Second order chiral phase transition in three flavor QCD?

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GF, Phys. Rev. D**105**, L071506 (2022) [arXiv:2201.07909]

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Introduction

- QCD Lagrangian:

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \bar{q}_i \big(i\gamma^\mu (D_\mu)_{ij} - m\delta_{ij} \big) q_j$$

$\longrightarrow$ $SU(3)$ gauge symmetry
$\longrightarrow$ $U_L(N_f) \times U_R(N_f)$ global (approx.) chiral symmetry
$\longrightarrow$ anomalous breaking of $U_A(1)$ axial symmetry

- At low temperatures: spontaneous breaking
$SU_L(N_f) \times SU_R(N_f) \longrightarrow SU_V(N_f)$

Introduction

- QCD Lagrangian:

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- At low temperatures: spontaneous breaking
$SU_L(N_f) \times SU_R(N_f) \longrightarrow SU_V(N_f)$
- Ginzburg-Landau paradigm for second order (or weakly first order) transitions:

i.) there exists a local order parameter Φ
ii.) the UV free energy can be expanded in terms of Φ
iii.) the free energy has to reflect all symmetries

Ginzburg–Landau analysis of the chiral transition

- GL theory for the chiral transition:
  $\longrightarrow$ gauge degrees of freedom are integrated out
  $\longrightarrow$ the emerging order parameter (Φ) is a $N_f \times N_f$ matrix
  $\longrightarrow$ it reflects chiral symmetry: $\Phi \to L\Phi R^\dagger$
- The most general free energy functional (no anomaly):

$$\Gamma = \int_x \Big[m^2 \,\mathrm{Tr}\,(\Phi^\dagger\Phi) + g_1 \big(\mathrm{Tr}\,(\Phi^\dagger\Phi)\big)^2 + g_2 \,\mathrm{Tr}\,(\Phi^\dagger\Phi\Phi^\dagger\Phi) + \dots + \mathrm{Tr}\,\big(\partial_i\Phi^\dagger\partial_i\Phi\big) + \dots \Big]$$

- Anomaly $\to$ Kobayashi–Maskawa–'t Hooft determinant:
  $\sim \det \Phi^\dagger + \det \Phi$
- Expansion of the full free energy leads to incorrect conclusions
  $\longrightarrow$ at T_C long wavelength fluctuations are important
  $\longrightarrow$ renormalization group is needed

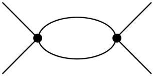
Ginzburg–Landau analysis of the chiral transition

- Pisarski & Wilczek analysis of the Ginzburg–Landau theory [1]:

  $\longrightarrow$ one-loop calculation of the β functions (no anomaly)

  $\longrightarrow$ counterterms for g_1, g_2:

$$\delta g_1, \delta g_2 \sim$$



- Results (ϵ expansion, $\epsilon = 4 - d$):

$$\beta_{g_1} = -\epsilon g_1 + \frac{N_f^2 + 4}{4\pi^2} g_1^2 + \frac{N_f}{\pi^2} g_1 g_2 + \frac{3 g_2^2}{4\pi^2}$$

$$\beta_{g_2} = -\epsilon g_2 + \frac{3}{2\pi^2} g_1 g_2 + \frac{N_f}{2\pi^2} g_2^2$$

- No infrared stable fixed point if $N_f > \sqrt{3}$

  $\Longrightarrow$ 2nd order transition cannot occur!

- Inclusion of the anomaly:

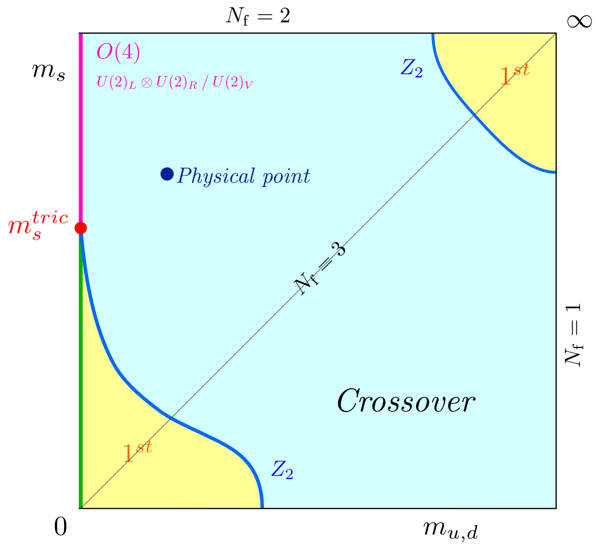
  $\longrightarrow$ $N_f = 2$: second order transition with $O(4)$ exponents

