

CLASSICAL VS. QUANTUM CORRECTIONS TO JET BROADENING IN A WEAKLY COUPLED QGP

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BASED ON 22???.XXXXX

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1 Physical Picture

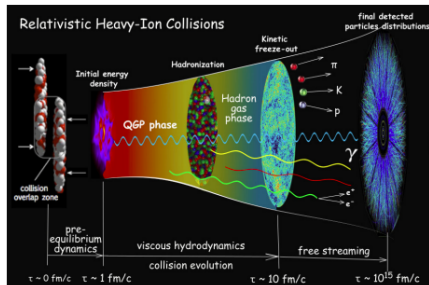
2 Single Scattering vs. Multiple Scattering

3 \hat{q} : Context and Motivation

4 Quantum Corrections to \hat{q}

HEAVY-ION COLLISIONS

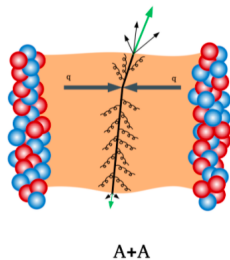
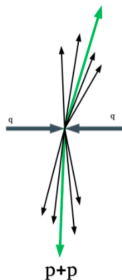
- Heavy nuclei are smashed together at the LHC and RHIC, liberating their constituents and forming the **Quark Gluon Plasma (QGP)**
- Short lifetime makes QGP extremely difficult to study, so what do we do?



[Heinz, 2013]

JET QUENCHING

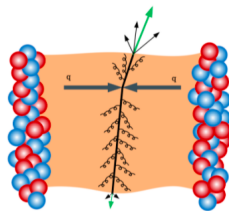
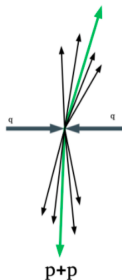
- A natural feature of gauge theories, **jets**, are structures of high energy, self-collimated final state particles
- Produced in both proton-proton collisions and heavy-ion collisions
 - ⇒ Nice probe for QGP
 - ⇒ **Jet Quenching**



M. Rybar/ATLAS Collaboration

JET QUENCHING

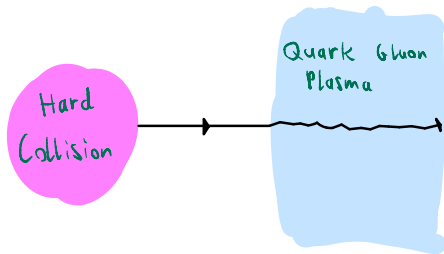
- **Transverse momentum broadening coefficient, \hat{q}** serves as key ingredient in characterising quenching
 - ▶ Relevant for computing in-medium splitting rates
 - ▶ Can be used for input to effective kinetic description of QGP



M. Rybar/ATLAS Collaboration

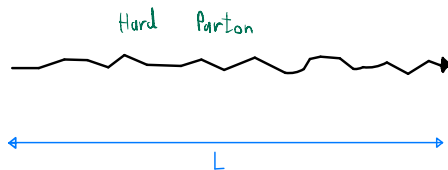
JET PROPAGATION

- Consider a nearly on shell, highly energetic (hard) parton with energy, E produced in a heavy ion collision
- Parton undergoes collisions with medium constituents while propagating through the plasma



ENERGY LOSS FROM ELASTIC COLLISIONS

- Hard parton picks up transverse momentum, $k_{\perp} \ll E$ from collisions with medium constituents
- View as diffusion process and define diffusion coefficient, \hat{q} as $k_{\perp}^2 \equiv \hat{q}L$



ENERGY LOSS FROM BREMSSTRAHLUNG

- The dominant mechanism for energy loss in the QGP is not the energy lost through these elastic collisions
- Instead, it comes from the bremsstrahlung induced from these elastic collisions



OUTLINE

1 Physical Picture

2 Single Scattering vs. Multiple Scattering

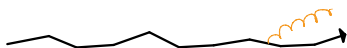
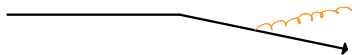
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HOW EXACTLY IS BREMSSTRAHLUNG TRIGGERED?

