

Chiral anomaly induces superconducting baryon crystal

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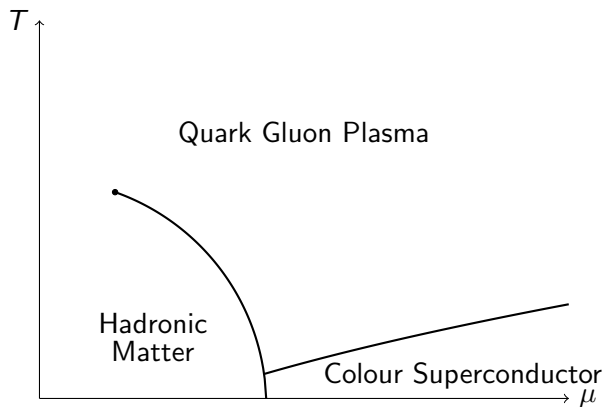
(based on arXiv:2206.01227)



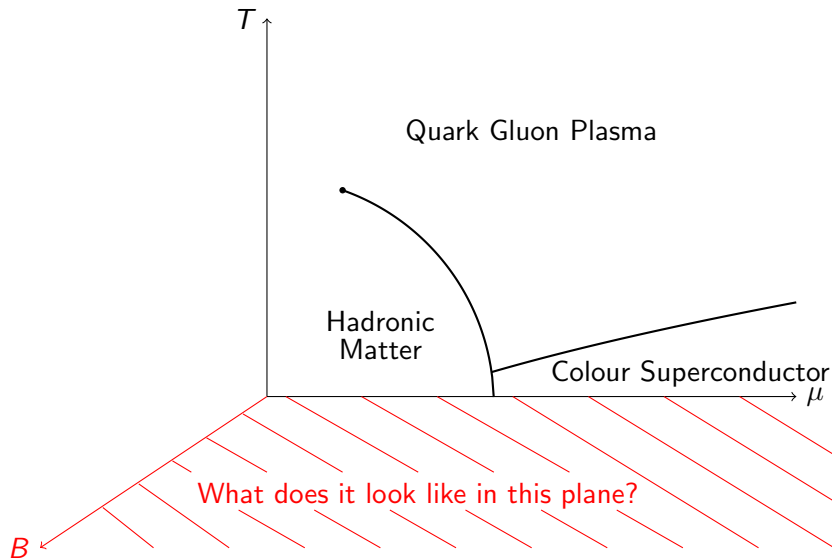
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Schematic QCD Phase Diagram

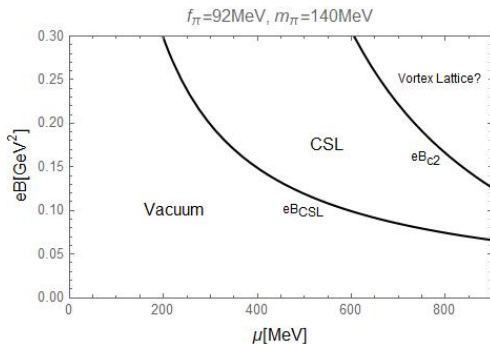


Schematic QCD Phase Diagram



The B - μ plane

- ▶ **Chiral anomaly** induces a coupling $\propto \mu B \nabla \pi^0$ ¹
- ▶ The ground state at $\mu \lesssim 1$ GeV and large B is an inhomogeneous phase of **neutral pions** (π^0) called the Chiral Soliton Lattice (CSL)²