  $\longrightarrow$ $N_f = 3$: first order transition

[1] R. D. Pisarski and F. Wilczek, Phys. Rev. D**29**, 338 (1984)

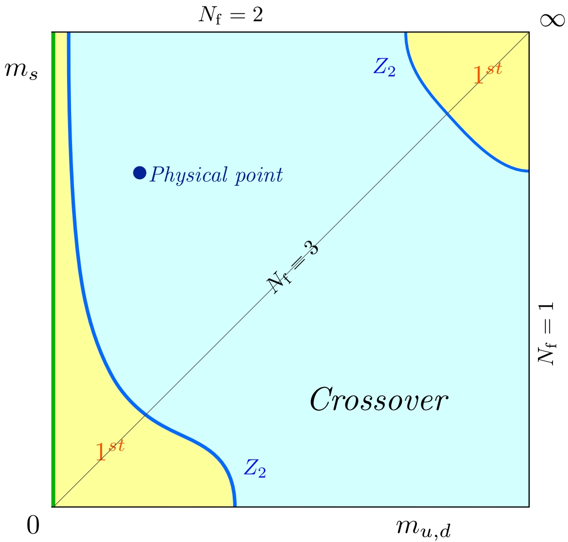
Ginzburg–Landau analysis of the chiral transition

Columbia plot with anomaly: [figure taken from F. Cuteri et. al, JHEP**11**, 141 (2021)]



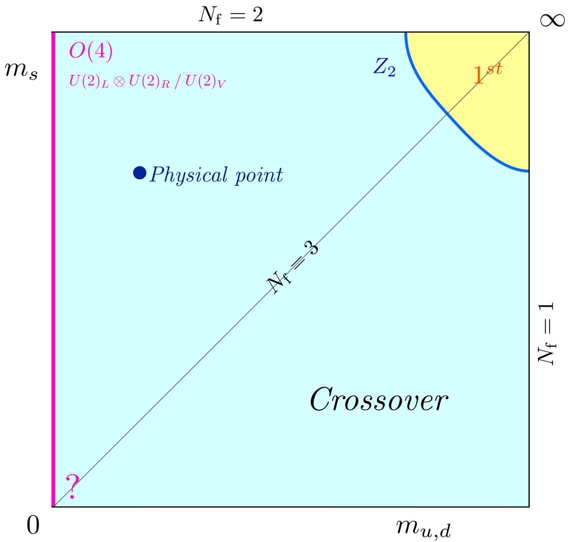
Ginzburg–Landau analysis of the chiral transition

Columbia plot without anomaly: [figure taken from F. Cuteri et. al, JHEP**11**, 141 (2021)]



Ginzburg–Landau analysis of the chiral transition

New, conjectured Columbia plot: [figure taken from F. Cuteri et. al, JHEP **11**, 141 (2021)]



Ginzburg–Landau analysis of the chiral transition

- Potential problems with the Pisarski & Wilczek analysis:
  $\longrightarrow$ it uses the field theoretical RG $\Longrightarrow$ valid only close to the Gaussian fixed point
  $\longrightarrow$ in $d = 3$ there are more (perturbatively) renormalizable operators!
  $\longrightarrow$ ϵ expansion is not reliable

- Example: superconducting phase transition
  $\longrightarrow$ Abelian Higgs model: ϵ expansion predicts a first order transition
  $\longrightarrow$ Monte Carlo simulations showed that the transition can be of second order
  $\longrightarrow$ IR fixed point is inaccessible in the ϵ expansion [2]
- Functional version of the Wilsonian Renormalization Group

[2] GF & T. Hatsuda, Phys. Rev. D**93**, 121701 (2016).
GF & T. Hatsuda, Phys. Rev. D**96**, 056018 (2017).

