Hybrid Hydrodynamic Attractor

based on arXiv:2006.09383 and upcoming work arXiv:2107.XXXXX,
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Strong and Electroweak Matter
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Motivation

- Matter produced in heavy-ion collisions involves both perturbative and non-perturbative degrees of freedom simultaneously and their dynamics cannot be factorized e.g. Jets travelling through quark-gluon plasma
- Most existing approaches work solely in either perturbative (weakly coupled) or non-perturbative (strongly coupled) frameworks
- Semi-holography describes both perturbative and non-perturbative dof in single framework

- To be consistent with Wilsonian RG, non-perturbative dynamics at a given energy scale should depend on perturbative dynamics only upto that energy scale

- We should be able to obtain effective macroscopic description of the combined system from coarse-grained descriptions of the sub-sectors

- This can be achieved by using democratic couplings arXiv: 1701.01229

- Only metric couplings are relevant for fluids

Semi-holography

- Full system lives in actual background metric:
$$g^{(B)}_{\mu\nu}$$
- first sub-sector lives in effective metric:
$$g_{\mu\nu}[\tilde{t}^{\gamma\delta}]$$
- second sub-sector lives in effective metric:
$$\tilde{g}_{\mu\nu}[t^{\alpha\beta}]$$
- subsector Ward Identities: $\nabla_\mu t^\mu{}_\nu = 0, \quad \tilde{\nabla}_\mu \tilde{t}^\mu{}_\nu = 0$

Metric Coupling Equations

$$
\begin{aligned}
g_{\mu\nu} = g^{(B)}_{\mu\nu} &+ \gamma g^{(B)}_{\mu\alpha} \tilde{t}^{\alpha\beta} g^{(B)}_{\beta\nu} \frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}} \\
&+ \gamma' g^{(B)}_{\mu\nu} \tilde{t}^{\alpha\beta} g^{(B)}_{\alpha\beta} \frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{g}_{\mu\nu} = g^{(B)}_{\mu\nu} &+ \gamma g^{(B)}_{\mu\alpha} t^{\alpha\beta} g^{(B)}_{\beta\nu} \frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} \\
&+ \gamma' g^{(B)}_{\mu\nu} t^{\alpha\beta} g^{(B)}_{\alpha\beta} \frac{\sqrt{-g}}{\sqrt{-g^{(B)}}}
\end{aligned}
$$

The individual ward identities $\nabla_\mu t^\mu{}_\nu = 0$, $\tilde{\nabla}_\mu \tilde{t}^\mu{}_\nu = 0$ and the coupling equations together imply $\nabla^{(B)}_\mu T^\mu{}_\nu = 0$, where

$$T^\mu{}_\nu = \frac{1}{2}\left((t^\mu{}_\nu + t_\mu{}^\nu)\frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} + (\tilde{t}^\mu{}_\nu + \tilde{t}_\mu{}^\nu)\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}} \right) + \Delta K \delta_\mu{}^\nu$$
$$=: T_1{}^\mu{}_\nu(\mathcal{E}_1, \mathcal{P}_1) + T_2{}^\mu{}_\nu(\mathcal{E}_2, \mathcal{P}_2) + T^\mu{}_{\nu,\mathrm{int}}$$

with

$$\Delta K = -\frac{\gamma}{2}\left(t^{\rho\alpha}\frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} \right) g^{(B)}_{\alpha\beta} \left(\tilde{t}^{\beta\sigma}\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}} \right) g^{(B)}_{\sigma\rho}$$
$$-\frac{\gamma'}{2}\left(t^{\alpha\beta}\frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} \right) g^{(B)}_{\alpha\beta} \left(\tilde{t}^{\sigma\rho}\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}} \right) g^{(B)}_{\sigma\rho}$$

- Full stress tensor is a polynomial of sub-system em-tensors

- Phenomenological description of full system can be obtained from hydrodynamic descriptions of the subsectors

Hydrodynamic Attractor with two fluids

- We study hydrodynamic attractors[1] in this framework
- Hydrodynamic Attractor is curve in phase space of the physical system to which all the initial conditions converge at late time
- We couple two MIS fluids with different transport coefficients using metric coupling and this system has two dimensional attractor surface
- the full system behaves as a single fluid
- universality of bottom-up thermalization
- Hydrodynamization times show interesting features

[1] arXiv: 2109.15081

The background metric is flat Minkowski metric in Bjorken flow

$$g^{(B)}_{\mu\nu} = diag(-1, 1, 1, \tau^2)$$

boost invariant ansatz for the effective metrics of subsectors:

$$g_{\mu\nu} = diag(-a^2, b^2, b^2, c^2)$$
$$\tilde{g}_{\mu\nu} = diag(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{c}^2)$$

a, b, c, $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ are functions of τ.

