Universality aspects of quantum corrections to transverse momentum broadening in QCD media

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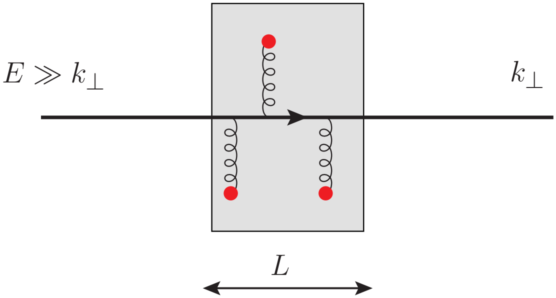
Ref: 2109.12041, 2203.09407

Transverse momentum broadening in QCD

- Physical system: a highly energetic parton propagating through a dense QCD medium.
- We compute the tranverse momentum distribution $\mathcal{P}(k_\perp)$ of the outgoing parton.

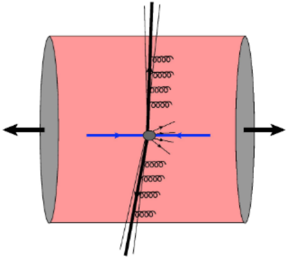


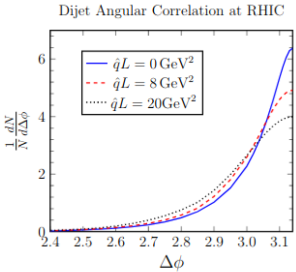
Goal

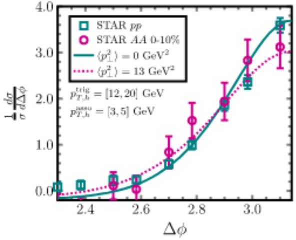
Compute the large L limit of the transverse momentum broadening distribution $\mathcal{P}(\boldsymbol{k}_\perp)$ including leading radiative corrections.

Why is TMB interesting?

- "Hot QCD": Dijet azimuthal angular distributions in heavy-ion collisions: access to the TMB and the medium properties.
- Ex: studies by Mueller, Wu, Xiao, Yuan 1604.04250 & Chen, Qin, Wei, Xiao, Zhang 1607.01932.





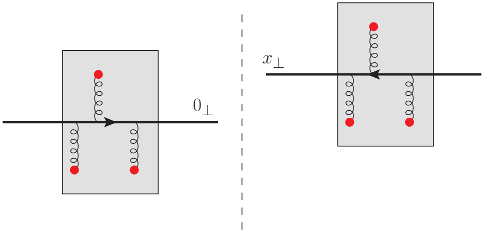


- "Cold QCD": fast probe of gluon distribution in large nuclei $L \propto A^{1/3} \gg 1$ at small-x.

TMB at "tree level"

- Forward scattering amplitude of an effective dipole with size $\boldsymbol{x}_\perp$,

$$\mathcal{S}(\boldsymbol{x}_\perp) = \frac{1}{N_c}\langle \mathrm{Tr}\, V^\dagger(\boldsymbol{x}_\perp) V(\boldsymbol{0}_\perp)\rangle\,, \qquad \text{with} \quad V(\boldsymbol{x}_\perp) = \mathcal{P} e^{ig\int_{-\infty}^{\infty} dx^+ A^-(x^+,\boldsymbol{x}_\perp)}$$



See talk by P. Schicho for NP determination of the collision kernel

- $\langle[\ldots]\rangle$ denotes average of the medium background field.
- Assuming independent multiple interactions,

$$\langle A^{-a}(x^+,\boldsymbol{k}_\perp) A^{-b*}(y^+,\boldsymbol{k}'_\perp)\rangle \propto g^2 n(x^+)\delta^{ab}\delta(x^+-y^+)\delta(\boldsymbol{k}_\perp-\boldsymbol{k}'_\perp)\times \overbrace{1/\boldsymbol{k}_\perp^4}^{\text{collision kernel}}$$

