Universitat de Barcelona

Institut de Ciències del Cosmos
Universitat de Barcelona

THERMAL EMISSION OF GRAVITATIONAL WAVES FROM WEAK TO STRONG COUPLING

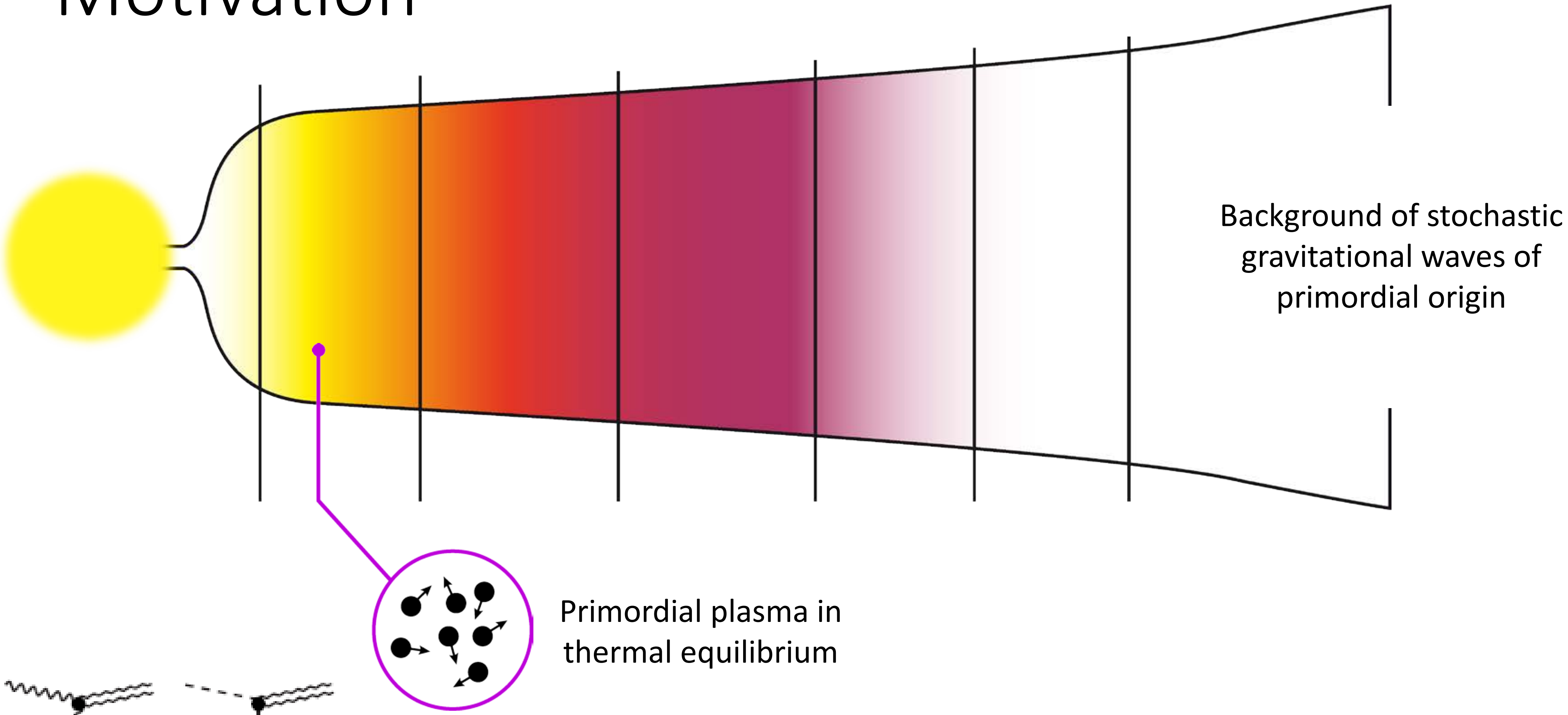
Lucía Castells-Tiestos

lucia.castells@icc.ub.edu

Based on 2202.05241 [hep-th]

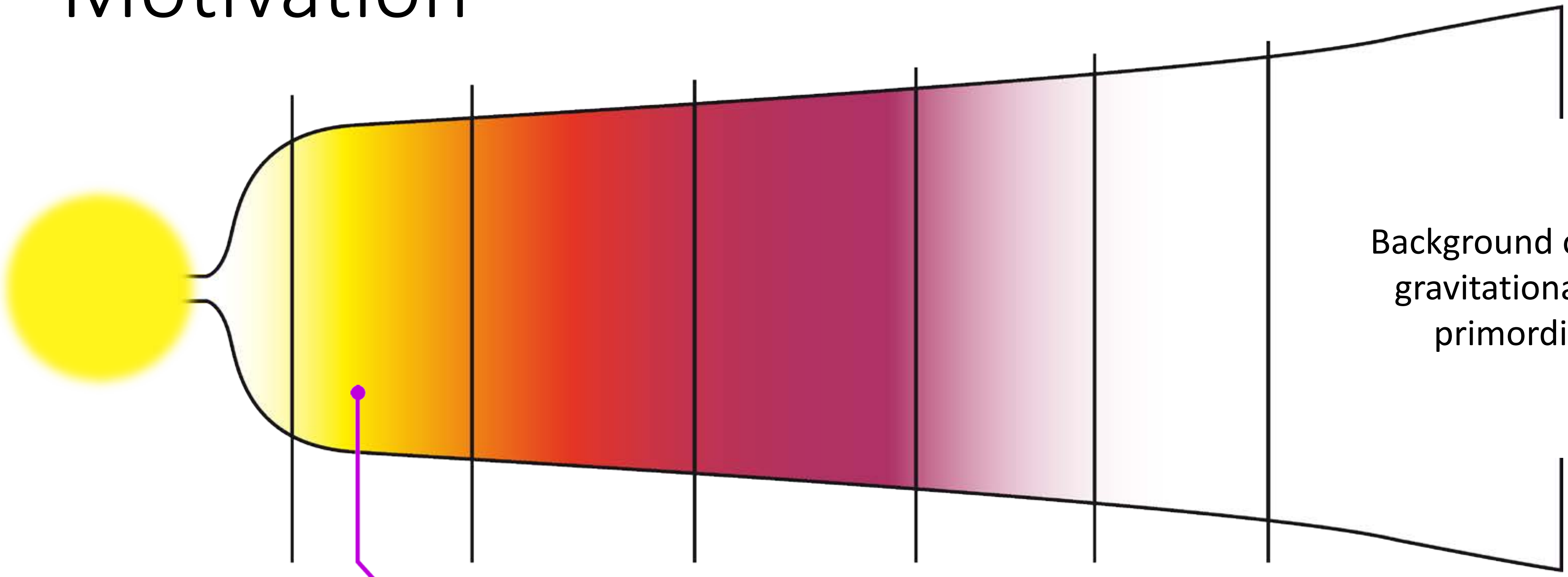
with Jorge Casalderrey-Solana

Motivation

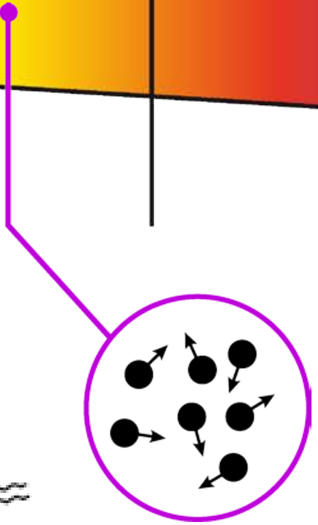


J. Ghiglieri et al, “Gravitational waves background from Standard Model Physics”, JHEP (2020)
A. Ringwald et al, “Gravitational waves as a Big Bang thermometer”, JCAP (2021)

Motivation



Background of stochastic gravitational waves of primordial origin



Primordial plasma in thermal equilibrium

Why do we care about this background?

- Max. temperature of the hot Big Bang
- Probe high-energy physics
- Dark Matter?

