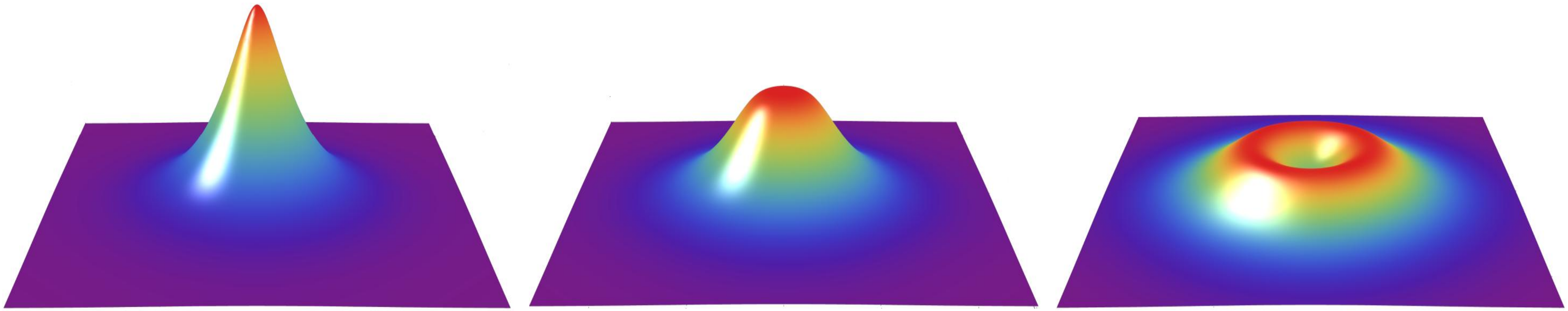
Evolutions in first-order viscous hydrodynamics

Yago Bea

University of Barcelona



With Hans Bantilan and Pau Figueras

Relativistic Hydrodynamics

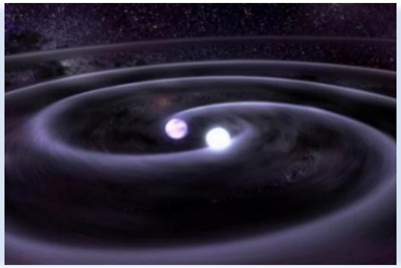
Hydrodynamics

Why hydrodynamics? → It describes interesting phenomena:

Hydrodynamics

Why hydrodynamics? ⟶ It describes interesting phenomena:

Neutron star mergers



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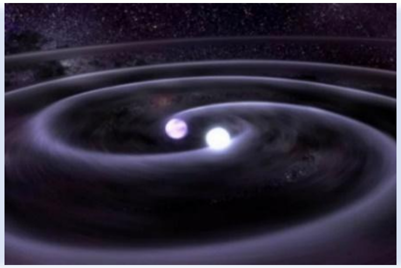
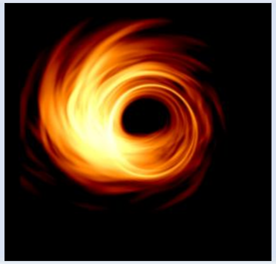
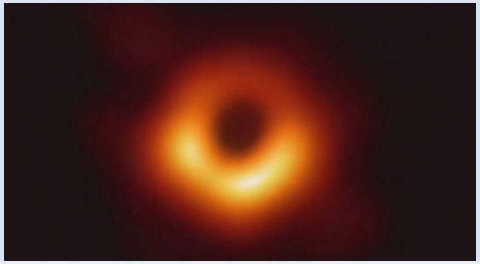


Black hole accretion disk





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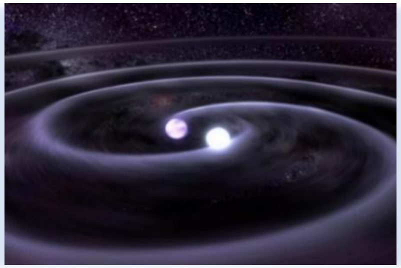

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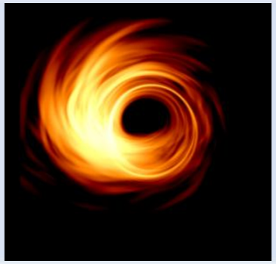
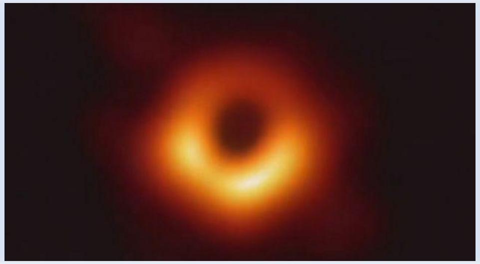
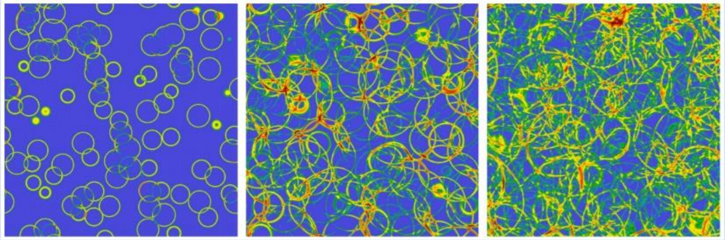




Early universe



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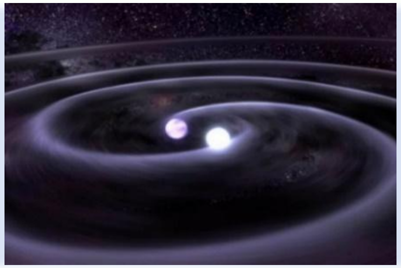

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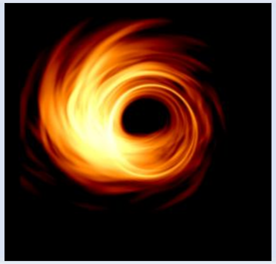

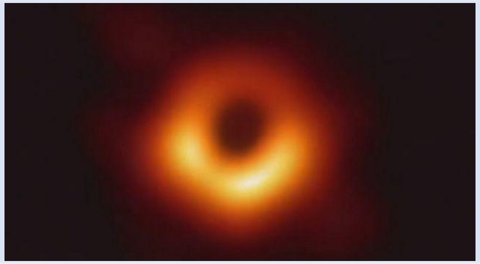

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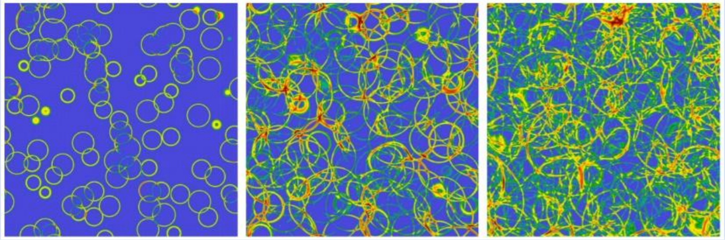
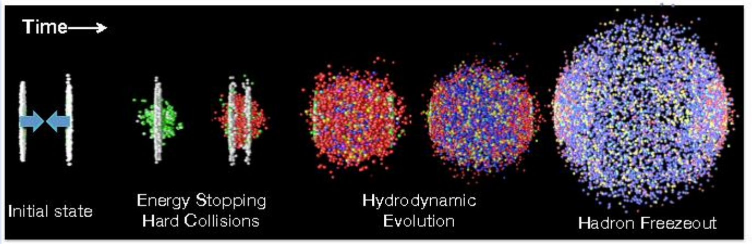


Quark-Gluon Plasma



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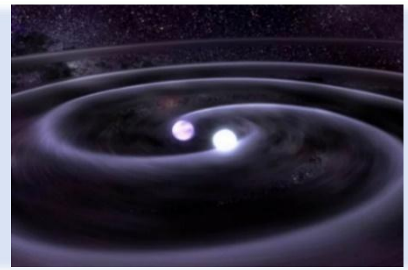

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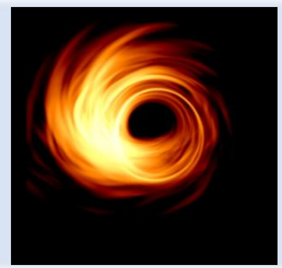

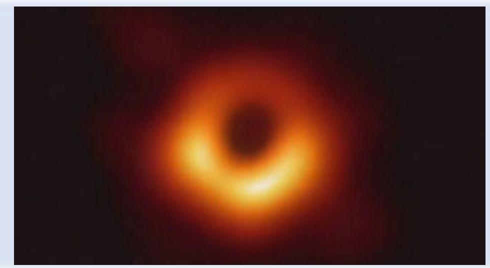

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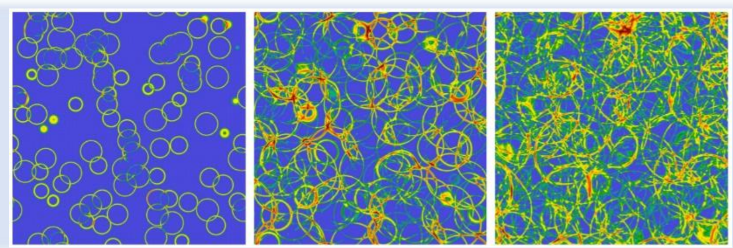

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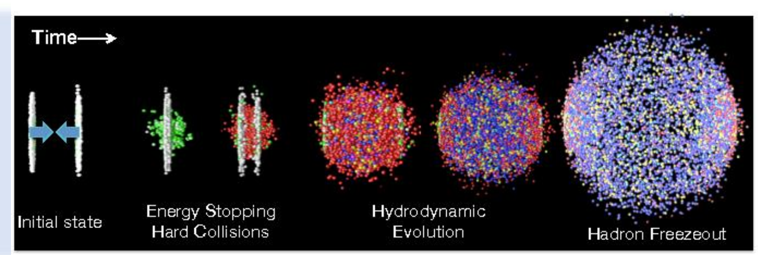


⟶ Relevant for groundbreaking research!

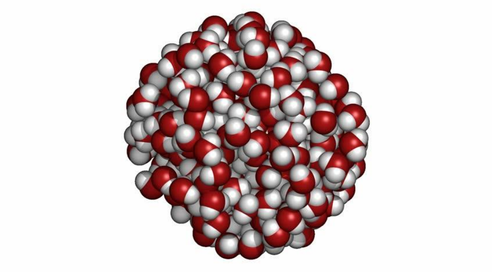
Hydrodynamics

What is hydrodynamics? ⟶ **Effective theory**

Hydrodynamics

What is hydrodynamics? → **Effective theory**

Water



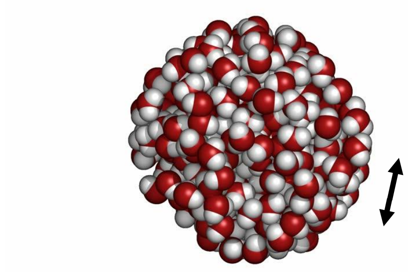
Complicated molecular dynamics

Collective description: hydrodynamics

Hydrodynamics

What is hydrodynamics? → **Effective theory**

Water



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Collective description: <u>hydrodynamics</u>

- Two scales well separated: $l_m \ll L$

Hydrodynamics

What is hydrodynamics? ⟶ **Effective theory**

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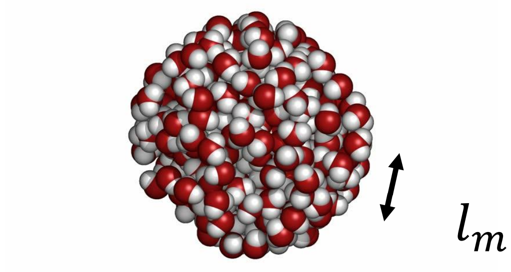

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- Two scales well separated: $l_m \ll L$

⟶ **Universality**

Hydrodynamics

What is hydrodynamics? ⟶ **Effective theory**

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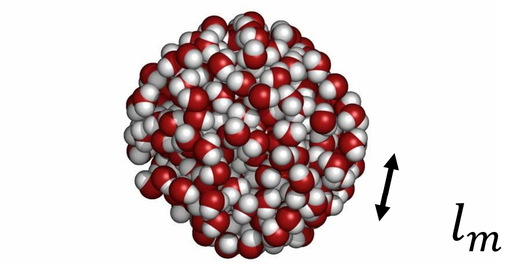

l_m

Complicated molecular dynamics

L

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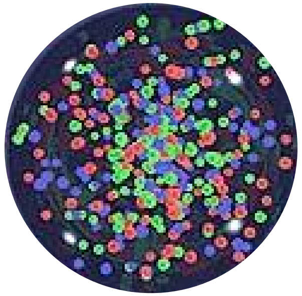
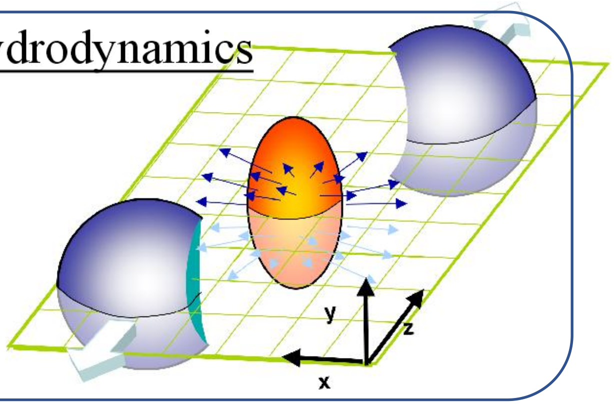
⟶ **Universality**

Quark-gluon plasma

Complicated partonic dynamics



Hydrodynamics



Causal hydrodynamics

Constitutive relations

$$T_{\mu\nu} = T_{\mu\nu}^{ideal} + \partial + \partial^2 + \ldots$$ **Gradient expansion**

0th order 1st order 2nd order

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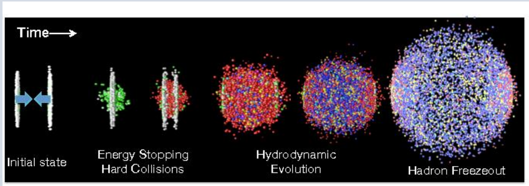

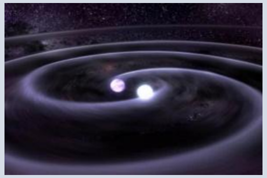

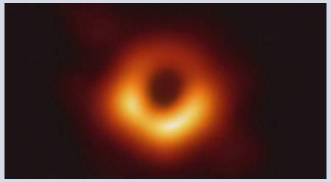

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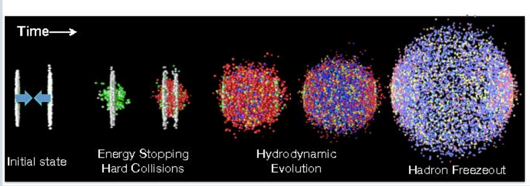

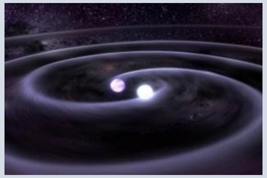

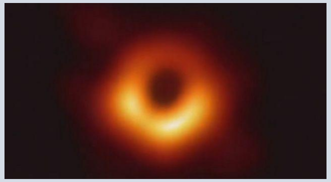

Ideal hydro ⟶ Well posed

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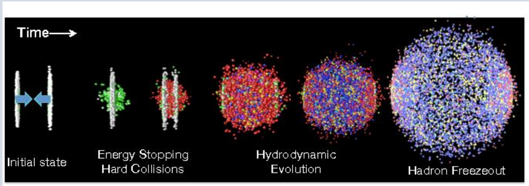

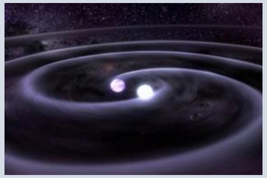

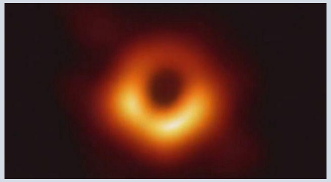

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Viscous hydro ⟶ Ill posed

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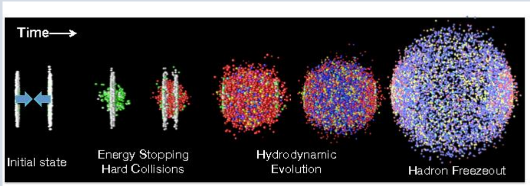

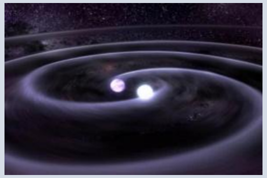

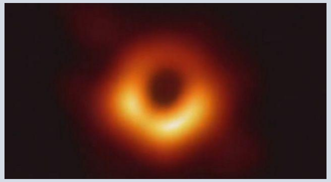

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Müller-Israel-Stewart (MIS) ⟶ Well posed

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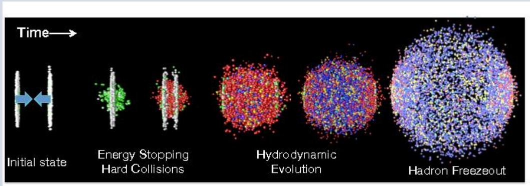

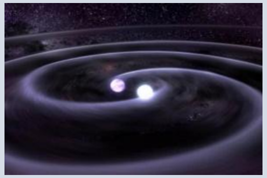

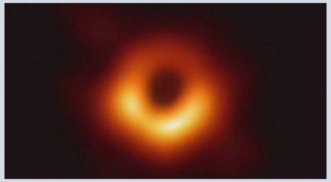

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Causal theories of viscous hydrodynamics:

- MIS
- BRSSS
- DNMR
- Divergence type
- …

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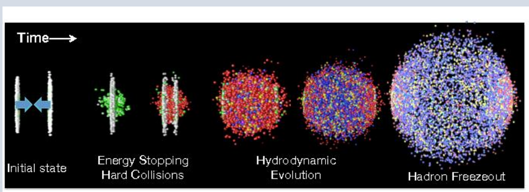

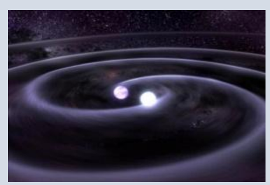
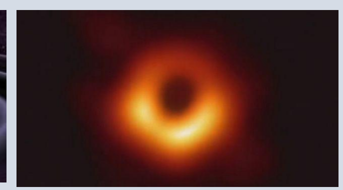

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- **BDNK**

Bemfica, Disconzi, Noronha ’17’19
Kovtun ’19

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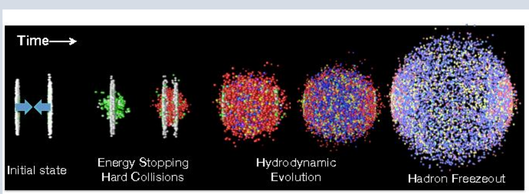

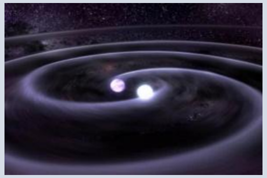

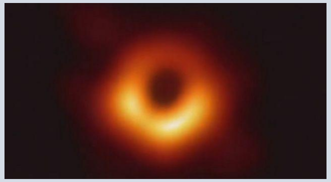

Real-time evolutions using this formulation

Bantilan, Bea, Figueras ’22

Pandya, Pretorius ’21

Pandya, Most, Pretorius ’22

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The BDNK equations

Hydro equations

- **Conformal theory**

Hydro equations

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- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^{\mu}\, u^{\nu} + p\, \Delta^{\mu\nu} \quad \Rightarrow \quad \nabla_{\mu} T^{\mu\nu} = 0$$

Hyperbolic!!

Hydro equations

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Hyperbolic!!

- First order hydro: **Landau frame**

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- Usual fix: **MIS-type**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \text{2nd order}$$

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⇓

New variable ↓ ($\Pi^{\mu\nu}$)

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New equation

Hydro equations

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⇒

Hyperbolic!!

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- First order hydro: **general frame**

$$T^{\mu\nu} = (\epsilon \qquad)\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} \qquad - \eta\, \sigma^{\mu\nu}$$

→ Include all 1st order terms compatible with Poincare symmetry

Hydro equations

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Not hyperbolic…

- First order hydro: **general frame**

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Hydro equations

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BDNK equations

Bemfica, Disconzi, Noronha ’17’19
Kovtun ’19

BDNK vs MIS

→ Why do we need another formulation of viscous hydrodynamics?

BDNK vs MIS

- → Why do we need another formulation of viscous hydrodynamics?
- → I will provide arguments why BDNK might be good alternative to MIS

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BDNK vs MIS

- Why do we need another formulation of viscous hydrodynamics?

- Non linear causality conditions for MIS unknown until 2020.

Nonlinear Constraints on Relativistic Fluids Far From Equilibrium

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Exploring theoretical uncertainties in the hydrodynamic description of relativistic heavy-ion collisions

Cheng Chiu[1,*] and Chun Shen[2,3,†]

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[2] *Department of Physics and Astronomy, Wayne State University, Detroit, Michigan, 48201, USA*
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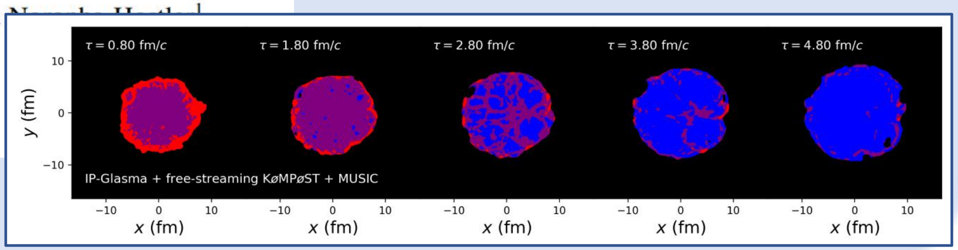
Causality violations in realistic simulations of heavy-ion collisions

Christopher Plumberg,[1] Dekrayat Almaalol,[2] Travis Dore,[1] Jorge Noronha,[1] and Jacquelyn
[1] Illinois Center for Advanced Studies of the Universe, Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA
[2] Department of Physics, Kent State University, Kent, OH 44242, USA
(Dated: March 31, 2021)



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Fábio S. Be

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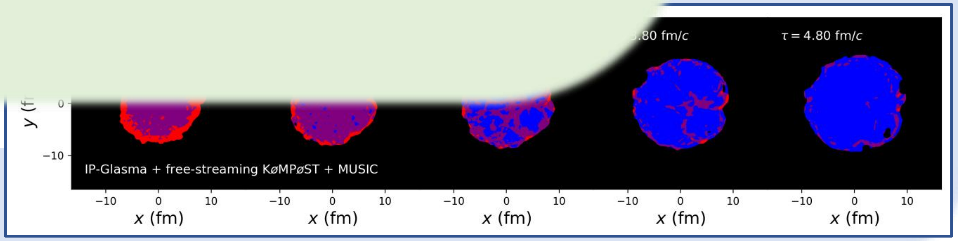

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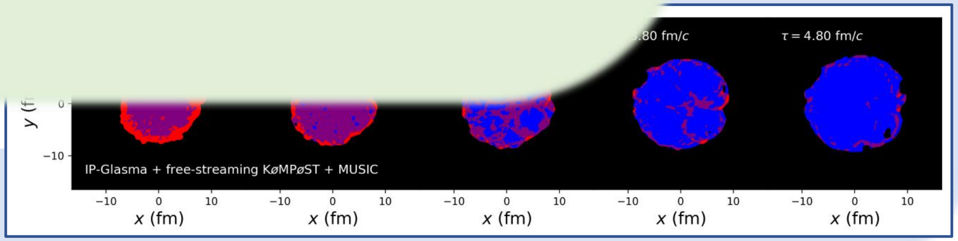

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Do not depend on evolved variables

Causality ensured all along the evolution!

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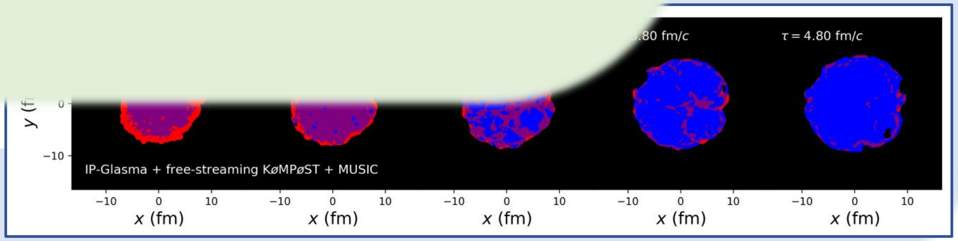

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$$\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d + \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] + \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} + (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,$$
$$\varepsilon + P + \Pi + \Lambda_d - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d - \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] - \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} - (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,$$

Do not depend on evolved variables

⬇

Causality ensured all along the evolution!

⟶ **BDNK might be good alternative to MIS!**

BDNK vs MIS

- Why do we need another formulation of viscous hydrodynamics?

