Dynamics of the $O(4)$ critical point in QCD

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with:

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SEWM22, Saclay, 21^{st} of June 2022

Critical dynamics and QCD

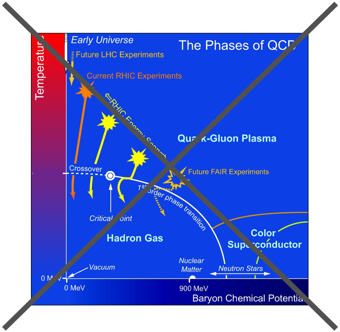
Numerical results

Disclaimer

Today

NOT about

(μ, T) Ising critical point



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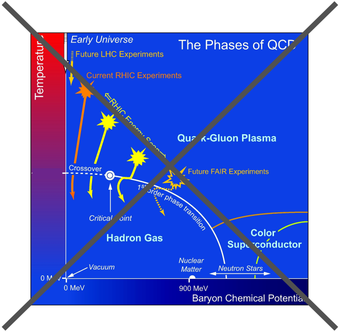

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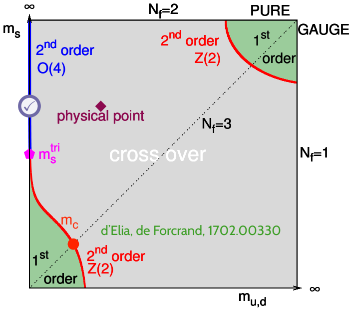
$m_u = 0, m_d = 0, m_s \neq 0$

deconfinement phase transition



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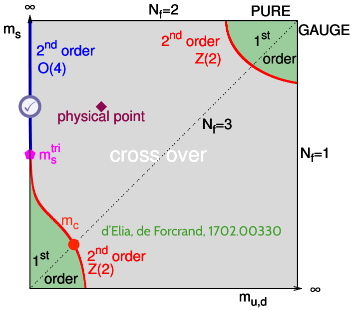

$N_f = 2$ chiral limit

• [Pisarski, Wilczek, 1984] $O(4)$ universality.

Spoiled by effective $U(1)_A$ restoration?

• [HotQCD, 2019, 2020]

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Lattice QCD

Strong evidence of 2^{nd} ord. phase trans.

(see [Philipsen, 2021] for a review)

• **Compatible with $O(4)$.**

($U(2) \times U(2)$ not excluded, but $U(1)_A$ not so restored at T_c [Kaczmarek, Mazur, Sharma, 2021])

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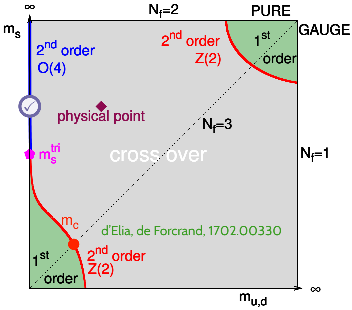

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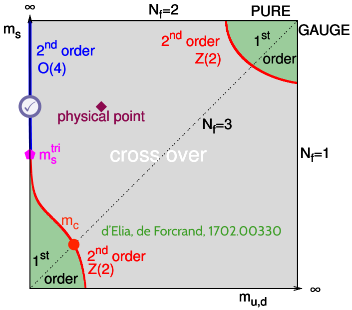

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Phys. m_u, m_d in critical region?

↓

Lat. QCD suggests yes

Potential **predictive power**

Phys. m_u, m_d in critical region?

$\downarrow$

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Potential **predictive power**

Static universality

Landau-Ginzburg:

$\phi = (\phi_0, \phi_1, \phi_2, \phi_3)$

Double-well $\downarrow$

$$F_\phi = \int \mathrm{d}x^3 \, \tfrac{1}{2}\partial_\mu \phi^a \partial_\mu \phi^a + \frac{m_0^2}{2}|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

$$+H\phi_0$$

$\uparrow$ Explicit break.

$H \sim m_q$

Order parameter:	$\langle\phi\rangle \sim q\bar{q}$
Broken phase, $m_0 < m_c$:	$\langle\phi\rangle \neq 0$
Restored, $m_0 > m_c$	$\langle\phi\rangle = 0$

What about dynamics?