This depends on the **quantum mechanical formation time, τ** associated with the radiated gluon and can be crudely separated into two cases:

- Case 1: Radiated gluon with **energy ω** is triggered by just collision with medium constituent
 - ▶ Known as Bethe-Heitler or **single-scattering regime**
- Case 2: Many collisions with smaller momentum exchange add up to trigger gluon radiation with **energy ω**
 - ▶ Known as harmonic oscillator or **multiple-scattering regime**
 - ▶ Requires LPM resummation



SINGLE SCATTERING

- Formation time of radiated particle is given parametrically as

$$\tau \sim \frac{\omega}{k_{\perp}^2} \quad (1)$$

- Let λ_{el} be the **mean free path** associated with collisions with the medium
- If $\tau \lesssim \lambda_{el}$, the so-called Bethe-Heitler spectrum turns out to be proportional to the **number of elastic collisions, N**

$$\omega \frac{dI}{d\omega} \simeq \frac{\alpha_S N_c}{\pi} N = \frac{\alpha_S N_c}{\pi} \frac{L}{\lambda_{el}} \quad (2)$$

MULTIPLE SCATTERING

- Now, assume that $\tau \gg \lambda_{el}$ and that radiated gluon undergoes transverse momentum kicks during formation time and picks up $k_{\perp}^2 \sim \hat{q}\tau_f$

$$\Rightarrow \tau = \sqrt{\frac{\omega}{\hat{q}}} \quad (3)$$

- Then, we can crudely say that if $N_{coh} = \tau/\lambda_{el}$ is the number of coherent collisions that

$$\omega \frac{dI}{d\omega} \simeq \frac{\alpha_S N_c}{\pi} \frac{N}{N_{coh}} = \frac{\alpha_S N_c}{\pi} L \sqrt{\frac{\hat{q}}{\omega}} \quad (4)$$

\Rightarrow As ω increases, spectrum suppressed \Rightarrow **LPM Effect**

THINGS TO NOTE

- Physics of these two regimes is very different
- **In multiple-scattering regime**, many collisions need to be resummed via BDMPS-Z/AMY formalisms [Baier et al., 1995, Zakharov, 1997, Arnold et al., 2003]
- Within this formalism, analytical solutions can be found if **Harmonic Oscillator Approximation (HOA)** is made, where potential describing soft interactions of hard partons with the medium $\propto \hat{q}_0 x_{\perp}^2$

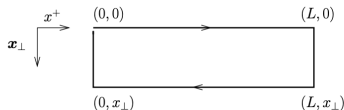
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HOW DO WE CALCULATE IT?

- Assume an infinitely long medium and send $E \rightarrow \infty$ so that the parton's behaviour eikonalizes
- \hat{q} can be related to the **transverse scattering rate, $\mathcal{C}(k_{\perp})$**

$$\hat{q}(\mu) = \int^{\mu} \frac{d^2 k_{\perp}}{(2\pi)^2} k_{\perp}^2 \mathcal{C}(k_{\perp})$$



[Ghiglieri and Teaney, 2015]

- $\mathcal{C}(k_{\perp})$ can in turn be related to a Wilson loop in the (x^+, x_{\perp}) plane [Casalderrey-Solana and Teaney, 2007, D'Eramo et al., 2011, Benzke et al., 2013]

LO AND NLO CONTRIBUTIONS TO \hat{q}

- Leading order contributions calculated by [Aurenche et al., 2002] and [Arnold and Xiao, 2008] coming from **soft scale** $k_{\perp} \sim gT$ and **hard scale** $k_{\perp} \sim T$ respectively
- $\mathcal{O}(g)$ **classical contributions** from soft scale calculated perturbatively by [Caron-Huot, 2009] and later on the lattice by [Panero et al., 2014, Moore and Schlusser, 2020, Moore et al., 2021]

LOGARITHMICALLY ENHANCED CORRECTIONS

- $\mathcal{O}(g^2)$ correction found to have double logarithmic $\sim \ln^2(L/\tau_{\min})$ and single logarithmic enhancements by [Liou et al., 2013](LMW) and separately by [Blaizot et al., 2014](BDIM)
- These are radiative, quantum corrections, argued to come from the single-scattering regime
- Both of these calculations were done
 - ▶ in a static-scatterer/random-colour field picture \Rightarrow justifies Instantaneous Approximation
 - ▶ using HOA \Rightarrow not well-suited to single-scattering regime

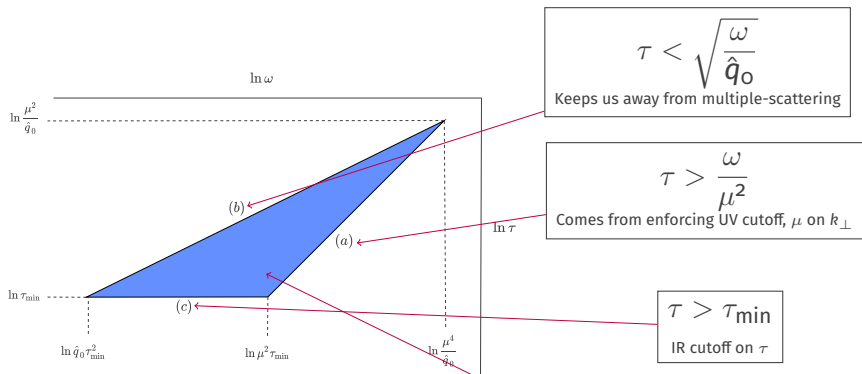
Which is larger: $K\mathcal{O}(g)$ or $\ln^2(\#)\mathcal{O}(g^2)$?