- Non linear

Nonlinea

Fábio S. Be

[1]Escola de Ciências

[2]

- The in ... evolution
- Constr

Exploring th

[2]Depart

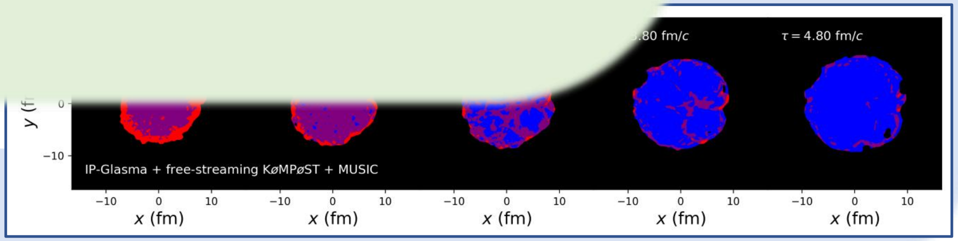
[3]RIK

Christopher Plu

[2]Department of Physics, Kent State University, Kent, OH 44242, USA

(Dated: March 31, 2021)

→ **Significant causality violations!**



BDNK $a_2 > 1\,, \quad a_1 > \dfrac{4\,a_2}{a_2 - 1}\,.$

MIS

$$(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{1}{2}\tau_{\pi\pi}|\Lambda_1| \geq 0$$

$$\varepsilon + P + \Pi - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}\Lambda_3 \geq 0,$$

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Do not depend on evolved variables

↓

Causality ensured all along the evolution!

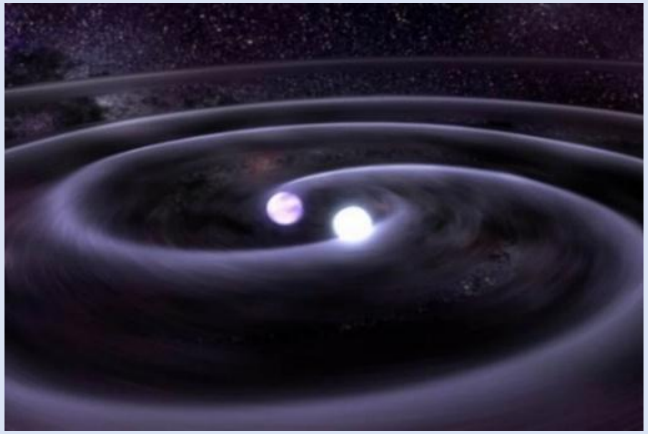
→ **BDNK might be good alternative to MIS!**

Evolve BDNK with realistic heavy-ion initial data.

Work in progress...

BDNK vs MIS

- What about applications in neutron star mergers? (and accretion discs, etc)



BDNK vs MIS

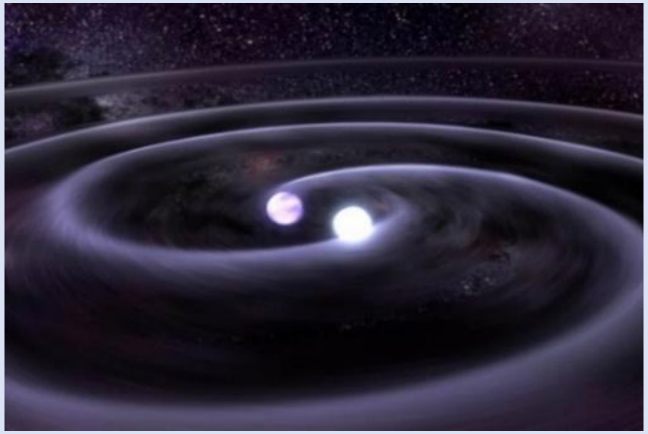
- What about applications in neutron star mergers?   (and accretion discs, etc)

⟶ Preliminar studies suggest that **viscosity might be relevant**

Rezzolla et al ‘17

Shibata et al‘20

Chabanov, Rezzolla, Rischke ‘21



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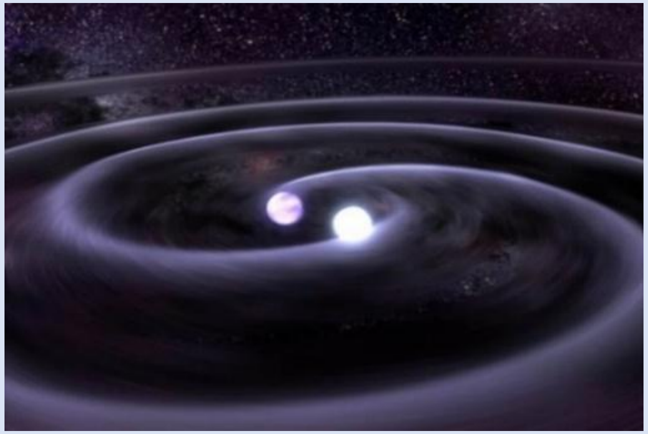

⟶ BDNK better behaved in the presence of shocks

Freistuhler ‘21
Pandya, Pretorius ‘21

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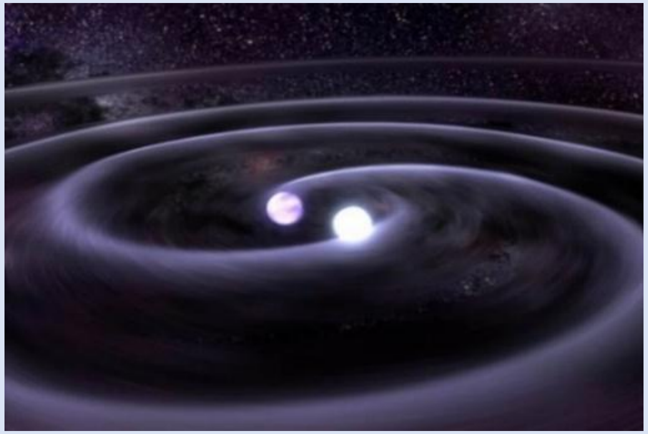

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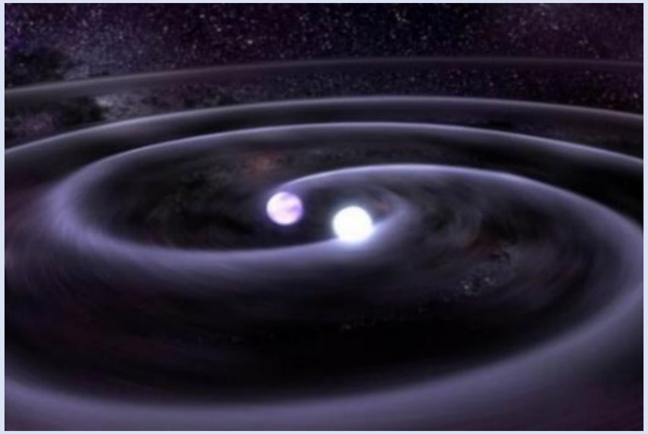

⟶ BDNK better behaved in the presence of shocks

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again, **BDNK might be good alternative to MIS!**

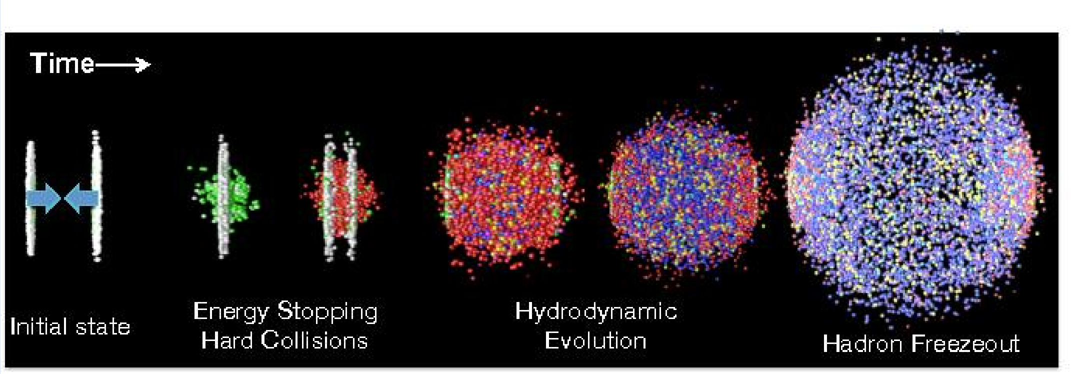
⟶ Underlying these arguments there is the fact that mathematical results are more achievable in BDNK than MIS

⟶ And the known mathematical results put BDNK in favour over MIS

Dynamical evolutions

Motivation

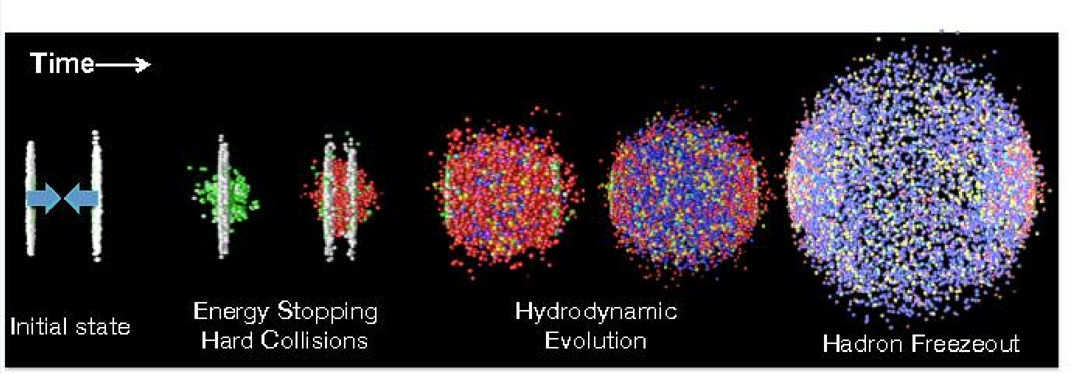
⟶ Motivation: In heavy-ion collisions dissipative terms comparable to ideal terms



⟶ Starts exploring the UV of the theory…

Motivation

→ Motivation: In heavy-ion collisions dissipative terms comparable to ideal terms



→ Starts exploring the UV of the theory…

→ We would like to explore the non-linear, and far from equilibrium regimes of the BDNK equations

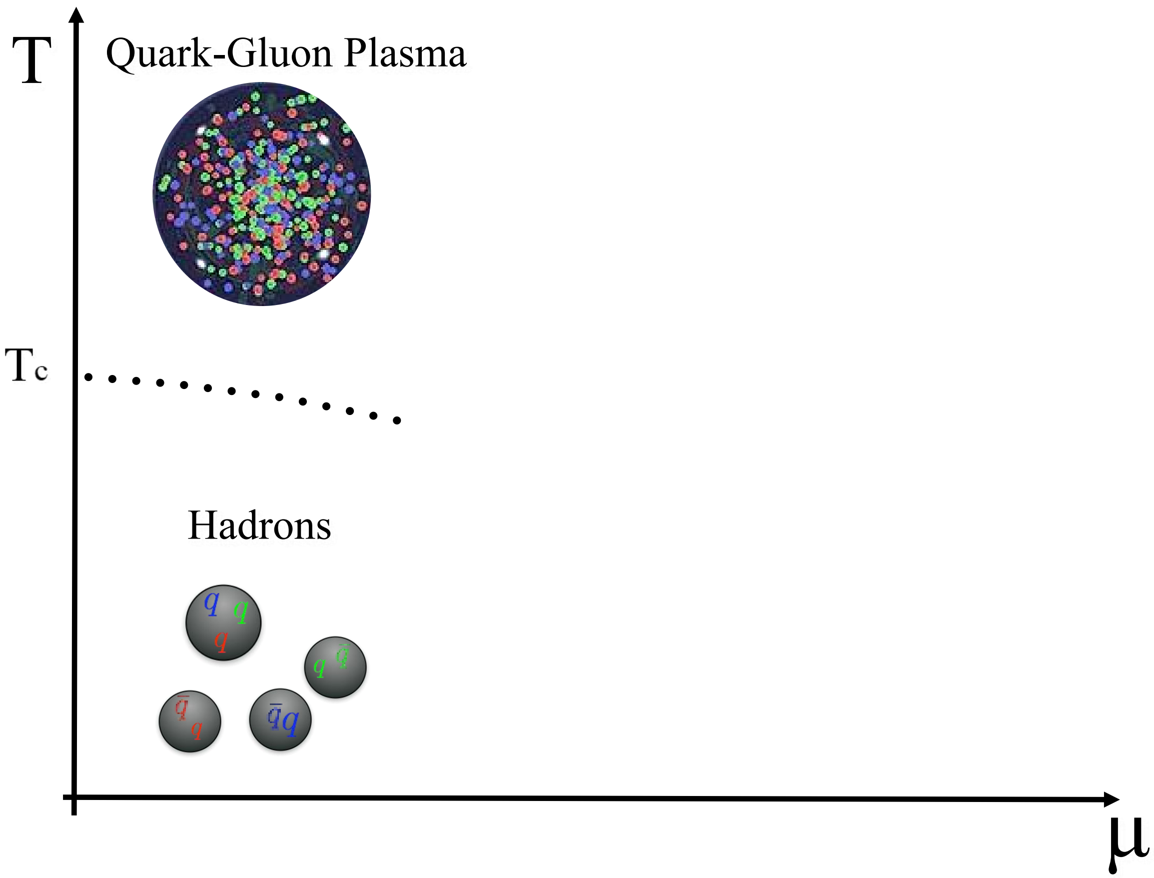
Holography

Holography

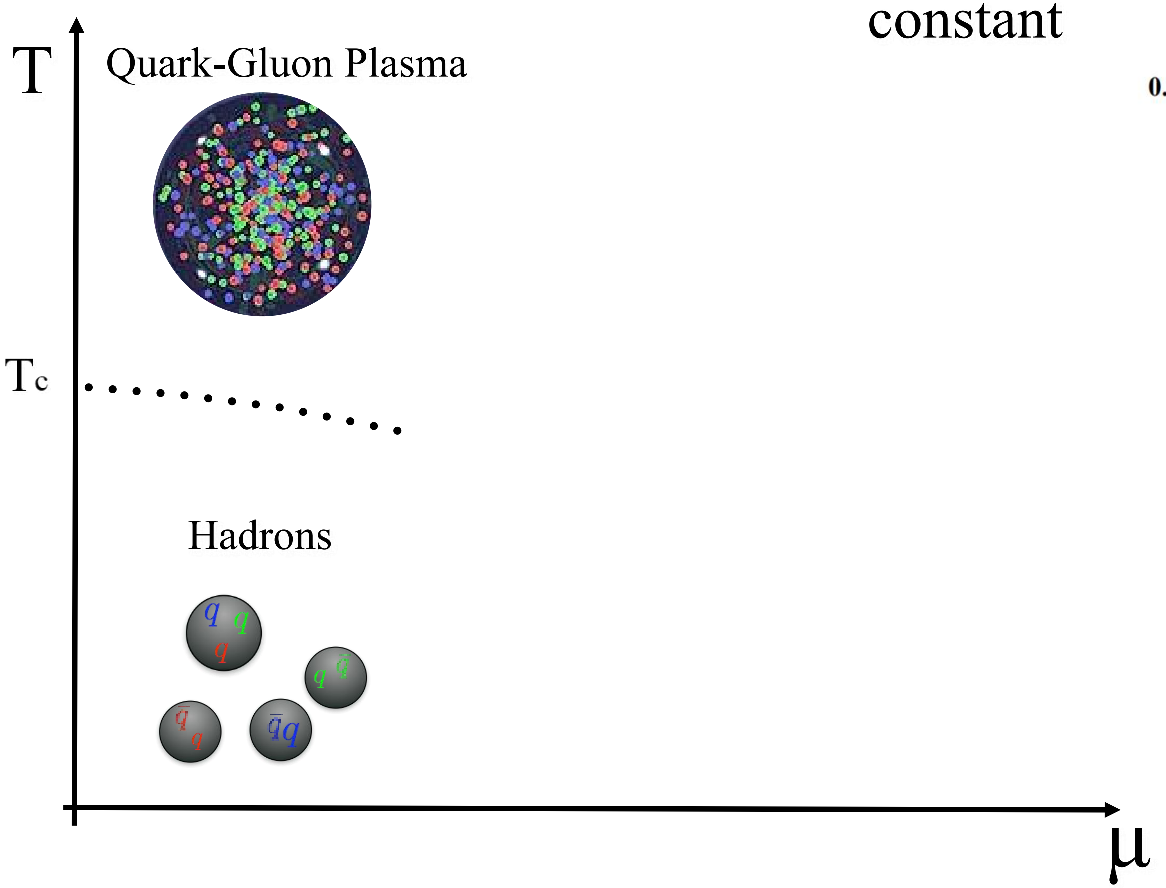
→ Excellent framework to study hydrodynamics.

→ Far from equilibrium strongly coupled field theories from first principles.