Dynamical universality

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IR dynamics

$\updownarrow$

Hydrodynamics

$\updownarrow$

EFT of slowly relaxing (conserved) variables

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One IR fixed point

↓ ↓ ↓ ↓ ↓

Different hydros

Need to specify the dynamics

Dynamical universality

IR dynamics

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Different hydros

Need to specify the dynamics

1. Statics

$$F_\phi = \int \mathrm{d}x^3 \, \tfrac{1}{2}\partial_\mu \phi^a \partial_\mu \phi^a + \frac{m_0^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + H\phi_0$$

2. Identify slow and critical d.o.f.

Cons. charges: $n_V^a \sim \bar{q}\gamma^0 T^a q$, $n_A^a \sim \bar{q}\gamma^0\gamma^5 T^a q$ $\rightarrowtail$ $n_{ab} \in \mathfrak{o}(4)$

$$F_j = \int \mathrm{d}x^3 \, \frac{1}{2\chi_0} n_{ab} n^{ab}$$

3. Is the order parameter conserved?

$\langle \phi \rangle \sim \bar{q}q \rightarrow$ No

4. Derive the E.o.M.

[Rajagopal, Wilczek, 1992]

[Grossi, Soloviev, Teaney, Yan, 2021]

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Poisson brackets

$\{field, F\}$, "closed dynamics"

∂_t field = ideal + dissipation/diffusion

Not conserved $\longrightarrow -\frac{\delta F}{\delta field}$

Conserved $\longrightarrow -\nabla^2 \frac{\delta F}{\delta field}$

$+$ noise

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$$\partial_t n_{ab} = -\nabla\cdot(\nabla\phi_{[a}\phi_{b]}) - H_{[a}\phi_{b]}$$
$$+D_0\nabla^2 n_{ab} + \partial_i\Xi^i_{ab}$$

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$$\langle\theta_a(x)\theta_b(x')\rangle = 2T_c\Gamma_0\,\delta_{ab}\,\delta^4(x-x')$$
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"Model G" of [Hohenberg, Halperin, 1977]

Solve this numerically

Compute $\langle O\rangle$: average over realizations

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0. Find phase transition (statics).

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1. Explore phase diagram

2. Broken phase and soft pions EFT

3. Critical behavior

See also [Schlichting et al.]

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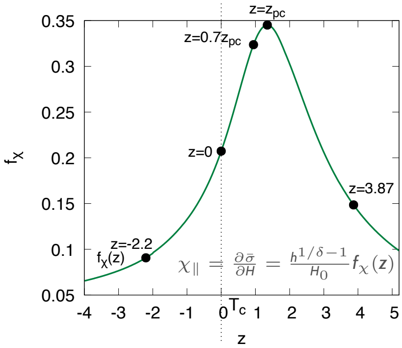
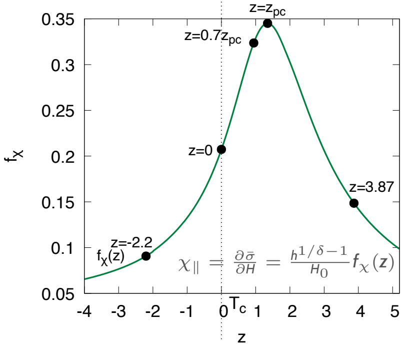


$\lambda, \Gamma, D, \chi_0$ fixed, $N_{lat} = 80$

$O(10^5)$ cpu hours

Observables



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$$\phi = (\sigma, \pi^a), \quad n^A_a = n_{0a}, \quad n^V_a = \tfrac{1}{2}\epsilon_{0abc} n_{bc}$$

$$G_{\sigma\sigma}(t,k) \equiv \tfrac{1}{V}\langle \sigma(t,\mathbf{k})\sigma(0,-\mathbf{k})\rangle_c$$

$$G_{\pi\pi}(t,k) \equiv \tfrac{1}{3V}\textstyle\sum_s \langle \pi_s(t,\mathbf{k})\pi_s(0,-\mathbf{k})\rangle_c$$

