Stochastic inflation from QFT and the parametric dependence of the effective noise amplitude

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The stochastic approach to computing cosmological observables

- Non-perturbative method to obtain effective IR dynamics of scalar fields in (near-)dS spacetime [Starobinsky 86', Starobinsky&Yokoyama 94']

- Based on a field split into long- and short-wavelength modes:

$$\phi(t,\mathbf{x}) = \phi_{IR}(t) + \phi_{UV}(t,\mathbf{x}) \qquad \begin{cases} \phi_{IR}: & \textbf{classical, non-linear evolution} \\ \phi_{UV}: & \textbf{quantum, linear evolution} \end{cases}$$

- Effective IR dynamics modelled as a classical stochastic process, described by a Langevin equation:

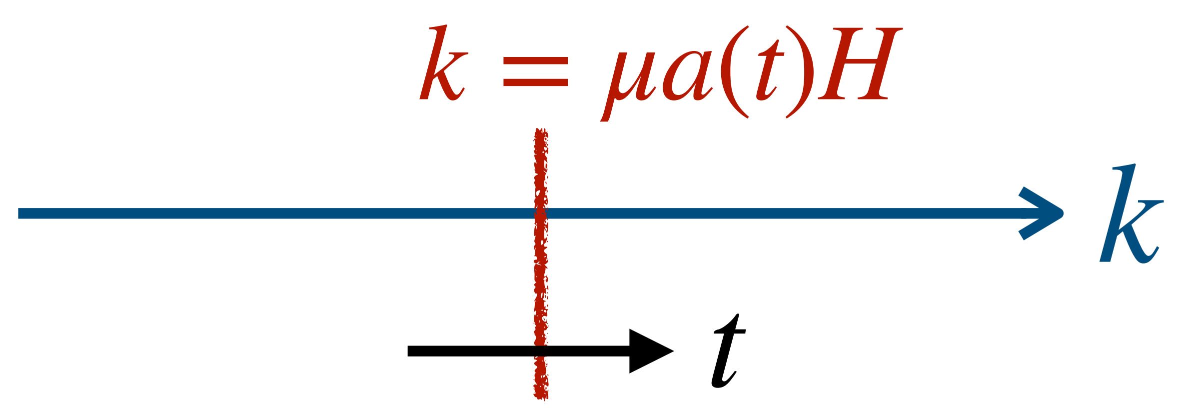
$$3H\dot{\phi}_{IR} + V'(\phi_{IR}) = \xi, \qquad < \xi(x)\xi(x') > = \frac{9H^5}{4\pi^2}\delta(t-t')\frac{\sin \mu a H|\mathbf{x}-\mathbf{x}'|}{\mu a H|\mathbf{x}-\mathbf{x}'|}$$

" μ small parameter"

- White noise ξ from the short-wavelength part of ϕ, stochastic average ~ quantum expectation value

The stochastic approach to computing cosmological observables

- Starobinsky's choice of field split:



$$\phi(x) = \phi_{IR}(t) + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\Theta(k - \mu a(t)H)\hat{\phi}_{\mathbf{k}}(t, \mathbf{x})$$

- Short-wavelength modes are redshifted into the IR domain, where they affect the IR modes ϕ_{IR} as white noise kicks

$$3H\dot{\phi}_{IR} + V'(\phi_{IR}) = \xi, \qquad < \xi(x)\xi(x') > = \frac{9H^5}{4\pi^2}\delta(t - t')\frac{\sin \mu aHr}{\mu aHr}$$

- Field split is a coarse graining procedure: ϕ_{IR} equal to value of ϕ averaged over some scale R by way of smoothing function

- Noise spatially localised on patches with space separation $r < (\mu aH)^{-1} \equiv R$ (coarse-graining scale)

The stochastic approach to computing cosmological observables

- Observables can be computed from the corresponding Fokker-Planck distribution and its late-time equilibrium solution,

$$\dot{P}(t,\phi_{IR}) = \frac{1}{3H}\partial_\phi[V'(\phi_{IR})P(t,\phi_{IR})] + \frac{H^3}{8\pi^2}\partial^2_\phi P(t,\phi_{IR}) \qquad \lim_{t\to\infty} P_{eq}(\phi_{IR}) \sim e^{-\frac{8\pi^2 V(\phi_{IR})}{3H^4}}$$

e.g. $< \phi_{IR}(t)^2 > = \int \mathrm{d}\phi_{IR} P_{\mathrm{eq}}(t,\phi_{IR})\phi^2_{IR}(t)$

$$< \phi_{IR}(t)^2 > \Big|_{V(\phi_{IR})=m\phi^2/2} = \frac{3H^4}{8\pi^2 m^2} \qquad < \phi_{IR}(t)^2 > \Big|_{V(\phi_{IR})=\lambda\phi^4/4} = \sqrt{\frac{3}{2\pi^2}}\frac{\Gamma(3/4)}{\Gamma(1/4)}\frac{H^2}{\sqrt{\lambda}}$$

