ELEMENTS

Bayesian constraints on the neutron-star equation of state with QCD input

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TU Darmstadt

SEWM 2022 (22.06.2022)

TG, Komoltsev, Kurkela, 2204.11877

SFB 1245

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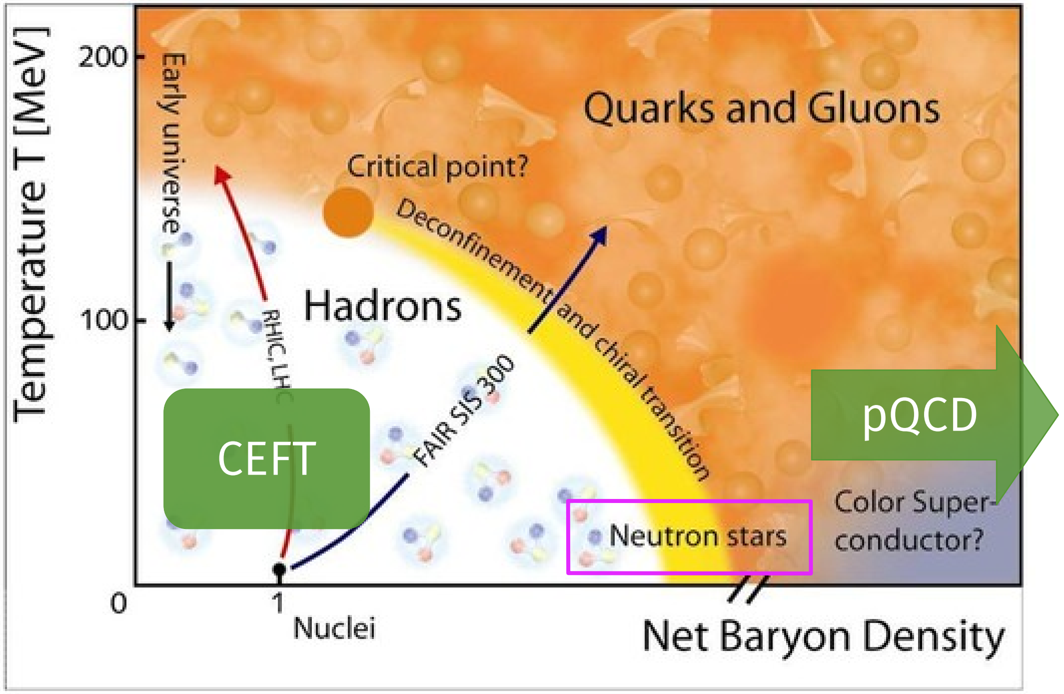
GSI

Motivation

- EOS of dense nuclear/QCD matter still unknown, requires input from fundamental theory + NS observations



Compressed Baryonic Matter (CBM) experiment

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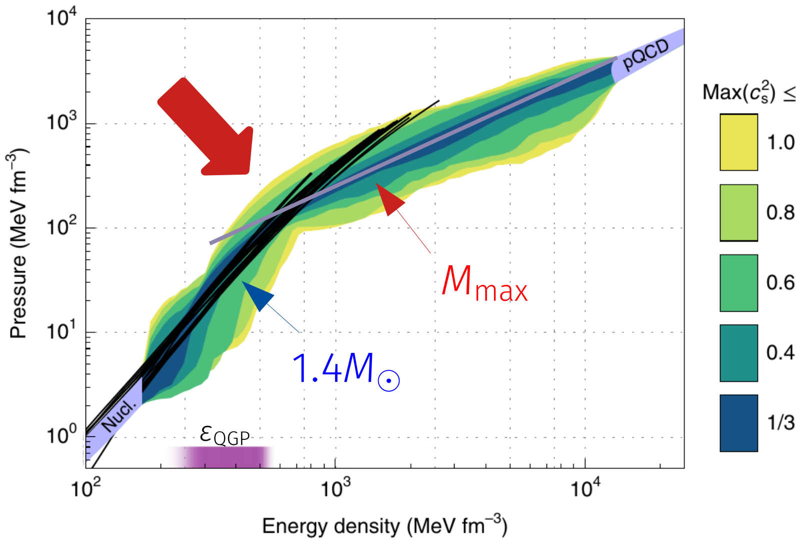
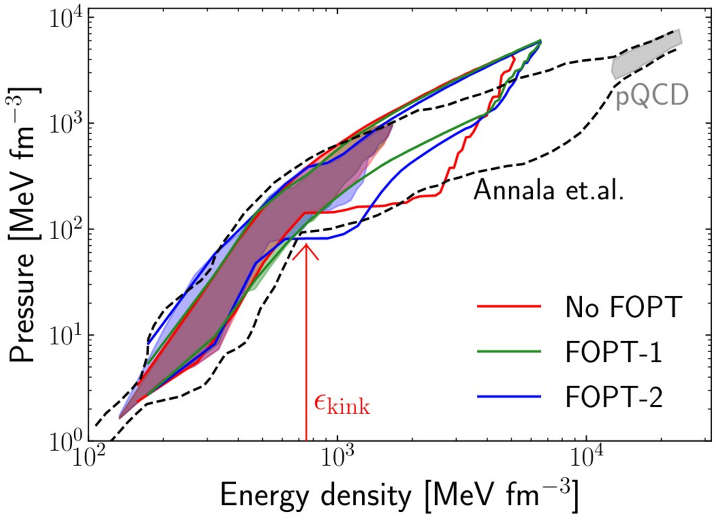
- Previous works with pQCD constraint see some softening transition along physical NS sequence, while other works without it do not



Visual summary of Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

VS.