Hard to say... But can definitely make a start by revisiting computation of quantum corrections

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DOUBLE LOGS FROM BDIM/LMW



$$\delta \hat{q}_0 = \frac{\alpha_S C_R}{\pi} \hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0 \tau^2}^{\mu^2 \tau} \frac{d\omega}{\omega} = \frac{1}{2} \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\min}} \propto \text{Area}$$

FROM STATIC SCATTERING TO A WEAKLY COUPLED QGP

Need to adapt BDIM/LMW result to weakly coupled QGP setting

$$\delta\hat{q}_0 = \frac{\alpha_s C_R}{\pi} \hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}$$

Consider $\mu < T$

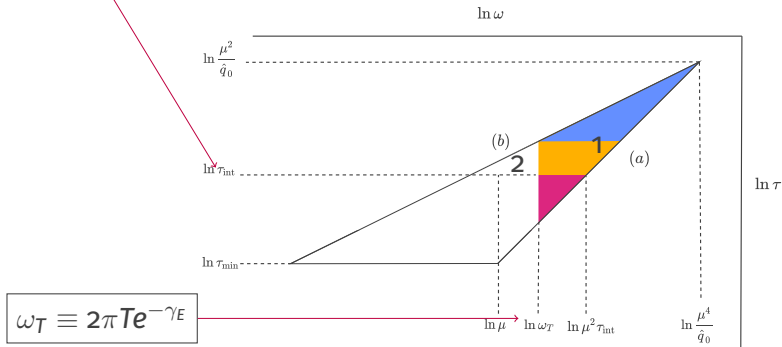
$$\begin{aligned} \delta\hat{q}_{1+2} &= \frac{\alpha_s C_R}{\pi} \int_{\tau_{\text{int}}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega} (1 + 2n_B(\omega)) \\ &= \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \left\{ \ln^2 \frac{\mu^2}{\hat{q}_0\tau_{\text{int}}} - \frac{1}{2} \ln^2 \frac{\omega_T}{\hat{q}_0\tau_{\text{int}}^2} \right\} \end{aligned}$$

$$n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$$

Introduce intermediate regulator τ_{int}

DOUBLE LOGS IN A WEAKLY COUPLED QGP

Intermediate regulator τ_{int}

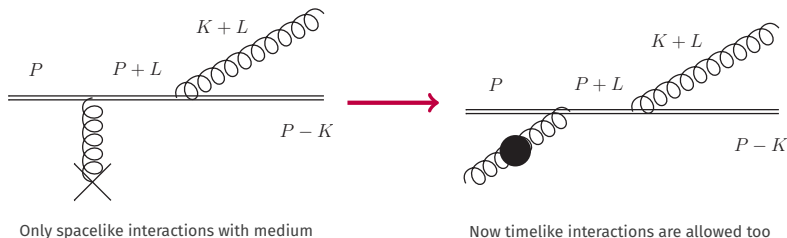


$$\omega_T \equiv 2\pi T e^{-\gamma E}$$

$$\delta \hat{q}_{1+2} = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \left\{ \ln^2 \frac{\mu^2}{\hat{q}_0 \tau_{\text{int}}} - \frac{1}{2} \ln^2 \frac{\omega_T}{\hat{q}_0 \tau_{\text{int}}^2} \right\}$$

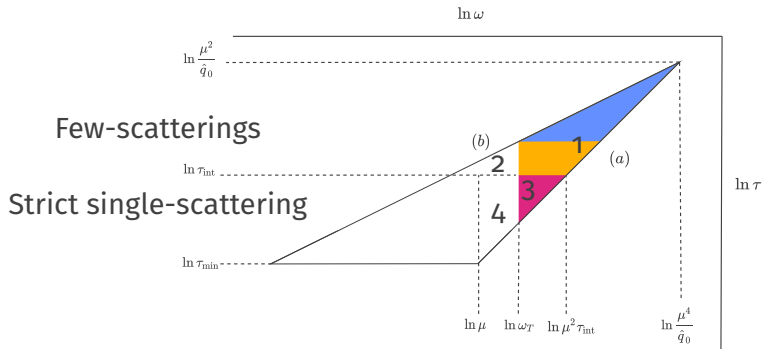
STRICT SINGLE-SCATTERING IN A WEAKLY COUPLED QGP

