All order resummed leading and next-to-leading logarithms for soft modes of dense QCD pressure

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Content

Motivations

- High and mid range of μ_B inaccessible by Lattice Simulations (with $T = 0$)
- Infrared divergences spoil naive perturbation theory
- Large scale dependence uncertainty in HTLpt calculations
- Renormalization Group properties can address these points

pQCD State-of-the-art at $T = 0, \mu \neq 0$

pQCD pressure for $N_f = 3, T = 0, \mu_B \neq 0$

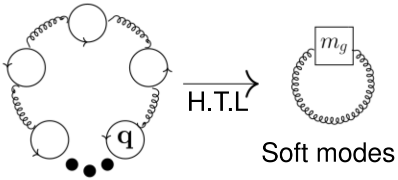
$$\mathcal{P}^{\mathrm{C.Q.M}} = \mathcal{P}_{\mathrm{f}} \left(1 - \frac{2}{\pi}\alpha_s - \frac{N_f}{\pi^2}\alpha_s^2 \ln \alpha_s - 0.874355\alpha_s^2 - 2d_A\alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln\left(\frac{M_h}{\mu}\right) \right)$$

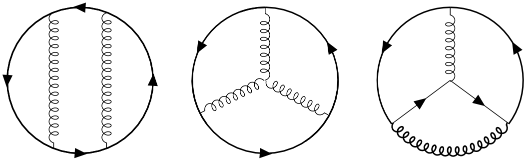
Ring (Soft)

Quarks (Hard)

Hard scale: μ , $M_h \sim$ renormalization scale

Soft scale: $m_E \sim \sqrt{\alpha_s}\mu$



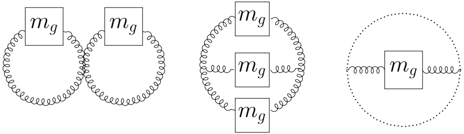


B.A Freedman and L.D. McLerran (1977), A. Vuorinen (2003)

Hard Thermal Loop

Hard Thermal Loop pressure (pure glue) at LO (equivalently NNLO for $m_g = m_E$)

$$\mathcal{P}^{\rm HTL}_{\rm LO} = \frac{d_A\, m_g^4}{(8\pi)^2}\left(\frac{1}{2\varepsilon} + C_{11} - \ln\left(\frac{m_g}{M}\right) + \varepsilon\left(\ln\left(\frac{m_g}{M}\right)^2 + C_{21}\ln\left(\frac{m_g}{M}\right) + C_{22}\right)\right)$$



C_{11}: S. Mogliacci et al. 2013
C_{21}, C_{22}: LF, J-L. Kneur (2021)

$$m_E^2 = 2\frac{\alpha_s}{\pi}\sum_f \mu_f$$

HTL at NLO (i.e NNNLO)

$$\mathcal{P}^{\rm HTL}_{\rm NLO} = \frac{N_c\, d_A\, \alpha_s m_g^4}{(8\pi)^2}\left(\frac{p_{-2}}{4\varepsilon^2} + \frac{p_{-1} - 2p_{-2}\ln\left(\frac{m_g}{M}\right)}{2\varepsilon} + 2p_{-2}\ln\left(\frac{m_g}{M}\right)^2 - 2p_{-1}\ln\left(\frac{m_g}{M}\right) + p_0\right)$$

T. Gorda, A. Kurkela, R. Paatelainen, S. Säppi and A. Vuorinen (GKPSV), PRL 127,162003 (2021)

Hard Thermal Loop: Factorization picture

HTL in factorization picture

$$\mathcal{P}^{\mathrm{HTL}}_{\mathrm{LO}} = \frac{d_A\, m_g^4}{(8\pi)^2}\left(\frac{1}{2\varepsilon} + C_{11} - \ln\left(\frac{m_g}{M_s}\right)\right)$$

$$\mathcal{P}^{\mathrm{HTL}}_{\mathrm{NLO}} = \frac{N_c\, d_A\, \alpha_s m_g^4}{(8\pi)^2}\left(\frac{p_{-2}}{4\varepsilon^2} + \frac{p_{-1} - 2p_{-2}\ln\left(\frac{m_g}{M_s}\right)}{2\varepsilon} + \cancel{2}^{\,1}\, p_{-2}\ln\left(\frac{m_g}{M_s}\right)^2 - 2p_{-1}\ln\left(\frac{m_g}{M_s}\right) + p_0\right)$$