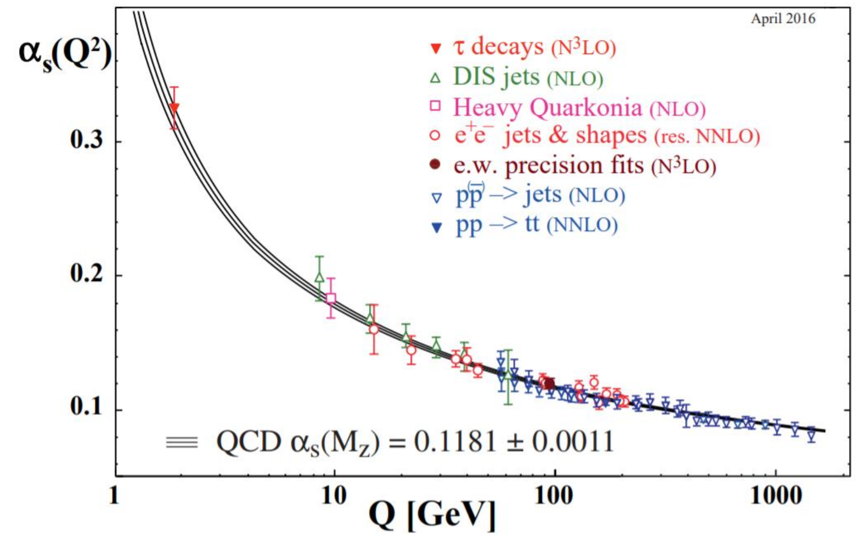
QCD & Holography



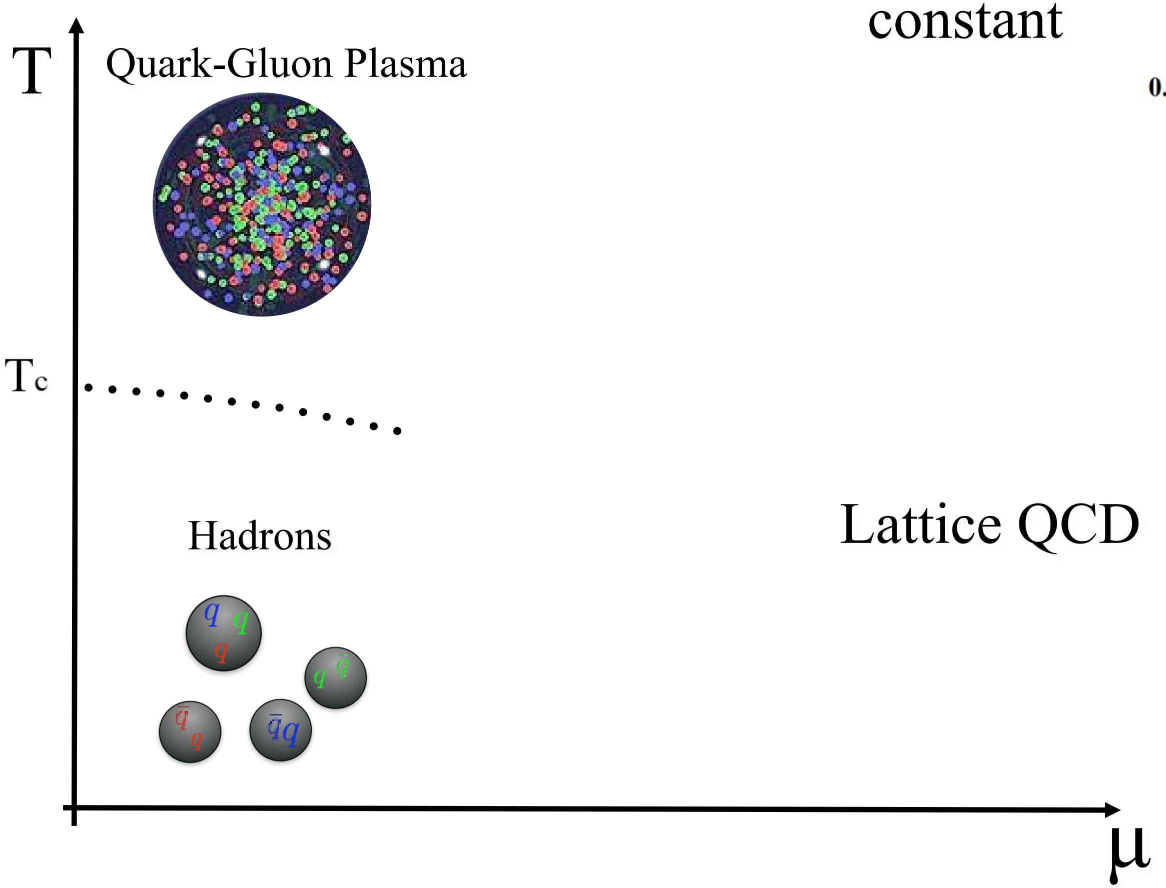
QCD & Holography



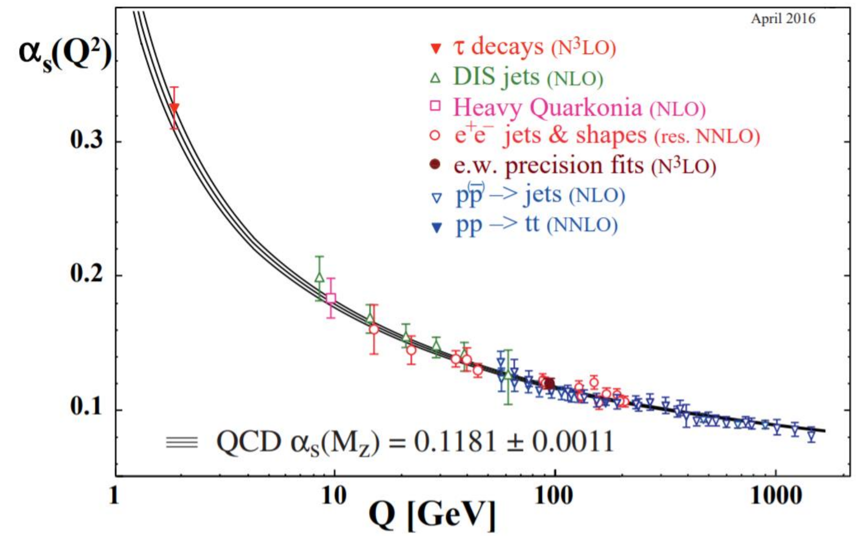
QCD
coupling
constant



QCD & Holography

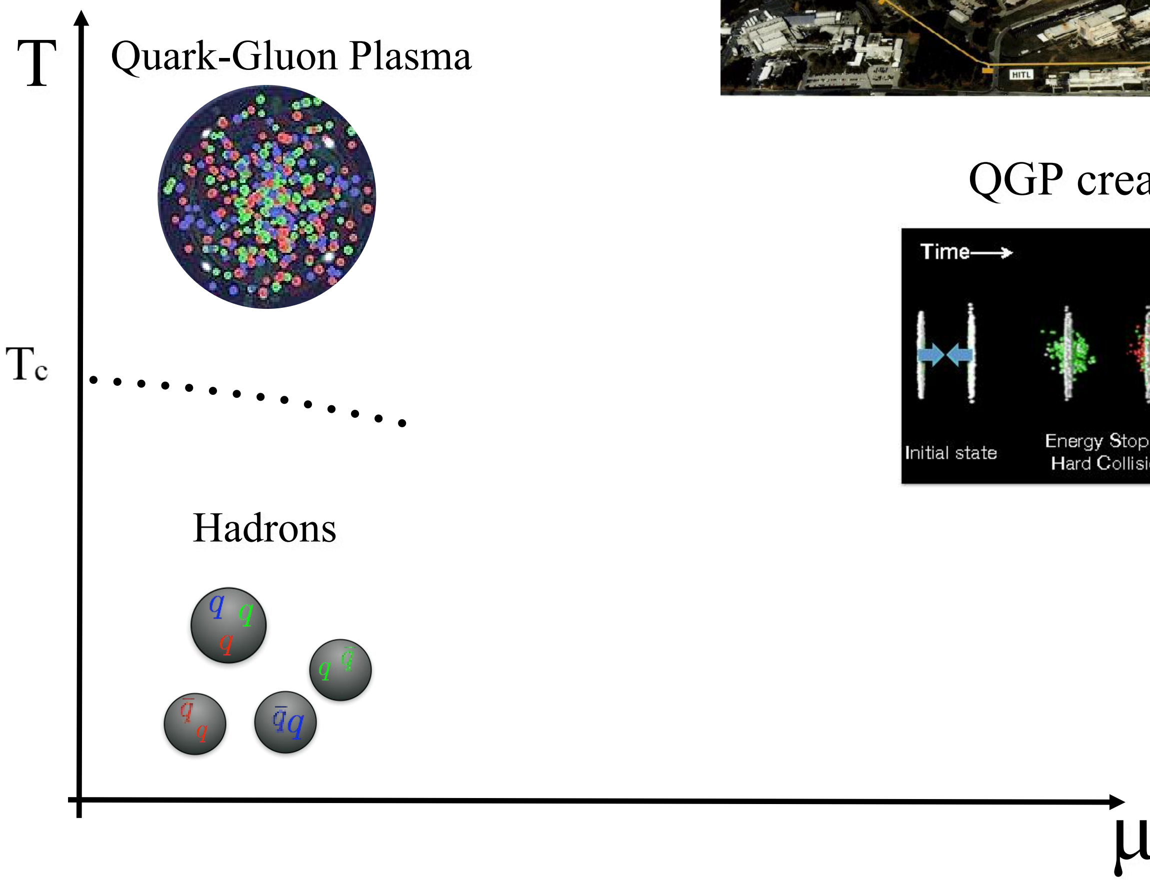


QCD coupling constant

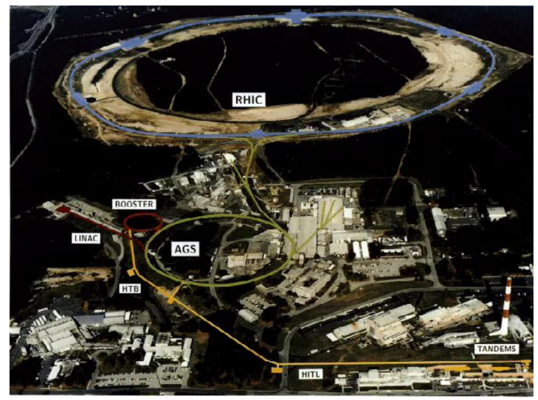


Lattice QCD

QCD & Holography

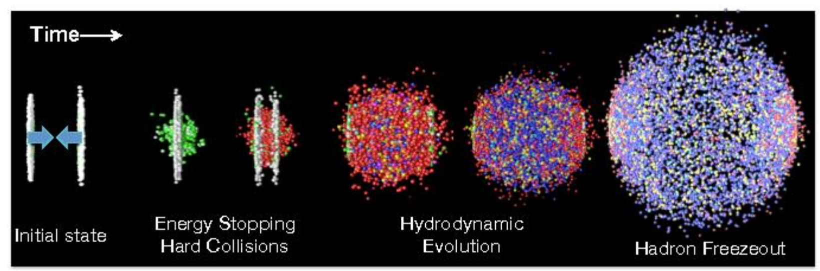


RHIC



LHC

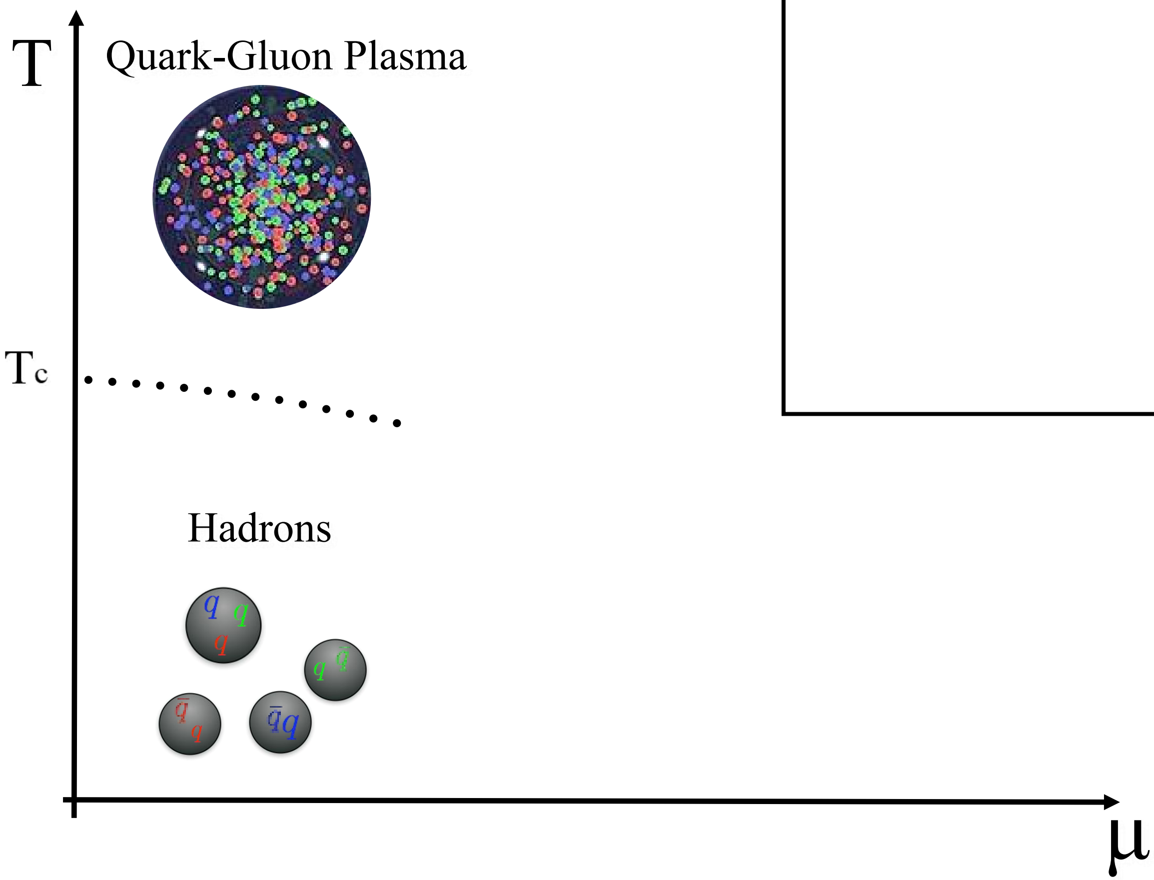
QGP created in heavy ion collision



- Perturbative QCD
- Lattice QCD

→ Limited applicability

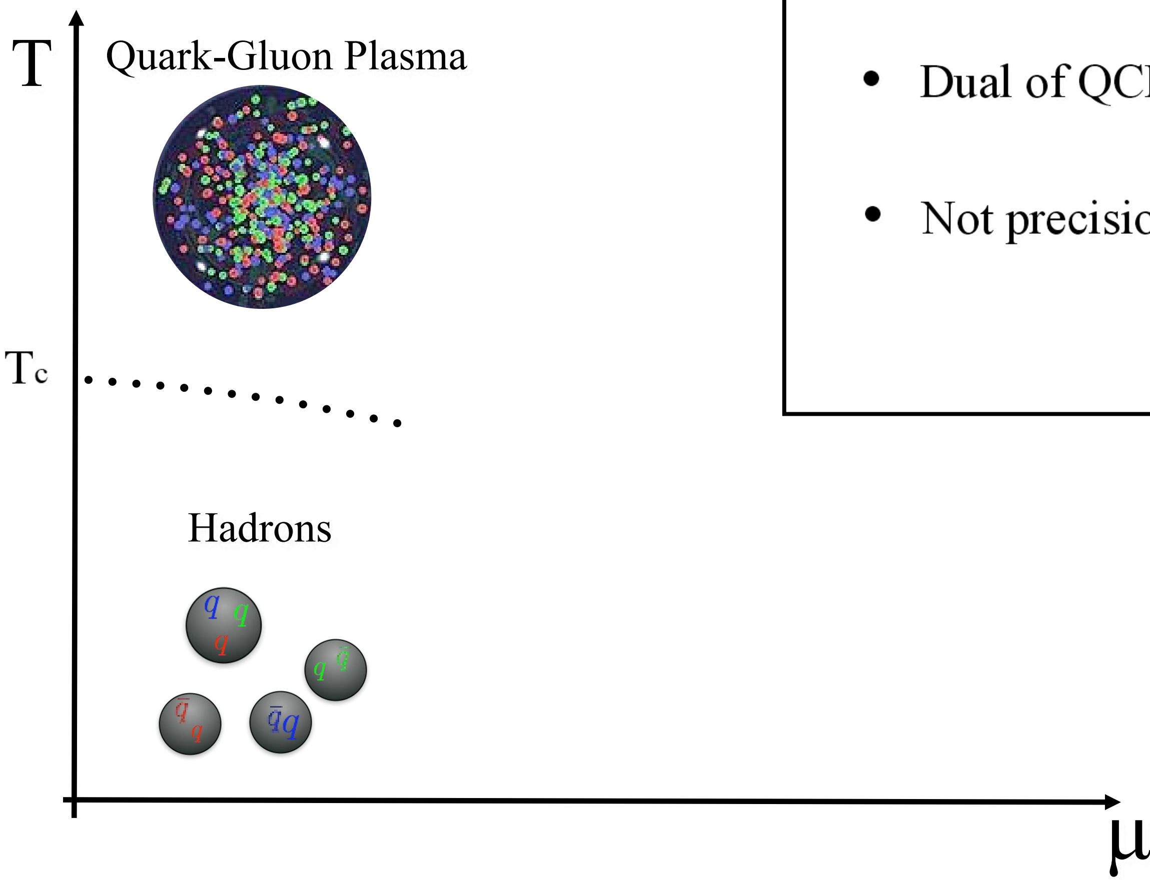
QCD & Holography



Holography

- Strongly coupled QFT
- Out of equilibrium physics

QCD & Holography



Holography

- Strongly coupled QFT
- Out of equilibrium physics
- Dual of QCD not known…
- Not precision holography

  → Qualitative aspects

Holography: Our model

- Gravity with Λ in 3+1 dim :

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda)$$

Holography: Our model

- CFT on Minkowski in 2+1 dim
- Decoupled sector of the stress tensor $T^{\mu\nu}$.

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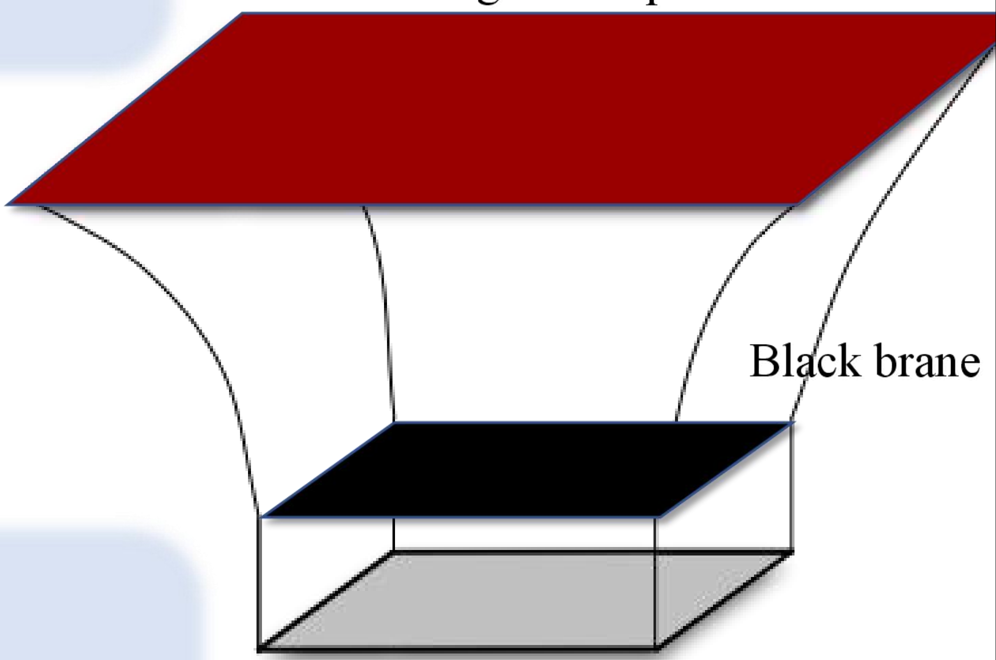

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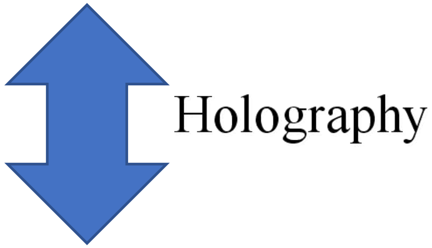

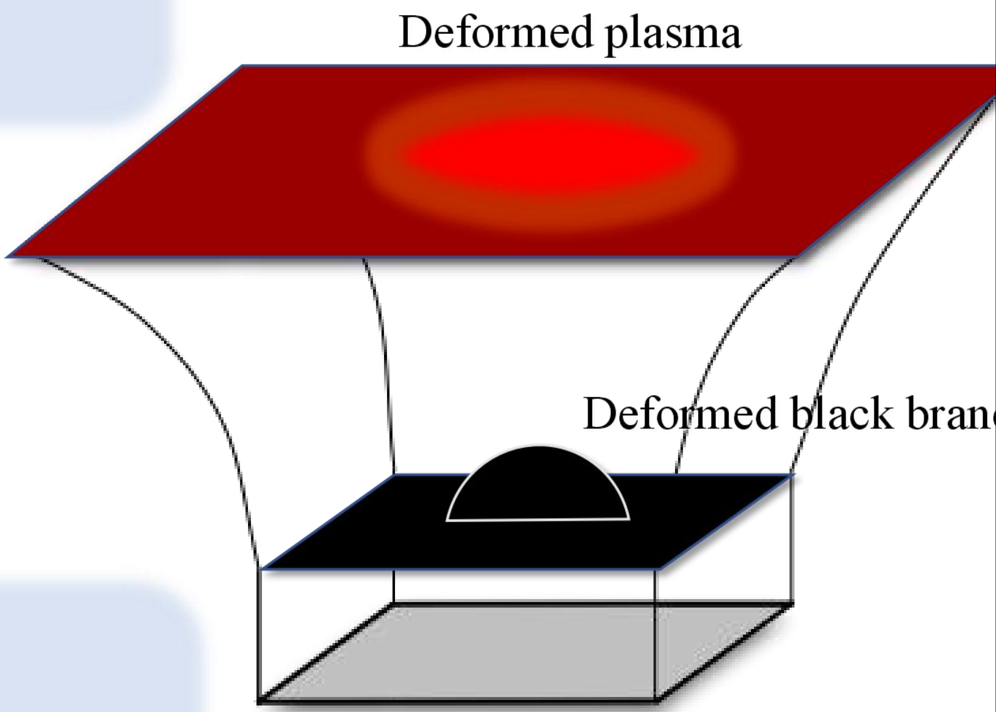

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Real-time quantum dynamics

Holography

Numerical Relativity

Dynamical classical gravity

- Gravity with Λ in 3+1 dim :

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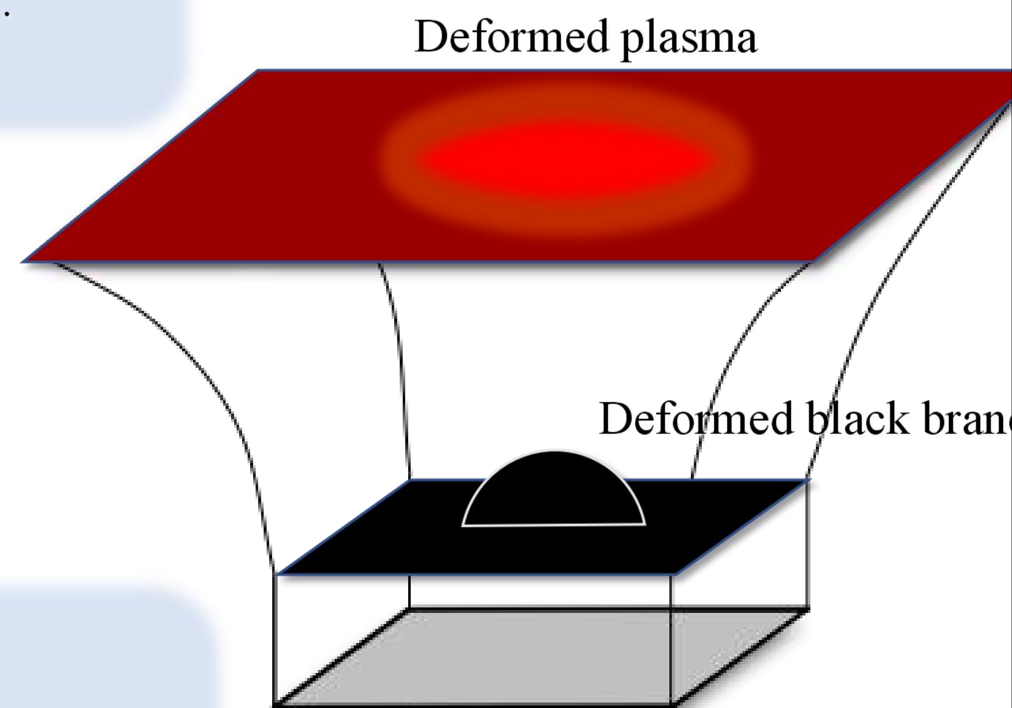
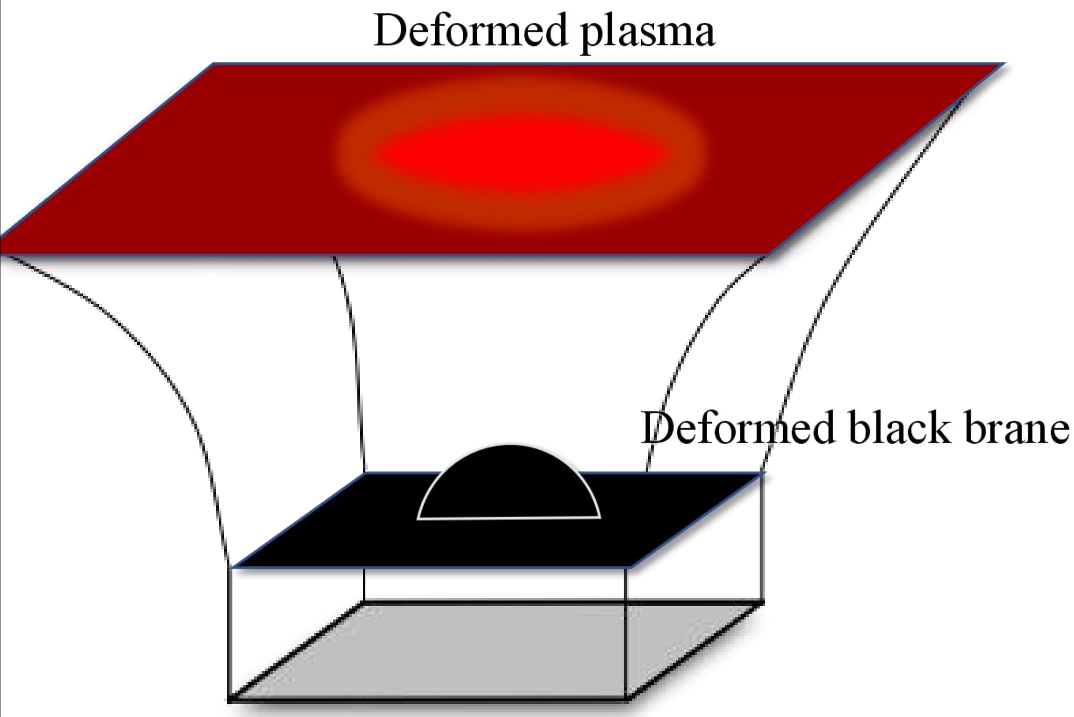


Holographic solution



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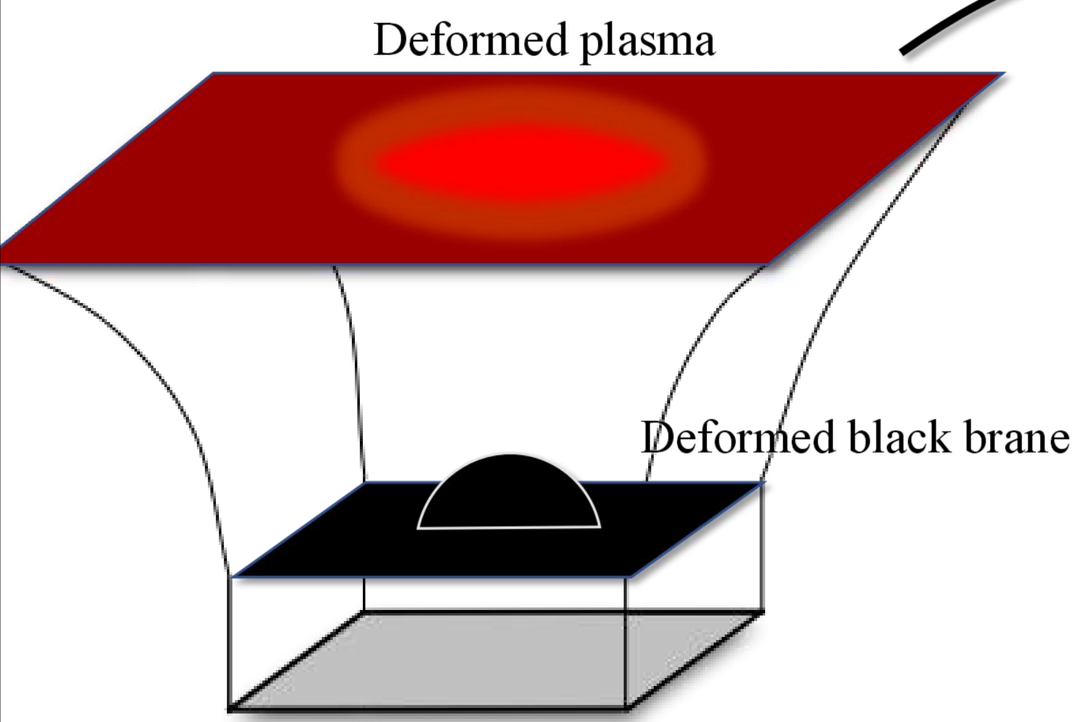
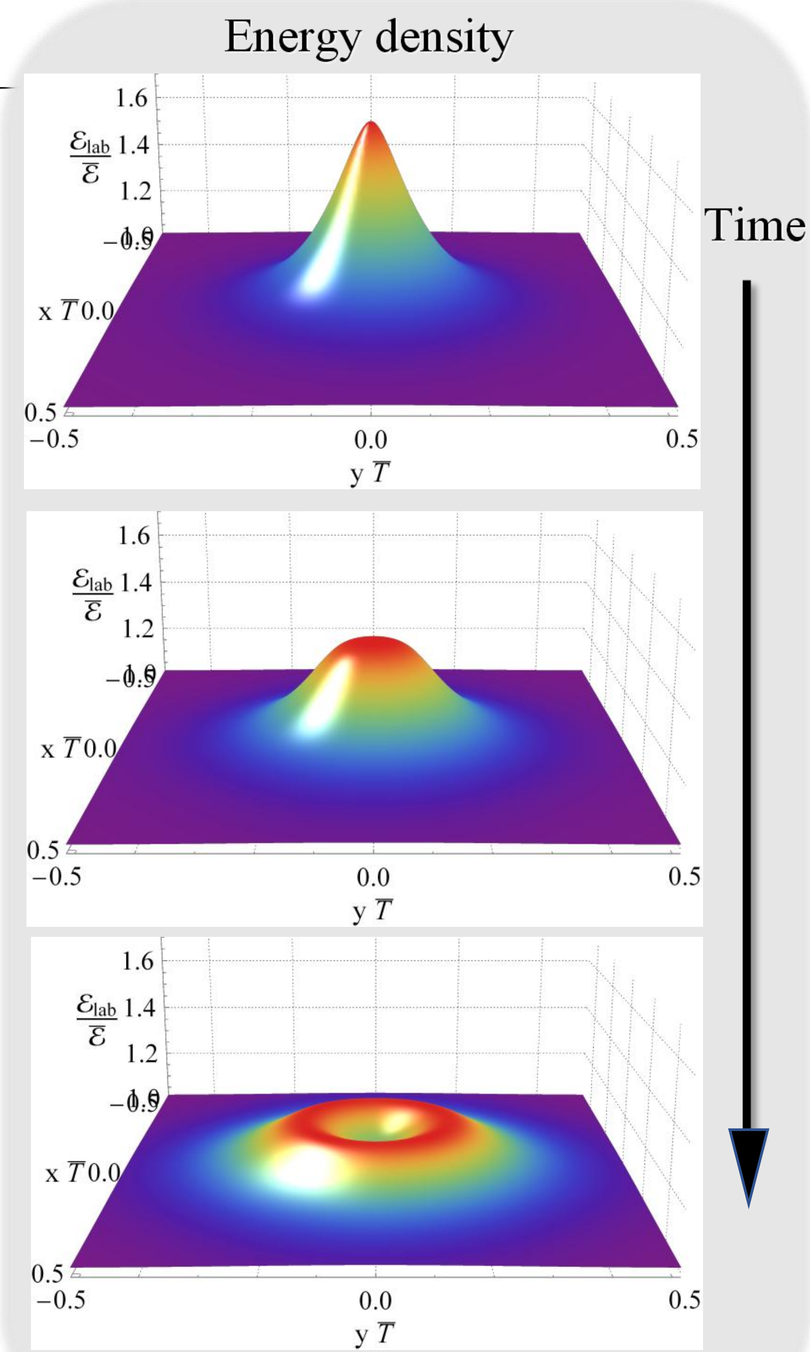
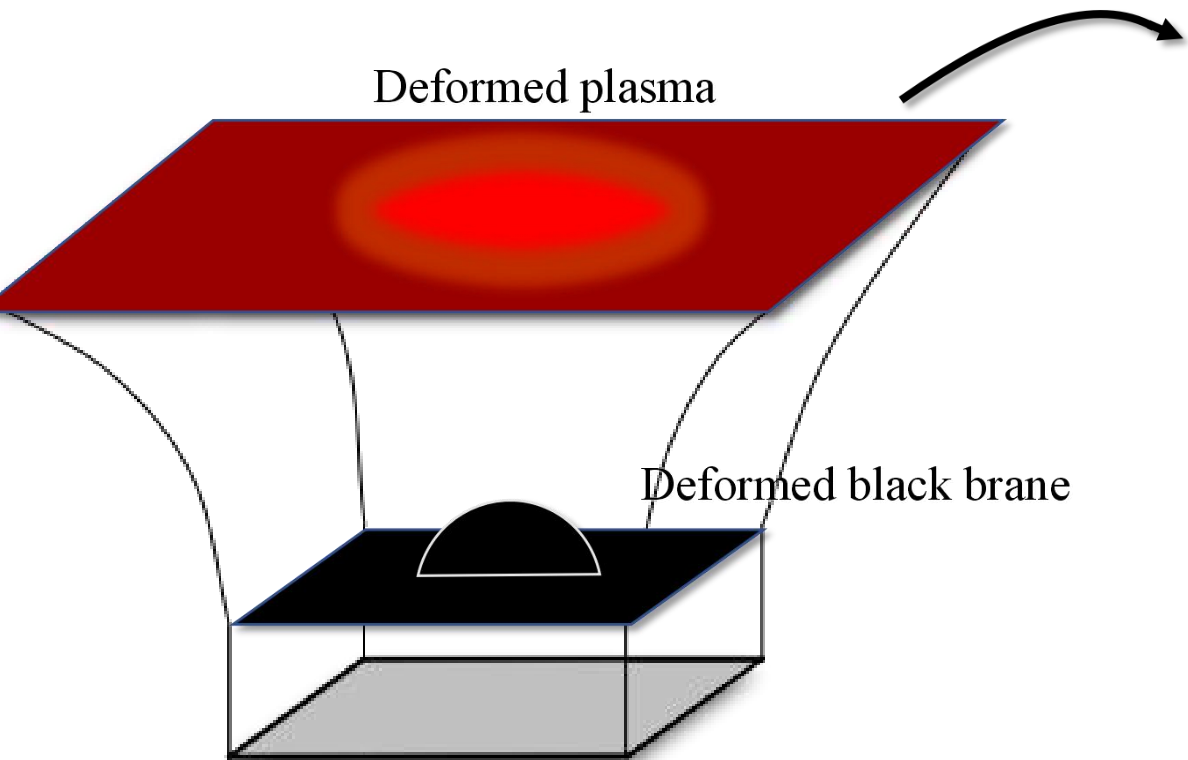
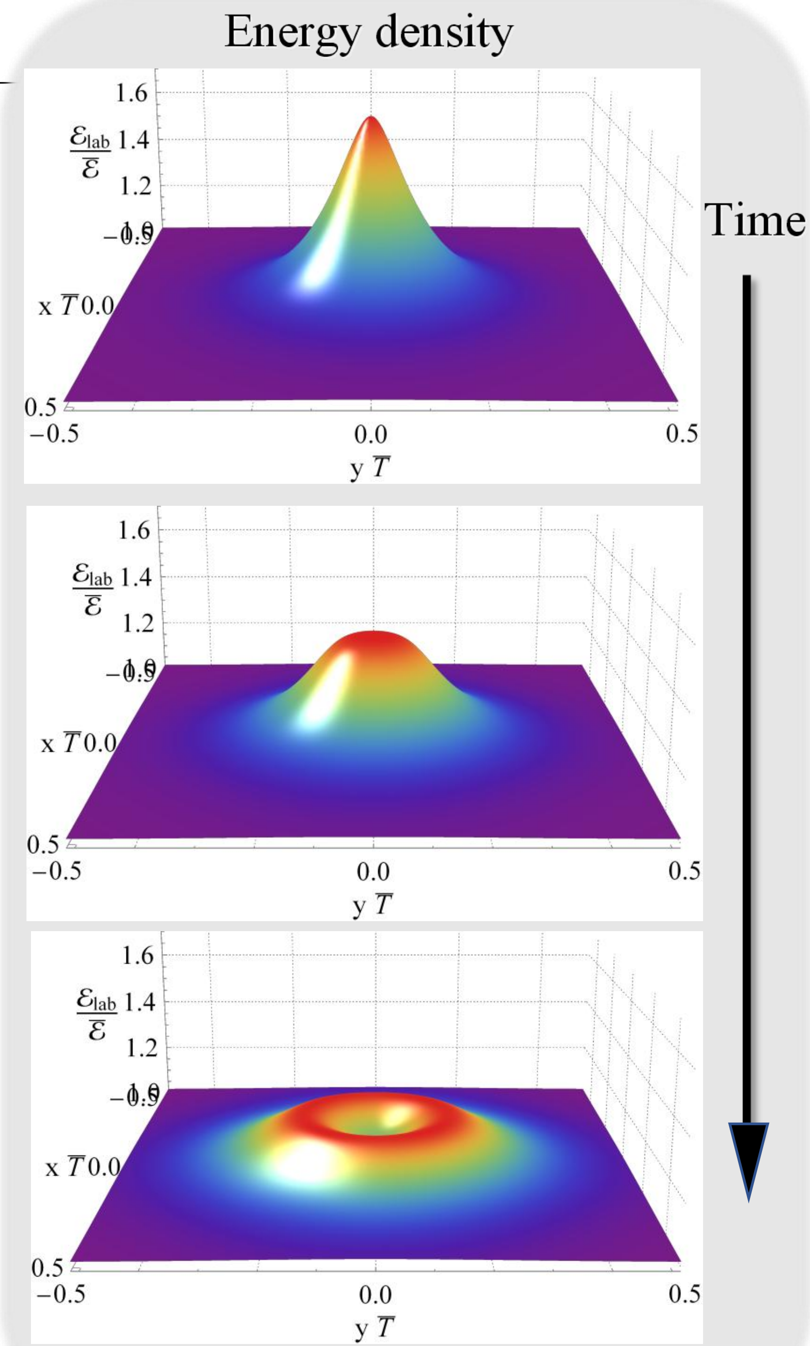




