SEWM 2022

Gravitational wave background from non-Abelian axion-like reheating

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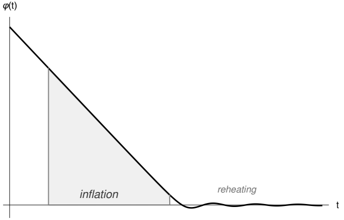
axion $\underbrace{\text{warm inflation}}$

- $\triangleright$ scalar field $\varphi = \varphi(t)$, $\vec{\nabla}\varphi \approx 0$
- $\triangleright$ self-interaction potential $V(\varphi)$
- $\triangleright$ medium at T, $T(0) \sim 0$
- $\triangleright$ friction Υ transfers energy from φ to medium
  $\Rightarrow$ many time scales
- $\triangleright$ How is φ coupled to the heat bath?



axion warm inflation

impose symmetry $\Rightarrow \varphi$ pseudoscalar

- Axion-like coupling
- periodic potential $V(\varphi)$

$$\mathcal{L} = \frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi - V(\varphi) - \varphi J + \mathcal{L}_{\text{bath}}$$

$$J = \frac{g^2}{f_a}\frac{\epsilon^{\mu\nu\rho\sigma}F^c_{\mu\nu}F^c_{\rho\sigma}}{64\pi^2}$$

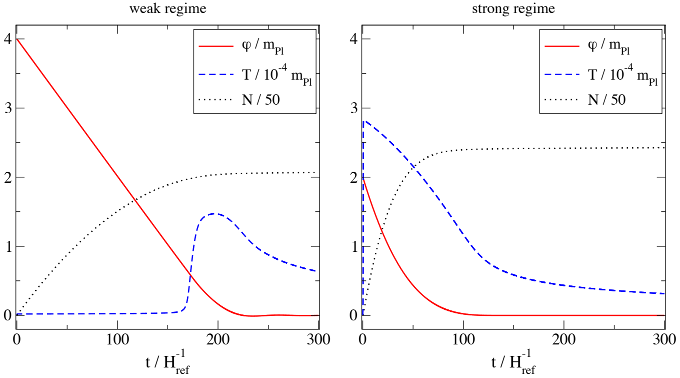
g YM coupling, $\alpha = \frac{g^2}{4\pi}$,

f_a decay constant, $c \in \{1, \ldots, N_c^2 - 1\}$

$\Rightarrow$ medium thermalizes fast

Benchmark solutions for $V(\varphi) \approx \frac{1}{2} m^2 \varphi^2$

M. Laine and S. Procacci, *JCAP* **06** (2021) 031 [arXiv:2102.09913v2]



Reheating:

$$e_r = e_\gamma + e_\nu + \underbrace{e_{\text{GW}}}_{\downarrow} + \ldots \quad \text{radiation energy density}$$

$$\Delta e_r \sim \Delta(N_{\text{eff}} - 3) \approx \begin{cases} 10^{-3} & \text{SM prediction}^1 \\ 10^{-1} & \text{exp. accuracy}^2 \end{cases}$$

assumption of thermal equilibrium justified: $\alpha^2 T > H$

- ▷ obtain lower bound for e_{GW}
- ▷ use local Minkowskian frame

[1] J. Ghiglieri, G. Jackson, M. Laine and Y. Zhu, *JHEP* **07** (2020) 092 [arXiv:2004.11392]

[2] A. Ringwald, J. Schütte-Engel and C. Tamarit, *JCAP* **03** (2021) 054 [arXiv:2011.04731]

GWb production rate:

$$\frac{\mathrm{d}e_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} \sim k^3 \underbrace{n_{\mathrm{B}}(k)}_{\text{distribution}} \; k\,\Gamma(k)$$

- ▷ interaction rate:

$$k\,\Gamma(k) \sim \frac{1}{m_{\mathrm{pl}}^2} \underbrace{\mathbb{L}^{\alpha\beta;\mu\nu}}_{\text{projection}} \, \mathrm{Im}\, G^{\mathrm{R}}_{\alpha\beta\mu\nu}(k,k)$$