UV divergences, they cancel against IR divergences in hard sector

M_s: Factorization scale

UV vs. IR cancellation argument

Hard Thermal Loop: EFT picture

Organization of a complete EFT calculation (ideally)

$$\mathcal{P}^{\rm Full} = \mathcal{P}_{\rm HTL} + \left(\mathcal{P}^{\rm Full} - \mathcal{P}_{\rm HTL}\right)\Big|_{m_g << \mu}$$

- $\mathcal{P}_{\rm QCD}$ and $\mathcal{P}_{\rm HTL}$ have different UV behaviors (counterterms, anomalous dimensions).
- $\left(\mathcal{P}_{\rm QCD} - \mathcal{P}_{\rm HTL}\right)\Big|_{m_g << \mu}$ is part of the *EFT matching* contribution. Their IR divergences cancel against IR divergences of $\mathcal{P}^{\rm Ring}_{\rm Hard}$.
- We consider here only the *soft* modes.
- α_s^2 order is already known exactly, thus we do not need to calculate this extra contribution.
- At α_s^3, hard contributions unknown but not needed for the LL, NLL: given by EFT RG.

RG definitions

(Massive) Renormalization group operator

$$M\frac{d}{dM} \equiv M\frac{\partial}{\partial M} + \beta(g^2)\frac{\partial}{\partial g^2} - \gamma_m^g(g^2)m_g\frac{\partial}{\partial m_g}$$

γ_m^g: Anomalous mass dimension of the gluon.

$$\gamma_m^g(g^2) = \gamma_0^g g^2 + \gamma_1^g g^4 + \ldots$$

$$\gamma_0^g = \frac{11N_c}{3(4\pi)^2} \equiv b_0^g$$

$$\longrightarrow \quad m_g^B \equiv m_g\,\mathcal{Z}_{m_g} \simeq m_g\left(1 - g^2\frac{\gamma_0^g}{2\varepsilon}\right) + \mathcal{O}(g^4)$$

Hard Thermal Loop: EFT picture

Renormalization of the EFT

$$\mathcal{P}^{\rm HTL}_{\rm LO} = \frac{d_A\, m_g^4}{(8\pi)^2}\left(\frac{1}{2\varepsilon} + C_{11} - \ln\left(\frac{m_g}{M_s}\right)\right)$$

$$\mathcal{P}^{\rm HTL}_{\rm NLO} = \frac{N_c\, d_A\, \alpha_s m_g^4}{(8\pi)^2}\left(\frac{p_{-2}}{4\varepsilon^2} + \frac{p_{-1} - 2p_{-2}\ln\left(\frac{m_g}{M_s}\right)}{2\varepsilon} + \cancel{2}^{\,1} p_{-2}\ln\left(\frac{m_g}{M_s}\right)^2 - 2p_{-1}\ln\left(\frac{m_g}{M_s}\right) + p_0\right)$$

Automatically

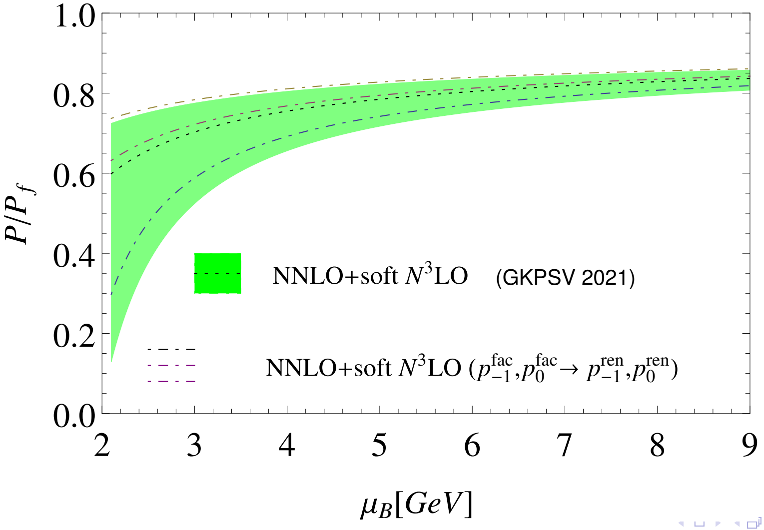
Renormalized by vacuum energy counterterms