Bantilan, Bea, Figueras '22

Holographic solution





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- We have a microscopic solution
- We want to test the aplicability **hydrodynamics**:
  - Check constitutive relations
  - Time evolutions in hydro

Hydro: Constitutive relations

$$T_{\mu\nu} = T_{\mu\nu}^{ideal} + \partial + \partial^2 + \ldots$$

Gradient expansion

0th 1st 2nd

Hydro: Constitutive relations

$$T_{\mu\nu} = T_{\mu\nu}^{ideal} + \partial + \partial^2 + \ldots$$ Gradient expansion

0th 1st 2nd

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\,\Delta^{\mu\nu} \quad -\eta\sigma^{\mu\nu} \quad -\eta\tau_\pi\left(\dot{\sigma}^{<\mu\nu>} + \frac{3}{2}\sigma^{\mu\nu}\nabla\cdot u\right) + \ldots$$

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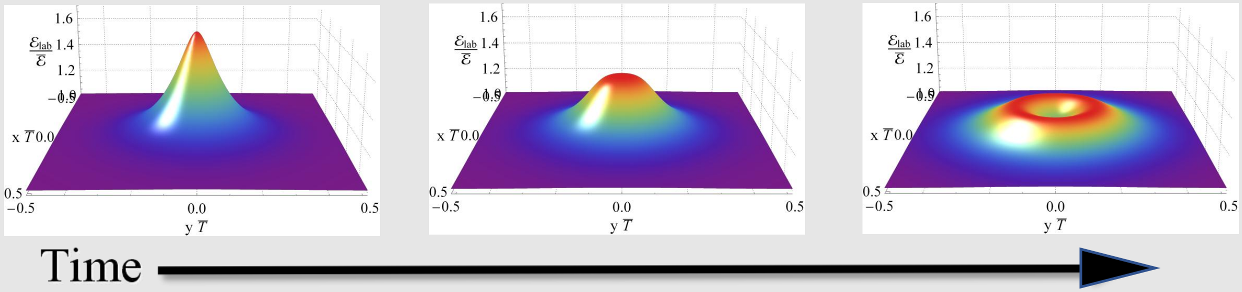

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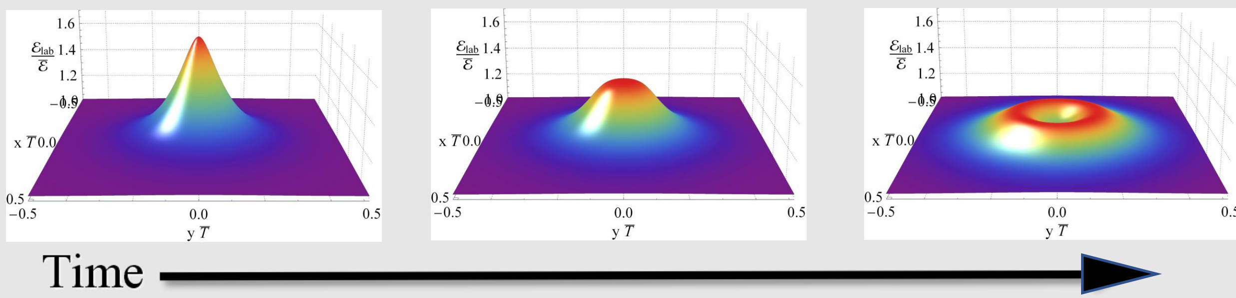
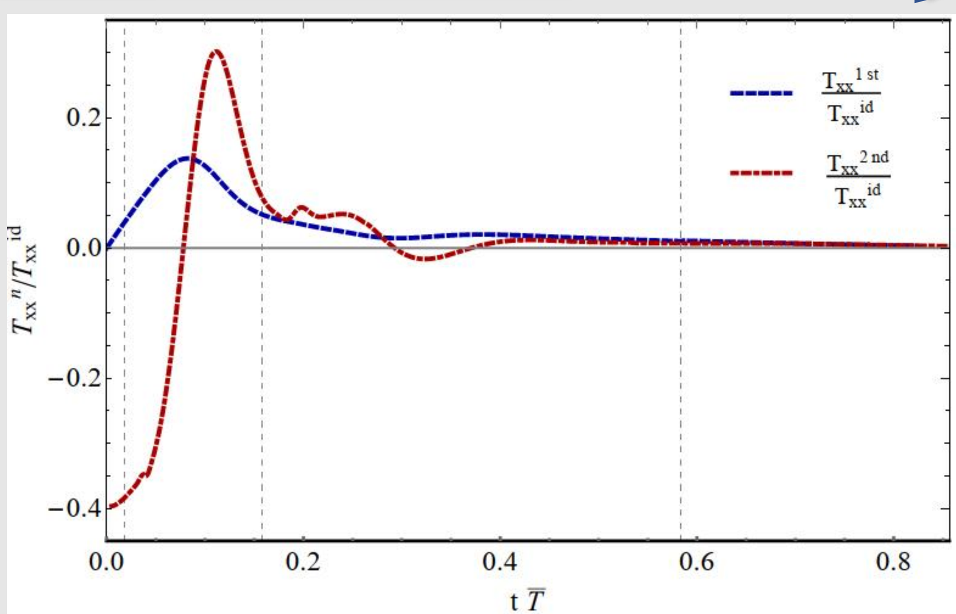


- Off center point
- T_{xx} component



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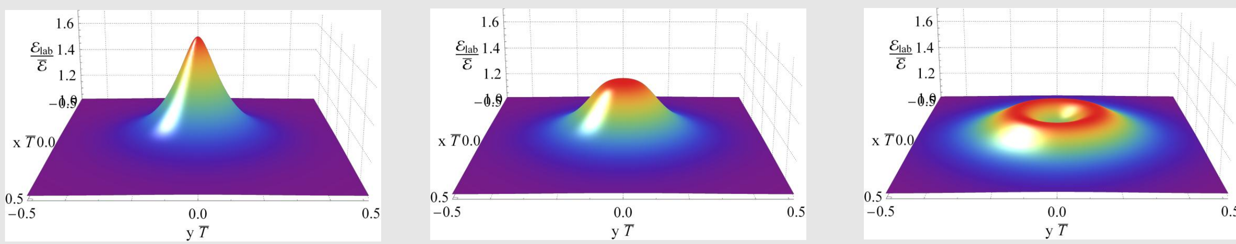
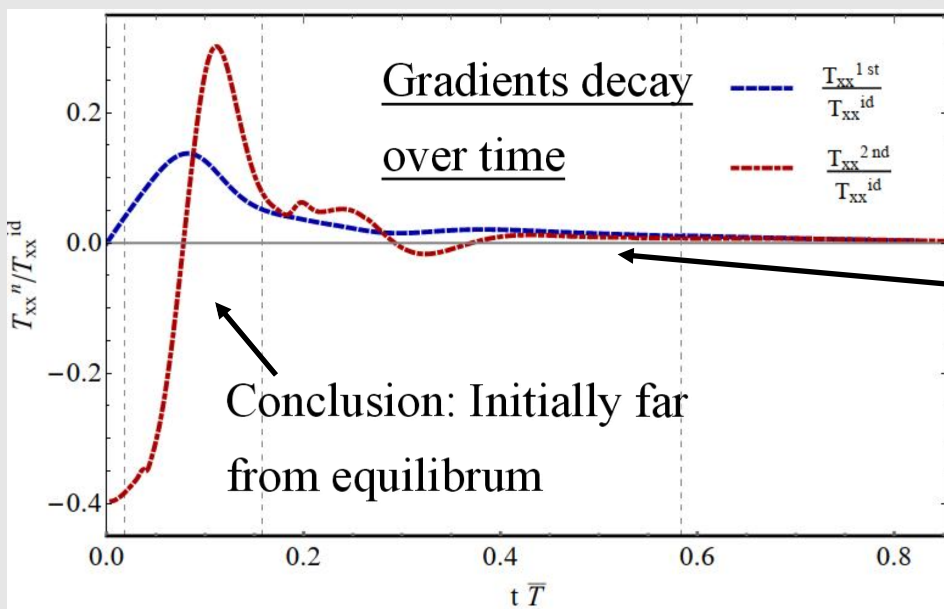


Time

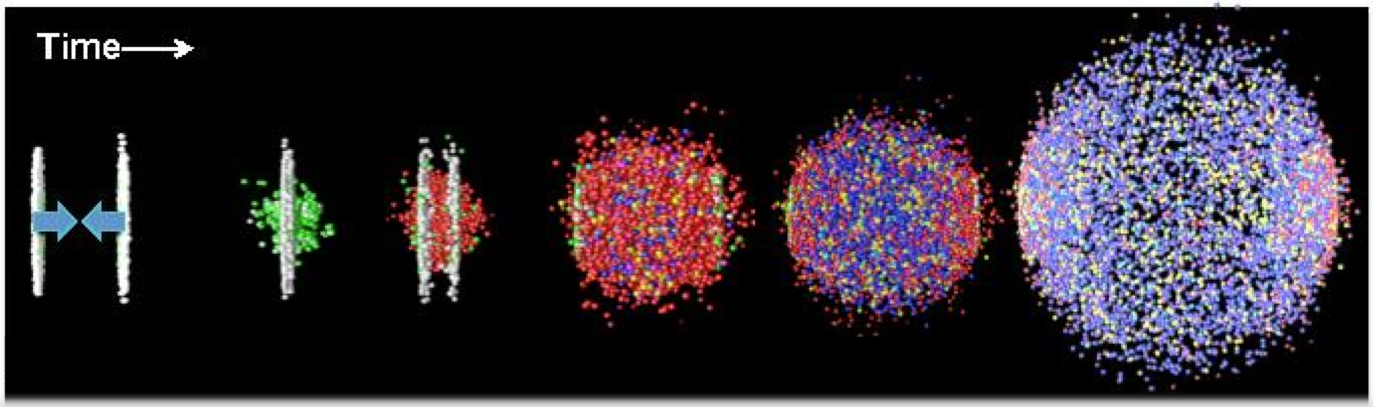
- Off center point
- T_{xx} component



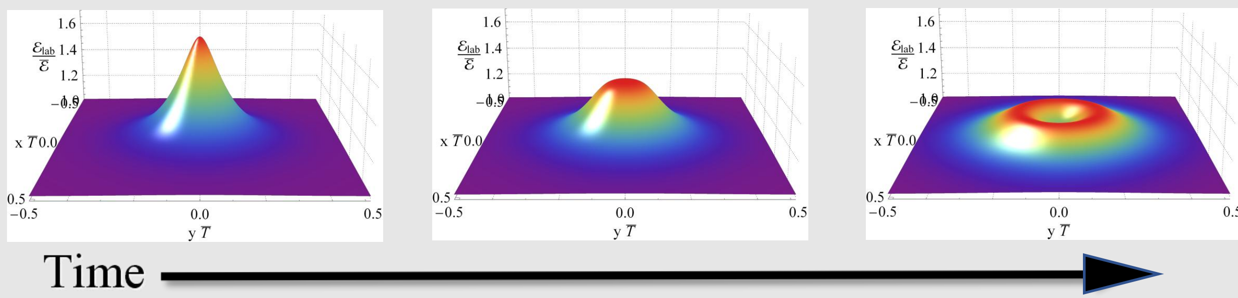
Hydrodynamizes around tT=0.5

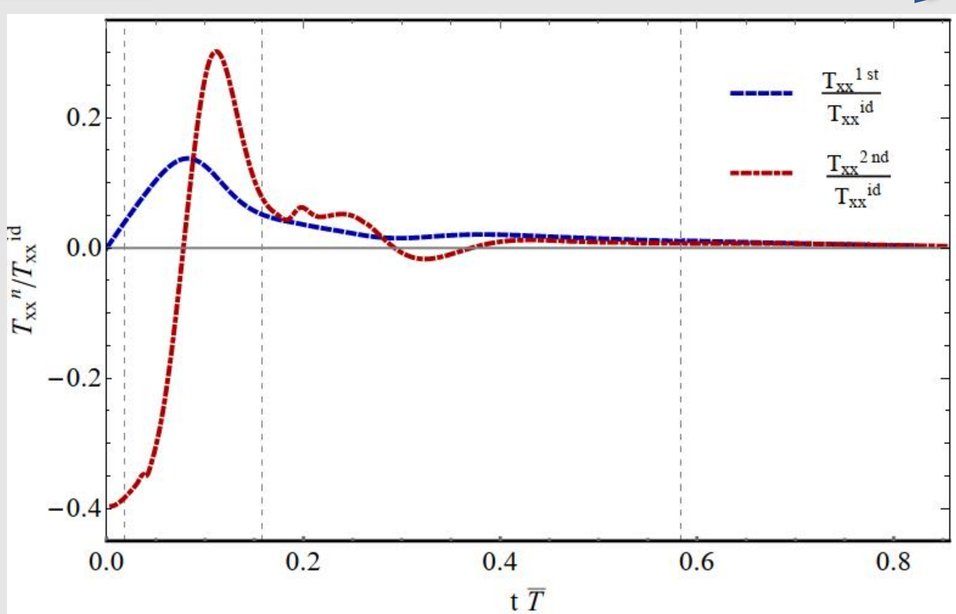
Hydro: Constitutive relations

Heavy-ion collision in QCD



Localized perturbation in our holographic theory





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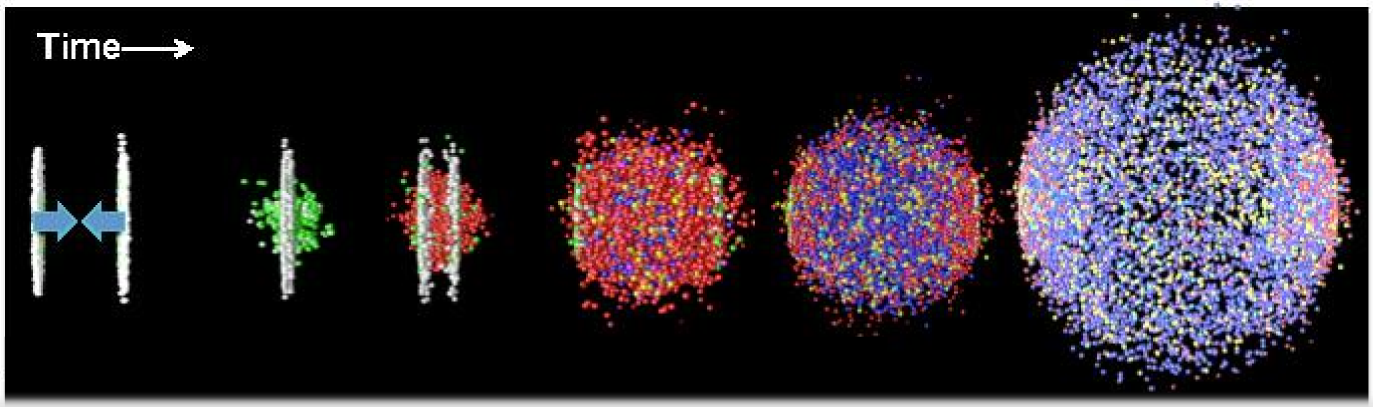

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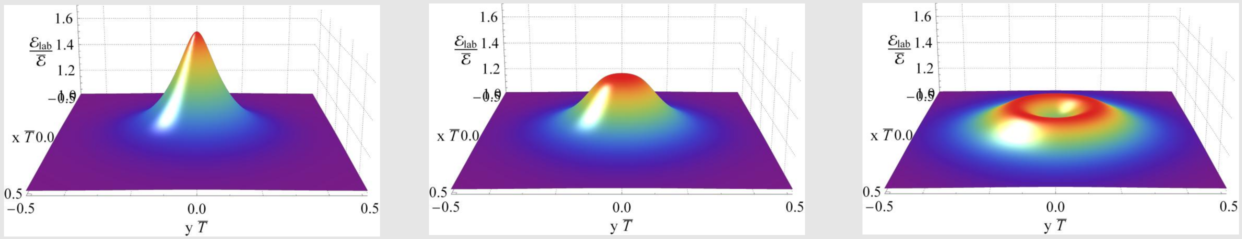
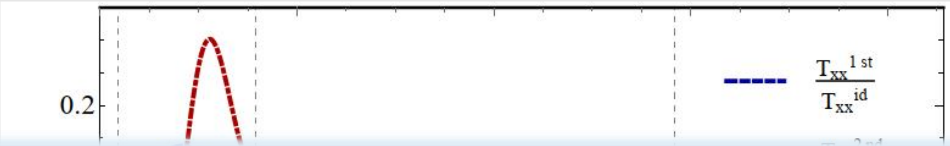


Time

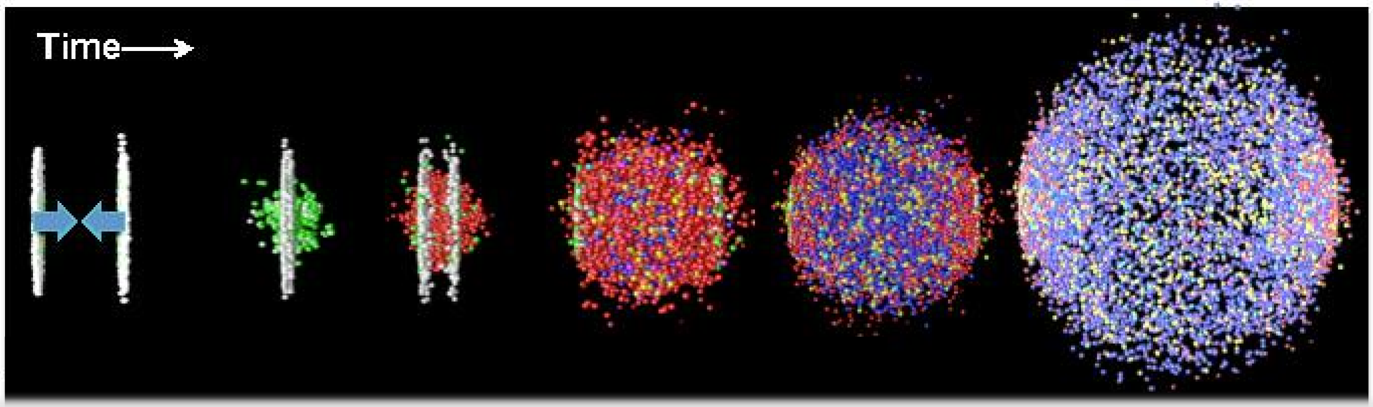


Theories and states are VERY DIFFERENT, but share one aspect:

→ Far from equilibrium and relaxes to a hydro regime

Hydro: Constitutive relations

Heavy-ion collision in QCD



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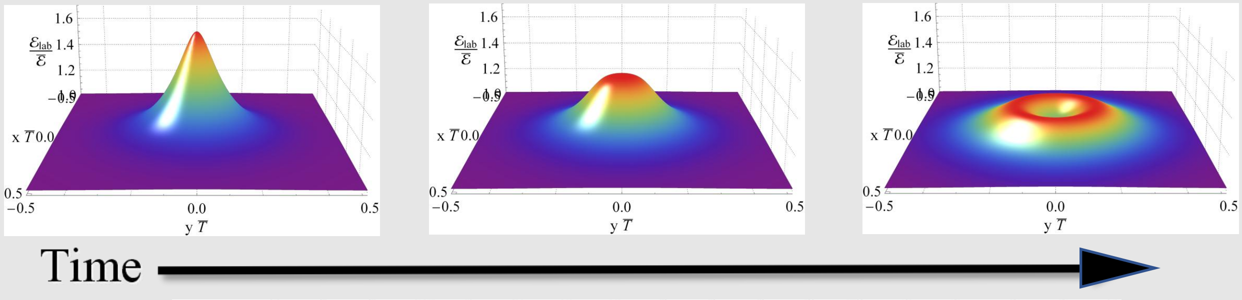

