New methods for studying the Electroweak phase transition

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Talk based on 2205.08815 , 2205.07241, and 2205.05145

Why care about phase transitions?

First-order phase transition $\Longrightarrow$ Electroweak Baryogenesis?

Baryon asymmetry

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \approx \underbrace{6 \times 10^{-10}}_{\text{Observation}} \gg \underbrace{10^{-20}}_{\text{Prediction}}$$

Gravitational Waves $\Longrightarrow$ Need accurate calculations

A classic tale about a hot topic

$$\left.\begin{array}{ll} \text{Effective Potential} : L \sim T\frac{d}{dT}V_A - T\frac{d}{dT}V_B & \to \alpha \\ \text{Nucleation Rate} : \Gamma \sim Ae^{-S_3/T} & \to \beta \end{array}\right\} \Omega_{\rm GW}$$

Effective field-theory to the rescue: Dimensional reduction

Phase transitions in a nutshell

Effective mass:
$m^2_{\text{eff}} = (m^2 + \underbrace{aT^2}_{\text{Thermal Mass}}) \ll m^2$

Fine-tuning $\implies \underbrace{bT^2}_{\text{2-loop Mass}} \approx m^2_{\text{eff}}$

RG $\implies \mu \frac{d}{d\log\mu} m^2_{\text{eff}} \approx m^2_{\text{eff}}$

Logarithms $\implies \log T^2/m^2_{\text{eff}} \gg 1$

Extreme uncertainties for $\Omega_{\text{GW}} \implies$ Can we trust theoretical calculations?

Solution: Integrate out $E \sim T$ modes (9508379,2104.04399)

No more large logs: $\log T^2/m^2_{\text{eff}} \to \underbrace{\log T^2/\mu^2}_{\text{Match at } \mu\sim T} + \underbrace{\log \mu^2/m^2_{\text{eff}}}_{\text{RG-evolution in the EFT}}$ ✓

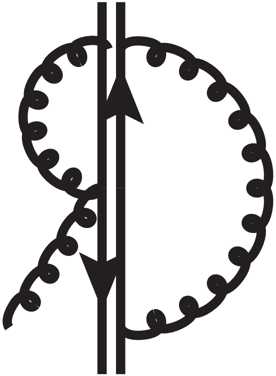
Two-loop thermal masses $\to$ From matching ✓

Thermally resummed couplings $\to$ From matching ✓

Simpler calculations $V_{\text{1-Loop}} \to -m^3_{\text{eff}}, \quad V_{\text{2-Loop}} \to \log \mu^2/m^2_{\text{eff}} + m^2_{eff}$ ✓

Get the high-temperature EFT in Mathematica within seconds!

`https://github.com/DR-algo/DRalgo` (2205.08815)



`DRalgo` : Automatic matching to two loops

- $\rightarrow$ Two-loop thermal masses ✓
- $\rightarrow$ Two-loop Debye masses ✓
- $\rightarrow$ One-loop thermal couplings ✓
- $\rightarrow$ Two-loop effective potential ✓
- $\rightarrow$ Beta functions at $T = 0$ ✓
- $\rightarrow$ Beta functions in the effective theory ✓

How does it work? (see Tuomas' talk)

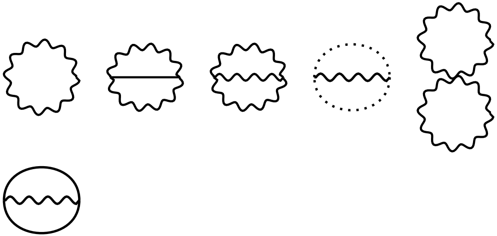
$$\left.\begin{array}{ll}
\text{Calculate effective couplings} & \to \lambda_{\text{eff}}(T), m^2_{\text{eff}}(T), \ldots \\
\text{Calculate 3d effective potential} & \to V^{3d}_{\text{eff}}(\phi) \to T_c \\
\text{Calculate 3d nucleation rate} & \to \Gamma \sim e^{-S_3} \to T_N \\
\text{Calculate latent heat} & \to \alpha \propto \frac{d}{dT} V^{3d}_{\text{eff}} = \frac{d\lambda_{\text{eff}}}{dT}\frac{dV^{3d}_{\text{eff}}}{d\lambda_{\text{eff}}} + \ldots \\
\text{Calculate phase-transition duration} & \to \beta \propto \frac{d}{dT} S_3 = \frac{d\lambda_{\text{eff}}}{dT}\frac{dS_3}{d\lambda_{\text{eff}}} + \ldots
\end{array}\right\} \Omega_{\text{GW}}$$