and *nonlocal divergences* by mass counterterms

Weinberg theorem proves renormalizability at massive NLO (i.e NNNLO)

$$p_{-1} \to p_{-1} - \frac{8\pi\,\gamma_0^g}{N_c}(C_{11} - \frac{1}{4}) \simeq p_{-1} - 0.5381 \quad , \quad p_0 \to p_0 - \frac{8\pi\,\gamma_0^g}{N_c}(C_{22} - \frac{C_{11}}{2}) \simeq p_0 - 0.9229$$

HTL pressure in EFT versus Factorization scheme



$M_h \in [\mu, 2\mu, 4\mu]$
$M_s = 0.275 m_E$
$m_g = m_E$

LL and NLL series

$$g^2 = (4\pi)\alpha_s$$

Soft pressure from power counting

$$\mathcal{P}^{soft} \sim m_g^4 \sum_{p=1}^{\infty} (g^2)^{p-1} \sum_{l=0}^{p} a_{p,l} \ln^{p-l}\left(\frac{m_g}{M_s}\right)$$

$$L \equiv \ln\left(\frac{m_g}{M_s}\right)$$

$$\begin{aligned} \mathcal{P}^{soft} = \; & a_{1,0}\, L + g^2\, a_{2,0}\, L^2 + \ldots \; LL \\ & + a_{1,1} + g^2\, a_{2,1}\, L + \ldots \; NLL \\ & + g^2\, a_{2,2} + \ldots \end{aligned}$$

Renormalization group property

The first order coefficient at which a series of (sub)leading logarithm appear is sufficient to determine all order coefficients upon requiring $M\frac{d\mathcal{P}}{dM} = 0$

LL and NLL series

LL series

$$-p\, a_{p,0} = \left(4\gamma_0^g + 2b_0^g(p-2)\right) a_{p-1,0}, \;\; p \geq 2$$

NLL series

$$(1-p)a_{p,1} = \left(4\gamma_0^g + 2b_0^g(p-2)\right) a_{p-1,1} + \left(4\gamma_1^g + 2b_1^g(p-3)\right) a_{p-2,0} + \gamma_0^g(p-1)a_{p-1,0}$$

$$\begin{aligned}\mathcal{P}^{soft} = \; & a_{1,0}\, L + g^2\, a_{2,0}\, L^2 + \ldots \; LL \\ & + a_{1,1} + g^2\, a_{2,1}\, L + \ldots \; NLL \\ & + g^2\, a_{2,2} + \ldots\end{aligned}$$

Prediction: $a_{2,0} = -2\gamma_0^g a_{1,0}$
Reproduce direct (GKPSV) calculation