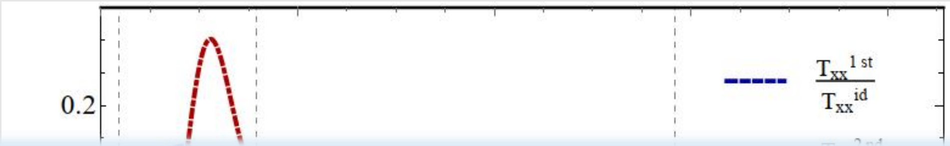

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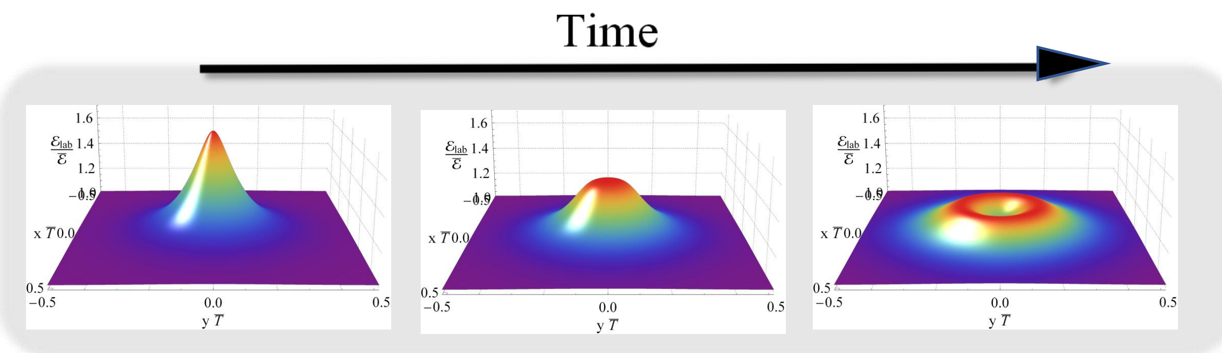
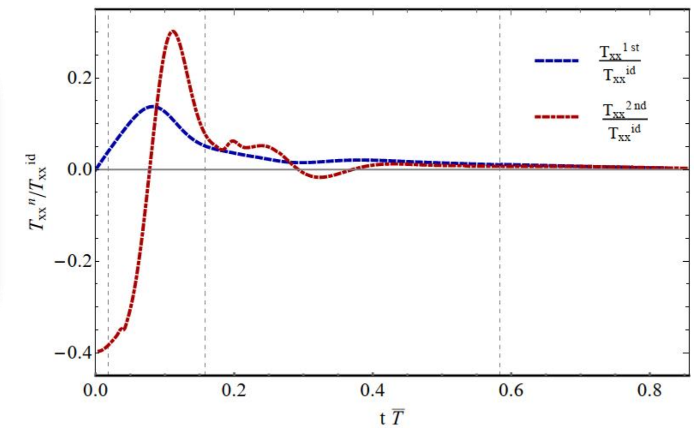
→ Far from equilibrium and relaxes to a hydro regime

→ We initialize the hydro codes at different times, in analogy to the QGP

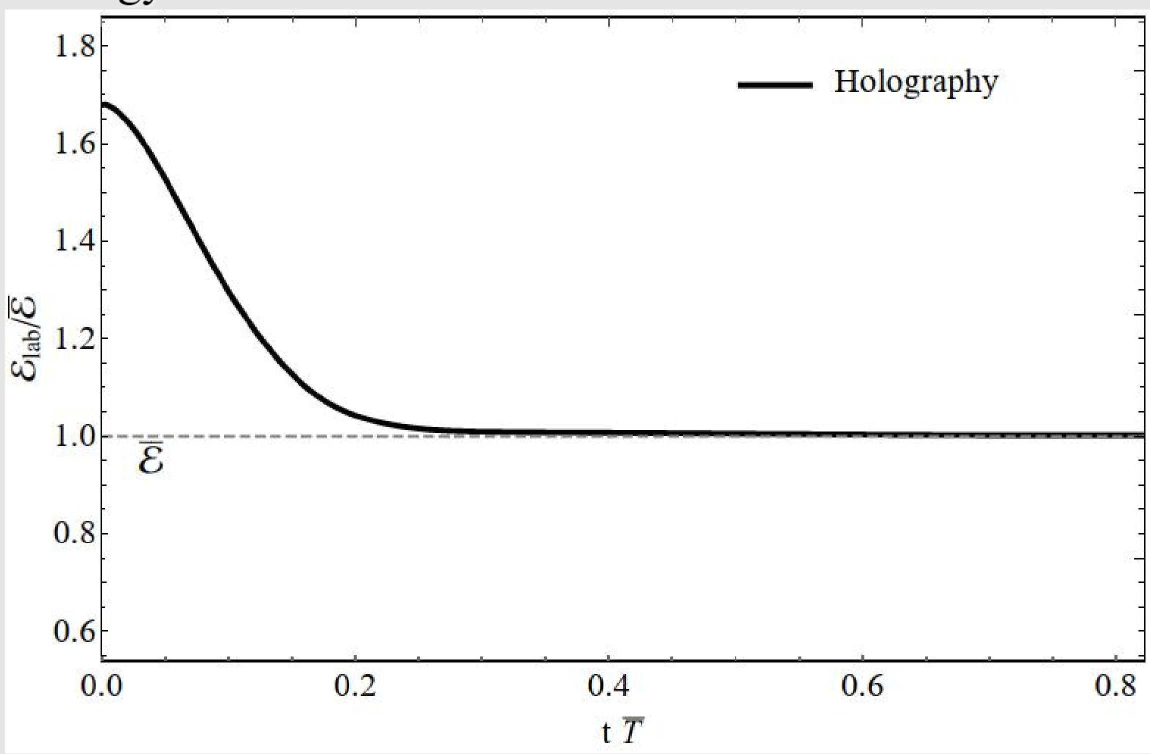
Evolutions: holography vs hydrodynamics

Evolutions: holography vs hydrodynamics





Energy at the center of the domain:



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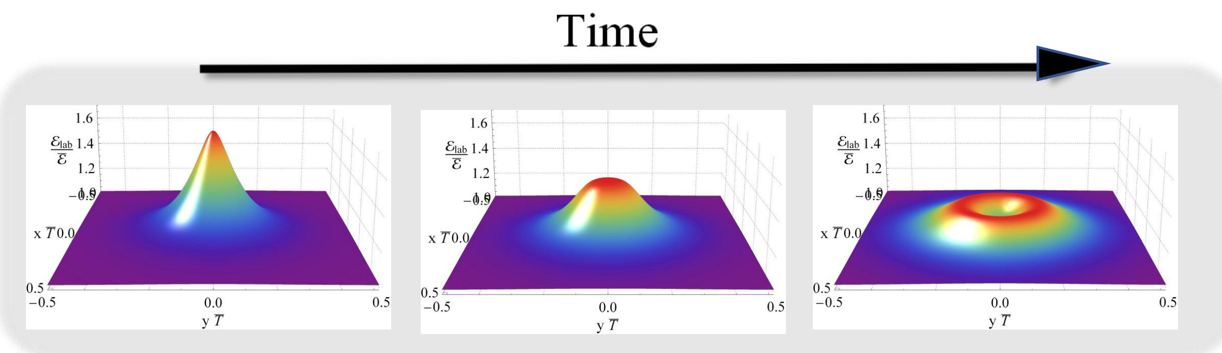
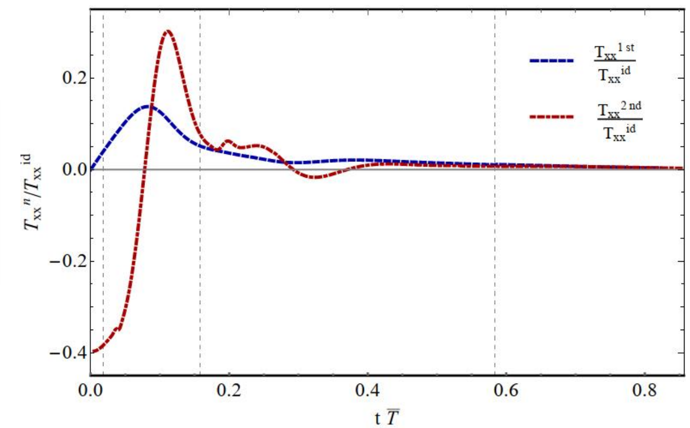
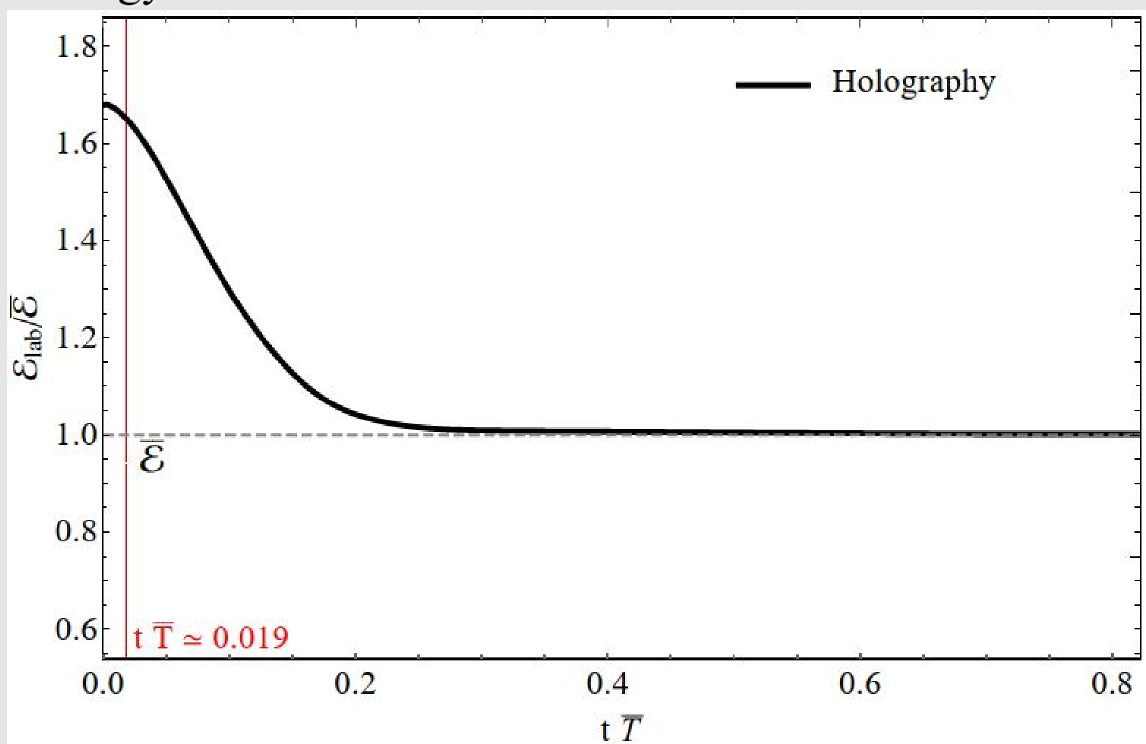
Evolutions: holography vs hydrodynamics





Energy at the center of the domain:



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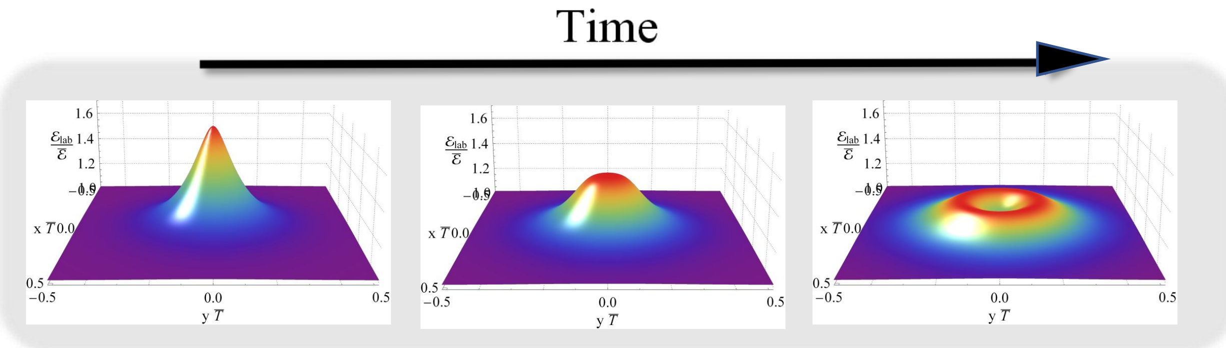
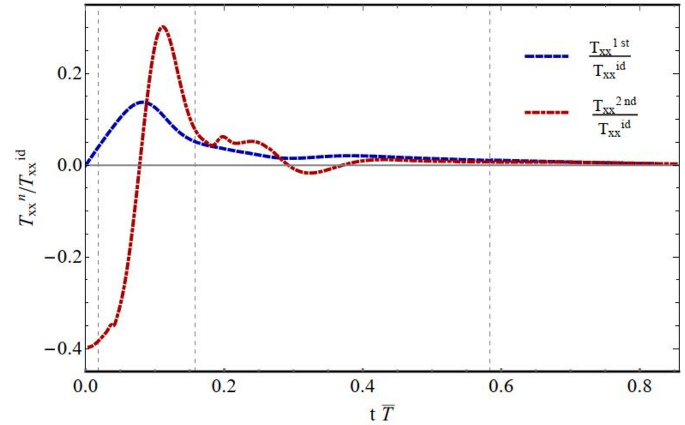
Evolutions: holography vs hydrodynamics





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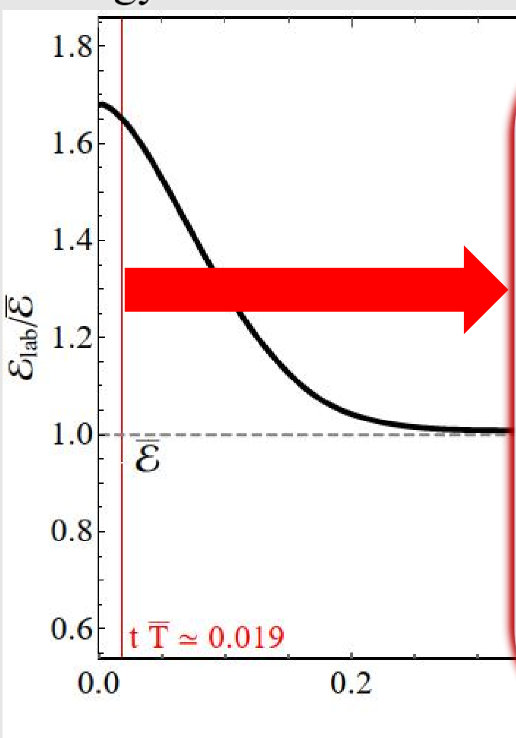

Initial data for hydro codes

MIS-type ε_{loc} u_x u_y Π_{xx} Π_{xy}

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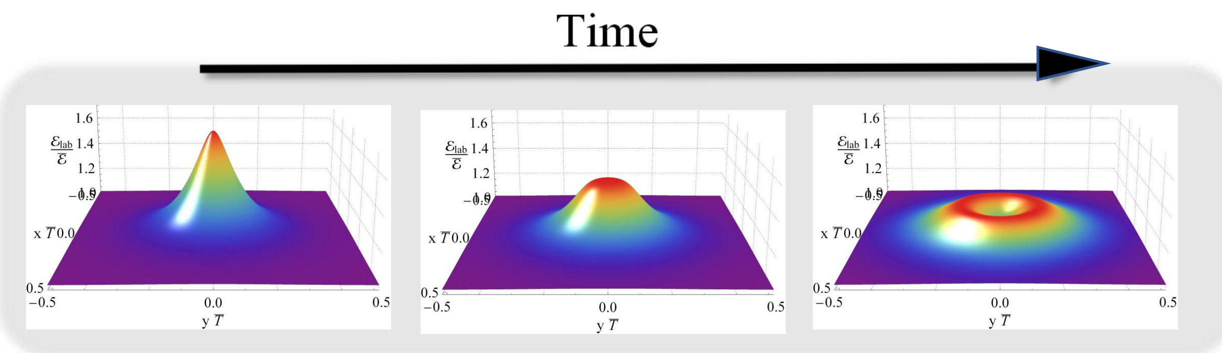
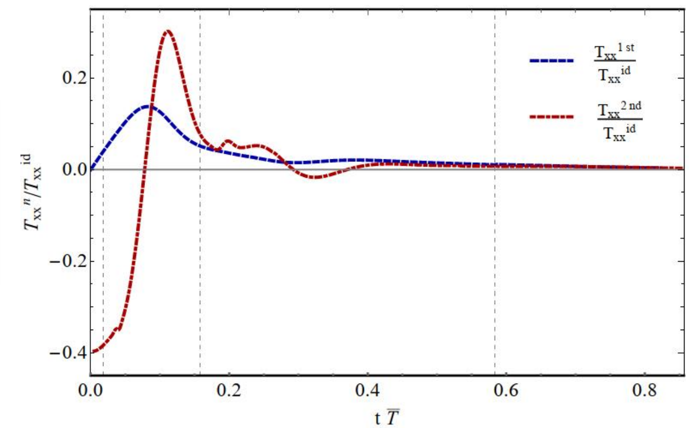
Evolutions: holography vs hydrodynamics





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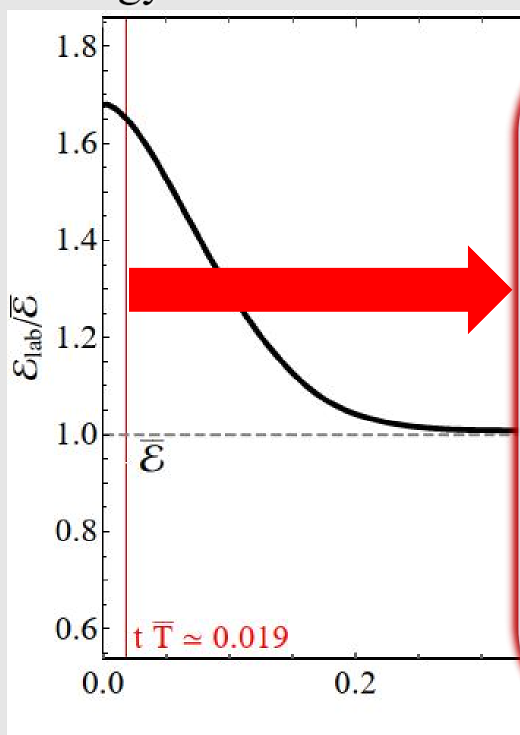

Initial data for hydro codes

MIS-type ε_{loc} u_x u_y Π_{xx} Π_{xy}

BDNK Change to causal frame $a_1 = 6, a_2 = 4$

ε_{loc} u_x u_y

$\partial_t \varepsilon_{loc}$ $\partial_t u_x$ $\partial_t u_y$

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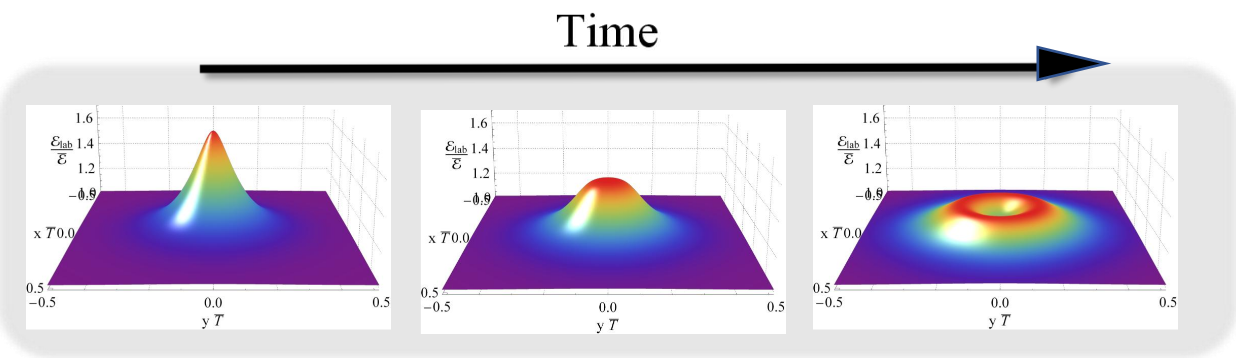
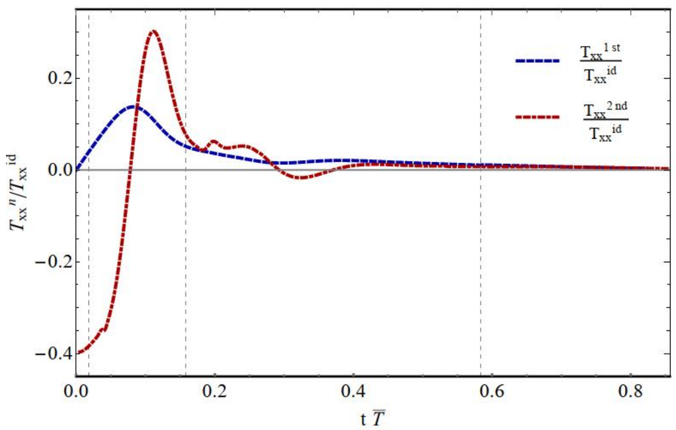
Evolutions: holography vs hydrodynamics





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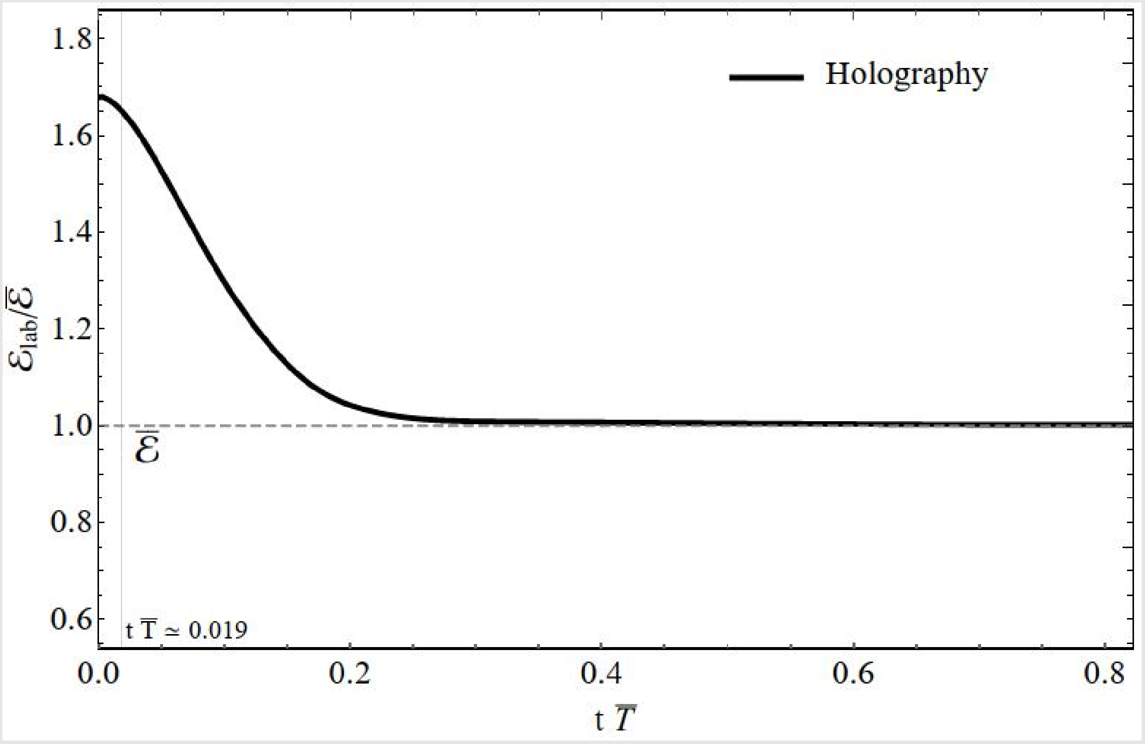

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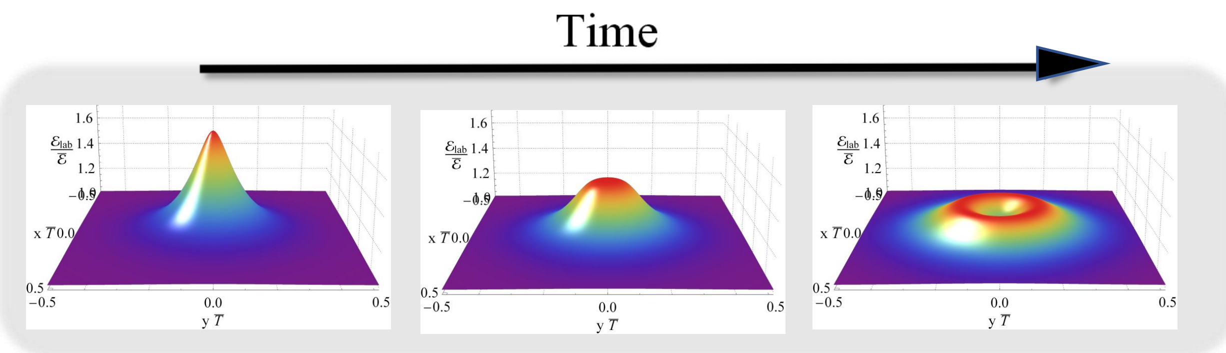
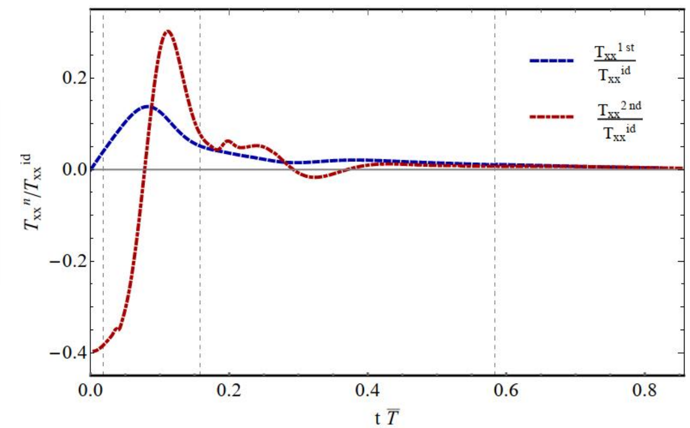
Evolutions: holography vs hydrodynamics