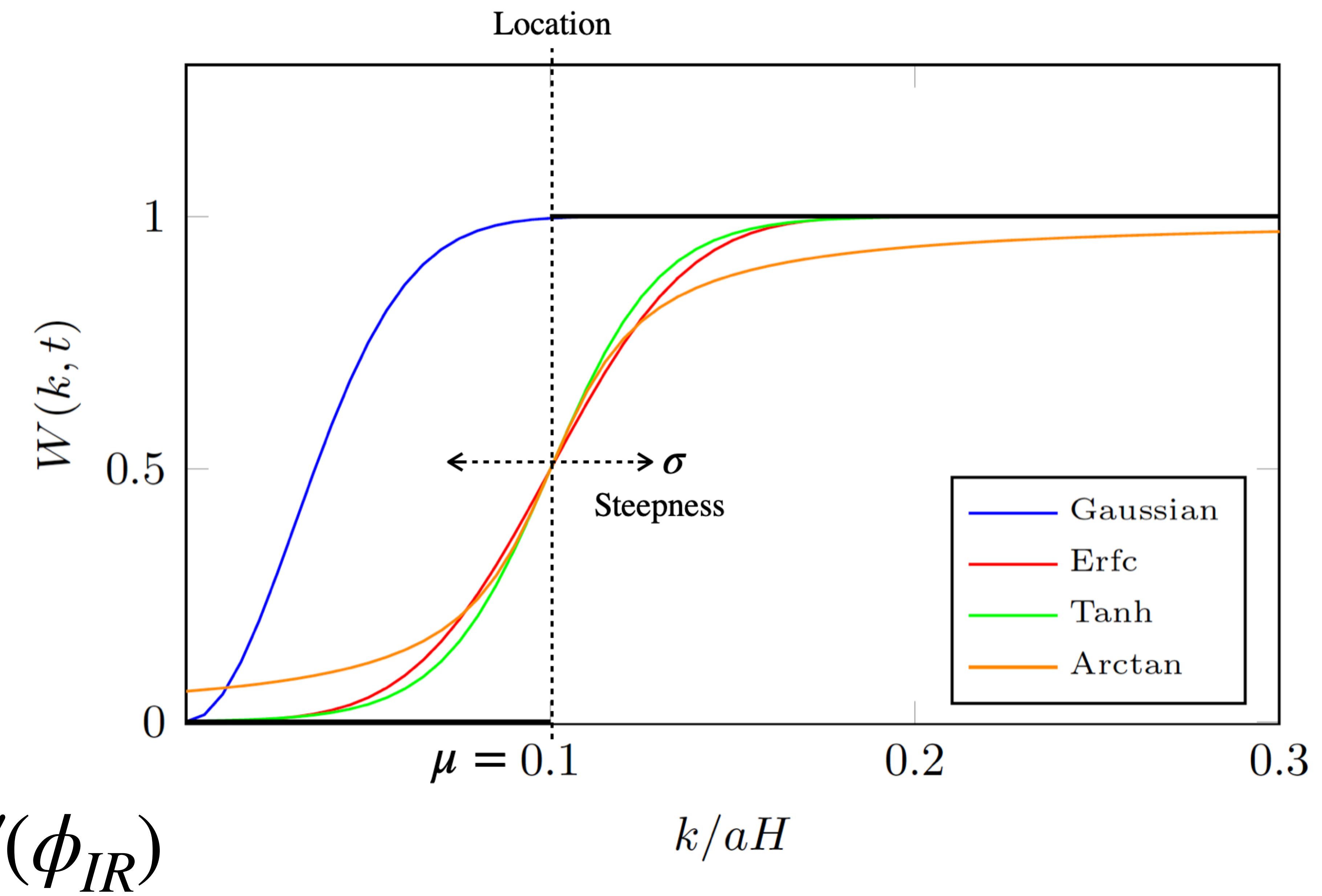
- Agrees (largely) with other QFT results at leading IR order *e.g.* large-N expansion [Serreau 11'], 2PI truncations [Garbrecht & Rigopolous 11', Beneke & Moss 12', Arai 12'], RG analyses [Burgess et al 09'] and Euclidean dS [Rajamaran 10'], …

- Many applications, but comparatively little work on the QFT basis of the stochastic formalism

Stochastic approach from QFT?

- Yes, can be derived from CTP path integral in Keldysh basis. Assume smooth transition between long- and short-wavelength modes:

$$\phi(x) = \phi_{IR}(t) + \int \frac{d^3\mathbf{k}}{(2\pi)^3} W(k,t)\hat{\phi}_{\mathbf{k}}(t,\mathbf{x})$$



- White noise ξ an effect from integrating out the short-wavelength part of ϕ $\rightarrow$ noise term via Hubbard-Stratonovich transformation

- IR effective EoM: $$0 = \ddot{\phi}_{IR} + 3H\dot{\phi}_{IR} - \frac{\nabla_{\mathbf{x}}^2}{a^2}\phi_{IR} - \xi + V'(\phi_{IR})$$

$$+ \int_{x'} Q_x Q_{x'} G_R(x,x') + \frac{\delta(t-t')}{a^3(t')}(W_{x'}G_0^{-1} + 2Q_{x'}W_{w'} + 2W_{x'}\dot{W}_{x'}\partial_{t'})\phi_{IR}(x')$$