- ▷ retarded correlator: $\mathcal{K}\cdot\mathcal{X} = \omega t - \mathbf{k}\cdot\mathbf{x}$

$$G^{\mathrm{R}}_{\alpha\beta\mu\nu}(\omega,k) = i\int_{\mathcal{X}} e^{i\mathcal{K}\cdot\mathcal{X}} \, \langle [T_{\alpha\beta}(\mathcal{X}), T_{\mu\nu}(0)] \rangle_T \, \theta(t)$$

$k \ll \pi T$: hydrodynamic fluctuations

$$T_{\mu\nu} \supset \partial_\mu \varphi \partial_\nu \varphi + T^r_{\mu\nu}$$

- macroscopic scales $\leftrightarrow$ collective phenomena
- correlator $\leftrightarrow$ shear viscosity
  $\Rightarrow$ weakly interacting particles contribute the most
- real time computation in Keldysh r/a basis

$$\Rightarrow \quad \frac{\mathrm{d}e_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} \overset{k \ll \pi T}{\approx} c_{\mathrm{IR}}(k, T)\, \frac{f_a^2 T^5}{m_{\mathrm{pl}}^2}$$

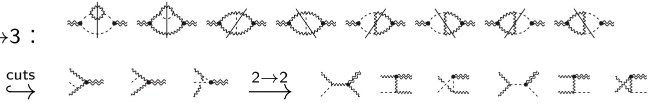
$k \sim \pi T$: Boltzmann regime

$$T_{\mu\nu} \supset \partial_\mu\varphi\partial_\nu\varphi - F^c_{\mu\alpha}F^{c\,\alpha}_{\nu}$$

$$\Downarrow$$

$$\mathbb{L}^{\alpha\beta;\mu\nu} G^{\mathrm{R}}_{\alpha\beta\mu\nu} \sim \bigoplus \underbrace{\mathrm{scat}_{n\to m}(g_1, \varphi, g_3)}_{\text{phase space}} \underbrace{\Theta_{n\to m}(\mathcal{P}_{g_1}, \mathcal{P}_{\varphi}, \mathcal{P}_{g_3})}_{|\text{matrix elements}|^2}$$

e.g. $1\to 3$:



$$\Rightarrow \quad \frac{\mathrm{d}e_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} \overset{k\sim\pi T}{\approx} c_{\mathrm{UV}}(k,\,T)\,\frac{T^9}{f_a^2 m_{\mathrm{pl}}^2}$$

for axion-like inflation we found:

$$\frac{\mathrm{d}e_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} \overset{k\ll\pi T}{\approx} c_{\mathrm{IR}}(k,T)\,\frac{f_a^2 T^5}{m_{\mathrm{pl}}^2}$$

$$\frac{\mathrm{d}e_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k} \overset{k\sim\pi T}{\approx} c_{\mathrm{UV}}(k,T)\,\frac{T^9}{f_a^2 m_{\mathrm{pl}}^2}$$

compare with SM results[3]

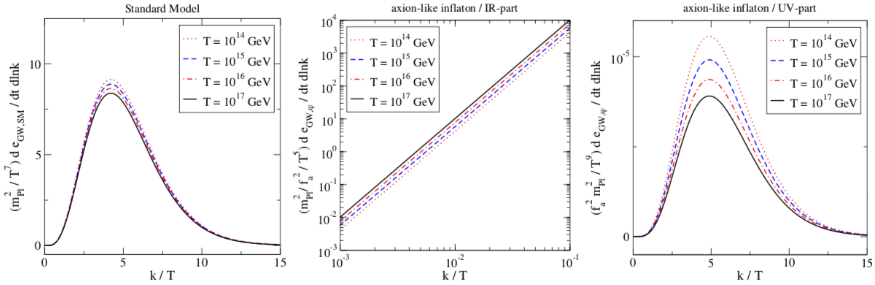
$$\left.\frac{\mathrm{d}e_{\mathrm{GW}}}{\mathrm{d}t\,\mathrm{d}\ln k}\right|_{\mathrm{SM}} \approx c_{\mathrm{SM}}(k,T)\,\frac{T^7}{m_{\mathrm{pl}}^2}$$

different parametric behaviours!