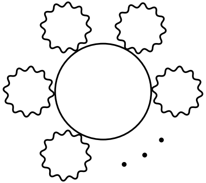
Radiative barriers in the High-T EFT (2205.0724)

Barrier from vector bosons

$$V_{\text{tree}}(\phi) = \frac{1}{2} m_3^2 \phi^2 + \frac{1}{4}\lambda_3 \phi^4 \to V_{\text{LO}}(\phi) = \frac{1}{2} m_3^2 \phi^2 - \frac{1}{16\pi} g_3^3 \phi^3 + \frac{1}{4}\lambda_3 \phi^4$$

$m_A^2 \sim g_3^2, \quad m_H \sim \lambda_3 \implies$ Only consistent if $\frac{m_H^2}{m_A^2} \sim x \ll 1$





The expansion is in powers of x—From lattice: continuous transition if $x \gtrsim 0.1$

Integrating out vectors bosons at 2-loops give V_{NLO}

Scalar-loop contribution appear first at NNLO

Strict perturbative expansion

Rewriting the potential with dimensionless variables

$$V_{\text{LO}}(\phi) = \frac{1}{2} y \phi^2 - \frac{1}{16\pi}\phi^3 + \frac{1}{4} x \phi^4, \quad x = \frac{\lambda_3}{g_3^2}, \quad y = \frac{m_3^2}{g_3^4}$$

Symmetric minima: $\phi_{\text{s}} = 0$ Broken minima $\phi_{\text{b}} \sim x^{-1} \neq 0$

How do we consistently include higher orders?

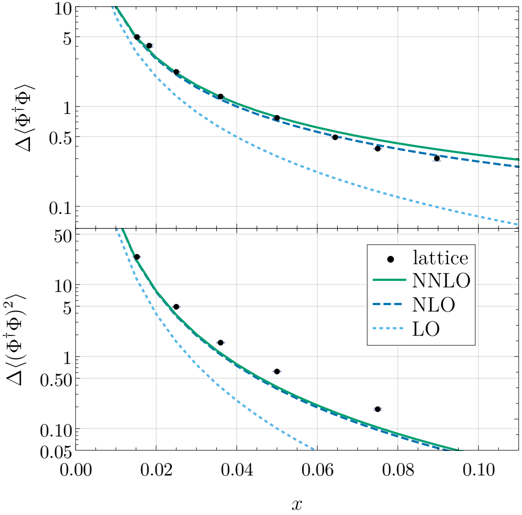
Minima coincide when $\Delta V(x, y_c) \equiv V_{\text{LO}}(\phi_{\text{b}}) - V_{\text{LO}}(\phi_{\text{s}}) = 0 \implies$ Critical mass y_c

Consistent expansion: $\phi_b = \phi_{\text{LO}} + x\phi_{\text{NLO}} + \ldots \implies$ Gauge invariance

Critical mass: $y_c = y_{\text{LO}} + x y_{\text{NLO}} + \ldots \implies$ Exact RG-invariance at every order

Observables: $\frac{d}{dy}\Delta V(x, y_c) \equiv \Delta\langle \Phi^\dagger \Phi \rangle, \quad \frac{d}{dx}\Delta V(x, y_c) \equiv \Delta\langle (\Phi^\dagger \Phi)^2 \rangle$

Comparison with Lattice (data from 2205.07238)



$$\Delta\left\langle\Phi^\dagger\Phi\right\rangle = \frac{1+\frac{51}{2}x+13\sqrt{2}x^{3/2}}{2(8\pi x)^2}$$

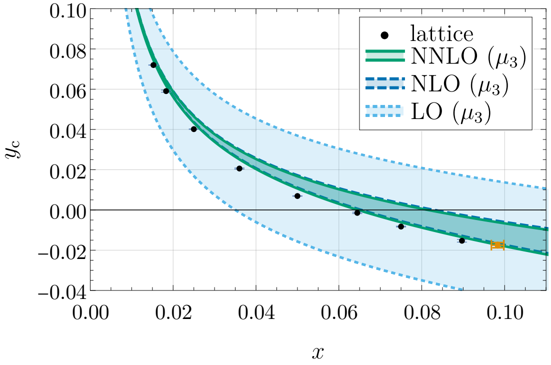
$$\Delta\left\langle(\Phi^\dagger\Phi)^2\right\rangle = \frac{1+51x+14\sqrt{2}x^{3/2}}{4(8\pi x)^4}.$$

