Pushing the HTL/HDL theory: power corrections and transport theory

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Breakdown of perturbation theory

Braaten and Pisarski; Frenkel and Taylor, 90'

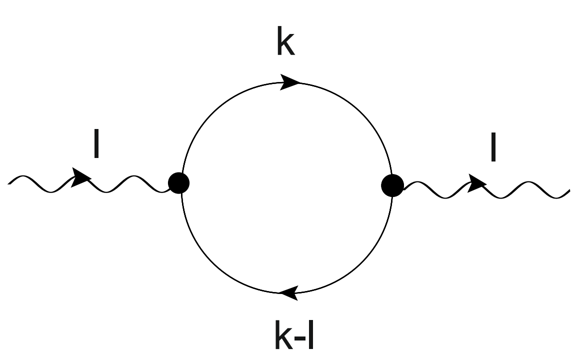
At high temperature: two relevant scales

hard $\sim T$

soft $\sim gT$

$g \ll 1$

One-loop thermal corrections hard thermal loops (HTLs) as relevant as the tree amplitudes for soft momenta (and they arise from hard loop momenta)



$$\Pi_{\rm HTL}(l) \sim g^2 T^2$$

$$\frac{\Pi_{\rm HTL}(l)}{l^2} \sim 1$$

for soft momentum

and have to be resummed into effective vertices and propagators

Hot QCD plasmas

$g << 1$

T	hard scale	classical on-shell particles transport equations
$g\,T$	soft scale	classical fields HTL EFT
$g^2T\ln(1/g)$	ultrasoft scale	Langevin type of eqs.
$g^2\,T$	magnetic scale	??

How to improve the previous picture?
let us start with QED

- Perturbative corrections (two-loop diagrams) $e^2 \times (\mathrm{HTL})$
- Power corrections (POW)

$$\left(\frac{k}{T}\right)^2 \times \mathrm{HTL} \qquad k \sim eT \Rightarrow e^2 \times \mathrm{HTL}$$

From one-loop diagrams: expanding (soft/hard) arXiv:1712.07949

From transport theory: pushing the gradient expansion, keeping quantum corrections arXiv:2107.03655

$$\Delta^2 = (\partial_x \cdot \partial_q)^2 \qquad (\hbar^2 \text{ corrections})$$

Using effective field theory methods (OSEFT): expansion done at the Lagrangian level arXiv:1603.05514

(I am ignoring possible effects of P and CP violation in this talk!)

Power corrections to the HTL photon polarization tensor

We use the RTF in the Keldysh representation

$$S_{R/A}(Q) = \frac{\not{Q}}{Q^2 \pm i\mathrm{sgn}(\mathrm{q}_0)\eta} \qquad S_S(Q) = -2\pi i\not{Q}\,(1-2n_F(|q_0|))\delta(Q^2)$$

$$\eta \to 0^+ \qquad Q^\mu = (q_0, \mathbf{q})$$

$$\Pi_R^{\mu\nu}(L) = -\frac{ie^2}{2}\int \frac{d^4q}{(2\pi)^4}\Big(\mathrm{Tr}[\gamma^\mu S_S(K)\gamma^\nu S_R(Q)] + \mathrm{Tr}[\gamma^\mu S_A(K)\gamma^\nu S_S(Q)]\Big)$$

$$K = Q - L$$

$$\Pi_R^{\mu\nu}(L) = e^2\int \frac{d^3q}{(2\pi)^3}\frac{1-2n_F(q)}{q}\left(\frac{2qv^\mu v^\nu - (v^\mu L^\nu + v^\nu L^\mu) + g^{\mu\nu}\,v\cdot L}{v\cdot L - \frac{L^2}{2q} + i\,\mathrm{sgn}(q-l_0)\eta}\right.$$
$$\left. - \frac{2q\tilde{v}^\mu\tilde{v}^\nu - (\tilde{v}^\mu L^\nu + \tilde{v}^\nu L^\mu) + g^{\mu\nu}\,\tilde{v}\cdot L}{\tilde{v}\cdot L + \frac{L^2}{2q} + i\,\mathrm{sgn}(q+l_0)\eta}\right)$$

$$v^\mu = (1, \mathbf{q}/q) \qquad \tilde{v}^\mu = (1, -\mathbf{q}/q)$$

Expanding the integrand: use dimensional regularization!