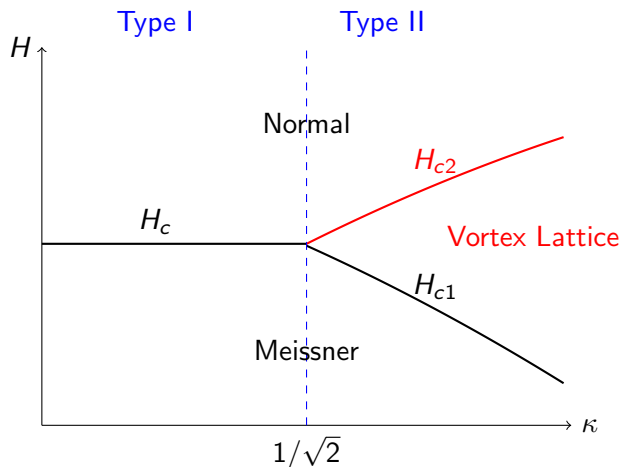


- ▶ At $B \geq B_{c2}$ the CSL becomes unstable to **charged pion** fluctuations

¹D. T. Son and M. A. Stephanov, Phys. Rev. D 77, 2008 (014021)

²T. Brauner and N. Yamamoto, JHEP, 132, 2017(04)

Superconductivity refresher



- ▶ The charged pion instability is reminiscent of the **Vortex Lattice-Normal transition**, but **inverted**

Lagrangian

We work in Chiral Perturbation Theory with two-flavours, starting from the Lagrangian

$$\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{WZW}},$$

where

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4} \text{Tr} \left[\nabla_\mu \Sigma^\dagger \nabla^\mu \Sigma \right] + \frac{m_\pi^2 f_\pi^2}{4} \text{Tr} \left[\Sigma + \Sigma^\dagger \right],$$

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

and the chiral anomaly is incorporated via the Wess-Zumino-Witten³ term

$$\begin{aligned} \mathcal{L}_{\text{WZW}} = & - \left(A_\mu^B - \frac{e}{2} A_\mu \right) \frac{\epsilon^{\mu\nu\rho\lambda}}{24\pi^2} \text{Tr} \left[(\Sigma \nabla_\nu \Sigma^\dagger) (\Sigma \nabla_\rho \Sigma^\dagger) (\Sigma \nabla_\lambda \Sigma^\dagger) \right. \\ & \left. + \frac{3ie}{4} F_{\nu\rho} \tau_3 \left(\Sigma \nabla_\lambda \Sigma^\dagger + \nabla_\lambda \Sigma^\dagger \Sigma \right) \right]. \end{aligned}$$

³J. Wess and B. Zumino, Phys. Lett. B 37, 1971 & E. Witten, Nucl. Phys. B 223, 1983

Free energy

From the Lagrangian we obtain the free energy

$$\mathcal{F} = \frac{1}{V} \int dV \left(|(\nabla - i(e\mathbf{A} + \nabla\alpha))\varphi|^2 - (\nabla\alpha)^2 |\varphi|^2 + \frac{(\nabla|\varphi|^2)^2}{2(f_\pi^2 - 2|\varphi|^2)} \right. \\ \left. - m_\pi^2 f_\pi \sqrt{f_\pi^2 - 2|\varphi|^2} \cos\alpha + \frac{f_\pi^2}{2} (\nabla\alpha)^2 + \frac{\mathbf{B}^2}{2} - \frac{e\mu}{4\pi^2} \mathbf{B} \cdot \nabla\alpha \right),$$

where φ (complex scalar field) and α (real scalar field) parameterise the pion fields, and the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$.

Equations of motion

From the Lagrangian/Free Energy we obtain the equations of motion for φ , \mathbf{A} and α

$$0 = \left[\mathcal{D} + \frac{\nabla^2 |\varphi|^2}{f_\pi^2 - 2|\varphi|^2} + \frac{(\nabla |\varphi|^2)^2}{(f_\pi^2 - 2|\varphi|^2)^2} + m_\pi^2 \cos \alpha \left(1 - \frac{f_\pi}{\sqrt{f_\pi^2 - 2|\varphi|^2}} \right) \right] \varphi,$$

$$\nabla \times \mathbf{B} = -ie (\varphi^* \nabla \varphi - \varphi \nabla \varphi^*) - 2e (\mathbf{eA} + \nabla \alpha) |\varphi|^2,$$

$$\nabla \cdot \left[\left(1 - \frac{2|\varphi|^2}{f_\pi^2} \right) \nabla \alpha \right] = m_\pi^2 \sqrt{1 - \frac{2|\varphi|^2}{f_\pi^2}} \sin \alpha,$$

respectively, where

$$\begin{aligned} \mathcal{D} \equiv \nabla^2 - i\nabla \cdot (\mathbf{eA} + \nabla \alpha) - 2i(\mathbf{eA} + \nabla \alpha) \cdot \nabla - (\mathbf{eA} + \nabla \alpha)^2 \\ + (\nabla \alpha)^2 - m_\pi^2 \cos \alpha. \end{aligned}$$

