

SYSTEM OF EVOLUTION EQUATIONS FOR QUARK AND GLUON JET QUENCHING WITH BROADENING

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IN COLLABORATION WITH : K. KUTAK^a, W. PŁACZEK^b,
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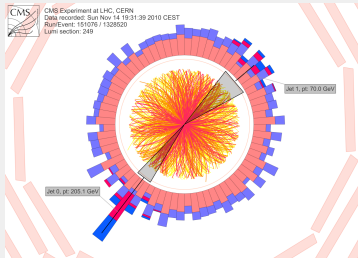
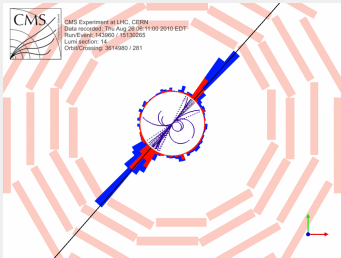
^bJagiellonian University,

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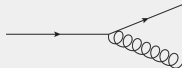


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JET QUENCHING \ DIJET ENERGY LOSS

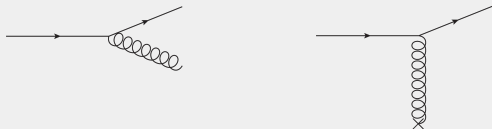


- Jet quenching observed through asymmetrical back to back dijet energy
- Understood as the result of the interaction of the jet with a quark-gluon plasma (QGP)
- Jet as hard probe of QGP



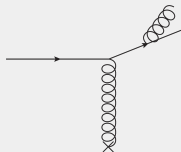
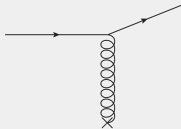
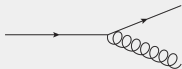
Possible processes

- Bremsstrahlung radiations (as in vacuum)



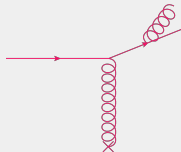
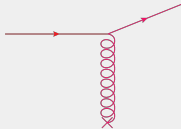
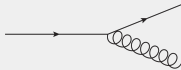
Possible processes

- Bremsstrahlung radiations (as in vacuum)
- Elastic scattering



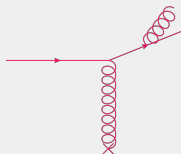
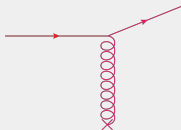
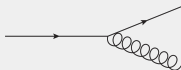
Possible processes

- Bremsstrahlung radiations (as in vacuum)
- Elastic scattering
- Inelastic scattering



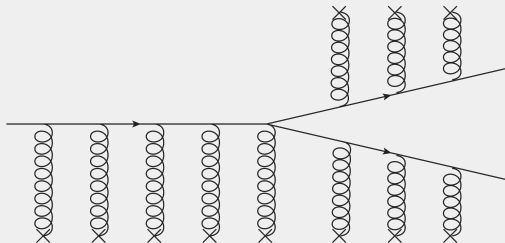
Possible processes

- Bremsstrahlung radiations (as in vacuum)
- Elastic scattering (collisions)
- Inelastic scattering → Medium induced radiations



Possible processes

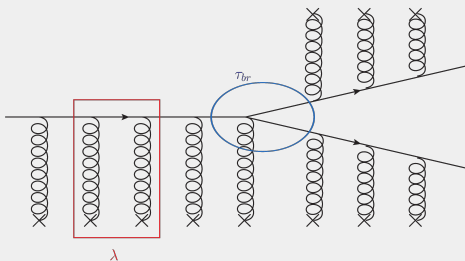
- Bremsstrahlung radiations (as in vacuum)
 - Elastic scattering (collisions)
 - Inelastic scattering → Medium induced radiations
- $P_{ij}(z)$
 - $\mathcal{C}(\mathbf{l})$
 - $\mathcal{K}_{ij}(z)$



- Static color charges
- Eikonal interaction

- Collinear radiation
- Main parameter : $\hat{q} = \frac{\Delta k_{\perp}^2}{\Delta l}$

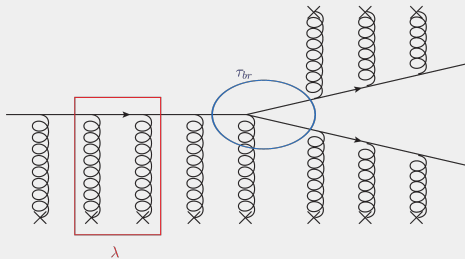
R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff. **RADIATIVE ENERGY LOSS OF HIGH-ENERGY QUARKS AND GLUONS IN A FINITE VOLUME QUARK - GLUON PLASMA.** *NUCL. PHYS. B*, **483:291-320, 1997**