[3] J. Ghiglieri, G. Jackson, M. Laine and Y. Zhu, *JHEP* **07** (2020) 092 [arXiv:2004.11392]

↷ plot coefficients c_{SM}, c_{IR}, and c_{UV}:

P. Klose, M. Laine and S. Procacci, *JCAP* **05** (2022) 021 [arXiv:2201.02317]



benchmark solutions reach $T_{\rm max} \simeq f_a/\alpha \sim 200\, f_a$

$\Rightarrow$ small contributions from c_{IR} $\lesssim$ from c_{UV} $\lesssim$ from c_{SM}

$\Rightarrow$ signal very demanding to observe

Reheating after axion-like inflation yields

- new sources of gravitational waves
- large difference between Abelian[4] vs. non-Abelian heat bath
- non-Abelian case: signal unlikely to exceed SM background

next step: more realistic potential(s)

[4] P. Adshead, J.T. Giblin Jr, M. Pieroni and Z.J. Weiner, *Phys. Rev. D* **101** (2020) 083534 [arXiv:1909.12842]

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Axion warm inflation with $V(\varphi) \approx \frac{1}{2}m^2\varphi^2$

$$\mathcal{L} = \frac{1}{2}\left(\partial^\mu\varphi\,\partial_\mu\varphi - m^2\varphi^2\right) - \varphi J + \mathcal{L}_{\text{bath}}$$

$$\downarrow$$

$$\langle \hat{J}(t)\rangle = -\int_0^t dt'\varphi(t') \underbrace{C_{\text{R}}(t-t')}_{\text{redarded correlator}} + \mathcal{O}(J^3)$$

$$\downarrow$$

$$\boxed{\ddot{\varphi} + \Upsilon\dot{\varphi} + m_{\text{T}}^2\,\varphi^2 \approx 0}$$

$$\Upsilon \approx \frac{\text{Im}\,C_{\text{R}}(m)}{m}\,, \qquad m_{\text{T}}^2 \approx m^2 - \text{Re}\ C_{\text{R}}(m)$$

Retarded pseudoscalar correlator $C_R \approx C_R^{vac} + C_R^{IR}$

▷ $\omega \gg \pi T$: vacuum part[5,6]

$$\mathrm{Im}\, C_R^{vac}(\omega) \sim \frac{\alpha^2 \omega^4}{f_a^2}$$

▷ $\omega \ll \alpha^2 T$: non-perturbative dynamics[7]

$$C_R^{IR}(\omega) \simeq -\frac{\omega \Delta \Upsilon_{IR}}{\omega + i\Delta}$$

Υ_{IR} = sphaleron rate, Δ = YM thermalization rate

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[5] S. Caron-Huot, *Phys. Rev. D* **79** (2009) 125009 [arXiv:0903.3958]

[6] A. L. Kataev, N. V. Krasnikov and A. A. Pivovarov, *Nucl. Phys. B.* **490** (1997) 505 (E) [arXiv:9612326]

[7] G. D. Moore and M. Tassler, *JHEP* **02** (2011) 105 [arXiv:1011.1167]

Computations of the GW production rate – IR

- ▷ retarded propagator of φ: $\epsilon_p^2 \equiv p^2+m^2$

$$\Pi^{\mathrm{R}}(\omega,\vec{p}) = \frac{1}{-\omega^2+\epsilon_p^2-i\omega\Upsilon} \;, \qquad \rho \equiv \mathrm{Im}\Pi^{\mathrm{R}} \;,$$

