Aspects of Renormalization in the 2PI Formalism

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Outline

Why 2PI?

- Strong coupling and high temperatures: usual perturbation theory converges poorly $\longrightarrow$ **need resummation methods**.
- 2PI formalism provides a non-perturbative approach in which the action is expressed in terms of the background field, and the corresponding resummed two-point function.
- Introduces a two-point source in the action which incorporates out-of-equilibrium correlators $\longrightarrow$ allows studying of out-of-equilibrium phenomena.

2PI Effective Action

(c.f. **J. M. Cornwall, R. Jackiw, and E. Tomboulis (1974)**)

$$\Gamma^{\text{2PI}}[\phi, G, \psi, D] = S_{\text{cl.}}[\phi, \psi] + \frac{i}{2}\,\mathrm{Tr}\left[\ln G^{-1}\right] + \frac{i}{2}\,\mathrm{Tr}\left[\tilde{G}_{\phi}^{-1} G\right]$$
$$- i\,\mathrm{Tr}\left[\ln D^{-1}\right] - i\,\mathrm{Tr}\left[\tilde{D}_{\psi}^{-1} D\right] + \underbrace{\Gamma_2^{\text{2PI}}[\phi, G, \psi, D]}_{\geq \text{2-loop}} + \text{const.}$$

where $\tilde{G}_{\phi}^{-1} = \frac{\delta^2 S_{\text{cl.}}}{\delta \phi^2}$ and $\tilde{D}_{\psi}^{-1} = \frac{\delta^2 S_{\text{cl.}}}{\delta \bar{\psi} \delta \psi}$.

- For practical calculations, Γ_2^{2PI} is evaluated up to a fixed loop order.
- Physical one- and two-point functions obtained using stationary conditions

$$\left.\frac{\delta \Gamma^{\text{2PI}}}{\delta \phi}\right|_{\overline{\phi}, \overline{G}, \overline{D}} = 0\,, \qquad \left.\frac{\delta \Gamma^{\text{2PI}}}{\delta G}\right|_{\overline{\phi}, \overline{G}, \overline{D}} = 0\,, \qquad \left.\frac{\delta \Gamma^{\text{2PI}}}{\delta D}\right|_{\overline{\phi}, \overline{G}, \overline{D}} = 0$$

Equations of Motion (EOMs)

The stationary conditions lead to EOMs for ϕ, G and D

$$(\Box_x + m^2)\phi(x) + \left.\frac{\delta\Gamma^{\text{2PI}}_{\text{int}}}{\delta\phi(x)}\right|_{\overline{\phi},\overline{G},\overline{D}} = 0$$

$$(\Box_x + m^2)G(x,z) + \int_y \overline{\Pi}(x,y)G(y,z) = \delta(x-z)$$

$$(i\partial\!\!\!/_x - M)D(x,z) + \int_y \overline{\Sigma}(x,y)D(y,z) = \delta(x-z)$$

where

$$\Gamma^{\text{2PI}}_{\text{int}} = S_{\text{int}} + \frac{i}{2}\,\mathrm{Tr}\left[\tilde{G}^{-1}_{\phi,\text{int}}G\right] - i\,\mathrm{Tr}\left[\tilde{D}^{-1}_{\psi,\text{int}}D\right] + \Gamma_2 + \text{const.}$$

with the self-energies

$$\overline{\Pi} = 2i\left.\frac{\delta\Gamma^{\text{2PI}}_{\text{int}}}{\delta G}\right|_{\overline{\phi},\overline{G}}, \qquad \overline{\Sigma} = i\left.\frac{\delta\Gamma^{\text{2PI}}_{\text{int}}}{\delta D}\right|_{\overline{\phi},\overline{G},\overline{D}}$$

Equations of Motion (EOMs)

$$(\Box_x + m^2)\phi(x) + \left.\frac{\delta\Gamma^{\text{2PI}}_{\text{int}}}{\delta\phi(x)}\right|_{\overline{\phi},\overline{G},\overline{D}} = 0$$

$$(\Box_x + m^2)G(x,z) + \int_y \overline{\Pi}(x,y)G(y,z) = \delta(x-z)$$

$$(i\partial\!\!\!/_x - M)D(x,z) + \int_y \overline{\Sigma}(x,y)D(y,z) = \delta(x-z)$$

Aim: Obtain the renormalized EOMs

General Aspects

Take a generic classical action

$$S_{\text{cl.}}[\phi,\psi] =$$
$$\int \left\{ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 + \bar{\psi}(i\partial\!\!\!/ - M)\psi - \frac{\alpha}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4(x) - g\bar{\psi}\psi\phi \right\}$$