$d = 3 + 2\epsilon$

$$\Pi^{\mu\nu}_{(1)}(L) = \quad 2e^2\nu^{3-d}\int \frac{d^dq}{(2\pi)^d}\frac{1-2n_F(q)}{q}\left(\frac{v^\mu v^\nu L^2}{(v\cdot L)^2} - \frac{v^\mu L^\nu + v^\nu L^\mu}{v\cdot L} + g^{\mu\nu}\right) \,,$$

This is the HTL

$$\Pi^{\mu\nu}_{(3)}(L) = \quad 2e^2\nu^{3-d}\int \frac{d^dq}{(2\pi)^d}\frac{1-2n_F(q)}{q^3}\frac{L^4}{4(v\cdot L)^2}\left(\frac{v^\mu v^\nu L^2}{(v\cdot L)^2} - \frac{v^\mu L^\nu + v^\nu L^\mu}{v\cdot L} + g^{\mu\nu}\right) \,,$$

First power correction

Higher order power corrections $\quad \dfrac{(1-2n_f(q))}{q}\left(\dfrac{L^4}{4q^2(v\cdot L)^2}\right)^n \times \mathrm{int}_{\mathrm{HTL}}$

which are increasingly every time more IR problematic!

First power correction (POW)

Note that if we keep only the thermal contribution, the integral is IR divergent in d=3

$$\nu^{-2\epsilon}\int_0^\infty dq q^{-1+2\epsilon}\, n_F(q) = \frac{1}{4\epsilon} + \frac{1}{2}\ln\left(\frac{\pi T e^{-\gamma_E}}{2\nu}\right) + \mathcal{O}(\epsilon)$$

The IR divergence of the thermal part is cancelled with the IR divergence of the vacuum!

$$1 - 2n_F(q) \approx \frac{q}{2T}$$

POW is IR finite!

But it contains the UV divergence of the vacuum, which is cured adding counterterms, as expected

$$\Pi^L_{\rm POW} = \frac{\alpha}{3\pi}\left[\frac{l^2}{\epsilon} + 2l^2\left(\ln\frac{\sqrt{\pi}Te^{-\gamma_E/2}}{2\nu} - 1\right) + (2l^2 - L^2)\left(1 - \frac{l_0}{2l}\ln\frac{l_0+l}{l_0-l}\right)\right]$$

$$\Pi^T_{\rm POW} = \frac{2\alpha L^2}{3\pi}\left[\frac{1}{2\epsilon} + \left(\ln\frac{\sqrt{\pi}Te^{-\gamma_E/2}}{2\nu} - 1\right) + \frac{1}{4} + \left(1 + \frac{L^2}{4l^2}\right)\left(1 - \frac{l_0}{2l}\ln\frac{l_0+l}{l_0-l}\right)\right]$$

Including chemical potential ; change

$$n_F(q) \to \frac{1}{2}\Big[n_F(q-\mu)+n_F(q+\mu)\Big]$$

Evaluate then

$$\frac{\nu^{-2\epsilon}}{2}\int_0^\infty dq q^{-1+2\epsilon}\Big[n_F(q-\mu)+n_F(q+\mu)\Big] = \frac{1}{4\epsilon}+\frac{1}{2}\ln\left(\frac{\pi T}{2\nu}\right)-\frac{1}{4}\Psi(T,\mu)+\mathcal{O}(\epsilon)$$

For T=0, the POW at finite chemical potential is obtained by

$$\ln\frac{\sqrt{\pi}e^{-\gamma^E/2}T}{2\nu} \quad \to \quad \ln\frac{e^{\gamma^E/2}\mu}{\sqrt{\pi}\nu}$$

Results have also been obtained by Gorda et al, 2022

Effective Lagrangian

Carignano, CM, Soto, '18

$$\mathcal{L}^{(1)}_{\rm HTL} = \frac{e^2}{2}\int \frac{d^3q}{(2\pi)^3}\left\{ \frac{2n_F(q)}{q}\left(F_{\rho\alpha}\frac{v^\alpha v^\beta}{(v\cdot\partial)^2}F^{\rho}{}_{\beta}\right) - \frac{2(n_F(q)+n_B(q))}{q}\left(\bar\psi\frac{v\cdot\gamma}{(iv\cdot D)}\psi\right)\right\}$$

