Debye mass effects in the dark sector in the Early Universe

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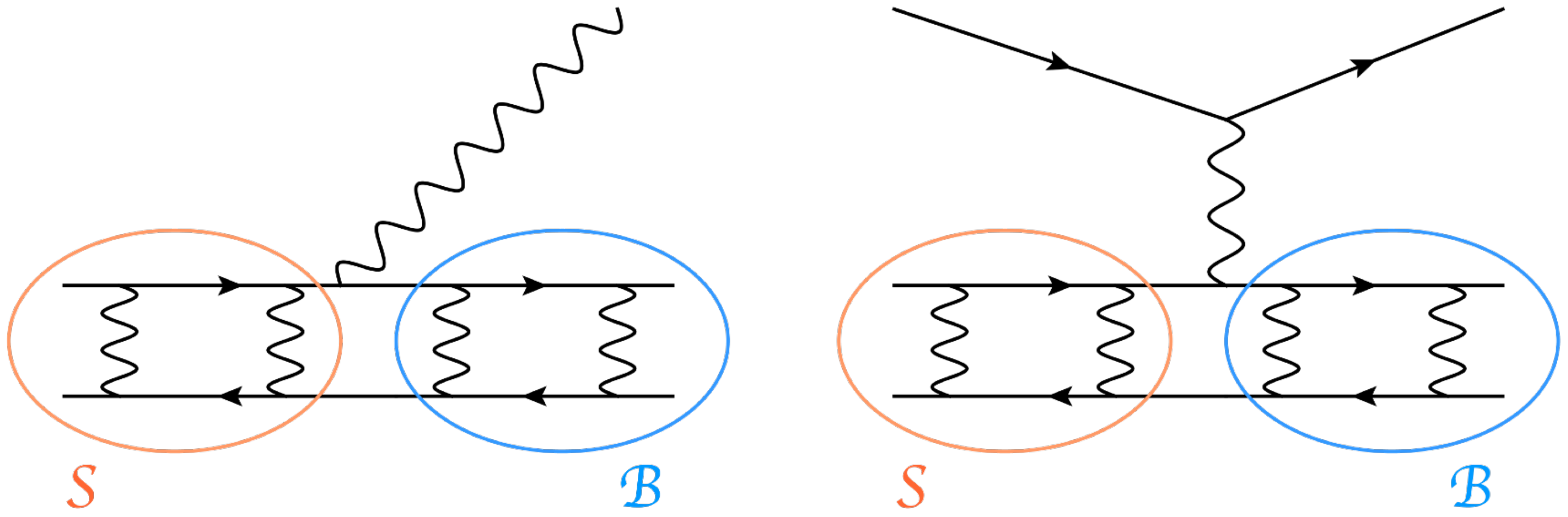
Outline

- Basic introduction
- Energy hierarchies (non-relativistic and thermal) and suitable EFTs
- Fixed order calculation
- Calculation based on exploiting hierarchy of scales (incl. HTL resummation)
- Dark Matter density evolution
- Conclusions and Future Work

Introduction and motivation

- Look at pairs of non-relativistic particles, which could form bound state:
  $U(1) : S(\chi\bar{\chi}) \leftrightharpoons B(\chi\bar{\chi})$
  $SU(3) : S(\chi\bar{\chi})_8 \leftrightharpoons B(\chi\bar{\chi})_1$

  when thermal effects are non-negligible



Applications:

- Dark matter bound states in early universe plasma

  (nucleosynthesis, relic abundances of DM)

- Heavy quarkonium production during heavy ion collision in QGP

  (properties of QGP, quarkonium suppression)

Model of the dark sector

$$\mathcal{L}_{DM} = \bar{X}(i\not{D} - m)X - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{n_f} \bar{f}_k(i\not{D} - m_f)f_k$$

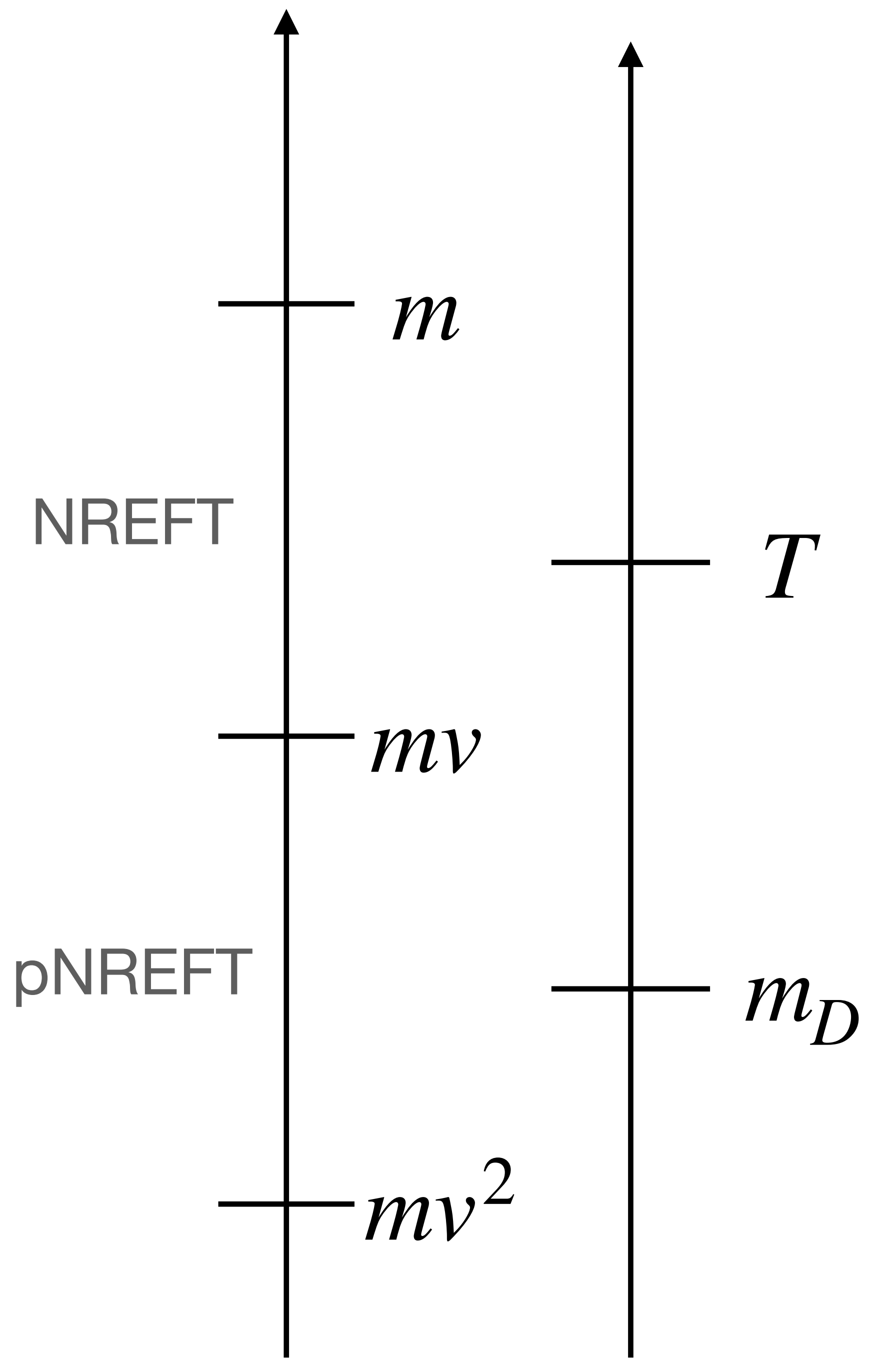
New part (comparing to prev. talk by G. Qerimi)

- Heavy dark matter fermion
- Dark photons
- Light d.o.f, fermions - enable scale m_D to appear

* Debye mass: $m_D^2 = n_f \frac{g^2T^2}{3}$

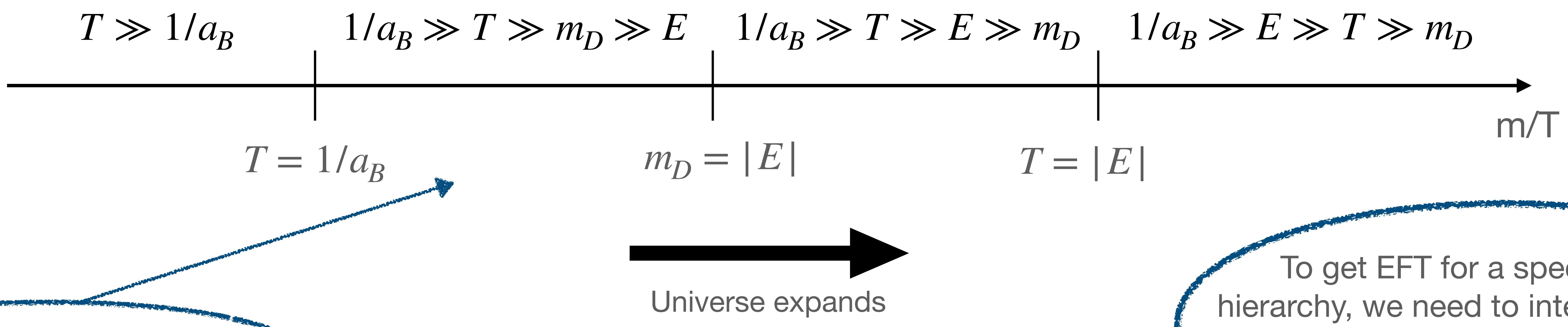
Energy scales present

- Two energy scales:
  - $T \gg m_D$ (thermal)
  - $m \gg mv \gg mv^2$ (non-relativistic)



- Cosmologically interesting: $m \gg T$; $T \gg m_D \sim gT$ (weakly interacting plasma)