J. Ghiglieri et al, “Gravitational waves background from Standard Model Physics”, JHEP (2020)
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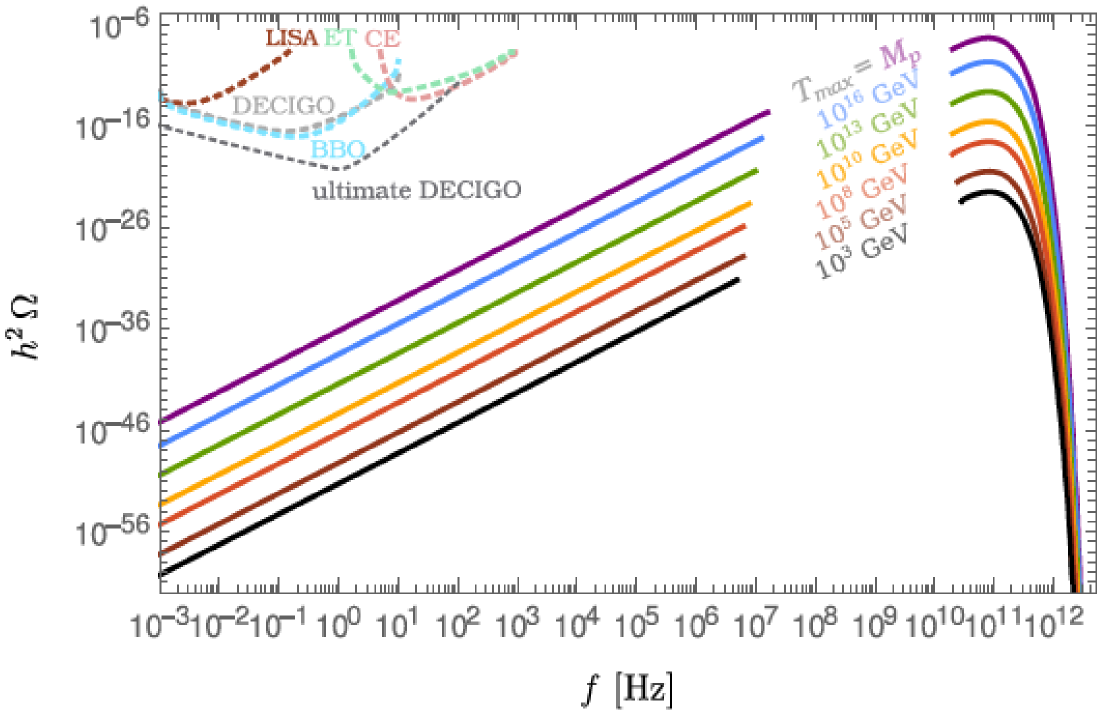
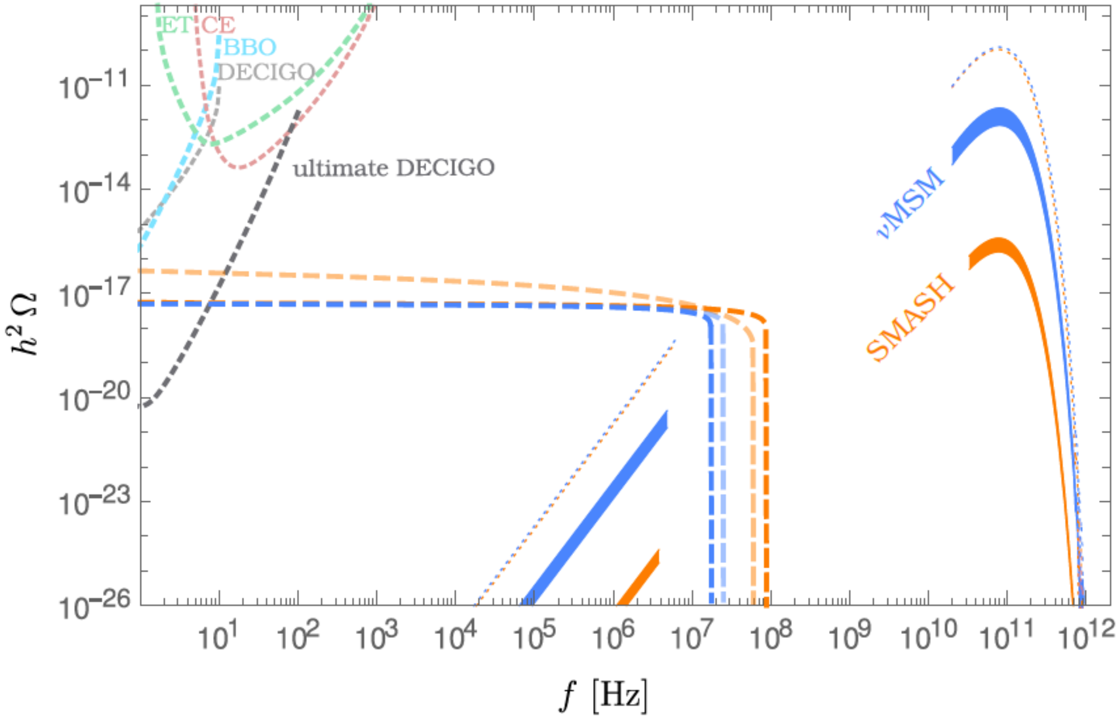
WHAT DO WE EXPECT FROM THIS BACKGROUND?

Expected behaviour

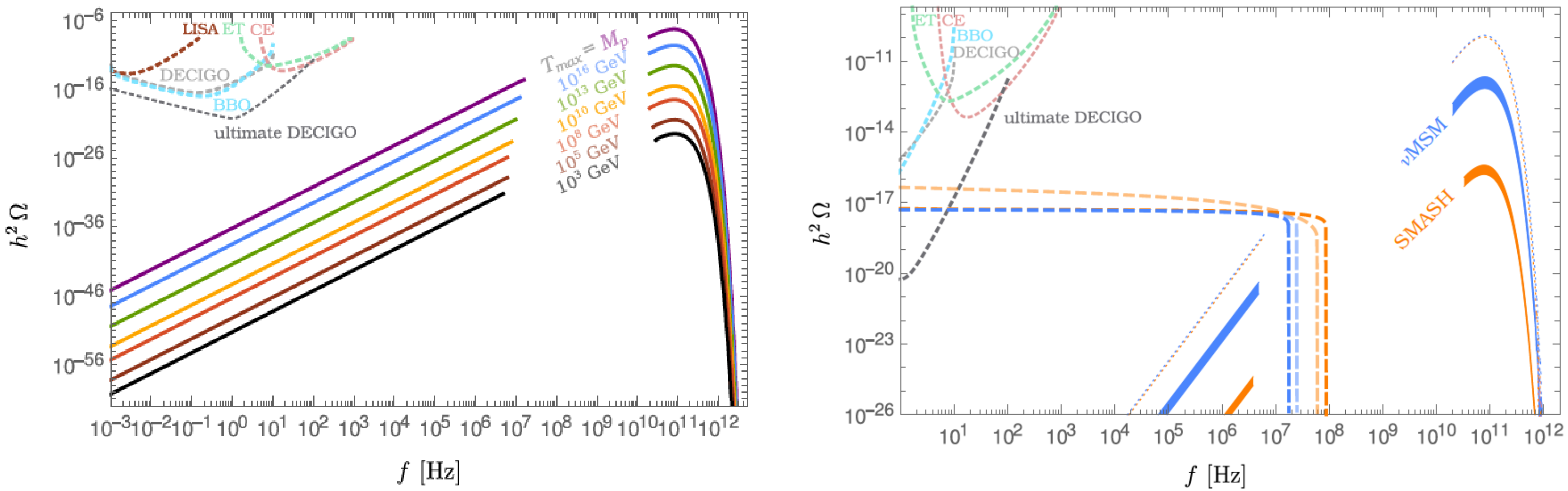




Energy density of gravitational waves from the primordial thermal plasma in the SM and two of its extensions: Neutrino Minimal SM (νMSM) and SM-Axion-Seesaw-Higgs (SMASH)

Figures from A. Ringwald et al, "Gravitational waves as a Big Bang thermometer", JCAP (2021)

Expected behaviour



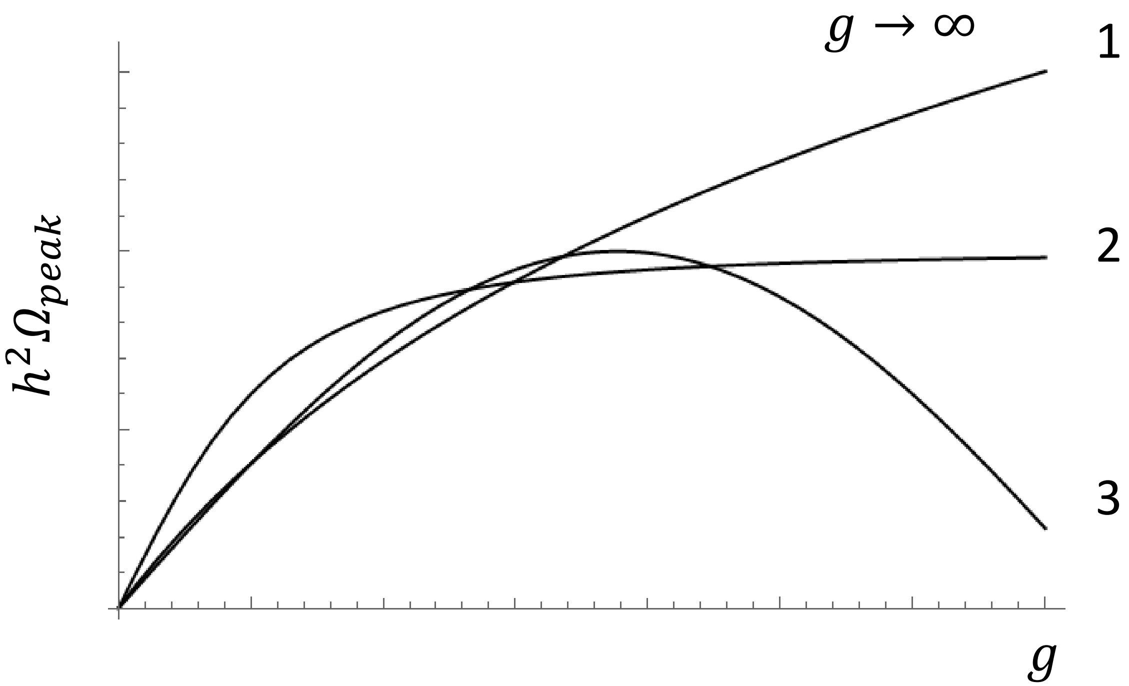
What if physics beyond the Standard Model is strongly-coupled?

