Bubble nucleation from vacuum initial conditions

Testing the classical-statistical approximation for quantum tunneling

[arxiv.2206.08691]

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Overview

Classical-statistical simulation as approximation for out-of-equilibrium QFT

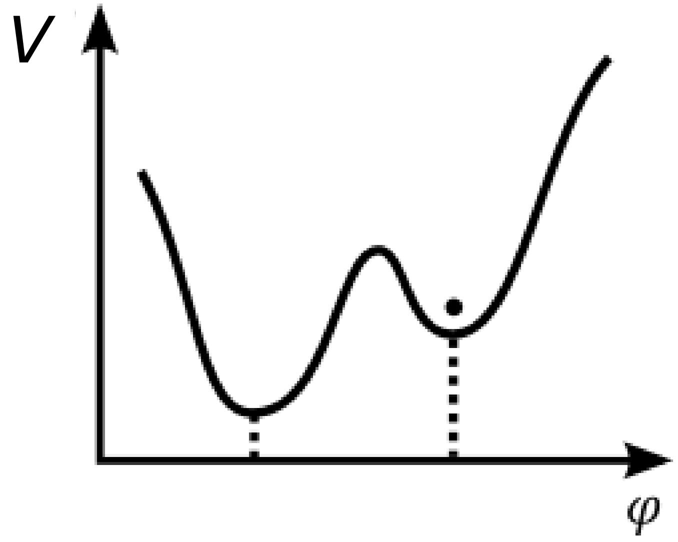
In particular:
initial vacuum fluctuations

This work:
Re-analysis of vacuum decay in a toy model

A group observed
CS ≈ QFT
In this model

1st order PT
Bubble nucleation
Vacuum / thermal state
Parameter space
dimensionality

1st order phase transition



- Transition from local minimum to global minimum
- Dynamics of phase transition:

Classical dynamics vs quantum tunneling

Bubble is formed – E_{crit} is required	No extra energy is required!
For a thermal state:	For vacuum state:
$$P \propto e^{-\frac{E_{crit}}{T}}$$	$$P \propto e^{-S_B}$$
"Bubble lives in space"	"Bubble lives in space-time" (Instanton)

Classical-statistical simulation

- Ensemble of initial conditions

$$\langle \phi_{\mathbf{k}} \phi_{\mathbf{k}'} \rangle = \frac{n_k + 1/2}{\omega_k} \delta^d_{\mathbf{k}-\mathbf{k}'} \qquad \langle \pi_{\mathbf{k}} \pi_{\mathbf{k}'} \rangle = (n_k + 1/2) \omega_k \delta^d_{\mathbf{k}-\mathbf{k}'}$$

- Evolved with classical equations of motions

$$\dot{\pi} = \nabla^2 \phi - V'(\phi) \qquad \dot{\phi} = \pi$$

- Observables obtained from averaging over independent configurations

$$\langle O \rangle = \frac{1}{N} \sum_i O_i$$

- (Only ?) Reliable if: $n_k >> 1$ (For appropriate observables, e.g. not $[\phi_{\mathbf{k}}, \pi_{\mathbf{k}}]$)

Classical-statistical simulation and "the half"

- Under certain circumstances $n_k >> 1$ does not need to be true at $t = 0$

  -> ok, if n_k grow large before self-interactions become important.

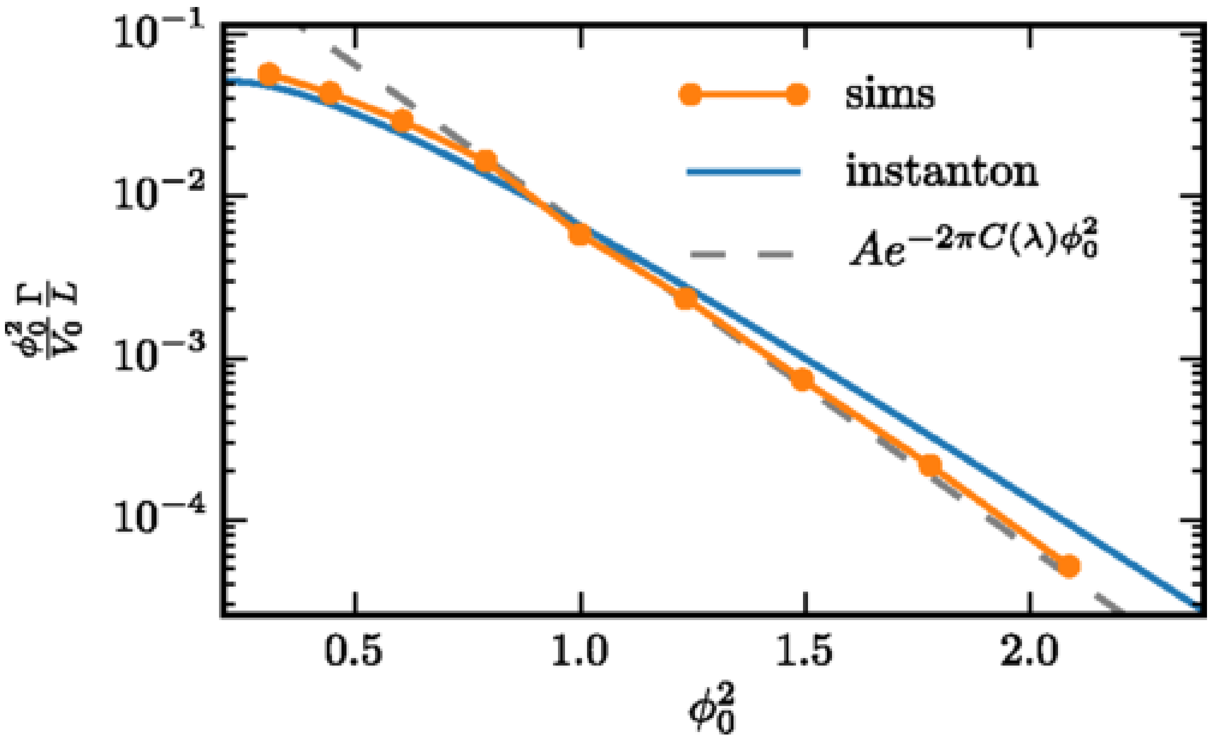
  (Relies on linearity of equation of motions)

- E.g. Weakly coupled scalar field in expanding background, resonant preheating, tachyonic preheating,...

- In general problematic: ½ does not stay put in CS!

Previous work on semiclassical vacuum decay

- Braden et al: A New Semiclassical Picture of Vacuum Decay [PhysRevLett.123.031601]



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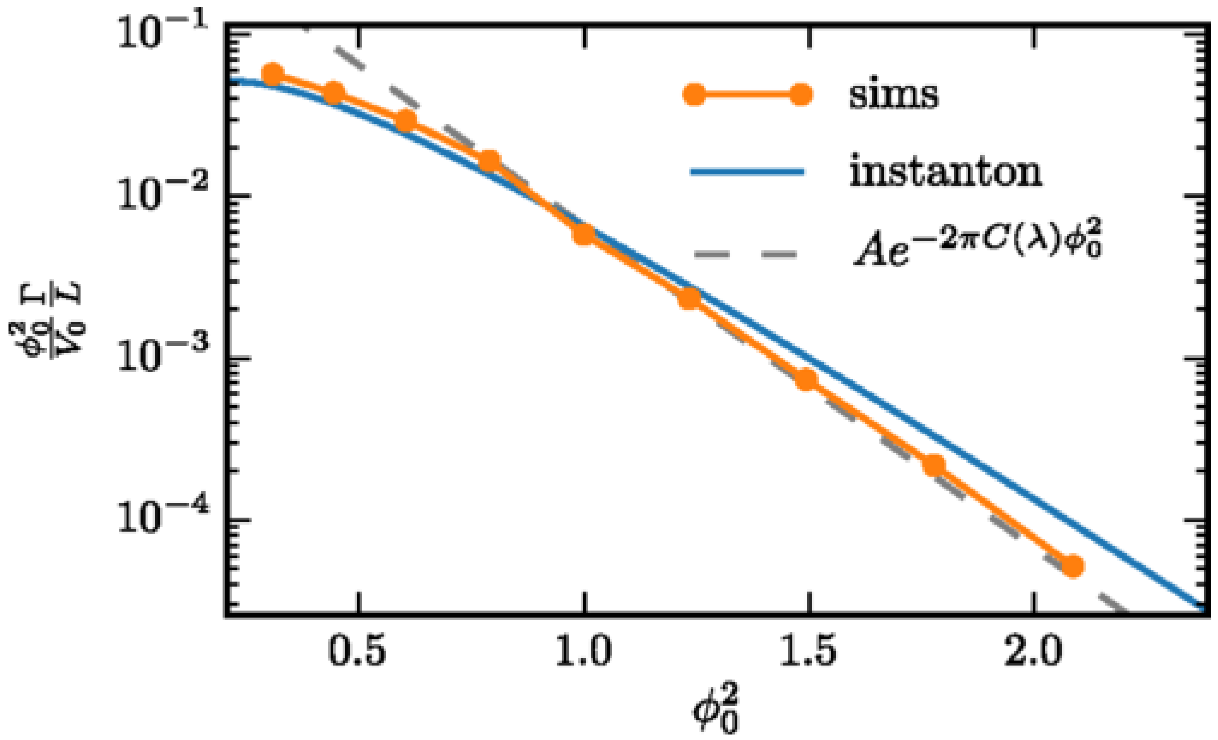

