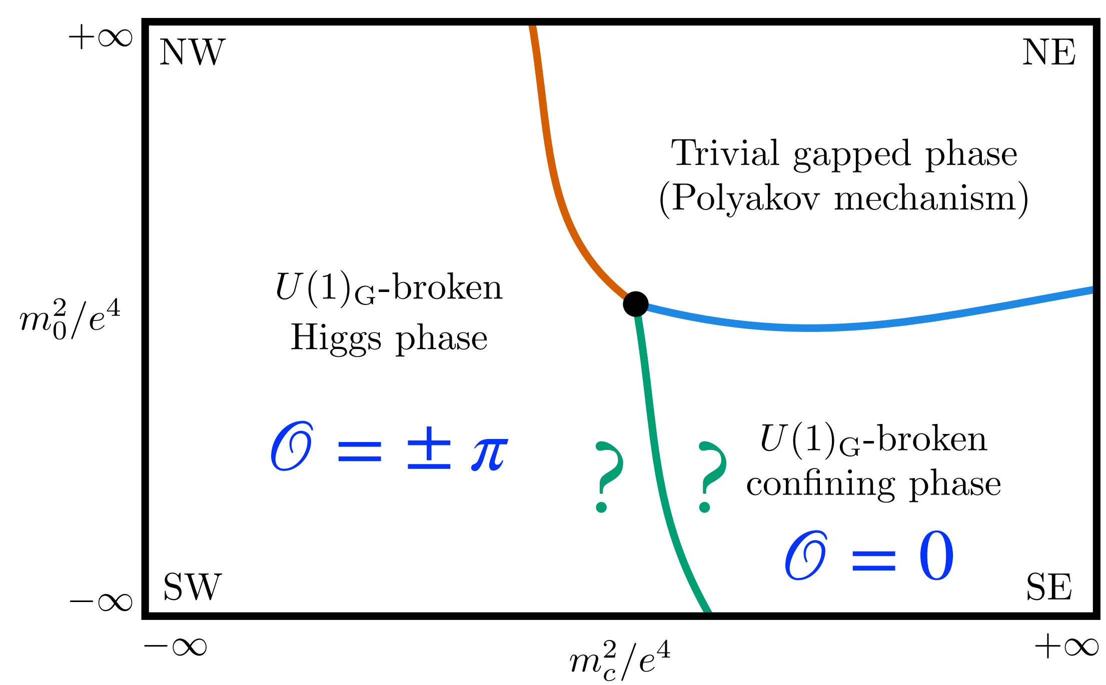

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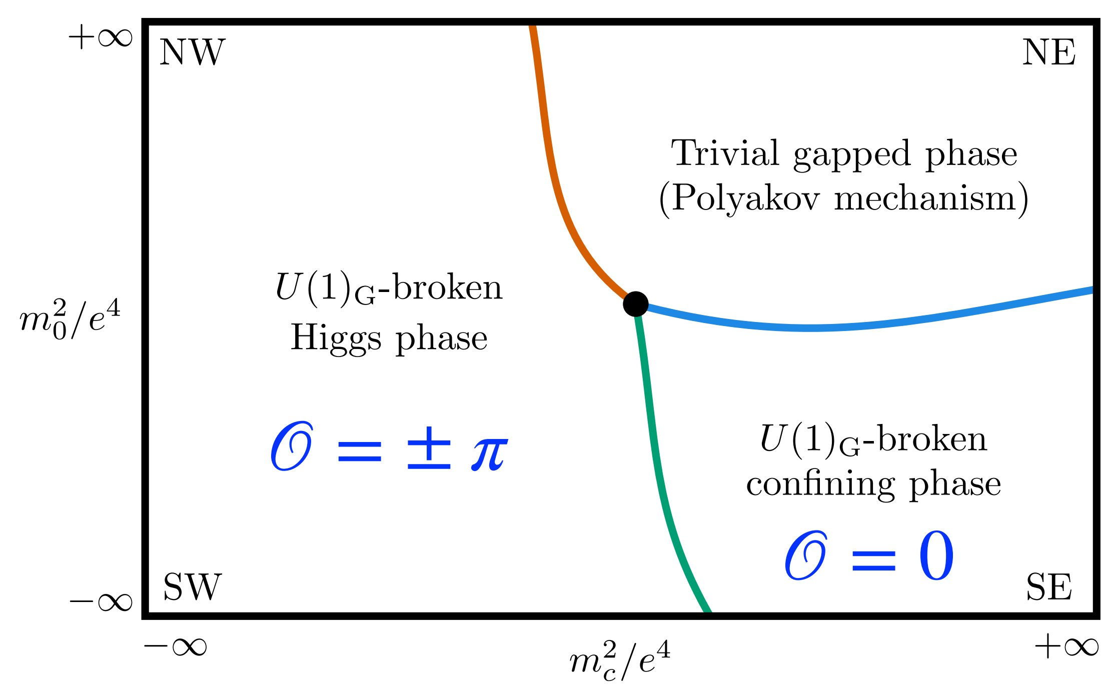

