Effective Field Theories for Dark Matter Pairs in the Early Universe

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Introduction

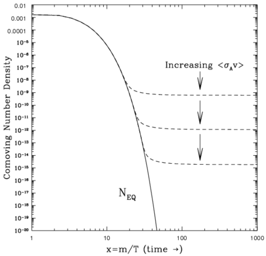
Thermal freeze-out of DM
(p)NREFTs for DM pairs

Motivation

Dark sector: particle-like DM (mass M) interacting via long-range mediator
Early universe ($T \gtrsim M$): heavy DM in thermal equilibrium with dark radiation (e.g. $X\bar{X} \leftrightarrow \gamma\gamma$)
Expanding universe ($T \lesssim M$): T cools down $\longrightarrow$ detailed balance lost

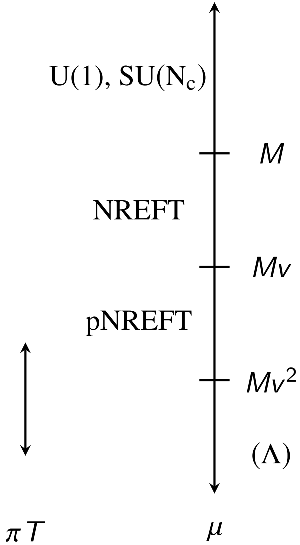
DM thermal freeze-out

- Evolution equation: $\dot{n} + 3Hn = -(n^2 - n_{\rm eq}^2)\langle\sigma_A v\rangle$
- Chemical freezout $H \sim n_{\rm eq}\langle\sigma_A v\rangle \quad \longrightarrow \quad T \sim M/25$



- $\Omega_X h^2 = 2.755 \times 10^8 \frac{m}{\rm GeV} Y_0 \simeq 0.12 \quad \longrightarrow \quad M \sim$ TeV

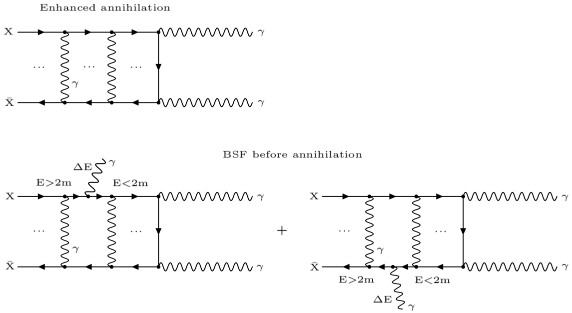
Hierarchy of Scales



- $\mathcal{L}_{\mathrm{U(1)}} = \bar{X}(i\not{D} - M)X - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\mathrm{portal}}$
- $\mathcal{L}_{\mathrm{SU(N_c)}} = \bar{X}(i\not{D} - M)X - \frac{1}{4}G^{A}_{\mu\nu}G^{A,\mu\nu} + \mathcal{L}_{\mathrm{portal}}$
- Weakly interacting ($\alpha \ll 1$) non-relativistic ($v \ll 1$) DM
- First integrate out $M \rightarrow$ NREFT
- Next integrate out $Mv \rightarrow$ pNREFT
- D.o.f.: singlet/adjoint DM bound/unbound states and ultrasoft dark gauge fields
- Dynamical thermal scales: $T \gg m_D \sim gT$

Energy Regime: $M \gg Mv \gg T \sim Mv^2$ $(\gg \Lambda)$

- Non-perturbative effects $\longrightarrow$ **Sommerfeld enhancement**
- Threshold effects $\longrightarrow$ **Bound-state formation** (BSF)



- Many more phenomena: bound-state dissociation, (de-)excitations, bremsstrahlung, thermal absorption, ...