Somasundaram, Tews, Margueron 2112.08157

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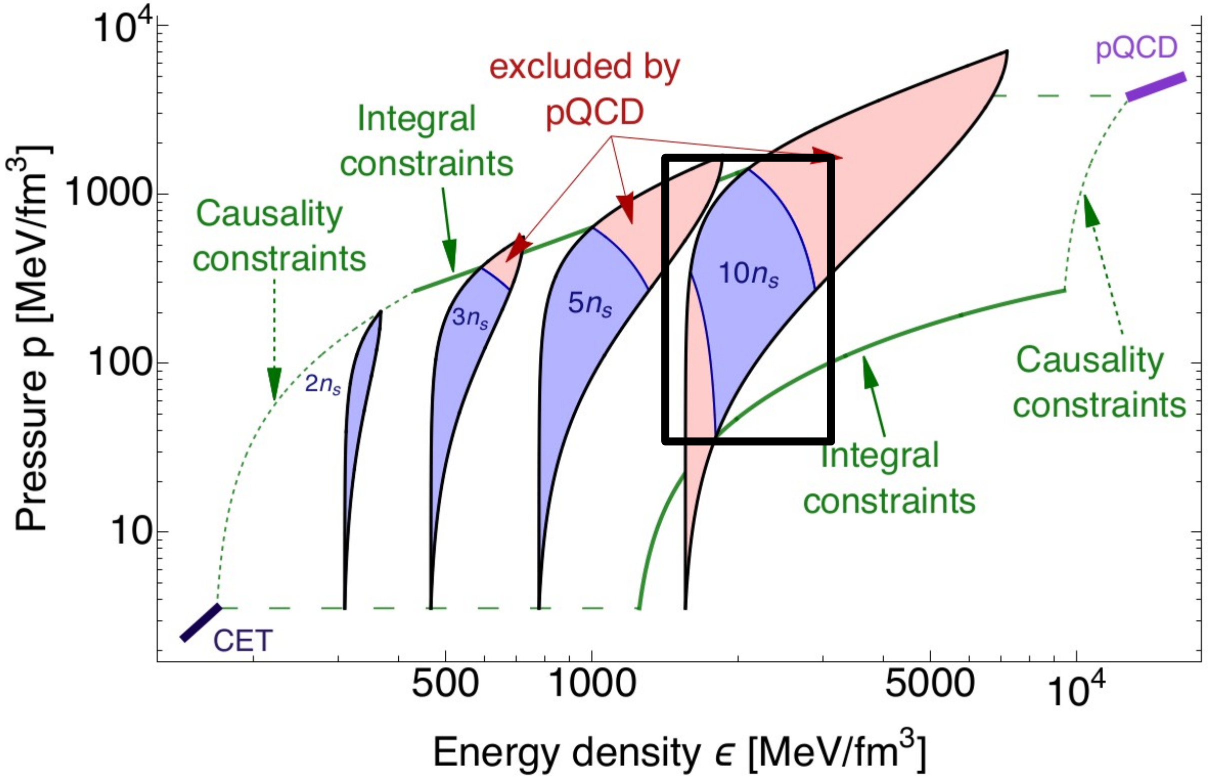
Question:	*Is softening a genuine (p)QCD prediction, or a result of interpolation through 2 orders of magnitude in density?*
Past weakness:	*Our past work has all been with hard cuts & not full measurement uncertainties*

How to feed down QCD input to lower densities

Komoltsev and Kurkela , PRL 128 (2022) (KoKu)



Want to use this $n = 10n_s$ region as high-density constraint

Setup

TG, Komoltsev, Kurkela, 2204.11877

- Use **Gaussian-Process regression** in auxiliary variable $\varphi(n) = -\ln\left(c_s^{-2}(n) - 1\right)$ to extend CEFT EOS to $10n_s$

  Similar to Landry & Essick Phys. Rev. D 99 (2019), but for function of n instead of ε

- *Condition* with low-density CEFT EOS

  95% CI matching spread of Hebeler, Lattimer, Pethick, Schwenk Astrophys. J. 773 (2013),

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- *Condition* with low-density CEFT EOS

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- Use hierarchical model, with:

$$\varphi(n) \sim \mathcal{N}\left(-\ln\left(\bar{c}_s^{-2} - 1\right), K(n, n')\right), \quad K(n, n') = \eta e^{-(n-n')^2/2l^2}$$

- With the hyperparameters themselves drawn from Gaussian distributions:

$$\bar{c}_s^2 \sim \mathcal{N}(0.5, 0.25^2), \; l \sim \mathcal{N}\left(1.0n_s, (0.25n_s)^2\right), \; \eta \sim \mathcal{N}(1.25, 0.25^2).$$

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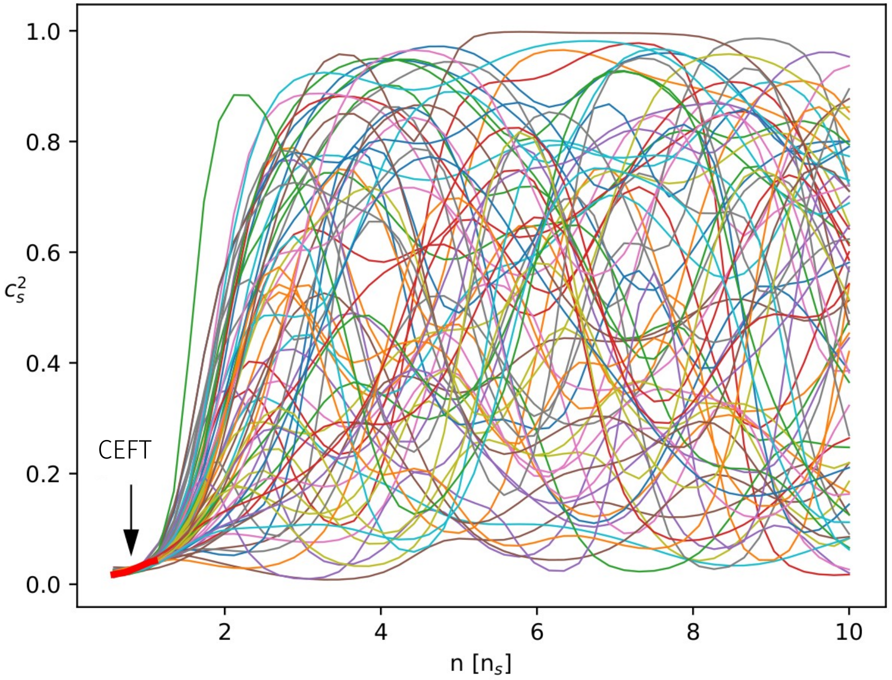

Setup

TG, Komoltsev, Kurkela, 2204.11877

1. Use **Gaussian-Process regression** in auxiliary variable $\varphi(n) = -\ln\left(c_s^{-2}(n) - 1\right)$ to extend CEFT EOS to $10n_s$

2. Fold in NS observations with full uncertainties
   - High-mass pulsars *(PSR J0348+0432 and PSR J1624-2230)*
     Approximate as Gaussians
   - GW170817
     Joint distribution on q and $\tilde{\Lambda}$
   - NICER measument *(PSR J0740+6620)*
     Joint distribution on M and R