Assume conformal equations of state for both the subsectors:

$$\epsilon = 3P, \quad \tilde{\epsilon} = 3\tilde{P}$$

stress tensors of subsectors:

$$t^\mu_\nu = \mathrm{diag}\left(-\epsilon, P, P, P\right) + \pi^\mu_\nu$$
$$\tilde{t}^\mu_\nu = \mathrm{diag}\left(-\tilde{\epsilon}, \tilde{P}, \tilde{P}, \tilde{P}\right) + \tilde{\pi}^\mu_\nu$$

$$\pi^\mu_\nu = \mathrm{diag}\left(0, \frac{\phi}{2}, \frac{\phi}{2}, -\phi\right),$$
$$\tilde{\pi}^\mu_\nu = \mathrm{diag}\left(0, \frac{\tilde{\phi}}{2}, \frac{\tilde{\phi}}{2}, -\tilde{\phi}\right)$$

EOMs for two subsectors

$$\nabla_\mu t^{\mu\nu} = 0, \quad \Big(\tau_\pi u^\alpha \nabla_\alpha + 1\Big)\pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$$
$$\tilde{\nabla}_\mu \tilde{t}^{\mu\nu} = 0, \quad \Big(\tilde{\tau}_\pi \tilde{u}^\alpha \tilde{\nabla}_\alpha + 1\Big)\tilde{\pi}^{\mu\nu} = -\tilde{\eta}\tilde{\sigma}^{\mu\nu}$$

We parametrize transport coefficients as follows

$$C_\eta = \frac{\eta}{s}, \quad C_\tau = \tau_\pi \epsilon^{1/4}$$
$$\tilde{C}_\eta = \frac{\tilde{\eta}}{\tilde{s}}, \quad \tilde{C}_\tau = \tilde{\tau}_\pi \tilde{\epsilon}^{1/4}$$

C_η, C_τ, $\tilde{C}_\eta$, $\tilde{C}_\tau$ are all dimensionless parameters which are given by the underlying microscopic theory

Strongly coupled (non-perturbative) sector ($\mathcal{N} = 4$ SYM values)

$$\tilde{C}_\tau = \frac{2 - \log(2)}{2\pi}, \quad \tilde{C}_\eta = \frac{1}{4\pi}$$

Weakly coupled (perturbative) sector

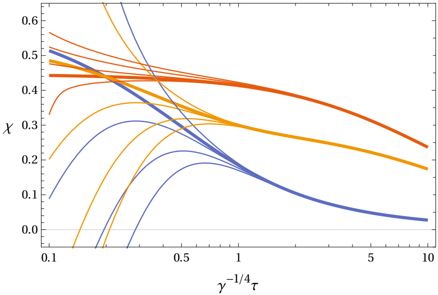
$$C_\tau = 5C_\eta, \quad C_\eta = 10\tilde{C}_\eta$$

dimensionless anisotropy variable: $\chi = \frac{\phi}{\epsilon+P}$, $\tilde{\chi} = \frac{\tilde{\phi}}{\tilde{\epsilon}+\tilde{P}}$

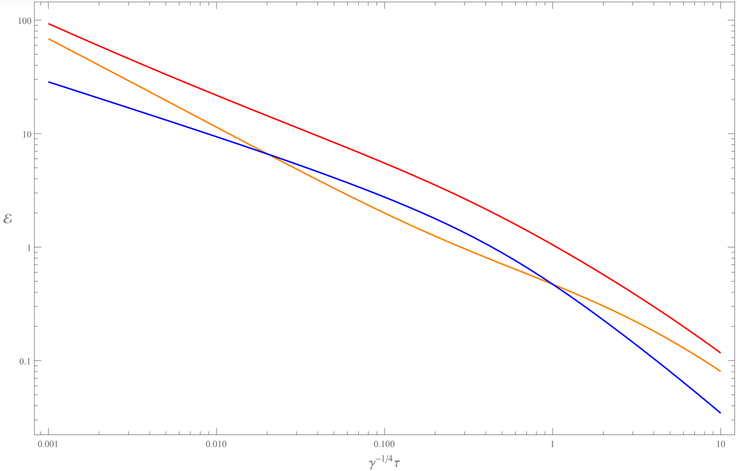
At $\tau \to 0$

$$\chi \to \sqrt{\frac{C_\eta}{C_\tau}} \approx 0.45 \text{ (weakly coupled/perturbative)}$$