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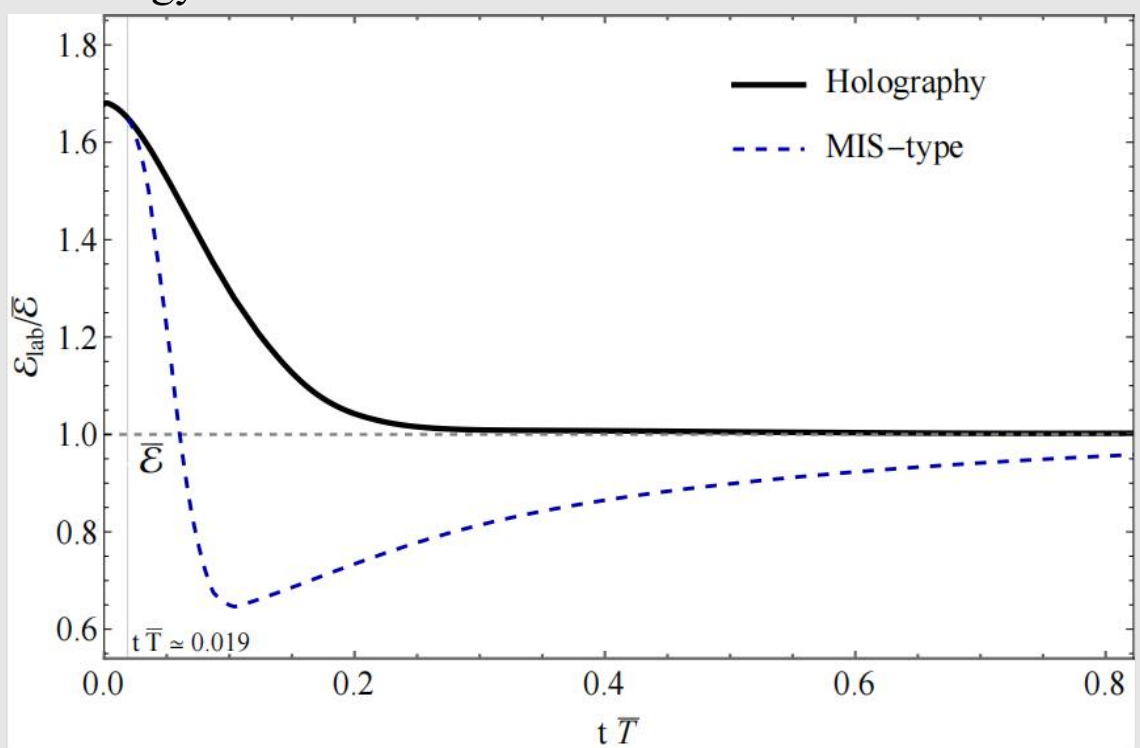

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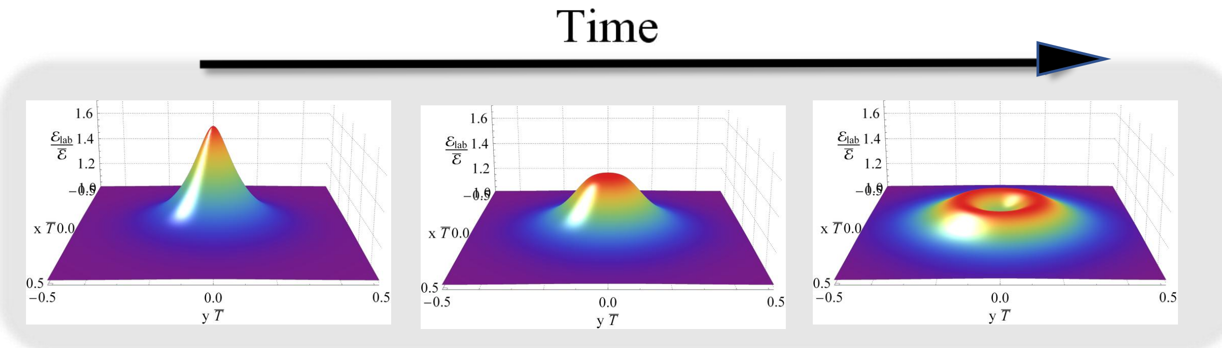
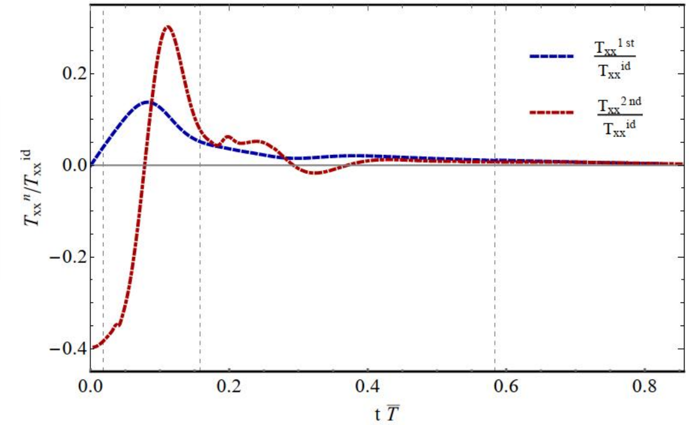
Evolutions: holography vs hydrodynamics





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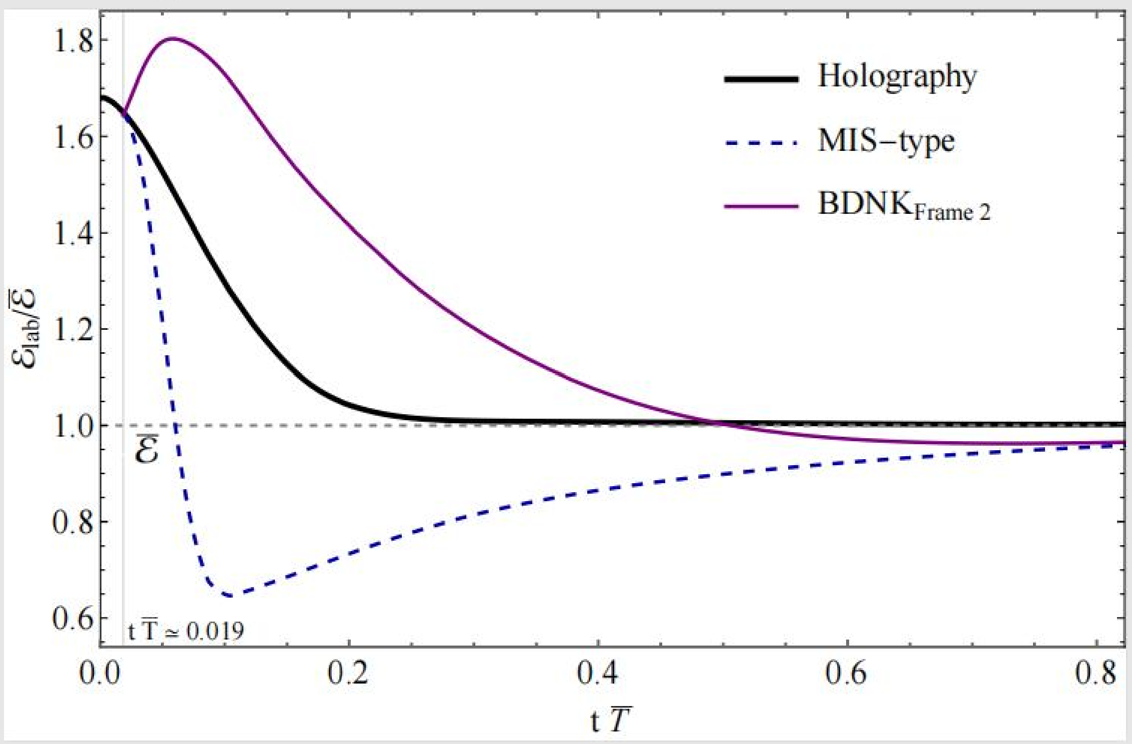

Bantilan, Bea, Figueras ‘22

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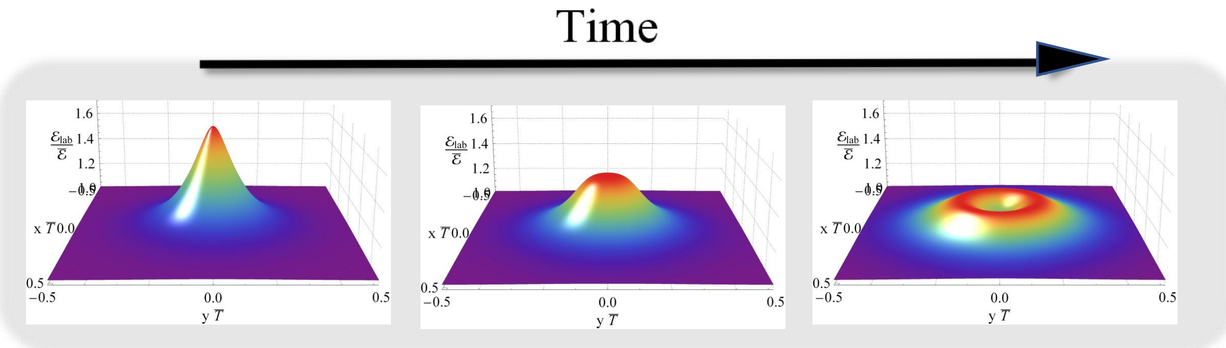

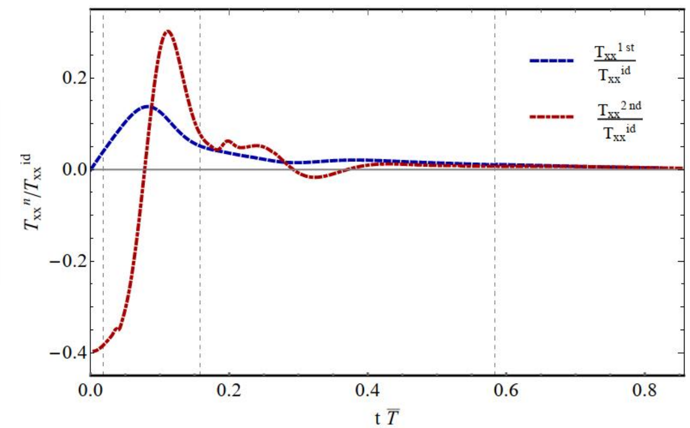

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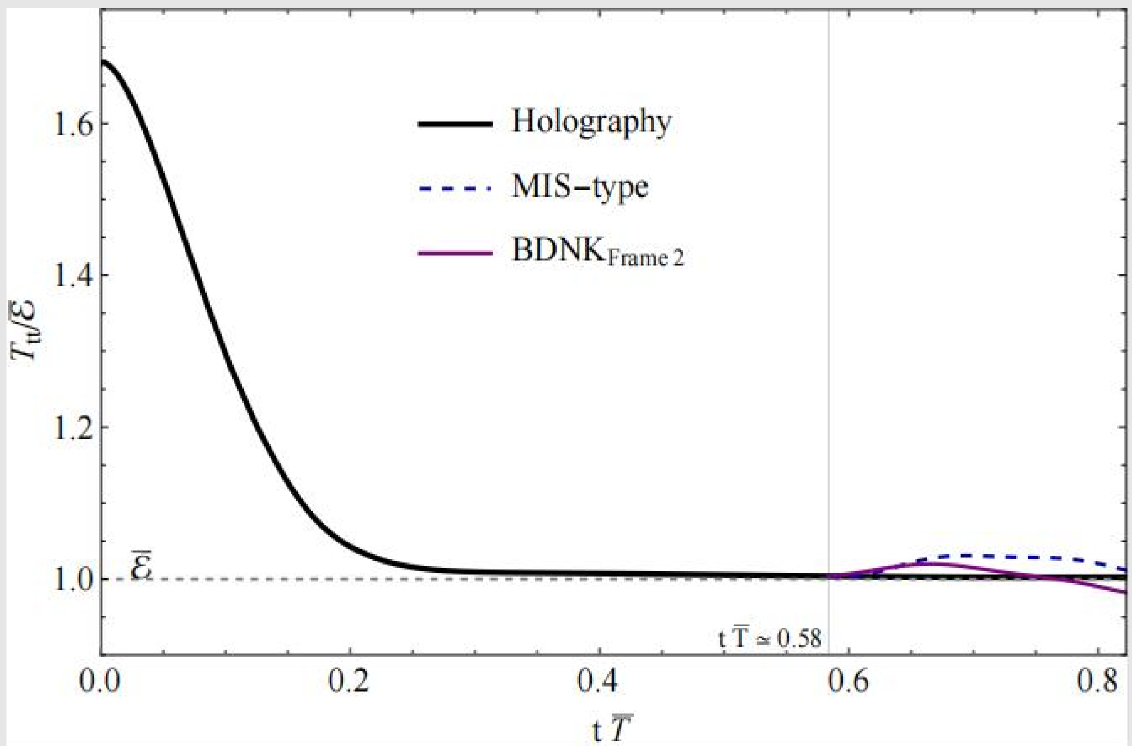
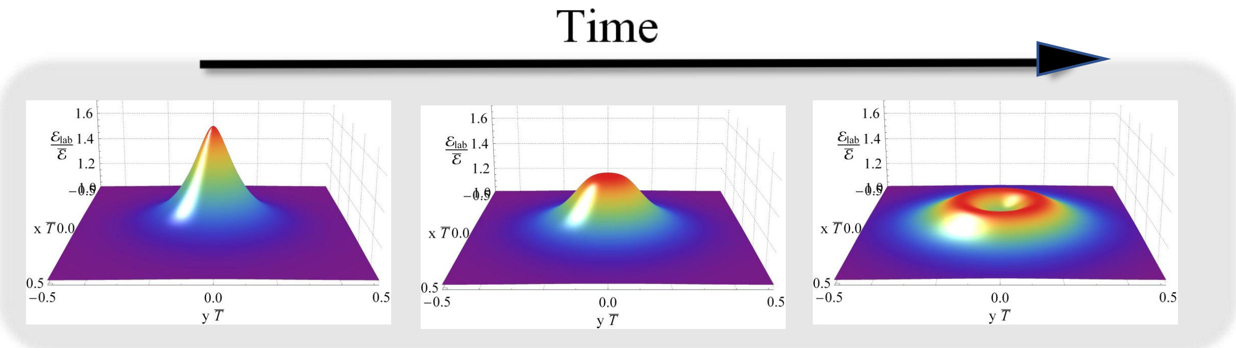
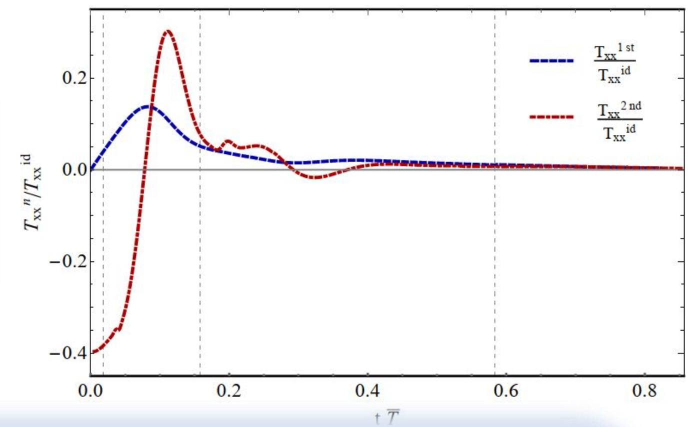


Bantilan, Bea, Figueras ‘22

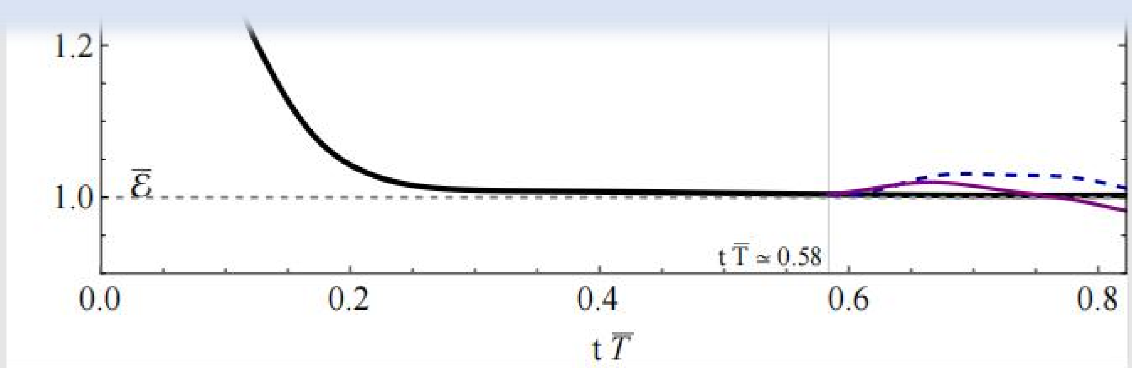
Evolutions: holography vs hydrodynamics





Main conclusions:

- Gradients dilute with time ⟶ hydro evolutions provide a better description at late times.
- Evolutions in BDNK: provides a physically sensible description of the system in the hydro regime, and compatible with MIS.



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Future directions

BDNK evolutions: Future

BDNK might be a very good alternative to MIS

BDNK evolutions: Future

BDNK might be a very good alternative to MIS

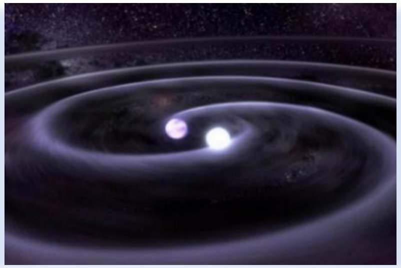
There is a full research program to be performed:

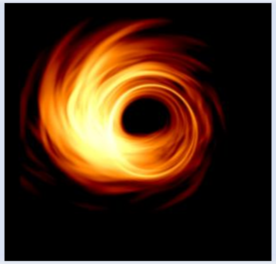
BDNK evolutions: Future

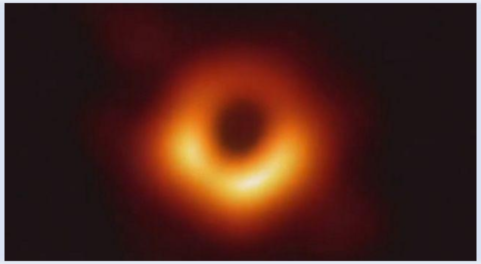
BDNK might be a very good alternative to MIS

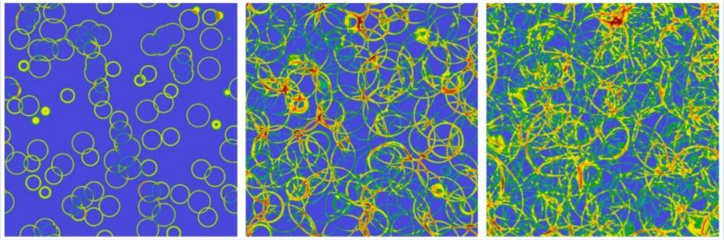
There is a full research program to be performed:

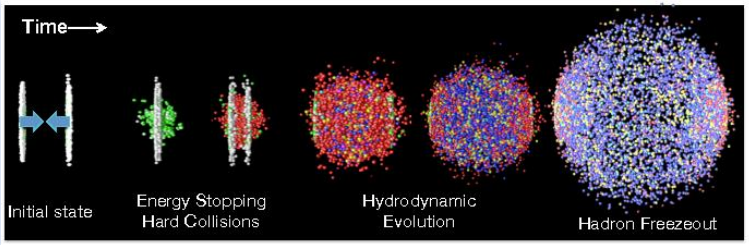
→ Implementing BDNK in these physical systems of interest











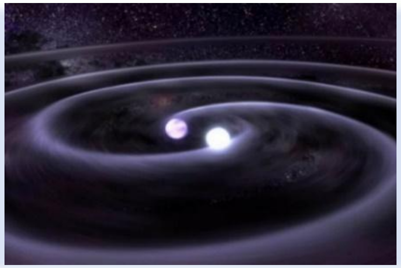
BDNK evolutions: Future

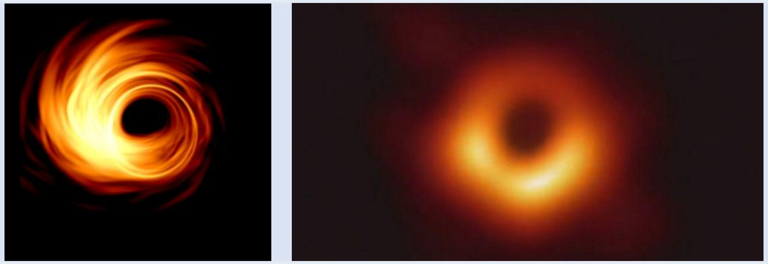
BDNK might be a very good alternative to MIS

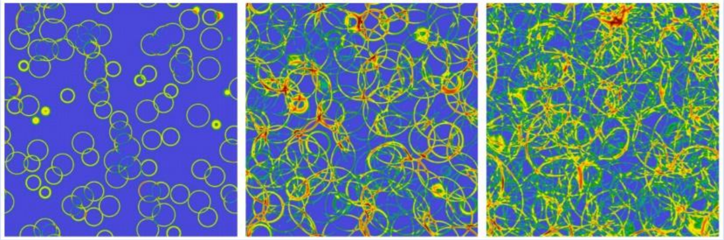
There is a full research program to be performed:

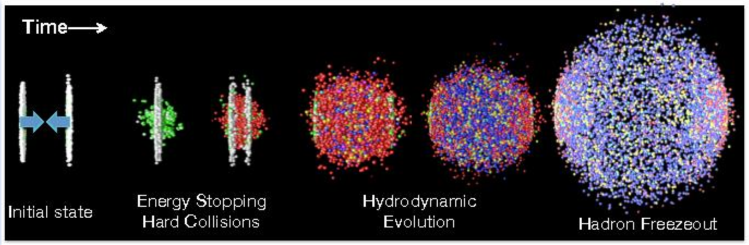
⟶ Implementing BDNK in these physical systems of interest

Work in progress...

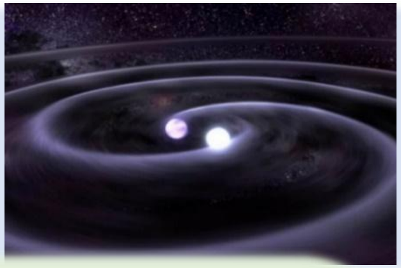




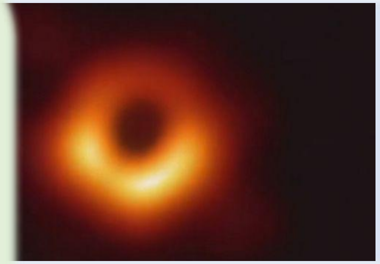




BDNK evolutions: Future



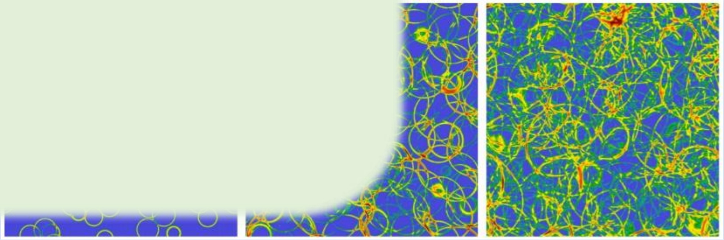
BDNK might be



There is a full r

→ Impleme
systems c

Thank you!