3. Fold in QCD input as constraint at $10n_s$

Setup: Bit more about QCD constraint/likelihood

TG, Komoltsev, Kurkela, 2204.11877

1. Define triplet of thermodynamic properties:

From TG, Kurkela, Paatelainen, Sappi, Vuorinen PRL 127 (2021), PRD 104 (2021)

$$\vec{\beta}_{\mathrm{QCD}}(X) = \{p_{\mathrm{QCD}}(\mu_H, X), n_{\mathrm{QCD}}(\mu_H, X), \mu_H\}, \quad X = \frac{3\bar{\Lambda}}{2\mu_H}$$

$X \in [1/2, 2]$ usually quantifies renormalization-scale dependence

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2. Create distribution on these properties at high density

$$P(\vec{\beta}_H) = \int d(\ln X) w(\log X) \delta^{(3)}(\vec{\beta}_H - \vec{\beta}_{\mathrm{QCD}}(X)), \quad w(\ln X) = 1_{[\ln(1/2), \ln(2)]}(\ln X)$$

suggested by Cacciari & Houdeau, JHEP 09, (2011)

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3. KoKu construction gives Δp_{min}, Δp_{max} between $10n_s$ and pQCD for each β_H:

$$P(\mathrm{QCD}\,|\,\mathrm{EoS}) = \int d\vec{\beta}_H P(\vec{\beta}_H) 1_{[\Delta p_{\mathrm{min}}, \Delta p_{\mathrm{max}}]}(\Delta p)$$

Perform by substituting in $P(\beta_H)$, performing Monte-Carlo integration over X

Setup: Bit more about QCD constraint/likelihood

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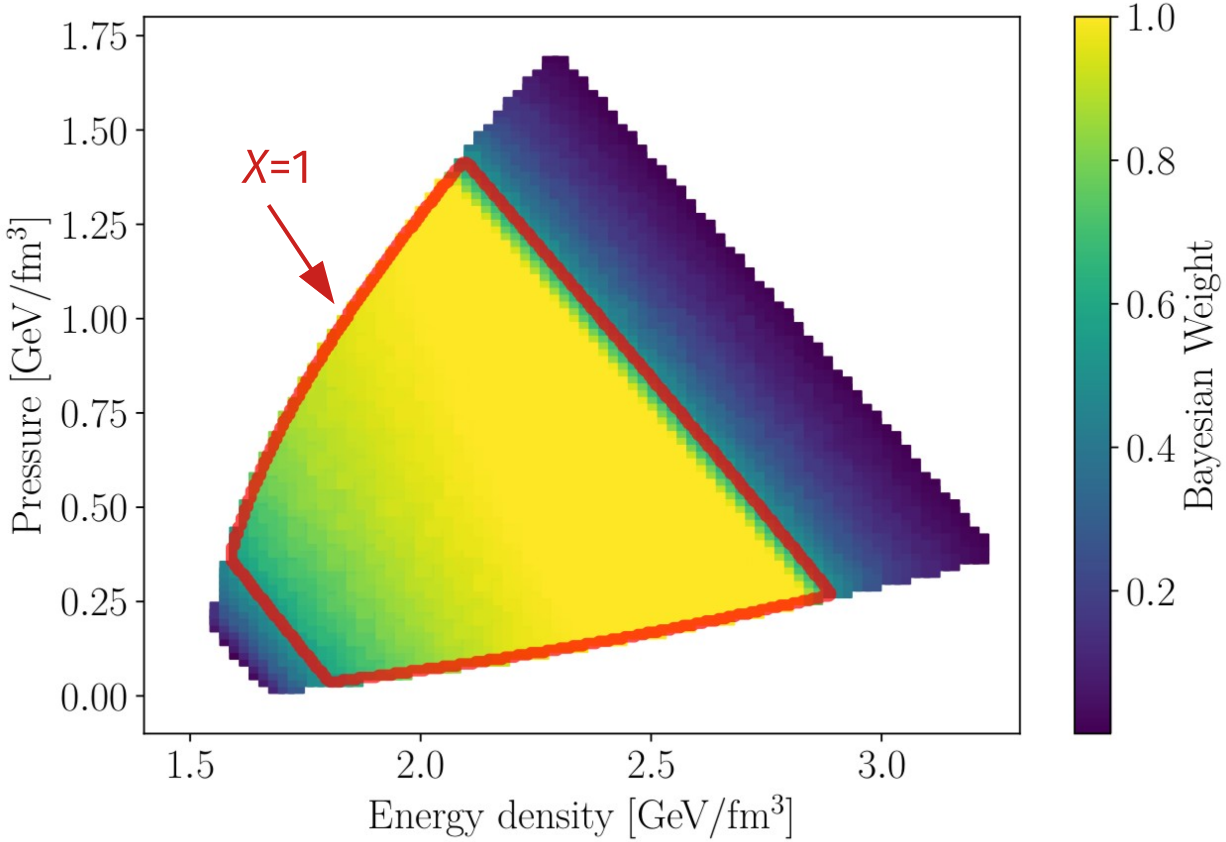
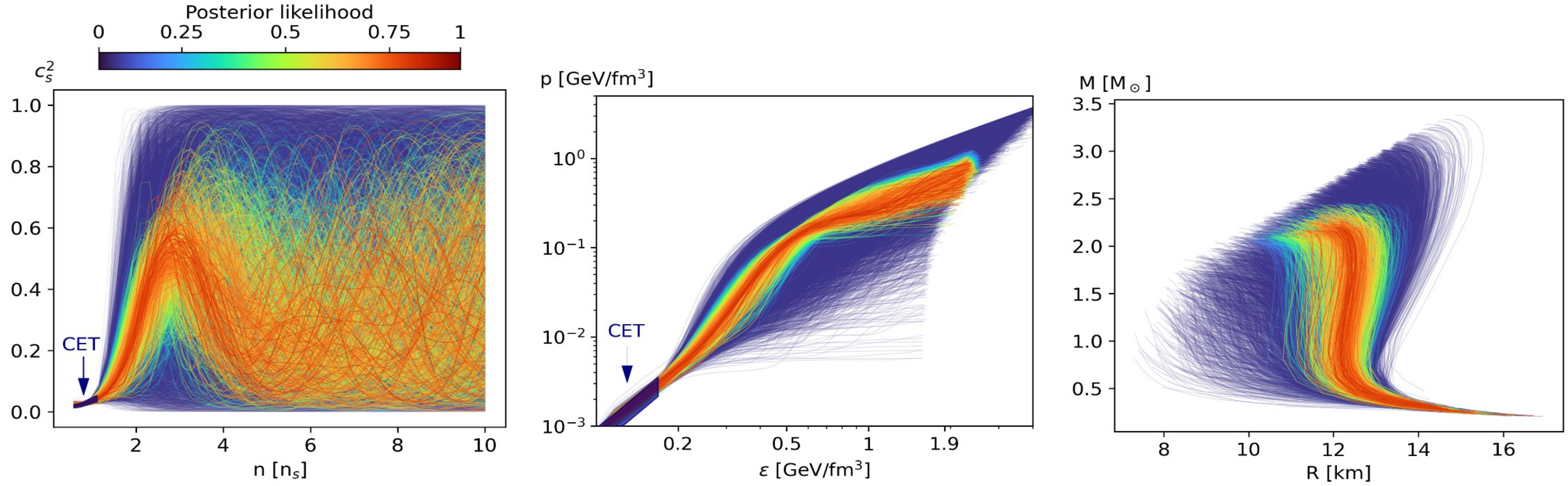
Weight of (ϵ, p) points at $n = 10n_s$