Functional Renormalization Group

- FRG generalizes the idea of the Wilsonian RG: fluctuations are taken into account at the level of the quantum effective action

$$Z[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi]+\int J\phi)} \quad \Rightarrow \quad \Gamma[\bar{\phi}] = -\log Z[J] - \int J\bar{\phi}$$

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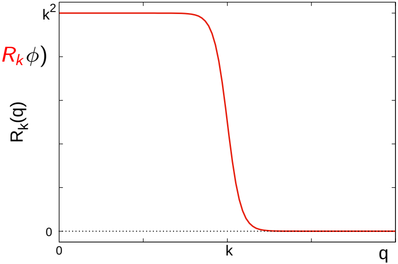
- Introduction of a flow parameter k and inclusion of fluctuations for which $q \gtrsim k$

$$Z_k[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi]+\int J\phi+\frac{1}{2}\int \phi R_k \phi)}$$

$\longrightarrow$ regulator: mom. dep. mass term suppressing low modes

$\longrightarrow$ take the $k \to 0$ limit



Functional Renormalization Group

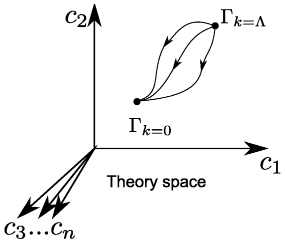
- Scale-dependent effective action:

$$\Gamma_k[\bar\phi] \;=\; -\log Z_k[J] - \int J\bar\phi - \frac{1}{2}\int \bar\phi R_k \bar\phi$$

$\longrightarrow k \approx \Lambda$: no fluctuations $\Rightarrow \Gamma_{k=\Lambda}[\bar\phi] = \mathcal{S}[\bar\phi]$

$\longrightarrow k = 0$: all fluctuations $\Rightarrow \Gamma_{k=0}[\bar\phi] = \Gamma[\bar\phi]$

- The scale-dependent effective action interpolates between classical- and quantum effective actions



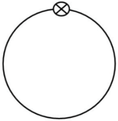
- The trajectory depends on R_k but the endpoint does not

- Choice of $R_k \leftrightarrow$ choice of scheme

Functional Renormalization Group

- Flow of the effective action is described by the Wetterich equation:

$$\partial_k \Gamma_k = \frac{1}{2} \int_q \int_p \mathrm{Tr}\left[\partial_k R_k(q,p)(\Gamma_k^{(2)} + R_k)^{-1}(p,q)\right] = \frac{1}{2}$$



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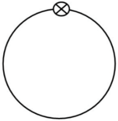

- Local potential approximation (LPA):

$$\Gamma_k[\bar\phi] = \int_x \left(\frac{Z_k}{2}\partial_i\bar\phi\partial_i\bar\phi + V_k(\bar\phi)\right)$$

$\longrightarrow$ leading order of the derivative expansion

$\longrightarrow$ equivalent statement: momentum dependence only in $\Gamma_k^{(2)}$

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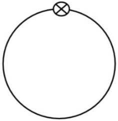

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$\longrightarrow$ equivalent statement: momentum dependence only in $\Gamma_k^{(2)}$

- Optimal flow equation for the effective potential ($Z_k \equiv 1$):

$$\begin{aligned} R_k(q,p) &= (k^2 - q^2)\Theta(k^2 - q^2)\delta(p+q) \\ \partial_k V_k &= \frac{k^4}{6\pi^2}\mathrm{Tr}\,(k^2 + V_k^{(2)})^{-1} \end{aligned}$$

Fixed points and stability

- How to build up the most general Ginzburg–Landau potential for three flavors in $d = 3$ in terms of renormalizable operators?
- Independent invariant tensors are needed:

$$I_1 = \mathrm{Tr}\,(\Phi^\dagger\Phi), \quad I_2 = \mathrm{Tr}\,(\Phi^\dagger\Phi - \mathrm{Tr}\,(\Phi^\dagger\Phi)/3)^2$$
$$I_3 = \mathrm{Tr}\,(\Phi^\dagger\Phi - \mathrm{Tr}\,(\Phi^\dagger\Phi)/3)^3$$

$\longrightarrow$ I_4, I_5, ... are not independent

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$\longrightarrow I_4, I_5, \ldots$ are not independent

- $U_A(1)$ breaking terms:

$$I_{\mathrm{det}} = \det\Phi^\dagger + \det\Phi, \quad \tilde{I}_{\mathrm{det}} = \det\Phi^\dagger - \det\Phi$$

$\longrightarrow \tilde{I}^2_{\mathrm{det}}$ could work but it is not independent