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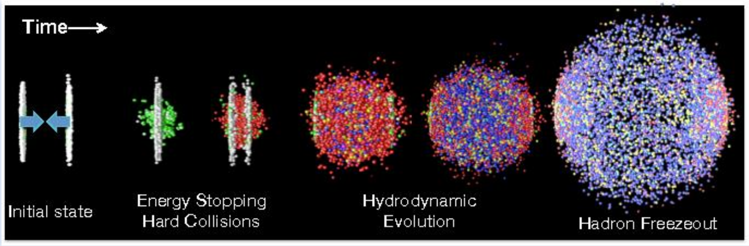
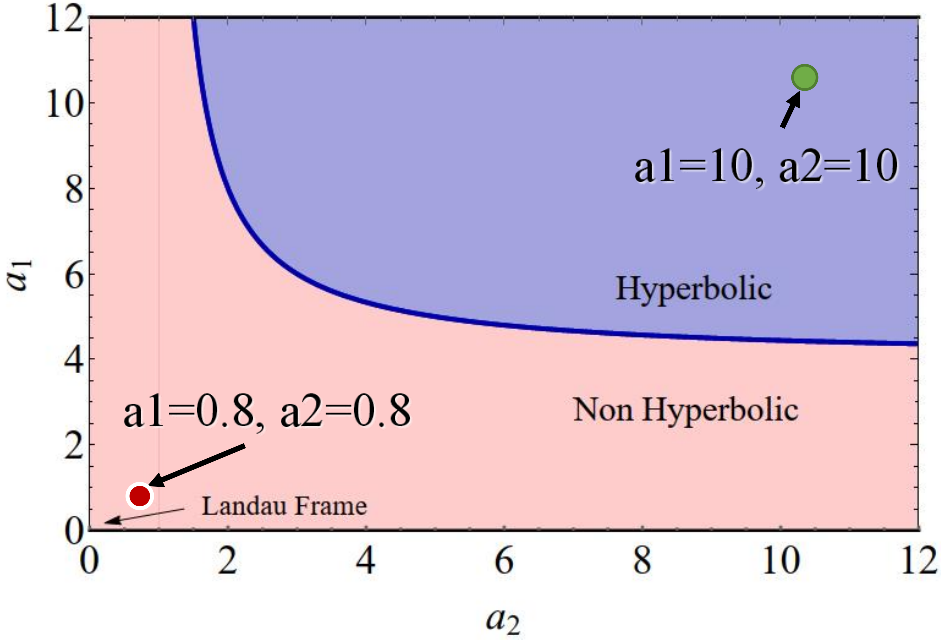


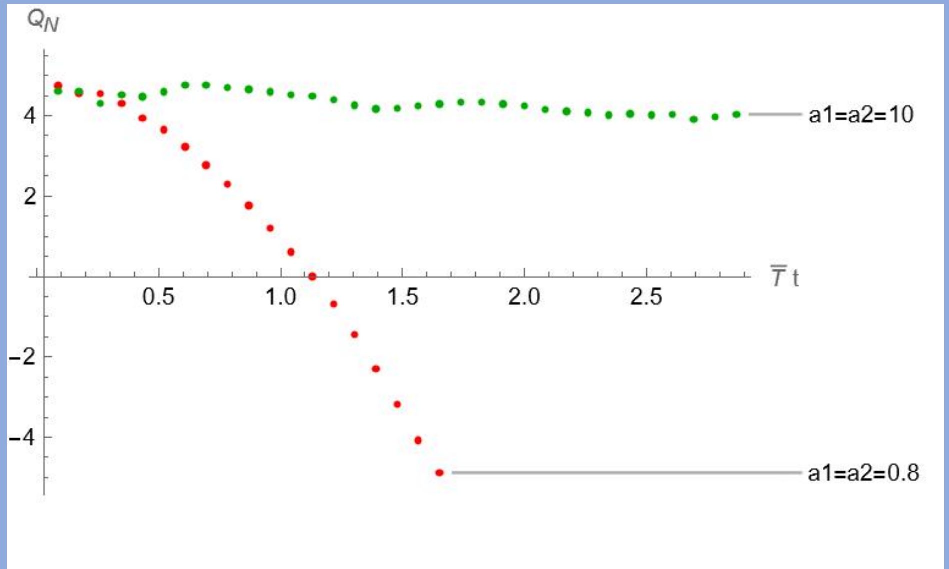
Thank you!

BDNK: Acausal region

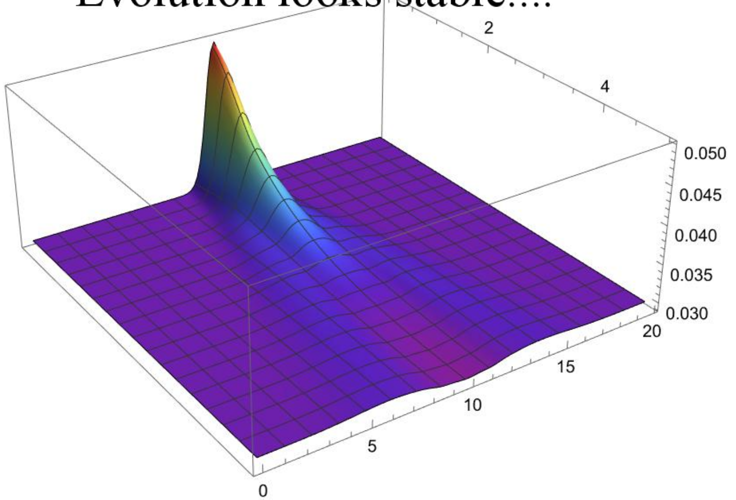
- BDNK eqs. are hyperbolic if:

$$a_2 > 1\,, \qquad a_1 > \frac{4\,a_2}{a_2 - 1}\,.$$





Evolution looks stable....

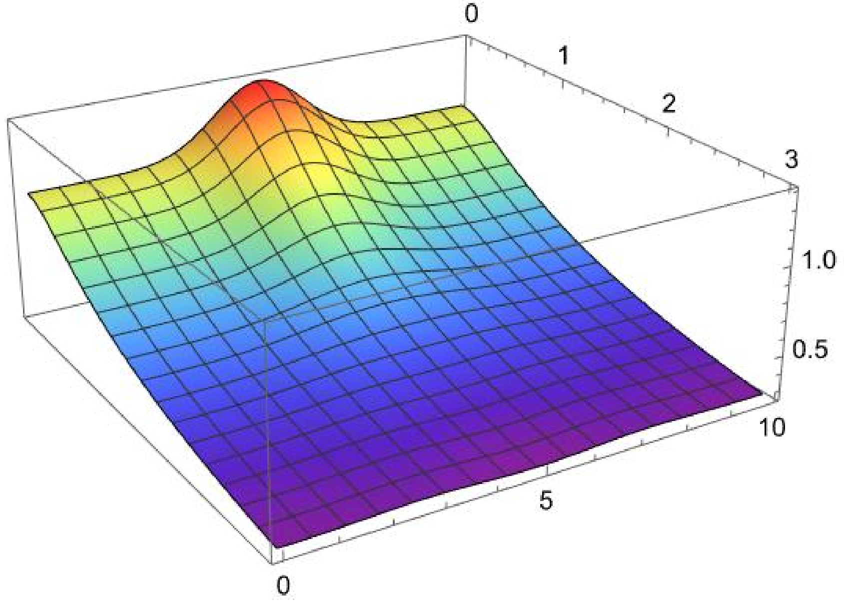


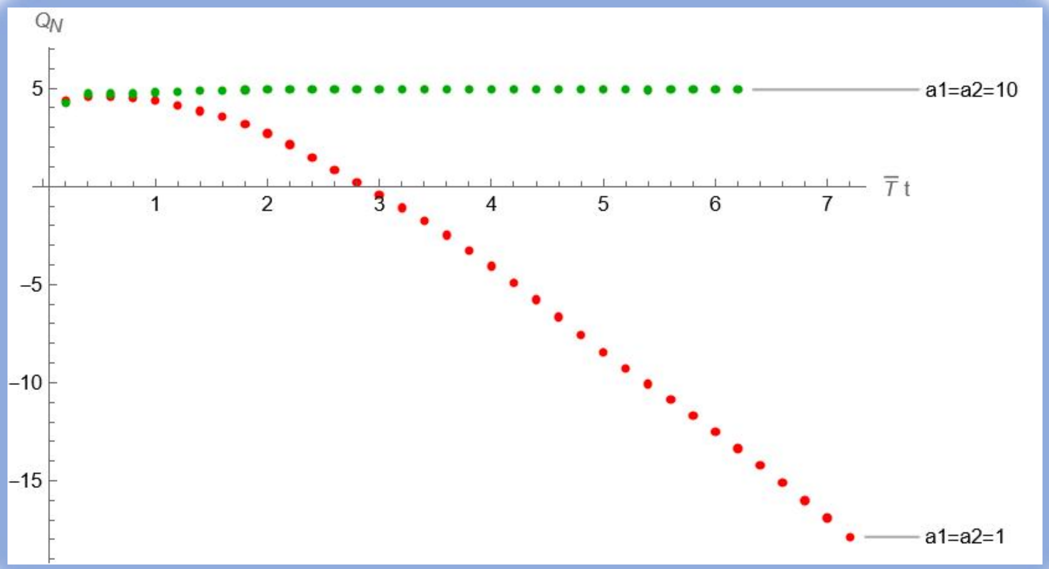
Backup slides: BDNK: 3+1, boost invariant

- 3+1 theory
- Boost invariant, 2+1 dynamics

⟶ Similar to hydro codes used to describe the QGP

Evolution looks stable....





⟶ Similar conclusions!

Hydro equations

- Conformal theory in 2+1 dimensions

- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

- First order hydro: **Landau frame**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} - \eta\sigma^{\mu\nu} \quad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

Not hyperbolic…

- Usual fix: **MIS-type**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \eta\tau_\pi \left(\dot{\sigma}^{<\mu\nu>} + \frac{3}{2}\sigma^{\mu\nu}\nabla \cdot u \right)$$

New variable

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi \left(\dot{\Pi}^{<\mu\nu>} + \frac{3}{2}\Pi^{\mu\nu}\nabla u \right)$$

New equation

Hyperbolic!!

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi \left(\dot{\Pi}^{<\mu\nu>} + \frac{3}{2}\Pi^{\mu\nu}\nabla u \right)$$

Backup slides: Hydro equations

- **Conformal theory**

- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} \Rightarrow \nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

- First order hydro: **Landau frame**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} - \eta\sigma^{\mu\nu} \Rightarrow \nabla_\mu T^{\mu\nu} = 0$$

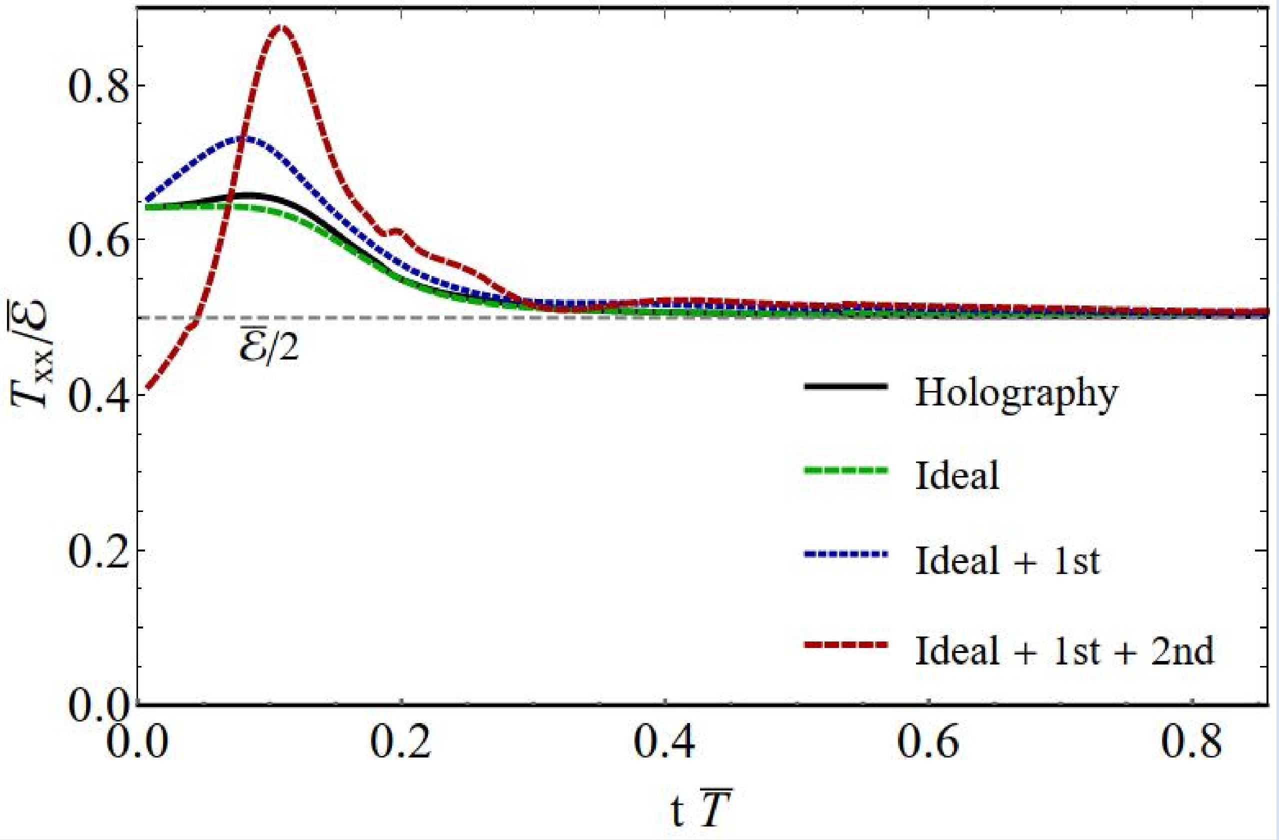
Not hyperbolic…

- First order hydro: **general frame**

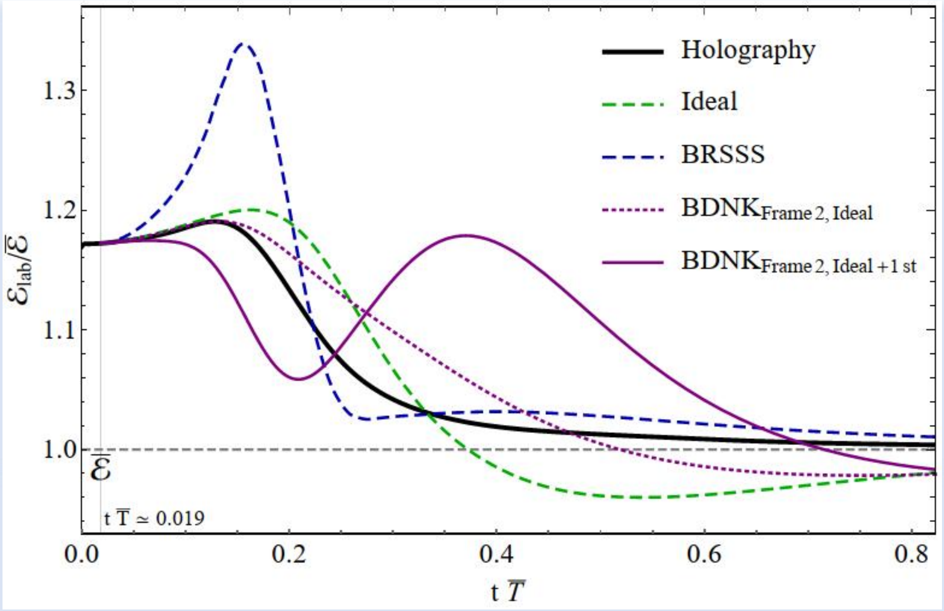
$$T^{\mu\nu} = \left[\epsilon + 2\, a_2 \eta \left(\frac{2}{3}\frac{\dot\epsilon}{\epsilon} + \nabla \cdot u\right)\right] \left(u^\mu\, u^\nu + \frac{\Delta^{\mu\nu}}{2}\right) + a_1 \eta \left[\left(\dot u^\mu + \frac{1}{3}\frac{\nabla_\perp^\mu \epsilon}{\epsilon}\right) u^\nu + (\mu \leftrightarrow \nu)\right] - \eta\, \sigma^{\mu\nu}$$

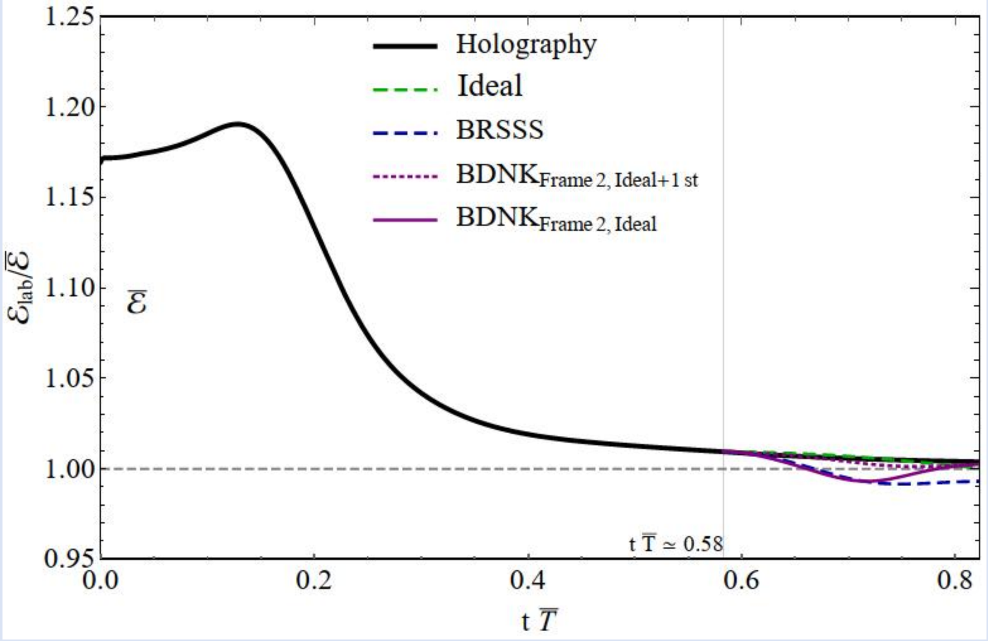
→ Include all 1st order terms compatible with Poincare symmetry

Backup slides: Constitutive relations

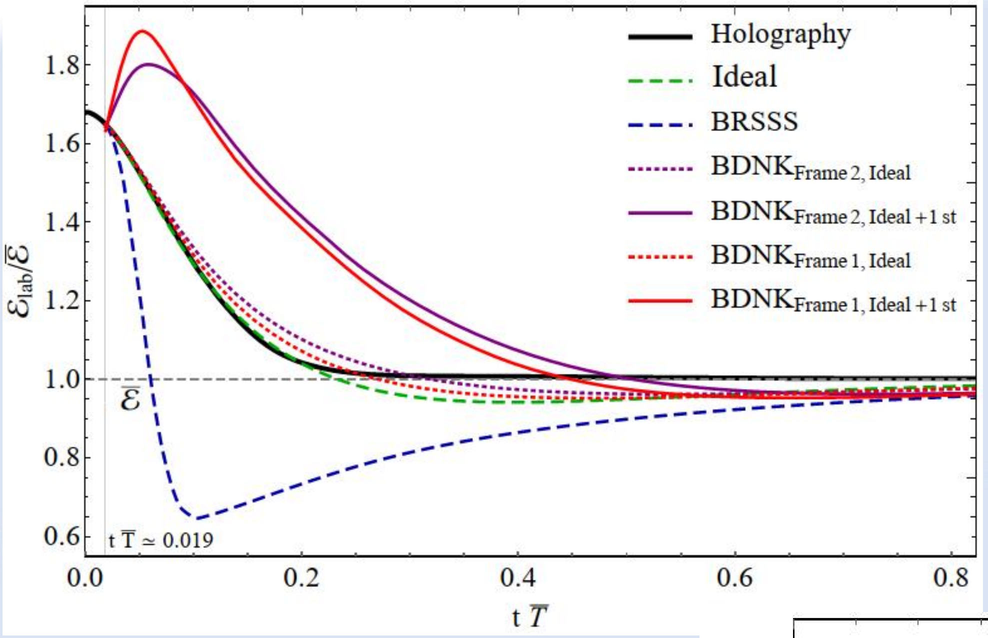


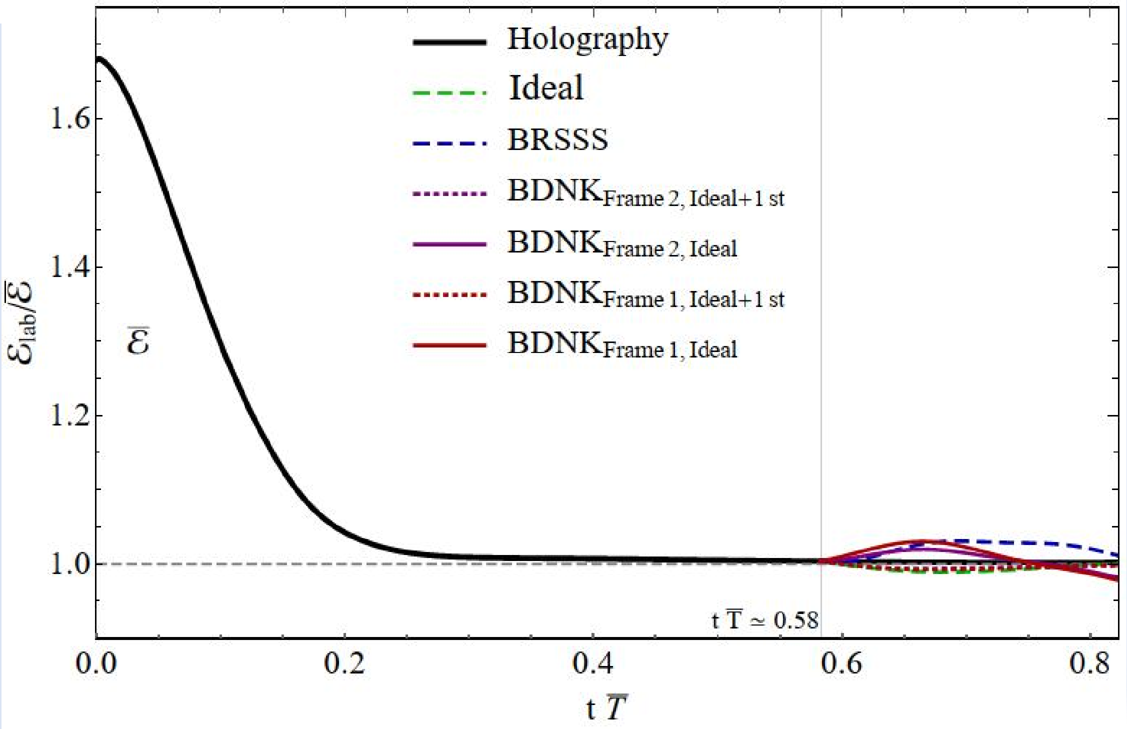
Backup slides: Off center





Backup slides: Frames 1 and 2





Hydro equations

- Conformal theory in 2+1 dimensions
- Ideal hydrodynamics

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu}$$

$\Rightarrow$ $\nabla_\mu T^{\mu\nu} = 0$ Hyperbolic!!

$$\frac{2}{3}\frac{\dot\epsilon}{\epsilon} + \nabla_\lambda u^\lambda = 0\,, \qquad \dot u^\mu + \frac{1}{3}\frac{\nabla_\perp^\mu \epsilon}{\epsilon} = 0\,.$$

- First order hydro: **Landau frame**

$$T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p\, \Delta^{\mu\nu} - \eta\sigma^{\mu\nu}$$

$\Rightarrow$ $\nabla_\mu T^{\mu\nu} = 0$ Not hyperbolic…

- First order hydro: **general frame**

$$T^{\mu\nu} = \left[\epsilon + 2\, a_2 \eta \left(\frac{2}{3}\frac{\dot\epsilon}{\epsilon} + \nabla\cdot u\right)\right]\left(u^\mu\, u^\nu + \frac{\Delta^{\mu\nu}}{2}\right) + a_1 \eta \left[\left(\dot u^\mu + \frac{1}{3}\frac{\nabla_\perp^\mu \epsilon}{\epsilon}\right) u^\nu + (\mu \leftrightarrow \nu)\right] - \eta\, \sigma^{\mu\nu}$$

$\Rightarrow$

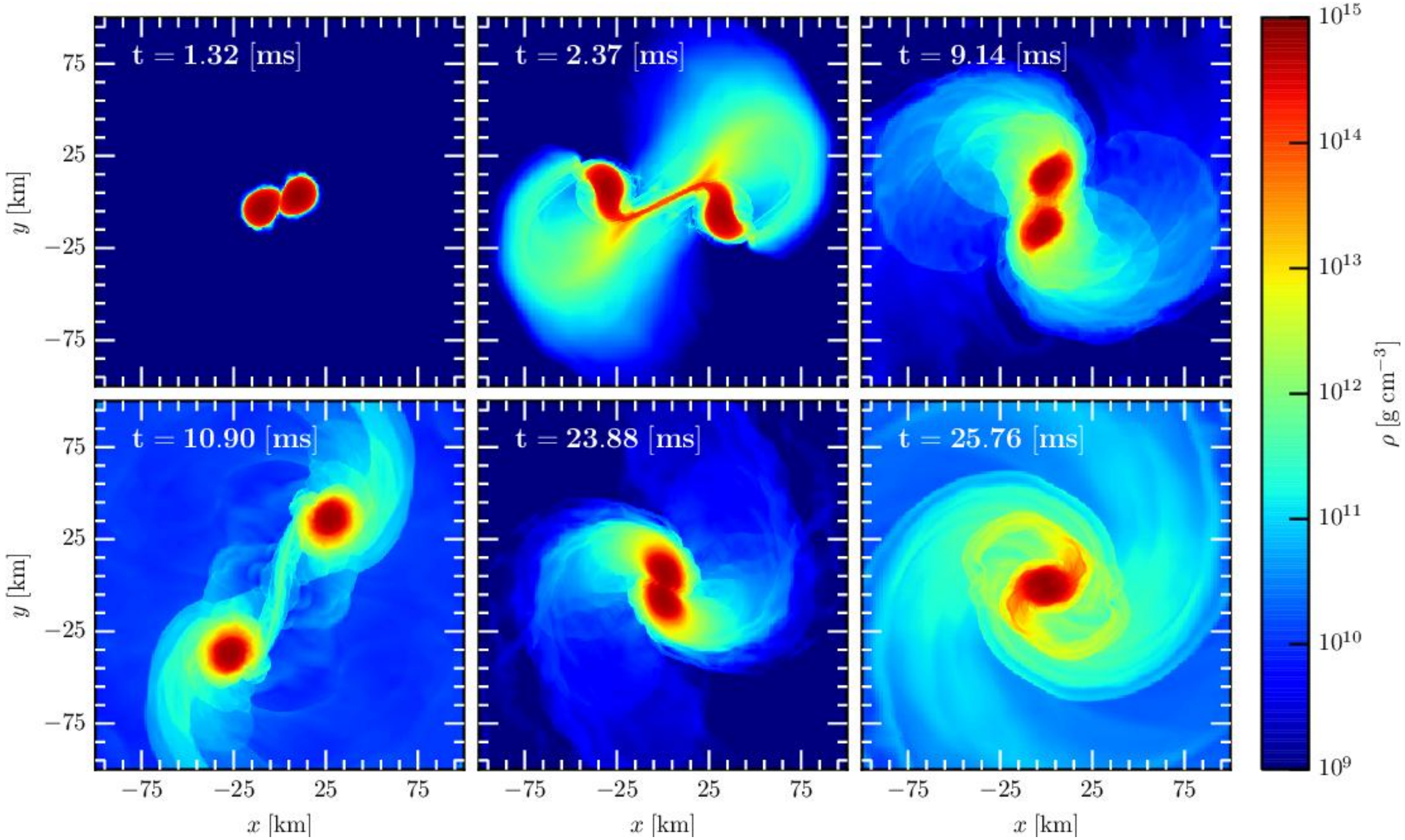
$$a_2 > 1\,, \qquad a_1 > \frac{4\, a_2}{a_2 - 1}\,.$$

$$\nabla_\mu T^{\mu\nu} = 0$$

Hyperbolic!!