Results

TG, Komoltsev, Kurkela, 2204.11877



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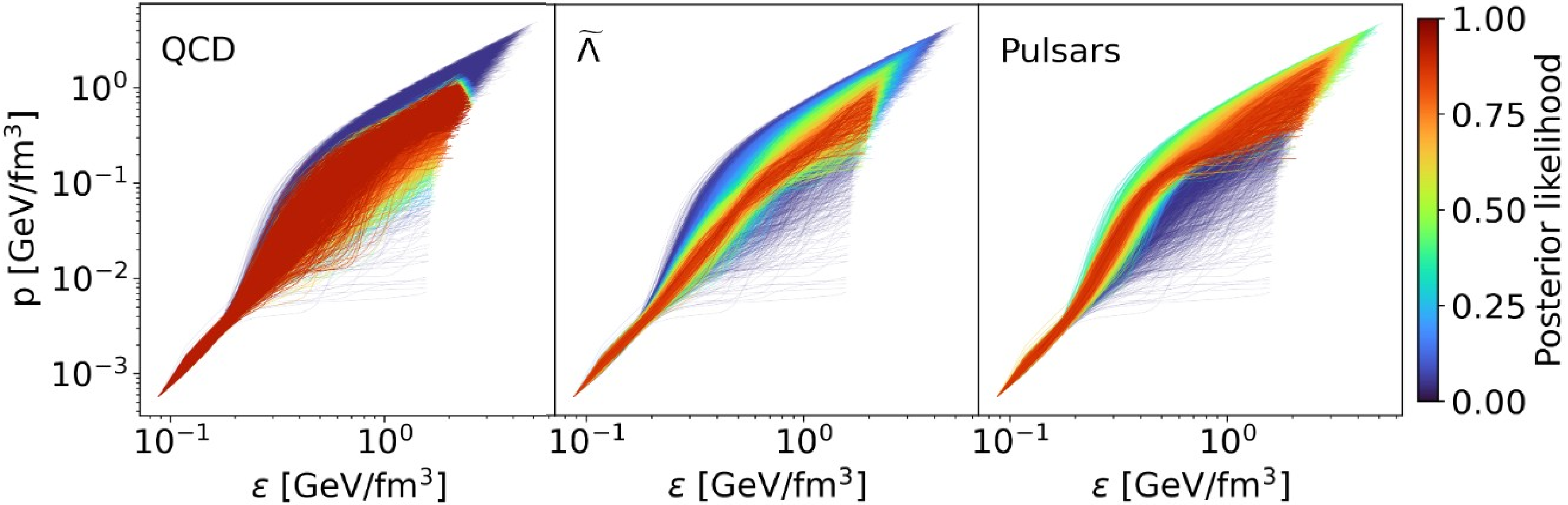

Results

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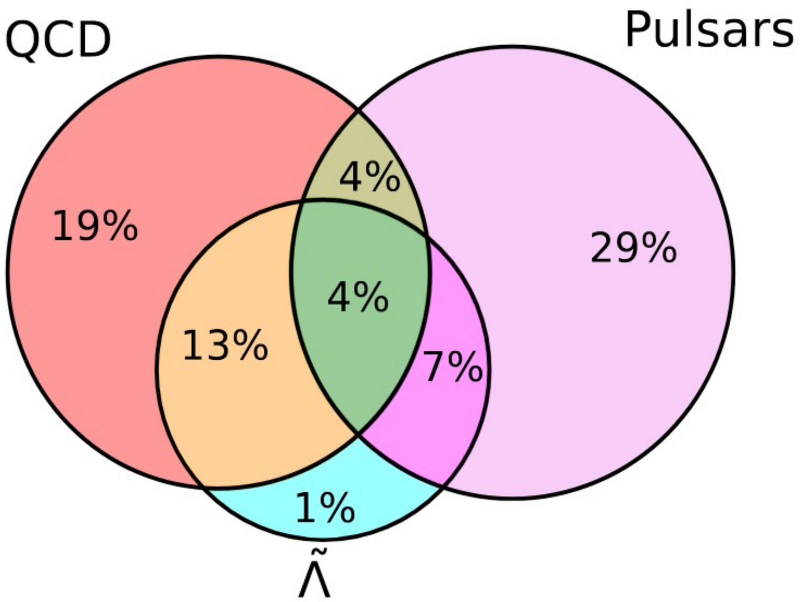
1. Inputs complementary