Non-Relativistic Effective Field Theories (NREFTs)

Annihilations processes

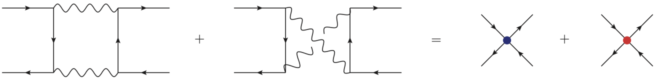
NREFTs

- If DM charged under U(1), integrating out scale M leads to **NRQED**, at order M^{-2}:

$$\mathcal{L}_{\mathrm{NRQED_{DM}}} = \psi^\dagger \left\{ iD^0 + \frac{\boldsymbol{D}^2}{2M} + c_F \frac{\boldsymbol{\sigma}\cdot g\boldsymbol{B}}{2M} + c_{\mathrm{D}} \frac{\boldsymbol{\nabla}\cdot g\boldsymbol{E}}{8M^2} + ic_{\mathrm{S}} \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times g\boldsymbol{E} - g\boldsymbol{E}\times\boldsymbol{D})}{8M^2} \right\} \psi$$
$$+ \chi^\dagger \left\{ iD^0 - \frac{\boldsymbol{D}^2}{2M} - c_F \frac{\boldsymbol{\sigma}\cdot g\boldsymbol{B}}{2M} + c_{\mathrm{D}} \frac{\boldsymbol{\nabla}\cdot g\boldsymbol{E}}{8M^2} + ic_{\mathrm{S}} \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times g\boldsymbol{E} - g\boldsymbol{E}\times\boldsymbol{D})}{8M^2} \right\} \chi$$
$$- \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{d_2}{M^2} F^{\mu\nu} \boldsymbol{D}^2 F_{\mu\nu} + \frac{d_s}{M^2} \psi^\dagger \chi \, \chi^\dagger \psi + \frac{d_v}{M^2} \psi^\dagger \boldsymbol{\sigma} \, \chi \chi^\dagger \boldsymbol{\sigma} \, \psi + \mathcal{L}_{\mathrm{portal}}$$

- Matching coefficients for pair annihilations:

$$\mathrm{Im}(d_s) = \pi\alpha(\mu_{\mathrm{M}})^2 \left[1 - \frac{\alpha(\mu_{\mathrm{M}})}{\pi} \left(5 - \frac{\pi^2}{4} \right) \right]$$
$$\mathrm{Im}(d_v) = \frac{4}{9}(\pi^2 - 9)\alpha(\mu_{\mathrm{M}})^3$$



- Annihilation cross section at NLO:

$$(\sigma^{\mathrm{NR}}_{\mathrm{ann}} v_{\mathrm{rel}})_{\mathrm{NLO}} = \frac{\pi\alpha(\mu_{\mathrm{M}})^2}{M^2} \left[1 + \frac{\alpha(\mu_{\mathrm{M}})}{\pi} \left(\frac{19}{12}\pi^2 - 17 \right) \right]$$

NREFTs

- If DM charged under SU(N_c):[1]

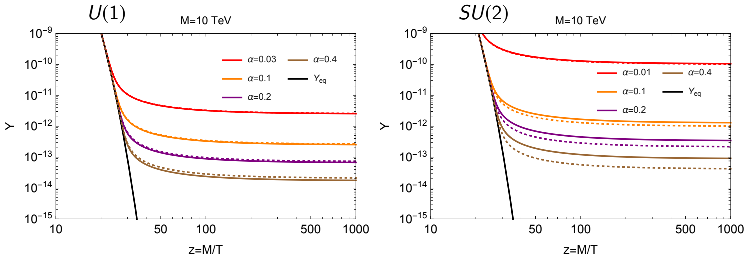
$$\mathcal{L}^{\psi\chi}_{\text{NREFT}_{\text{DM}}} \sim \frac{f_1(^1S_0)}{M^2}\psi^\dagger\chi\,\chi^\dagger\psi + \frac{f_1(^3S_1)}{M^2}\psi^\dagger\boldsymbol{\sigma}\,\chi\chi^\dagger\boldsymbol{\sigma}\,\psi + \frac{f_N(^1S_0)}{M^2}\psi^\dagger T^A\chi\,\chi^\dagger T^A\psi + \frac{f_N(^3S_1)}{M^2}\psi^\dagger T^A\boldsymbol{\sigma}\,\chi\chi^\dagger\boldsymbol{\sigma}\,T^A\psi$$