Time scales

For a radiated parton of energy ω and transverse momentum k_{\perp}

- Formation time : $\tau_f \sim \frac{2\omega}{k_{\perp}^2}$
- Branching time : $\tau_{br}(\omega) \sim \sqrt{\frac{2\omega}{\hat{q}}}$
- $\tau_{br}(\omega_c) \sim L \rightarrow$ limit on soft emission
- $\tau_{br}(\omega_{BH}) \sim \lambda \rightarrow$ Bethe-Heitler spectrum (incoherent collision)



Limit studied

$$\omega_{BH} \ll \omega \lesssim \omega_c$$

Where medium-induced radiation dominates

Formulation

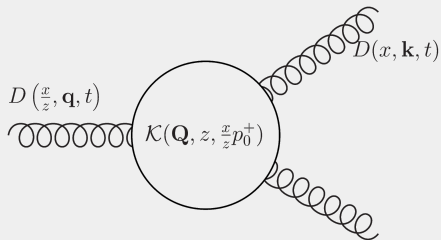
$$\begin{aligned} \frac{\partial}{\partial t} D(x, \mathbf{k}, t) = & \alpha_s \int_0^1 dz \int \frac{d^2 q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right] \\ & + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t) \end{aligned}$$

J.-P. Blaizot, F. Dominguez, E. Iancu, and Y. Mehtar-Tani. **PROBABILISTIC PICTURE FOR MEDIUM-INDUCED JET EVOLUTION. *JHEP*, 06:075, 2014**

Formulation

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \alpha_s \int_0^1 dz \int \frac{d^2 q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right] \\ + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t)$$

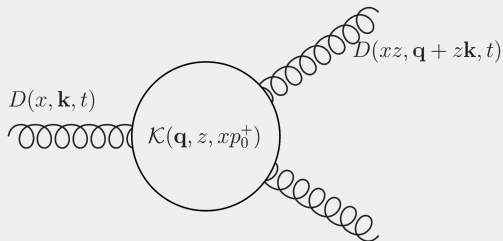
$$\mathbf{Q} = \mathbf{k} - z\mathbf{q}$$



Formulation

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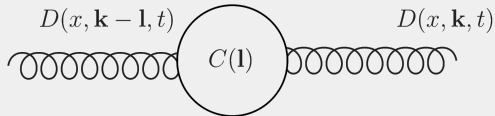
J.-P. Blaizot, F. Dominguez, E. Iancu, and Y. Mehtar-Tani. **PROBABILISTIC PICTURE FOR MEDIUM-INDUCED JET EVOLUTION. JHEP, 06:075, 2014**



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J.-P. Blaizot, F. Dominguez, E. Iancu, and Y. Mehtar-Tani. **PROBABILISTIC PICTURE FOR MEDIUM-INDUCED JET EVOLUTION. JHEP, 06:075, 2014**

$$\frac{\partial}{\partial t} D(x, \mathbf{k}, t) = \text{Diagram 1} - \text{Diagram 2} + \text{Diagram 3}$$

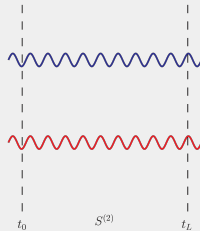
The diagram shows the evolution equation for the gluon distribution function $D(x, \mathbf{k}, t)$. It consists of three terms:

- Diagram 1:** A circle containing the kernel \mathcal{K} on the left, connected to a wavy line representing the gluon distribution $D(x, \mathbf{k}, t)$ on the right.
- Diagram 2:** A wavy line representing $D(x, \mathbf{k}, t)$ on the left, connected to a circle containing the kernel \mathcal{K} on the right.
- Diagram 3:** A circle containing the kernel \mathcal{C} on the left, connected to a wavy line representing $D(x, \mathbf{k}, t)$ on the right.