resample proportional to likelihood

Results

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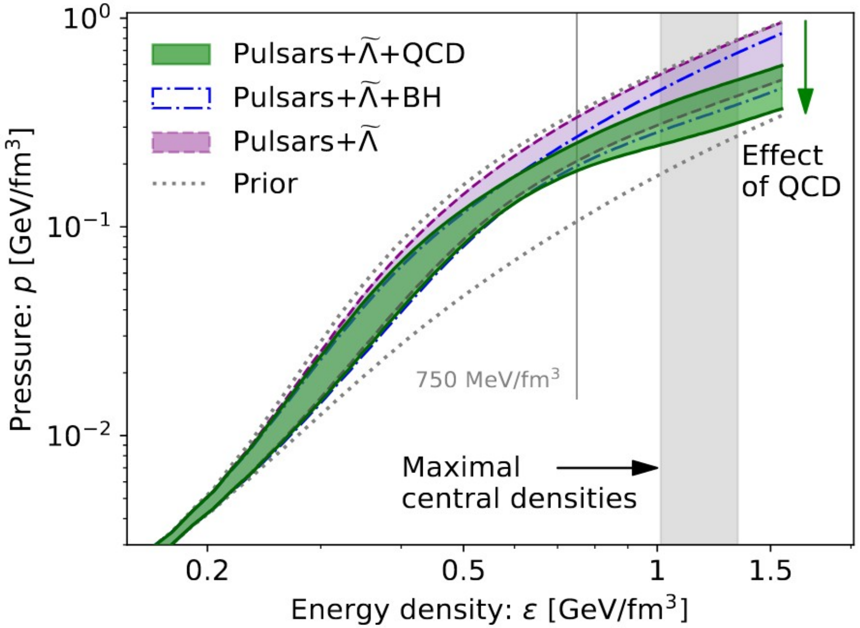
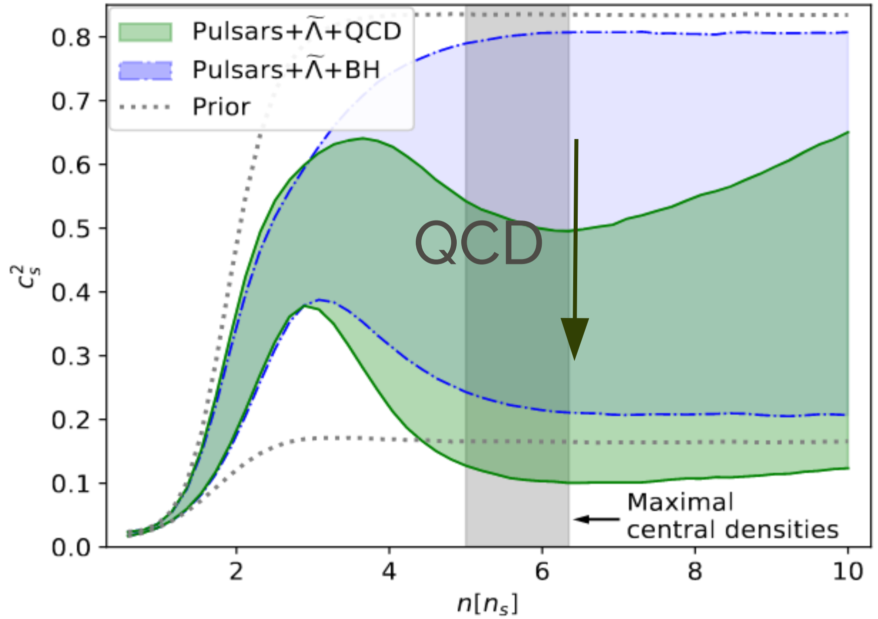
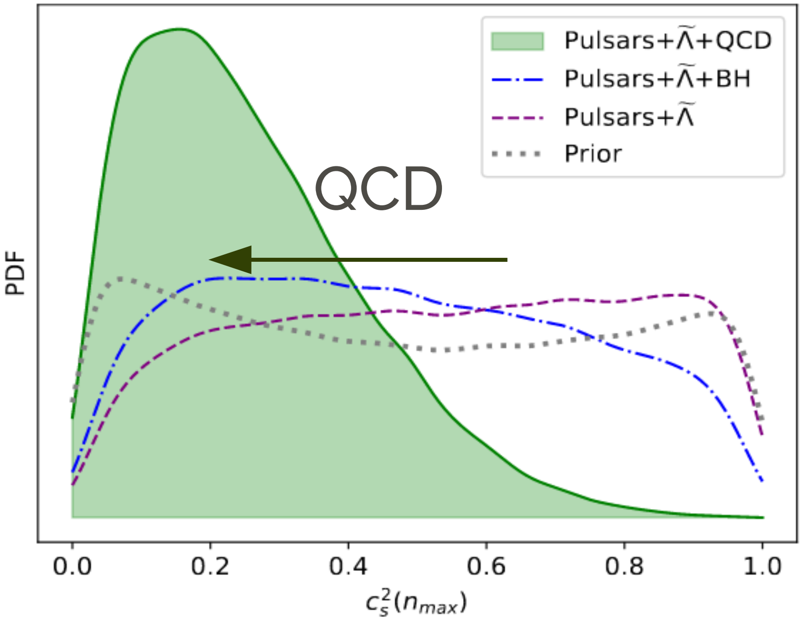
1. Inputs complementary

2. *QCD input softens the EOS*







Results

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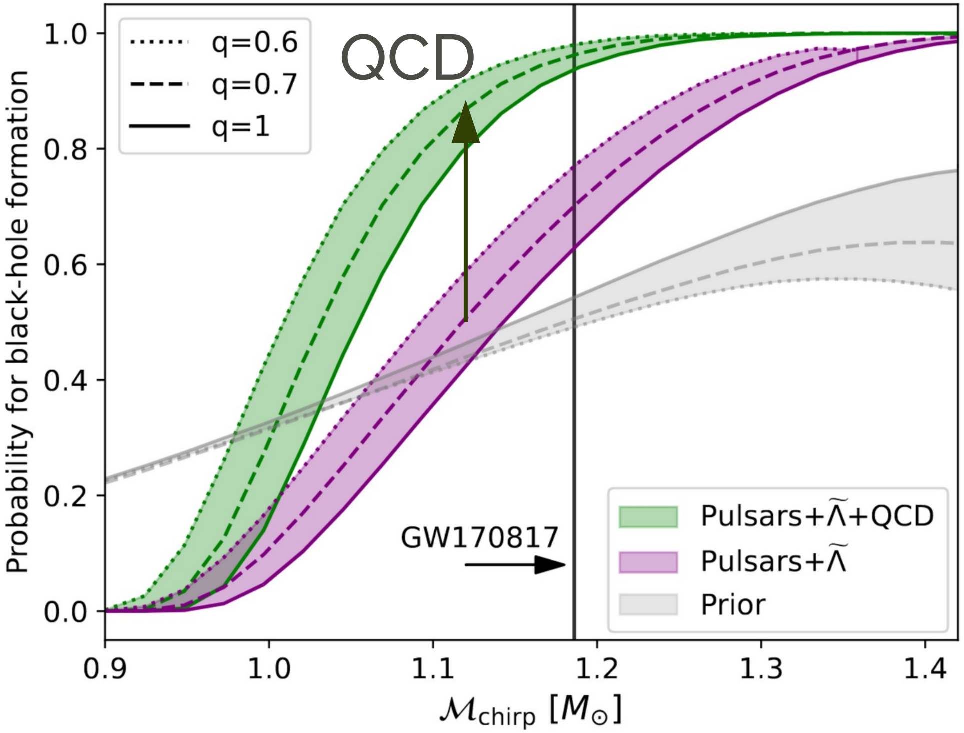
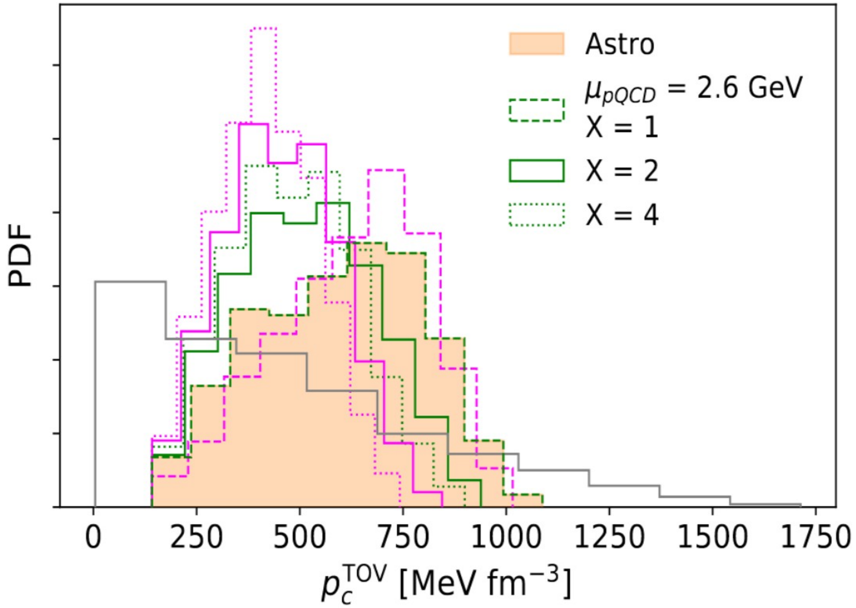
1. Inputs complementary
2. *QCD input softens the EOS*
3. QCD input implies BH formation