- Compute $\mathcal{C}(k_{\perp})$ using HTL resummation instead of Random Colour Approximation
- Investigate which logs are produced by *soft, collinear* modes through a *semi-collinear* process [Ghiglieri et al., 2013, Ghiglieri et al., 2016]



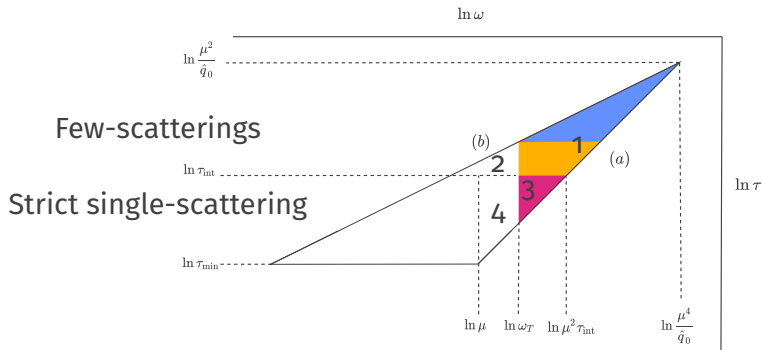
⇒ Going beyond instantaneous approximation

STRICT SINGLE-SCATTERING CONTRIBUTION



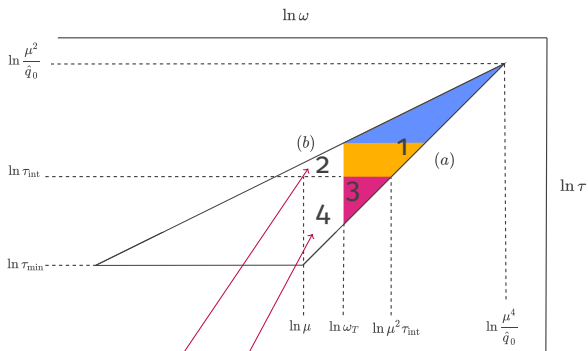
$$\delta \hat{q}_{3+4} = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{\mu^2 \tau_{\text{int}}}{\omega_T} + \text{subleading logs}$$

TOTAL DOUBLE LOG CONTRIBUTION



$$\delta \hat{q}_{DLog} = \delta \hat{q}_{1+2} + \delta \hat{q}_{3+4} = \frac{\alpha_S C_R}{4\pi} \hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0 \omega_T} \propto \text{Area}_{1+3}$$

DOUBLE LOGS IN A WEAKLY COUPLED QGP



Why is it that region 2 and 4 do not contribute to the double Logs?

VACUUM AND QUANTUM CORRECTION CANCELLATION

First, note that

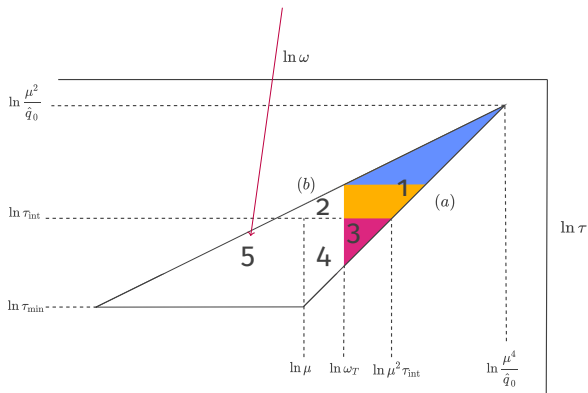
$$\lim_{\frac{\omega}{T} \rightarrow 0} \left(1 + 2n_B(\omega) \right) = 1 + \frac{2T}{\omega} - 1 \quad (5)$$

The absence of the IR scale in any logarithms can then be seen by looking at the following integral, with $\nu_{IR} \ll T \ll \nu_{UV}$

$$\begin{aligned} \int_{\nu_{IR}}^{\nu_{UV}} \frac{d\omega}{\omega} \left(\underbrace{1}_{\text{vacuum}} + \underbrace{2n_B(\omega)}_{\text{thermal}} \right) &= \underbrace{\ln \frac{\nu_{UV}}{\nu_{IR}}}_{\text{vacuum}} + \underbrace{\frac{2T}{\nu_{IR}} - \ln \frac{2\pi T}{\nu_{IR} e^{\gamma_E}}}_{\text{thermal}} + \dots \\ &= \frac{2T}{\nu_{IR}} + \ln \frac{\nu_{UV} e^{\gamma_E}}{2\pi T} + \dots \end{aligned} \quad (6)$$

RELATION TO SOFT CORRECTIONS

Region of phase space from which $\mathcal{O}(g)$ corrections emerge



Classical $\frac{T}{\nu_{\text{IR}}}$ corrections cancel against those extracted from [Caron-Huot, 2009]!