- As the Universe expanse, temperature drops:

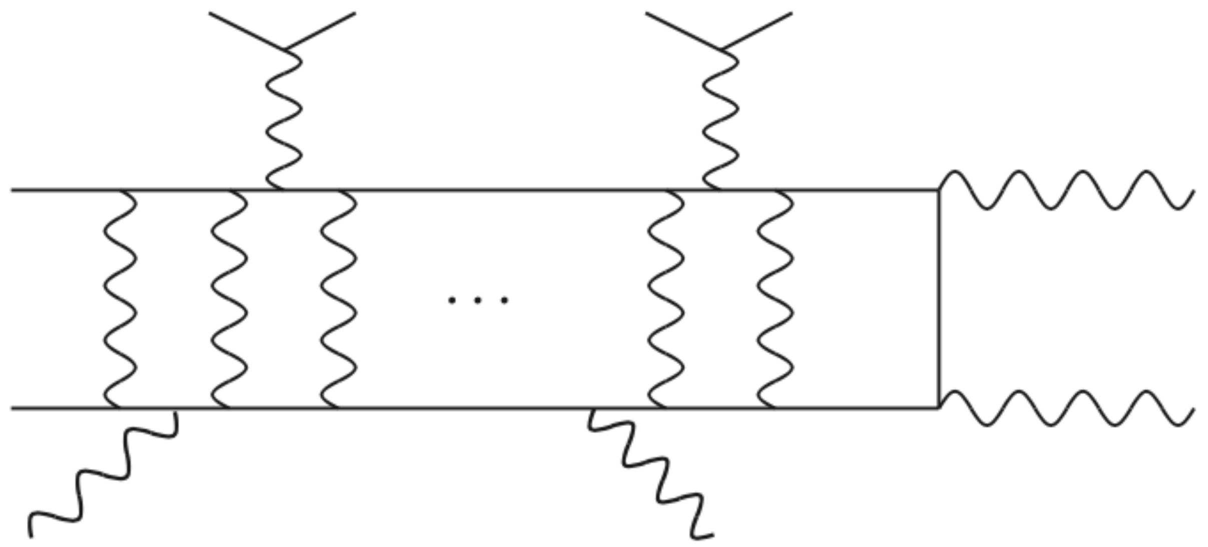


Freeze-out usually occurs here

To get EFT for a specific energy hierarchy, we need to integrate out scale by scale, starting from the highest (m scale)

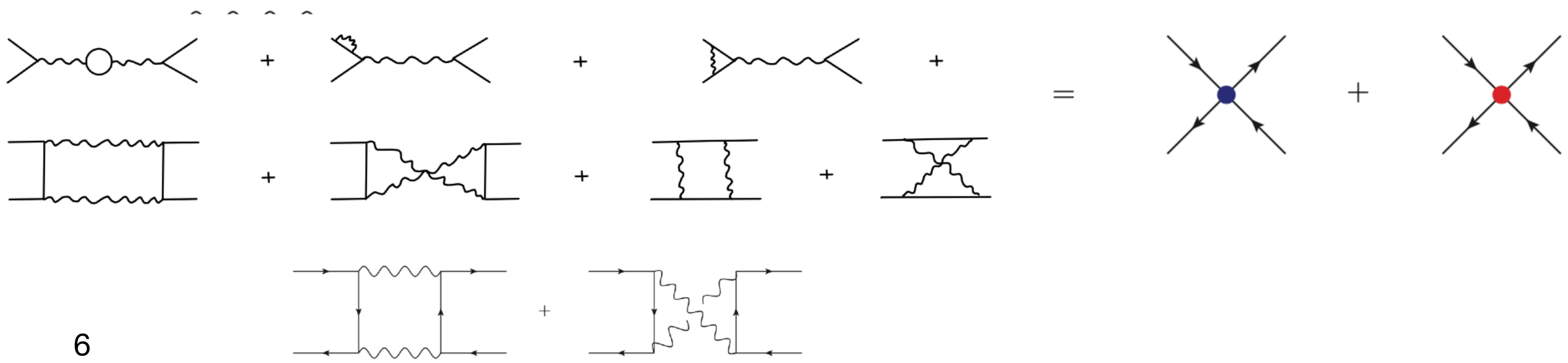
Pair description in EFT: NRQED

$$\mathcal{L}_{DM} = \bar{X}(i\not{D} - m)X - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{n_f} \bar{f}_k(i\not{D} - m_f)f_k$$



Integrating out hard scale $m \gg mv \sim p \sim \frac{1}{a_B}$

$$\begin{aligned}\mathcal{L}_{NRQED_{DM}} =& \psi^\dagger \left(iD_0 + \frac{\vec{D}^2}{2m} + c_F \frac{\vec{\sigma} g\vec{B}}{2m} + c_D \frac{\vec{\Delta} g\vec{E}}{8m^2} + ic_S \frac{\vec{D}\times g\vec{E} - g\vec{E}\times\vec{D}}{8m^2} \right)\psi \\ &+ \chi^\dagger \left(iD_0 - \frac{\vec{D}^2}{2m} - c_F \frac{\vec{\sigma} g\vec{B}}{2m} + c_D \frac{\vec{\Delta} g\vec{E}}{8m^2} + ic_S \frac{\vec{D} g\vec{E} - g\vec{E}\vec{D}}{8m^2} \right)\chi \\ &- \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{d_s}{m^2}F^{\mu\nu}\vec{D}^2 F_{\mu\nu} + \frac{d_s}{m^2}\psi^\dagger\chi\chi^\dagger\psi + \frac{d_v}{m^2}\psi^\dagger\vec{\sigma}\chi\chi^\dagger\vec{\sigma}\psi \\ &+ \sum_{i=1}^{n_f} \bar{f}_k(i\not{D} - m_f)f_k + o(1/m^2)\end{aligned}$$



Pair description in EFT: pNRQED

Integrating out soft scale $mv \gg mv^2 \sim E$

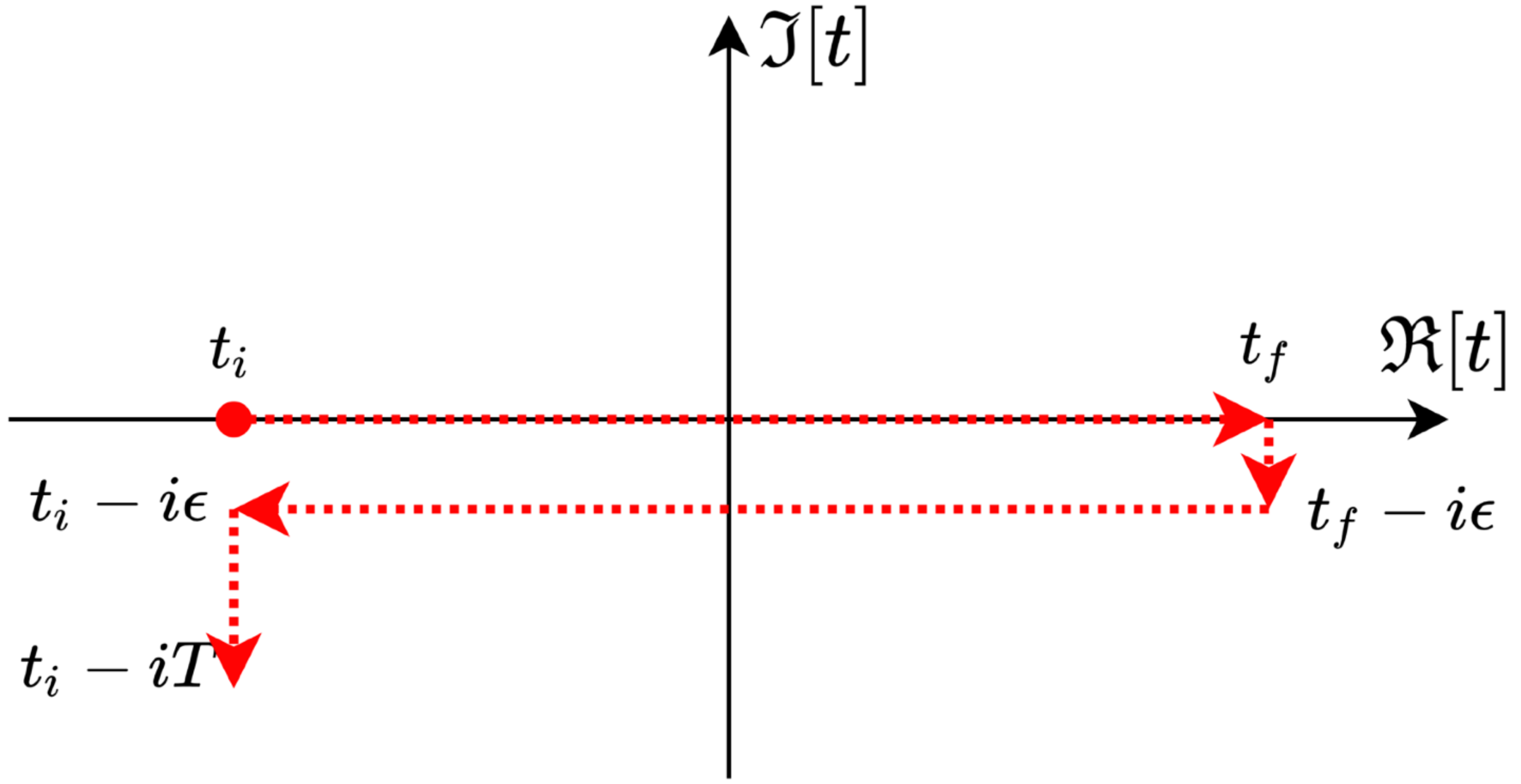
$$\mathcal{L}_{pNRQED_{DM}} = \int d\vec{r}\phi^\dagger(t,r,R)\left(i\partial_0 - H(r,p,P,s_1,s_2) + g\vec{r}\vec{E}(R,t)\right)\phi(t,r,R) + \\ + \frac{1}{4}F^{\mu\nu}(R,t)F_{\mu\nu}(R,t) + \mathcal{L}_{lightfermions}$$

$$H(r,p,P,s_1,s_2) = \frac{p^2}{m} + \frac{P^2}{4m} - \frac{p^4}{4m^3} + \mathcal{O}(1/m^3) + V(r,p,P,s_1,s_2)$$
$$V(r,p,P,s_1,s_2) = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \mathcal{O}(1/m^3)$$