Figures from A. Ringwald et al, “Gravitational waves as a Big Bang thermometer”, JCAP (2021)

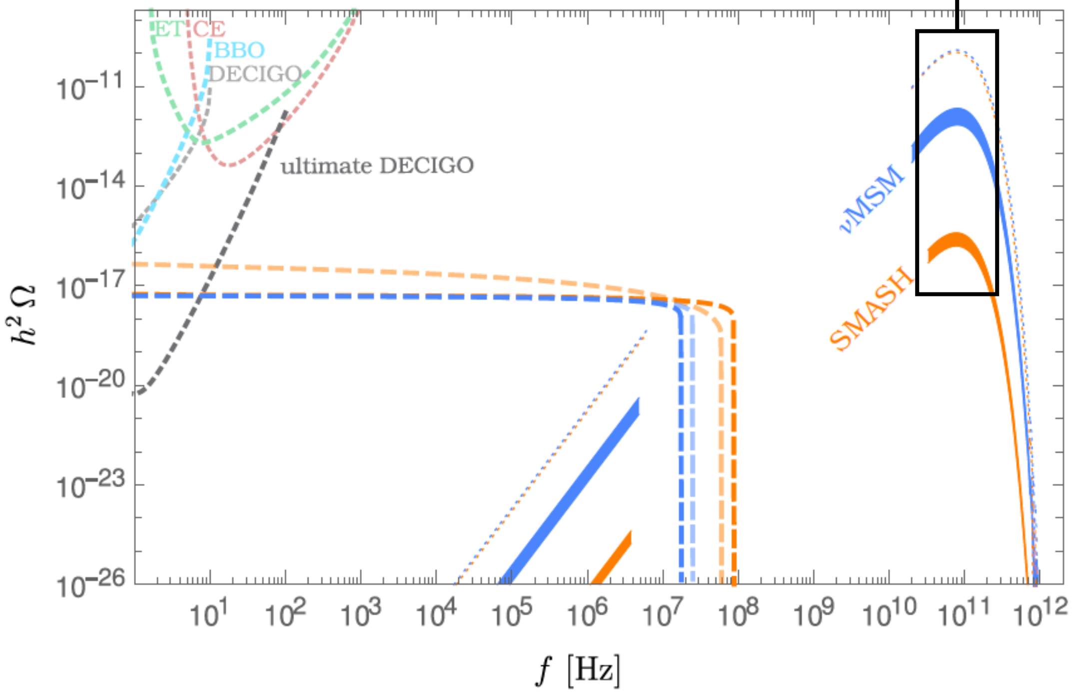
HOW DOES THE SIGNAL DEPEND ON THE COUPLING?

Coupling dependence

How does the peak of the spectrum change with coupling in the weak field limit?



Peak magnitude of the spectrum



Do we expect the peak of the spectrum to…

1. diverge at large coupling?
2. grow and converge?
3. grow until a maximum and decrease?

Coupling dependence

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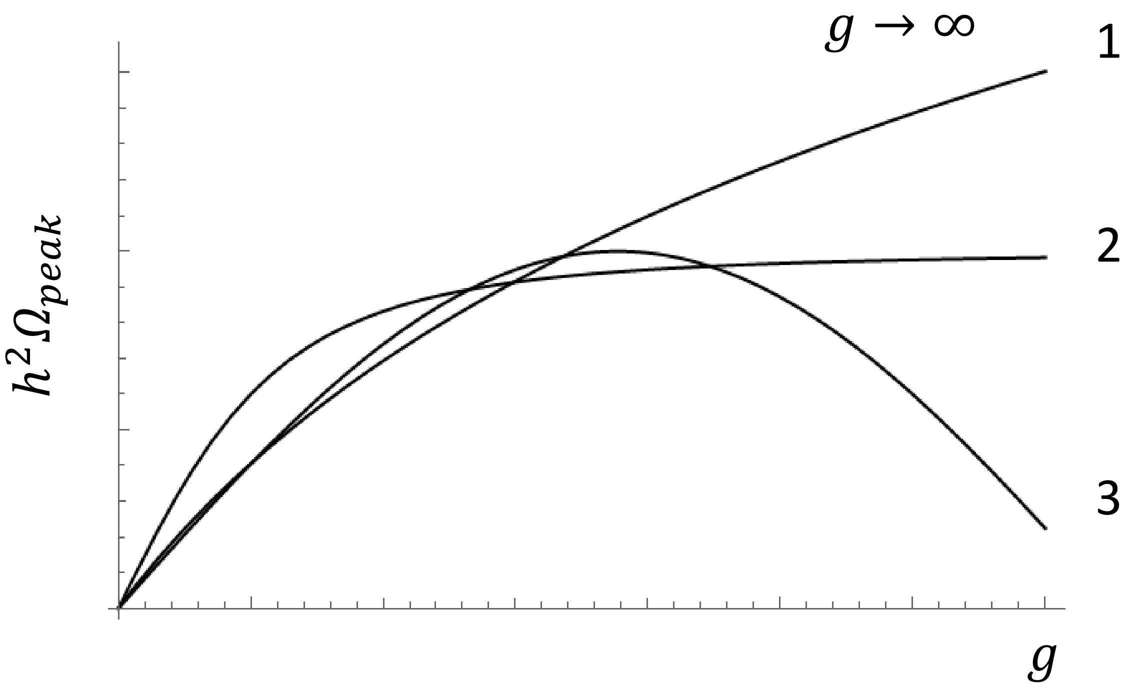

Do we expect the peak of the spectrum to…

1. diverge at large coupling?
2. grow and converge?
3. grow until a maximum and decrease?

We want to compare the weakly-coupled and the strongly-coupled regimes **within the same theory.**

Coupling dependence

Candidate theory with which to access both two regimes:

$\boldsymbol{\mathcal{N} = 4}$ $\boldsymbol{SU(N_c)}$ **Supersymmetric Yang-Mills theory** in the $N_c \to \infty$ limit and with respect to the ‘t Hooft coupling constant $\lambda \equiv g^2 N_c$

$$\mathcal{L}_{\mathrm{SYM}} = \mathrm{Tr}\left[-\frac{1}{2}G_{\mu\nu}^2 + \left(D_\mu \Phi_A\right)^2 + i\bar{\psi}_i \gamma^\mu D_\mu \psi_i - \frac{1}{2}g^2 (i[\Phi_A, \Phi_B])^2 \right.$$
$$\left. - ig\bar{\psi}_i \left[\alpha_{ij}^p X_p + i\beta_{ij}^q \gamma_5 Y_q \,, \psi_j\right]\right] + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{gh}} + \Delta\mathcal{L}_{\mathrm{SYM}}$$

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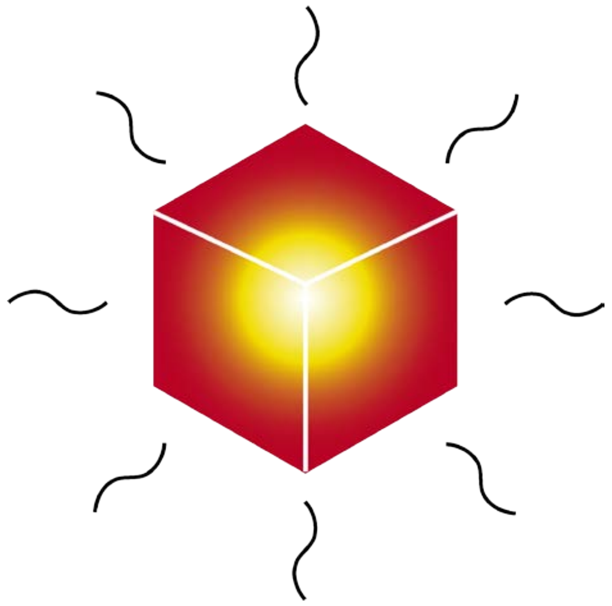
$\boldsymbol{\mathcal{N}=4}$ SYM allows us to bound the magnitude of the thermal spectrum by moving from $\boldsymbol{\lambda \to 0}$ to $\boldsymbol{\lambda \to \infty}$.

PRODUCTION RATE OF THERMAL GRAVITATIONAL WAVES

Gravitational waves from a static thermal source

Energy production rate of thermal gravitational radiation:

$$\frac{d\rho_{GW}}{dt\,d^3k} = \frac{4\pi G}{(2\pi)^3}\Lambda_{ijmn}\int \boldsymbol{d^4x\, e^{i(\omega t-\mathbf{kx})}\langle T_{ij}(0,0)T_{mn}(t,\mathbf{x})\rangle}$$



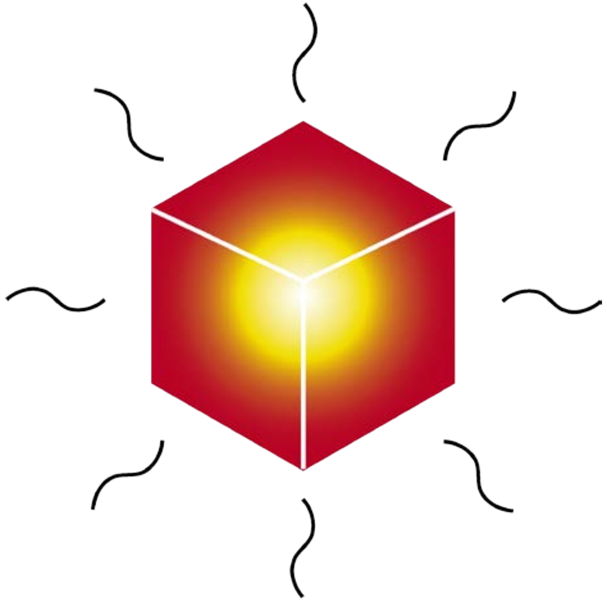
under light-like condition $\omega = k$ and Λ_{ijmn} the projector onto spin-2 modes.