Comparison with other recent work

Somasundaram, Tews, Margueron (2204.14039) perform ultra-conservative analysis with QCD input, ***broadly consistent with our results***:

- Apply QCD input exactly at n_{TOV}
- *Find QCD input constraints for most X values* – only small range near $X = 1/2$ not constraining beyond astro



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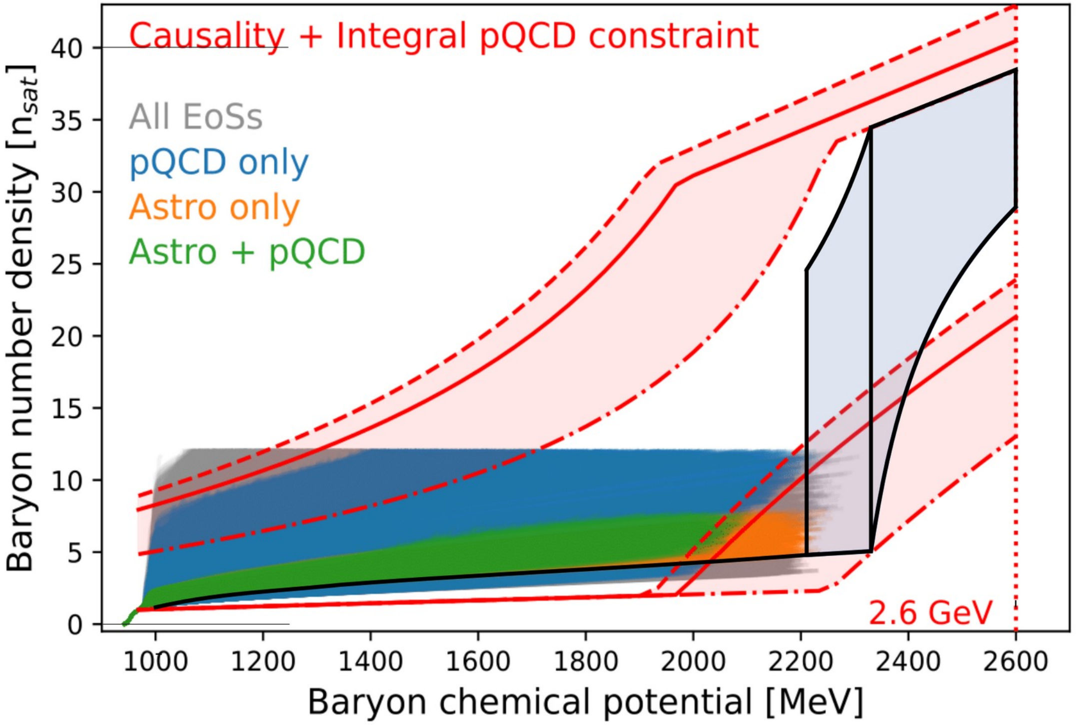
- Apply QCD input exactly at n_{TOV}
- *Find QCD input constraints for most X values* – only small range near $X = 1/2$ not constraining beyond astro
- These EOSs with $X \approx 1/2$ need *very specific behaviour beyond n_{TOV}* to reach pQCD

1. PT at n_{TOV} of $\Delta n = 20 n_s\ (\Delta n / n = 4)$, or
2. PT at $n_{\mathrm{TOV}} + 0.2 n_s$ of $\Delta n = 30 n_s\ (\Delta n / n = 6)$



c.f. Fujimoto + 2205.03882 for signatures of such PTs

Summary

- *Should use QCD input in analysis of NS-EOS inference; it impacts the inference!*

Jupyter notebook available on Github: OKomoltsev/QCD-likelihood-function

- QCD input at $10n_s$ *drives softening* in TOV stars / at high densities, *as indicated in hard-cut analysis*
- QCD input *complementary* to NS observational inputs
- QCD input *implies BH formation* for most NS-NS mergers

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Thanks for your attention!