- Annihilation cross section for color-singlet at NLO:

$$(\sigma^{\text{NR}}_{\text{ann}}v_{\text{rel}})_{\text{NLO}} = \frac{C_F}{2N_c}\frac{\pi\alpha_s(\mu_{\text{M}})^2}{M^2}\left[1+\frac{\alpha_s(\mu_{\text{M}})}{72N_c\pi}\left(1044+400N_c^2-3\pi^2(35+2N_c^2)\right)\right]$$

- Annihilation cross section for color-adjoint at NLO:

$$(\sigma^{\text{NR}}_{\text{ann}}v_{\text{rel}})_{\text{NLO}} = \left(2C_F-\frac{3}{4}N_c\right)\frac{\pi\alpha_s(\mu_{\text{M}})^2}{M^2} \times\left[1+\frac{\alpha_s(\mu_{\text{M}})}{72N_c\pi}\left(\frac{N_c^4(796-42\pi^2)+36(\pi^2-20)+N_c^2(1499\pi^2-16144)}{N_c^2-4}\right)\right]$$



[1] A. Vairo, arXiv:0311303v2 (2004).

potential Non-Relativistic Effective Field Theories (pNREFTs)

Enhanced pair annihilations, decays
Electric dipole transitions at finite T

pNREFTs

- Integrating out the scale Mv in NRQED by expansion in r $\rightarrow$ **pNRQED**:

$$\mathcal{L}_{\text{pNRQED}_{\text{DM}}} = \int d^3r \phi^\dagger(t,\boldsymbol{r},\boldsymbol{R}) \left[i\partial_0 - H + g\,\boldsymbol{r}\cdot\boldsymbol{E}(\boldsymbol{R},t)\right]\phi(t,\boldsymbol{r},\boldsymbol{R}) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

with:

$$H(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) = \frac{\boldsymbol{p}^2}{M} + \frac{\boldsymbol{P}^2}{4M} + \frac{\boldsymbol{p}^4}{4M^3} + \ldots + V(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) + \ldots$$

$$V(\boldsymbol{r},\boldsymbol{p},\boldsymbol{P},\boldsymbol{S}_1,\boldsymbol{S}_2) = V^{(0)} + \frac{V^{(1)}}{M} + \frac{V^{(2)}}{M^2} + \ldots,\quad V^{(0)} = -\alpha(\mu_{1/r})/r$$

- For SU(N$_c$): $S = S1_c/\sqrt{N_c}\,,\; O = O^A T^A/\sqrt{T_F}$

$$\mathcal{L}_{\text{pNREFT}_{\text{DM}}} = \int d^3r \text{Tr}\left[S^\dagger(t,\boldsymbol{r},\boldsymbol{R})\left[i\partial_0 - H_s\right]S(t,\boldsymbol{r},\boldsymbol{R}) + \int d^3r O^\dagger(t,\boldsymbol{r},\boldsymbol{R})\left[iD_0 - H_o\right]O(t,\boldsymbol{r},\boldsymbol{R})\right]$$
$$+ \text{Tr}\left[V_A(r)g(S^\dagger\boldsymbol{r}\cdot\boldsymbol{E}\,O + \text{c.c.}) + \frac{V_B(r)}{2}g(O^\dagger\boldsymbol{r}\cdot\boldsymbol{E}\,O + \text{c.c.})\right] - \frac{1}{4}G^A_{\mu\nu}G^{A,\mu\nu}$$

$$V_s^{(0)} = -C_F\frac{\alpha_s(\mu_{1/r})}{r},\quad V_o^{(0)} = \frac{1}{2N_c}\frac{\alpha_s(\mu_{1/r})}{r}$$