The energy density carried by thermal gravitational waves depends on the equilibrium correlator of the energy-momentum tensor in field theory.

Gravitational waves from a static thermal source

Energy production rate of thermal gravitational radiation*:

$$\frac{d\rho_{GW}}{dt d^3k} = \frac{-8\pi G n_B(k)}{(2\pi)^3} \Lambda_{ijmn} \mathrm{Im}\langle T_{ij}(k) T_{mn}(-k)\rangle_R$$



under light-like condition $\omega = k$ and Λ_{ijmn} the projector onto spin-2 modes.

Computation of the correlator in $\mathcal{N} = 4$ SYM:

- Weak coupling limit $\lambda \to 0$: Perturbation theory
- Strong coupling limit $\lambda \to \infty$: ~~Perturbation theory~~ **Holography**

* Via KMS relations

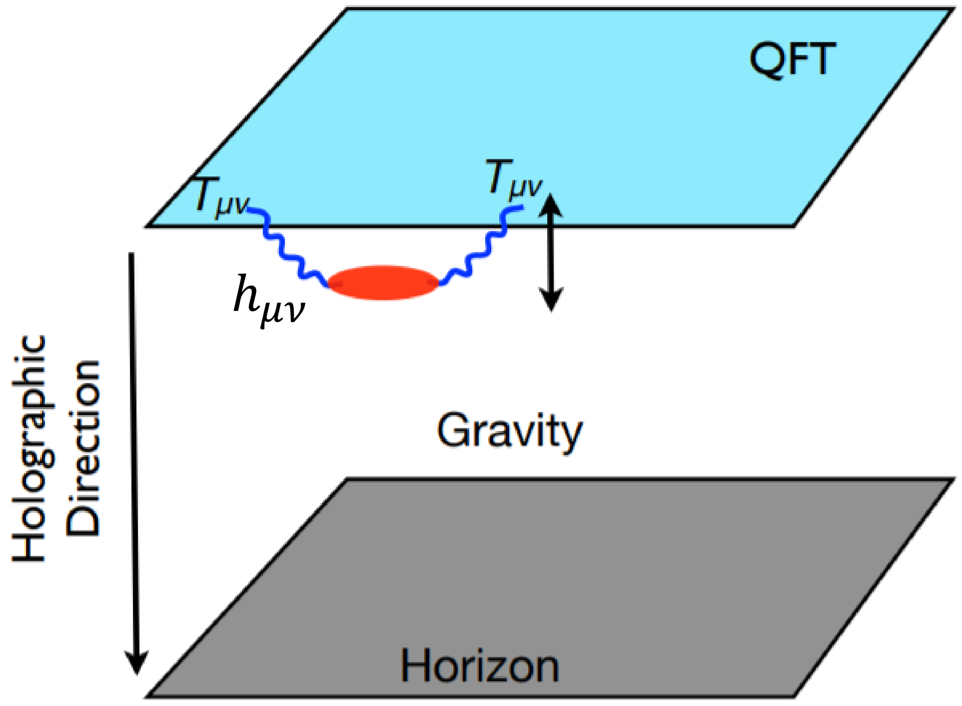
THERMAL EMISSION RATE AT STRONG COUPLING

Gauge/gravity duality: AdS/CFT correspondence

Strongly-coupled, non-perturbative quantum field theories with conformal invariance

Holography

Weakly-coupled gravity theories with black hole horizons in asymptotically Anti de Sitter spacetime



J.M. Maldacena, "Adv. Theor. Math. Phys." 2 (1998), pp. 231-252.

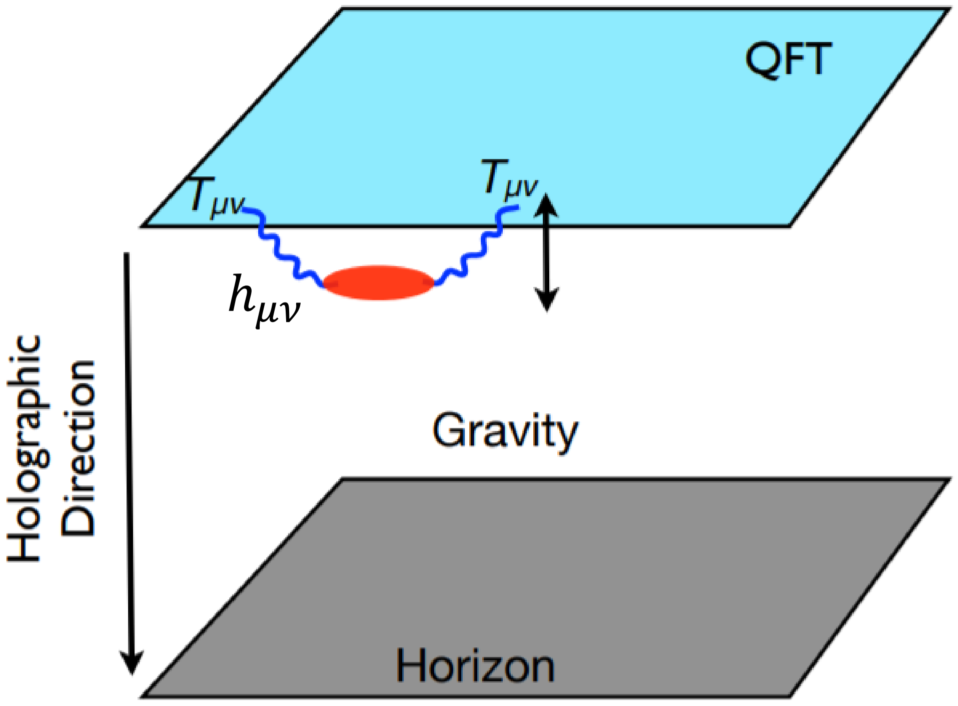
Gauge/gravity duality: AdS/CFT correspondence

Strongly-coupled, non-perturbative quantum field theories with conformal invariance

Holography ↕

Weakly-coupled gravity theories with black hole horizons in asymptotically Anti de Sitter spacetime

Gauge/gravity duality allows us to compute correlation functions in the field theory in terms of the gravity prescription.



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Gauge/gravity duality: AdS/CFT correspondence

Energy-momentum tensor $T_{\mu\nu}$ in the CFT

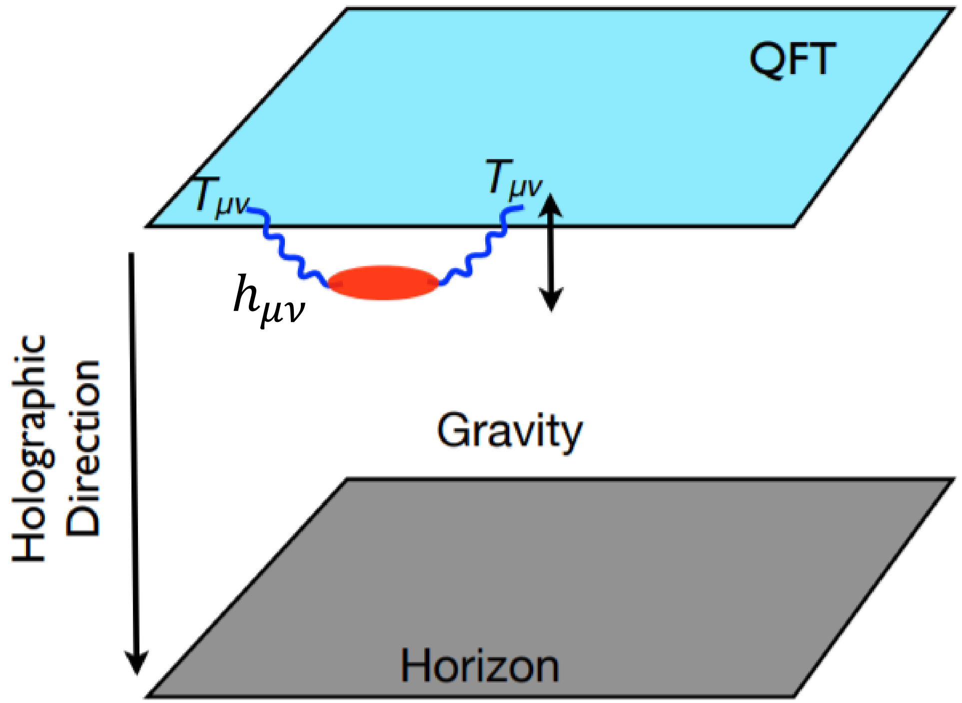
$\updownarrow$

Gravitational fluctuations $h_{\mu\nu}$

We need to consider fluctuations $\boldsymbol{h_{\mu\nu}}$ over the gravitational background:

$$h_{\mu\nu}(t,x,u) = h_{\mu\nu}(u)e^{-ik(t-z)}$$

Restrict to the **spin-2** fluctuating field h_y^x subject to the infalling boundary condition.