TMB at "tree level" and saturation scale $Q_s(L)$

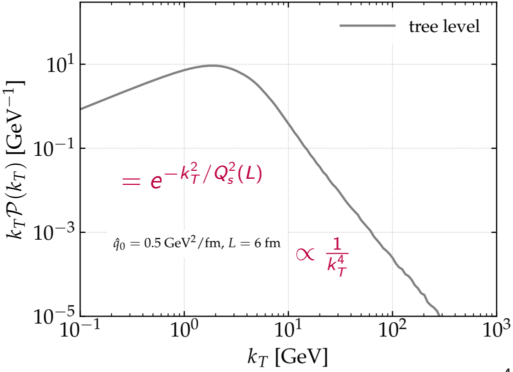
$\Rightarrow$ LO $\hat{q}$ given by $\hat{q}^{(0)}(1/\boldsymbol{x}_\perp^2) = \hat{q}_0 \ln \frac{1}{\boldsymbol{x}_\perp^2 \mu^2}$, $\mu \sim m_D$

$\Rightarrow$ Fourier transform of the dipole S-matrix

$$\mathcal{P}^{(0)}(\boldsymbol{k}_\perp) = \int d^2\boldsymbol{x}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp} e^{-\frac{1}{4}\hat{q}(1/\boldsymbol{x}_\perp^2)L\boldsymbol{x}_\perp^2}$$

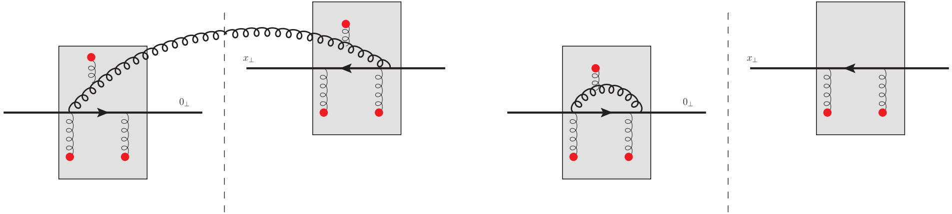
- Q_s emergent momentum scale
- Transition between the unitarity bound $\mathcal{S} \sim 1$ and the dilute regime $\mathcal{S} \ll 1$.
- At tree-level,

$$Q_s^2(L) \simeq \hat{q}_0 L$$



TMB at one loop in a dense QCD medium

- Computation at one-loop in $\alpha_s(p_T) \ll 1$, but to all-orders in $\alpha_s n$.



- Typical order of magnitude of the NLO correction to $\hat{q}$: Liou, Mueller, Wu, 1304.7677

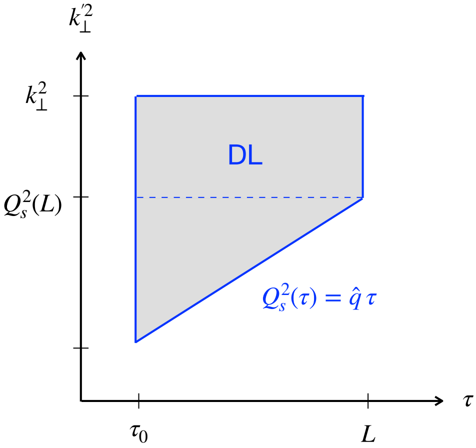
$$\hat{q}^{(1)}(L, 1/\boldsymbol{x}_\perp^2) \sim \frac{\alpha_s N_c}{\pi} \int_{\tau_0}^{L} \frac{d\tau'}{\tau'} \int_{Q_s^2(\tau)}^{1/\boldsymbol{x}_\perp^2} \frac{d\boldsymbol{k}_\perp'^2}{\boldsymbol{k}_\perp'^2} \times \hat{q}_0$$

See also talk by E. Weitz

- Double log enhancement: $Q_s^2(L) = \hat{q}_0 L \left(1 + \frac{\bar{\alpha}_s}{2} \ln^2(L/\tau_0) + \ldots\right)$ at NLO.