- Contact terms responsible for s-wave annihilation:

$$\delta\mathcal{L}^{\text{ann}}_{\text{pNRQED}_{\text{DM}}} = \frac{i}{M^2}\int d^3r \phi^\dagger\delta^3(\boldsymbol{r})\left[2\text{Im}(d_s) - \boldsymbol{S}^2\left(\text{Im}(d_s) - \text{Im}(d_v)\right)\right]\phi + \ldots$$

$$\delta\mathcal{L}^{\text{ann}}_{\text{pNREFT}_{\text{DM}}} = i\frac{N_c}{M^2}\int d^3r\, S^\dagger\delta^3(\boldsymbol{r})\left[2\text{Im}(f_1(^1S_0)) - \boldsymbol{S}^2\left(\text{Im}(f_1(^1S_0)) - \text{Im}(f_1(^3S_1))\right)\right]S + \ldots$$
$$+ O^\dagger\delta^3(\boldsymbol{r})\left[2\text{Im}(f_N(^1S_0)) - \boldsymbol{S}^2\left(\text{Im}(f_N(^1S_0)) - \text{Im}(f_N(^3S_1))\right)\right]O + \ldots$$

Annihilations Revisited

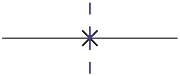
- U(1) case: Sommerfeld-enhanced annihilation cross section at NLO

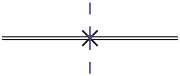
$$(\sigma^{\rm NR}_{\rm ann} v_{\rm rel})^{\rm SE}_{\rm NLO}(\boldsymbol{p}) = (\sigma^{\rm NR}_{\rm ann} v_{\rm rel})_{\rm NLO} |\psi_{\boldsymbol{p}}(\boldsymbol{r})|^2 = \frac{\pi\alpha(\mu_{\rm M})^2}{M^2}\left[1 + \frac{\alpha(\mu_{\rm M})}{\pi}\left(\frac{19}{12}\pi^2 - 17\right)\right] \frac{2\pi\alpha(\mu_{1/r})/v_{\rm rel}}{1-e^{-2\pi\alpha(\mu_{1/r})/v_{\rm rel}}}$$

- Decay width of bound-states at NLO

$$\Gamma_{{\rm pd},n} = \frac{4{\rm Im}(d_s)}{M^2}|\psi_n(0)|^2 = \frac{M\alpha(\mu_{1/r})^3\alpha(\mu_{\rm M})^2}{2n^3}\left[1 - \frac{\alpha(\mu_{\rm M})}{\pi}\left(5 - \frac{\pi^2}{4}\right)\right]$$

$$\Gamma_{{\rm od},n} = \frac{4{\rm Im}(d_v)}{M^2}|\psi_n(0)|^2 = \frac{2(\pi^2-9)M\alpha(\mu_{\rm M})^3}{9\pi n^3}\alpha(\mu_{1/r})^3$$





- SU(N$_c$) case: Unbound color-singlet annihilations

$$(\sigma^{\rm NR}_{\rm ann} v_{\rm rel})^{\rm SE}_{\rm NLO}(\boldsymbol{p}) = (\sigma^{\rm NR}_{\rm ann} v_{\rm rel})^{s}_{\rm NLO} |\psi^{s}_{\boldsymbol{p}}(\boldsymbol{r})|^2$$

- Adjoint field annihilations

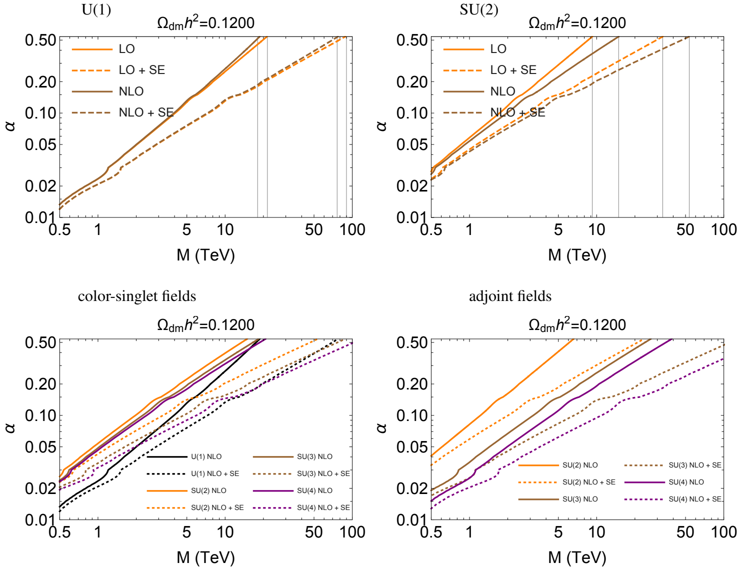
$$(\sigma^{\rm NR}_{\rm ann} v_{\rm rel})^{\rm SE}_{\rm NLO}(\boldsymbol{p}) = (\sigma^{\rm NR}_{\rm ann} v_{\rm rel})^{o}_{\rm NLO} |\psi^{o}_{\boldsymbol{p}}(\boldsymbol{r})|^2$$