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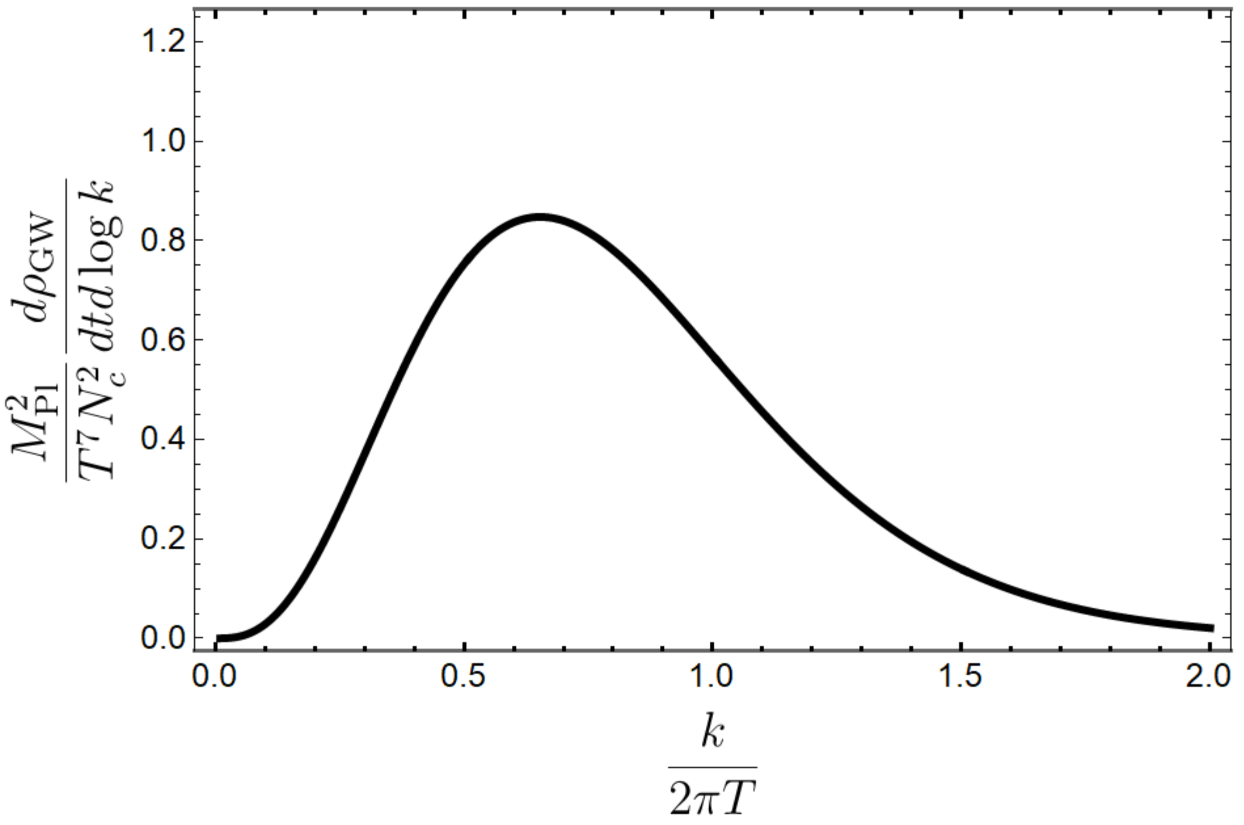
Emission rate at strong coupling



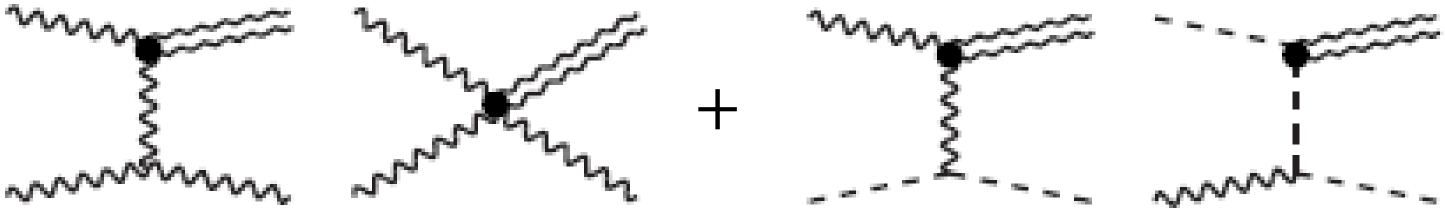
Energy production rate emitted by strongly-coupled $\mathcal{N} = 4$ SYM matter

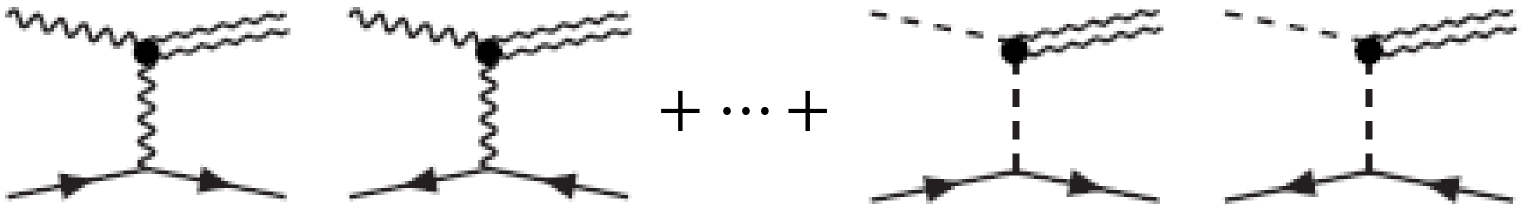
THERMAL EMISSION RATE AT WEAK COUPLING

Perturbative analysis

Energy-momentum tensor correlation function from standard **perturbation theory**:

$$G^{<}_{xy;xy}(k) \equiv \int d^4x\, e^{i(kt-\mathbf{kx})} \langle T_{xy}(0,0) T_{xy}(t,\mathbf{x}) \rangle$$

$= \ HTL +$  $+ \cdots$

$+$  $+ \cdots$

J. Ghiglieri et al, "Gravitational waves background from Standard Model Physics: Complete leading order". JHEP (2020).

Perturbative analysis

Generalization to an arbitrary theory with gauge fields, real scalars and Weyl fermions:

$$G^{<}_{xy;xy}(k) \;\to\; \hat{\eta}\left(T, \frac{k}{T} \equiv \hat{k}\right) \simeq \hat{\eta}_{HTL}(T, \hat{k}) + \sum_{n=1}^{\mathcal{N}_g} g_n(T)^2 N_n \left(\frac{1}{2} T_{n,\mathrm{Ad}}\, \hat{\eta}_{gg}(\hat{k}) + \sum_{\hat{\imath}} T_{n,\hat{\imath}}\, \hat{\eta}_{sg}(\hat{k}) + \frac{1}{2}\sum_{\hat{\alpha}} T_{n,\hat{\alpha}}\, \hat{\eta}_{fg}(\hat{k})\right) + \frac{1}{4}\sum_{i\alpha\beta} \left|\mathcal{Y}^{i}_{\alpha\beta}(T)\right|^2 \hat{\eta}_{sf}(\hat{k}), \qquad m_D \ll T$$

A. Ringwald et al, “Gravitational waves as a Big Bang thermometer”, JCAP (2021)

Perturbative analysis

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$\mathcal{N} = 4$ Super Yang-Mills at weak coupling $g \to 0$

$$\mathcal{L}_{\mathrm{SYM}} = \mathrm{Tr}\left[-\frac{1}{2} G_{\mu\nu}^2 + \left(D_\mu \Phi_A\right)^2 + i\bar{\psi}_i \gamma^\mu D_\mu \psi_i - \frac{1}{2} g^2 (i[\Phi_A, \Phi_B])^2 - ig\bar{\psi}_i \left[\alpha^p_{ij} X_p + i\beta^q_{ij}\gamma_5 Y_q\,, \psi_j\right]\right] + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{\mathrm{gh}} + \Delta\mathcal{L}_{\mathrm{SYM}}$$