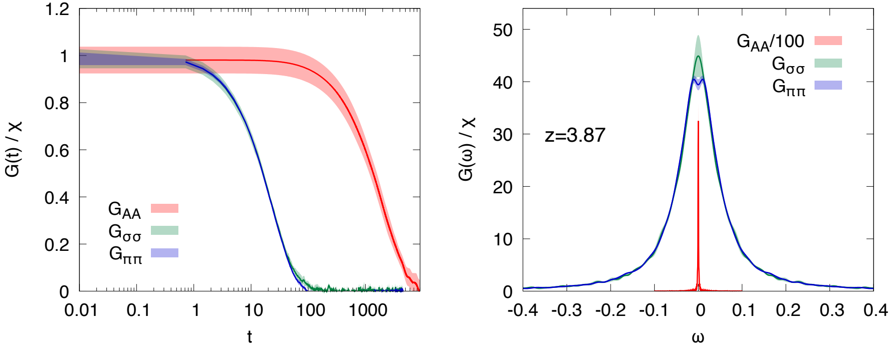
$$G_{AA}(t,k) \equiv \tfrac{1}{3V}\textstyle\sum_s \langle n^s_A(t,\mathbf{k}) n^s_A(0,-\mathbf{k})\rangle_c$$

$$G(t) \equiv G(t, k=0)$$

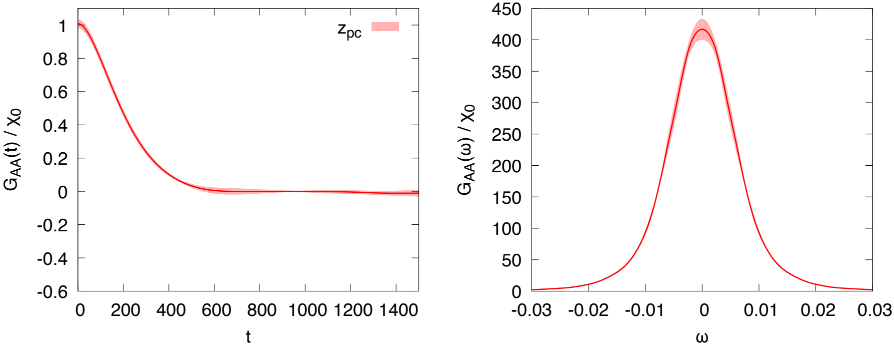
$$G(t) = \tfrac{1}{2\pi}\int \mathrm{d}\omega G(\omega) e^{i\omega t}$$

$$\rho(\omega) = \omega G(\omega)$$

Restored phase



Across the transition



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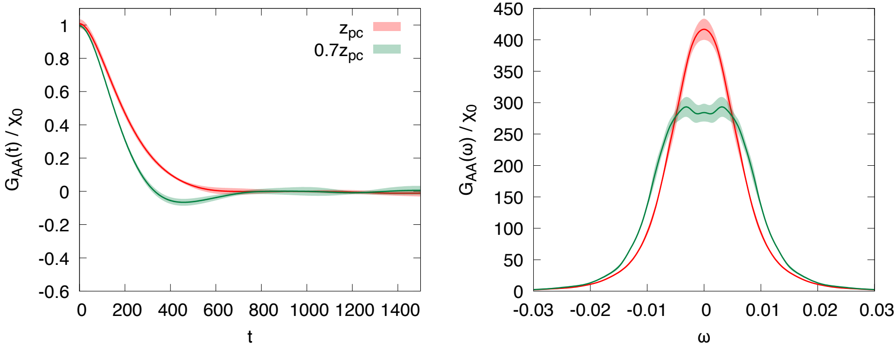

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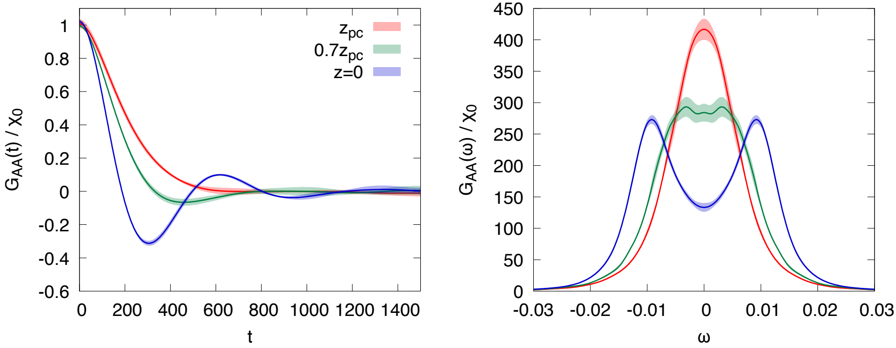