In d spatial dimensions

$$v^\mu = q^\mu/|\mathbf{q}|$$

$$\mathcal{L}^{(3)\gamma}_{\rm HTL} = \frac{e^2\nu^{3-d}}{4}\int \frac{d^dq}{(2\pi)^d}\frac{1-2n_F(q)}{q^3}\left\{F_{\rho\alpha}\frac{v^\alpha v^\beta}{(v\cdot\partial)^4}\partial^4 F_{\beta}{}^{\rho}\right\}$$

$$\mathcal{L}^{(3)\,\psi}_{\rm HTL} = \frac{e^2\nu^{3-d}}{4}(d-1)\left[\int \frac{d^dq}{(2\pi)^d}\frac{n_F(q)+n_B(q)}{q^3}\left\{\bar\psi\, D^2\frac{v\cdot\gamma}{(iv\cdot D)^3}D^2\,\psi\right\}\right.$$

$$\left. + \int \frac{d^dq}{(2\pi)^d}\frac{1+2n_B(q)}{2q^3}\left\{\bar\psi\left(D^2(iD\cdot\gamma)\frac{1}{(iv\cdot D)^2} + + \frac{1}{(iv\cdot D)^2}(iD\cdot\gamma)D^2\right)\psi\right\}\right] + \mathcal{O}(e^3)$$

Power corrections = gradient expansion in transport theory

Transport equations can be derived from quantum field theory

typically, a gradient expansion is carried out

The leading term corresponds to classical physics

(and recall classical physics ~ HTL)

Subleading terms corresponds to pure quantum corrections,
and we see those give the POW!

Quantum corrections to classical transport

A Dirac particle does not move along a straight line with constant velocity, but carries out a dancing motion (**Zitterbewegung**) with the speed of light centered on a point that does move uniformly

ZB oscillations might become problematic when deriving quantum corrections to classical transport equations!

Introduce a coarse graining that eliminates these oscillations

Rafelski, Shin 93

(for example, with a Gaussian … but this introduces one scale in the problem)

Or alternatively …. diagonalize the dynamics, separating particles
nd antiparticles degrees of freedom (use OSEFT = on-shell effective field theory)

Wigner function: quantum analogue of the classical distribution function

(but it is a 4x4 matrix in Dirac space)

$$W(X,q) = \int \frac{d^4s}{(2\pi)^4} e^{-iq\cdot s} W(x,y) = \int \frac{d^4s}{(2\pi)^4} e^{-iq\cdot s} \langle \bar{\psi}(x) U(x,y) \psi(y) \rangle$$

$$X = \frac{x+y}{2} \qquad s = x - y$$

$$U\left(X + \frac{s}{2}, X - \frac{s}{2}\right) = \exp\left\{-ies^{\mu} \int_0^1 dz\, A_{\mu}\left(X - \frac{s}{2} + zs\right)\right\}$$

$$\not{D}_y W(x,y) = W(x,y) \not{D}_x^{\dagger} = 0 \qquad D_x^{\mu} = \partial_x^{\mu} + ieA^{\mu}(x)$$

$$(D_x^2 \mp D_y^{*2}) W(x,y) - \frac{e}{2}\left[F_{\mu\nu}(x)\sigma^{\mu\nu} W(x,y) \mp F_{\mu\nu}(y) W(x,y) \sigma^{\mu\nu}\right] = 0$$

Carry out the Wigner transformation and the gradient expansion in $\partial_X \equiv \partial$

(in principle you can keep all terms …

Vasak, Gyulassy and Elze, 1987

$$(q \cdot \partial - eq \cdot F \cdot \partial_q) W(X,q) + \frac{ie}{4} F_{\mu\nu}[\sigma^{\mu\nu}, W] - \frac{e}{8} \Delta F_{\mu\nu}(X)\{\sigma^{\mu\nu}, W\} =$$

$$\left[\frac{e}{24} q^\mu (\Delta^2 F_{\mu\nu}) \partial_q^\nu + \frac{e}{12}(\Delta F^{\lambda\nu}) \partial_\lambda^q (\partial_\nu - eF_{\nu\beta}\partial_q^\beta)\right] W + \frac{ie}{32} \Delta^2 F_{\mu\nu}[\sigma^{\mu\nu}, W] + \mathcal{O}(\partial^3)$$