Double logarithmic phase space

- Phase space in terms of $\boldsymbol{k}_\perp$ and τ of the gluon.
- Unlike DGLAP or BFKL double log, non-linear saturation bound: $Q_s^2(\tau) \simeq \hat{q}_0 \tau$.
- Constrains the emission to be triggered by a single scattering.



See also talk by E. Weitz

Liou, Mueller, Wu, 1304.7677, Blaizot, Mehtar-Tani, 1403.2323, Iancu 1403.1996, Blaizot, Dominguez 1901.01448

Resummation of the leading radiative corrections

- Resummation to all orders via the evolution equation

$$\frac{\partial \hat{q}(\tau, \boldsymbol{k}_\perp^2)}{\partial \tau} = \int_{Q_s^2(\tau)}^{\boldsymbol{k}_\perp^2} \frac{d\boldsymbol{k}_\perp'^2}{\boldsymbol{k}_\perp'^2} \bar{\alpha}_s(\boldsymbol{k}_\perp'^2)\ \hat{q}(\tau, \boldsymbol{k}_\perp'^2)$$

with $Q_s^2(\tau) \equiv \hat{q}(\tau, Q_s^2(\tau))\tau$.

- Exponentiation of the double logarithmic corrections.

$$\mathcal{P}(\boldsymbol{k}_\perp) = \int \mathrm{d}^2\boldsymbol{x}_\perp\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\ \exp\left[-\frac{1}{4}\left(\hat{q}^{(0)} + \alpha_s\hat{q}^{(1)} + \ldots\right) L\,\boldsymbol{x}_\perp^2\right]$$

cf Liou, Mueller, Wu, 1304.7677, Blaizot, Mehtar-Tani, 1403.2323, Iancu 1403.1996

Asymptotic limit of TMB at fixed coupling

- Large system size limit of $\hat{q}(L, \boldsymbol{k}_\perp^2)$

$$\frac{\hat{q}(L, \boldsymbol{k}_\perp^2)L}{Q_s^2(L)} \underset{L\to\infty}{=} \begin{cases} e^{2\beta \ln\left(\frac{\boldsymbol{k}_\perp^2}{Q_s^2(L)}\right)} & \text{if } \boldsymbol{k}_\perp^2 \leq Q_s^2(L) \\ e^{\beta \ln\left(\frac{\boldsymbol{k}_\perp^2}{Q_s^2(L)}\right)} \left[1 + \beta \ln\left(\frac{\boldsymbol{k}_\perp^2}{Q_s^2(L)}\right)\right] & \text{else} \end{cases}$$

with

$$Q_s^2(L) = \hat{q}_0 L \left(\frac{L}{\tau_0}\right)^{c-1}$$

Here $\beta = (c-1)/(2c)$ and $c = 1 + 2\sqrt{\bar{\alpha}_s + \bar{\alpha}_s^2} + 2\bar{\alpha}_s$.

$\Longrightarrow$ **extended geometric scaling** $\boldsymbol{k}_\perp^2 \ll Q_s^4/\mu^2$. PC, Mehtar-Tani 2109.12041

- Similar to geometric scaling for gluon distribution at small x: $\ln(1/x) \leftrightarrow \ln(L/\tau_0)$.

Lévy flights

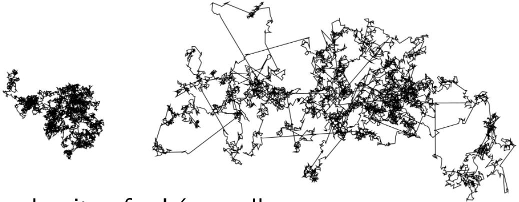
- At large time $L \gg \tau_0$, near the peak

$$\mathcal{S}(\boldsymbol{x}_\perp, L) \approx \exp\left(-\frac{1}{4}(|\boldsymbol{x}_\perp|Q_s(L))^{2-4\beta}\right) , \qquad \beta \simeq \sqrt{\bar{\alpha}_s}$$