Thermal field theory in RTF

Key points

- We use Real-Time formalism (i.e. Keldysh contour, double d.o.f., but “usual" Feynman diagram formalism)
- Matrix structure of propagators
- In propagators, there is a thermal part, which put’s particles on-shell $\sim \delta(p^2 - m^2)$



Bound state formation/dissociation from TFT

- The bound-state formation cross section, could be inferred as (i.e. optical theorem):

$$\sigma_{BSF} v_{rel} = < p, l \,|\, \frac{\Sigma_S^{>}}{i} \,|\, p, l > \; = -2 < p, l \,|\, \Im[\Sigma_S] \,|\, p, l >$$

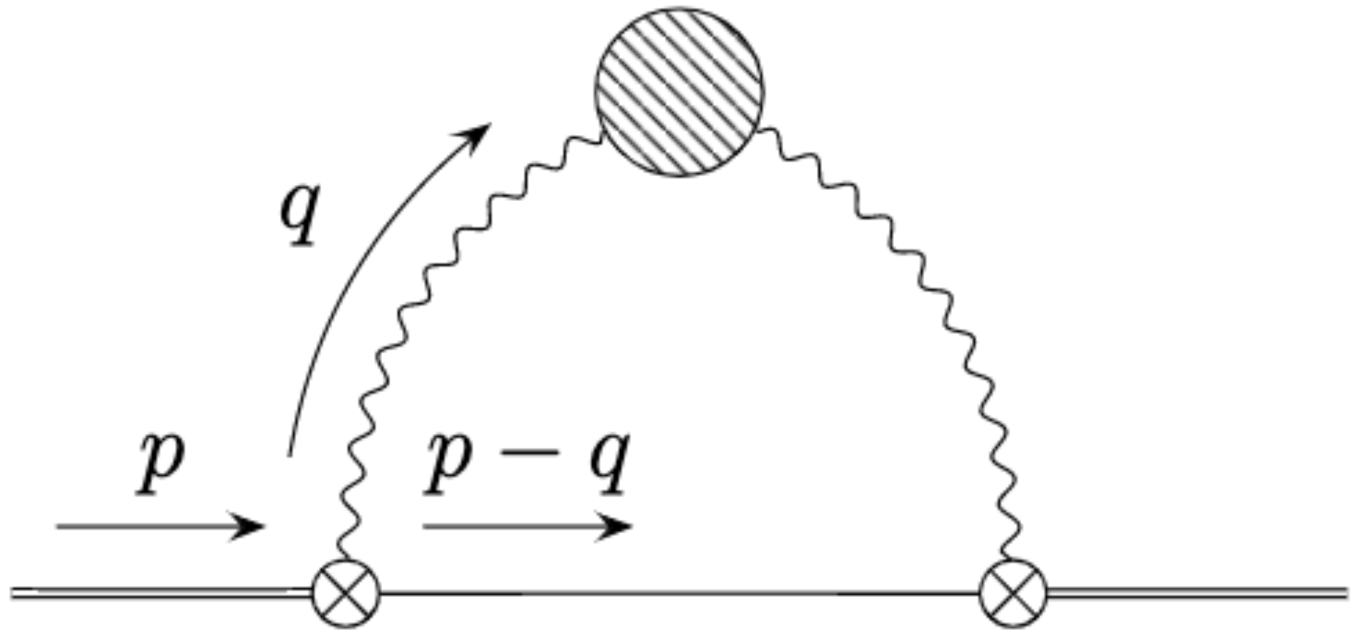
Pair's self-energy

Self-energy in pNRQED:

$$\Sigma_{S/B} = -ig^2 \int_0^{\infty} dt e^{it(p_0 - h^{(0)})} r^i r^j \left\langle E^i(t,0) E^j(0,0) \right\rangle =$$

$$= -ig^2 \frac{\mu^{4-d}}{d-1} r_i^2 \int \frac{d^d q}{(2\pi)^d} \frac{i}{p_0 - q_0 - h^{(0)} + i\epsilon} \left\langle \vec{E}(q) \vec{E}(0) \right\rangle$$

$$= -ig_d^2 \frac{\mu^{4-d}}{d-1} r_i^2 \int \frac{d^d q}{(2\pi)} \frac{i}{p_0 - q_0 - h^{(0)} + i\epsilon} [q_0^2 D_{ii}(q) + \vec{q}^{\,2} D_{00}(q)]$$