Gives corresponding **2PI Effective Action** up to 2-loop order

$$\Gamma^{\text{2PI}}[\phi,G,D] =$$
$$S_{\text{cl.}}[\phi,\psi] - \int \left\{ \frac{1}{2}\left(\Box + m^2\right) G - \frac{1}{2}\alpha\,\phi\,G - \frac{1}{8}\lambda G^2 - \frac{1}{4}\lambda\,\phi^2\,G \right.$$
$$\left. + \operatorname{Tr}\left[\left(i\partial\!\!\!/ - M\right) D\right] - g\,\phi\,\operatorname{Tr}[D] \right\} - \int\!\!\int \left[\frac{i}{12}\alpha^2\,G^3 + \frac{i}{2}g^2 G\,\operatorname{Tr}\left[D^2\right] \right]$$

General Aspects

- Define renormalized fields and propagators from the bare ones in usual way

$$\phi = Z_{\phi,2}^{\frac{1}{2}}\phi_R\,, \qquad G = Z_{\phi,0}\, G_R\,,$$
$$\psi = Z_{\psi,2}^{\frac{1}{2}}\psi_R\,, \qquad D = Z_{\psi,0}\, D_R\,,$$

- The mass and coupling constant counterterms are accordingly

$$Z_{\phi,2}m^2 = m_R^2 + \delta m_2^2\,, \quad Z_{\phi,2}^{\frac{i}{2}} Z_{\phi,0}^{\frac{3-i}{2}}\,\alpha = \alpha_i + \delta\alpha_{R,i} \quad (i = 0,1,2,3)$$
$$Z_{\phi,0}m^2 = m_R^2 + \delta m_0^2\,, \quad Z_{\phi,2}^{j/2} Z_{\phi,0}^{(4-j)/2}\lambda = \lambda_{R,j} + \delta\lambda_j \quad (j = 0,2,4)\,,$$
$$Z_{\psi,0}M = M_R + \delta M_0 \qquad Z_{\psi,0} Z_{\phi,2}^{\frac{k}{2}} Z_{\phi,0}^{\frac{1-k}{2}}\, g = g_{R,k} + \delta g_k \quad (k = 0,1)$$

- Need a tadpole counterterm to cancel loop-induced contributions linear in ϕ

$$-\int_x \delta t_1\, \phi_R(x)\,.$$

Renormalized EOMs

Finally, can write down **renormalized EOMs**

$$\left[(1+\delta Z_{\phi,2})\Box + \widehat{m}_2^2(x)\right]\phi_R(x) = -\delta t_1 - \frac{(\alpha_1+\delta\alpha_1)}{2}G_R(x,x) - (g_1+\delta g_1)\,\mathrm{Tr}[D_R(x,x)]\,,$$

$$\left[(1+\delta Z_{\phi,0})\Box + \widehat{m}_0^2(x)\right]G_R(x,y) = \delta(x-y) - \int_z \Pi(x,z)G_R(z,y)\,,$$

$$\left[i\,(1+\delta Z_{\psi,0})\,\partial\!\!\!/_x - \widehat{M}_0(x)\right]D_R(x,y) = \delta(x-y) - \int_z \Sigma(x,z)D_R(z,y)\,,$$

- Self-energies have been split into a local part (absorbed into mass term) and a non-local part ("memory integrals").
- Renormalized EOMs can be solved numerically with initial conditions.
- However, need to determine the unknown counterterms.

2PI Kernels

- Resummed nature of 2-point functions "mixes" different orders in perturbation theory $\longrightarrow$ usual BPHZ procedure to determine divergences does not really work for 2PI.
- Renormalization of coupling constants needed to properly take into account sub-divergences appearing in the renormalization of the 2-point functions.
- Thus, besides renormalizing couplings in $S_{\text{cl.}}$, one also needs to renormalize couplings between one-point functions and two-point functions, and between just two-point functions.
- **Solution:** define vertex functions resummed using Bethe-Salpeter equations $\longrightarrow$ allows for consistent renormalization with finite number of counterterms.