Backup slides

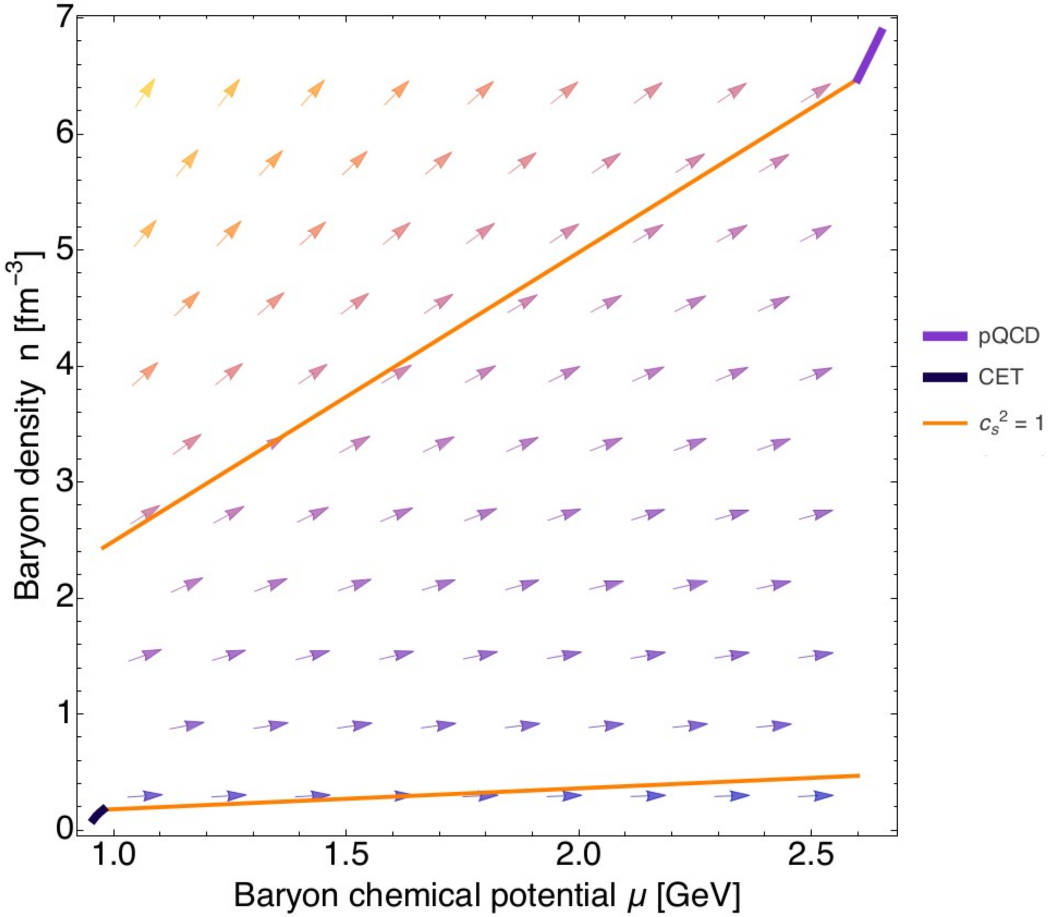
How to feed down QCD input to lower densities

Komoltsev and Kurkela, PRL 128 (2022) (KoKu)

1. Stability

2. Causality

3. Consistency



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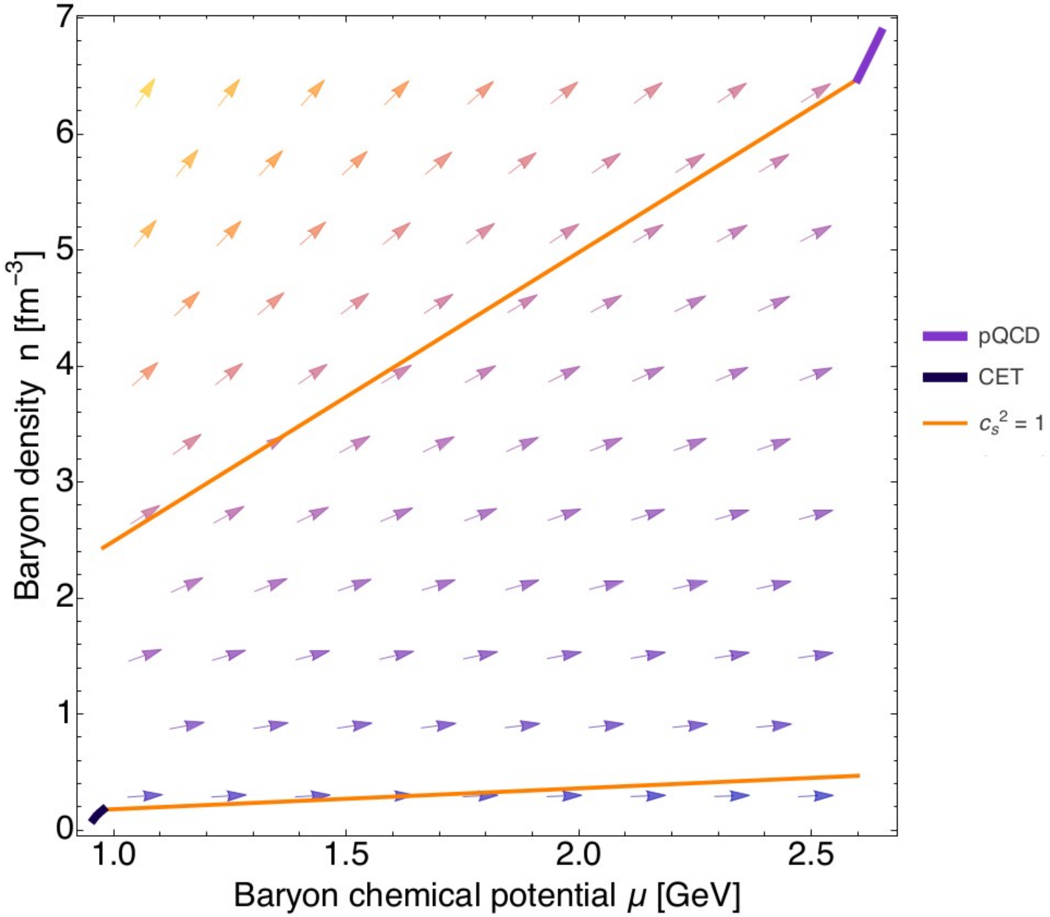