$\Delta \equiv \partial \cdot \partial_q$ Spatial derivatives only act on F

Note: if you restore dimensions - this carries an $\hbar$

We assume

$\Delta \ll 1$ $\frac{\ell}{q} \ll 1$

$$W = \mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\alpha V_\alpha + \gamma^5\gamma^\alpha A_\alpha + \frac{1}{2}\sigma^{\mu\nu}S_{\mu\nu}$$

Transport equation for the vectorial components

$$\left\{\left[q\cdot\partial - eq\cdot F\cdot\partial_q + \frac{e}{24}q^\mu(\Delta^2 F_{\mu\nu})\partial_q^\nu + \frac{e}{12}(\Delta F^{\lambda\nu})\partial_\lambda^q(\partial_\nu - eF_{\nu\sigma}\partial_q^\sigma)\right]g^{\alpha\beta} - e\left(1-\frac{1}{8}\Delta^2\right)F^{\alpha\beta}\right\}V_\beta = 0$$

Dispersion relation is also modified

$$\left[Q^2 - \frac{e}{6}q^\mu\Delta F_{\mu\nu}\partial_q^\nu - \frac{e}{12}\partial^\mu F_{\mu\nu}\partial_q^\nu - \frac{1}{4}\left(\partial_\mu - eF_{\mu\nu}\partial_q^\nu\right)^2\right]V^\alpha + \frac{e}{2}\Delta F^{\alpha\beta}V_\beta = 0$$

$$Q^2 = q_\mu q^\mu$$

HTL and POW from transport theory

Close to thermal equilibrium

$$V^\mu = V^\mu_{(0)} + e\Big(V^\mu_{\rm HTL} + V^\mu_{\rm pow} + \dots\Big) + e^2(\dots)$$

$$V^\alpha_{(0)} = q^\alpha \, 2\pi \, sgn(q^0)\, \delta(Q^2) f_F(q^0) \qquad f_F = [1 + \exp(q^0/T)]^{-1}$$

Linearize the transport equation around the thermal equilibrium solution

$$q \cdot \partial V^\alpha_{\rm HTL} = e\Big[g^\alpha{}_\beta q \cdot F \cdot \partial_q + F^\alpha{}_\beta\Big] V^\beta_{(0)}$$

$$q \cdot \partial V^\alpha_{\rm pow} + e\Big[\frac{1}{24} q^\mu (\Delta^2 F_{\mu\nu}) \partial^\nu_q g^{\alpha\beta} + \frac{1}{8} \Delta^2 F^{\alpha\beta}\Big] V^{(0)}_\beta = 0 \, ,$$

The electromagnetic current

$$j^\mu = 4e \int \frac{d^4q}{(2\pi)^4} V^\mu$$

$$j^\mu_{\rm HTL} = 2e^2 \int_q \frac{1-2f_F(q)}{q} \left[\frac{v_\lambda}{v\cdot\partial} F^{\lambda\mu} - \partial_\nu \frac{v^\mu v_\lambda}{(v\cdot\partial)^2} F^{\lambda\nu} \right]$$

$$j^\mu_{\rm pow} = 4e^2 \int_q (1-2f_F(q)) \left(\frac{\partial^4}{4(q\cdot\partial)^2} \right) \left[\frac{q_\lambda}{q\cdot\partial} F^{\lambda\mu} - \partial_\nu \frac{q^\mu q_\lambda}{(q\cdot\partial)^2} F^{\lambda\nu} \right],$$

$$\Pi^{\mu\nu} = \frac{\delta j^\mu}{\delta A_\nu}$$

We reproduce the HTL and POW:

UV divergence (which is a vacuum effect) is also reproduced!

New pieces: corrections to the soft propagation.