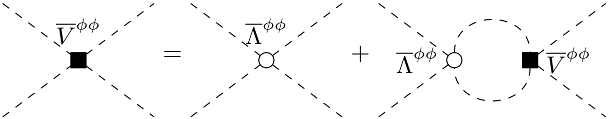
2PI Kernels

Example of such a vertex function

$$\overline{V}_R^{\phi\phi} = \overline{\Lambda}_R^{\phi\phi} + \frac{i}{2}\int \overline{\Lambda}_R^{\phi\phi}\, G_R^2\, \overline{V}_R^{\phi\phi}$$

where we use the (scalar) 2PI kernel

$$\overline{\Lambda}_R^{\phi\phi} \equiv 4\frac{\delta^2\Gamma_{\text{int}}^{\text{2PI}}}{\delta G_R^2}$$



Procedure

- Identify the vertices involving two-point functions.
- Using stationary condition for G and D, obtain **gap equations**

$$G_R^{-1} = G_0^{-1} - \overline{\Pi}\,, \qquad D_R^{-1} = D_0^{-1} - \overline{\Sigma}$$

- Use the vertex functions as aid to solve the gap equation(s) for the two-point functions.
- Work in momentum space to employ usual techniques.

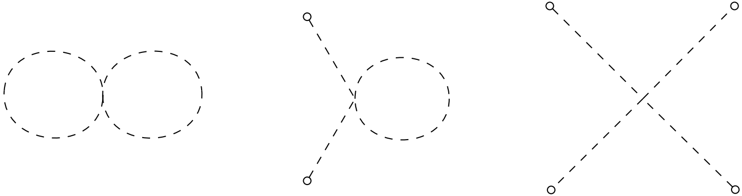
Hartree Approximation

J.-P. Blaizot, E. Iancu and U. Reinosa (2003, 2004); A. Pilaftsis and D. Teresi (2013, 2017), M. Carrington et al. (2015, 2016).......

Focus on obtaining counterterms in **broken phase** ($\phi_R \neq 0$).

$$\Gamma^{\text{2PI}}_{\text{int}}[\phi_R, G_R] =$$

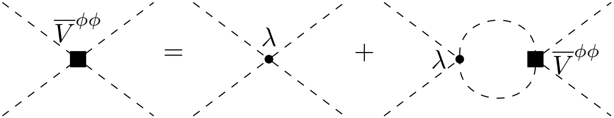
$$\int \Big\{ -\frac{1}{2}(\delta Z_{\phi,0}\Box + \delta m_0^2)G_R - \frac{1}{2}\phi_R(\delta Z_{\phi,2}\Box + \delta m_2^2)\phi_R - \left[\frac{1}{8}(\lambda_R + \delta\lambda_0)G_R^2 + \frac{1}{4}(\lambda_R + \delta\lambda_2)G_R\phi_R^2 + \frac{1}{4!}(\lambda_R + \delta\lambda_4)\phi_R^4\right] \Big\}$$



Vertex Counterterms

$$\overline{\Lambda}_R^{\phi\phi} = -(\lambda_R + \delta\lambda_0) \quad \longrightarrow \quad \text{momentum independent!}$$

- Set $\overline{V}_R^{\phi\phi}(\tilde{p}) = -\lambda_R$ at some fixed point $\tilde{p}$



Vertex Counterterms

$$\overline{\Lambda}_R^{\phi\phi} = -(\lambda_R + \delta\lambda_0) \quad \longrightarrow \quad \text{momentum independent!}$$

- Set $\overline{V}_R^{\phi\phi}(\tilde{p}) = -\lambda_R$ at some fixed point $\tilde{p}$
- Obtain the counterterm

$$\boxed{\delta\lambda_0 = -\lambda_R + \frac{\lambda_R}{1 - \frac{i}{2}\lambda_R I(\tilde{p})}} \quad \text{with} \quad I(p) = \int_q G_R(q) G_R(p+q)$$

For $\delta\lambda_2$, use **auxiliary vertex function**

$$V_R = \Lambda_R + \frac{i}{2}\int \Lambda_R G_R^2 \overline{V}_R^{\phi\phi}, \quad \text{with} \quad \Lambda_R \equiv 4\frac{\delta^3\Gamma_{\text{int}}^{\text{2PI}}}{\delta G \delta^2 \phi_R}$$

Obtain $\boxed{\delta\lambda_2 = \delta\lambda_0}$ $\longrightarrow$ peculiar feature of Hartree

Gap Equation

In Hartree, we have

$$G_R^{-1}(p^2) = -i(p^2 - m_R^2) + \frac{i}{2}(\lambda_R + \delta\lambda_0)\int_q G_R(q) + \frac{i}{2}(\lambda_R + \delta\lambda_2)\phi_R^2 - i(\delta Z_0\, p^2 - \delta m_0^2)$$

Impose **on-shell** renormalization conditions,

$$i\frac{\partial}{\partial p^2}G_R^{-1}(p^2)\Big|_{p^2=m_R^2} = 1 - i\frac{\partial}{\partial p^2}\overline{\Pi}(p^2)\Big|_{p^2=m_R^2} \stackrel{!}{=} 1$$

$$iG_R^{-1}(p^2)\Big|_{p^2=m_R^2} = -i\overline{\Pi}(p^2)\Big|_{p^2=m_R^2} \stackrel{!}{=} 0$$