BDNK equations

Viscosity can be relevant



BDNK: Integration method

Recall: BDNK eqs. are 2nd order in time

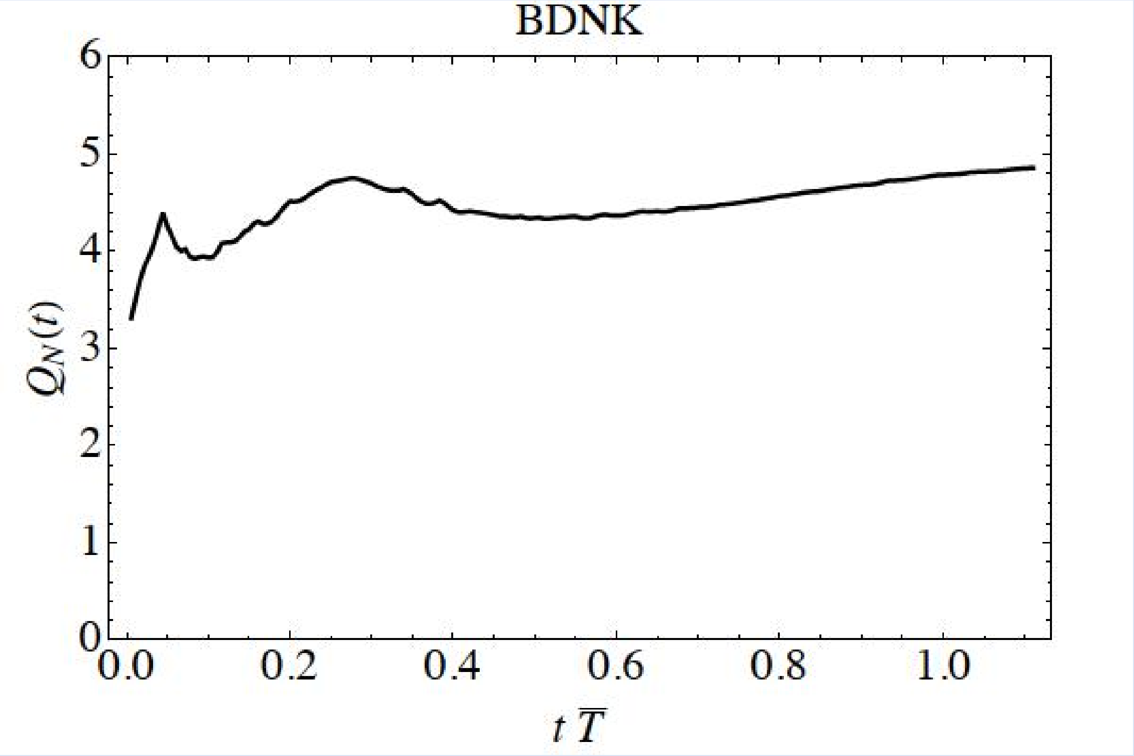
- Reduce to 1st order. RK4 unstable ⟶ Unstable…
- Implicit integration method ⟶ STABLE!!
- Explicit integration method RKNG34 ⟶ STABLE and FASTER!!

⟶ We use RKNG34 in our simulations

BDNK: Convergence tests

- Convergence test for BDNK
- Performed for evolutions of Gaussian profiles

Quantity capturing the convergence order



BDNK: We are not the first ones

- We are not the first ones performing time evolution using BDNK:

A numerical exploration of first-order relativistic hydrodynamics

Alex Pandya* and Frans Pretorius†
Department of Physics, Princeton University, Princeton, New Jersey 08544, USA.
(Dated: April 5, 2021)

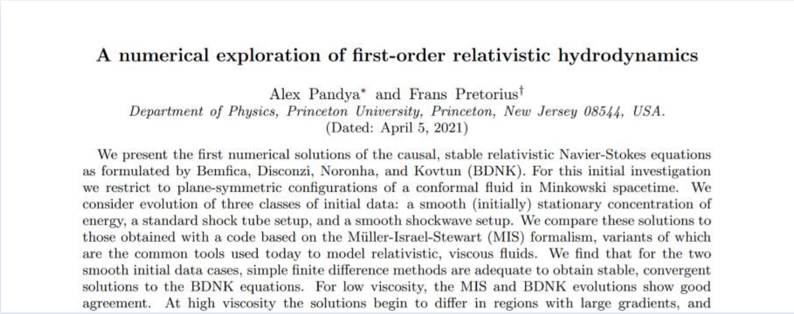
We present the first numerical solutions of the causal, stable relativistic Navier-Stokes equations as formulated by Bemfica, Disconzi, Noronha, and Kovtun (BDNK). For this initial investigation we restrict to plane-symmetric configurations of a conformal fluid in Minkowski spacetime. We consider evolution of three classes of initial data: a smooth (initially) stationary concentration of energy, a standard shock tube setup, and a smooth shockwave setup. We compare these solutions to those obtained with a code based on the Müller-Israel-Stewart (MIS) formalism, variants of which are the common tools used today to model relativistic, viscous fluids. We find that for the two smooth initial data cases, simple finite difference methods are adequate to obtain stable, convergent solutions to the BDNK equations. For low viscosity, the MIS and BDNK evolutions show good agreement. At high viscosity the solutions begin to differ in regions with large gradients, and

- This is a first exploration, but still many open question.
- Our works only partially overlap, and they are complementary

⟶ With our work and Pretorius paper, we are paving the way to the implementation in relevant physical systems.

BDNK: We are not the first ones

- Main differences:



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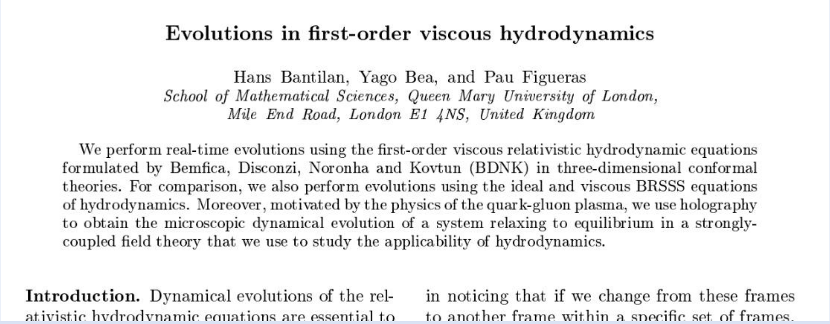


Evolutions in first-order viscous hydrodynamics

Hans Bantilan, Yago Bea, and Pau Figueras
School of Mathematical Sciences, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom

We perform real-time evolutions using the first-order viscous relativistic hydrodynamic equations formulated by Bemfica, Disconzi, Noronha and Kovtun (BDNK) in three-dimensional conformal theories. For comparison, we also perform evolutions using the ideal and viscous BRSSS equations of hydrodynamics. Moreover, motivated by the physics of the quark-gluon plasma, we use holography to obtain the microscopic dynamical evolution of a system relaxing to equilibrium in a strongly-coupled field theory that we use to study the applicability of hydrodynamics.

Introduction. Dynamical evolutions of the rel-

in noticing that if we change from these frames

- 1+1 dynamics
- 3+1 theory
- Motivated by neutron star mergers
- Integration method: conservative methods (HRSC)

- 2+1 dynamics
- 2+1 theory
- Motivated by heavy-ion collisions
- Integration method: Explicit RKNG34
- Microscopic solution

→ Our works only partially overlap, and they are complementary

→ With our work and Pretorius paper, we are paving the way to the implementation in relevant physical systems.

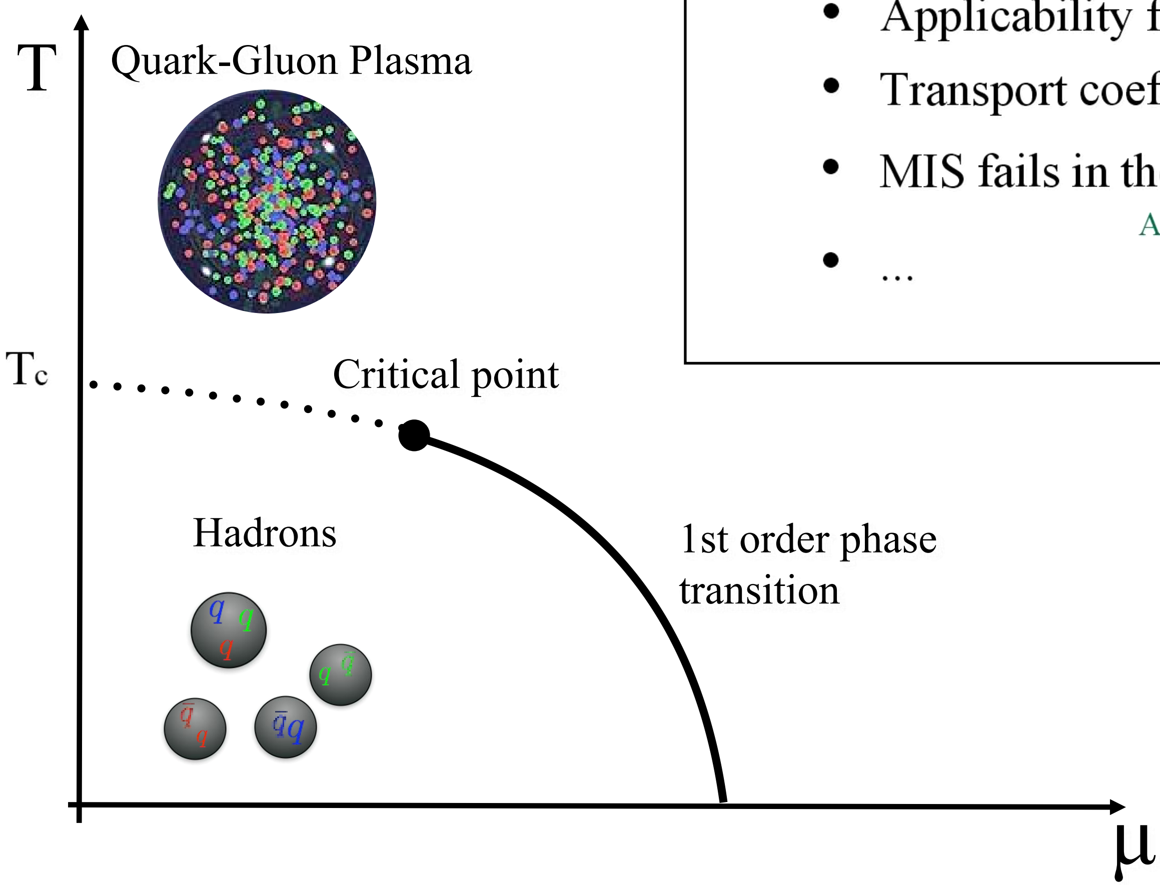
QCD & Holography

<u>What have we learned from holography so far?</u>

Chesler, Yaffe, Casalderrey, Mateos, Heller, van der Schee, …

- Early hydrodynamization times
- Applicability with large gradients
- Applicability for small systems
- Transport coefficients
- MIS fails in the presence of a phase transition

  Attems, Bea, Mateos, Casalderrey, Triana, Zilhao ’19, ‘20
- …



Holography: Our model

- CFT on Minkowski in 2+1 dim
- Decoupled sector of the stress tensor $T^{\mu\nu}$.

Real-time quantum dynamics

Holography

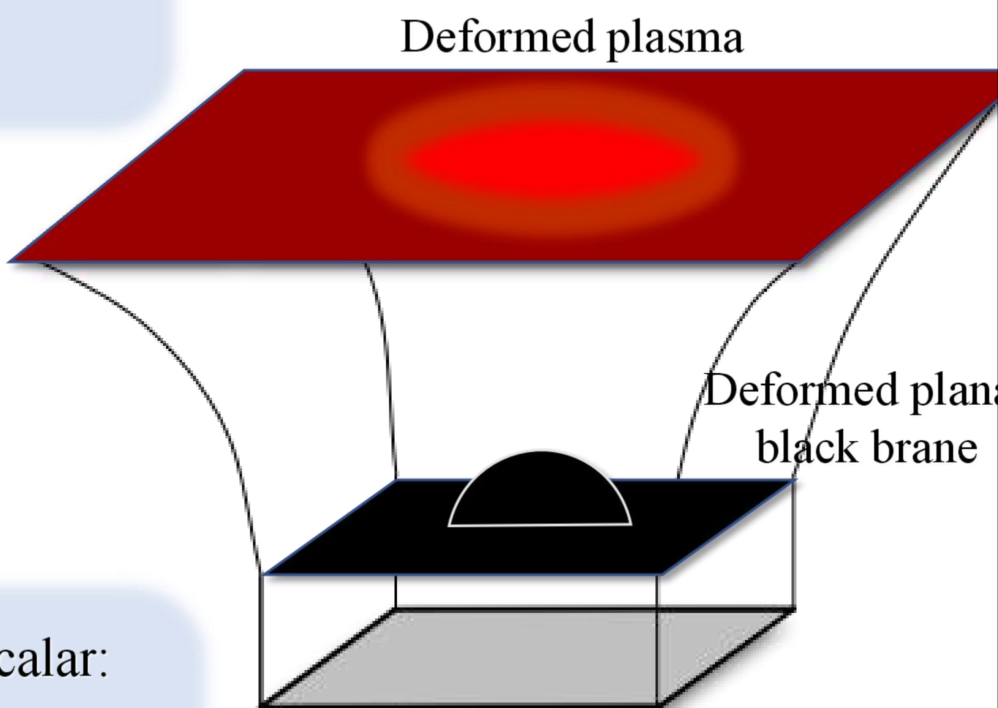
Numerical Relativity

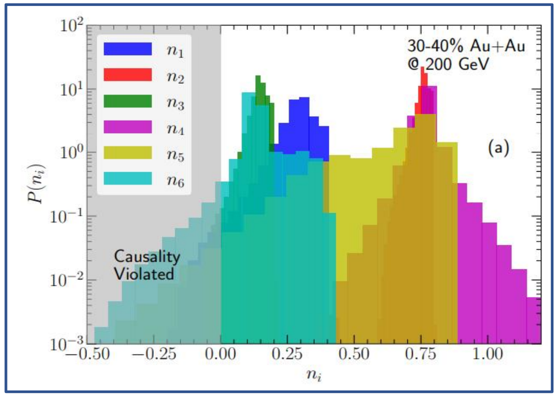
Dynamical classical gravity

- Gravity with Λ in 3+1 dim plus massless scalar:

$$S \sim \int d^{3+1}x \sqrt{-g}\,(R - 2\Lambda + (\partial\phi)^2)$$

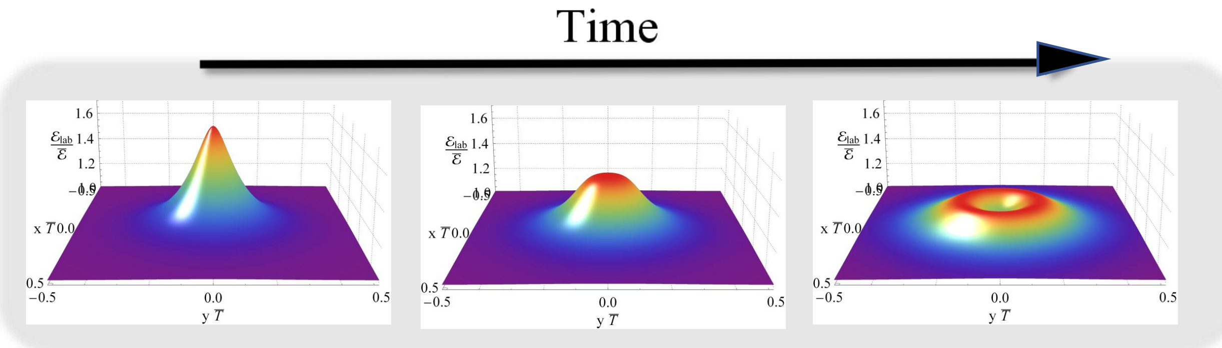
- We focus on the Poincare patch of AdS.

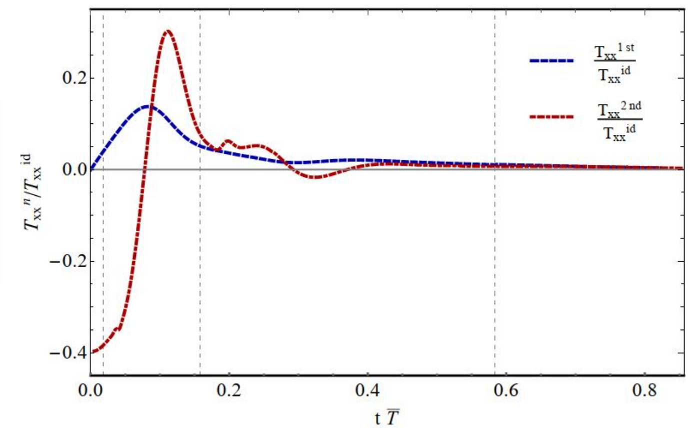




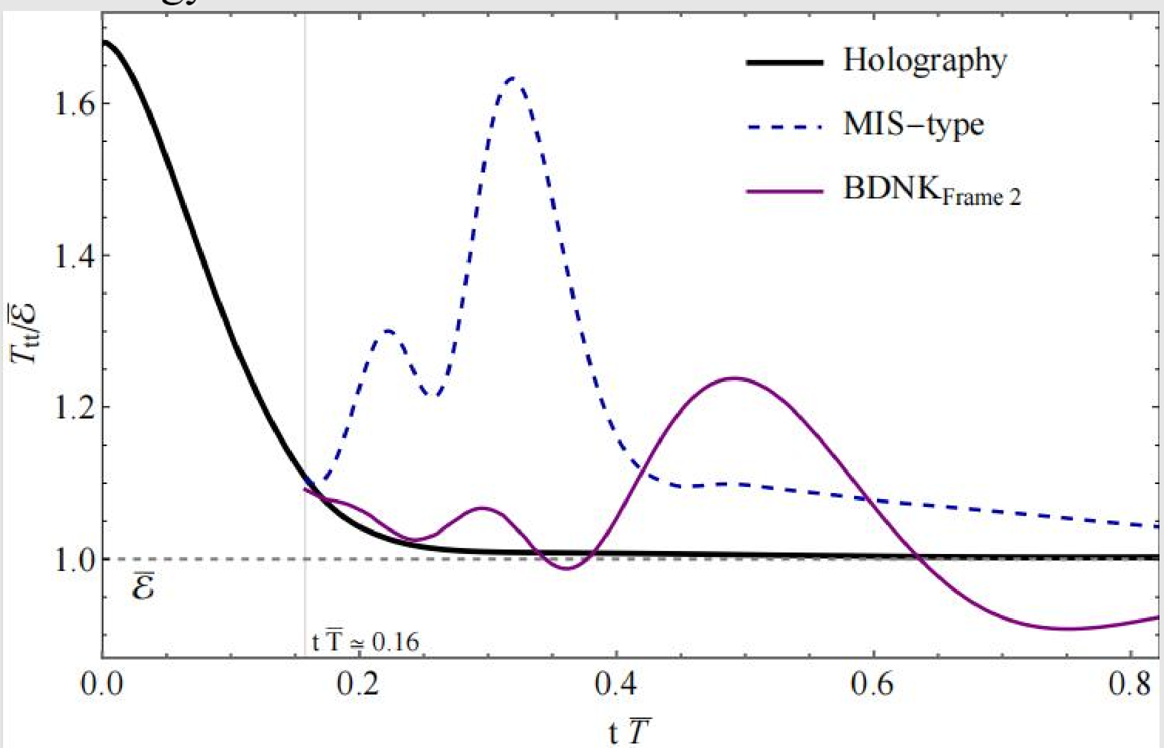
30-40% Au+Au
@ 200 GeV
(a)
Causality Violated
n_1
n_2
n_3
n_4
n_5
n_6
$P(n_i)$
n_i

Evolutions: holography vs hydrodynamics





Energy at the center of the domain:



Bantilan, Bea, Figueras '22

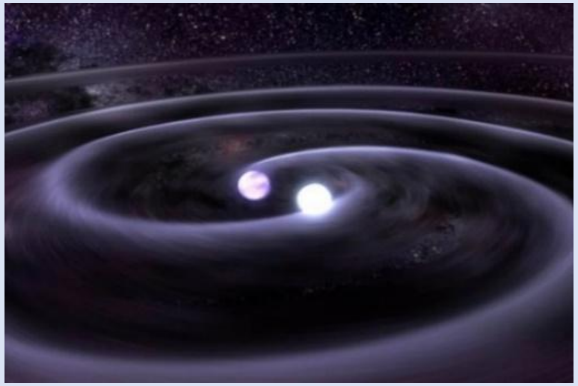
Motivation: astrophysical systems

Neutron star mergers

State of the art
- Ideal hydrodynamics

Beyond state of the art
- **Viscous** hydrodynamics

Shibata et al'20
Chabanov, Rezzolla, Rischke '21



→ More realistic scenarios, closer to astrophysical systems.

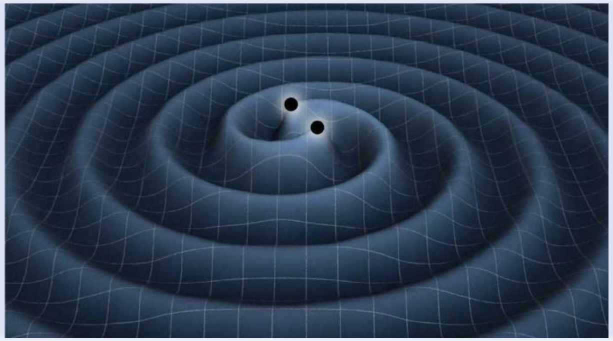
→ New era of precision gravitational waves (LISA, …)

Motivation: astrophysical systems

Motivation: astrophysical systems

Black hole mergers

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu} \Rightarrow R_{\mu\nu} = 0$$



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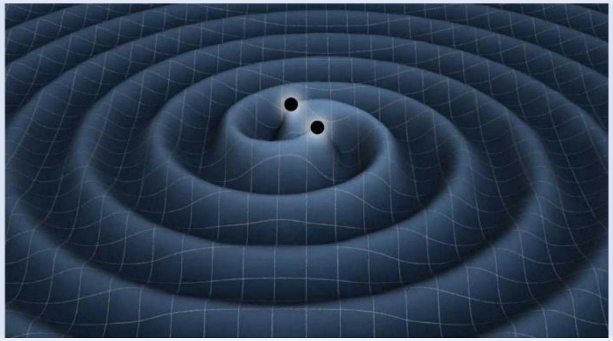
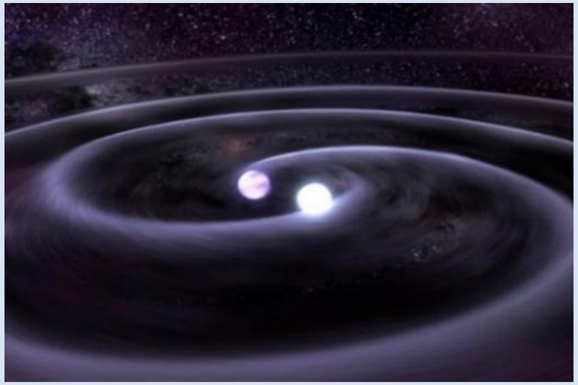


Neutron star mergers

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa T_{\mu\nu}$$

Matter must be specified



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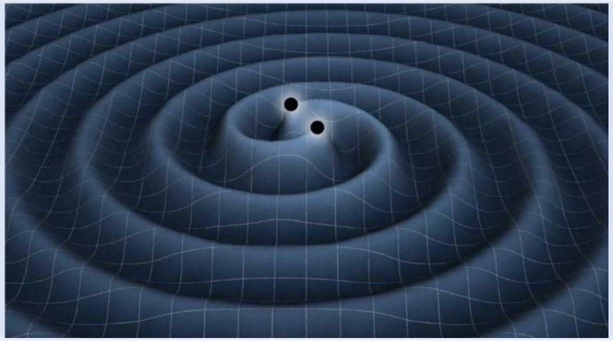

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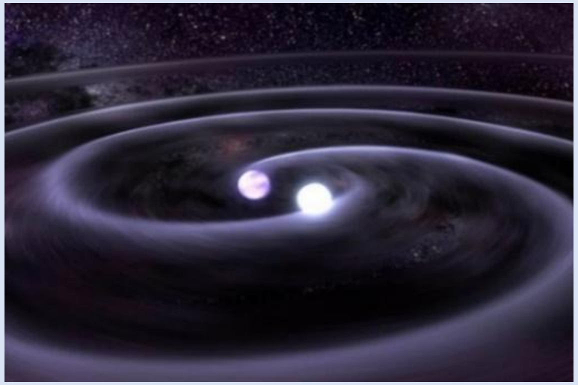

Ideally: solve gravity coupled to QCD

→ At the moment not feasible