- ▷ retarded correlator of $T_{\mu\nu}$:

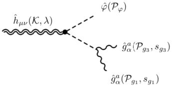
$$\begin{aligned}\mathrm{Im}\; G^{\mathrm{R}}_{xy;xy}(\omega,k)\Big|_{\mathbf{k}=k\,\mathbf{e}_z} = \int_{\mathcal{P}_1,\mathcal{P}_2} \bar{\delta}(\mathcal{K}-\mathcal{P}_1-\mathcal{P}_2)[1+n_{\mathrm{B}}(\omega_1)+n_{\mathrm{B}}(\omega_2)] \\ \times\{\rho_{,x,x}(\mathcal{P}_1)\rho_{,y,y}(\mathcal{P}_2)+\rho_{,y,y}(\mathcal{P}_1)\rho_{,x,x}(\mathcal{P}_2) \\ +2\rho_{,x,y}(\mathcal{P}_1)\rho_{,y,x}(\mathcal{P}_2)\}\end{aligned}$$

$$\Big\Downarrow \; \omega,k,\Upsilon \ll \pi T$$

$$\frac{\mathrm{d}e}{\mathrm{d}t\,\mathrm{d}\ln k} \approx \frac{k^3\Upsilon}{2\pi^3 m_{\mathrm{pl}}^2}\int_0^\infty dp\,\left(\frac{p^3}{\epsilon_p}\right)^2 n_{\mathrm{B}}(\epsilon_p)[1+n_{\mathrm{B}}(\epsilon_p)]\;\underbrace{\mathcal{F}\Big(k,\frac{p}{\epsilon_p}k,\Upsilon\Big)}_{\sim\begin{cases}\Upsilon^{-2} & k\ll\Upsilon \\ k^{-2} & k\gg\Upsilon\end{cases}} \;.$$

Computations of the GW production rate – UV

$\Theta_{1\to3}(\mathcal{P}_{g_1},\mathcal{P}_{\varphi},\mathcal{P}_{g_3})$ = 

$\hookrightarrow$ 

- ▷ on shell: $\mathcal{K}^2=\mathcal{P}^2_{g_1,g_3}=0\,,\ \ \mathcal{P}^2_\varphi=m^2\,;\ \ \mathcal{Q}_i\equiv\mathcal{K}-\mathcal{P}_i\,,\ \ s_{13}\equiv Q^2_\varphi=(\mathcal{P}_{g_1}+\mathcal{P}_{g_3})^2$

- ▷ vertices:  $= -8i\frac{c_\chi g^2}{f_a}\delta^{ab}\epsilon_{\alpha\beta\gamma\delta}\mathcal{P}^\gamma_{g_1}\mathcal{P}^\delta_{g_3}$,

 $= 2i\mathcal{P}^\alpha_\varphi\mathcal{Q}^\beta_\varphi$,

 $= -2i\delta^{ab}[\mathcal{Q}^\alpha\mathcal{P}^\beta\eta^{\gamma\delta}+(\mathcal{Q}\cdot\mathcal{P})\eta^{\alpha\gamma}\eta^{\beta\delta}-\eta^{\alpha\gamma}\mathcal{P}^\beta\mathcal{Q}^\delta-\eta^{\alpha\delta}\mathcal{Q}^\beta\mathcal{P}^\gamma]$

- ▷ polarization sums: $\sum_\lambda\to\mathbb{L}_{\mu\nu;\rho\sigma}(\mathcal{K})\,,\quad \sum_{s_1,s_3}\to\mathbb{P}^{\mathrm{T}}_{\mu\nu}(\mathcal{P}_{g_{1,3}})$

$$\Rightarrow\ \ldots\ \Rightarrow\ \ldots\ \Rightarrow\ \Theta_{1\to3}\sim\frac{s^4_{13}+m^8}{(s_{13}-m^2)^2}\ \xrightarrow{m\approx0}\ s^2_{13}\ .$$