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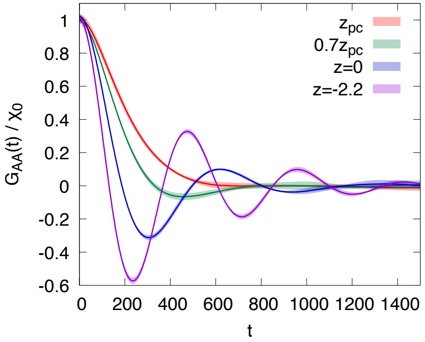

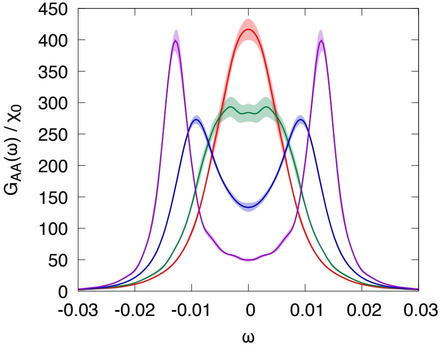

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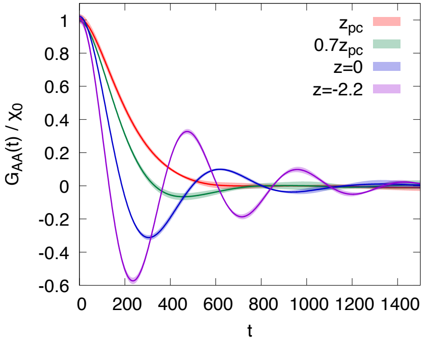
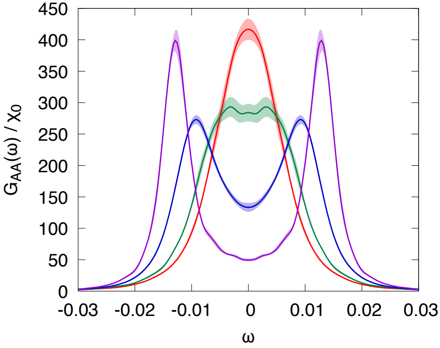




Emergence of pion waves!

And criticality in all of that?

Critical behavior

Scaling hypothesis:

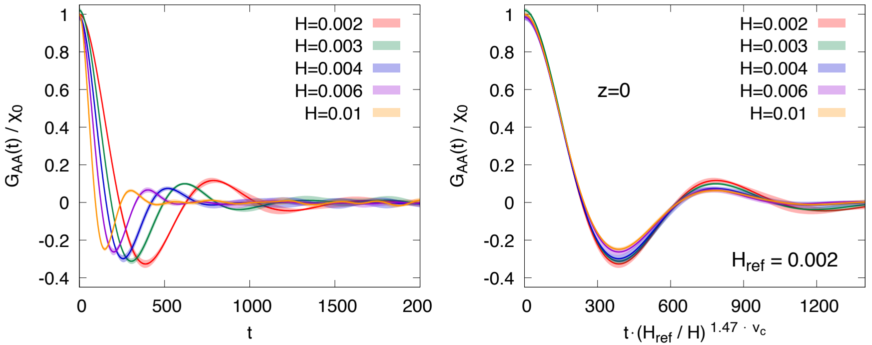
$$\frac{G_{\sigma\sigma}(t,H)}{\chi_{\parallel}} = Y_{\sigma}\left(\xi^{-\zeta}t\right) = \tilde{Y}_{\sigma}\left(H^{\zeta\nu_c}t\right)$$

$$\frac{G_{\pi\pi}(t,H)}{\chi_{\perp}} = Y_{\pi}\left(\xi^{-\zeta}t\right) = \tilde{Y}_{\pi}\left(H^{\zeta\nu_c}t\right)$$

$$\frac{G_{AA}(t,H)}{\chi_{0}} = Y_{A}\left(\xi^{-\zeta}t\right) = \tilde{Y}_{A}\left(H^{\zeta\nu_c}t\right)$$