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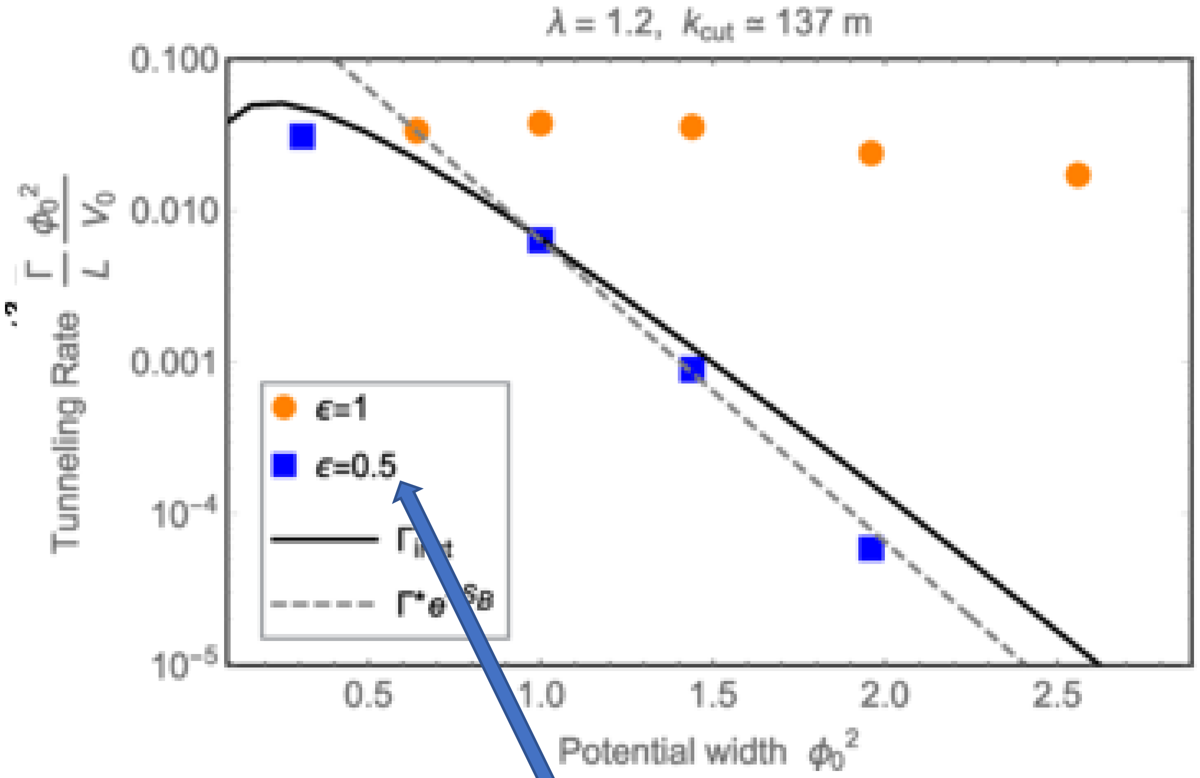
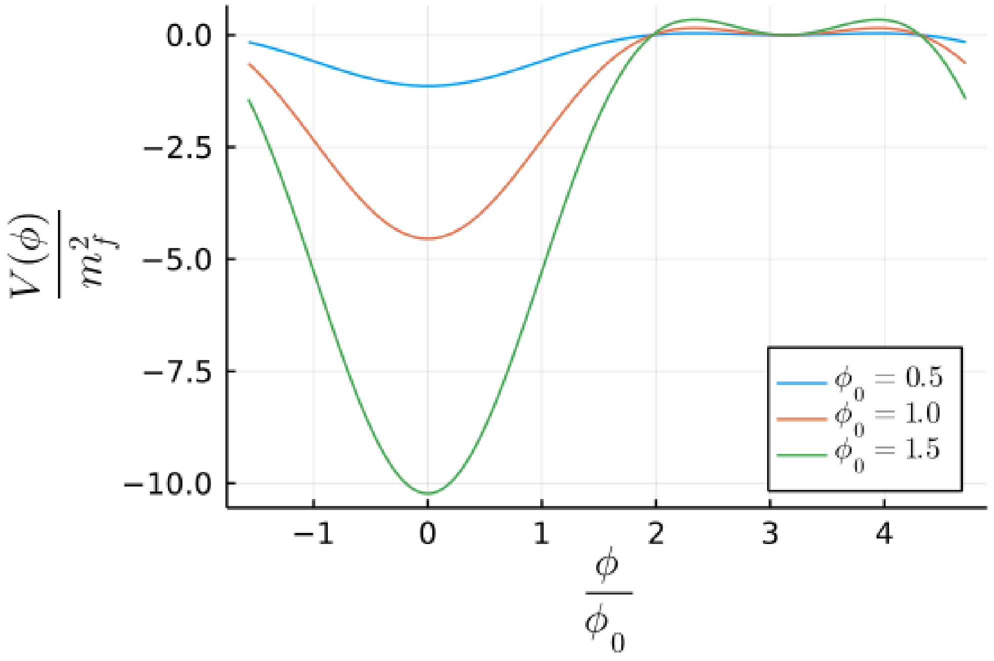


Changes 1/2 to 1/8

The toy model

$$S = \int dx^{d+1} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V_0 \Big(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2} \sin^2\left(\frac{\phi}{\phi_0}\right) - 1 \Big) \right]$$



$$m_f^2 = \frac{d^2 V}{d\phi^2}\bigg|_{\phi=\phi_{\text{local}}} = \frac{V_0}{\phi_0^2}(-1 + \lambda^2)$$

Parameters:

$\lambda = 1.2 \quad m_f \quad \phi_0$

Initial data

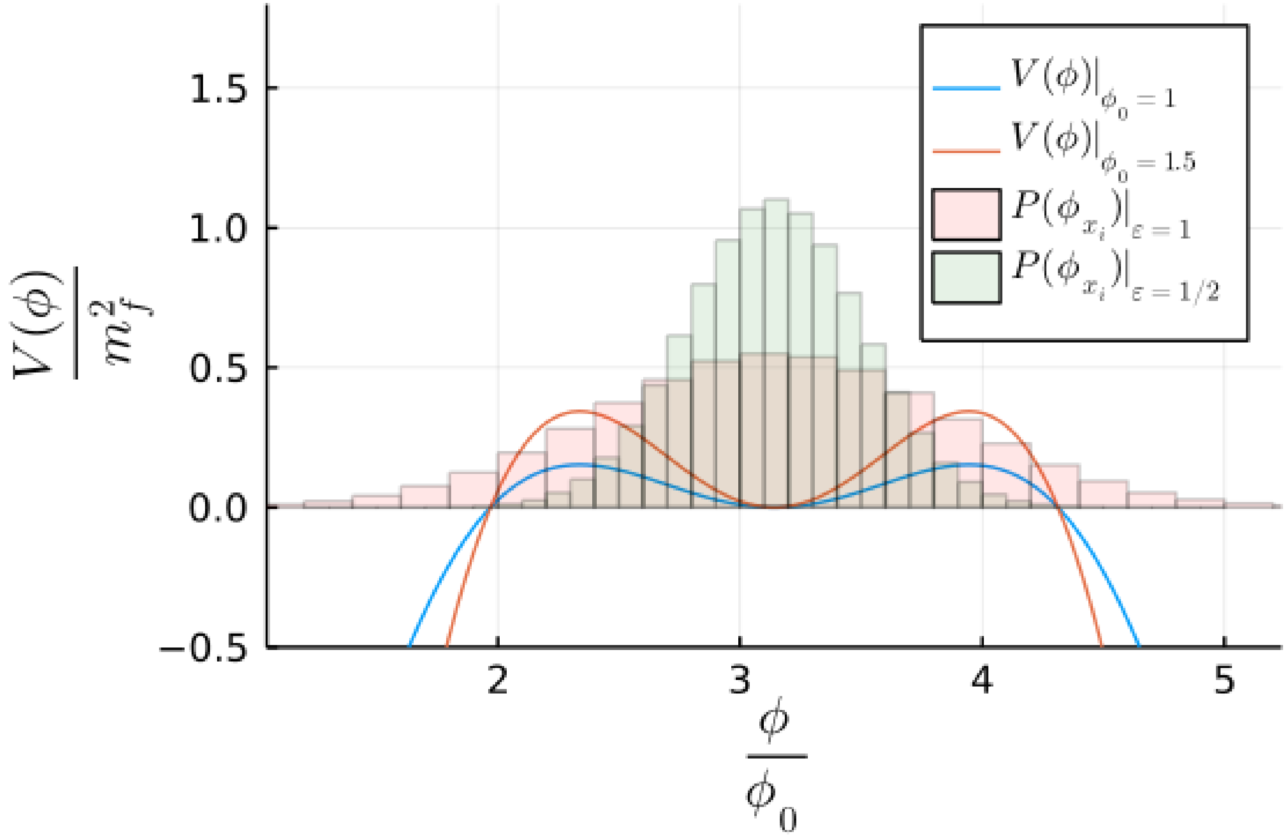
$$\langle \phi_{\mathbf{k}} \phi_{\mathbf{k}'} \rangle = \epsilon^2 \frac{1}{2\omega_k} \delta^d_{\mathbf{k}-\mathbf{k}'}$$

$$\langle \pi_{\mathbf{k}} \pi_{\mathbf{k}'} \rangle = \epsilon^2 \frac{\omega_k}{2} \delta^d_{\mathbf{k}-\mathbf{k}'}$$