- $\Longrightarrow$ the TMB distribution satisfies a fractional Fokker-Planck equation

$$\frac{\partial \mathcal{P}(L, \boldsymbol{k}_\perp)}{\partial L} = \nu \frac{\partial^\gamma \mathcal{P}(L, \boldsymbol{k}_\perp)}{\partial |\boldsymbol{k}_\perp|^\gamma} , \qquad \gamma = 2 - 4\beta$$

Brownian motion



Levy flight

- Equation for the prob. density of a Lévy walker, e.g.
  - $\dot{v} = -\mu v + \eta^\gamma(t)$
  - $\eta^\gamma(t)$ Lévy stable noise ($\gamma = 2$ is the standard white Gaussian noise).

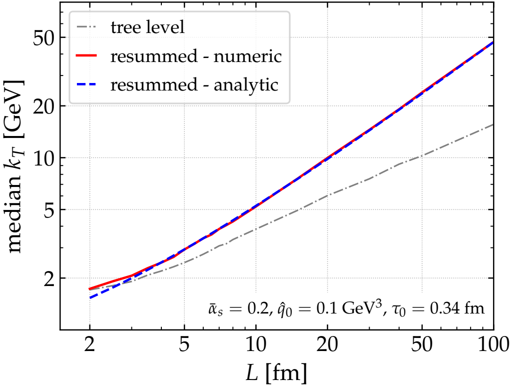
Superdiffusion in momentum space

- $Q_s^2(L) = \hat{q}_0 L \left(\frac{L}{\tau_0}\right)^{c-1}$

- The median of the distribution scales like

$$\mathcal{M} \sim L^{1/2+\sqrt{\bar{\alpha}_s}}$$

- $\Rightarrow$ super-diffusive behaviour. NLO corrections yields super-diffusion in momentum space.

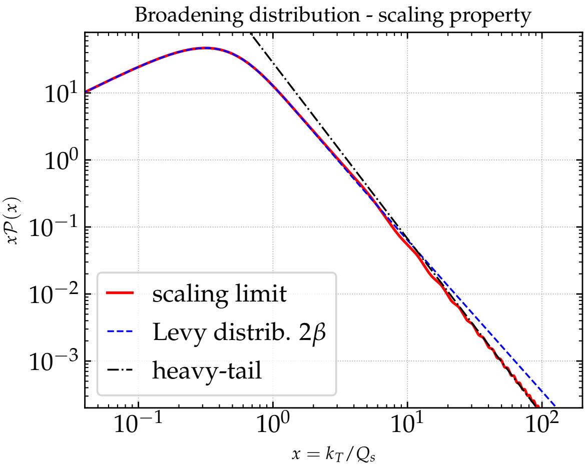


Heavy-tailed distribution

- $\hat{q} \simeq e^{\beta \ln(\boldsymbol{k}_\perp^2/Q_s^2)}$ at large k_T.
- Fourier transform of the "stretched" exponential $\exp(-[...]\boldsymbol{x}_\perp^\gamma)$ with $\gamma \simeq 2 + 2\sqrt{\bar{\alpha}_s} > 2$
- Heavy tailed distribution

$$\mathcal{P}(\boldsymbol{k}_\perp) \propto \frac{1}{k_T^{4-2\sqrt{\bar{\alpha}_s}}}$$

PC, Mehtar-Tani 2109.12041



Beyond the asymptotic limit

- We have determined the limit $L \to \infty$ of the TMB distribution.
- What about the sub-asymptotic corrections?
- Are they universal = independent of the initial conditions?
- Can they be used down to realistic values of L?

Wave front propagation into unstable state

- We borrow techniques from front propagation into unstable sate.

  Ebert, van Saarloos, 0003181, Brunet, Derrida, 0005362

- Similar to the traveling wave interpretation of the solutions to BK.