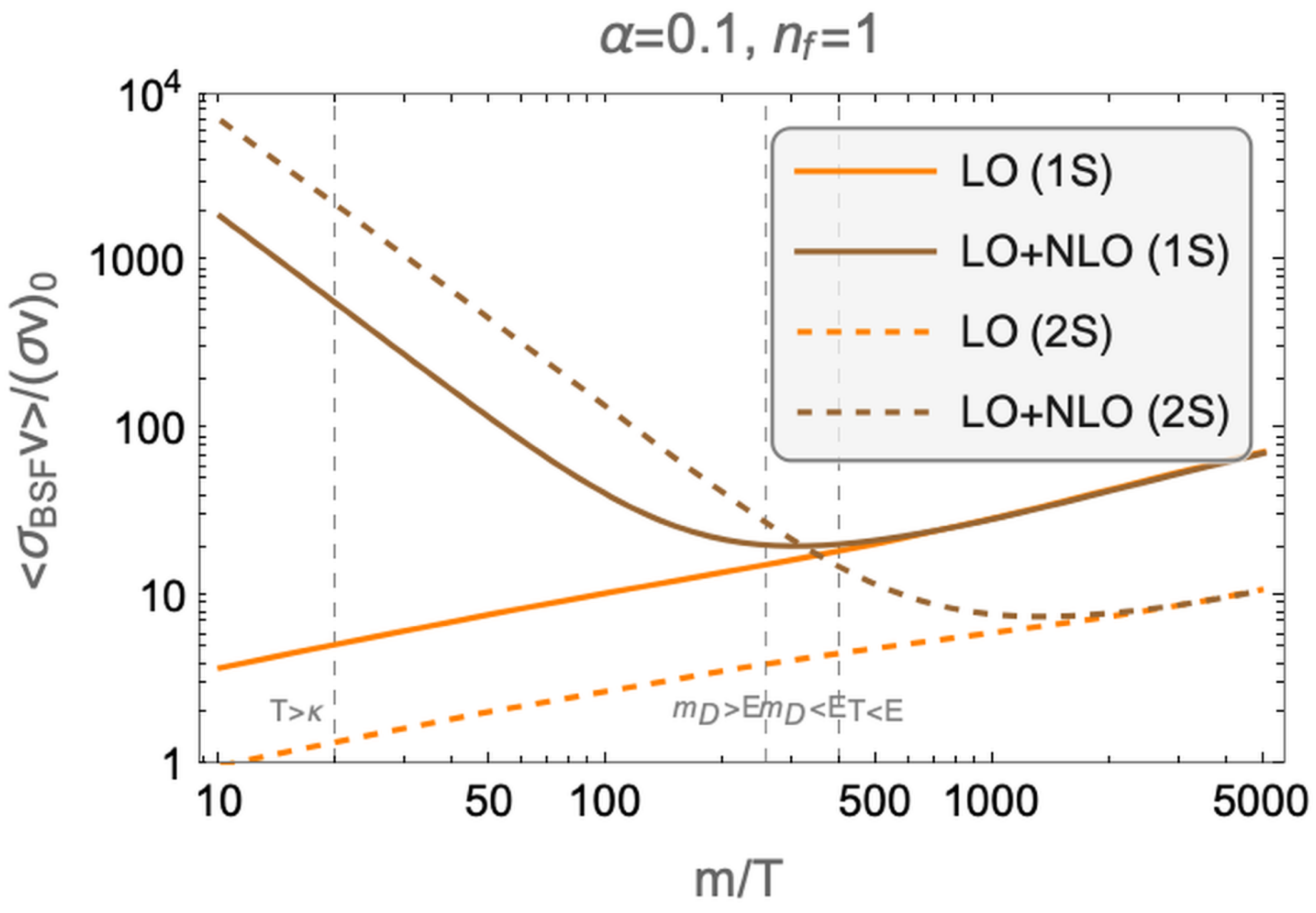
BSF at fixed order*

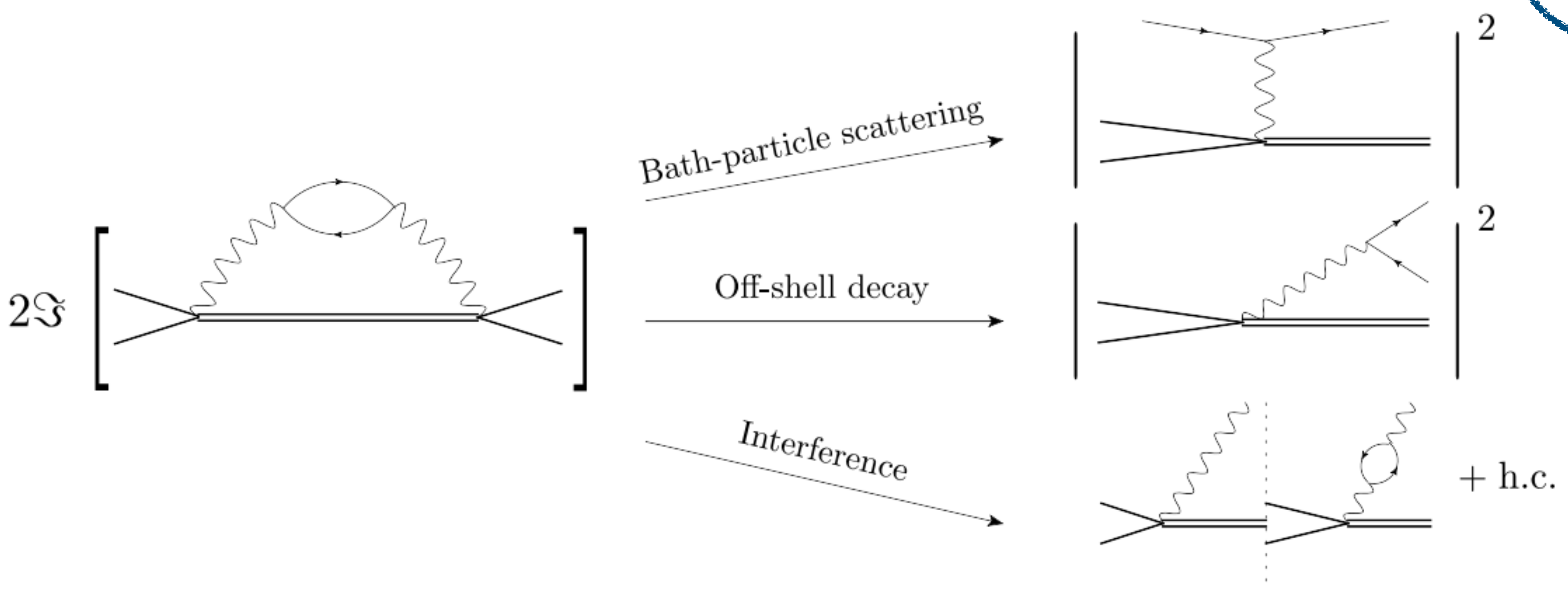
*in EE propagator

- LO

- NLO (2002.07145 by Binder, Harz et al.)

No hierarchy between scales E and T
m_D scale - not addressed





$m \gg mv \gg T \gg m_D \gg E$

Exploiting the large scale separation

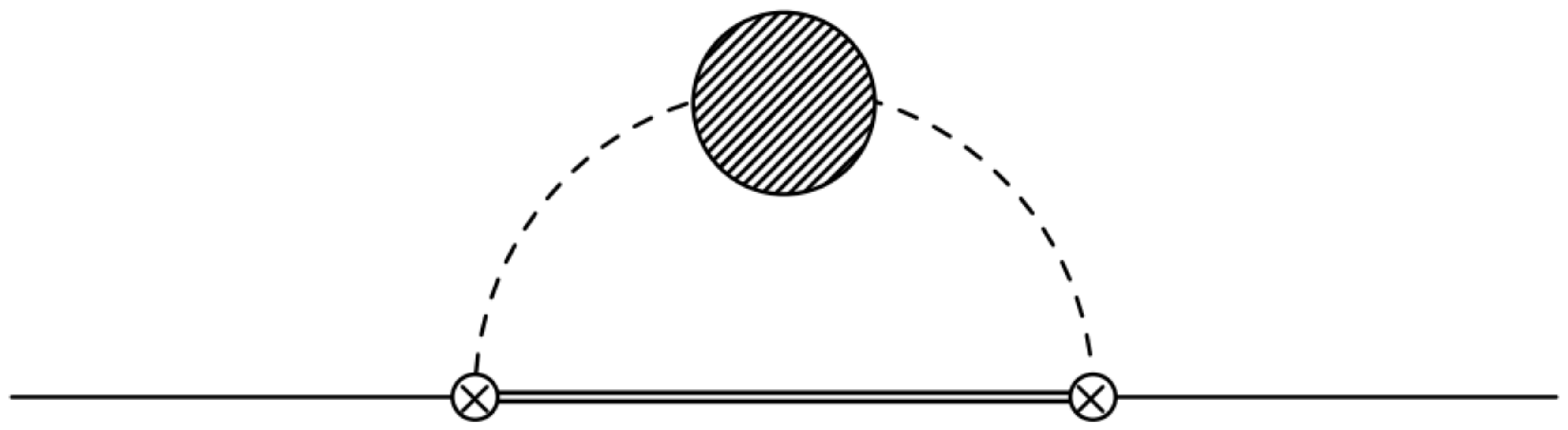
When integrating out each of the scales, one can use hierarchy to expand objects
(i.e. in powers of T/E)

- After integrating out scale M and Mv, we start with $pNRQED_{DM}$ at $T = 0$

$$\mathscr{L}_{pNREFT} = \mathscr{L}_{light} + \int d^3r \phi^+[i\partial_0 - \hat{h}]\phi + \phi^+ \vec{r} g \vec{E} \phi$$

- Integrate out scale T

  The photon sector is modified to HTL one.



  Imaginary correction to self-energy comes from the imaginary part (symmetric) of photon self-energy

$$\Im[\Sigma^T] = \frac{1}{6}\alpha r_i^2 T m_D^2 (\frac{1}{\epsilon} + \gamma_E - \ln(\frac{T}{\pi\mu}) + \frac{2}{3} - 4ln2 - 2\frac{\zeta'(2)}{\zeta(2)})$$

IR divergent,
will cancel-out with UV divergence from lower scale

EFT $(M \gg Mv \gg T \gg m_D \gg E)$

- Integrate out scale m_D in similar way (where the photon propagator is the HTL resummed one)

$$\Im[\Sigma^{m_D}] = -\frac{1}{6}\alpha r_i^2 T m_D^2 \left(\frac{1}{\epsilon} - \gamma_E + \ln(\frac{\pi\mu^2}{m_D^2}) + \frac{5}{3} \right)$$

- Contribution from scale E is the LO BSF via photon emission

$$\Im[\Sigma^{E}] = \frac{1}{6}\alpha r_i^2 T m_D^2 \left(4\frac{\Delta E^3}{T m_D^2} \left(\frac{T}{\Delta E} + \frac{1}{2} \right) \right)$$

- All-together, we have:

$$\Im[\Sigma] = \Im[\Sigma^T] + \Im[\Sigma^{m_D}] + \Im[\Sigma^E] = \frac{1}{6}\alpha r_i^2 T m_D^2 (2\gamma_E + \ln(\frac{m_D^2}{T^2}) - 1 - 4ln2 - 2\frac{\zeta'(2)}{\zeta(2)} + 4\frac{\Delta E^3}{T m_D^2}(\frac{T}{\Delta E} + \frac{1}{2}))$$