Gap Equation

In Hartree, we have

$$G_R^{-1}(p^2) = -i(p^2 - m_R^2) + \frac{i}{2}(\lambda_R + \delta\lambda_0)\int_q G_R(q) + \frac{i}{2}(\lambda_R + \delta\lambda_2)\phi_R^2 - i(\delta Z_0\, p^2 - \delta m_0^2)$$

Impose **on-shell** renormalization conditions, obtain

$$\delta Z_0 = 0$$

$$\delta m_0^2 = -\frac{i}{2}(\lambda_R + \delta\lambda_0)\int_q G_R(q) - \frac{i}{2}(\lambda_R + \delta\lambda_2)\phi_R^2$$

$$\Longrightarrow \boxed{G_R^{-1}(p^2) = -i(p^2 - m_R^2) = G_0^{-1}(p^2)}$$

Thus, can obtain $\delta m_0^2 = -m_R^2$

Field Counterterms

- Remaining counterterms related to the field; take field derivatives of the 2PI Effective Action, i.e.

$$\Gamma^{(n)}(x_1, \ldots, x_n) = \frac{\delta^n \Gamma^{\text{2PI}}}{\delta\phi_1 \ldots \delta\phi_n}$$

- Careful! G depends on ϕ, need to use **chain-rule** $\longrightarrow$ leads to a system of coupled integral equations.

Example: Field Two-Point Function

$$\Gamma^{(2)} = iG_0^{-1} + \frac{\delta^2\Gamma^{\text{2PI}}_{\text{int}}}{\delta^2\phi_R} + \int \frac{\delta^2\Gamma^{\text{2PI}}_{\text{int}}}{\delta\phi_R\delta G_R} G_R \frac{\delta\overline{\Pi}}{\delta\phi_R} G_R$$

Use the same on-shell renormalization conditions used for G; obtain

$$\delta Z_2 = -\frac{\lambda_R^2\phi_R^2}{128\pi^2}\frac{\dot{B}_0(m_R^2, m_R^2, m_R^2)}{1 - \frac{\lambda_R}{32\pi^2}\left[B_0(\tilde{p}^2, m_R^2, m_R^2) - B_0(m_R^2, m_R^2, m_R^2)\right]}$$

$$\delta m_2^2 = -m_R^2 + m_R^2\delta Z_2 - \frac{\lambda_R\phi_R^2}{4}\frac{1}{1 - \frac{\lambda_R}{32\pi^2}\left[B_0(\tilde{p}^2, m_R^2, m_R^2) - B_0(m_R^2, m_R^2, m_R^2)\right]} - \frac{(\lambda_R + \delta\lambda_4)\,\phi_R^2}{2}$$

For $\phi_R = 0$ (unbroken phase), have

$$\boxed{\delta Z_2 = \delta Z_0} \quad \text{and} \quad \boxed{\delta m_2^2 = \delta m_0^2}$$

(Consistent with **J. Berges et al (2005)**)

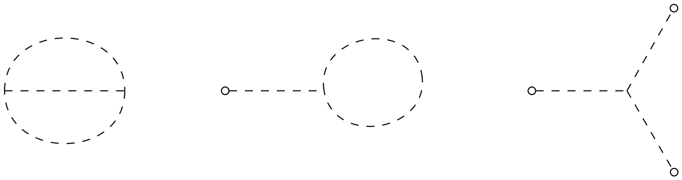
With a Trilinear Interaction

- Example of a memory integral
- Will use to demonstrate subtleties involving a momentum-dependent kernel

$$\Gamma_{\text{int}}[\phi_R, G_R] =$$

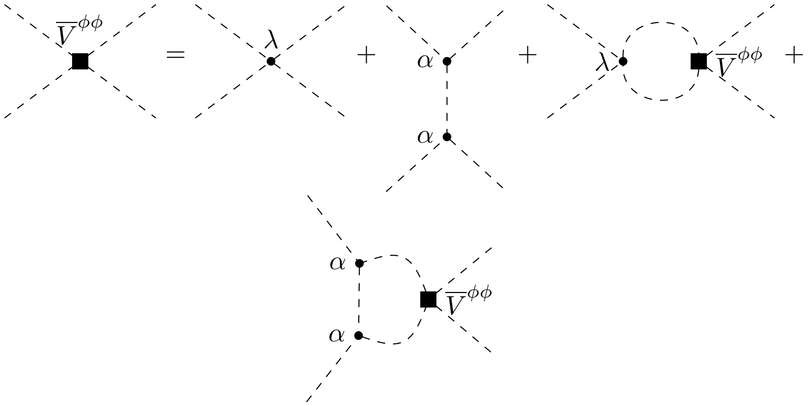
$$\Gamma_{\text{int, Hartree}} - \int \left\{ \delta t_1 \phi_R - \left[\frac{1}{2}(\alpha_R + \delta\alpha_1)\phi_R G_R + \frac{1}{3!}(\alpha_R + \delta\alpha_3)\phi_R^3 \right] \right\}$$

$$- \int \int \left[\frac{i}{12} (\alpha_R + \delta\alpha_0)^2 \, G_R^3 \right]$$