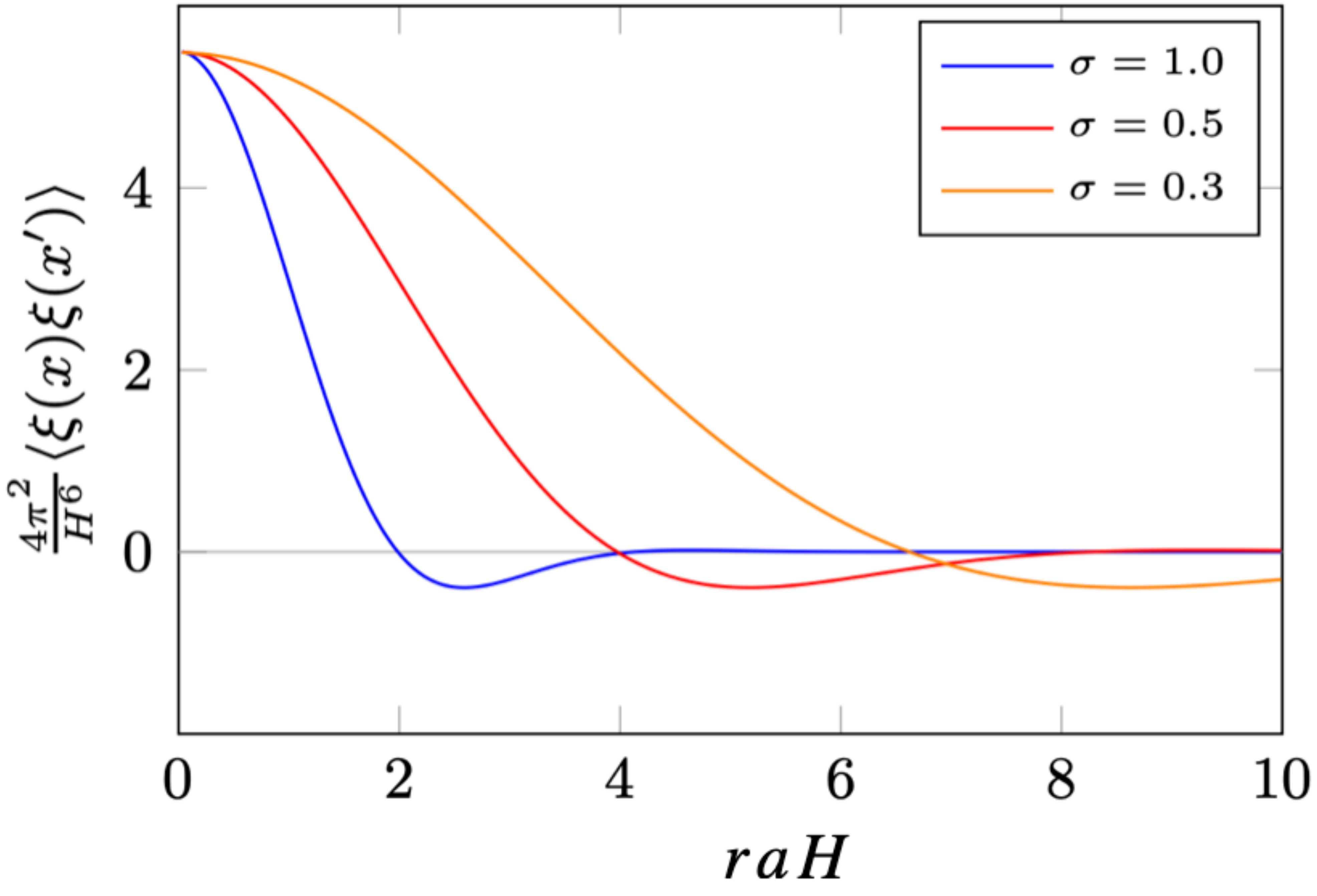
- Noise purely a result of time-dependent field split — present also in non-interacting case