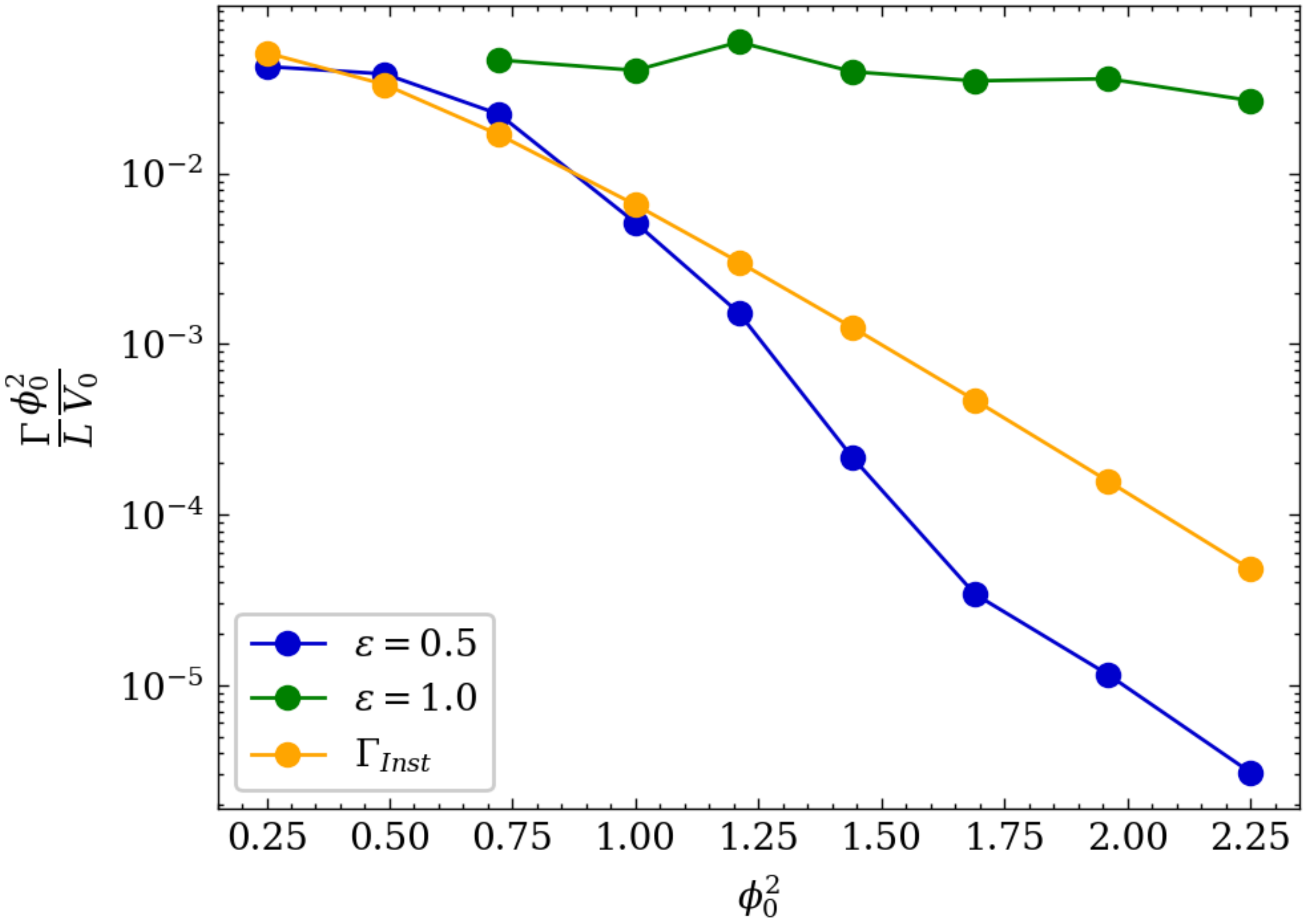
Evolution

$$\dot{\pi} = \nabla^2 \phi - V'(\phi)$$

$$\dot{\phi} = \pi$$



1+1 dimensions



- Initial vacuum fluctuations

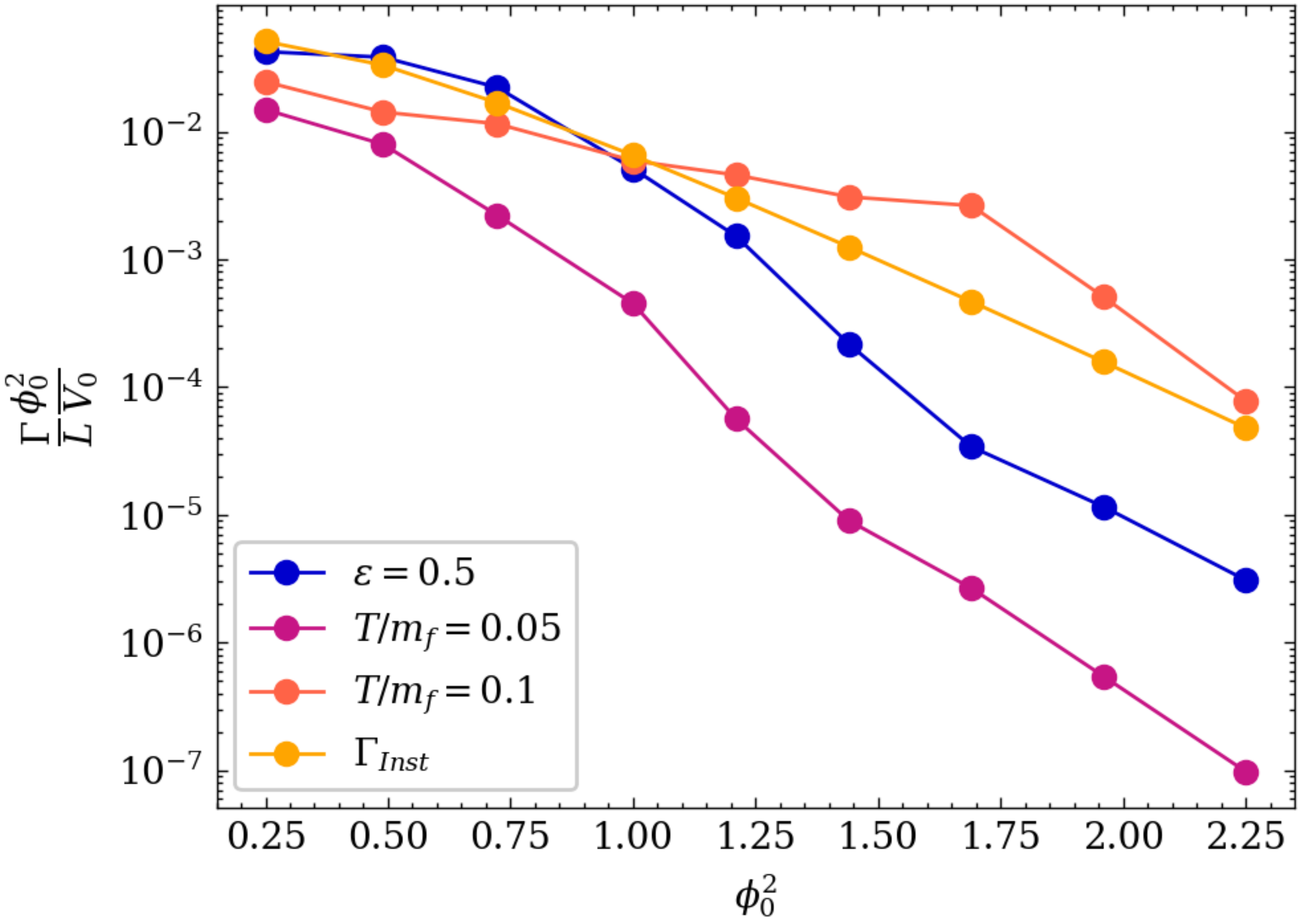
$$\langle \phi_{\mathbf{k}} \phi_{\mathbf{k}'} \rangle = \epsilon^2 \frac{1}{2\omega_k} \delta^d_{\mathbf{k}-\mathbf{k}'}$$

$$\langle \pi_{\mathbf{k}} \pi_{\mathbf{k}'} \rangle = \epsilon^2 \frac{\omega_k}{2} \delta^d_{\mathbf{k}-\mathbf{k}'}$$

$$\langle \phi \rangle = \pi \phi_0$$

- Reproduces result of [PhysRevLett.123.031601] [PhysRevD.102.076003]

1+1 dimensions



- Initial thermal state

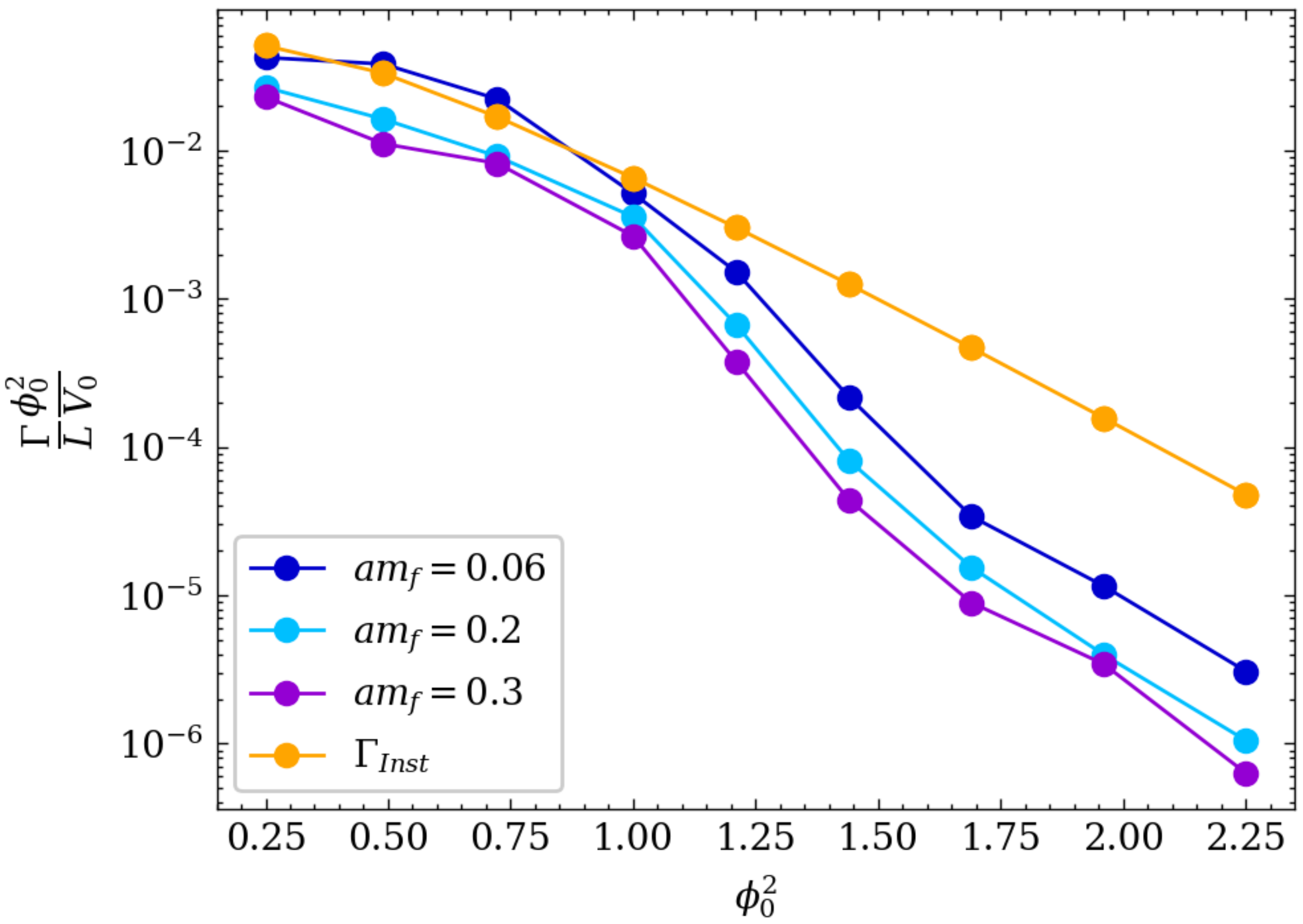
$$\langle \phi_{\mathbf{k}} \phi_{\mathbf{k}'} \rangle = \frac{T}{\omega_k^2} \delta^d_{\mathbf{k}-\mathbf{k}'}$$

$$\langle \pi_{\mathbf{k}} \pi_{\mathbf{k}'} \rangle = T \delta^d_{\mathbf{k}-\mathbf{k}'}$$

$$\langle \phi \rangle = \pi \phi_0$$

- $T/m_f = 0.1$ Approximates Instanton as well!

1+1 dimensions



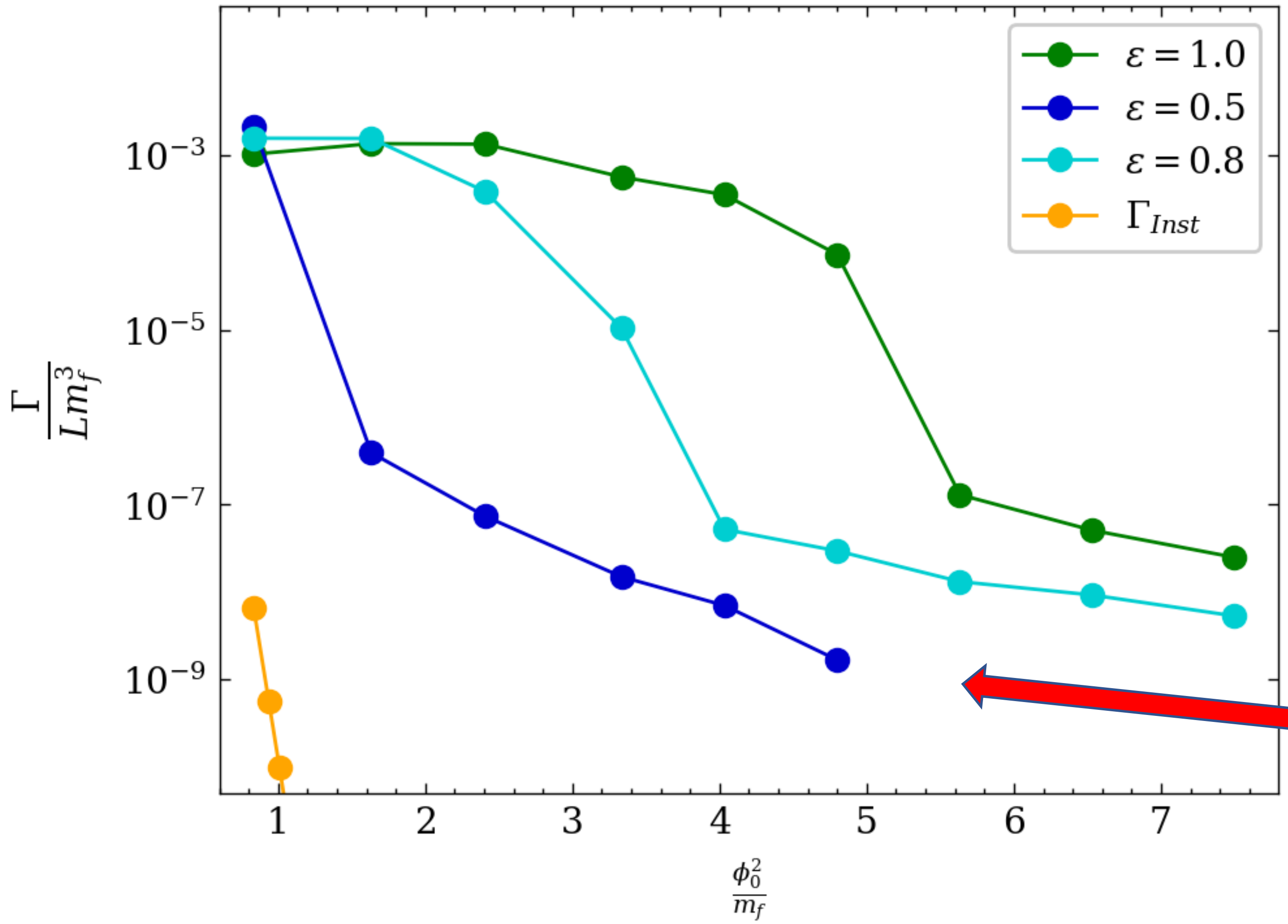
- Cut off variation:

-> Reduced lattice spacing a
-> Increased momentum cut off
-> more energy available
-> higher transition rates

- So far:

-> quantum ½ is not the point
-> parameters to tune result:
ε, T, m_f

2+1 dimensions



- Initial vacuum fluctuations

$$\langle \phi_{\mathbf{k}} \phi_{\mathbf{k}'} \rangle = \epsilon^2 \frac{1}{2\omega_k} \delta^d_{\mathbf{k}-\mathbf{k}'}$$

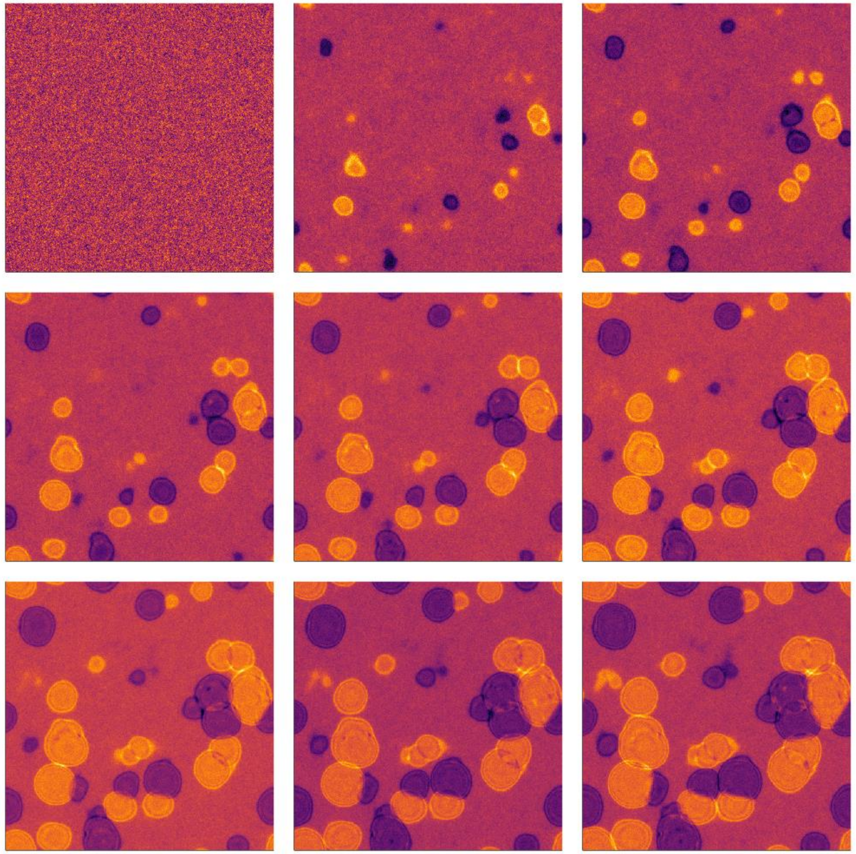
$$\langle \pi_{\mathbf{k}} \pi_{\mathbf{k}'} \rangle = \epsilon^2 \frac{\omega_k}{2} \delta^d_{\mathbf{k}-\mathbf{k}'}$$

$$\langle \phi \rangle = \pi \phi_0$$

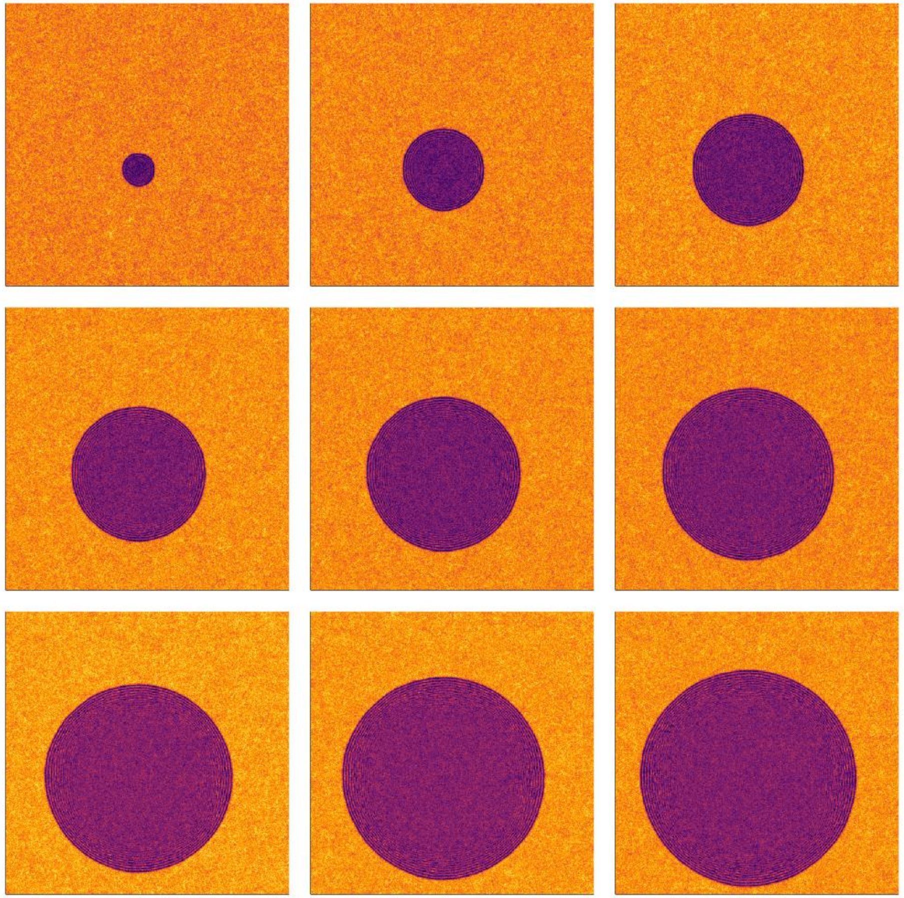
-> CS orders of magnitude off Instanton result

No transition occurs!

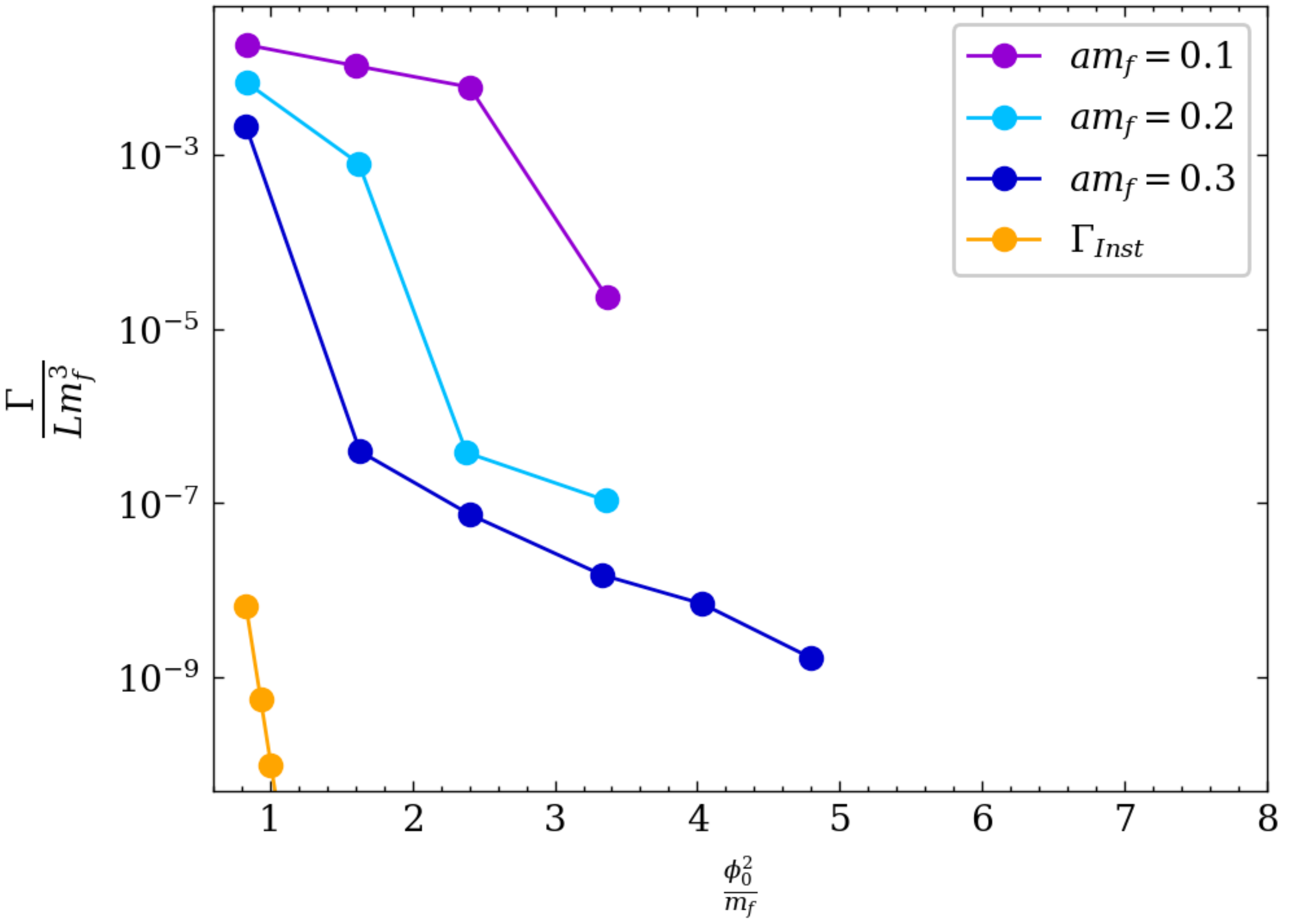
2+1 dimensions: Configurations



High / low
Transition rate



2+1 dimensions



- Cut off variation:

-> Reduced lattice spacing a
-> Increased momentum cut off
-> more energy available
-> higher transition rates

-> cut off dependence much higher than in 1+1 dimensions

Conclusion

- Parametrical agreement for vacuum decay in 1+1 dimensions exist
  - But not for the ½ initial condition – it requires tuning with fudge-factor or the cut-off
  - And it also appears for instance for a thermal initial state – tuning T

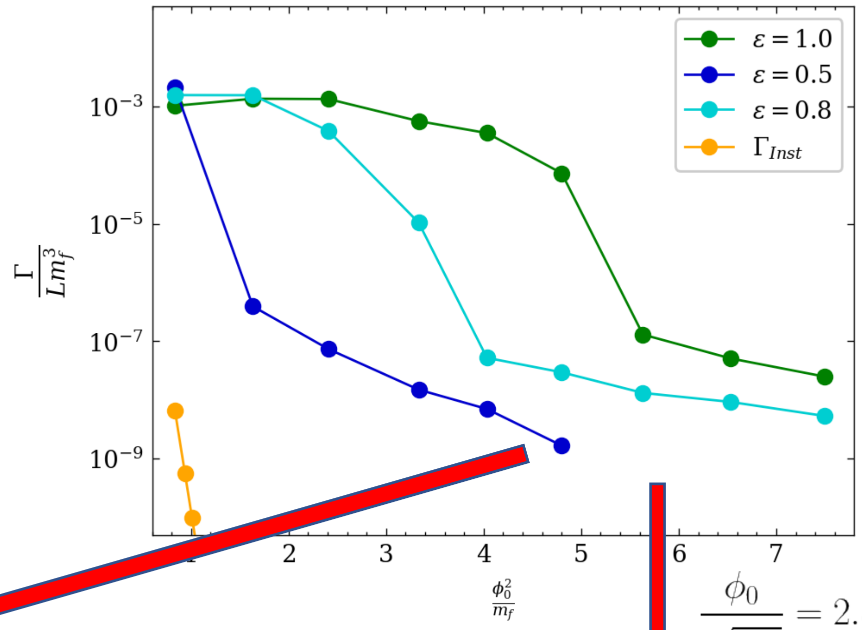
- In 2+1 dimensions, no such agreement exists

- 3+1 dimensional simulations of this type take too long to complete. Other methods need to be used. Gould, Güyer, Rummukainen (hep-lat/0101018) / Moore, Rummukainen, Tranberg (hep-lat/0101018)

- There is no basis to claiming that classical-statistical simulations can approximate quantum bubble nucleation/false vacuum decay.
- For a non-vacuum initial state, the classical rate is much larger than the instanton rate, and classical-statistical methods (or stochastic methods) may be used.