atomic superfluids

- superfluidity due to Bose-condensed neutral spinless atoms
- can vortices carry non-zero magnetic flux $\Phi_B \equiv \left\langle \oint d\ell \cdot A \right\rangle_{\text{vortex}}$?
  - no symmetry requires Φ_B to vanish
  - long distance EFT: neutral condensate + E&M, $S = S_\phi + S_{\text{EM}} + S_{\phi,\,\text{EM}}$

$$S_\phi = \int dt\, d^3x \left[\phi^\dagger \left(i\partial_t + \mu + \tfrac{\nabla^2}{2M}\right)\phi - \frac{f_4\, a}{M}\,|\phi|^4 + \cdots\right]$$

$$S_{\text{EM}} = \tfrac{1}{2}\int dt\, d^3x\, \left(\mathbf{E}^2 - c^2\mathbf{B}^2 + \cdots\right)$$

$$\begin{aligned} S_{\phi,\,\text{EM}} = \int dt\, d^3x\, \Big[& b\, e\, a^2\, (\boldsymbol{\rho}\cdot\mathbf{E} + \boldsymbol{\omega}\cdot\mathbf{B}) \\ & + \tfrac{1}{2} c_E\, a^3\, \left(n\,\mathbf{E}^2 - 2\,\mathbf{j}\cdot(\mathbf{E}\times\mathbf{B})\right) \\ & - \tfrac{1}{2} c_M\, e^4 a^3\, \left(n\,\mathbf{B}^2 - \tfrac{2}{c^2}\,\mathbf{j}\cdot(\mathbf{E}\times\mathbf{B})\right) + \cdots \Big] \end{aligned}$$

M = atomic mass, Z = nuclear charge

a = atomic charge radius

$n = \phi^\dagger\phi$ = atom density

$\mathbf{j} = \frac{i}{2M}((\nabla\phi^\dagger)\phi - \phi^\dagger\nabla\phi)$ = atom number flux

$\rho = \nabla n$ = density gradient

$\omega = \nabla\times\mathbf{j}$ = vorticity

$c_E, c_M \approx$ dielectric, diamagnetic suscecptibilities: $\epsilon/\epsilon_0 = 1 + c_E\, a^3\bar{n}$, $\mu_0/\mu = 1 + c_M\,(e^2/c)^2\, a^3\,\bar{n}$

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b = dimensionless $O(1)$ coefficient = $Z/6$ for low density

vortex magnetic effect

- $b \neq 0 \Rightarrow$ vorticity sources magnetic flux
- underlying physics: density gradient induced polarization
  - neutral atoms: electrostatic potential $\propto$ particle density
    - $\rho(\mathbf{r}) = -\nabla^2(\frac{Zea^2}{6} f(\mathbf{r}))$, $f(\mathbf{r}) \approx$ smeared $\delta^3(\mathbf{r})$, so $\tilde{\rho}(\mathbf{k}) = \frac{Ze\,a^2}{6}\mathbf{k}^2 + O(\mathbf{k}^4)$
    - $\Phi(\mathbf{x}) = (-\nabla^2)^{-1}\rho/\epsilon_0 = \frac{Zea^2}{6\epsilon_0}\sum_i f(\mathbf{x}-\mathbf{x}_i) \approx \frac{Zea^2}{6\epsilon_0} n(\mathbf{x})$ A.M. Kosevich, 2005
  - density gradient $\Rightarrow$ electric polarization $\mathbf{P} = \frac{1}{6}Zea^2\nabla n$
  - rotating polarization $\Rightarrow$ magnetization $\mathbf{M} = \mathbf{P}\times\mathbf{v}$
- superfluid vortex: $\nabla n \propto \hat{r}$, $\mathbf{v}_s = (\hbar/M)\,\hat{\theta}/r$, $\mathbf{M} \propto \hat{z}$
  - magnetic flux $\Phi_B = \mu_0 \int d\Sigma\cdot\mathbf{M} = \frac{\pi}{3}\mu_0\hbar\,Zea^2\,n/M = \frac{2}{3}Z\alpha\,\lambda_C\,na^2\,\Phi_0$

$\Phi_0 = \pi\hbar/e$ = superconducting flux quantum

observability?

$$\frac{\Phi_B}{\Phi_0} = 8\pi\alpha^2\, b\, n\, a^2 a_{\rm B}\, \frac{m_e}{M} = 7\times 10^{-7}\, \frac{b}{A} \left(\frac{a_{\rm B}}{a}\right) n a^3$$

bosonic atoms,
atomic number A

- largest effect in dense superfluid, at limit of EFT validity
- superfluid helium:
  - helium charge radius $a \approx a_{\rm B}$, diluteness parameter $na^3 \approx 0.0034$

$$\frac{\Phi_B}{\Phi_0} \approx 1\times 10^{-10}$$

  - potentially observable:
    - quantum-limited SQUID noise $\sim 45\times 10^{-9}\, \Phi_0/\sqrt{Hz}$

conclusions

- vortices, gauge holonomies, effective field theory, macroscopic electrodynamics, dense QCD, cold atoms: combining old ingredients can make interesting new stories…

The end