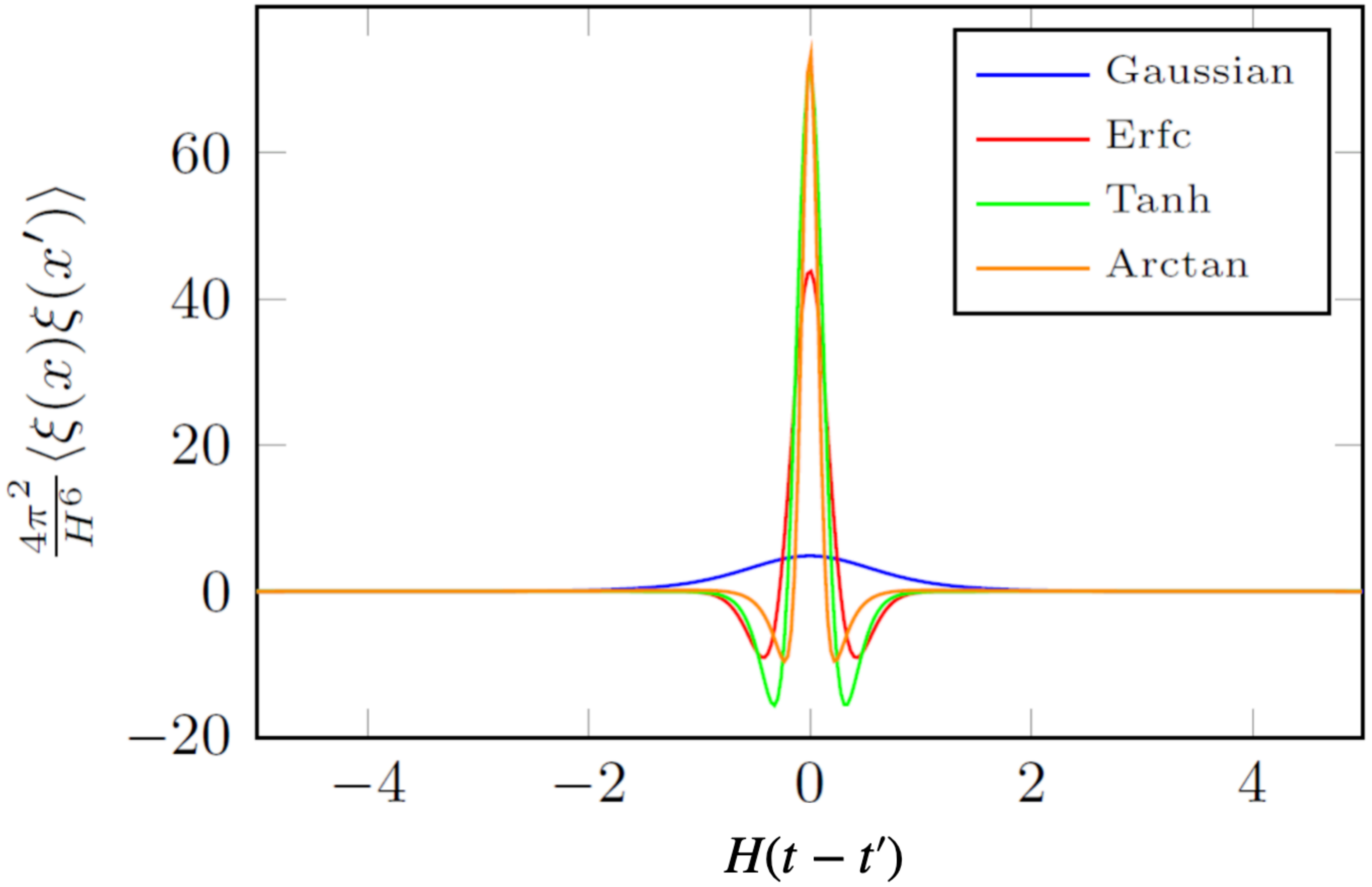
Effective noise

- Noise correlation $$< \xi(x)\xi(x') > = \int_{\mathbf{k}} Q_t Q_{t'} G_F(t,t'), \qquad Q_t = -\ddot{W}(k,t) - \dot{W}(k,t)(3H + 2\partial_t)$$

Exponentially decaying for spatial separations $r > (\sigma a H)^{-1}$ and around equal time arguments