Latent heat: $L \approx 4\times\Delta\left\langle\Phi^\dagger\Phi\right\rangle$

Large corrections from NLO ✓

NNLO correction under control

Result for the critical mass



$$y_c = \frac{1 - \frac{51}{2} x \log \tilde{\mu}_3 - 2\sqrt{2} x^{3/2}}{2(8\pi)^2 x}$$

Expect expansion to fail when $y_c \approx 0$

Radiative corrections to the nucleation rate

Thermal escape

$\Gamma = \underbrace{\Gamma_{\text{stat}}}_{\text{Boltzmann factor}} \times \underbrace{\Gamma_{\text{dyn}}}_{\text{Damping}}$

Effective action: $\Gamma_{\text{stat}} = e^{-S_{\text{eff}}} \sim T^3 e^{-S_{\text{LO}}}$

Higher-order: $S_{\text{eff}} = S_{\text{LO}} + S_{\text{NLO}} + \ldots$

$S_{\text{NLO}} \sim R^3$ and $S_{\text{LO}} \sim R^2$

$\rightarrow$ Trouble with large bubbles

Corrections to the bounce are important

Functional determinant

$S_{\text{NLO}} = \frac{1}{2}\sum_i \operatorname{Tr}\log\left[-\nabla^2 + M_i^2[\phi_B]\right]$ $\rightarrow$ Calculate numerically

Straightforward to calculate in the effective theory

Recent lattice results for a radiative barrier in 2205.07238

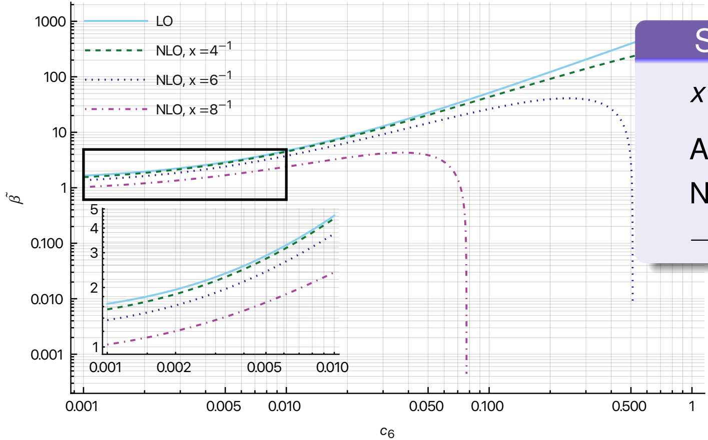
Example: Dimension-6 operator

$$V(\phi) = \frac{1}{2} m_3^2 \phi^2 - \frac{1}{4}\lambda_3 \phi^4 + \frac{1}{32} c_6 \phi^6, \quad y = \frac{m_3^2}{\lambda_3^2}$$

$S_{\text{eff}}(y_N) = S_{\text{LO}} + S_{\text{NLO}} = 126 \implies$ Nucleation mass y_N

$\beta_N / H_N \sim \tilde{\beta} = \frac{d}{dy} S_{\text{eff}}(y_N)$—Observable in the effective 3d theory

Results for $\tilde{\beta}$ (2205.05145)



Sizeable radiative corrections

$$x = \frac{\lambda_3}{g_3^2} \approx \frac{\lambda}{g^2}, \qquad c_6 = T^2 c_{6,4d}$$

- Absolute upper bound $c_6 \lesssim x^3$
- NLO can change $\tilde{\beta}$ by a factor of 2
- $\rightarrow$ Corrections propagate to GWs

Calculating 1-loop corrections are not only doable, but straightforward

Summary

The Electroweak phase transition is a hot topic

$\rightarrow$ Uncertainties for common methods span orders of magnitude

$\rightarrow$ High-temperature effective theory key to reduce RG-scale dependence

$\rightarrow$ EFT construction has been automatized

$\rightarrow$ Calculations simpler in the EFT

Strict perturbative expansions are simple and consistent

$\rightarrow$ 3-loop corrections straightforward to include for the effective potential

$\rightarrow$ 1-loop corrections straightforward to include for the nucleation rate