Momentum-dependent Kernel

$$\boxed{\overline{\Lambda}_R^{\phi\phi} = -(\lambda_R + \delta\lambda_0) - 2i\alpha_R^2 \, G_R}$$



Momentum-dependent Kernel

$$\overline{\Lambda}_R^{\phi\phi} = -(\lambda_R + \delta\lambda_0) - 2i\alpha_R^2 G_R$$

Obtain

$$\boxed{\delta\lambda_0 = -\lambda_R + \frac{\lambda_R + \alpha_R^2 I_2(\tilde{p}_2, \tilde{p}, \tilde{p}_3, \tilde{p}_4)}{1 + \frac{i}{2} I_1(\tilde{p}, \tilde{p}_3, \tilde{p}_4)}}$$

where

$$I_1(p,k,r) = \int_q G_R(q) G_R(p+q) \overline{V}_R^{\phi\phi}(q+p, -q, k, r)$$

$$I_2(l,p,k,r) = \int_q G_R(l+q) G_R(q) G_R(p+q) \overline{V}_R^{\phi\phi}(q+p, -q, k, r)$$

Gap Equation with a Momentum-Dependent Kernel

$$G_R^{-1}(p) = -i(p^2 - m_R^2) + \frac{i}{2}(\lambda_R + \delta\lambda_0)\int_q G_R(q) - \frac{\alpha_R^2}{2}\int_q G_R(q)G_R(p+q)$$
$$+ \frac{i}{2}(\lambda_R + \delta\lambda_2)\phi_R^2 + i(\alpha_R + \delta\alpha_1)\phi_R - i(\delta Z_0\, p^2 - \delta m_0^2)$$

Use on-shell conditions to obtain counterterms

$$\delta Z_0 = \frac{i\alpha_R^2}{2}\int_q G_R(q)\left[\frac{\partial}{\partial p^2}G_R(p+q)\right]\Bigg|_{p^2=m_R^2}$$
$$\delta m_0^2 = -\frac{1}{2}(\lambda_R + \delta\lambda_0)\int_q G_R(q) - \frac{i\alpha_R^2}{2}\left[\int_q G_R(q)G_R(p+q)\right]\Bigg|_{p^2=m_R^2}$$
$$- \frac{1}{2}(\lambda_R + \delta\lambda_2)\phi_R^2 - (\alpha_R + \delta\alpha_1)\phi_R + \delta Z_0 m_R^2$$

More difficult to determine the form of G.

Solution to Gap Equation

Trick: plug back in the counterterms

$$G_R^{-1}(p^2) =$$

$$-i(p^2 - m_R^2) - \frac{\alpha_R^2}{2}\left\{\int_q G_R(q)G_R(p+q) - \left[\int_q G_R(q)G_R(\tilde{p}+q)\right]\Big|_{\tilde{p}^2=m_R^2}\right\}$$

$$+\frac{\alpha_R^2}{2}(p^2 - m_R^2)\left\{\frac{\partial}{\partial k^2}\left[\int_q G_R(q)G_R(k+q)\right]\right\}\Big|_{k^2=m_R^2}$$

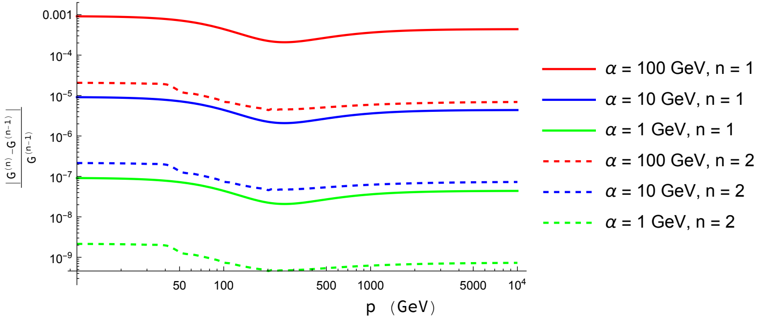
- Only α coupling appears.
- Have a consistent integral equation for $G \longrightarrow$ solve iteratively, initializing with G_0.
- Shift to Euclidean space for the numerics.