Input for NLL series, $a_{2,1} \propto p_{-1}^{ren}$

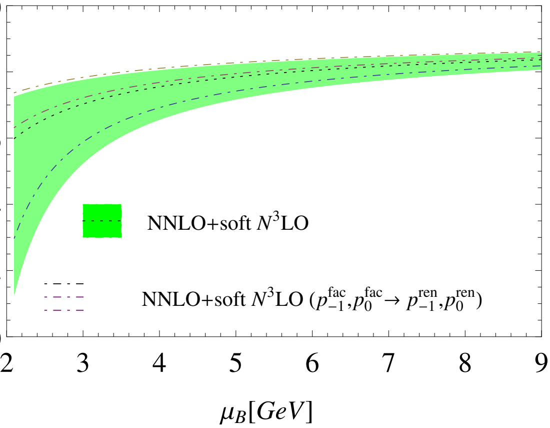
Resummation formula

LL series

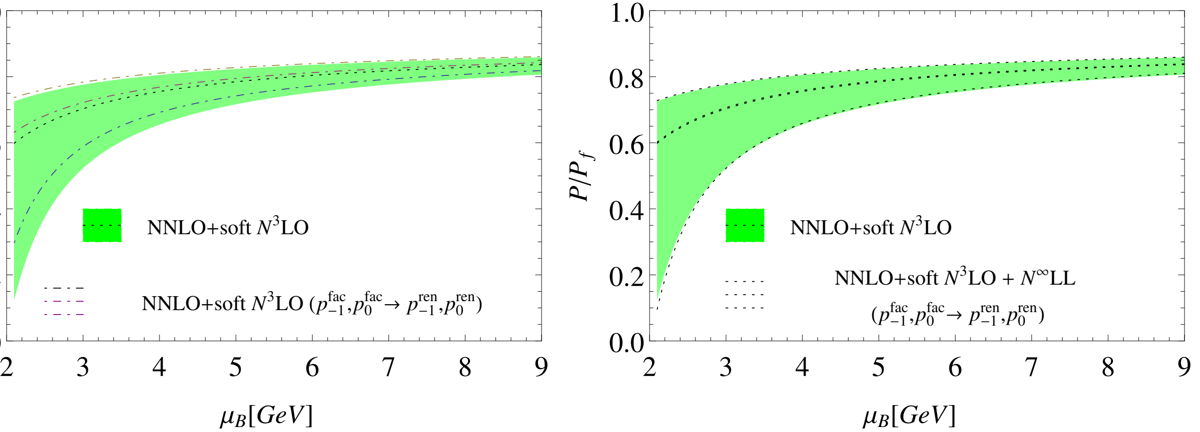
$$\mathcal{P}_{\mathrm{LL}}^{\mathrm{sum}} = \frac{a_{1,0} g^2 \ln\left(\frac{m_g}{M}\right) m_g^4}{g^2} f_1^{1-4\left(\frac{\gamma_0^g}{2b_0^g}\right)} = \frac{a_{1,0} g^2 \ln\left(\frac{m_g}{M}\right) m_g^4}{g^2} f_1^{-1},$$

$$f_1 = 1 + 2b_0^g g^2 \ln\frac{m_g}{M}.$$

LL resummation



LL resummation



Vacuum energy anomalous dimension: $\hat{\Gamma}^g$

NNLO vacuum energy counterterm

$$\Delta\mathcal{E}_0^{(2)} = \frac{d_A m_g^4}{(8\pi)^2}\left\{\frac{1}{2\varepsilon} + N_c\left(\frac{g^2}{4\pi}\right)\left(-\frac{11}{24\pi\varepsilon^2} + \left(\frac{p_{-1}}{2\varepsilon} - \frac{11}{6\pi\varepsilon}(C_{11} - \frac{1}{4})\right)\right)\right\} \equiv \mathcal{Z}_0^g(g^2)m_g^4$$

At LO, same counterterm as in HTL (J.O Andersen et al.(2002))

Vacuum energy anomalous dimension: $\hat{\Gamma}^g$

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At LO, same counterterm as in HTL (Andersen et al. (2002))

$$-\frac{d\mathcal{P}_{\mathrm{HTL}}}{d\ln M} = \frac{d\mathcal{E}_0}{d\ln M} \equiv \hat{\Gamma}_0^g(g^2)\, m_g^4 = \frac{d}{d\ln M}\left(m_g^4 \sum_{k\geq 0} s_k^g g^{2k-2}\right) \to \mathcal{E}_0^B \equiv M^{-2\varepsilon}\left(\mathcal{E}_0(g^2) - m_g^4 \mathcal{Z}_0^g(g^2)\right)$$