$$\tilde{\chi} \to \sqrt{\frac{\tilde{C}_\eta}{\tilde{C}_\tau}} \approx 0.62 \text{ (strongly coupled/non-perturbative)}$$



strongly coupled, weakly coupled, total system



Energy Densities

strongly coupled sector, weakly coupled sector, full system

Bottom-up thermalization

At early times, energy in the weakly coupled sector is always greater than the energy in the strongly coupled sector.

At early times (near $\tau = 0$), the subsystem energy densities have the following behaviour:

$$\mathcal{E}_1 := (ab^2c/\tau)\epsilon \sim \tau^{4(\sqrt{\frac{C_\eta}{C_\tau}}-1)/3}, \quad \mathcal{E}_2 := (\tilde{a}\tilde{b}^2\tilde{c}/\tau)\tilde{\epsilon} \sim \tau^{4(2\sqrt{\frac{\tilde{C}_\eta}{\tilde{C}_\tau}}-\sqrt{\frac{C_\eta}{C_\tau}}-1)/3}$$

$$\mathcal{E}_2/\mathcal{E}_1 \sim \tau^{8(\sqrt{\frac{\tilde{C}_\eta}{\tilde{C}_\tau}}-\sqrt{\frac{C_\eta}{C_\tau}})/3} \qquad \sqrt{\frac{\tilde{C}_\eta}{\tilde{C}_\tau}} > \sqrt{\frac{C_\eta}{C_\tau}}$$

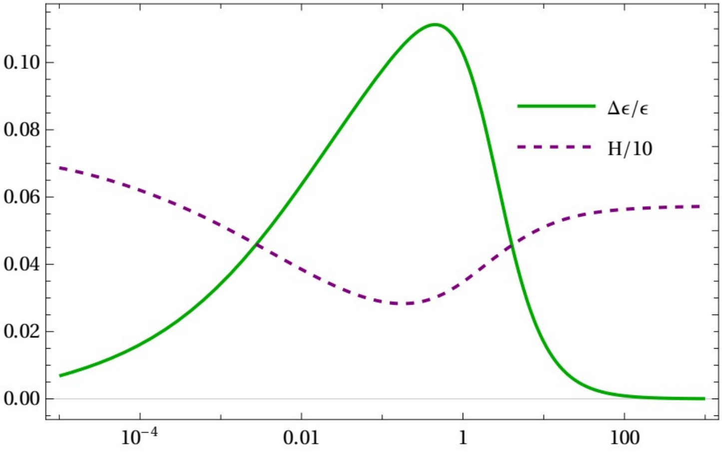
Single fluid with effective η/s

- At late times, variables $\epsilon, \phi, \tilde{\epsilon}, \tilde{\phi}$ admit hydrodynamic expansions involving two parameters $\alpha := lim_{\tau\to\infty}\epsilon\tau^{4/3}$ and $\beta := lim_{\tau\to\infty}\tilde{\epsilon}\tau^{4/3}$.
- The two sub-sectors do not equilibrate but the full system can be described as a single fluid

$$\left(\frac{\eta}{s}\right)^{\text{full}} = C_\eta^{eff} := \lim_{\tau\to\infty} H(\tau)$$

$$C_\eta^{eff} = \frac{C_\eta \alpha^{4/3} + \tilde{C}_\eta \beta^{4/3}}{(\alpha+\beta)^{4/3}}$$

$$H(\tau) = \frac{C_\eta \epsilon^{4/3} + \tilde{C}_\eta \tilde{\epsilon}^{4/3}}{(\epsilon+\tilde{\epsilon})^{4/3}}$$