ζ : dyn. scaling exponent

Critical line



$$\zeta = 1.47 \pm 0.01(\textit{stat.})$$

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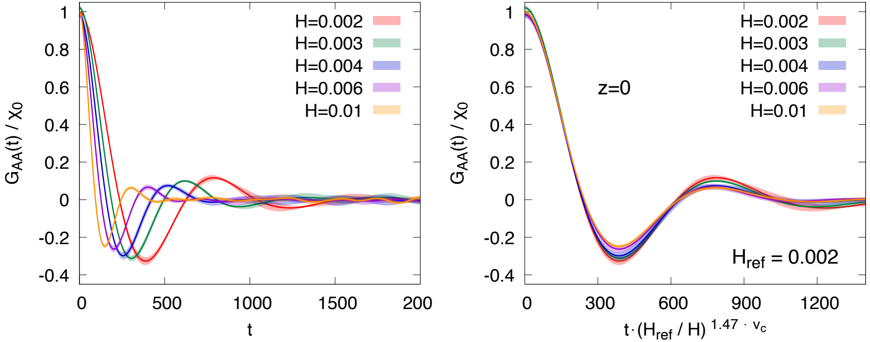

$$\zeta = 1.47 \pm 0.01(\textit{stat.})$$

Compare: $\zeta^{th} = 1.5$

Outlooks

- Finite k

- Expansion rate

  ↓

Kibble-Zurek mechanism

- Pheno. prediction

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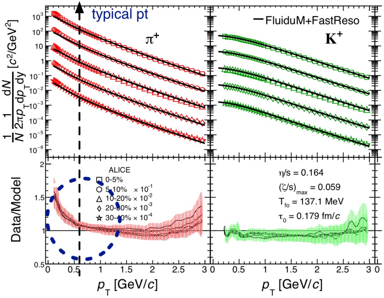
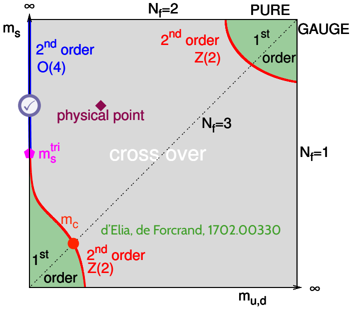
A recent ordinary hydro fit from Devetak et al 1909.10485



- $m_u, m_d = 0$ deconfinement P.T. dynamical universality class
- Studied pion waves
- Observed critical behavior
- Extracted dyn. critical exponent



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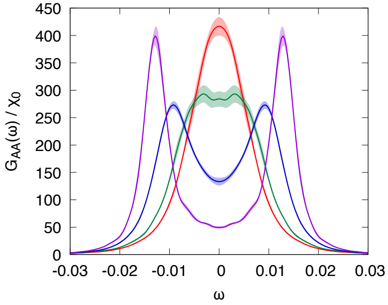

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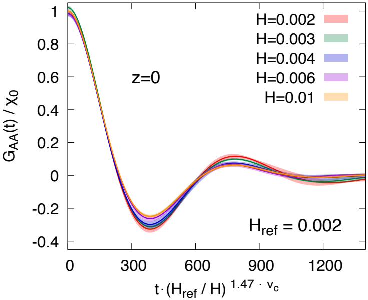

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Thanks!

Backup

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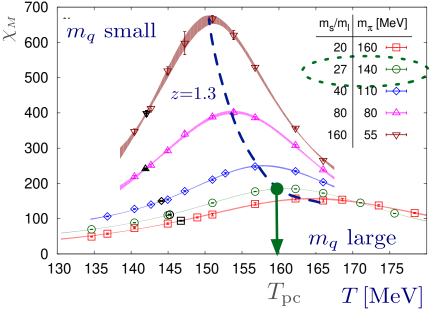
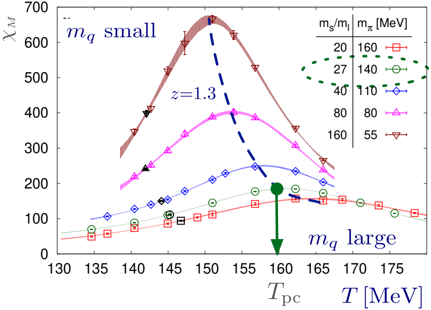
HotQCD, PRL 123 (2019) 6, 062002