- Decay width of bound color-singlet

$$\Gamma_{{\rm pd},n} = C_F^4 \frac{M\alpha(\mu_{1/r})^3\alpha(\mu_{\rm M})^2}{4N_c n^3}\left[1 + \frac{\alpha_s(\mu_{\rm M})}{72N_c\pi}\left(1044 + 400N_c^2 - 3\pi^2(35 + 2N_c^2)\right)\right]$$

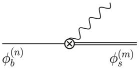
$$\Gamma_{{\rm od},n} = \frac{\pi^2-9}{9\pi n^3} C_F^4 (N_c^2 - 4)\left(\frac{N_c}{2} - C_F\right)^2 M\alpha_s(\mu_{\rm M})^3 \alpha_s(\mu_{1/r})^3$$

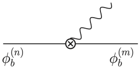
Parameter Space

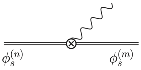


Electric dipole transitions at finite T

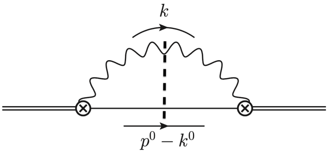
$$M \gg Mv \gg T \sim Mv^2 \ (\gg \Lambda)$$







Formation of Bound-States



- Electric correlator at finite T in **real-time formalism**:

$$\langle \boldsymbol{E}(t)\boldsymbol{E}(0)\rangle_T = \int \frac{dk^0}{2\pi}\, e^{-ik^0 t} \int \frac{d^3k}{(2\pi)^3}\, \left[k_0^2 D_{ii}(k) + \boldsymbol{k}^2 D_{00}(k)\right]$$

- LO thermal gauge field propagator in Coulomb gauge:

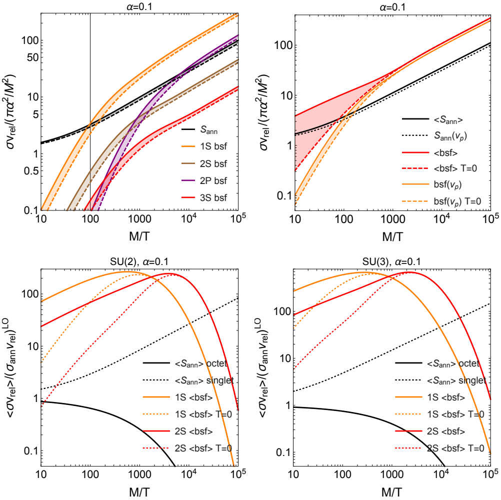
Longitudinal part: $D_{00}^{(0)}(|\boldsymbol{k}|) = \begin{pmatrix} \frac{i}{|\boldsymbol{k}|^2} & 0 \\ 0 & \frac{-i}{|\boldsymbol{k}|^2} \end{pmatrix}$

Transverse part: $$D_{ij}^{(0)}(k_0, |\boldsymbol{k}|) = \left(\delta_{ij} - \frac{k^i k^j}{|\boldsymbol{k}|^2}\right) \left\{ \begin{pmatrix} \frac{i}{k_0^2 - |\boldsymbol{k}|^2 + i\eta} & \Theta(-k_0) 2\pi\delta(k_0^2 - |\boldsymbol{k}|^2) \\ \Theta(k_0) 2\pi\delta(k_0^2 - |\boldsymbol{k}|^2) & \frac{-i}{k_0^2 - |\boldsymbol{k}|^2 - i\eta} \end{pmatrix} + 2\pi\delta(k_0^2 - |\boldsymbol{k}|^2)\, n_B(|k_0|) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$$