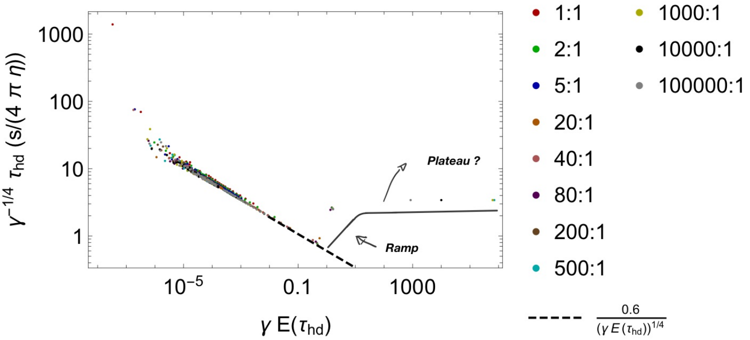


Interaction Measure and Effective Shear Viscosity

Hydrodynamization

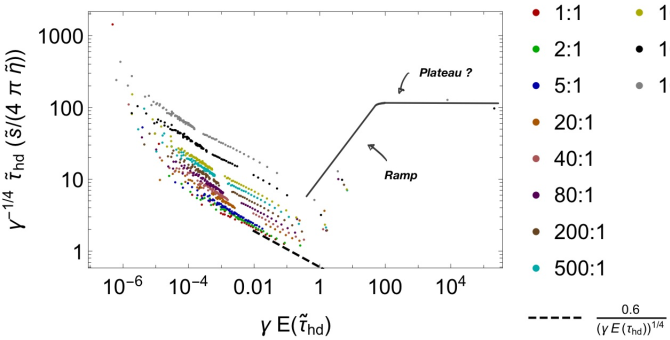
- Anisotropy, $A = \frac{P_\perp - P_L}{P} \sim 6\chi$
- Hydrodynamizaton criterion: $\frac{|\Delta P_L|}{P} := \frac{|\phi - \phi_{1st}|}{P} < 0.1$, $\tau > \tau_{hd}$
- If more energy in perturbative sector at initial time, then universality in perturbative sector and non-perturbative sector is non-universal, vice versa if more energy in the non-perturbative sector at initial time
- Hydrodynamization time vs total energy density plot shows conformality at intermediate energies

Hydrodynamization time - perturbative sector

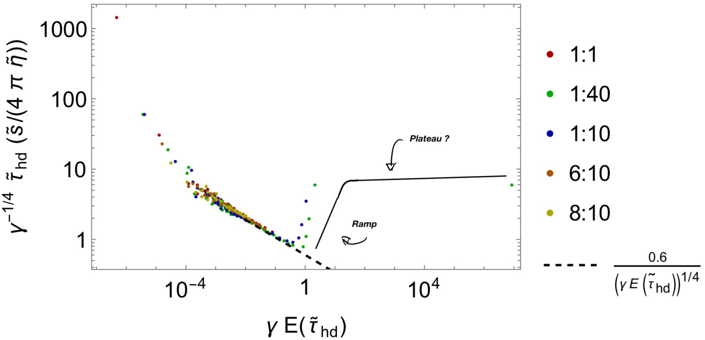


Hydrodynamization time v/s total energy density at hydrodynamization time for various ratios (> 1) of initial energy densities for perturbative sector

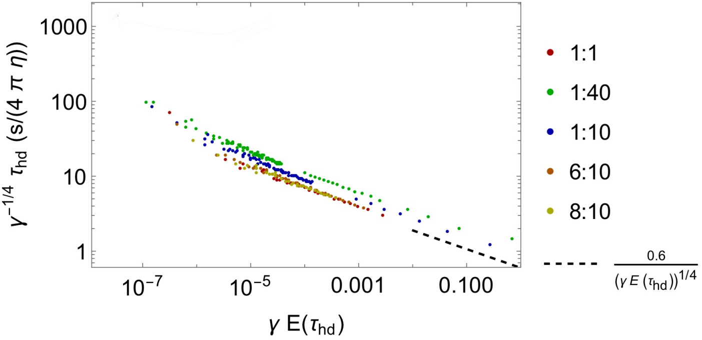
Hydrodynamization time - non-perturbative sector