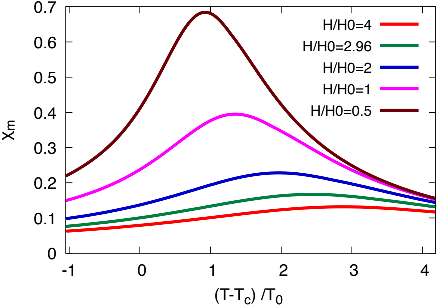


Lattice data

HotQCD, PRL 123 (2019) 6, 062002

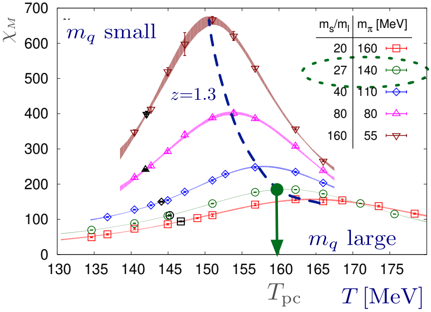


$O(4)$ scaling, from Engels et al., Nucl.Phys.B 655 (2003) 277-299

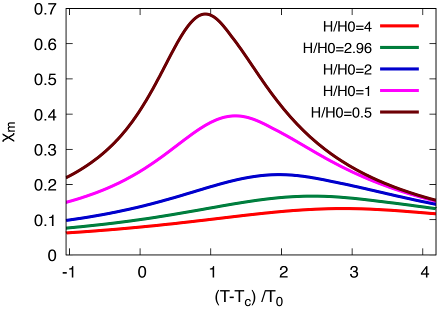


Lattice data

HotQCD, PRL 123 (2019) 6, 062002



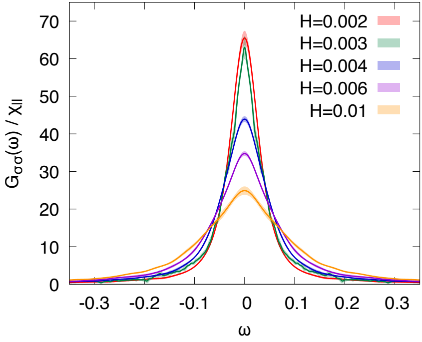
$O(4)$ scaling, from Engels et al., Nucl.Phys.B 655 (2003) 277-299

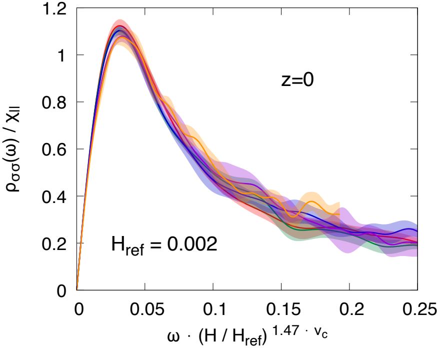


Potential **predictive power**

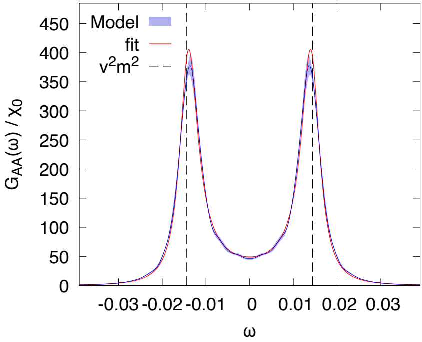
[Rajagopal, Wilczek, 1992]

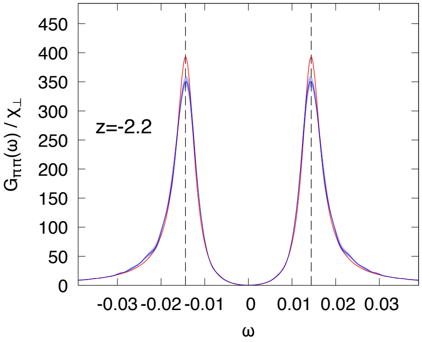
Critical line





Broken phase and pion EFT





Pions EFT in the broken phase [Grossi, Soloviev, Teaney, Yan, 2020]:

$$G_{\pi\pi}(\omega) = \frac{2\chi_\perp \Gamma m^2 \omega^2}{(-\omega^2+m_p^2)^2+\omega^2(\Gamma m^2)^2} \qquad G_{AA}(\omega) = \frac{2\chi_0 \Gamma m^2 m_p^2}{(-\omega^2+m_p^2)^2+\omega^2(\Gamma m^2)^2}$$

Broken phase and pion EFT

Gell-Mann Oakes Renner

$\uparrow$

EFT pred. (”GOR”):

$$\left(m_p^{EFT}\right)^2 = \frac{H\bar{\sigma}}{\chi_0}$$

Num. result:

$$\frac{H\bar{\sigma}}{\chi_0} \cdot \frac{1}{\left(m_p^{fit}\right)^2} = 1.011 \pm 0.001(stat.)$$

Pion EFT works surpisingly well!