Formulation

$$\begin{aligned} \frac{\partial}{\partial t} D(x, \mathbf{k}, t) = & \alpha_s \int_0^1 dz \int \frac{d^2 q}{(2\pi)^2} \left[2\mathcal{K}(\mathbf{Q}, z, \frac{x}{z} p_0^+) D\left(\frac{x}{z}, \mathbf{q}, t\right) - \mathcal{K}(\mathbf{q}, z, x p_0^+) D(x, \mathbf{k}, t) \right] \\ & + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} \mathcal{C}(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t) \end{aligned}$$

Studied in : E. Blanco, K. Kutak, W. Płaczek, M. Rohrmoser, and R. Straka. **MEDIUM INDUCED QCD CASCADES: BROADENING AND RESCATTERING DURING BRANCHING.** *JHEP*, 04:014, 2021



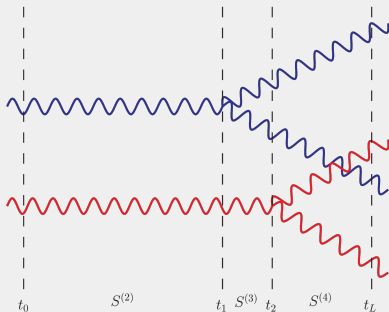
Collision Kernel

$$C(\mathbf{l}) = \left[w(\mathbf{l}) - (2\pi)^2 \delta^{(2)}(\mathbf{l}) \int \frac{d^2\mathbf{q}}{(2\pi)^2} w(\mathbf{q}) \right]$$

For a weakly coupled QGP at high temperature T

$$w(\mathbf{l}) = \frac{g^4 n N_c}{l^2 (l^2 + m_D^2)}, \quad m_D^2 = \left(1 + \frac{N_f}{6}\right) g^2 T^2, \quad n = m_D^2 \frac{T}{g^2} \propto T^3$$

Miklos G. and X.-N. Wang. **MULTIPLE COLLISIONS AND INDUCED GLUON BREMSSTRAHLUNG IN QCD. NUCL. PHYS. B, 420:583–614, 1994**



Splitting Kernel

$$\mathcal{K}(\mathbf{Q}, z, p_0^+) = \frac{P_{gg}(z)}{z(1-z)p_0^+} \text{Re} \int_0^\infty d\Delta t \int \frac{d^2\mathbf{P}}{(2\pi)^2} \frac{d^2\mathbf{l}}{(2\pi)^2} (\mathbf{P} \cdot \mathbf{Q}) \tilde{S}_{ij}^{(3)}(\mathbf{P}, \mathbf{Q}, \mathbf{l}, z, \Delta t, t)$$

J.-P. Blaizot, F. Dominguez, E.d Iancu, and Y. Mehtar-Tani. **MEDIUM-INDUCED GLUON BRANCHING**. *JHEP*, 01:143, 2013

Integrated

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) \theta(z - x) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

J.-P. Blaizot, F. Dominguez, E. Iancu, and Y. Mehtar-Tani. **PROBABILISTIC PICTURE FOR MEDIUM-INDUCED JET EVOLUTION. *JHEP*, 06:075, 2014**

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$$\text{Stopping time : } t^* = \frac{\tau_{br}(\omega)}{\bar{\alpha}} = \frac{1}{\bar{\alpha}} \sqrt{\frac{\omega}{\bar{q}}}, \quad \bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

Integrated

$$\frac{\partial}{\partial t} D(x, t) = \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, t\right) \theta(z-x) - \frac{z}{\sqrt{x}} D(x, t) \right]$$

Collinear branching

$$\begin{aligned} \frac{\partial}{\partial t} D(x, \mathbf{k}, t) &= \frac{1}{t^*} \int_0^1 dz \mathcal{K}(z) \left[\frac{1}{z^2} \sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \frac{\mathbf{k}}{z}, t\right) \theta(z-x) - \frac{z}{\sqrt{x}} D(x, \mathbf{k}, t) \right] \\ &+ \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C(\mathbf{l}) D(x, \mathbf{k} - \mathbf{l}, t) \end{aligned}$$

J.-P. Blaizot, F. Dominguez, E. Iancu, and Y. Mehtar-Tani. **PROBABILISTIC PICTURE FOR MEDIUM-INDUCED JET EVOLUTION. JHEP, 06:075, 2014**

Studied in : K. Kutak, W. Płaczek, and R. Straka. **SOLUTIONS OF EVOLUTION EQUATIONS FOR MEDIUM-INDUCED QCD CASCADES. EUR. PHYS. J. C, 79(4):317, 2019**