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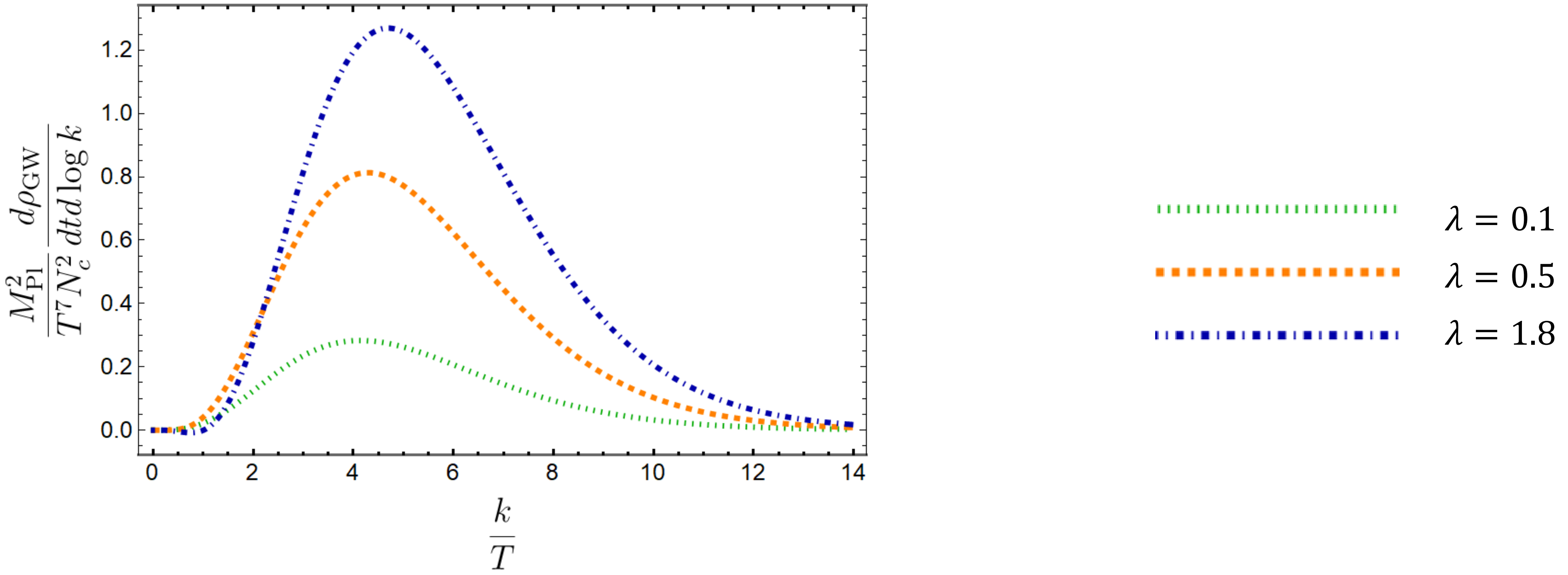
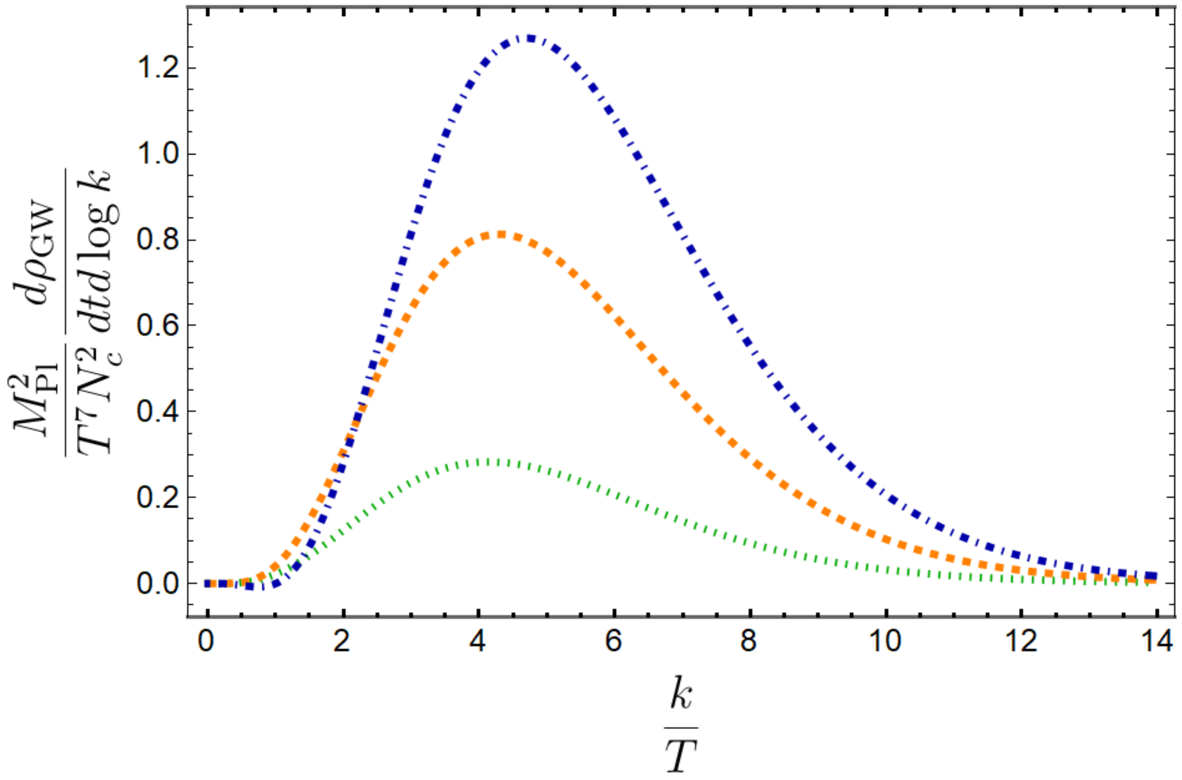
Emission rate at weak coupling