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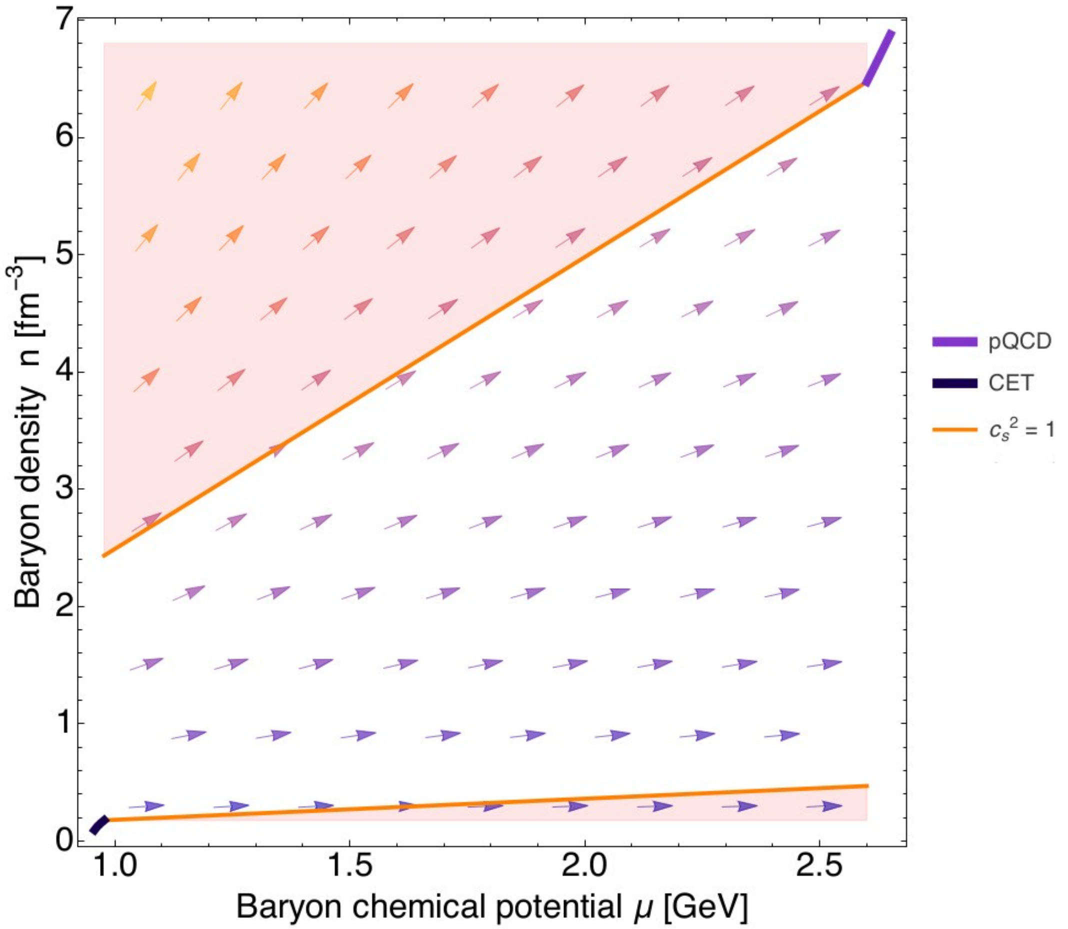

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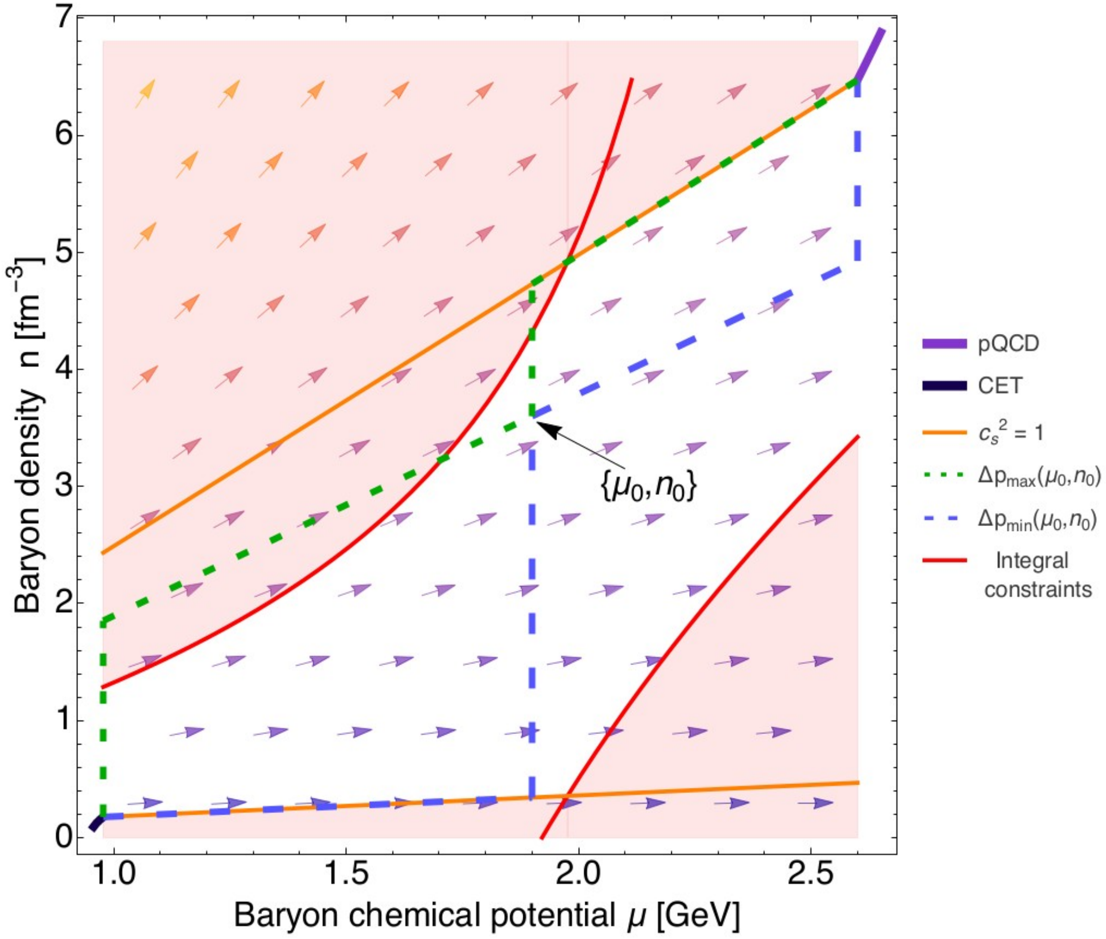
3. Consistency

$$\int_{\mu_{\rm CET}}^{\mu_{\rm QCD}} d\mu\, n(\mu) = p_{\rm QCD} - p_{\rm CET}$$

Fixed!



How to feed down QCD input to lower densities

Komoltsev and Kurkela, PRL 128 (2022) (KoKu)

- Full thermodynamic matching [in (p, n, μ)] places nontrivial constraints on the EOS between, e.g., $10n_s$ and n_{QCD}.

- In particular, the following integral is fixed:

$$\int_{\mu}^{\mu_{\mathrm{QCD}}} \mathrm{d}\mu' \, n(\mu') = p_{\mathrm{QCD}} - p \equiv \Delta p$$

- Provides new ***integral constraints*** on the EOS between CEFT and (p)QCD:

  Given (p, n, μ), KoKu construction provides Δp_{min}, Δp_{max}; must be compared with fixed Δp

- Allows QCD input to be applied ***below pQCD densities***