System of Equations

$$\begin{aligned} \frac{\partial}{\partial t} D_g(x, \mathbf{k}, t) = & \int_0^1 dz \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \alpha_s \left\{ 2\mathcal{K}_{gg} \left(\mathbf{Q}, z, \frac{x}{z} p_0^+ \right) D_g \left(\frac{x}{z}, \mathbf{q}, t \right) + \mathcal{K}_{gq} \left(\mathbf{Q}, z, \frac{x}{z} p_0^+ \right) \sum_i D_{q_i} \left(\frac{x}{z}, \mathbf{q}, t \right) \right. \\ & \left. - \left[\mathcal{K}_{gg}(\mathbf{q}, z, x p_0^+) + \mathcal{K}_{qg}(\mathbf{q}, z, x p_0^+) \right] D_g(x, \mathbf{k}, t) \right\} + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C_g(\mathbf{l}) D_g(x, \mathbf{k} - \mathbf{l}, t) \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial t} D_{q_i}(x, \mathbf{k}, t) = & \int_0^1 dz \int \frac{d^2 \mathbf{q}}{(2\pi)^2} \alpha_s \left\{ \mathcal{K}_{qq} \left(\mathbf{Q}, z, \frac{x}{z} p_0^+ \right) D_{q_i} \left(\frac{x}{z}, \mathbf{q}, t \right) + \frac{1}{N_F} \mathcal{K}_{qg} \left(\mathbf{Q}, z, \frac{x}{z} p_0^+ \right) D_g \left(\frac{x}{z}, \mathbf{q}, t \right) \right. \\ & \left. - \mathcal{K}_{qq}(\mathbf{q}, z, x p_0^+) D_{q_i}(x, \mathbf{k}, t) \right\} + \int \frac{d^2 \mathbf{l}}{(2\pi)^2} C_q(\mathbf{l}) D_{q_i}(x, \mathbf{k} - \mathbf{l}, t) \end{aligned}$$

Splitting kernels

$$\mathcal{K}_{ij}(\mathbf{Q}, z, p_0^+) = \frac{2P_{ij}(z)}{z(1-z)p_0^+} \sin\left(\frac{\mathbf{Q}^2}{2k_{\text{br}}^2}\right) \exp\left(-\frac{\mathbf{Q}^2}{2k_{\text{br}}^2}\right)$$

with $k_{\text{br}}^2 = \sqrt{z(1-z)p_0^+ f_{ij}(z) \hat{q}}$, $\hat{q} = \frac{\hat{q}}{N_c}$ and

$$\begin{aligned} f_{gg}(z) &= (1-z)C_A + z^2C_A, & f_{qg}(z) &= C_F - z(1-z)C_A, \\ f_{gq}(z) &= (1-z)C_A + z^2C_F, & f_{qq}(z) &= zC_A + (1-z)^2C_F \end{aligned}$$

Collision kernels

$$w_g(\mathbf{l}) = \frac{N_c g^4 n}{\mathbf{l}^2(\mathbf{l}^2 + m_D^2)}, \quad w_q(\mathbf{l}) = \frac{C_F g^4 n}{\mathbf{l}^2(\mathbf{l}^2 + m_D^2)},$$

Method

- Convert the BDIM equation in a Voltera-type equation
- Solve it by iteration
- Calculate the iterative solution through a MCMC algorithm

Programs

BDIM equation solve with 2 MC programs :

- MINCAS → solution for D
- TMDICE → solution for F

$$F_a(x, \mathbf{k}, t) := \frac{d^3 N_a}{dx d^2 \mathbf{k}}, \quad \text{and } D_a(x, \mathbf{k}, t) := x F_a(x, \mathbf{k}, t)$$

M. Rohrmoser. **THE TMDICE MONTE CARLO SHOWER PROGRAM AND ALGORITHM FOR JET-FRAGMENTATION VIA COHERENT MEDIUM INDUCED RADIATIONS AND SCATTERING.** *COMPUT. PHYS. COMMUN.*, 276:108343, 2022

Other MC programs

- P. Caucal, E. Iancu., A. H. Mueller, and G. Soyez. **A NEW PQCD BASED MONTE CARLO EVENT GENERATOR FOR JETS IN THE QUARK-GLUON PLASMA. POS, HARDPROBES2018:028, 2019**
- JEWEL : K. C. Zapp. **JEWEL 2.0.0: DIRECTIONS FOR USE. EUR. PHYS. J. C, 74(2):2762, 2014**

Specificities

With our approach, we aim at :