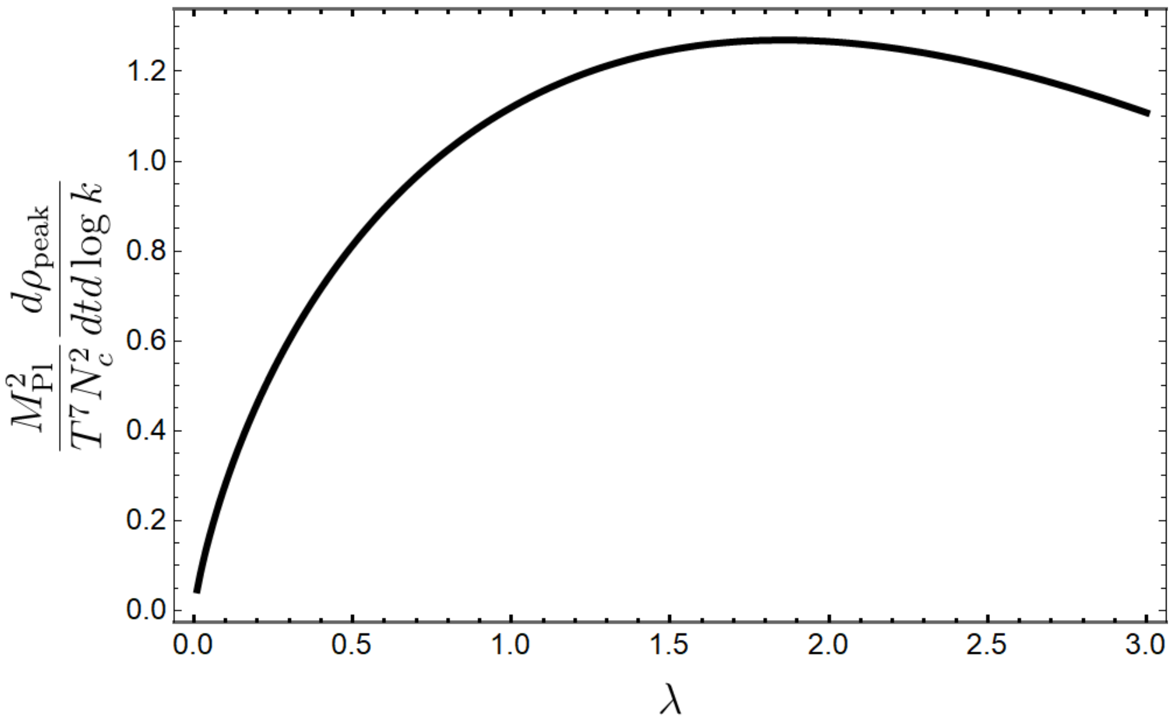
Energy production rate at weak coupling

Emission rate at weak coupling

$\lambda_{peak} = 1.8$

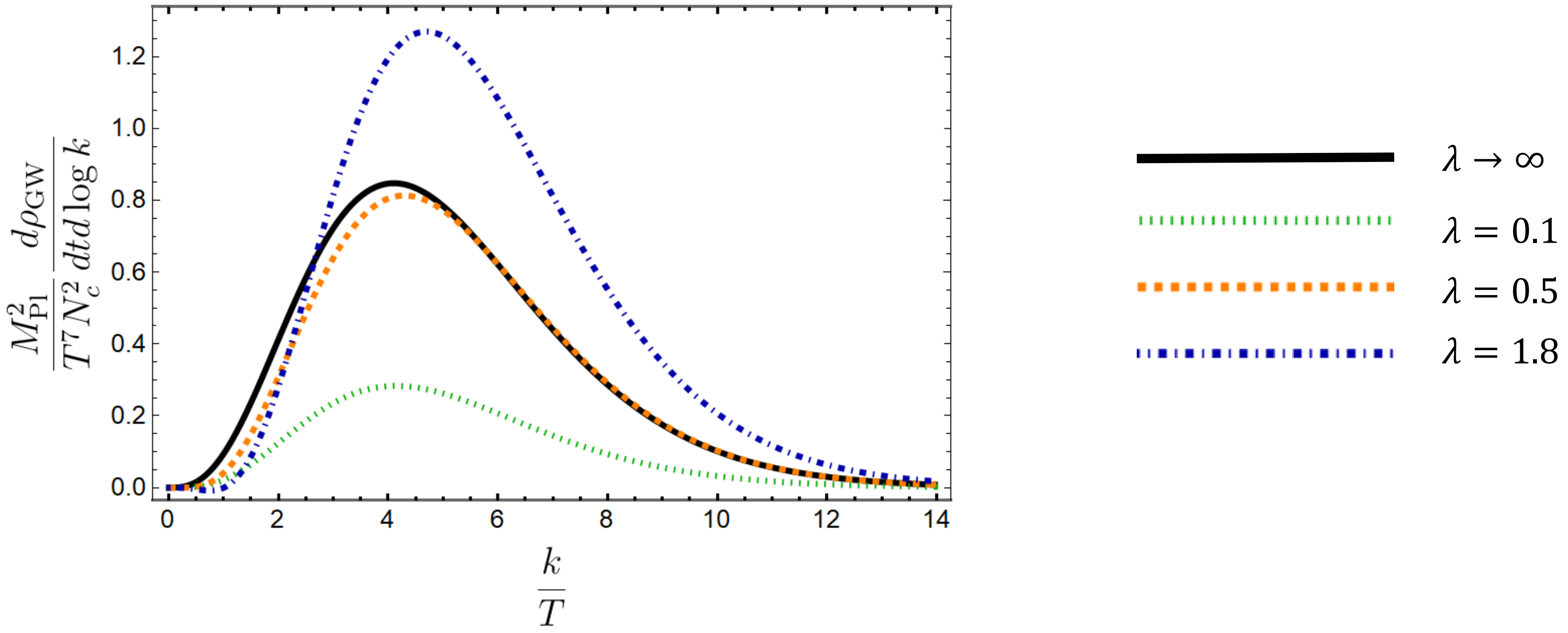


Energy production rate at weak coupling



Peak magnitude of the spectrum with respect to λ

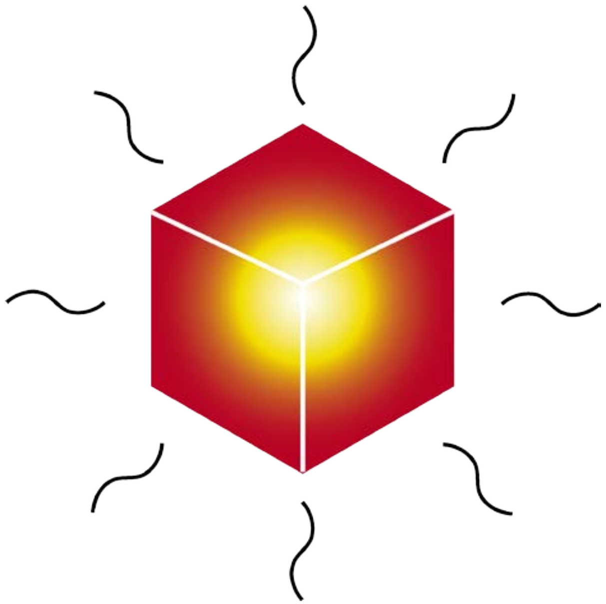
Emission rate from a static source



Comparison between the strongly-coupled and the weakly-coupled regimes

EVOLUTION WITH THE EXPANSION OF THE UNIVERSE

Cosmological evolution



Emission from a static thermal equilibrium state at temperature T

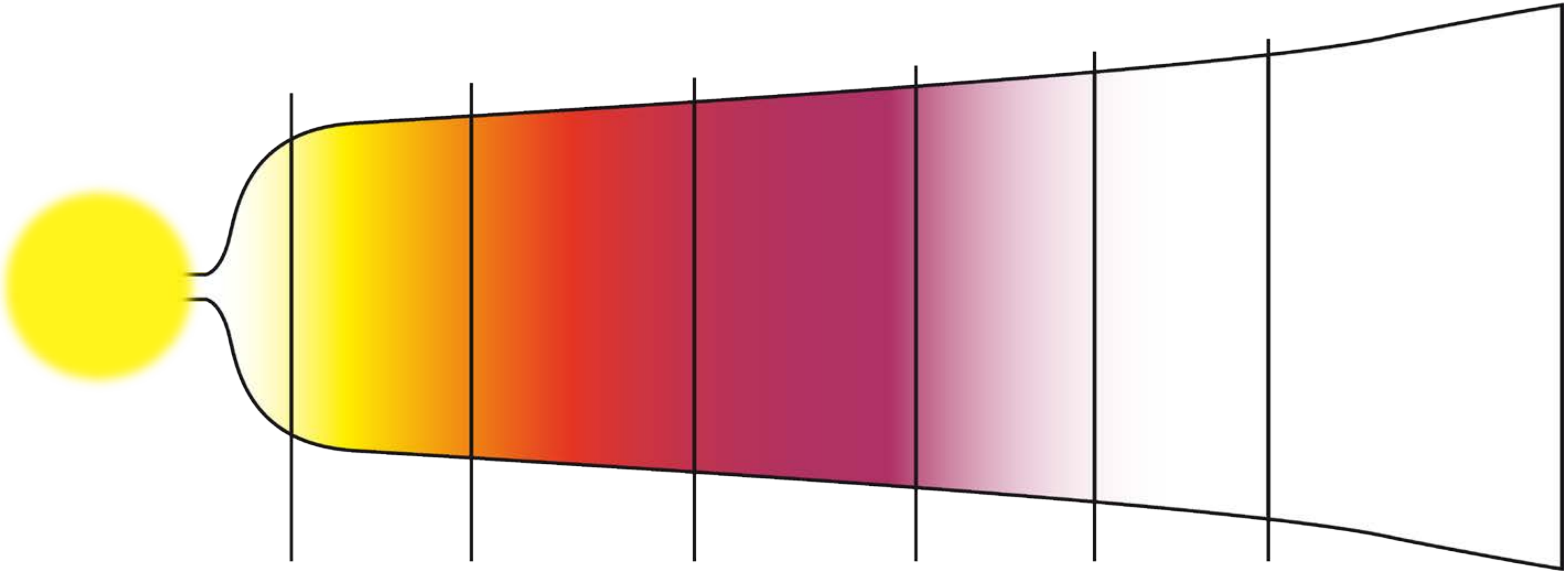
$\longleftrightarrow$

Emission from a box at fixed T

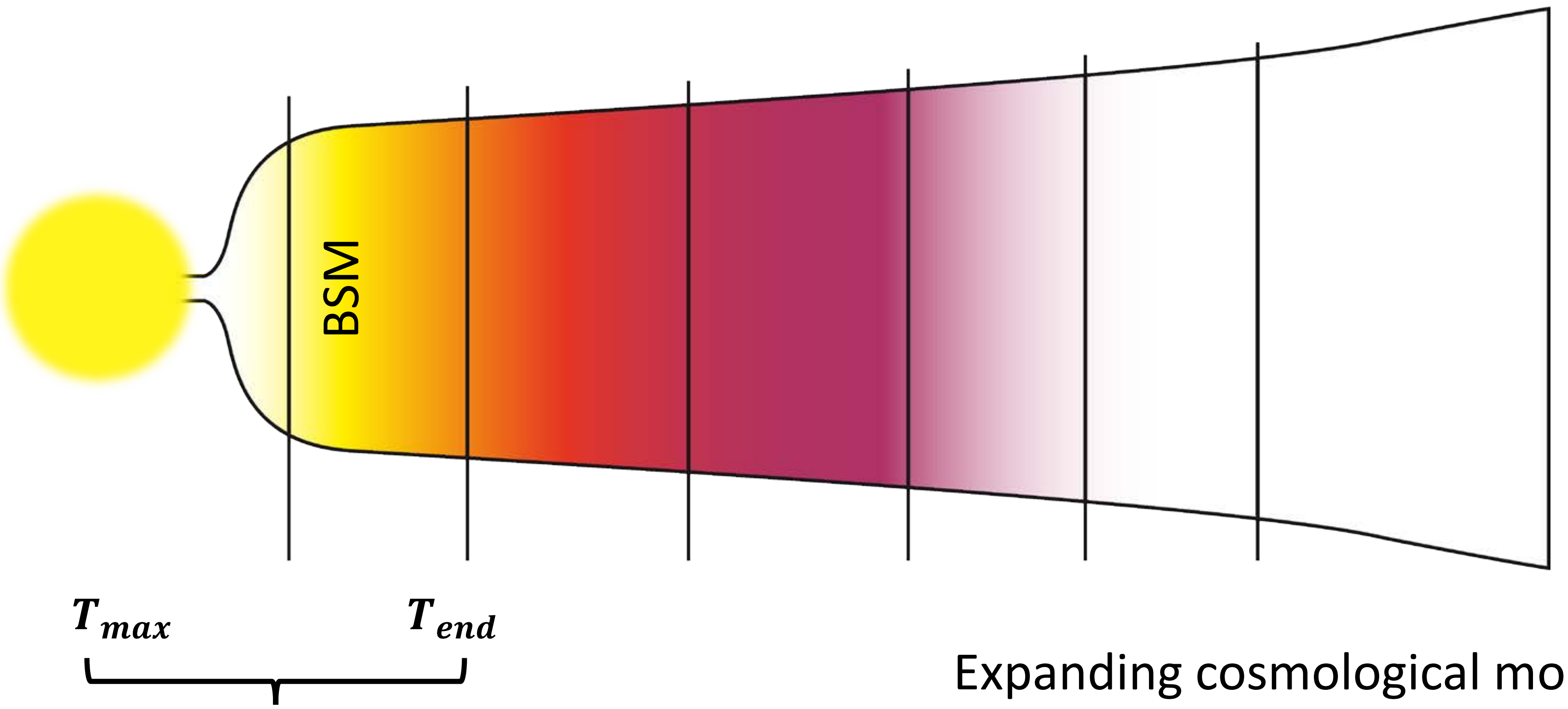
Emission from a thermal plasma in the Universe