$\longrightarrow \tilde{I}^2_{\mathrm{det}} = I^2_{\mathrm{det}} + 4I_1^3/27 - 2I_1 I_2/3 + 4I_3/3$

$\longrightarrow \det\Phi^\dagger \cdot \det\Phi = (I^2_{\mathrm{det}} - \tilde{I}^2_{\mathrm{det}})/4$

Fixed points and stability

- The most general Ginburg–Landau potential (9 couplings!):

$$
\begin{aligned}
V_k[\Phi] = {} & m_k^2 I_1 + a_k I_{\text{det}} + g_{1,k} I_1^2 + g_{2,k} I_2 \\
& + b_k I_1 I_{\text{det}} + \lambda_{1,k} I_1^3 + \lambda_{2,k} I_1 I_2 + a_{2,k} I_{\text{det}}^2 + g_{3,k} I_3 + \mathcal{O}(\phi^7)
\end{aligned}
$$

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$$V_k[\Phi] = m_k^2 I_1 + a_k I_{\rm det} + g_{1,k} I_1^2 + g_{2,k} I_2 + b_k I_1 I_{\rm det} + \lambda_{1,k} I_1^3 + \lambda_{2,k} I_1 I_2 + a_{2,k} I_{\rm det}^2 + g_{3,k} I_3 + \mathcal{O}(\phi^7)$$

- Optimized flow: $\partial_k V_k = \frac{k^4}{6\pi^2}\,\mathrm{Tr}\,(k^2 + V_k^{(2)})^{-1}$
- Left hand side:

$$\partial_k V_k = \partial_k m_k^2 I_1 + \partial_k a_k I_{\rm det} + \partial_k g_{1,k} I_1^2 + \partial_k g_{2,k} I_2 + \partial_k b_k I_1 I_{\rm det} + \partial_k \lambda_{1,k} I_1^3 + \partial_k \lambda_{2,k} I_1 I_2 + \partial_k a_{2,k} I_{\rm det}^2 + \partial_k g_{3,k} I_3$$

- Right hand side? Need to compatible with the lhs via $V_k^{(2)}$
  $\longrightarrow \Phi = \sum_{a=0}^{8} \phi^a T^a \equiv \sum_{a=0}^{8} (s^a + i\pi^a) T^a$
  $\longrightarrow V_k^{(2)}$ depends on the fields, not invariants!
  $\longrightarrow [k^2 + V_k^{(2)}]$: 18×18 matrix, in practice cannot be inverted

Fixed points and stability

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$$V_k[\Phi] = m_k^2 I_1 + a_k I_{det} + g_{1,k} I_1^2 + g_{2,k} I_2 + b_k I_1 I_{det} + \lambda_{1,k} I_1^3 + \lambda_{2,k} I_1 I_2 + a_{2,k} I_{det}^2 + g_{3,k} I_3 + \mathcal{O}(\phi^7)$$

- Trick: we need flows of couplings, Φ is not important!

$\longrightarrow$ free to choose Φ at each level of the expansion

$\longrightarrow$ requirement: $[k^2 + V_k^{(2)}]$ is easily invertable

$\longrightarrow$ e.g. $\Phi = s_0 T^0 \Rightarrow I_1 = s_0^2/2,\ I_{det} = s_0^3/3\sqrt{6},\ \ldots$

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- Problem: invariants need to be disentangled from each order

  $\longrightarrow$ $\mathcal{O}(\phi^2)$: 1 invariant I_1

  $\longrightarrow$ $\mathcal{O}(\phi^3)$: 1 invariant I_{det}

  $\longrightarrow$ $\mathcal{O}(\phi^4)$: 2 invariants $I_1^2,\ I_2$

  $\longrightarrow$ $\mathcal{O}(\phi^5)$: 1 invariants $I_1 I_{\text{det}}$

  $\longrightarrow$ $\mathcal{O}(\phi^6)$: 4 invariants $I_1^3,\ I_1 I_2,\ I_{\text{det}}^2,\ I_3$