- non collinear splitting / broadening
- both low and high x region

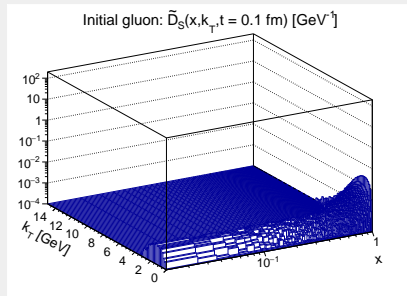
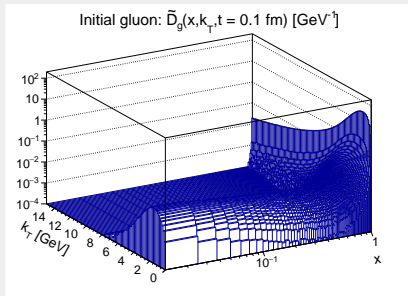
Parameters

- $x_{\min} = 10^{-4}$
- $\epsilon = 10^{-6}$
- $l_{\min} = 0.1 \text{ GeV}$
- $N_C = 3$
- $N_F = 3$
- $\alpha_S = \pi/10$
- $E = 100 \text{ GeV}$
- $n = 0.243 \text{ GeV}^3$
- $\hat{q} = 1 \text{ GeV}^2/\text{fm}$
- $m_D = 0.993 \text{ GeV}$

Singlet distribution

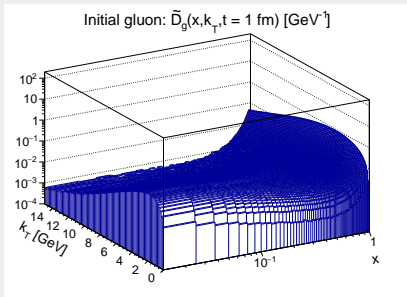
$$D_S(x, \mathbf{k}, t) = \sum_{i=1}^{N_f} (D_{q_i}(x, \mathbf{k}, t) + D_{\bar{q}_i}(x, \mathbf{k}, t))$$

Time : 0.1fm

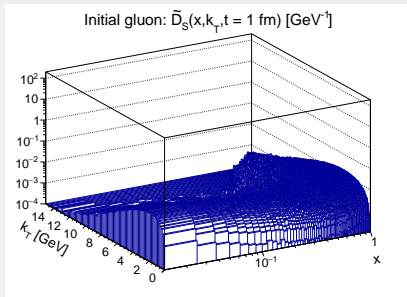


Time : 1fm

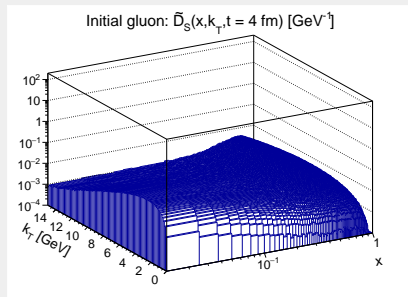
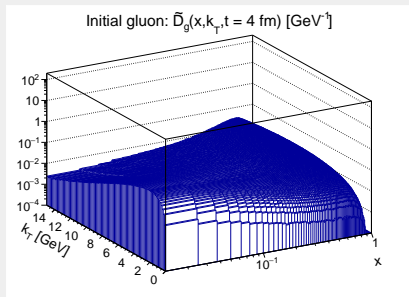
Initial gluon: $\tilde{D}_g(x, k_T, t = 1 \text{ fm}) [\text{GeV}^{-1}]$



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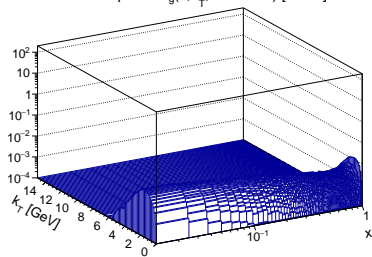