Renormalization group invariant (RGI) pressure

$$\mathcal{P}_{\mathrm{RGI}}^{\mathrm{HTL}} \equiv \mathcal{P}^{\mathrm{HTL}} - m_g^4 \sum_{k\geq 0} s_k^g g^{2k-2} = \mathcal{P}^{\mathrm{HTL}} - m_g^4 \frac{s_0^g}{g^2} - m_g^4 s_1^g + \ldots$$

Matching the perturbative QCD pressure

$$\mathcal{P}^{\rm HTL}(g^2(M), m(M)) = \mathcal{P}^{\rm HTL}(M_0) + \int_{g^2(M_0)}^{g^2(M)} dx \left\{ \frac{-\hat{\Gamma}_0^g(x)}{\beta(x)} \exp\left[-4\int_{g^2(M_0)}^{x} dy \frac{\gamma_m(y)}{\beta(y)}\right]\right\} m_g^4(M_0)$$

Boundary term

$$\mathcal{P}^{\rm HTL}_{\rm RGI,B.C} \equiv \mathcal{P}^{\rm HTL}_{\rm RGI} + m_g^4(M_0)\left(\frac{s_0^g}{g(M_0)^2} + s_1^g\right)$$

$$s_0^g = -\frac{a_{1,0}}{2(b_0^g - 2\gamma_0^g)} = \frac{-d_A}{2(8\pi)^2\, b_0^g},$$

$$s_1^g = a_{1,1} + \frac{a_{2,1}}{4\gamma_0^g} + \frac{a_{1,0}}{4} + \frac{s_0^g}{2\gamma_0^g}(b_1^g - 2\gamma_1^g).$$

To match the perturbative pressure of full QCD, M_0 must be of the order of the hard scale $\mathcal{O}(\mu)$. Natural choice: $M_0 = 2\mu$.

Collecting the pieces

RGI pressure

$$\mathcal{P}_{\rm RGI}^{\rm C.Q.M} = \mathcal{P}_{\rm f}\left(1 - \frac{2}{\pi}\alpha_s - \frac{N_f}{\pi^2}\alpha_s^2 \ln\alpha_s - 0.874355\alpha_s^2 - 2d_A\alpha_s^2\frac{(11N_c - 2N_f)}{3(4\pi)^2}\ln\left(\frac{M_h}{\mu}\right)\right)$$

← Full N^2LO

Collecting the pieces

RGI pressure

$$\mathcal{P}_{\rm RGI}^{\rm C.Q.M} = \mathcal{P}_{\rm f}\left(1 - \frac{2}{\pi}\alpha_s - \frac{N_f}{\pi^2}\alpha_s^2 \ln\alpha_s - 0.874355\alpha_s^2 - 2d_A\alpha_s^2\frac{(11N_c - 2N_f)}{3(4\pi)^2}\ln\left(\frac{M_h}{\mu}\right)\right) \leftarrow \text{Full } N^2LO$$

$$+\frac{N_c\, d_A\, \alpha_s m_E^4}{(8\pi)^2}\left(p_{-2}\ln\left(\frac{m_g}{M_s}\right)^2 - 2p_{-1}^{\rm ren}\ln\left(\frac{m_g}{M_s}\right) + p_0^{\rm ren}\right) \leftarrow \text{Soft N}^3\text{LO}$$

Collecting the pieces

RGI pressure

$$\mathcal{P}_{\text{RGI}}^{\text{C.Q.M}} = \mathcal{P}_{\text{f}} \left(1 - \frac{2}{\pi}\alpha_s - \frac{N_f}{\pi^2}\alpha_s^2 \ln \alpha_s - 0.874355\alpha_s^2 - 2 d_A \alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln\left(\frac{M_h}{\mu}\right)\right) \leftarrow \text{Full } N^2LO$$

$$+ \frac{N_c\, d_A\, \alpha_s m_E^4}{(8\pi)^2} \left(p_{-2} \ln\left(\frac{m_g}{M_s}\right)^2 - 2 p_{-1}^{\text{ren}} \ln\left(\frac{m_g}{M_s}\right) + p_0^{\text{ren}} \right) \leftarrow \text{Soft N}^3\text{LO}$$