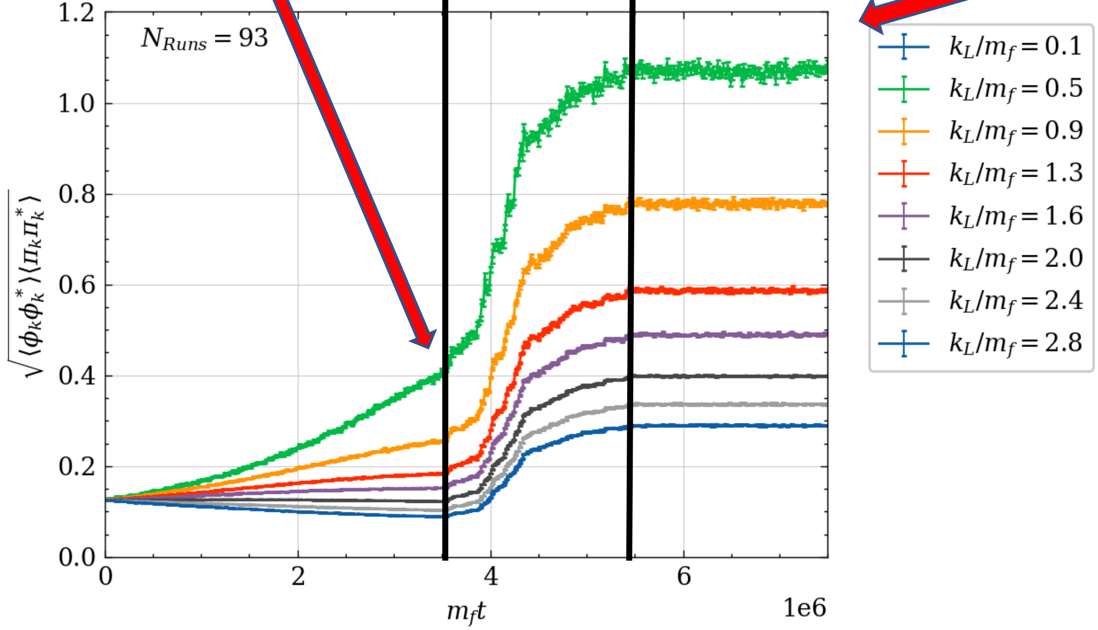
Backup

Occupation numbers

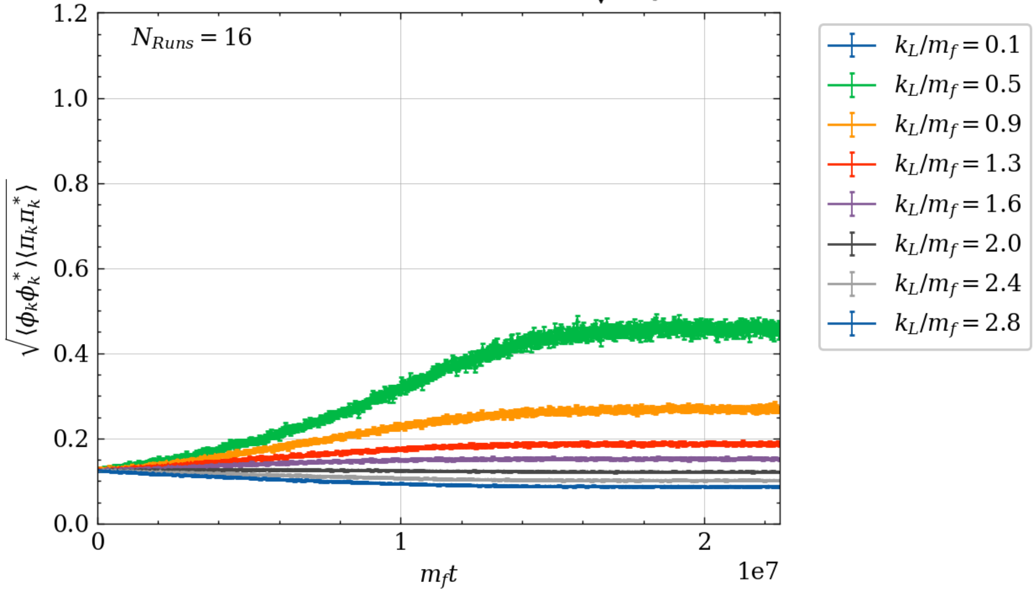


Time when individual transitions take place

No quantum half state any more!



$\frac{\phi_0}{\sqrt{m_f}} = 2.56, \epsilon = 0.5$

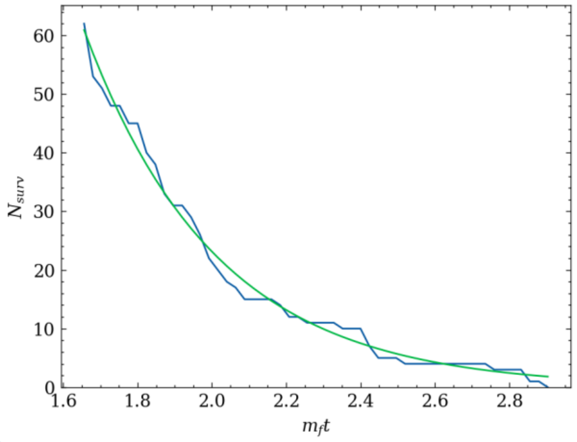


Order parameter

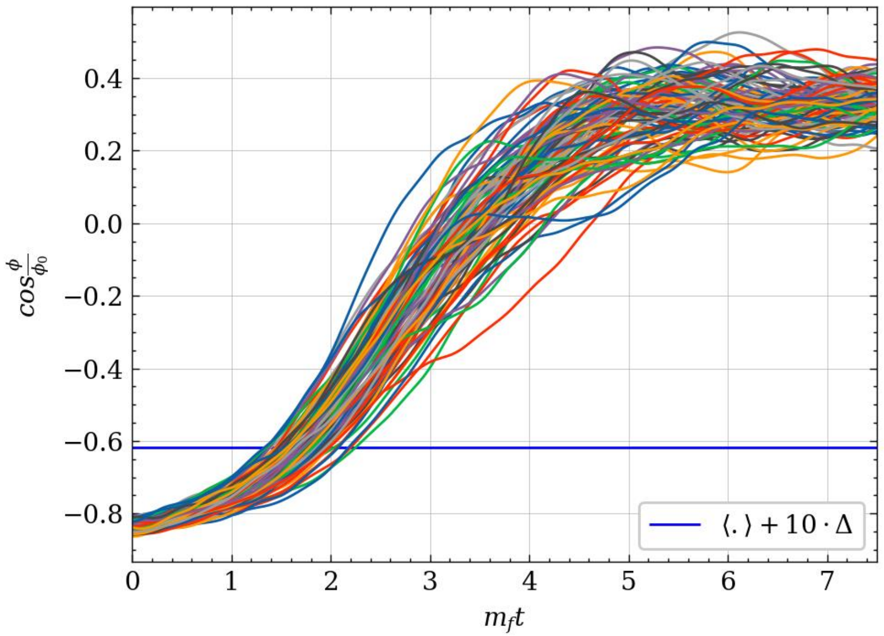
- Lattice average: $\langle \cos(\phi/\phi_0) \rangle$

- Definition of transitioned configuration

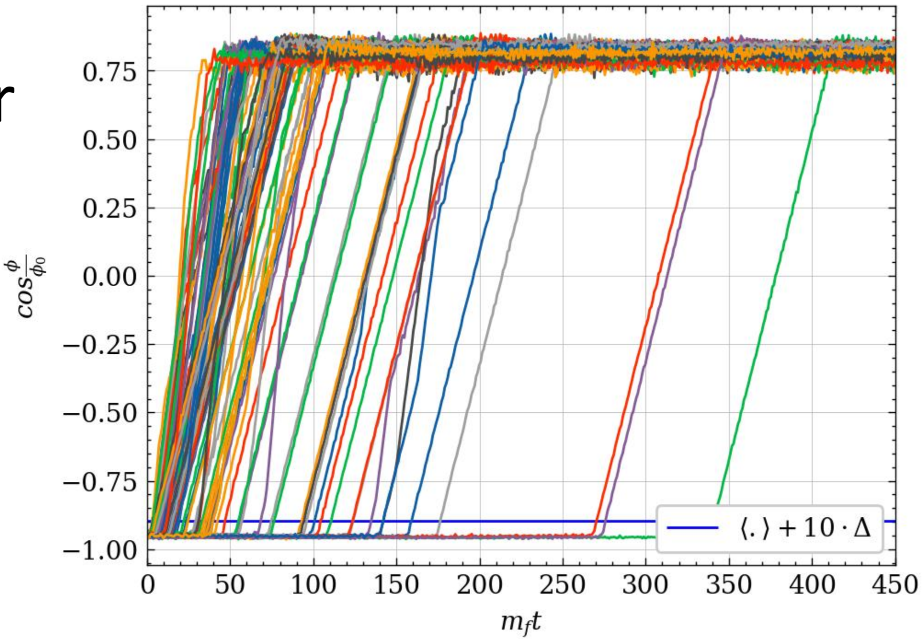
$$\langle \cos(\phi/\phi_0) \rangle > \langle \cos(\phi/\phi_0) \rangle_{t=0} + 10\Delta_{t=0}$$