$$W(k,t) = 1 - e^{-(k/\sigma a H)^2/2}$$





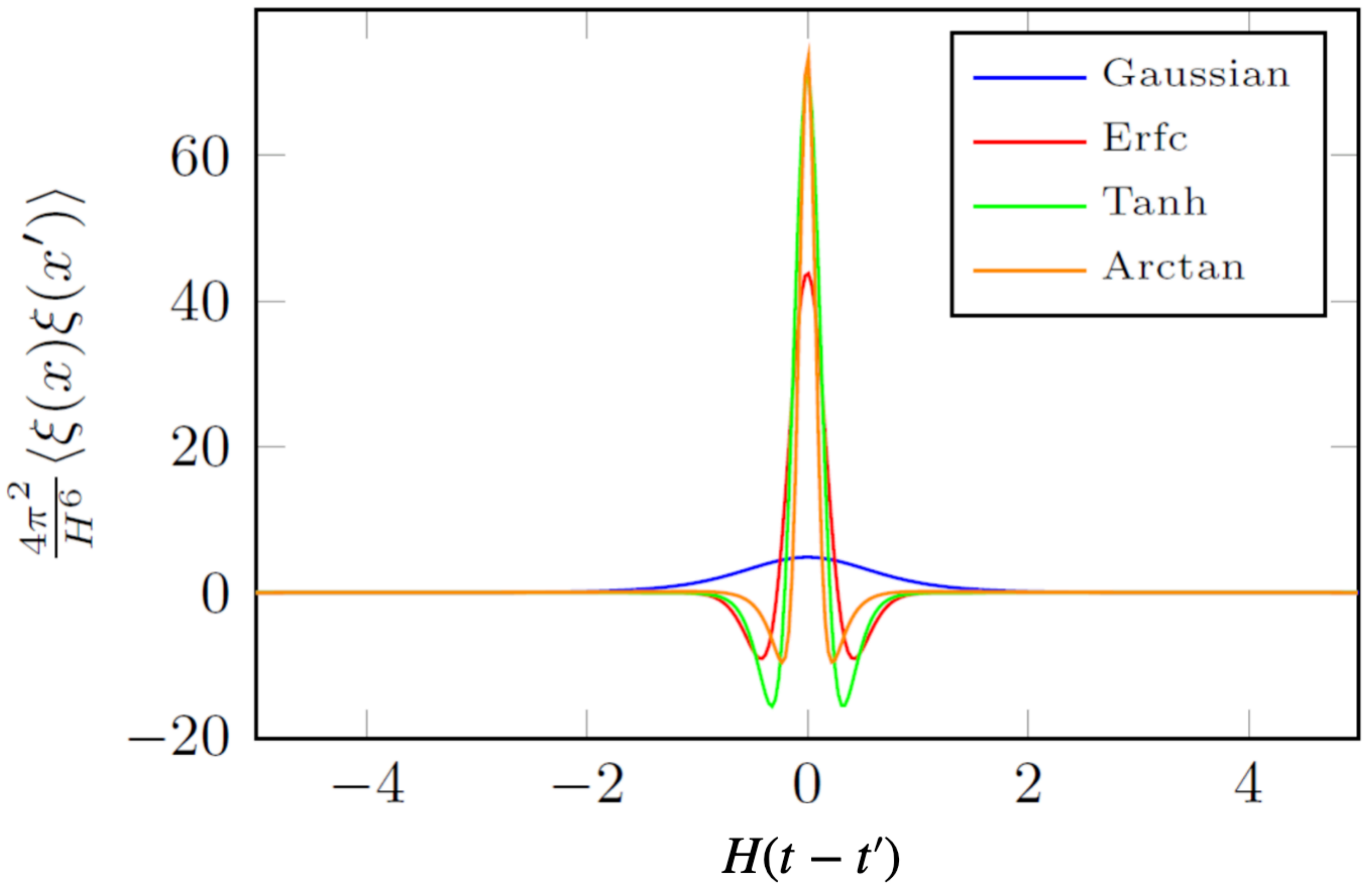
Effective noise

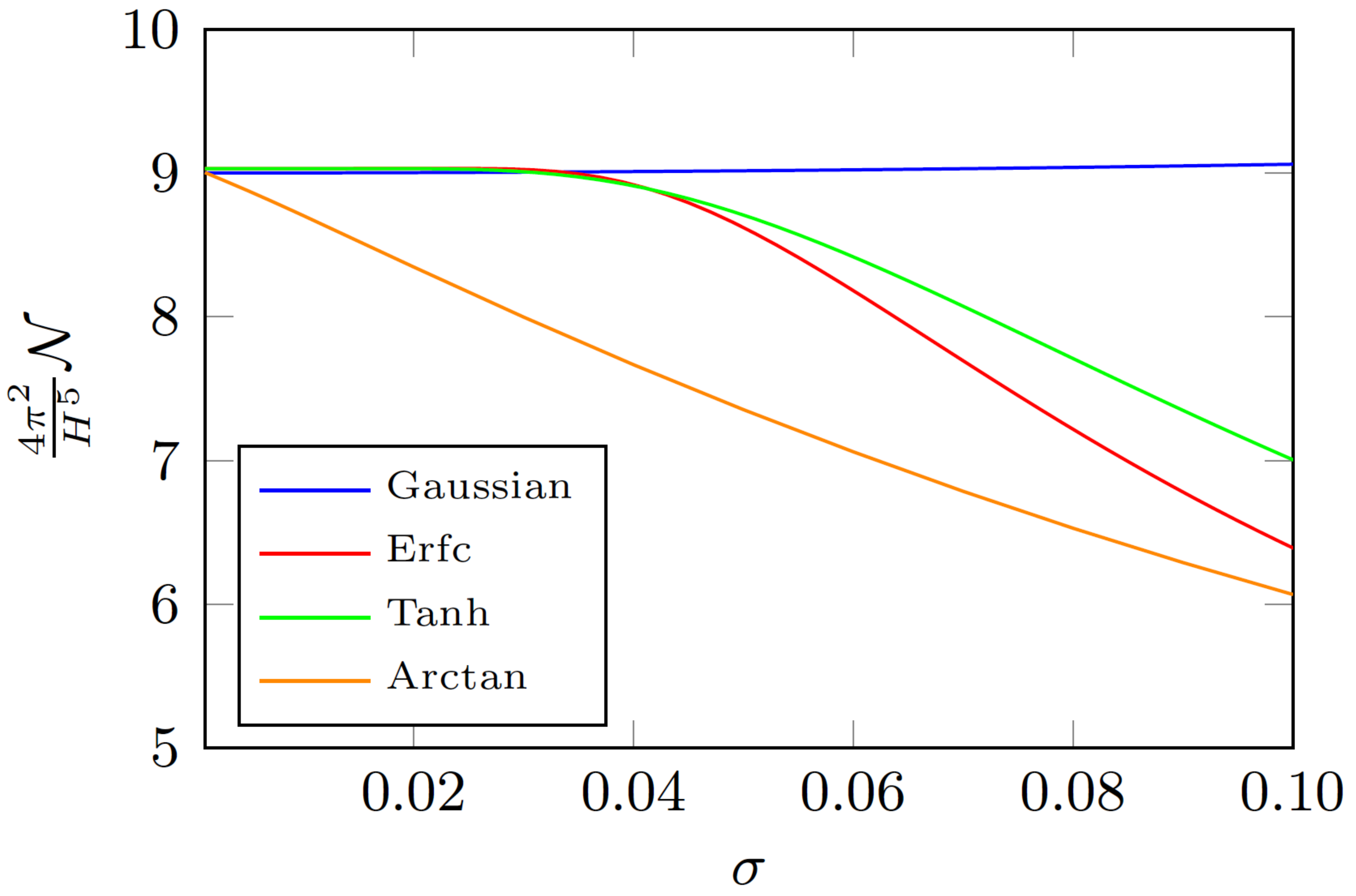
- Prescription: interpret stochastic noise as smoothed-out δ-function in time
  → noise amplitude modulated by some function f dependent on all ingoing parameters:

$$< \xi^2(x) > = \frac{9H^5}{4\pi^2} f(\mu, \sigma, \epsilon_M, \epsilon_H, \ldots) \equiv \mathcal{N}$$