Applying Abrikosov's expansion near B_{c2}

- ▶ To find solutions analytically near B_{c2} , we follow **Abrikosov's** original paper⁴ and expand in the small parameter $\epsilon \sim \sqrt{B - B_{c2}}$ like so

$$\varphi = \varphi_0 + \delta\varphi + \dots, \quad \mathbf{A} = \mathbf{A}_0 + \delta\mathbf{A} + \dots, \quad \alpha = \alpha_0 + \delta\alpha + \dots,$$

where

$$\mathbf{A}_0, \alpha_0 \sim \epsilon^0, \quad \varphi_0 \sim \epsilon^1, \quad \delta\mathbf{A}, \delta\alpha \sim \epsilon^2, \quad \delta\varphi \sim \epsilon^3$$

- ▶ We expand the free energy and equations of motion (with $\mathbf{B}_0 = \nabla \times \mathbf{A}_0$ and $\delta\mathbf{B} = \nabla \times \delta\mathbf{A}$)
- ▶ For simplicity, we solve the equations of motion **in the chiral limit** (i.e. $m_\pi = 0$)

⁴A.A. Abrikosov, JETP, 5, p.1174, 1957(06)

Solutions of the expanded equations of motion

- ▶ The solutions at ϵ^0 are

$$\mathbf{B}_0 = B_{c2} \hat{\mathbf{e}}_z, \quad \alpha_0(z) = \frac{e\mu}{4\pi^2 f_\pi^2} B_{c2} z$$

- ▶ The equation for φ_0 is the same as the Schrödinger equation for a particle in a magnetic field with the usual Landau level result⁵
- ▶ Choosing the ground state solution,

$$\varphi_0(x, y) = \sum_{n=-\infty}^{\infty} C_n e^{inqy} e^{-\frac{eB_{c2}}{2} \left(x - \frac{nq}{eB_{c2}}\right)^2},$$

where relations between different C_n determine the configuration of the lattice

⁵See M. Tinkham, Introduction to Superconductivity. Dover Publications, New York, 2004

Solutions of expanded equations of motion

- ▶ The correction to the magnetic field becomes

$$\delta \mathbf{B}(x, y) = [\langle B \rangle - B_{c2} + e (\langle |\varphi_0(x, y)|^2 \rangle - |\varphi_0(x, y)|^2)] \hat{\mathbf{e}}_z,$$

where we've introduced a spatial average defined as

$$\langle f(x, y, z) \rangle = \frac{1}{V} \int f(x, y, z) dV,$$

for a function $f(x, y, z)$ over the volume V

- ▶ The other equation at order ϵ^2 is solved by

$$\delta \alpha(z) = \frac{e\mu}{4\pi^2 f_\pi^2} (\langle B \rangle - B_{c2}) z$$

Free energy result

We do not solve the $\delta\varphi$ equation but use it instead to show that

$$e\langle|\varphi_0|^2\rangle = \frac{\langle B\rangle - B_{c2}}{(2\kappa^2 - 1)\beta + 1}, \quad \text{where} \quad \beta = \frac{\langle|\varphi_0|^4\rangle}{\langle|\varphi_0|^2\rangle^2},$$

and $2\kappa^2 = B_{c2}/ef_\pi^2$.

With \mathcal{F} expanded to fourth order, we obtain

$$\Delta\mathcal{F} = -\frac{1}{2} \frac{(\langle B\rangle - B_{c2})^2}{(2\kappa^2 - 1)\beta + 1},$$

which is the difference in free energy between our constructed phase and the “CSL” phase.