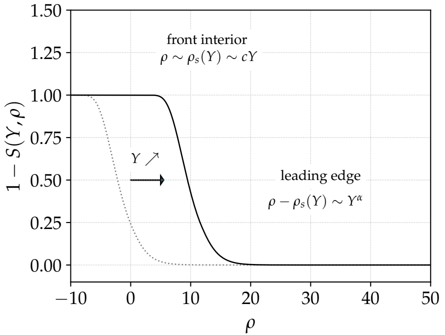
  Munier, Peschanski, 0310357 - Beuf, 1008.0498

- Typical example reaction-diffusion process: Fisher-Kolmogoroff-Petrovsky-Piscounoff eq.

$$\partial_t \phi = \partial_x^2 \phi + \phi - \phi^k$$

- Universality of the wave-front velocity $\dot{\rho}_s$:

$$\dot{\rho}_s = c + \text{corrections}$$



"At late time, the universal properties of the front are determined by the linearized dynamics of arbitrarily small perturbations about the unstable state."

Van Saarloos, "Front propagation into unstable states," Physics Reports 386 2-6

Leading edge expansion

- Diffusive deviation from the asymptotic limit, with we consider.

$$\hat{q}(Y,\rho) = \hat{q}_0 e^{\rho_s(Y)-Y} e^{\beta x}\left[\, Y^{\alpha} G\left(\frac{x}{Y^{\alpha}}\right) + \ldots\right]$$
$$\dot{\rho}_s(Y) = c + \delta\dot{\rho}_s(Y)$$

- Diffusion power characteristics of the universality class of the evolution equation.
- Homogeneity conditions fix the power α.
- $\alpha = 1/2$ for fixed coupling, $\alpha = 1/6$ for running coupling

Results for fixed coupling

- For fixed coupling, we find the pre-asymptotic behaviour

$$\frac{\hat{q}(L,\boldsymbol{k}_\perp^2)L}{Q_s^2(L)} = \begin{cases} \exp\left(\beta x - \frac{\beta x^2}{4cY}\right)\left[1+\beta x - \frac{3x}{c(1+c)Y}\left(1+\frac{\beta(c+4)x}{6}\right)+\mathcal{O}\left(\frac{1}{Y^2}\right)\right] & \text{if } x \geq 0 \\ \exp\left(2\beta x - \frac{3}{c(1+c)}\frac{x}{Y}+\mathcal{O}\left(\frac{1}{Y^2}\right)\right) & \text{if } x < 0\,. \end{cases} \tag{1}$$

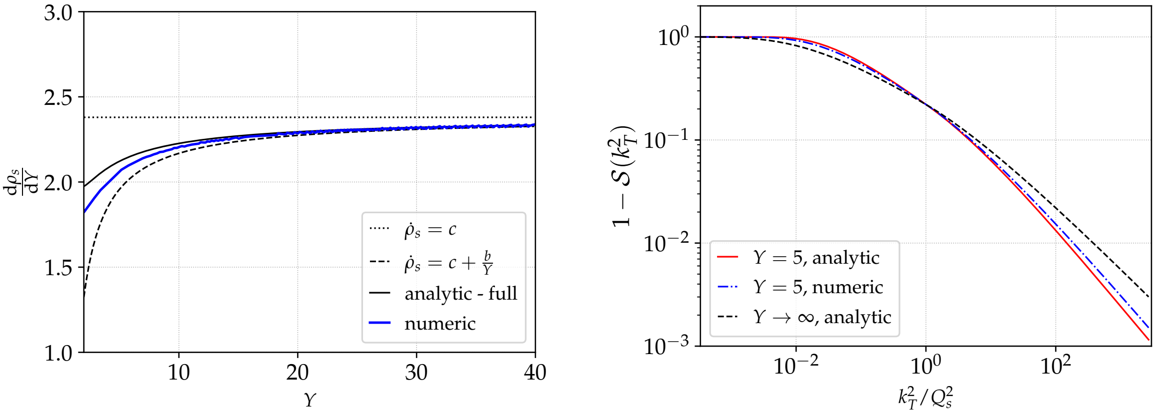
with

$$\rho_s(Y) = cY - \frac{3c}{(1+c)}\ln(Y) - \frac{6c\sqrt{2\pi(c-1)}}{(1+c)^2}\frac{1}{\sqrt{Y}} + \mathcal{O}(Y^{-1})$$

PC, Mehtar-Tani 2109.12041

- $x = \ln(\boldsymbol{k}_\perp^2/Q_s^2(L))$, $Y = \ln(L/\tau_0)$.