Time : 4fm

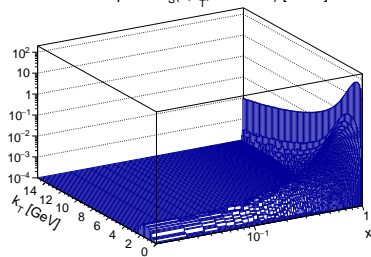


Time : 0.1fm

Initial quark: $\tilde{D}_g(x, k_T, t = 0.1 \text{ fm}) [\text{GeV}^{-1}]$

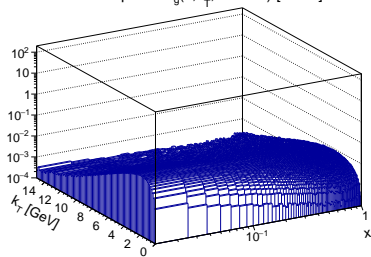


Initial quark: $\tilde{D}_S(x, k_T, t = 0.1 \text{ fm}) [\text{GeV}^{-1}]$

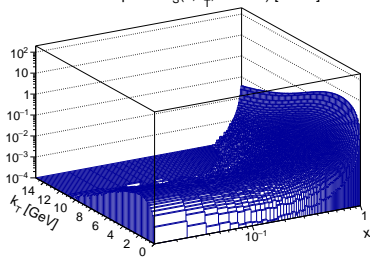


Time : 1fm

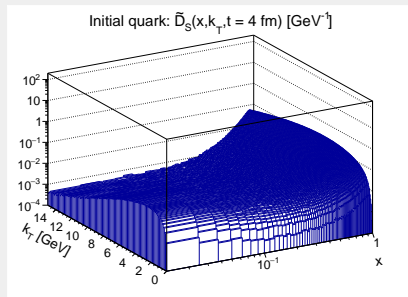
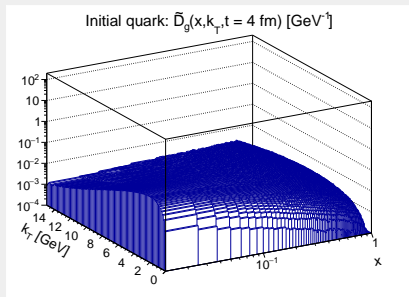
Initial quark: $\tilde{D}_g(x, k_T, t = 1 \text{ fm}) [\text{GeV}^{-1}]$

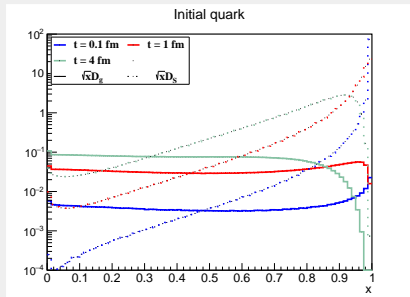
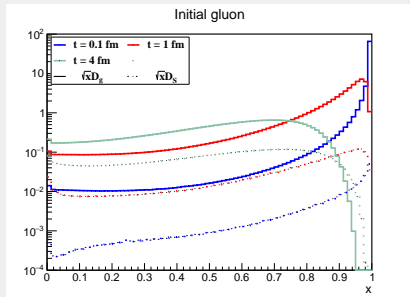


Initial quark: $\tilde{D}_g(x, k_T, t = 1 \text{ fm}) [\text{GeV}^{-1}]$

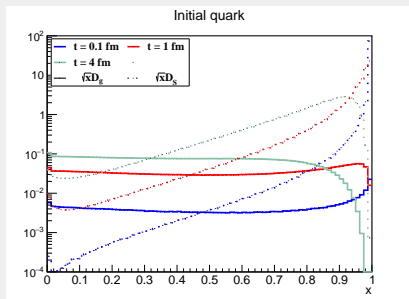
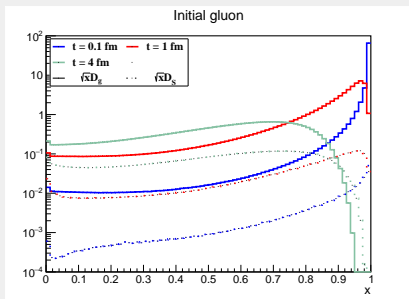


Time : 4fm





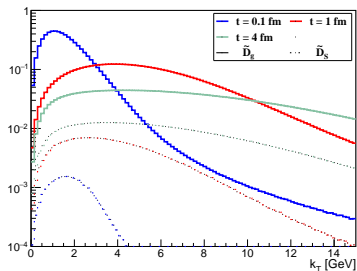
$$D(x, t) = \int d^2\mathbf{k} D_I(x, \mathbf{k}, t)$$



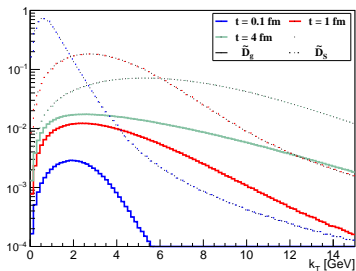
$\frac{1}{\sqrt{x}}$ scaling at low x as shown in :