We have analytically constructed a phase which is preferred above B_{c2} !

β and lattice configurations

- ▶ To minimise \mathcal{F} we must **minimise** β which can vary depending on the periodicity condition $C_n = C_{n+N}$, where N is an integer
- ▶ To explore a continuum of geometries⁶, we set $N = 2$ and $C_0 = \pm iC_1$

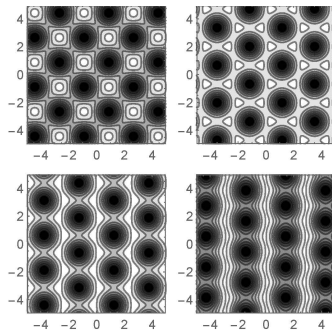
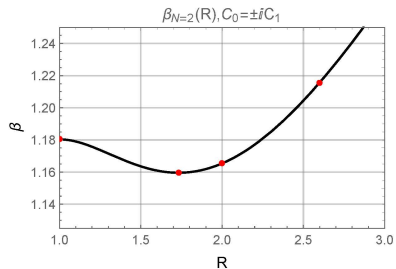


Figure: $R = q^2/\pi$. *Left:* Red dots correspond to contour plots on the right. *Right:* $|\varphi_0|^2$ in the x - y plane. Dark regions correspond to vortices.

⁶W.H. Kleiner, L.M. Roth and S.H. Autler, Phys. Rev. 133 5A A1226, 1964

Charged pion condensate and baryon number density

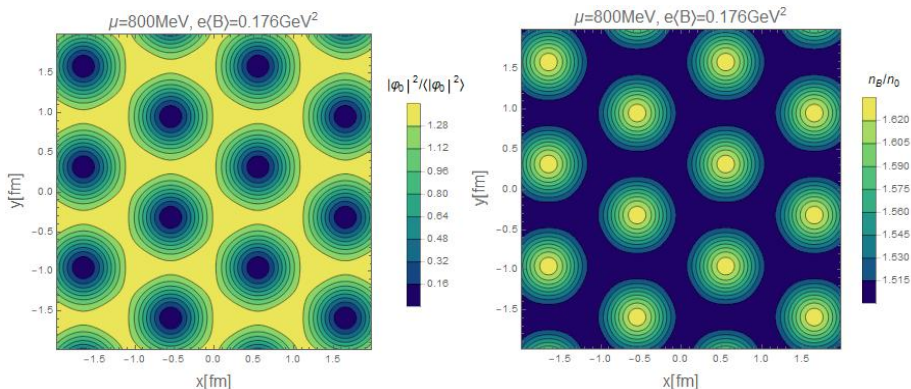


Figure: Charged pion vortex lattice (left) and local baryon number density (right).

A superconducting crystal with “baryon tubes”!

Summary

- ▶ The **preferred** phase above B_{c2} is a **charged pion vortex lattice** coexisting with a neutral pion superflow
- ▶ Further analysis of β parameter reveals the most preferred configuration of vortices is **hexagonal**
- ▶ The **baryon number density** is not only **non-zero** but also **inhomogeneous**, exhibiting the periodic hexagonal configuration of the charged pion vortex lattice
- ▶ Our phase is a **2D superconducting baryon crystal**

Outlook

- ▶ In the more physical scenario where $m_\pi \neq 0$, a **3D crystalline structure** is expected
- ▶ Our calculation is confined near B_{c2} - what does the lattice look like **away from the transition?**
- ▶ At $\mu \sim 1 \text{ GeV}$ **actual baryons** are expected to emerge, thus their inclusion would lead to a more realistic calculation
- ▶ The CSL⁷ and charged pion superconductivity⁸ also emerge in the **B - μ_I plane** - can we extend our results to this plane?⁹

⁷T. Brauner, G. Filios, and H. Kolečová, JHEP 12, p. 29, 2019

⁸P. Adhikari, T. D. Cohen, and J. Sakowitz, Phys. Rev. C 91, 2015 (4)

⁹M. S. Grønli and T. Brauner, Eur. Phys. J. C 82, 2022(4) 