Some plots



- Sub-asymptotic corrections enable one to have a good agreement with the numeric.
- Analytic results can be systematically improved.

Running coupling and single logarithmic corrections in the $L \to \infty$ limit

- The single log corrections have been computed first by Liou, Mueller, Wu

Liou, Mueller, Wu 1304.7677, Arnold 2111.05348, Arnold, Gorda, Iqbal 2112.05161

$$\hat{q}(L, Q_s^2) = \hat{q}_0 \left[1 + \frac{\bar{\alpha}_s}{2} \ln^2\left(\frac{L}{\tau_0}\right) + \bar{\alpha}_s \left(2\ln(2) - \gamma_E - \frac{1}{3}\right) \ln\left(\frac{L}{\tau_0}\right) + \mathcal{O}(\alpha_s)\right]$$

- The asymptotic limit is not sensitive to the details of the non-linearities.
- To single log accuracy, contributions from NLL BFKL kernel $\sim B_g/\gamma^2$:

$$\frac{\partial \hat{q}(Y,\rho)}{\partial Y} = \int_{\rho_s}^{\rho} d\rho' \alpha_s(\rho')\hat{q}(Y,\rho') + \bar{\alpha}_s^2 B_g \int_{\rho_s}^{\rho} d\rho' \int_{\rho_s}^{\rho'} d\rho'' \hat{q}(Y,\rho'')$$

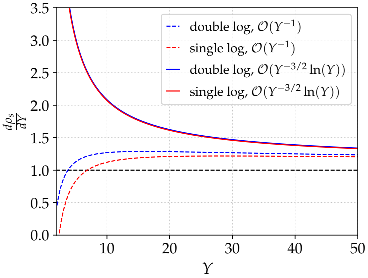
- $B_g = -11/12 - n_f/(6N_c^3)$ finite part of the DGLAP gluon splitting function (from BKFL-DGLAP duality)

Running coupling and single logarithmic corrections in the $L \to \infty$ limit

- Final result, exact to all orders in pQCD (but for $Y = \ln(L/\tau_0) \to \infty$...)
- All universal terms in the asymptotic expansion of $Q_s^2(L)$

$$\frac{d\ln(Q_s^2(L))}{d\ln L} = 1 + \frac{4b_0}{(4b_0Y)^{1/2}} + \frac{2\xi_1 b_0}{(4b_0Y)^{5/6}} + (1 - 8b_0 + 4b_0B_g)\frac{1}{4Y} - \frac{7\xi_1^2 b_0}{270}\frac{1}{(4b_0Y)^{7/6}}$$
$$-(5 + 1944b_0)\frac{\xi_1 b_0}{81}\frac{1}{(4b_0Y)^{4/3}} - b_0^2(2 - 16b_0 + 8b_0B_g)\frac{\ln(Y)}{(4b_0Y)^{3/2}} + \mathcal{O}\left(\frac{1}{Y^{3/2}}\right)$$

- In agreement with Iancu, Triantafyllopoulos 1405.3525 for the linearized equation.



PC, Mehtar-Tani, 2203.09407

Summary

- Study of the effect of radiative corrections on transverse momentum broadening in a dense QCD medium for large system sizes.
- TMB satisfies extended geometric scaling.
- Radiative corrections yield super-diffusive behaviour in momentum space, and a heavy tail with power index smaller than the typical Rutherford behaviour.
- The DLA non-linear evolution equations share similar mathematical properties as equations for wave front propagation into unstable states.
- Enable to compute the universal behaviour of the TMB distribution, valid down to realistic values of the system size.