- $$G^{\mathrm{DM}}(p_0) = \begin{pmatrix} \frac{i}{p_0 - h^{(0)} + i\eta} & 0 \\ 2\pi\delta(p_0 - h^{(0)}) & \frac{-i}{p_0 - h^{(0)} - i\eta} \end{pmatrix} + 2\pi\delta(p_0 - h)\, n_B(p_0 + 2M) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

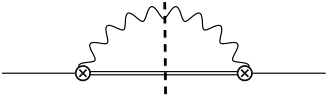
Formation of Bound-States[2]

- U(1): $(\sigma_{\rm bsf} v_{\rm rel})(\boldsymbol{p}) = -2\langle \boldsymbol{p}|{\rm Im}[\Sigma(p^0)]|\boldsymbol{p}\rangle = \frac{4}{3}\alpha(\mu_E)\sum_n \left[1 + n_B(\Delta E_n^p)\right] |\langle n|\boldsymbol{r}|\boldsymbol{p}\rangle|^2(\Delta E_n^p)^3$
- SU(N_c): $(\sigma_{\rm bsf} v_{\rm rel})(\boldsymbol{p}_{\rm o}) = \frac{4}{3}C_F\alpha_s(\mu_E)\sum_n \left[1 + n_B(\Delta E_n^{p_{\rm o}})\right] |\langle n|\boldsymbol{r}|\boldsymbol{p}_{\rm o}\rangle|^2(\Delta E_n^{p_{\rm o}})^3$



[2] K. Petraki et al, arXiv:1407.7874v2 (2015), arXiv:1611.01394v2 (2017); T. Binder et al, arXiv:2002.07145 (2020), arXiv:2107.03945 (2021); M. Garny, J. Heisig, arXiv:2112.01499 (2021)

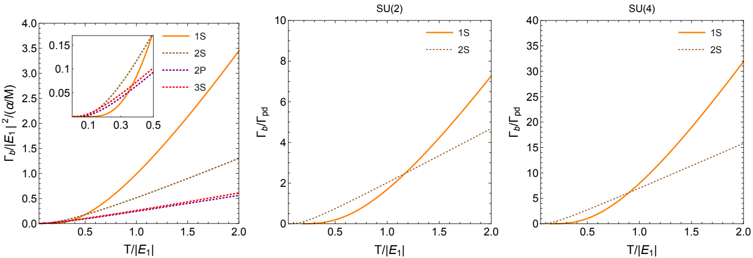
Bound-State Dissociation[3]



- U(1): $\Gamma_n^T = -2\langle n|\mathrm{Im}[\Sigma(p^0)]|n\rangle = \int_{|\boldsymbol{k}|\geq|E_n|} \frac{d^3k}{(2\pi)^3}\, n_B(|\boldsymbol{k}|)\sigma_{\mathrm{dis}}(\boldsymbol{k})$

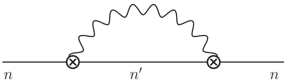
$$\sigma_{\mathrm{dis}}(\boldsymbol{k}) = \frac{4}{3}\alpha(\mu_E)\frac{M^{\frac{3}{2}}}{2}|\boldsymbol{k}|\sqrt{|\boldsymbol{k}|+E_n}\;|\langle n|\boldsymbol{r}|\boldsymbol{p}\rangle|^2\Big|_{|\boldsymbol{p}|=\sqrt{M(|\boldsymbol{k}|+E_{n,\ell,m})}}$$

- Similarly for SU(N_c)