Solutions to Gap Equation

$$G_R^{-1\,(n)}(p^2) =$$

$$-\,i(p^2 - m_R^2) - \frac{\alpha_R^2}{2}\left\{\int_q G_R^{(n-1)}(q)G_R^{(n-1)}(p+q) - \left[\int_q G_R^{(n-1)}(q)G_R^{(n-1)}(\tilde{p}+q)\right]\Bigg|_{\tilde{p}^2=m_R^2}\right\}$$

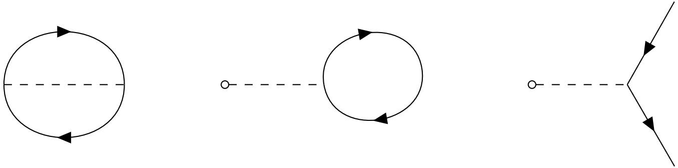
$$+\,\frac{\alpha_R^2}{2}(p^2 - m_R^2)\left\{\frac{\partial}{\partial k^2}\left[\int_q G_R^{(n-1)}(q)G_R^{(n-1)}(k+q)\right]\right\}\Bigg|_{k^2=m_R^2}$$



$m_R = 100\,\text{GeV},\ \lambda_R = 0.8$

Yukawa Theory

$$
\begin{aligned}
&\Gamma_{\text{int}}[\phi_R, G_R, \psi_R, D_R] = \\
&\Gamma_{\text{int, Hartree}} + \int \Big\{ (i\delta Z_{\psi,0}\not{\partial} - \delta M_0) D_R + \overline{\psi}_R (i\delta Z_{\psi,2}\not{\partial} - \delta M_2)\psi_R(x) \\
&- \frac{i}{2}\int_x \int_y (g_R + \delta g_0)^2 G_R(x,y)\,\mathrm{Tr}[D_R(x,y)D_R(y,x)] - \int_x \delta t_1 \phi_R(x) \\
&- \int_x \left[(g_R + \delta g_1)\phi_R(x)\,\mathrm{Tr}[D_R(x,x)] + (g_R + \delta g_3)\phi_R(x)\overline{\psi}_R(x)\psi_R(x) \right]
\end{aligned}
$$



Coupled Gap Equations

Obtain a system of coupled equations for D and G

$$G_R^{-1}(p^2) =$$

$$-i(p^2 - m_R^2) - \frac{i}{2}(\lambda_R + \delta\lambda_0)\int_q G_R(q)$$

$$- g_R^2 \int_q \mathrm{Tr}\left[D_R(q) D_R(p+q)\right] - \frac{i}{2}(\lambda_R + \delta\lambda_2)\phi_R^2 - i(\delta Z_{\phi,0}\, p^2 - \delta m_0^2)$$

$$D_R^{-1}(p) =$$

$$-i(\not{p} - M_R) + i(g_R + \delta g_1)\phi_R + g_R^2 \underbrace{\int_q D_R(p+q) G_R(q)}_{X(p^2)\not{p} + Y(p^2)} - i(\delta Z_{\psi,0}\,\not{p} - \delta M_0)$$

Coupled Gap Equations

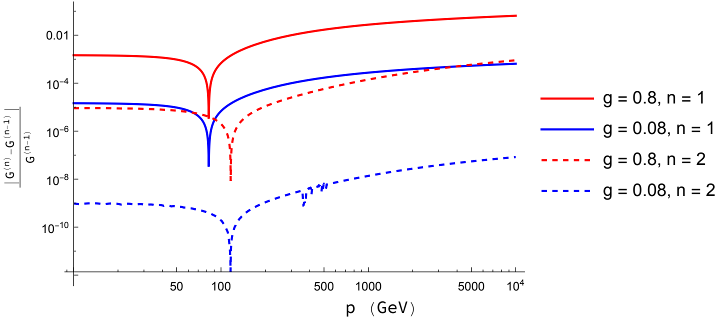
Use **same trick** as before to simplify the system of equations

$$G_R^{-1}(p^2) =$$
$$-i(p^2 - m_R^2) - g_R^2 \left\{ \int_q \mathrm{Tr}\left[D_R(q)D_R(p+q)\right] - \left[\int_q \mathrm{Tr}\left[D_R(q)D_R(k+q)\right]\right]\Big|_{k^2=m_R^2} \right\}$$
$$+ ig_R^2(p^2 - m_R^2)\left\{\frac{\partial}{\partial k^2}\left[\int_q \mathrm{Tr}\left[D_R(q)D_R(k+q)\right]\right]\right\}\Big|_{k^2=m_R^2}$$

$$D_R^{-1}(p) = -i(\not{p} - M_R) + \not{p}\left\{g_R^2\left[X(p^2) - X(M_R^2)\right]\right\} + g_R^2\left[Y(p^2) - Y(M_R^2)\right]$$
$$- 2g_R^2 M_R(\not{p} - M_R)\left\{\frac{\partial}{\partial p^2}\left[M_R X(p^2) + Y(p^2)\right]\right\}\Big|_{p^2=M_R^2}$$