$$+ m_E^4\, g(M_h)^4\, a_{3,0} \ln^3\left(\frac{m_E}{M_s}\right) + m_E^4\, g(M_h)^6\, a_{4,0} \ln^4\left(\frac{m_E}{M_s}\right) + \ldots \leftarrow \text{LL series}$$

$$+ m_E^4\, g(M_h)^4 a_{3,1} \ln^2\left(\frac{m_E}{M_s}\right) + m_E^4\, g(M_h)^6\, a_{4,1} \ln^3\left(\frac{m_E}{M_s}\right) + \ldots \leftarrow \text{NLL series}$$

Collecting the pieces

RGI pressure

$$
\begin{aligned}
\mathcal{P}_{\mathrm{RGI}}^{\mathrm{C.Q.M}} = \mathcal{P}_{\mathrm{f}} \Bigg(1 - \frac{2}{\pi}\alpha_s - \frac{N_f}{\pi^2}\alpha_s^2 \ln \alpha_s - 0.874355\alpha_s^2 - 2 d_A \alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln\left(\frac{M_h}{\mu}\right)\Bigg) \quad &\leftarrow \text{Full } N^2LO \\
+ \frac{N_c\, d_A\, \alpha_s m_E^4}{(8\pi)^2} \left(p_{-2} \ln\left(\frac{m_g}{M_s}\right)^2 - 2 p_{-1}^{\mathrm{ren}} \ln\left(\frac{m_g}{M_s}\right) + p_0^{\mathrm{ren}} \right) \quad &\leftarrow \text{Soft N}^3\text{LO} \\
+ m_E^4\, g(M_h)^4\, a_{3,0} \ln^3\left(\frac{m_E}{M_s}\right) + m_E^4\, g(M_h)^6\, a_{4,0} \ln^4\left(\frac{m_E}{M_s}\right) + \ldots \quad &\leftarrow \text{LL series} \\
+ m_E^4\, g(M_h)^4 a_{3,1} \ln^2\left(\frac{m_E}{M_s}\right) + m_E^4\, g(M_h)^6\, a_{4,1} \ln^3\left(\frac{m_E}{M_s}\right) + \ldots \quad &\leftarrow \text{NLL series} \\
- s_0^g \left(\frac{m_E^4(M_h)}{g(M_h)^2} - \frac{m_E^4(M_0)}{g^2(M_0)} \right) - s_1^g \left(m_E^4(M_h) - m_E^4(M_0) \right) \quad &\leftarrow \text{RGI restoring terms}
\end{aligned}
$$

Compact formula

RGI pressure

$$\mathcal{P}_{\rm RGI}^{\rm C.Q.M} = \mathcal{P}_{\rm f}\left(1 - \frac{2}{\pi}\alpha_s - \frac{N_f}{\pi^2}\alpha_s^2 \ln\alpha_s - 0.874355\alpha_s^2 - 2d_A\alpha_s^2\frac{(11N_c - 2N_f)}{3(4\pi)^2}\ln\left(\frac{M_h}{\mu}\right)\right) - \frac{s_0^4\, m_g^4}{g^2\, f_2^{4A_0-1}}[R(f_2)]^B\left(1 - \frac{a'_{1,1}\, g^2}{s_0^g\, f_2} - \frac{a_{2,2}\, g^4}{s_0^g\, f_2^2}\right) - \left(a_{1,1} + a_{1,0}\ln\left(\frac{m_g}{M_s}\right)\right) + \frac{m_g^4(M_0)}{g^2(M_0)}s_0^g + m_g^4(M_0)s_1^g$$