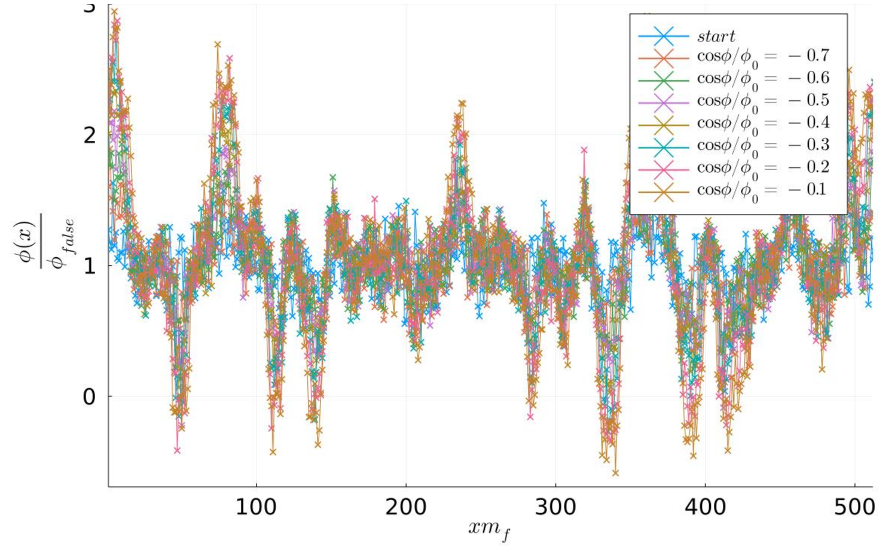
Ensemble average of 100 simulations

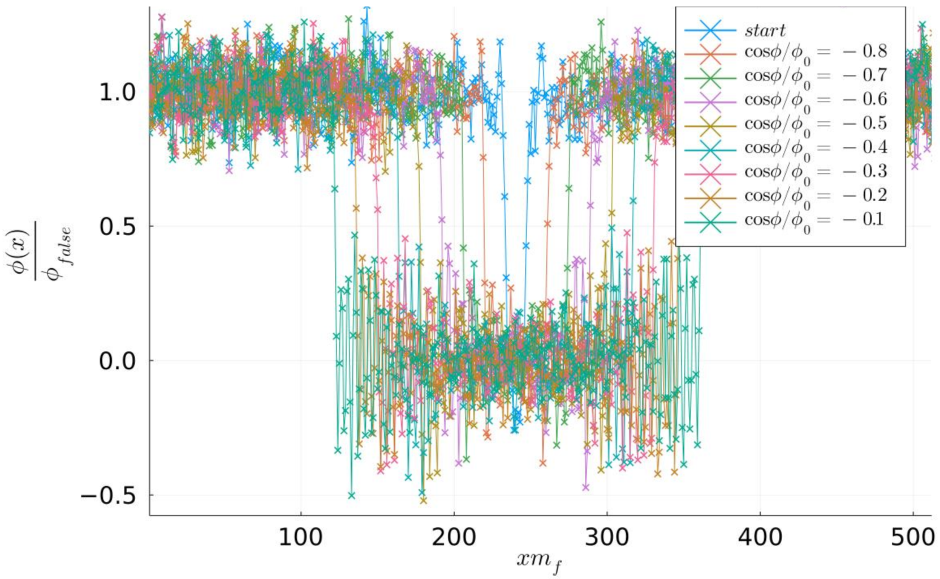


Taking a look at order parameter & configurations:

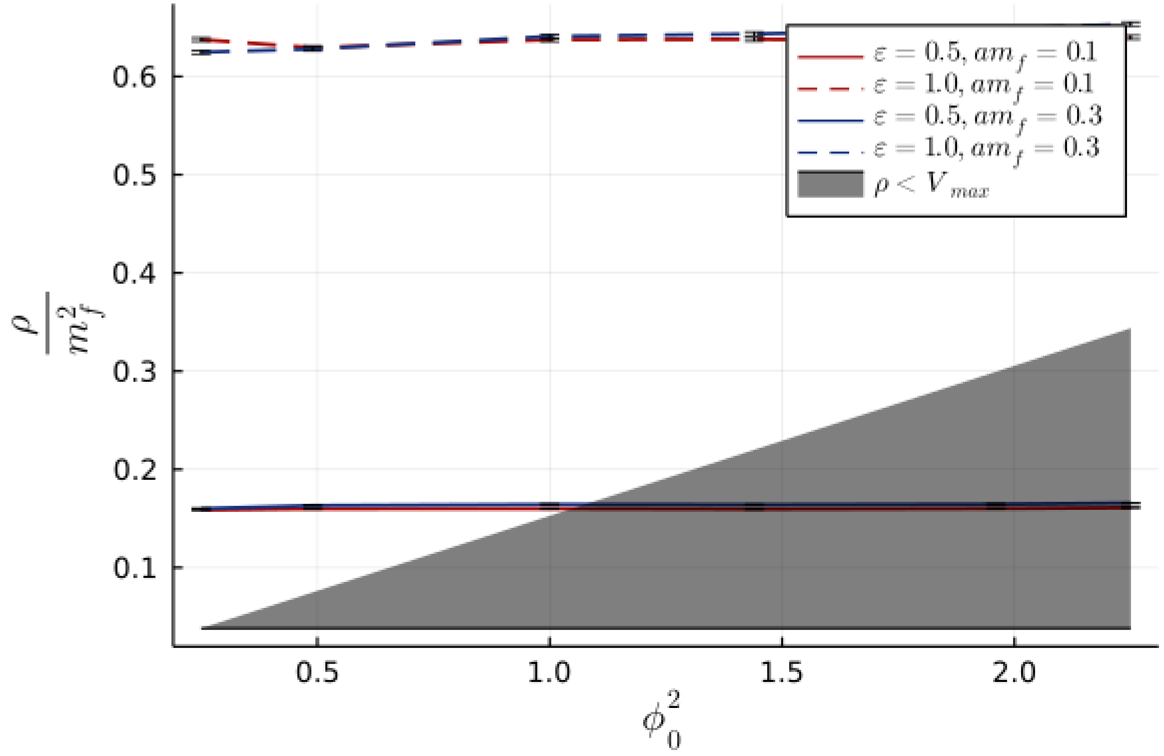


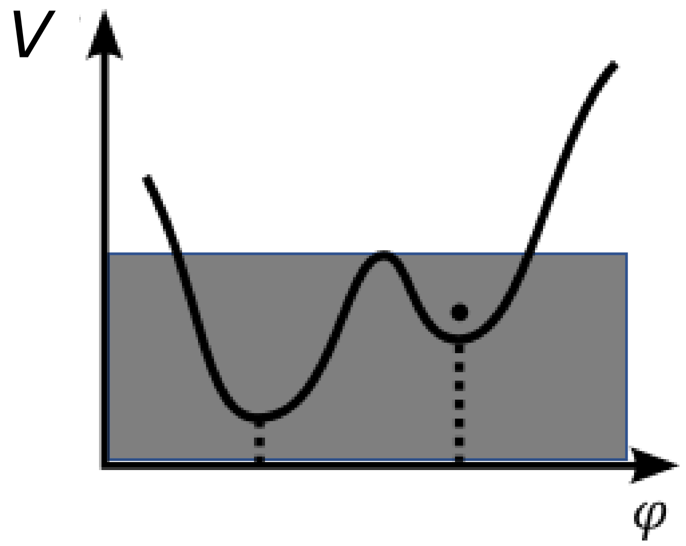
High / low Transition rate





Average energy density in 1+1 dimensions



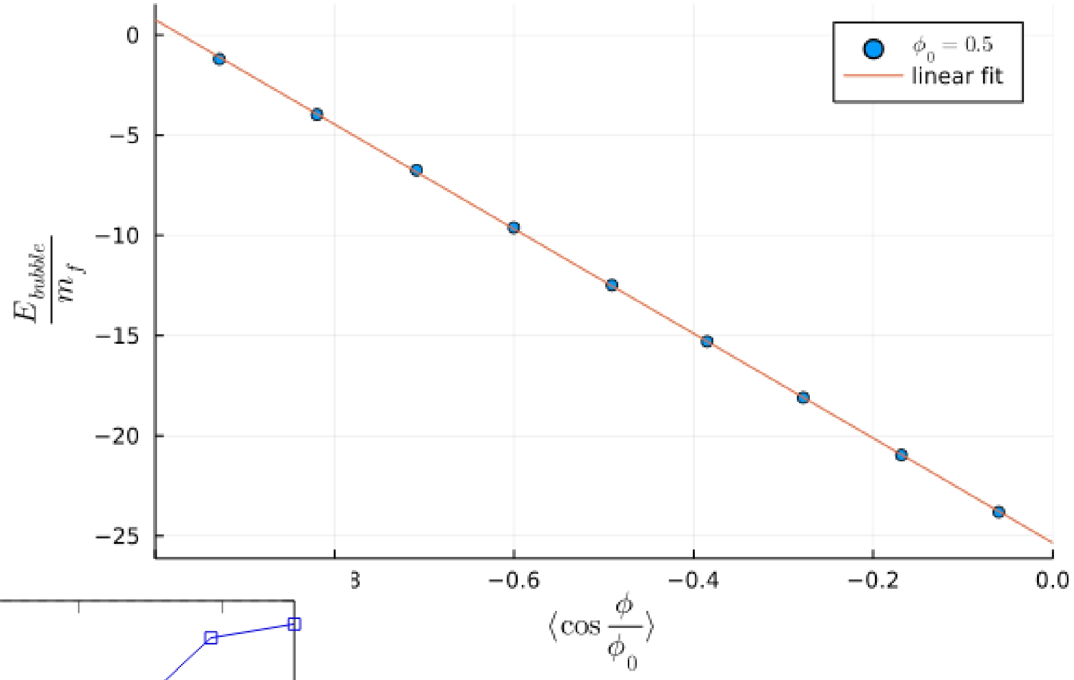


Bubble nucleation E_{crit}

Simple model for bubble nucleation:

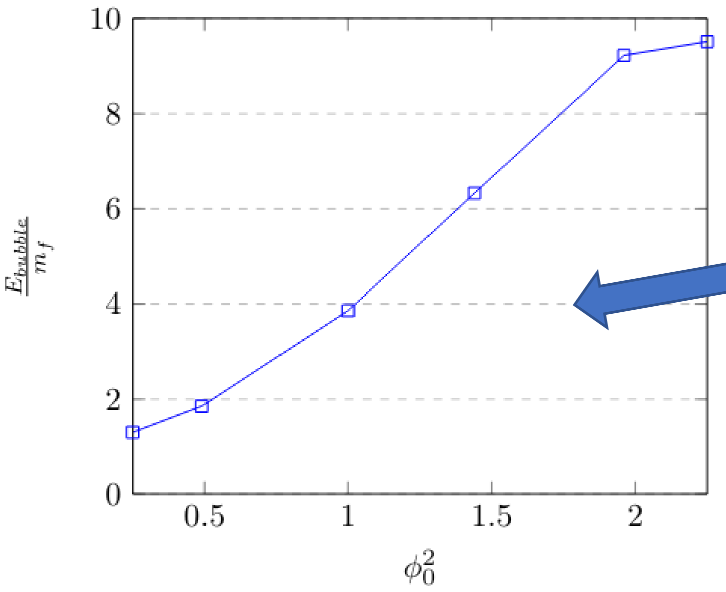
$$E_1 = 2\sigma + 2R\Delta V$$

$$\langle \cos\frac{\phi}{\phi_0} \rangle = \frac{4R - N_x}{N_x}$$



Energy to form a "critical bubble":