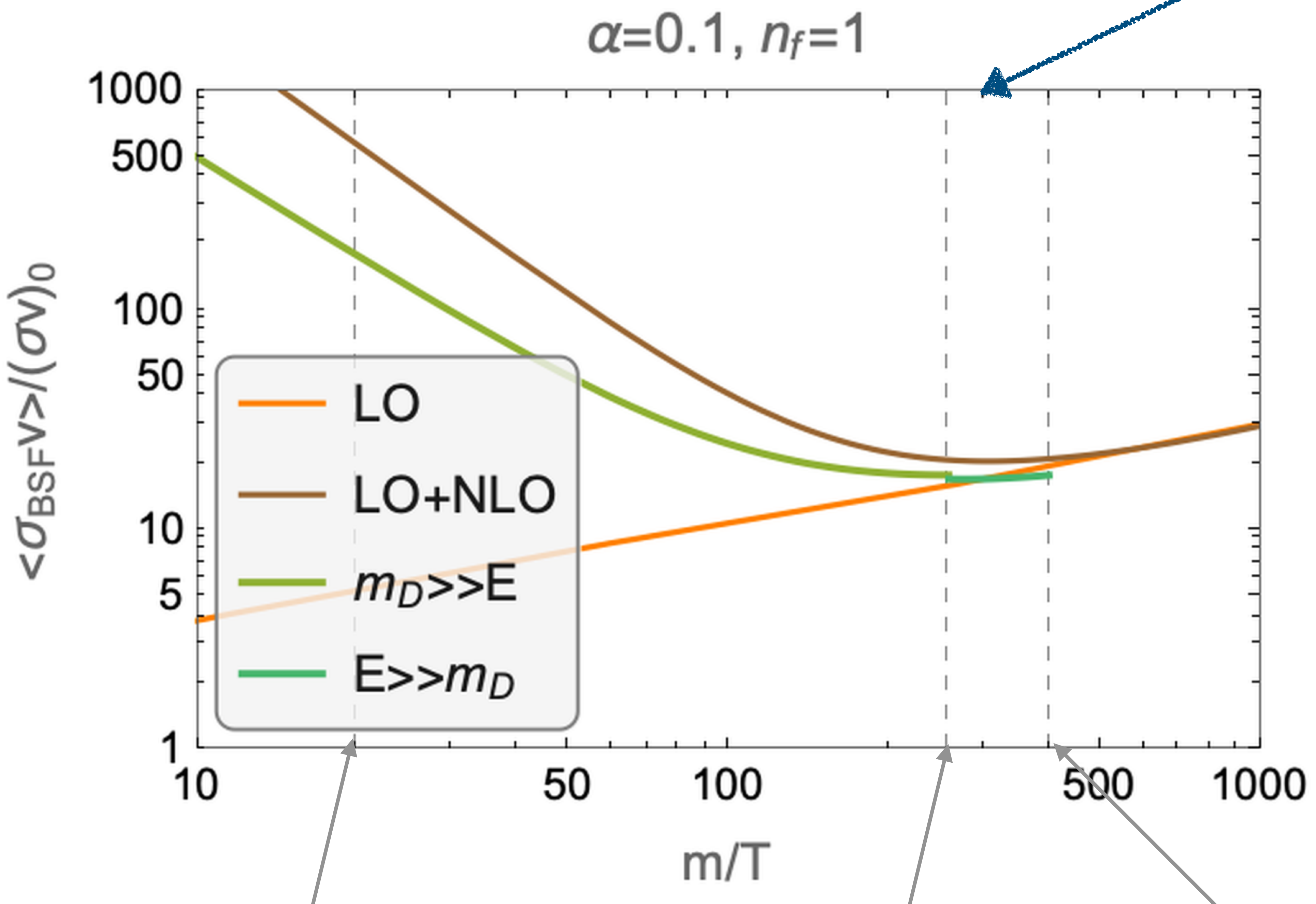
$$\Downarrow$$

$$(\sigma_{BSF} v_{rel})_{T \gg m_D \gg E}$$

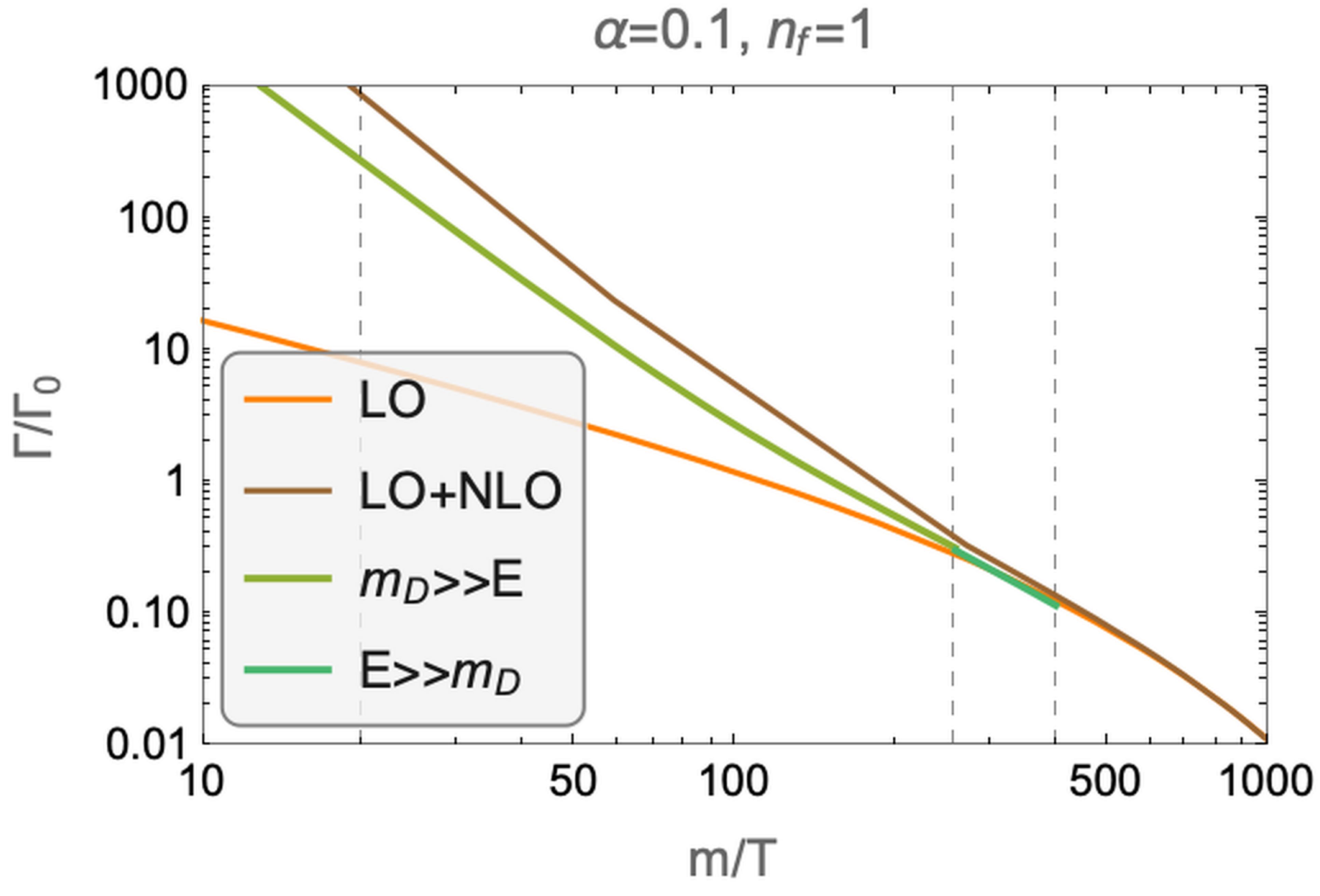
which is NOT divergent anymore

Bound State Formation: Results

In the same manner $E \gg m_D$ is treated

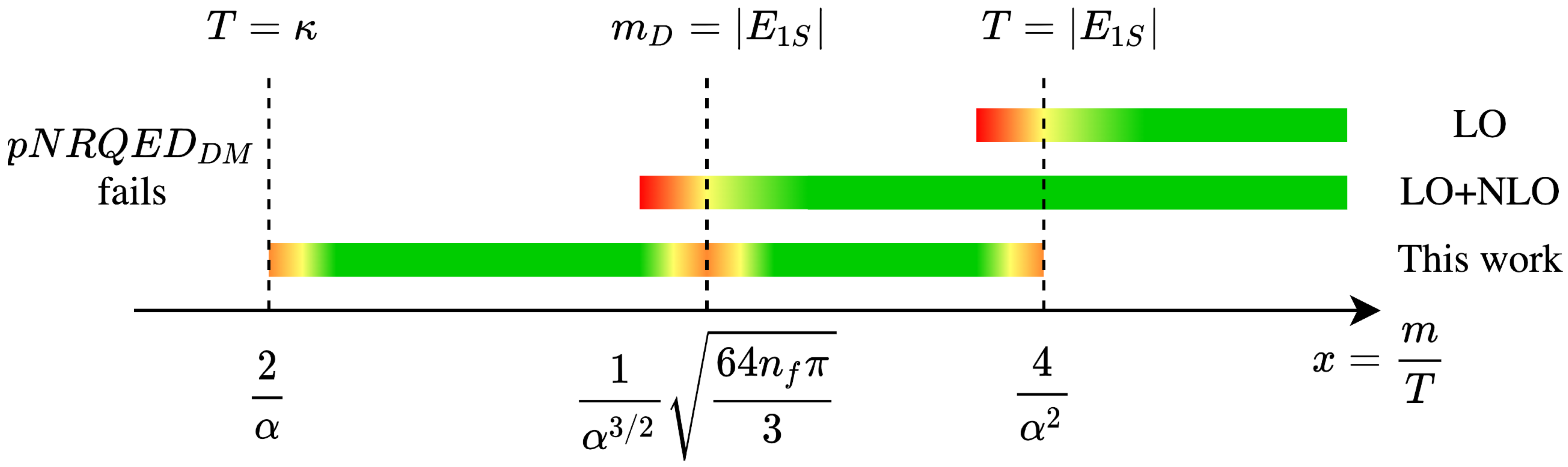


$T = 1/a_B$ $m_D = |E_{1S}|$ $T = |E_{1S}|$



Higher n states are affected “longer” by effects of resummation

Bound State Formation: regions of validity



Dark Matter Density Evolution

Boltzmann equations - as the classic simplification of OQF

$$\frac{dn_f}{dt} + 3Hn_f = -\langle \sigma_{ann} v_{rel} \rangle (n_f^2 - n_{f,eq}^2) - \sum_B \left(\langle \sigma_{BSF}^B v_{rel} \rangle n_f^2 - \Gamma_B^{ion} n_B \right)$$