Y. Mehtar-Tani and S. Schlichting. **UNIVERSAL QUARK TO GLUON RATIO IN MEDIUM-INDUCED PARTON CASCADE.** *JHEP*, 09:144, 2018

Initial gluon

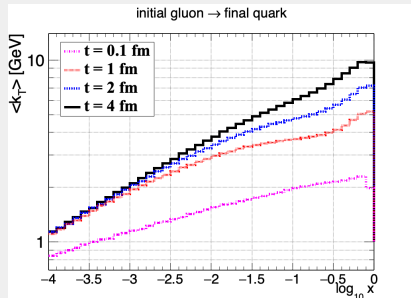
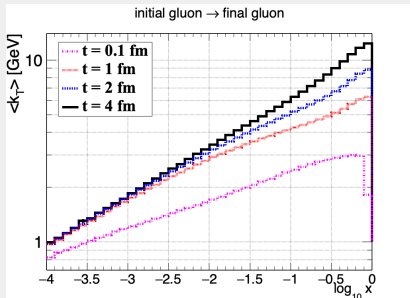


Initial quark



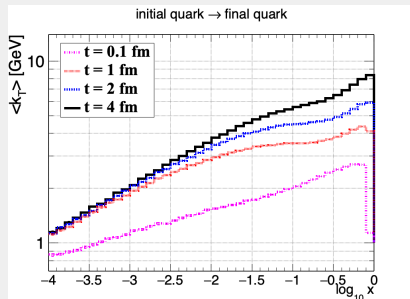
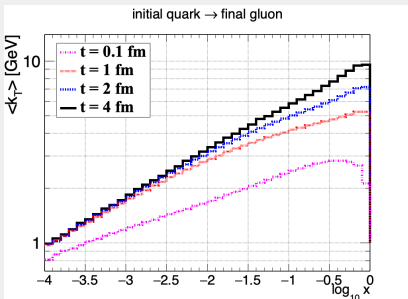
$$\tilde{D}(x, k_T, t) = \int_0^{2\pi} r \, d\phi \, k_T D_I(x, \mathbf{k}, t)$$

Initial Gluon



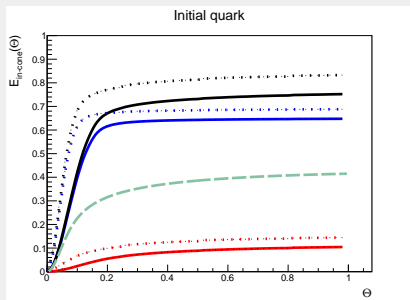
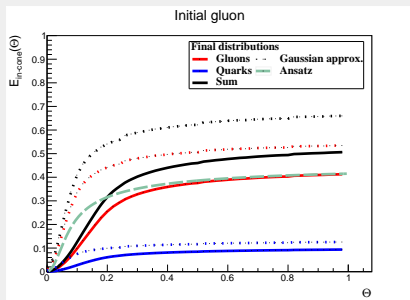
$$\langle k_T \rangle = \frac{\int d^2\mathbf{k} |\mathbf{k}| D(x, \mathbf{k}, t)}{\int d^2\mathbf{k} D(x, \mathbf{k}, t)} = \frac{\int_0^\infty dk_T k_T^2 D(x, k_T, t)}{\int_0^\infty dk_T k_T D(x, k_T, t)}$$

Initial Quark



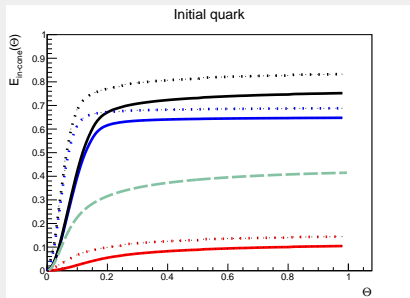
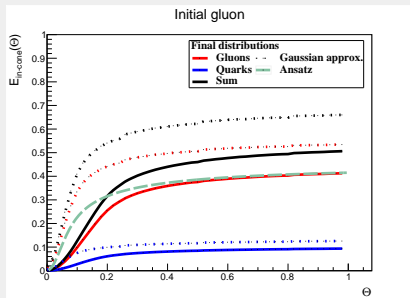
$$\langle k_T \rangle = \frac{\int d^2\mathbf{k} |\mathbf{k}| D(x, \mathbf{k}, t)}{\int d^2\mathbf{k} D(x, \mathbf{k}, t)} = \frac{\int_0^\infty dk_T k_T^2 D(x, k_T, t)}{\int_0^\infty dk_T k_T D(x, k_T, t)}$$