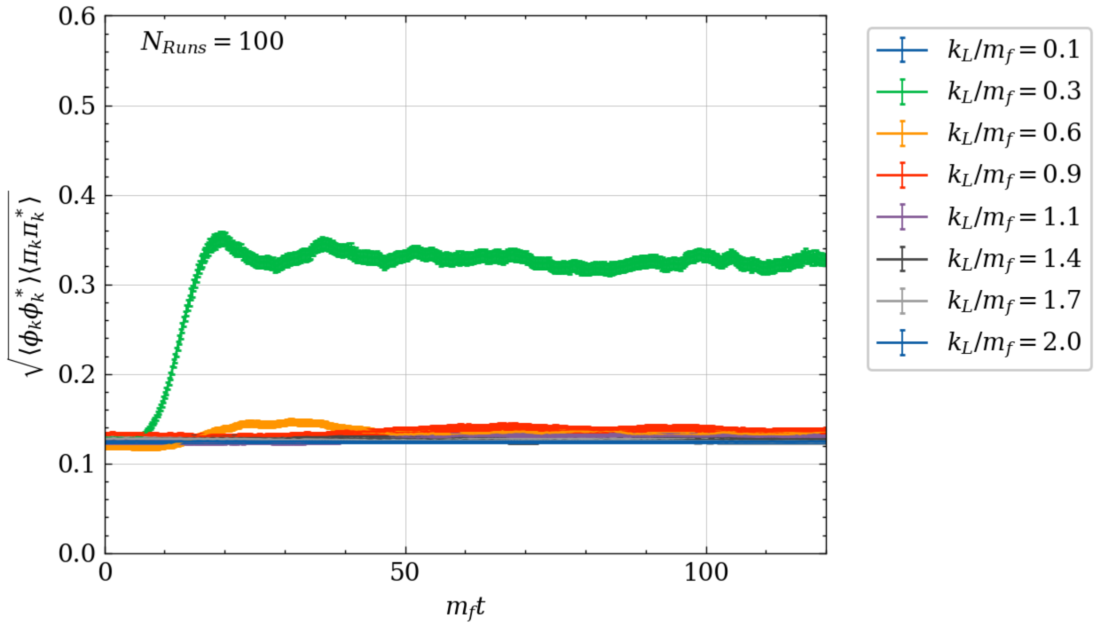
$$E_{bubble} = 2\sigma$$



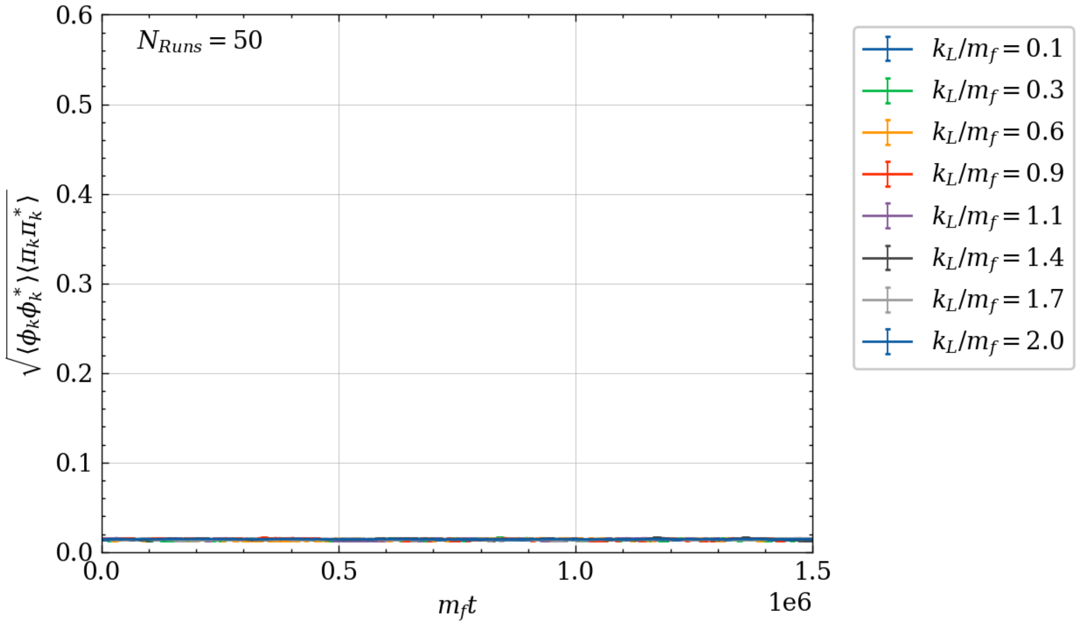
No classical transition possible – only quantum tunneling!

Occupation numbers in 1+1 dimensions

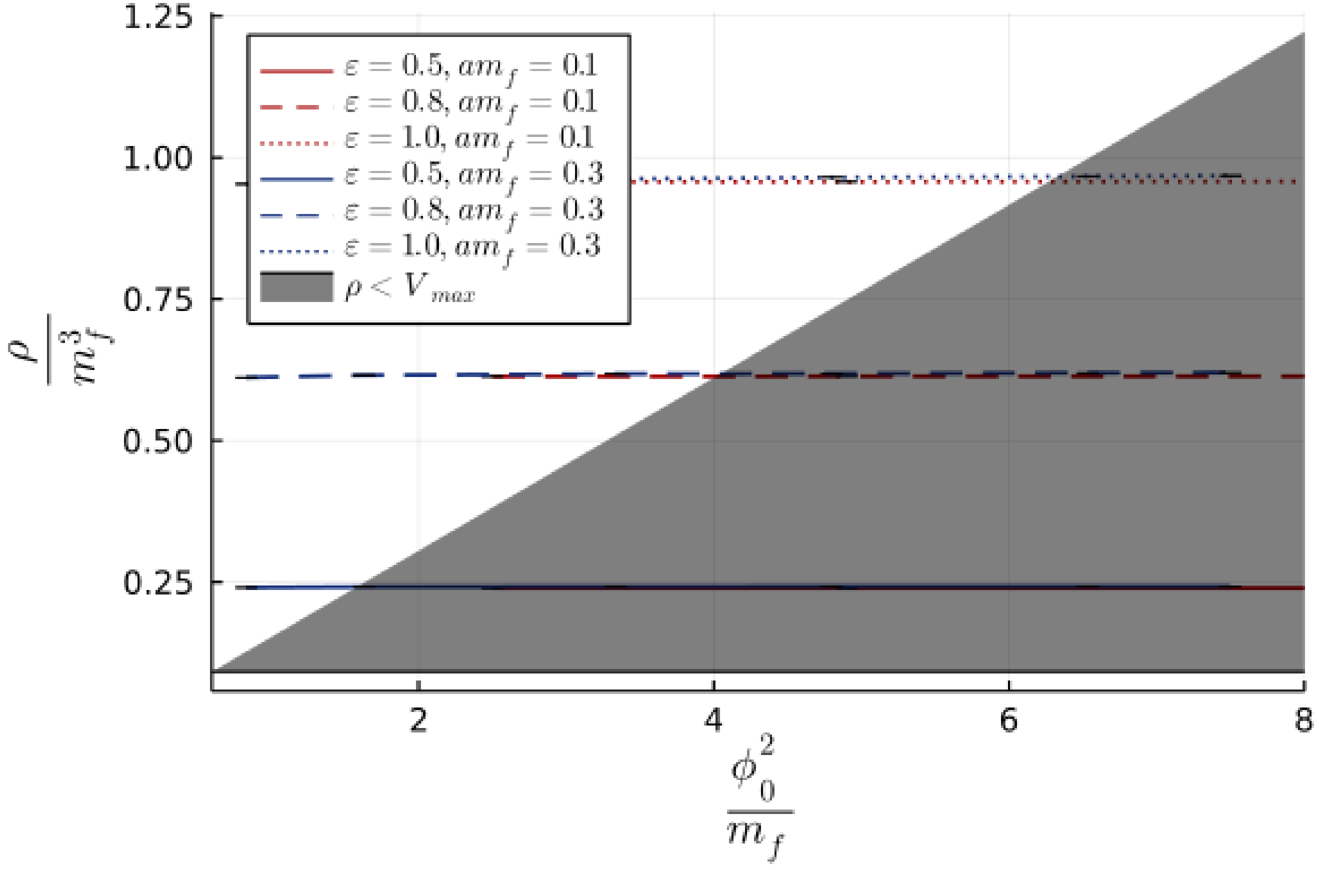
Representative of bubbling system

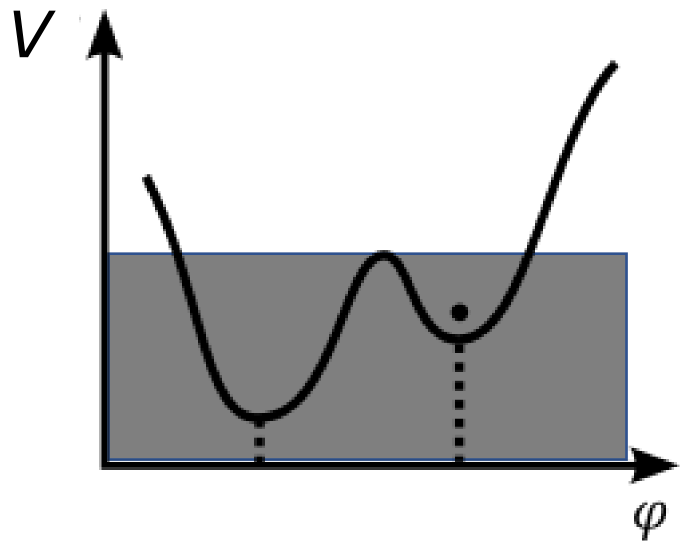


Representative of system with too little energy to bubble.



Average energy density in 2+1 dimensions





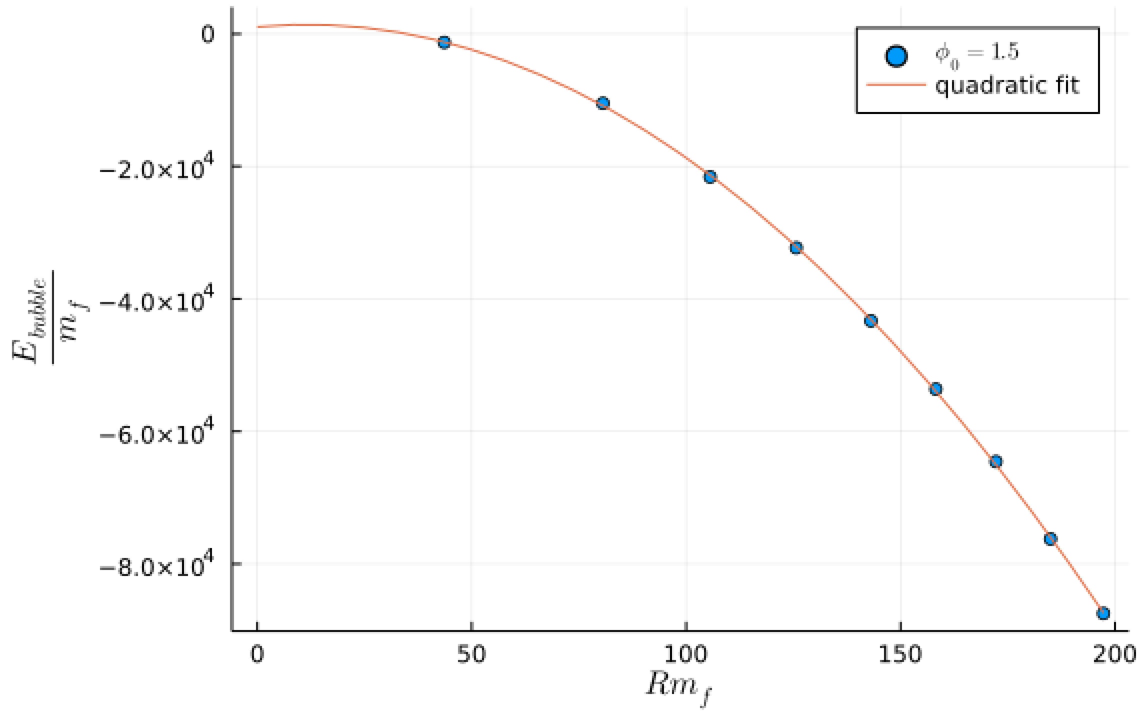
Bubble nucleation E_{crit}

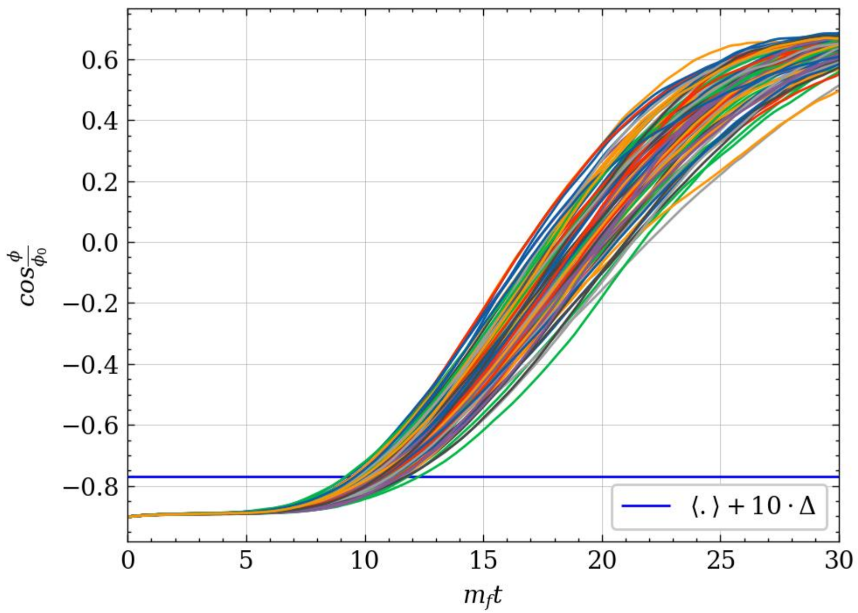
Simple model for bubble nucleation:

$$E_2 = 2\pi R\sigma + \pi R^2 \Delta V$$

$$E_{\text{crit},2} = \frac{\pi\sigma^2}{\Delta V}$$

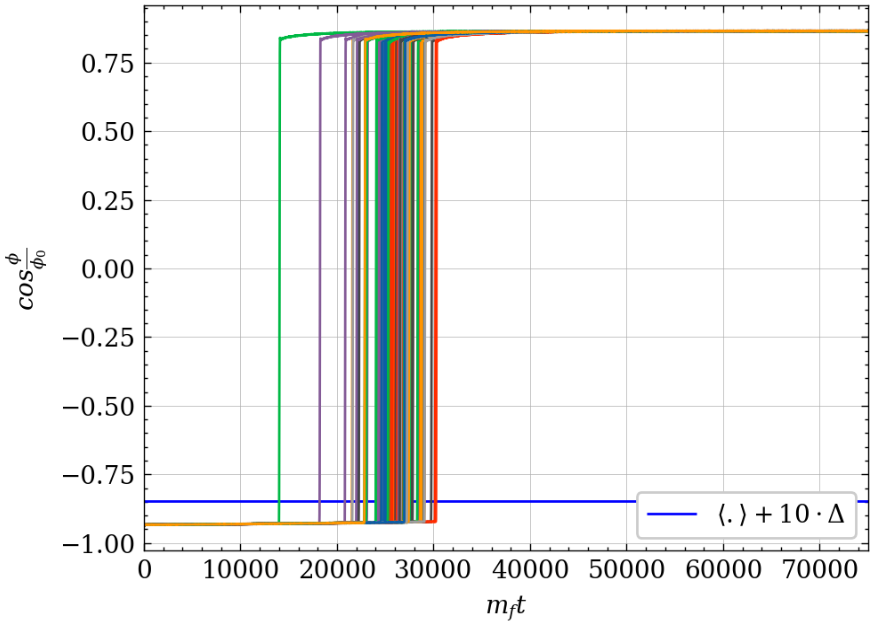
$$R_{\text{crit},2} = -\frac{\sigma}{\Delta V}.$$

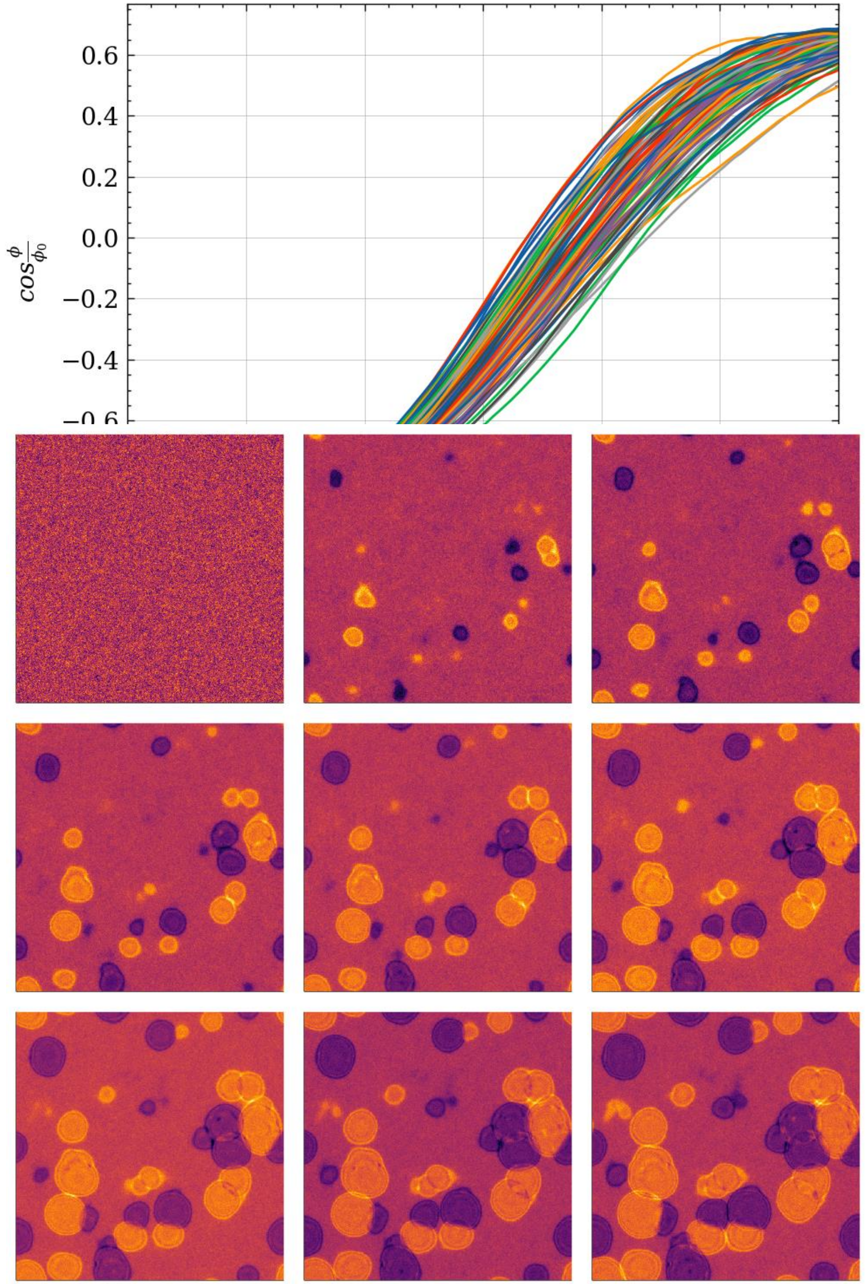




Taking a look at order parameter & configurations:

High / low Transition rate





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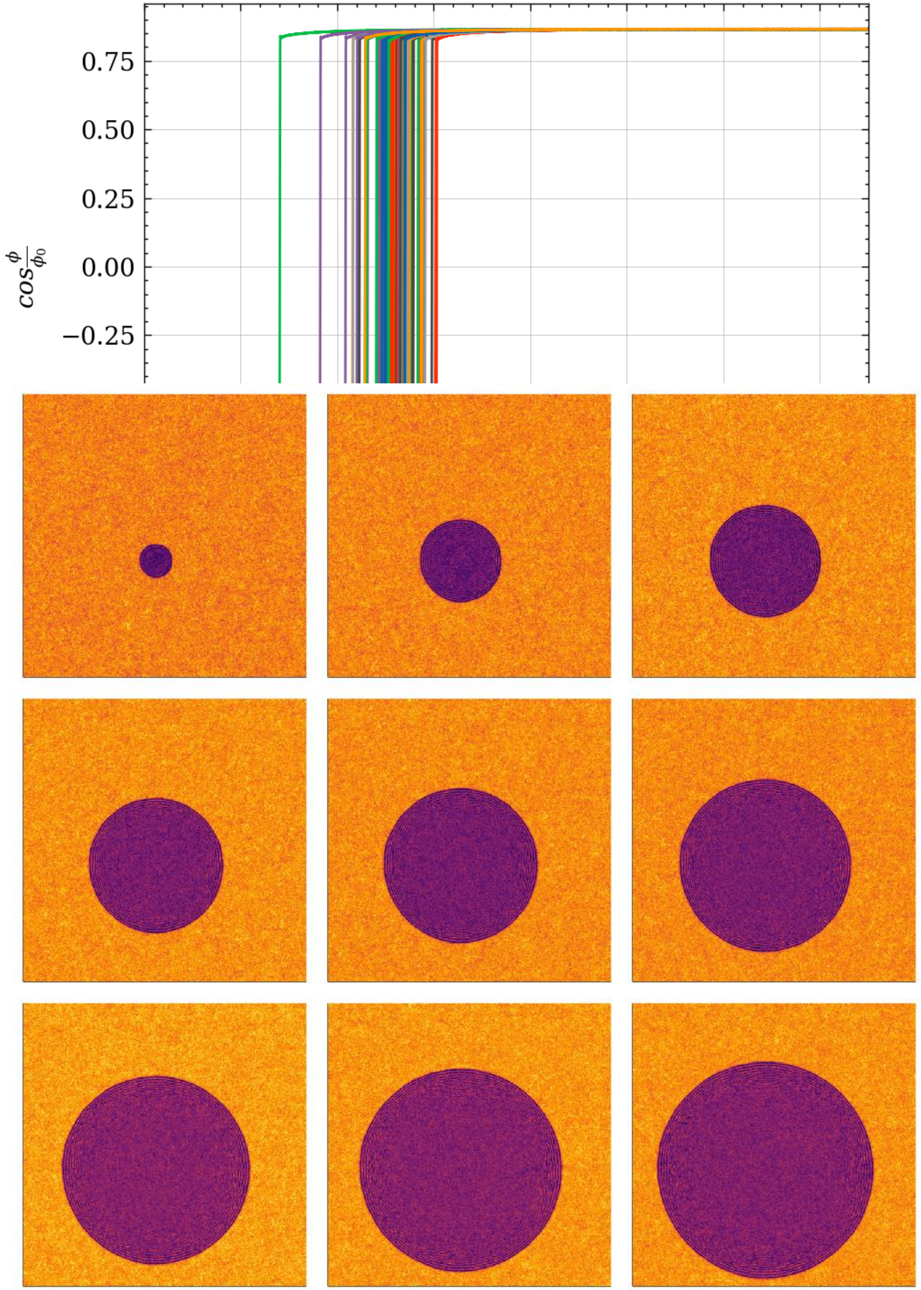

Instanton calculation

- Bounce solution $\left(\partial_r^2 + \frac{d}{r}\partial_r\right)\phi(r) = \frac{dV}{d\phi}$ $\phi(r=\infty) = \phi_{\text{local}}, \partial_r\phi(0) = 0$
- Rate in 1+1 dimensions $\frac{\Gamma}{L} = 2m_f^2\left(\frac{S_B}{2\pi}\right)e^{-S_B}$
- Generalization to 2+1 dimensions

$$\frac{\Gamma}{L^2} = 2m_f^3\left(\frac{S_B}{2\pi}\right)^{3/2}e^{-S_B}$$