$\Longrightarrow$

Consider the expansion of the Universe

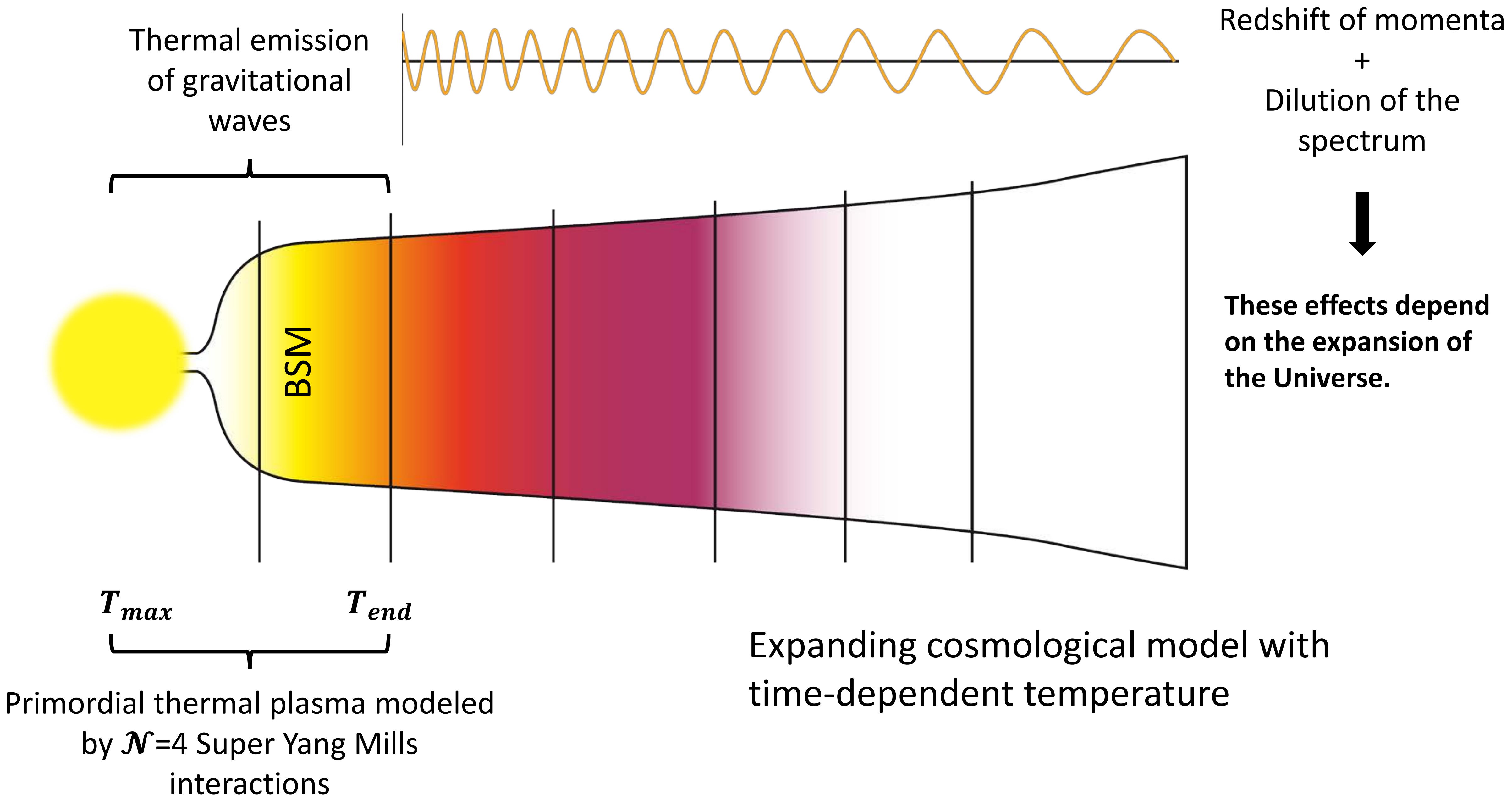


Expanding cosmological model with
time-dependent temperature



Primordial thermal plasma modeled by $\mathcal{N}$=4 Super Yang Mills interactions

Expanding cosmological model with time-dependent temperature


Thermal emission of gravitational waves
Redshift of momenta
+
Dilution of the spectrum
These effects depend on the expansion of the Universe.
BSM
T_{max}
T_{end}
Primordial thermal plasma modeled by $\mathcal{N}$=4 Super Yang Mills interactions
Expanding cosmological model with time-dependent temperature

Cosmological evolution

The expansion of the Universe changes the spectrum by three effects:

- Rate of change of temperature dT/dt.
- Dilution of the fraction of energy of gravitational waves.
- Redshift of the frequency.

These effects depend on the number of effective degrees of freedom g_{*s}:

$$g_{*s}(T) = \frac{s(T)}{\frac{2\pi^2}{45}T^3}$$

Cosmological evolution

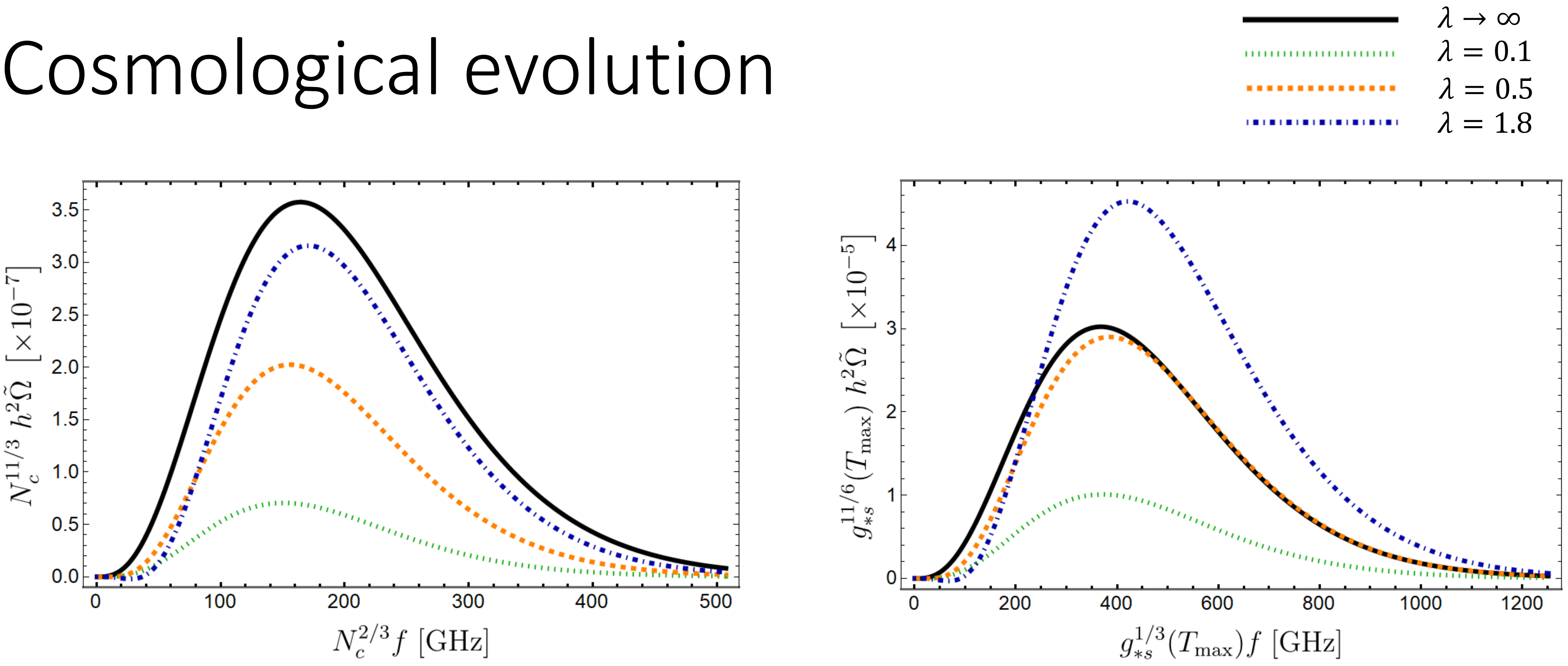
Present-day energy density per unit time and per logarithmic current frequency:

$$\Omega(f) \equiv \Omega(f; \lambda, \boldsymbol{g}_{*s})$$

- Weak coupling $\lambda \to 0$: $g_{*s,WC} = 15N_c^2$
- Strong coupling $\lambda \to \infty$: $g_{*s,SC} = \frac{45}{4}N_c^2$

The expansion of the Universe makes the spectra in the two regimes different even if the emission from the fixed plasma were the same.

Cosmological evolution



Actual spectra

Spectra after rescaling by the number of effective degrees of freedom

Current energy density after convolution with the expansion of the Universe, with $\tilde{\Omega} \equiv \frac{M_{Pl}}{T_{max}} \frac{\Omega}{N_c^2}$

Conclusions

- We have compared the strong coupling limit with a extrapolation of the leading-order perturbative calculation to intermediate values of the coupling.
- For the values of the coupling considered we have found no qualitative difference in the shape of the GW spectrum between weak and strong coupling.
- In particular, we have found a quantitative agreement between the strongly-coupled and the weakly-coupled static spectra for the limit value $\lambda = 0.5$.

Note for future work:

This analysis can help us build intuition about how to access strongly-coupled plasma dynamics via gauge/gravity duality. It could be extended to other more realistic, non-conformal theories.