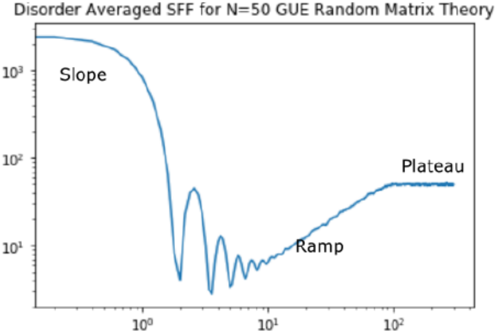
Hydrodynamization time v/s total energy density at hydrodynamization time for various ratios (>1) of initial energy densities for non-perturbative sector



hydrodynamization time in non-perturbative sector v/s total energy density at initial time for various ratios (< 1) of initial energy densities



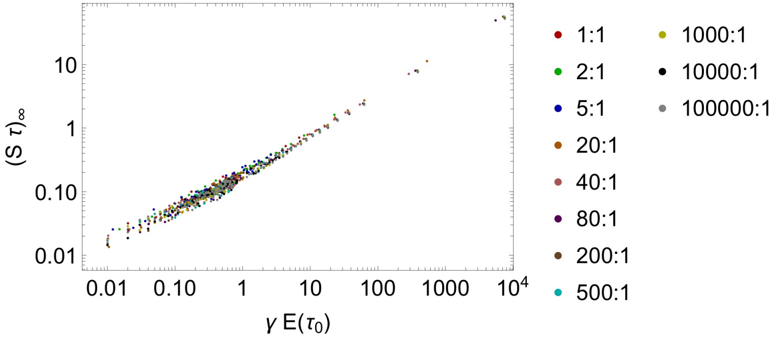
hydrodynamization time in perturbative sector v/s total energy density at initial time for various ratios (< 1) of initial energy densities



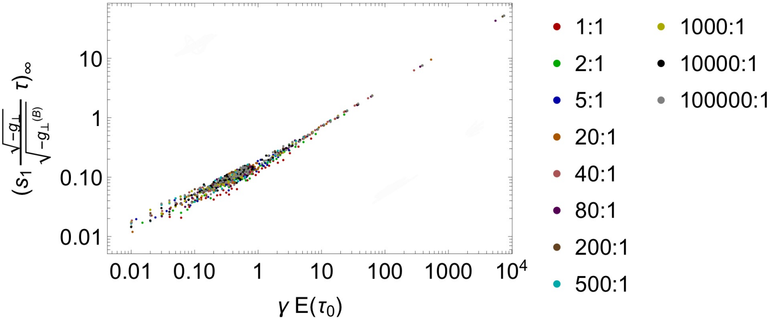
Spectral Form Factor [2]

[2] arXiv: 2012.01436

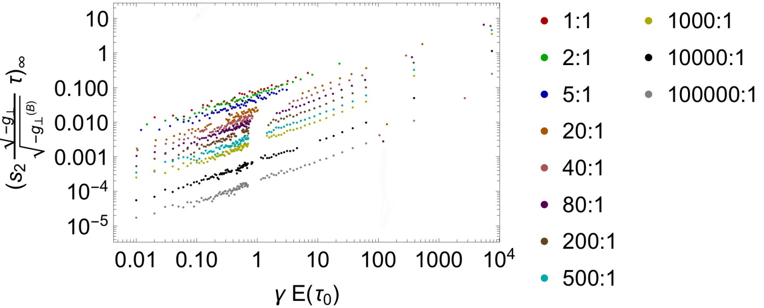
Entropy density



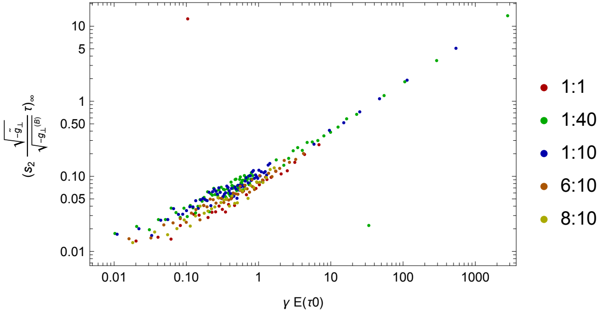
Total entropy density at final time v/s total energy density at initial time for various ratios (> 1) of initial energy densities