Fixed points and stability

$$\beta_{m^2} \equiv k\partial_k \bar{m}_k^2 = -2\bar{m}_k^2 - \frac{4}{9\pi^2}\frac{15\bar{g}_{1,k}+4\bar{g}_{2,k}}{(1+\bar{m}_k^2)^2} + \frac{4}{3\pi^2}\frac{\bar{a}_k^2}{(1+\bar{m}_k^2)^3},$$

$$\beta_a \equiv k\partial_k \bar{a}_k = -\frac{3\bar{a}_k}{2} - \frac{4}{\pi^2}\frac{\bar{b}_k}{(1+\bar{m}_k^2)^2} + \frac{4}{3\pi^2}\frac{\bar{a}_k(3\bar{g}_{1,k}-4\bar{g}_{2,k})}{(1+\bar{m}_k^2)^3},$$

$$\beta_{g_1} \equiv k\partial_k \bar{g}_{1,k} = -\bar{g}_{1,k} - \frac{1}{9\pi^2}\frac{2\bar{a}_{2,k}+99\bar{\lambda}_{1,k}+16\bar{\lambda}_{2,k}}{(1+\bar{m}_k^2)^2} + \frac{4}{27\pi^2}\frac{24\bar{a}_k\bar{b}_k+117\bar{g}_{1,k}^2+48\bar{g}_{1,k}\bar{g}_{2,k}+16\bar{g}_{2,k}^2}{(1+\bar{m}_k^2)^3}$$
$$-\frac{16}{9\pi^2}\frac{\bar{a}_k^2(6\bar{g}_{1,k}+\bar{g}_{2,k})}{(1+\bar{m}_k^2)^4} + \frac{8}{9\pi^2}\frac{\bar{a}_k^4}{(1+\bar{m}_k^2)^5},$$

$$\beta_{g_2} \equiv k\partial_k \bar{g}_{2,k} = -\bar{g}_{2,k} + \frac{1}{3\pi^2}\frac{\bar{a}_{2,k}-5\bar{g}_{3,k}-13\bar{\lambda}_{2,k}}{(1+\bar{m}_k^2)^2} - \frac{4}{3\pi^2}\frac{\bar{a}_k\bar{b}_k-6\bar{g}_{1,k}\bar{g}_{2,k}-4\bar{g}_{2,k}^2}{(1+\bar{m}_k^2)^3} + \frac{4}{3\pi^2}\frac{\bar{a}_k^2(3\bar{g}_{1,k}+5\bar{g}_{2,k})}{(1+\bar{m}_k^2)^4}$$
$$+\frac{2}{3\pi^2}\frac{\bar{a}_k^4}{(1+\bar{m}_k^2)^5},$$

$$\beta_b \equiv k\partial_k \bar{b}_k = -\frac{\bar{b}_k}{2} + \frac{4}{9\pi^2}\frac{\bar{b}_k(66\bar{g}_{1,k}-4\bar{g}_{2,k})+3\bar{a}_k(5\bar{a}_{2,k}+9\bar{\lambda}_{1,k}-4\bar{\lambda}_{2,k})}{(1+\bar{m}_k^2)^3}$$
$$+\frac{8}{3\pi^2}\frac{-3\bar{a}_k^2\bar{b}_k-18\bar{a}_k\bar{g}_{1,k}^2+12\bar{a}_k\bar{g}_{1,k}\bar{g}_{2,k}+4\bar{a}_k\bar{g}_{2,k}^2}{(1+\bar{m}_k^2)^4} + \frac{32}{9\pi^2}\frac{\bar{a}_k^3(3\bar{g}_{1,k}-\bar{g}_{2,k})}{(1+\bar{m}_k^2)^5},$$

$$\beta_{\lambda_1} \equiv k\partial_k \bar{\lambda}_{1,k} = \frac{8}{27\pi^2}\frac{9\bar{b}_k^2+3\bar{a}_{2,k}\bar{g}_{1,k}+24\bar{g}_{1,k}(9\bar{\lambda}_{1,k}+\bar{\lambda}_{2,k})+4\bar{g}_{2,k}(9\bar{\lambda}_{1,k}+4\bar{\lambda}_{2,k})}{(1+\bar{m}_k^2)^3}$$
$$-\frac{4}{81\pi^2}\frac{72\bar{a}_k\bar{b}_k(9\bar{g}_{1,k}+\bar{g}_{2,k})+4(297\bar{g}_{1,k}^3+108\bar{g}_{1,k}^2\bar{g}_{2,k}+72\bar{g}_{1,k}\bar{g}_{2,k}^2+16\bar{g}_{2,k}^3)+9\bar{a}_k^2(2\bar{a}_{2,k}+45\bar{\lambda}_{1,k}+4\bar{\lambda}_{2,k})}{(1+\bar{m}_k^2)^4}$$
$$+\frac{32}{81\pi^2}\frac{\bar{a}_k^2(15\bar{a}_k\bar{b}_k+171\bar{g}_{1,k}^2+36\bar{g}_{1,k}\bar{g}_{2,k}+8\bar{g}_{2,k})}{(1+\bar{m}_k^2)^5} - \frac{80}{81\pi^2}\frac{\bar{a}_k^4(15\bar{g}_{1,k}+\bar{g}_{2,k})}{(1+\bar{m}_k^2)^6} + \frac{8}{9\pi^2}\frac{\bar{a}_k^6}{(1+\bar{m}_k^2)^7},$$