Time : 4fm



$$E_{\text{in-cone}}(\Theta) = \int_0^1 dx \int_0^{xE \sin \Theta} dk_T \tilde{D}(x, k_T, t)$$

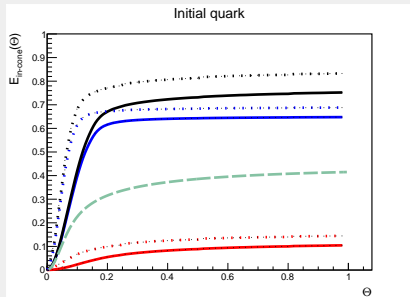
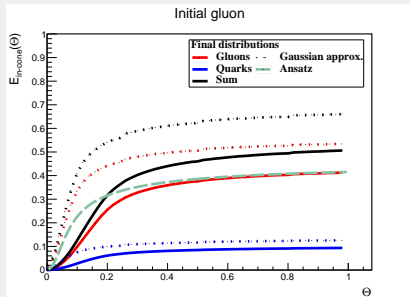
Time : 4fm



Gaussian approximation

$$D_G(x, \mathbf{k}, t) = D(x, t) \frac{4\pi}{\hat{q}t} \exp\left(-\frac{\mathbf{k}^2}{\hat{q}t}\right)$$

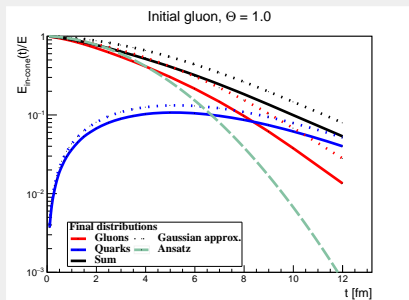
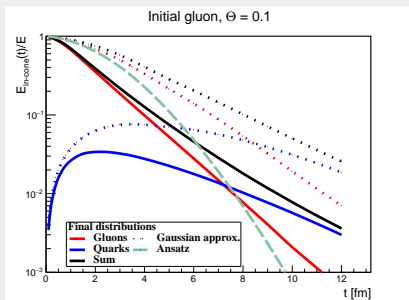
Time : 4fm



Analytical Ansatz

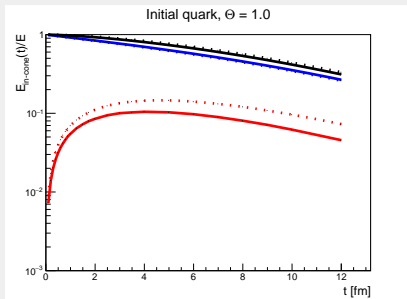
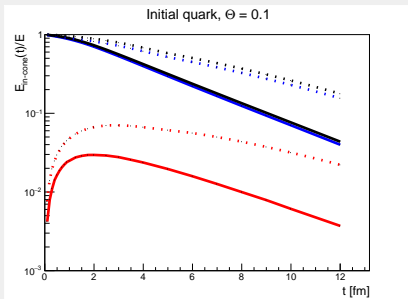
$$D_A(x, \mathbf{k}, t) = \frac{t/t_*}{\sqrt{x(1-x)^3}} \exp\left(-\pi \frac{(t/t_*)^2}{1-x}\right) \frac{4\pi}{\hat{q}t} \exp\left(-\frac{\mathbf{k}^2}{\hat{q}t}\right)$$

Initial Gluon



$$E_{\text{in-cone}}(\Theta) = \int_0^1 dx \int_0^{xE \sin \Theta} dk_T \tilde{D}(x, k_T, t)$$

Initial Quark



$$E_{\text{in-cone}}(\Theta) = \int_0^1 dx \int_0^{xE \sin \Theta} dk_T \tilde{D}(x, k_T, t)$$

CONCLUSION








- We derived in-medium splitting kernel accounting for broadening for BDIM evolution equation of both quarks and gluons \rightarrow BDMPS-Z beyond eikonal approximation
- We solved this evolution equation through MCMC methods (with MINCAS and TMDICE)
- The study of the solutions has shown that :
 - ▶ gluons broaden more than quarks in time
 - ▶ quarks are more collimated than gluons
 - ▶ quarks dominate at late time
 - ▶ the $\langle k_T \rangle$ distribution is universal at low- x and late times

Outlook

- Vacuum shower
- Dynamics of the medium (through time or temperature dependence of its parameters)

THANKS FOR YOUR ATTENTION!

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