Free DM particles

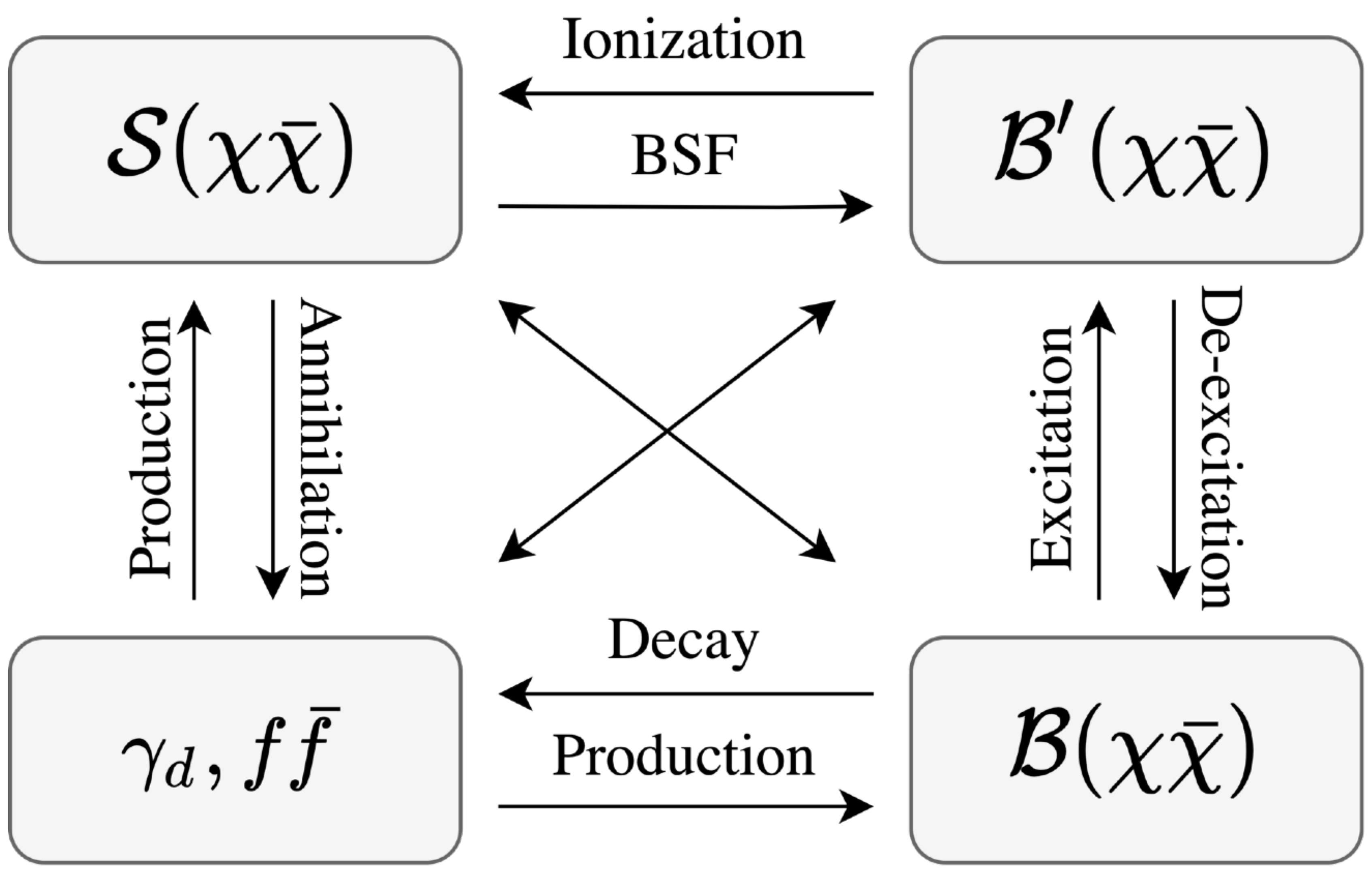
$$\frac{dn_B}{dt} + 3Hn_B = \langle \sigma_{BSF}^B v_{rel} \rangle n_f^2 - \Gamma^{ion} n_B - \Gamma_B^{dec}(n_B - n_{B,eq}) - \sum_{B \neq B'} (\Gamma_{B \to B'}^{exc} n_B - \Gamma_{B' \to B}^{exc} n_B')$$

Bound state DM

Assumptions (1503.07142): $\Gamma_{dec} \gg H$
(Bound states - close to equilibrium)
+ detailed balance equation
+ neglecting (de)excitations

$$\frac{dn_f}{dt} + 3Hn_f = -\left(\langle \sigma_{ann} v_{rel} \rangle + \sum_B \langle \sigma_{BSF}^B v_{rel} \rangle \frac{\Gamma_B^{dec}}{\Gamma_B^{dec} + \Gamma_B^{ion}} \right) (n_f^2 - n_{f,eq}^2)$$

Effective cross-section



Decay and Annihilation

Decay and Annihilation: Directly from $pNRQED_{DM}$ Lagrangian (imaginary part)

$$\delta\mathcal{L}^{annih}_{pNRQED_{DM}} = \frac{i}{m^2}\int d\vec{r}\phi^\dagger\delta(\vec{r})(2\Im[d_s] - \vec{S}^2(2\Im[d_s] - 2\Im[d_v]))\phi$$

2 spin states:
para-, orthodarkonium

$$\langle\sigma_{ann}v_{rel}\rangle = \frac{Im(d_s) + 3Im(d_v)}{m^2}S(\alpha/v_{rel}) = /LO/ = \frac{\alpha^2\pi(1+n_f)}{m^2}\frac{2\pi\alpha/v_{rel}}{1-e^{2\pi\alpha/v_{rel}}}$$

Sommerfeld enhancement factor

$$\Gamma^{dec}_{1S,pd} = \frac{4\Im[d_s]}{m^2}|\psi_{100}(0)|^2 = \frac{m\alpha^5}{2} + \mathcal{O}(\alpha^6)$$

$$\Gamma^{dec}_{1S,od} = \frac{4\Im[d_v]}{m^2}|\psi_{100}(0)|^2 = \frac{n_f}{3}\frac{m\alpha^5}{2} + \mathcal{O}(\alpha^6)$$

New contribution: decay into light d.o.f.

Dark Matter Density Evolution

Effective cross-section: asymptotic regimes

$$\left\langle \sigma_{eff} v_{rel} \right\rangle = \left\langle \sigma_{ann} v_{rel} \right\rangle + \sum_B \left\langle \sigma^B_{BSF} v_{rel} \right\rangle \frac{\Gamma^{dec}_B}{\Gamma^{dec}_B + \Gamma^{ion}_B}$$

- Ionisation equilibrium $T \gg E, \Gamma_{ion} \gg \Gamma_{dec}$

$$\left\langle \sigma_{eff} v_{rel} \right\rangle \approx \left\langle \sigma_{ann} v_{rel} \right\rangle + \sum_B \Gamma^{dec}_B \frac{n^{eq}_B}{(n^{eq}_f)^2}$$

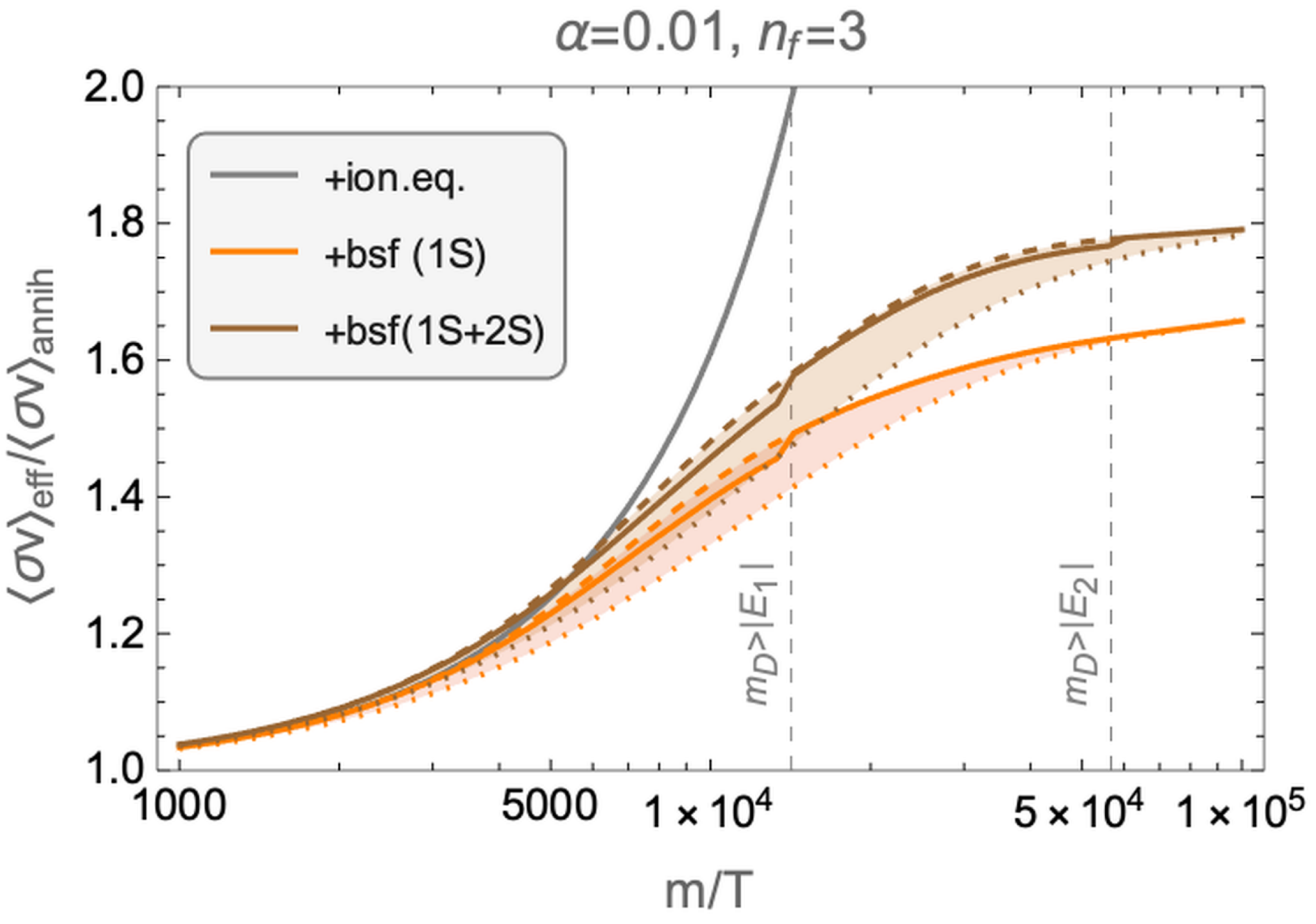
- $T < E, \Gamma_{dec} \gg \Gamma_{ion}$