$$\beta_{\lambda_2} \equiv k\partial_k \bar{\lambda}_{2,k} = \frac{2}{9\pi^2}\frac{2\bar{g}_{2,k}(25\bar{g}_{3,k}+54\bar{\lambda}_{1,k}+44\bar{\lambda}_{2,k}-2\bar{a}_{2,k})-9\bar{b}_k^2-6\bar{g}_{1,k}(\bar{a}_{2,k}-5\bar{g}_{3,k}-28\bar{\lambda}_{2,k})}{(1+\bar{m}_k^2)^3}$$
$$+\frac{1}{3\pi^2}\frac{36\bar{a}_k\bar{b}_k(2\bar{g}_{1,k}+\bar{g}_{2,k})-8\bar{g}_{2,k}(36\bar{g}_{1,k}^2+21\bar{g}_{1,k}\bar{g}_{2,k}+7\bar{g}_{2,k}^2)+\bar{a}_k^2(6\bar{a}_{2,k}+5\bar{g}_{3,k}+36\bar{\lambda}_{1,k})}{(1+\bar{m}_k^2)^4}$$
$$+\frac{8}{27\pi^2}\frac{9\bar{a}_k^3\bar{b}_k+180\bar{a}_k^2\bar{g}_{1,k}^2+132\bar{a}_k^2\bar{g}_{1,k}\bar{g}_{2,k}+26\bar{a}_k^2\bar{g}_{2,k}^2}{(1+\bar{m}_k^2)^5} + \frac{20}{9\pi^2}\frac{\bar{a}_k^4(3\bar{g}_{1,k}+2\bar{g}_{2,k})}{(1+\bar{m}_k^2)^6},$$

$$\beta_{a_2} \equiv k\partial_k \bar{a}_{2,k} = \frac{4}{3\pi^2}\frac{6\bar{b}_k^2+15\bar{a}_{2,k}\bar{g}_{1,k}-8\bar{a}_{2,k}\bar{g}_{2,k}}{(1+\bar{m}_k^2)^3} + \frac{16}{\pi^2}\frac{\bar{a}_k\bar{b}_k(\bar{g}_{2,k}-3\bar{g}_{1,k})}{(1+\bar{m}_k^2)^4} + \frac{16}{3\pi^2}\frac{\bar{a}_k^2(9\bar{g}_{1,k}^2+2\bar{g}_{2,k})}{(1+\bar{m}_k^2)^5},$$

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$$+\frac{1}{\pi^2}\frac{4\bar{a}_k\bar{b}_k\bar{g}_{2,k}+8\bar{g}_{2,k}^2(\bar{g}_{2,k}-9\bar{g}_{1,k})+\bar{a}_k^2(\bar{g}_{3,k}+8\bar{\lambda}_{2,k}-2\bar{a}_{2,k})}{(1+\bar{m}_k^2)^4} + \frac{16}{9\pi^2}\frac{3\bar{a}_k^3\bar{b}_k+2\bar{a}_k^2\bar{g}_{2,k}(7\bar{g}_{2,k}-12\bar{g}_{1,k})}{(1+\bar{m}_k^2)^5}$$
$$+\frac{20}{9\pi^2}\frac{\bar{a}_k^4(5\bar{g}_{2,k}-6\bar{g}_{1,k})}{(1+\bar{m}_k^2)^6} + \frac{2}{\pi^2}\frac{\bar{a}_k^6}{(1+\bar{m}_k^2)^7}.$$