For soft momentum (schematically) $L \sim eT$

$$\Pi^{(1-loop)} \sim e^2T^2(1 + \frac{L^2}{T^2} + \frac{L^4}{T^4} + \cdots)$$

HTL

POW

The first POW competes with 2-loops coming from hard scales for soft momenta

$$\Pi^{(2-loops)} \sim \alpha^2 T^2$$

Nothing new: loop expansion =/= perturbative expansion

But POW are the leading correction to the HTL for $eT \ll L \ll T$

Two-loops corrections have been also computed

Carignano, Carrington, Soto 2019; Gorda et al 2022

$$\Pi_I = \Pi_I^{\rm htl} + \Pi_I^{\rm pow\cdot corr} + \Pi_I^{\rm 2loop} \,, \qquad I = L, T$$

In Feynman gauge

$$\Pi_L^{\rm htl}(l_0, \boldsymbol{l}) = \frac{e^2 T^2}{3}\left(1 - \frac{l_0}{2|\boldsymbol{l}|}\ln\left(\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right)\right)$$

$$\Pi_L^{\rm pow}(l_0, \boldsymbol{l}) = -\frac{e^2}{4\pi^2}\left(\boldsymbol{l}^2 - \frac{l_0^2}{3}\right)\left(1 - \frac{l_0}{2|\boldsymbol{l}|}\ln\left(\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right)\right)$$

$$\Pi_L^{\rm 2loop}(l_0, \boldsymbol{l}) = \frac{e^4 T^2 L^2}{8\pi^2 \boldsymbol{l}^2}$$

$$\Pi_T^{\rm htl}(l_0, \boldsymbol{l}) = \frac{e^2 T^2}{3}\frac{l_0}{4\boldsymbol{l}^3}\left(2|\boldsymbol{l}|l_0 - L^2 \ln\left(\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right)\right)$$

$$\Pi_T^{\rm pow}(l_0, \boldsymbol{l}) = \frac{e^2}{4\pi^2}\left(\frac{l_0^2}{2} + \frac{l_0^4}{6\boldsymbol{l}^2} - \frac{2\boldsymbol{l}^2}{3} - \frac{l_0^3}{12\boldsymbol{l}^3}\left(2\boldsymbol{l}^2 + l_0^2 - \frac{3\boldsymbol{l}^4}{l_0^2}\right)\ln\left(\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right)\right)$$

$$\Pi_T^{\rm 2loop}(l_0, \boldsymbol{l}) = -\frac{e^4 T^2}{16\pi^2}\frac{l_0}{|\boldsymbol{l}|}\ln\left(\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right)$$

Taking the renormalization scale as $\nu = T e^{-\gamma_E/2 - 1}\sqrt{\pi}/2$

On-going related work

arXiv:2106.08904

If the fermions have a mass ~ soft there are corrections that compete with the perturbative and POW

$$\Pi_I = \Pi_I^{\rm HTL} + \Pi_I^{m} + \Pi_I^{\rm pow} + \Pi_I^{\rm 2loop} \qquad\qquad I = L, T$$

$m \ll T$ computed with OSEFT and kinetic theory

$$\Pi^{\rm L}_{\rm m}(l_0, \boldsymbol{l}) = \frac{e^2 m^2}{2\pi^2} \frac{\boldsymbol{l}^2}{l_0^2 - \boldsymbol{l}^2}$$

$$\Pi^{\rm T}_{\rm m}(l_0, \boldsymbol{l}) = \frac{e^2 m^2}{2\pi^2} \frac{l_0}{2|\boldsymbol{l}|} \ln\left(\frac{l_0 + |\boldsymbol{l}|}{l_0 - |\boldsymbol{l}|}\right)$$

Intermediate IR divergencies in the computation appear, but they finally cancel -
use a regulator respectful with gauge symmetry, as DR!

Corrections to the photon screening mass

$$m_S^2 = \frac{e^2T^2}{3}\left(1 - \frac{e^2}{8\pi^2} - \frac{1}{2\pi^2}\frac{m^2}{T^2}\right)$$

Discussion

I have discussed some of the corrections (POW) to the soft photon physics in high T plasmas, and how these can be recovered from transport theory

This should open the door to the computation of many properties in plasmas, mainly to the dynamics

Results for QCD will soon appear …