Scalar Two-Point Function

$$G_R^{-1\,(n)}(p^2) =$$
$$-\,i(p^2 - m_R^2) - g_R^2 \left\{ \int_q \mathrm{Tr}\left[D_R^{(n-1)}(q) D_R^{(n-1)}(p+q) \right] - \left[\int_q \mathrm{Tr}\left[D_R^{(n-1)}(q) D_R^{(n-1)}(k+q) \right] \right] \Big|_{k^2=m_R^2} \right\}$$
$$+\, i g_R^2 (p^2 - m_R^2) \left\{ \frac{\partial}{\partial k^2} \left[\int_q \mathrm{Tr}\left[D_R^{(n-1)}(q) D_R^{(n-1)}(k+q) \right] \right] \right\} \Big|_{k^2=m_R^2} .$$



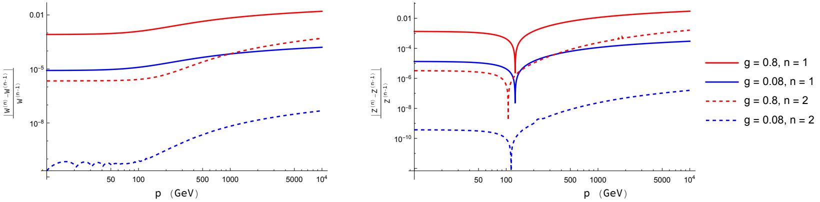
$m_R = 100$ GeV, $M_R = 60$ GeV, $\lambda = 0.8$

Fermionic Two-Point Function

$$D_R^{-1}(p) = -i\left(W(p^2)\not{p} - Z(p^2)\right)$$

$$W(p^2) = 1 + i(g_R + \delta g_0)^2[X(p^2) - X(M_R^2)] - 2i(g_R + \delta g_0)^2 M_R \left\{\frac{\partial}{\partial p^2}[M_R X(p^2) + Y(p^2)]\right\}\Big|_{p^2=M_R^2}$$

$$Z(p^2) = M_R - i(g_R + \delta g_0)^2[Y(p^2) - Y(M_R^2)] - 2i(g_R + \delta g_0)^2 M_R^2 \left\{\frac{\partial}{\partial p^2}[M_R X(p^2) + Y(p^2)]\right\}\Big|_{p^2=M_R^2}.$$



$m_R = 100$ GeV, $M_R = 60$ GeV, $\lambda = 0.8$

Conclusions and Outlook

- 2PI provides a systematic approach to study phenomenon in strong coupled systems.
- Carried out renormalization in the broken phase, relevant for the calculation of the effective potential.
- For momentum dependent kernels, demonstrated an **iterative procedure** to solve gap equations which converges sufficiently fast.
- Can investigate models such as the SM and scalar extensions of the SM (singlet, SU(2) doublets, etc.) in a similar manner

Thank you for your attention!

Backup: 2PI Generating Functional

- Begin with generating functional

$$Z[J_1, J_2] = \int \mathcal{D}\varphi \, \mathsf{exp}\left(iS[\varphi] + i \int_x J_1(x)\varphi(x) + \frac{i}{2}\int_{x,\,y} \varphi(x) J_2(x,y)\varphi(y) \right)$$

- Define $W[J_1, J_2] = -i \ln Z[J_1, J_2]$ to obtain the macroscopic field and the connected propagator

$$\phi(x) = \frac{\delta W[J_1, J_2]}{\delta J_1(x)}\,, \qquad G(x,y) = 2\frac{\delta W[J_1, J_2]}{\delta J_2(x,y)} - \phi(x)\phi(y)\,.$$

- More intuitive to describe the system using ϕ and G $\longrightarrow$ perform Legendre transformation w.r.t. sources $J_{1,2}$

$$\Gamma^{\text{2PI}}[\phi, G] = W[J_1, J_2] - \int_x \frac{\delta W[J_1, J_2]}{\delta J_1(x)} J_1(x) - \int_{x\,y} \frac{\delta W[J_1, J_2]}{\delta J_2(x,y)} J_2(x,y)$$