Fixed points and stability

- Fixed points: $\beta_i = 0 \ \forall \ i$
- First step: solve for marginal couplings
  $\longrightarrow \beta_{\lambda_1} = \beta_{\lambda_2} = \beta_{a_2} = \beta_{g_3} \equiv 0$
  $\longrightarrow \lambda_1, \lambda_2, a_2, g_3$ are plugged into the remaining β functions
- Second step: solve for relevant couplings
- Third step: check stability matrix $\partial\beta_i/\partial\omega_j$
  $(\{\omega_j\} : m^2, g_1, g_2, a, b)$

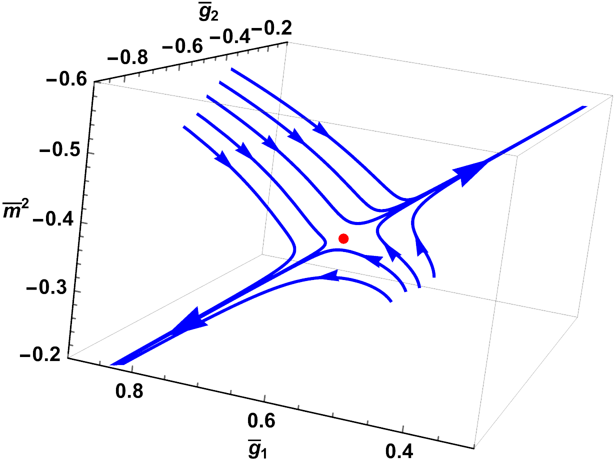
m^2	g_1	g_2	a	b	# of RD
0	0	0	0	0	5
-0.31496	0.43763	0	0	0	3
-0.38262	0.59726	-0.62042	0	0	2
-0.01786	0.09163	-0.14148	-0.11900	0.39087	4

Fixed points and stability

m^2	g_1	g_2	a	b	# of RD
0	0	0	0	0	5
-0.31496	0.43763	0	0	0	3
-0.38262	0.59726	-0.62042	0	0	2
-0.01786	0.09163	-0.14148	-0.11900	0.39087	4

- No fixed point with one relevant direction
  $\longrightarrow$ first order transition?
- BUT the third one has a block diagonal stability matrix:
  $\longrightarrow (m^2, g_1, g_2) \oplus (a, b)$
- Both a and b are related to the $U_A(1)$ anomaly!
  $\longrightarrow$ without anomaly the no. of relevant directions is 1 !

Fixed points and stability



- If the $U_A(1)$ symmetry is recovered at T_c, the transition is of second order!
- Temperature eigenvalue leads to $\nu \approx 0.83$

Fixed points and stability

- Lessons from the ϵ-expansion:

  $\longrightarrow$ if the $U_A(1)$ symmetry is recovered at T_c:
  first order transition for $N_f = 2, 3$

  $\longrightarrow$ if the anomaly is present at T_c:
  second order for $N_f = 2$, first order for $N_f = 3$

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  first order

- Increasing evidence of a second order transition for $N_f = 3$:
  $\longrightarrow$ F. Cuteri, O. Philipsen, A. Sciarra, JHEP **11**, 141 (2021)
  $\longrightarrow$ L. Dini et al., Phys. Rev. D**105**, 034510 (2022)

- If the transition is of second order, RG hints that the $U_A(1)$ axial symmetry is recovered at T_c!

Summary

- Order of the chiral transition for $N_f = 3$ flavors
  $\longrightarrow$ common wisdom: first order irrespectively of the anomaly
  $\longrightarrow$ based on perturbation theory, ϵ expansion of RG flows
- Functional Renormalization Group
  $\longrightarrow$ RG flows can be extracted directly in $d = 3$
  $\longrightarrow$ optimization of the RG is important
  $\longrightarrow$ LPA approximation
- Reanalysis of the Ginzburg–Landau theory
  $\longrightarrow$ in $d = 3$ there are 9 renormalizable operators
  $\longrightarrow$ 2 new fixed points in the system
  $\longrightarrow$ without the anomaly, one of them describes a second order transition

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  $\longrightarrow$ in $d = 3$ there are 9 renormalizable operators
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  $\longrightarrow$ without the anomaly, one of them describes a second order transition
- Questions to be asked:
  $\longrightarrow$ transition order for $N_f \neq 3$?
  $\longrightarrow$ improvement of the RG truncation?
  (wavefunction renormalization, higher derivative terms)