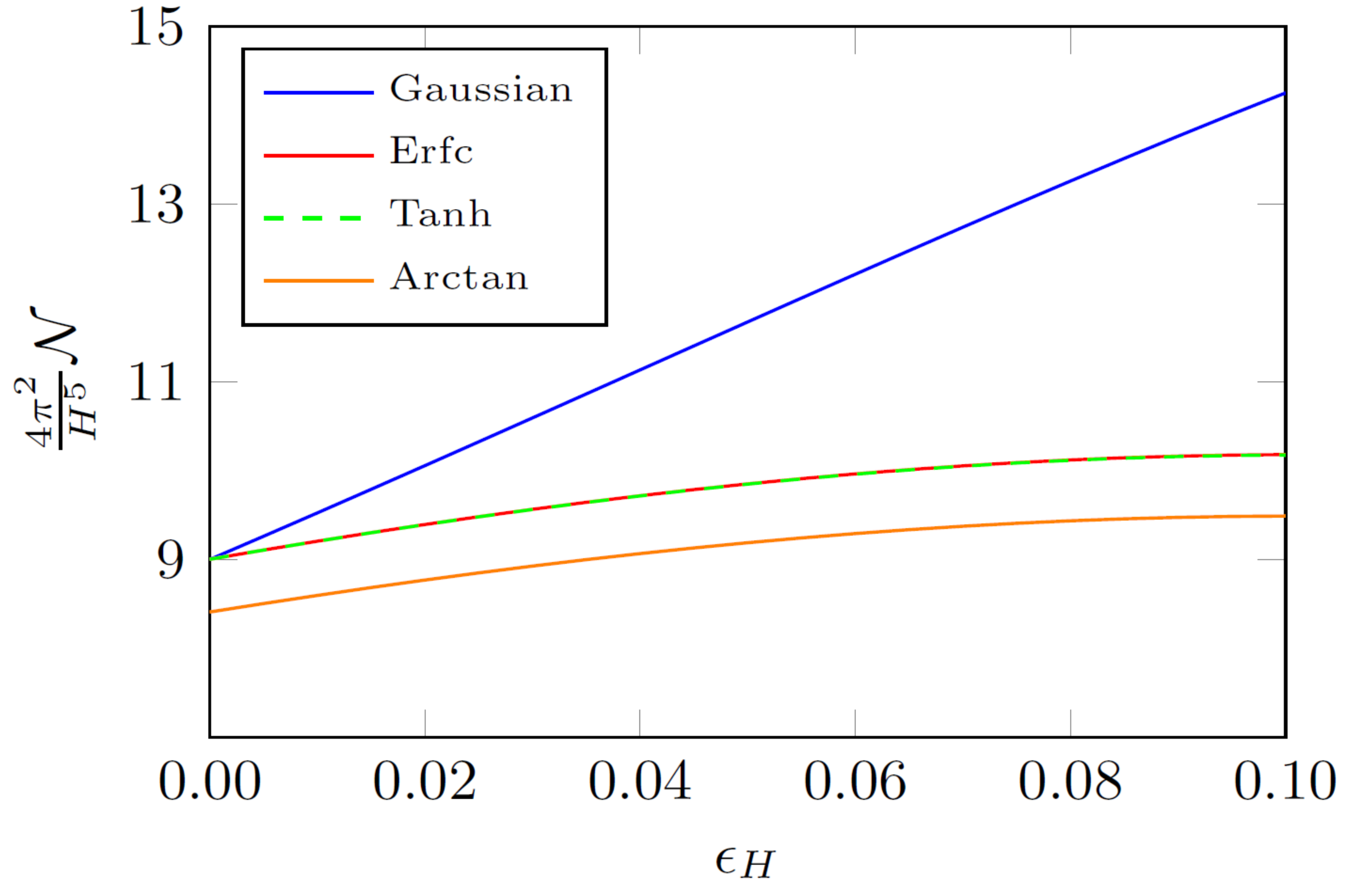
$$\epsilon_M \equiv \frac{m^2}{3H^2}$$

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2}$$

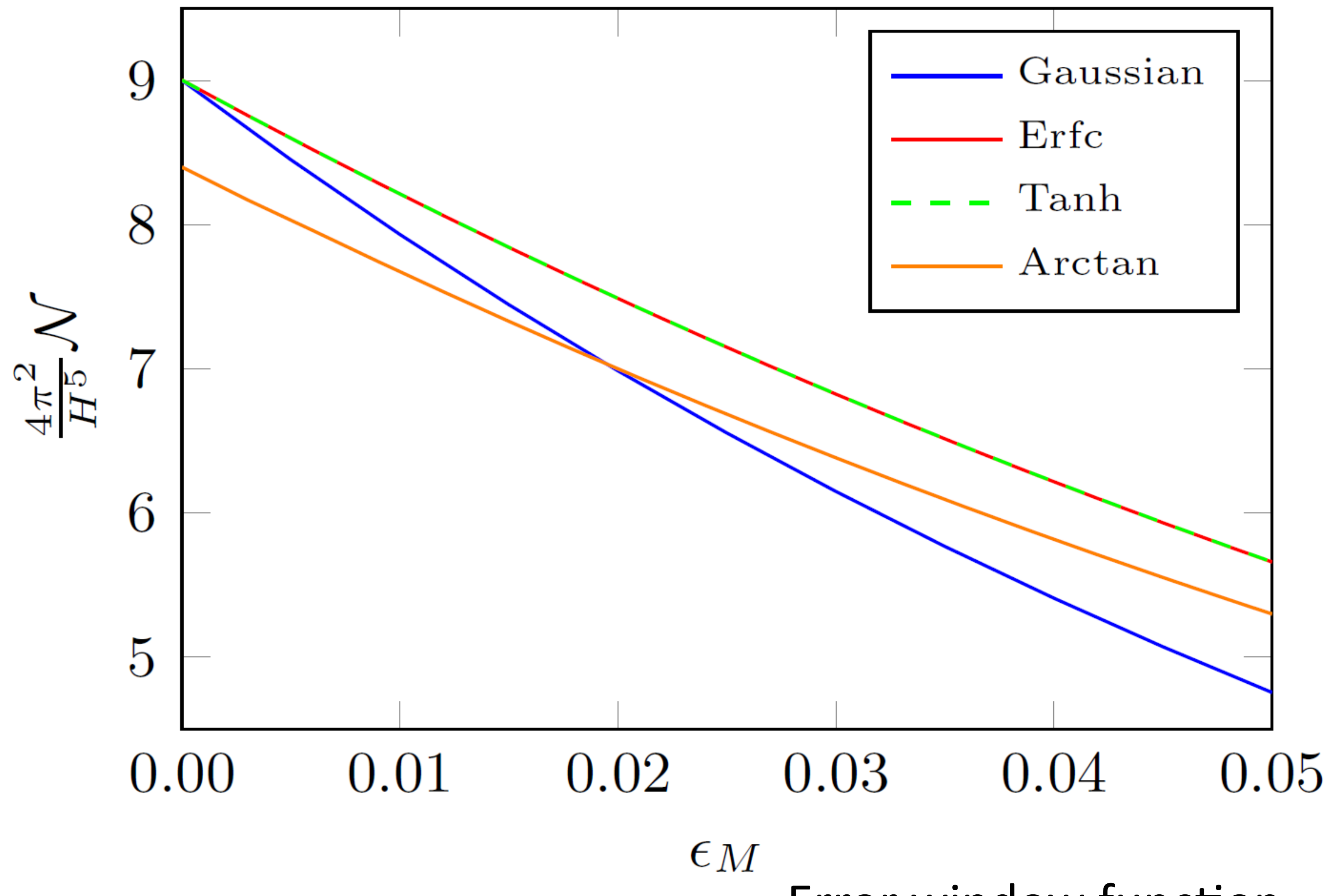




Effective noise

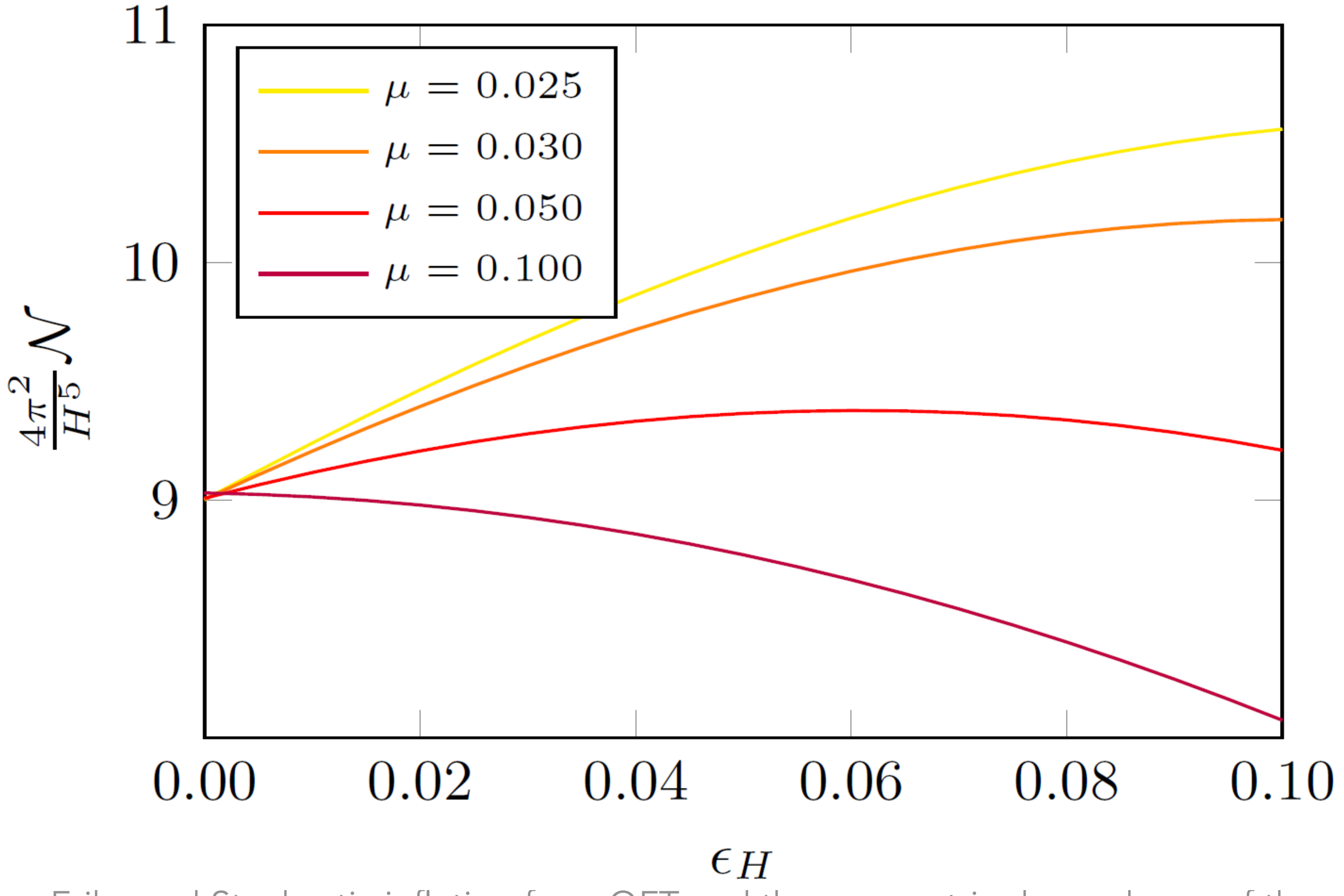


$\sigma = 0.005, \quad \mu = 0.03$

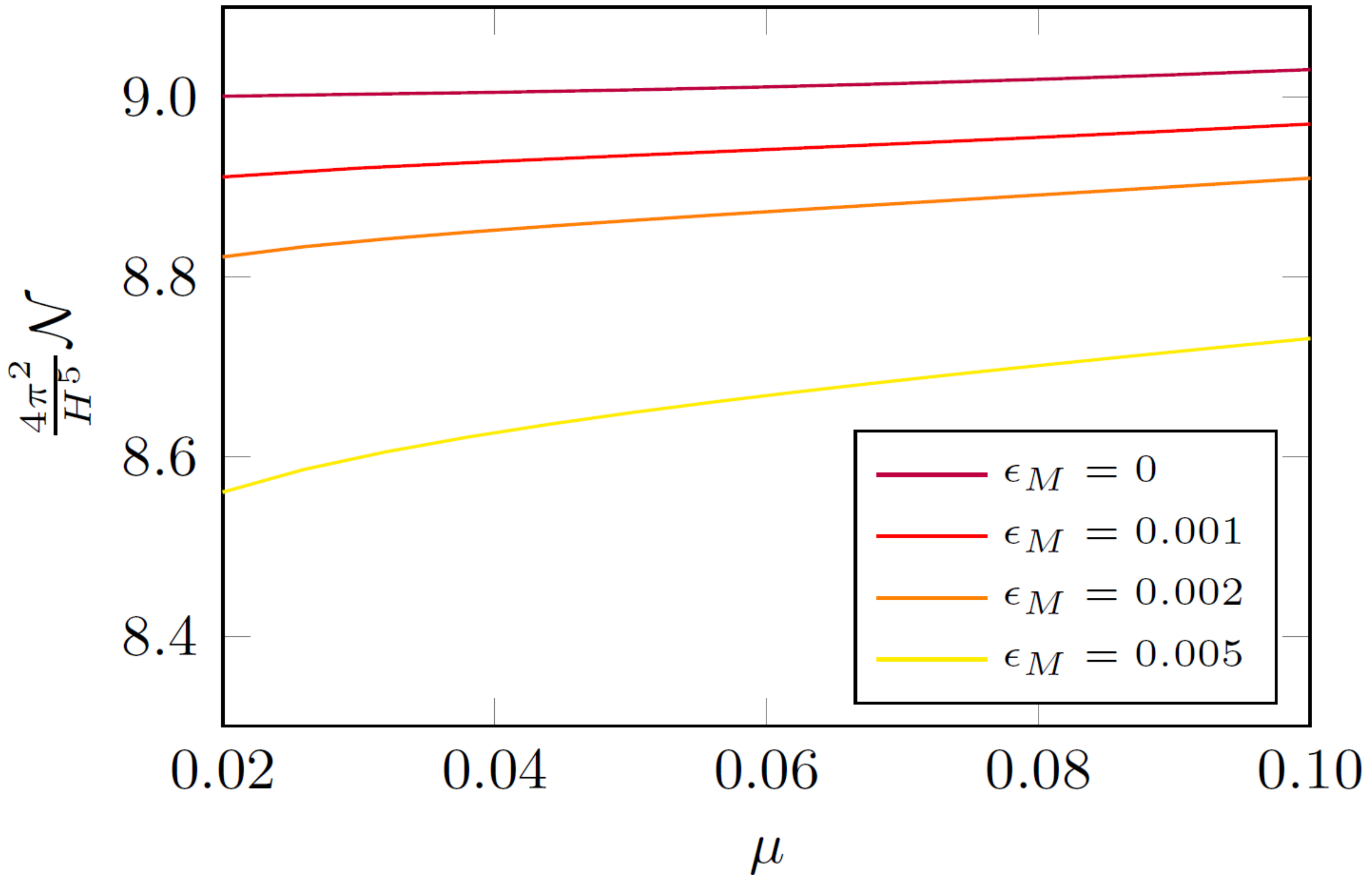


$$\epsilon_M \equiv \frac{m^2}{3H^2}$$

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2}$$



Error window function



Noise amplitude effect on observables

- Standard Starobinsky Langevin equation and noise amplitude;

$$3H\dot{\phi}_{IR} + V'(\phi_{IR}) = \xi, \qquad < \xi^2(x) > = \frac{9H^5}{4\pi^2}$$