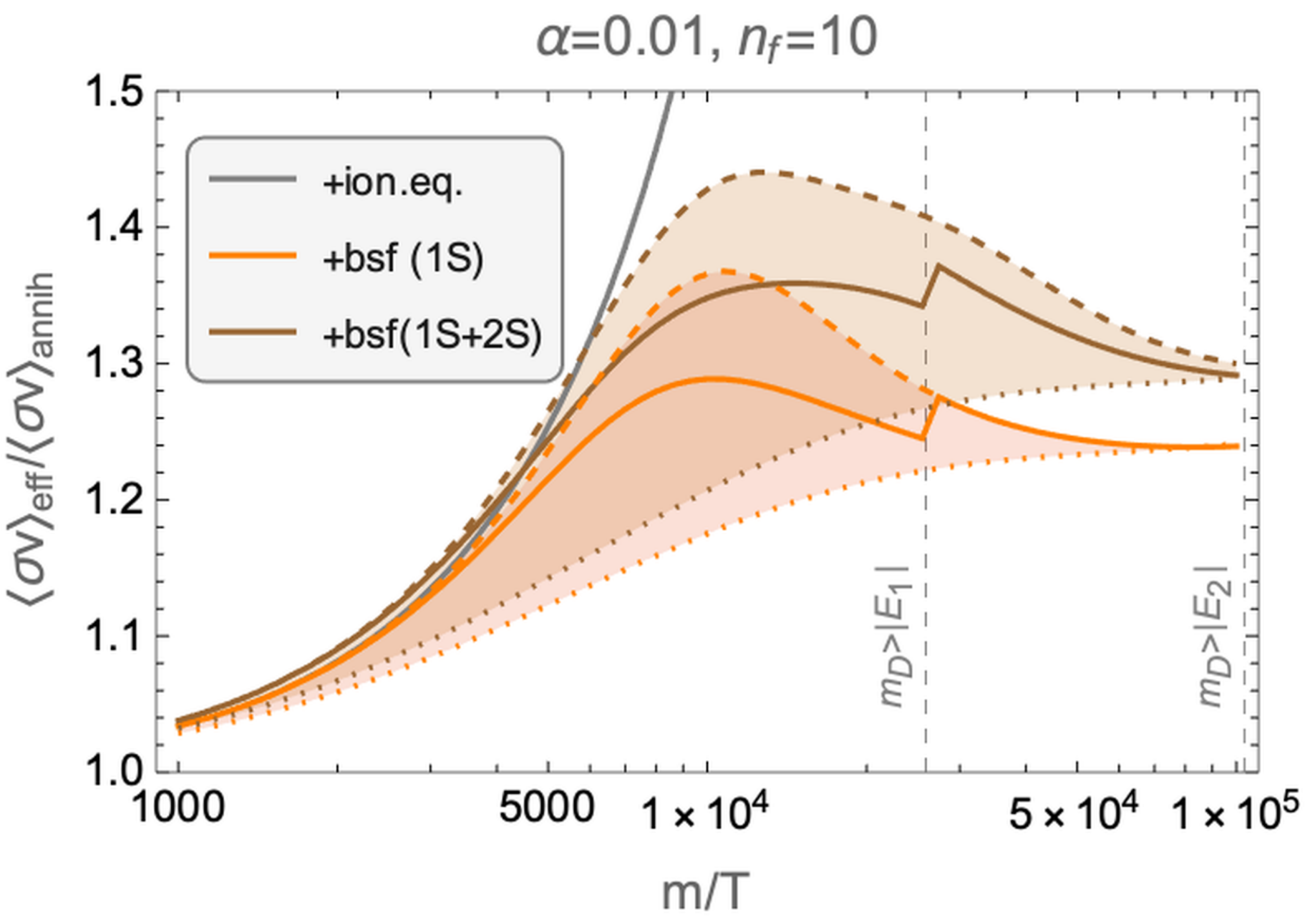
$$\left\langle \sigma_{eff} v_{rel} \right\rangle \approx \left\langle \sigma_{ann} v_{rel} \right\rangle + \sum_B \left\langle \sigma^B_{BSF} v_{rel} \right\rangle$$

Dark Matter Density Evolution

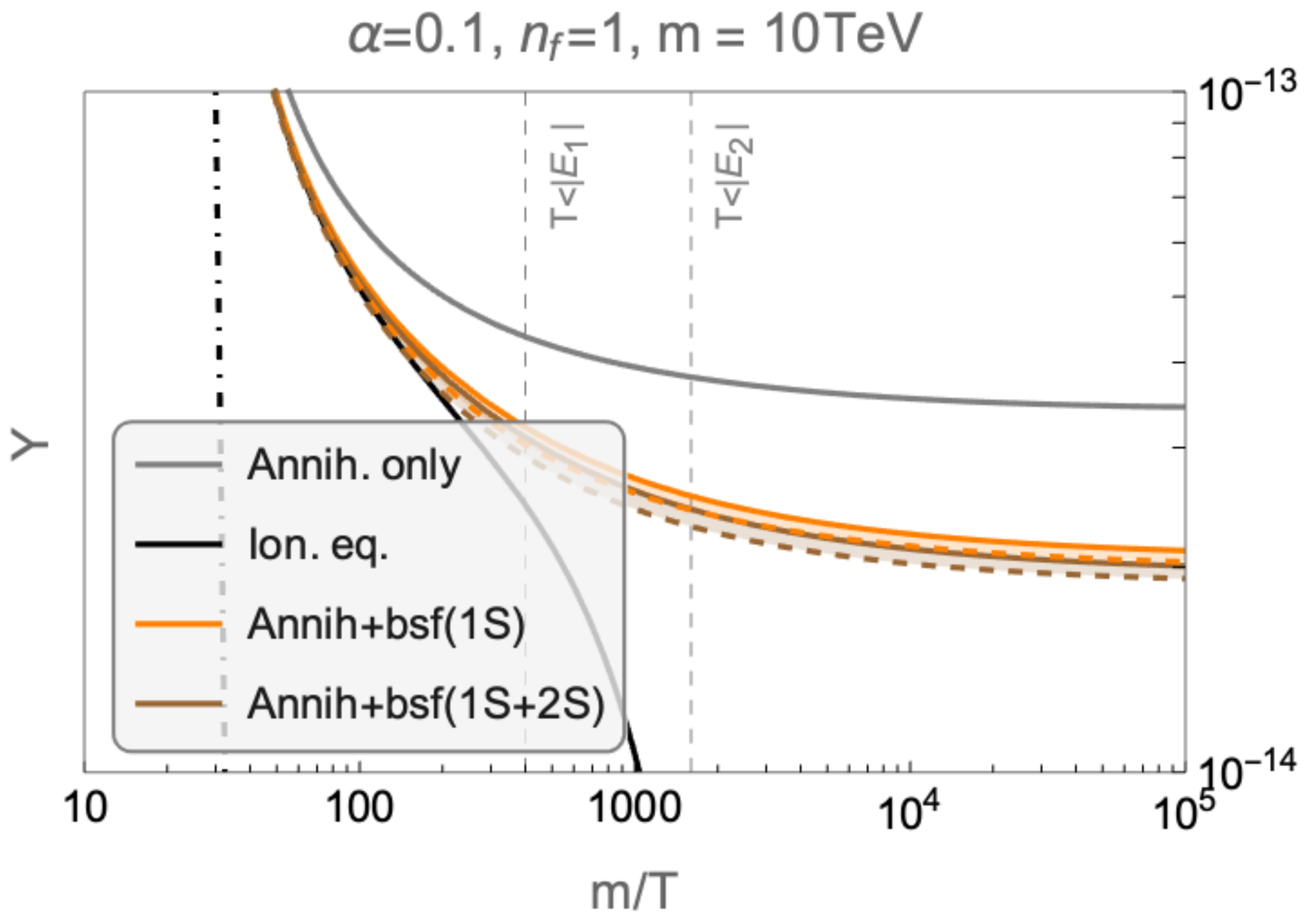
Effective cross-section

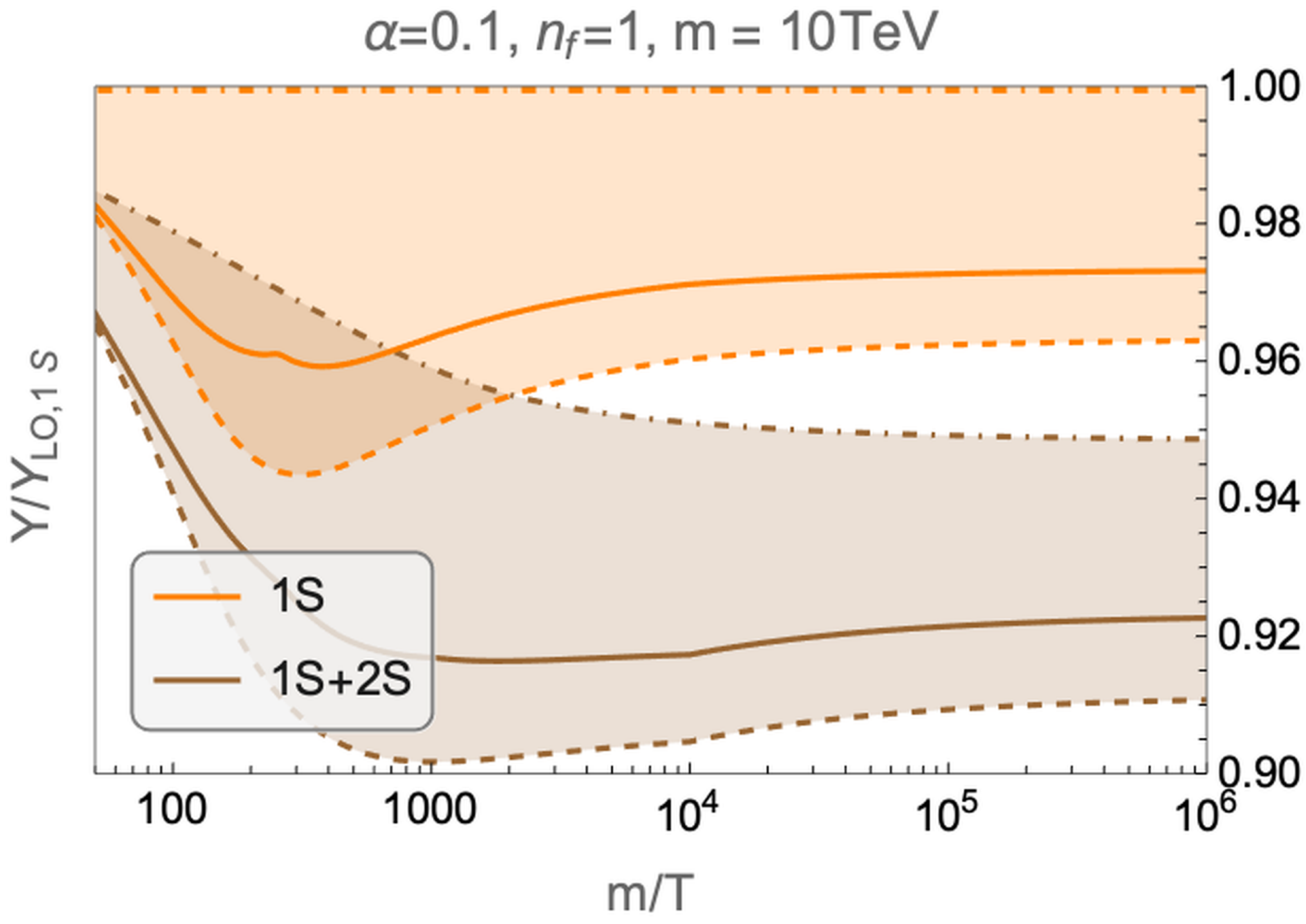
$$\left\langle \sigma_{eff} v_{rel} \right\rangle = \left\langle \sigma_{ann} v_{rel} \right\rangle + \sum_B \left\langle \sigma^B_{BSF} v_{rel} \right\rangle \frac{\Gamma^{dec}_B}{\Gamma^{dec}_B + \Gamma^{ion}_B}$$