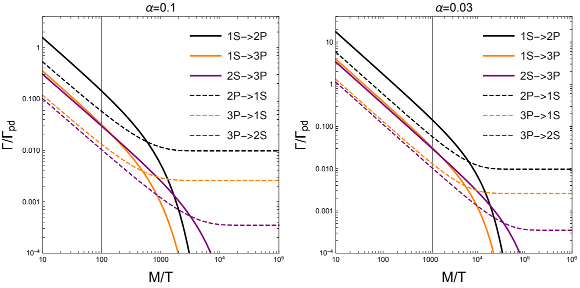


[3] N. Brambilla et al, arXiv:1007.4156 (2010), arXiv:1109.5826 (2011), T. Binder et al, arXiv:2107.03945 (2021); M. Garny, J. Heisig, arXiv:2112.01499 (2021)

Bound-to-Bound Transitions



- Only via charge-neutral dark photons
- Excitations:
  $\Gamma^{T,n}_{\text{ex.}} = \sum_{n'>n} \frac{4}{3}\alpha(\mu_E) \left|\Delta E_n^{n'}\right|^3 n_B\left(\left|\Delta E_n^{n'}\right|\right) |\langle\, n' \,|\boldsymbol{r}|\, n\,\rangle|^2$
- Deexcitations:
  $\Gamma^{T,n}_{\text{de-ex.}} = \sum_{n'<n} \frac{4}{3}\alpha(\mu_E) \left(\Delta E_n^{n'}\right)^3 \left[1 + n_B\left(\Delta E_n^{n'}\right)\right] |\langle\, n' \,|\boldsymbol{r}|\, n\,\rangle|^2$



Continuum-Continuum Transitions

- Abelian case:

  Bremsstrahlung
  $(\sigma v_{\rm rel})^T_{\rm em.}(\boldsymbol{p}) = \frac{4}{3}\alpha(\mu_E)\int_{|\boldsymbol{p}'|<|\boldsymbol{p}|}\frac{d^3p'}{(2\pi)^3}(\Delta E^p_{p'})^3\left[1+n_B(\Delta E^p_{p'})\right]|\langle\boldsymbol{p}|\boldsymbol{r}|\boldsymbol{p}'\rangle|^2$

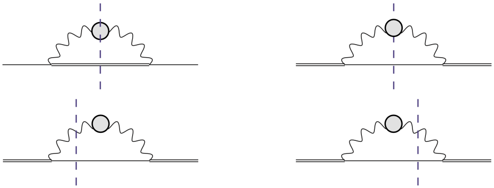
  Thermal absorption
  $(\sigma v_{\rm rel})^T_{\rm abs.}(\boldsymbol{p}) = \frac{4}{3}\alpha(\mu_E)\int_{|\boldsymbol{p}'|>|\boldsymbol{p}|}\frac{d^3p'}{(2\pi)^3}|\Delta E^p_{p'}|^3\left[1+n_B(|\Delta E^p_{p'}|)\right]|\langle\boldsymbol{p}|\boldsymbol{r}|\boldsymbol{p}'\rangle|^2$

- In SU(N_c) more processes:
  octet-octet, octet-unbound singlet, unbound singlet-unbound singlet

- Responsible for keeping the pairs in kinetic equilibrium

Conclusions and Outlook

- Non-relativistic heavy DM pairs in a weakly coupled thermal medium
- Hierarchy of scales $M \gg Mv \gg T \sim Mv^2$ ($\gg \Lambda$)
- pNREFTs at finite T describe any relevant ultrasoft processes + annihilations at desired order
- For $Mv \sim T \gg Mv^2$: T needs to be integrated out
  → Modifications by HTL effects (see A. Dashko's talk)
  Coulomb potential modified by screening thermal Debye mass
  Landau damping becomes important



- For $T \sim M$: No formation of bound-states in the plasma
- OQS: General framework for real-time dynamics of DM in a much larger reservoir[4] (see M. Escobedo's talk)

Thank you!

[4] N. Brambilla et al, arXiv:1711.04515v2 (2018), arXiv:2205.10289 (2022); X. Yao, T. Mehen, arXiv:1811.07027v3 (2019)