Robust methods are needed for accurate predictions

Thank You

Backup slides

DRalgo example: Standard-Model with nF fermion families

Effective Couplings: Lb, Lf $\sim \log \mu / T$ (matching scale $\mu \sim T$)

Out[∘]=
$$\left\{\mathrm{gw3d}^2 \to \frac{\mathrm{gw}^4\, T\,(43\,\mathrm{Lb} - 8\,\mathrm{Lf}\,\mathrm{nF} + 4)}{96\,\pi^2} + \mathrm{gw}^2\, T,\ \mathrm{gY3d}^2 \to \mathrm{gY}^2\, T - \frac{\mathrm{gY}^4\, T\,(3\,\mathrm{Lb} + 40\,\mathrm{Lf}\,\mathrm{nF})}{288\,\pi^2},\ \mathrm{gs3d}^2 \to \frac{\mathrm{gs}^4\, T\,(33\,\mathrm{Lb} - 4\,\mathrm{Lf}\,\mathrm{nF} + 3)}{48\,\pi^2} + \mathrm{gs}^2\, T,\right.$$
$$\left.\lambda 1\mathrm{H3d} \to \frac{T\left(24\,\lambda 1\mathrm{H}\left(3\,\mathrm{gw}^2\,\mathrm{Lb} + \mathrm{gY}^2\,\mathrm{Lb} - 4\,\mathrm{Lf}\,\mathrm{yt}^2\right) + (2 - 3\,\mathrm{Lb})\left(3\,\mathrm{gw}^4 + 2\,\mathrm{gw}^2\,\mathrm{gY}^2 + \mathrm{gY}^4\right) + 256\,\pi^2\,\lambda 1\mathrm{H} - 192\,\lambda 1\mathrm{H}^2\,\mathrm{Lb} + 48\,\mathrm{Lf}\,\mathrm{yt}^4\right)}{256\,\pi^2}\right\}$$

One-loop scalar masses

Out[∘]=
$$\left\{\mathrm{m23d} \to \frac{1}{16}\, T^2\left(3\,\mathrm{gw}^2 + \mathrm{gY}^2 + 8\,\lambda 1\mathrm{H} + 4\,\mathrm{yt}^2\right) + \mathrm{m2}\right\}$$

Two-loop Debye masses

Out[∘]=
$$\left\{\mu\mathrm{sqSU2} \to \frac{\mathrm{gw}^2\left(T^2\left(\mathrm{gw}^2\left(86\,\mathrm{Lb}\,(2\,\mathrm{nF} + 5) - 32\,(\mathrm{Lf} - 1)\,\mathrm{nF}^2 + (44 - 80\,\mathrm{Lf})\,\mathrm{nF} + 207\right) - 3\left(6\left(8\,\mathrm{gs}^2\,\mathrm{nF} - 4\,\lambda 1\mathrm{H} + \mathrm{yt}^2\right) + \mathrm{gY}^2\,(4\,\mathrm{nF} - 3)\right)\right) + 144\,\mathrm{m2}\right)}{1152\,\pi^2},\right.$$
$$\mu\mathrm{sqSU3} \to \frac{\mathrm{gs}^2\, T^2\left(4\,\mathrm{gs}^2\,(33\,\mathrm{Lb}\,(\mathrm{nF} + 3) + \mathrm{nF}\,(-4\,\mathrm{Lf}\,(\mathrm{nF} + 3) + 4\,\mathrm{nF} + 3) + 45) - 27\,\mathrm{gw}^2\,\mathrm{nF} - 11\,\mathrm{gY}^2\,\mathrm{nF} - 36\,\mathrm{yt}^2\right)}{576\,\pi^2},$$
$$\left.\mu\mathrm{sqU1} \to -\frac{\mathrm{gY}^2\left(T^2\left(18\left(88\,\mathrm{gs}^2\,\mathrm{nF} - 36\,\lambda 1\mathrm{H} + 33\,\mathrm{yt}^2\right) + 81\,\mathrm{gw}^2\,(4\,\mathrm{nF} - 3) + \mathrm{gY}^2\left(6\,\mathrm{Lb}\,(10\,\mathrm{nF} + 3) + 800\,(\mathrm{Lf} - 1)\,\mathrm{nF}^2 + 60\,(4\,\mathrm{Lf} + 17)\,\mathrm{nF} - 45\right)\right) - 1296\,\mathrm{m2}\right)}{10\,368\,\pi^2}\right\}$$