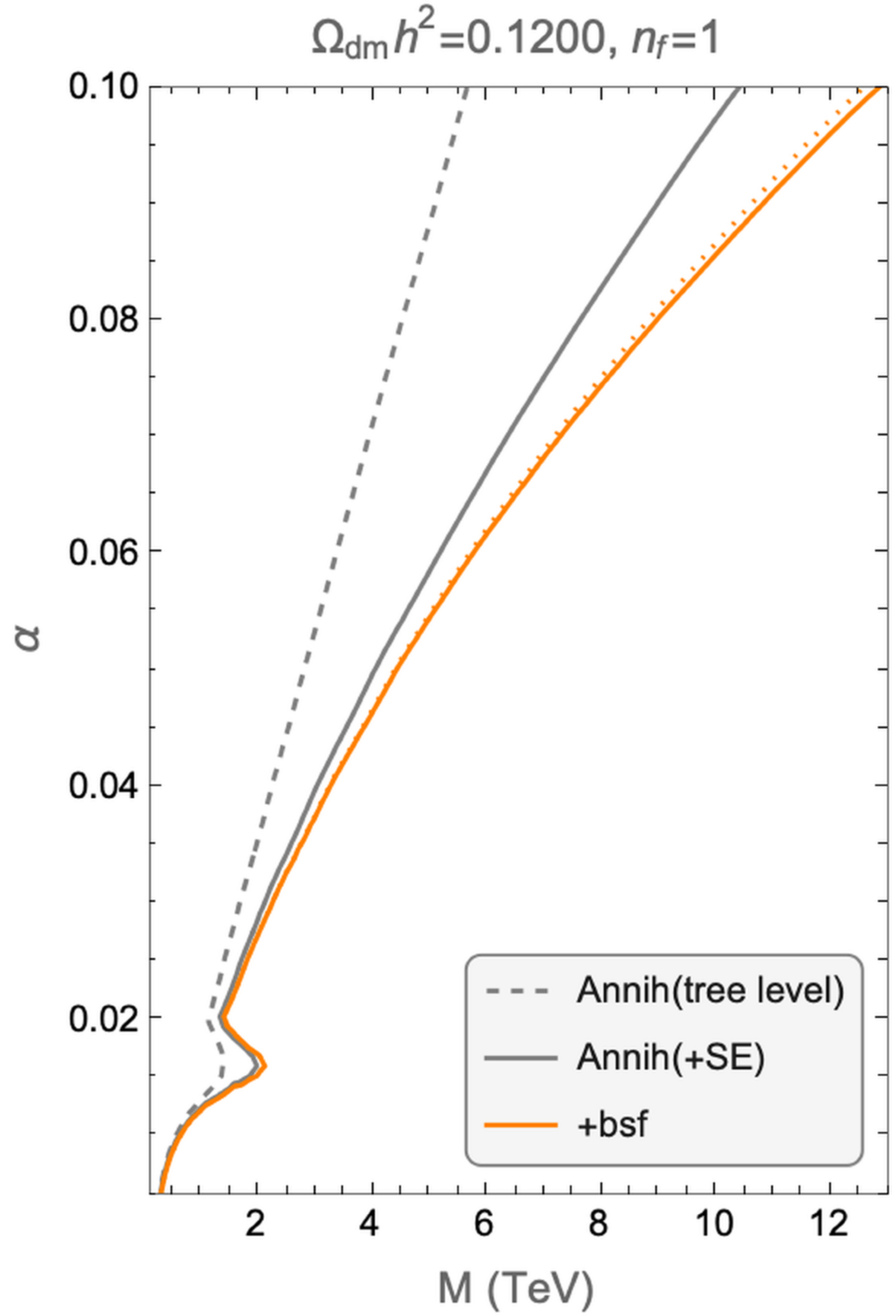


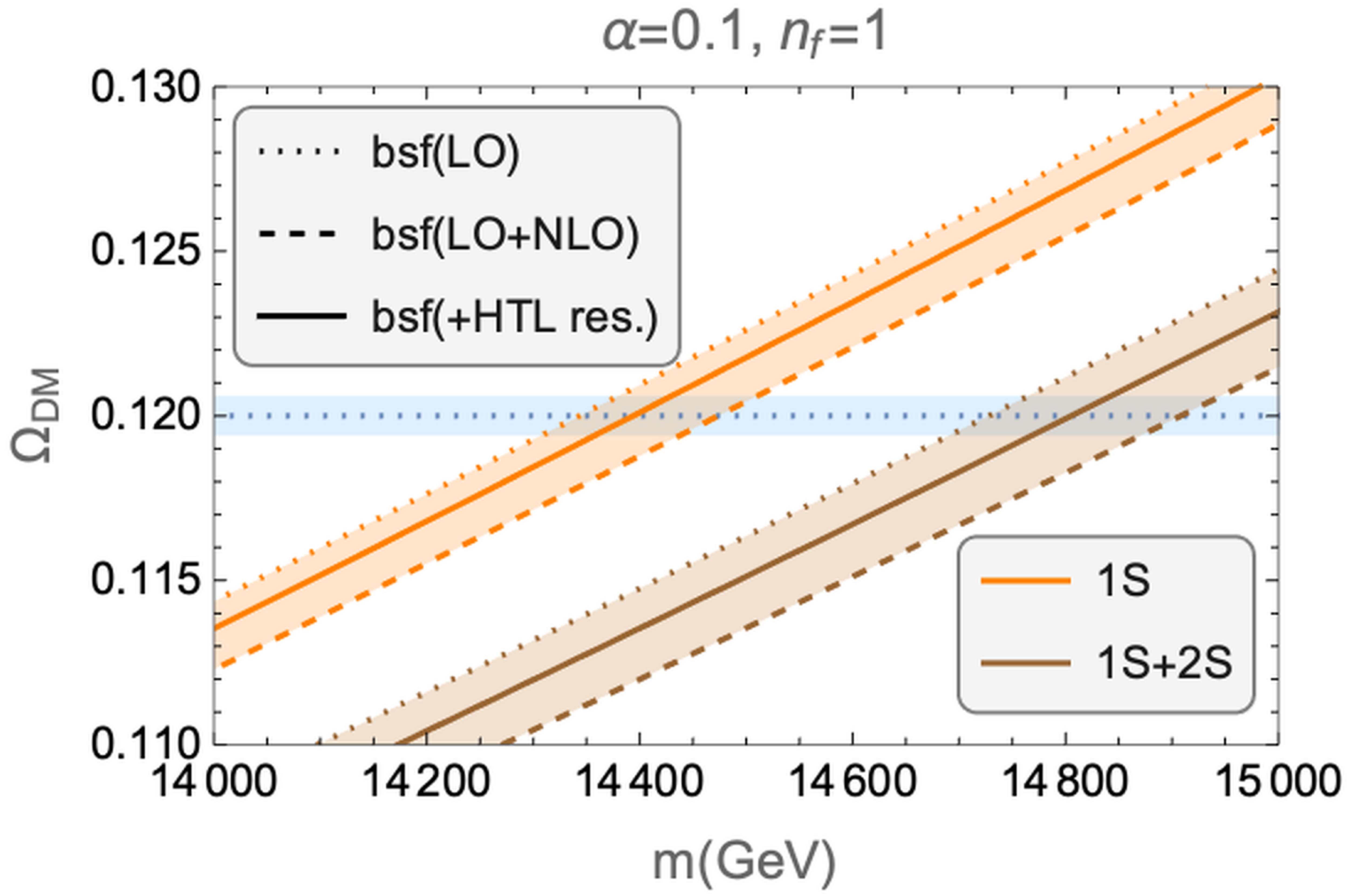
Results: yield





Results: parameters space(s)





Conclusions and Future Work

- The presence of Debye mass scale affects the evolutions of the dark matter in the Early Universe.
- As for NLO contribution, these corrections are more relevant for stronger coupling and larger number of light d.o.f.
- The effects are of the same order as the NLO correction.

- Study the case of $m_D \sim E$ in more detail.
- Explore $T \approx m_D$ (exactly where we expect the effect to be the strongest).
- Include higher n states.

Thank you for your attention