recovered in limit of **sharp cut-off window function, that is located far in the IR, for vanishing mass and pure dS expansion**

- Example smooth approximate step window function (error function) gives

$$f(\mu, \sigma, \epsilon_M, \epsilon_H) \approx \frac{(1-\epsilon_H)^4}{1-\frac{1}{2}\epsilon_H}\left(\frac{\mu}{1-\epsilon_H}\right)^{2\epsilon_M - 2\epsilon_H}\left[1-\left(\frac{\mu}{\sigma}\right)^2\left(\frac{5}{4}+\epsilon_M-\epsilon_H\right)+\mathcal{O}\left((\mu/\sigma)^4\right)\right] \longrightarrow 1 \quad \text{as} \quad \begin{cases}\sigma \to 0 \\ \epsilon_M, \epsilon_H \to 0\end{cases}$$

- Enters observables via Fokker-Planck distribution

$$< O(\phi_{IR}(t)) > = \int \mathrm{d}\phi_{IR} O(\phi_{IR}) e^{-\frac{8\pi^2 V(\phi_{IR})}{3f(\mu,\sigma,\epsilon_M,\epsilon_H,\ldots)H^4}}$$

Summary & conclusions

- Stochastic inflation non-perturbative method to compute observables in cosmology:
  IR dynamics = classical stochastic process

- Not well-understood how it compares to QFT methods — region of validity?

- Stochastic dynamics obtainable from QFT: dependent on all ingoing parameters, including details of the window function separating the long- and short-wavelength modes

- Starobinsky's Langevin equation recovered only for $k \sim 0$ mode

Outlook: more QFT comparisons: stochastic vs quantum evolution? interactions?

Thank you

Backup : parametric dependence of effective noise