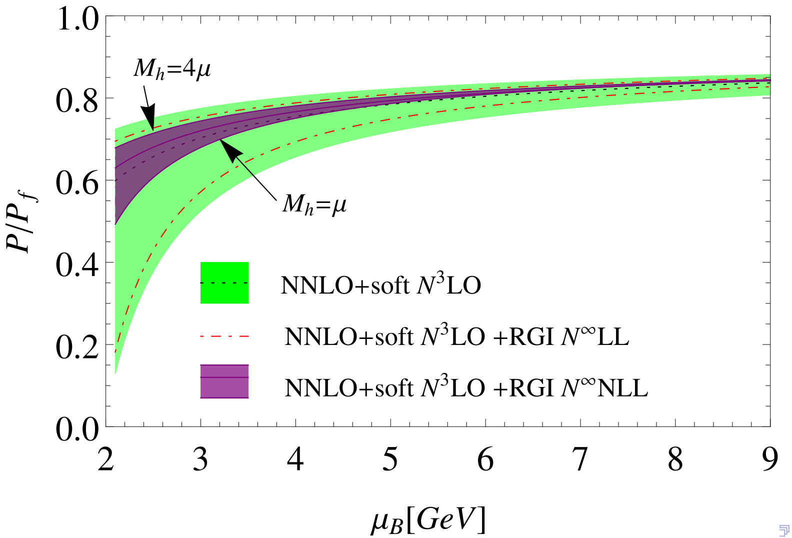
$$a'_{1,1} = a_{1,1} - s_1^g$$

To avoid double counting

$$R(f_2) = (1 + g^2\, b_1^g/(b_0^g f_2))/(1 + g^2\, b_1^g/b_0^g), \qquad A_0 = \gamma_0^g/(2b_0^g), \quad A_1 = \gamma_1^g/(2b_1^g)$$

$$f_2 \simeq 1 + \left[2b_0^g\, g^2 + 2(b_1^g - \gamma_0^g b_0^g)g^4\right]\ln\frac{m_g}{M} + \mathcal{O}(g^6), \qquad B = 4(A_1 - A_0), \; C = b_1^g/(2(b_0^g)^2)$$

Complete RGI pressure results



$M_h \in [\mu, 2\mu, 4\mu]$
$M_s = 0.275 m_E$
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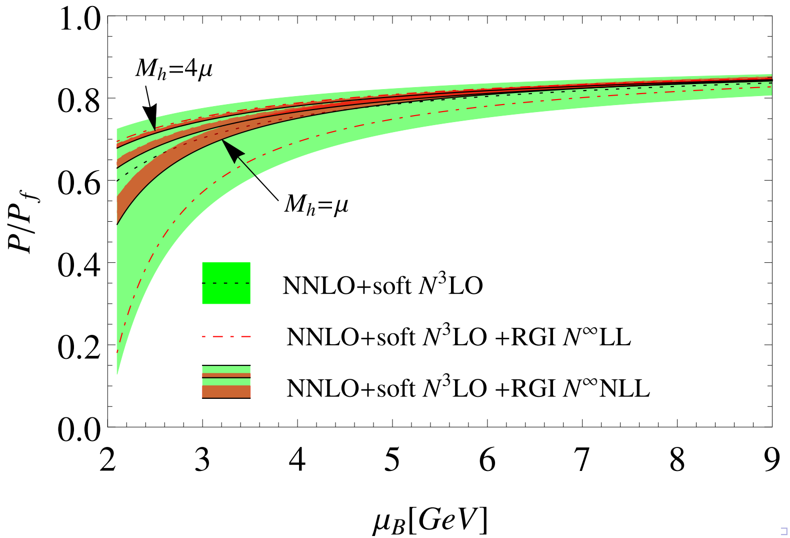
Complete RGI pressure results



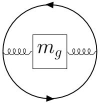
$M_h \in [\mu, 4\mu]$
$M_s \in [1/2, 2] \times 0.275 m_E$
$m_g = m_E$

LF, J-L. Kneur (2021)

Conclusion and perspectives

- Our RGI construction exhibits substantially improved remnant scale dependence.

- Evaluation of the HTL fermion loop diagram (beyond pure glue)
- Add these logs to the resummation



(Plus higher orders)

- Effects of non-zero quark masses
- Application to neutron stars equation-of-state with expected reduced scale dependence uncertainties.

Thank you for your attention !