- Take HOA so as to postpone dealing with neighbouring region where single scattering and multiple scattering simultaneously become important
- We find that all parts of our final expression, which come from relaxing the instantaneous approximation are subleading
- If we consider $\mu > T$, we can show that our results become closer to the BDIM/LMW double logs

SUMMARY OF RESULTS/CONCLUSIONS

- Double and single logarithmic corrections computed within the setting of a weakly coupled QGP
- Emended BDIM/LMW result so that it includes thermal corrections
- Can show how our result fits with respect to these emended corrections as well as the soft corrections coming in at $\mathcal{O}(g)$
- Still would like to better understand the phase space boundary between single-scattering and multiple-scattering

OUTLOOK – GOING BEYOND HARMONIC OSCILLATOR APPROXIMATION

HOA not well-suited to single-scattering

⇒ So how can we go beyond it?

$$\delta \hat{q}_{\text{DLog}} = \frac{\alpha_s C_R}{4\pi} \hat{q}_0 \ln^2 \# \longrightarrow \frac{\alpha_s C_R}{4\pi} \hat{q}(\rho) \ln^2 \#$$

where $\hat{q}(\rho) \propto \ln \frac{\rho^2}{m_D^2}$

ρ separates us from neighbouring region
with simultaneously single-scattering and multiple scatterings

Need to solve transverse momentum-dependent LPM equation
without HOA in order to shed light on how this
region could be dealt with

THANKS FOR LISTENING!

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**RADIATIVE ENERGY LOSS OF HIGH-ENERGY QUARKS IN FINITE SIZE NUCLEAR
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RELATING $\mathcal{C}(k_{\perp})$ TO THE WILSON LOOP

Wilson loop defined, in the $x^- = 0$ plane in as

$$\langle W(x_{\perp}, 0) \rangle = \frac{1}{N_c} \text{Tr} \langle [0, x_{\perp}]_- \mathcal{W}^{\dagger}(x_{\perp}) [x_{\perp}, 0]_+ \mathcal{W}(0) \rangle, \quad (7)$$

where

$$\mathcal{W}(x_{\perp}) = \mathcal{P} \exp \left(ig \int_{-\frac{L}{2}}^{\frac{L}{2}} dx^+ A^-(x^+, x_{\perp}) \right) \quad (8)$$

One can show that [D'Eramo et al., 2011, Benzke et al., 2013]

$$\lim_{L \rightarrow \infty} \langle W(x_{\perp}) \rangle = \exp(-\mathcal{C}(x_{\perp})L) \quad (9)$$

where

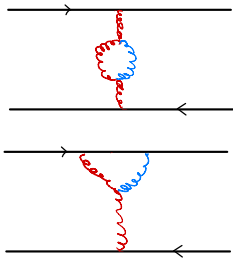
$$\mathcal{C}(x_{\perp}) = \int \frac{d^2 k_{\perp}}{(2\pi)^2} (1 - e^{i\mathbf{x}_{\perp} \cdot \mathbf{k}_{\perp}}) \mathcal{C}(k_{\perp}) \quad (10)$$

PARAMETRIC FORM OF \hat{q}

$$\hat{q} \sim \alpha_s^2 T^3 \left\{ C_1 \ln \frac{T}{m_D} + C_2 \ln \frac{\mu}{T} + C_3 + Kg + \alpha_s (C_5 \ln^2(\#) + C_6 \ln \#' + \dots) \right\} \quad (11)$$

SOME WILSON LOOP DIAGRAMS

- Can think of sticking together amplitude and conjugate amplitude to get diagrams on the right
- **Black lines** represent hard parton in the amplitude and conjugate amplitude
- **Red gluons** are bremsstrahlung, represented by thermal propagators
- **Blue gluons** are those that are exchanged with the medium and are represented by Hard Thermal Loop propagators



WHERE DO THESE DIAGRAMS COME FROM?