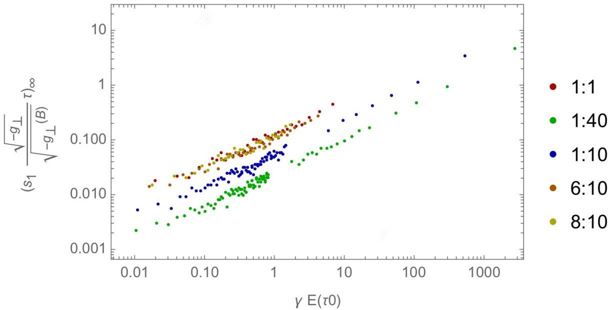
Entropy density in perturbative sector at final time v/s total energy density at initial time for various ratios (> 1) of initial energy densities



Entropy density in non-perturbative sector at final time v/s total energy density at initial time for various ratios (> 1) of initial energy densities



Entropy density in non-perturbative sector at final time v/s total energy density at initial time for various ratios (< 1) of initial energy densities



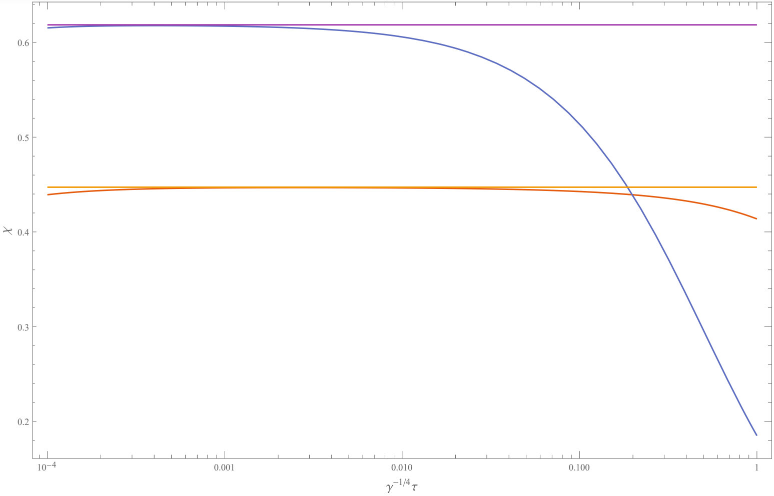
Entropy density in perturbative sector at final time v/s total energy density at initial time for various ratios (< 1) of initial energy densities

Conclusions

- Hybrid two fluid model in combination with MIS equations provides a model for non-equilibrium dynamics of two component system with different amounts of self interactions
- Hybrid system exhibits a two dimensional attractor surface ruled by curves. Any initial condition evolves to one of these curves on the attractor surface.
- Bottom-up thermalization is universal as long as one of the systems is weakly coupled and another is strongly coupled.
- At later times weakly coupled system dominates again as in QGP to hadron gas crossover

Conclusions

- Full system behaves as a single fluid at late times even though the two subsectors never equilibrate. EoS and shear viscosity of the full system are determined by the curve on the attractor surface to which the system evolves at late time.
- Universality in perturbative (non-perturbative) sector, non-perturbative (perturbative) sectors remembers initial conditions if more energy in perturbative (non-perturbaive) sector at initial time
- Our model is successful in capturing only certain features of heavy ion collisions and open to further generalizations



Attractor, $\tilde{\sigma} \approx 0.62$ (strongly coupled), $\sigma \approx 0.45$ (weakly coupled)

The equilibrium of the hybrid system is described by coupling two perfect fluids. (J. High Energ. Phys. 2018, 54 (2018))
Extend this model to capture non-equilibrium dynamics
Describe each sub-sector by MIS theory (arXiv:2006.09383)

- In MIS theory, $\pi^{\mu\nu}$ is promoted to an independent dynamical variable (on the same footing as $\mathcal{E}$ and u^μ).
- $\pi^{\mu\nu}$ satisfies the following relaxation type equation:

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi u^\alpha \nabla_\alpha \pi^{\mu\nu}$$

  τ_π is new transport coefficient called as relaxation time.
- Two sets of Equations of motion for the fluid:

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= 0 && \text{Ward Identity} \\ (1 + \tau_\pi u^\alpha \nabla_\alpha)\pi^{\mu\nu} &= -\,\eta\sigma^{\mu\nu} && \text{MIS equation} \end{aligned}$$