Backup: Fermionic 2PI Kernels

U. Reinosa (2006)

With fermions, need to be more careful $\longrightarrow$ have more kernels

$$\overline{\Lambda}^{\psi\psi}_{R\,(\alpha\beta),(\gamma\delta)} \equiv -\frac{\delta^2 \Gamma^{\text{2PI}}_{\text{int}}}{\delta D^{\beta\alpha}_R \, \delta D^{\gamma\delta}_R} \,,$$

$$\overline{\Lambda}^{\psi\phi}_{R\,(\alpha\beta)} \equiv -2\frac{\delta^2 \Gamma^{\text{2PI}}_{\text{int}}}{\delta D^{\beta\alpha}_R \, \delta G_R} \,.$$

Former resummed using

$$\overline{V}^{\psi\psi}_{R\,(\alpha\beta),(\gamma\delta)} = \overline{\Lambda}^{\psi\psi}_{R\,(\alpha\beta),(\gamma\delta)} + i\int \overline{\Lambda}^{\psi\psi}_{R\,(\alpha\beta),(\sigma\rho)} D_{R\,\sigma\mu} \overline{V}^{\psi\psi}_{R\,(\mu\nu),(\gamma\delta)} D_{R\,\nu\rho}$$

Need to take into account additional divergent structures; define modified scalar kernel

$$\tilde{\Lambda}^{\phi\phi}_R = \overline{\Lambda}^{\phi\phi}_R - i\int \text{Tr}\left[D_R \overline{\Lambda}^{\phi\psi}_R \, D_R \overline{\Lambda}^{\psi\phi}_R\right] + i\int\int D_{R\,\nu\beta} \, \overline{\Lambda}^{\phi\psi}_{R,\,\beta\alpha} \, D_{R\,\alpha\mu} \overline{V}^{\psi\psi}_{R\,(\mu\nu),(\rho\sigma)}$$

Backup: $\delta\lambda_4$ in Hartree

$$
\begin{aligned}
\Gamma^{(4)}(p_1,p_2,p_3,p_4) &= -(\lambda_R+\delta\lambda_4)+\lambda_R\left[J(p_1+p_2)+J(p_1-p_3)+J(p_1-p_4)\right]\\
&+\lambda_R^3\phi_R^2\big[J(p_1+p_2)J(p_3)J(p_4)C_0(p_1+p_2,p_4)+J(p_1-p_3)J(p_2)J(p_4)C_0(p_1-p_3,p_4)\\
&\quad+J(p_1-p_4)J(p_2)J(p_3)C_0(p_1-p_4,p_3)+J(p_1)J(p_2)J(p_3+p_4)C_0(p_1,p_3+p_4)\\
&\quad+J(p_1)J(p_3)J(p_2-p_4)C_0(p_1,p_2-p_4)+J(p_1)J(p_4)J(p_2-p_3)C_0(p_1,p_2-p_3)\big]\\
&-\lambda_R^4\phi_R^4\,J(p_1)J(p_2)J(p_3)J(p_4)\big[D_0(p_2,p_1+p_2,p_3)+D_0(p_2,p_2-p_3,p_3)+D_0(p_2,p_2-p_4,p_4)\big]\\
&+\lambda_R^5\phi_R^4\,J(p_1)J(p_2)J(p_3)J(p_4)\big[J(p_1+p_2)C_0(p_1,p_1+p_2)C_0(p_3+p_4,p_4)\\
&\quad+J(p_1-p_3)C_0(p_1,p_1-p_3)C_0(p_4-p_2,p_4)+J(p_1-p_4)C_0(p_1,p_1-p_4)C_0(p_3-p_2,p_3)\big]
\end{aligned}
$$

where we have introduced

$$
J(p)\equiv\left[1-\frac{\lambda_R}{2}\Big(B_0(\tilde{p}^2,m_R^2,m_R^2)-B_0(p^2,m_R^2,m_R^2)\Big)\right]^{-1}
$$

Backup: Loop Integrals in Euclidean Space

In four-dimensional Euclidean spacetime, one obtains the following formula for the integral over two propagators

$$\int_q G_R^{\mathrm{E}}(||q||) G_R^{\mathsf{E}}(||p+q||)$$
$$= \frac{1}{8\pi^3 p^2} \int_0^\Lambda dq\, q\, G_R^{\mathrm{E}}(q) \int_{|(q-||p||)|}^{|(q+||p||)|} du\, u\, \sqrt{-\lambda(u^2, q^2, ||p||^2)}\, G_R^{\mathrm{E}}(u)$$

and

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$