Development of transverse flow for small and large systems in conformal kinetic theory

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based on PRD 105 (2022) 014031 and WiP

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CRC-TR 211
Strong-interaction matter under extreme conditions

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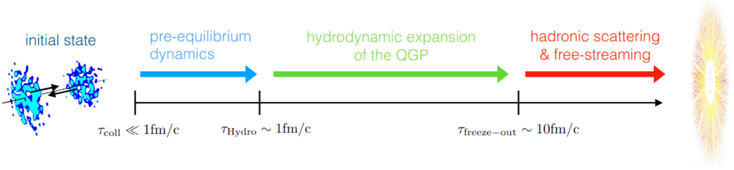
Introduction

Standard modelling of heavy ion collisions

Spacetime evolution dominated by hydrodynamic phase



initial state

pre-equilibrium dynamics

hydrodynamic expansion of the QGP

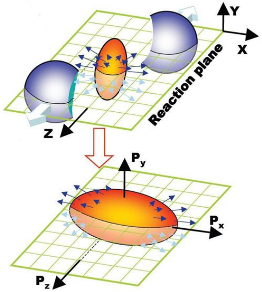
hadronic scattering & free-streaming

$\tau_{\mathrm{coll}} \ll 1\mathrm{fm/c}$

$\tau_{\mathrm{Hydro}} \sim 1\mathrm{fm/c}$

$\tau_{\mathrm{freeze-out}} \sim 10\mathrm{fm/c}$

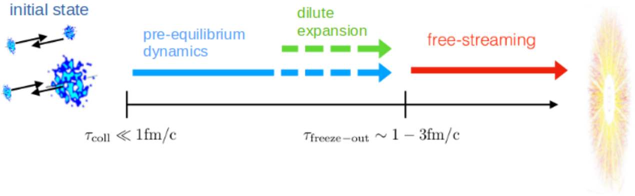
- early stage requires non-equilibrium description, but system quickly equilibrates
- strongly interacting QGP leaves imprints of thermalization and collectivity in final state observables
  - anisotropic flow explained via anisotropic pressure gradients in hydrodynamic evolution
- transport description after hadronization



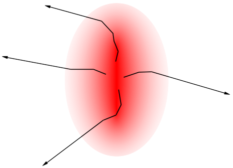
Hiroshi Masui (2008)

Small systems

Very dilute, hydrodynamics not necessarily applicable



- still collective behaviour is observed!
- collectivity can also be explained in kinetic theory, a microscopic description which does not rely on equilibration
  - anisotropic flow explained via individual scatterings being more likely in some directions
- limit of large interaction rate is hydrodynamics!



Aim

Case study in simplified kinetic theory description on full range from small to large system size with comparison to hydrodynamics based on transverse flow

Theoretical Description

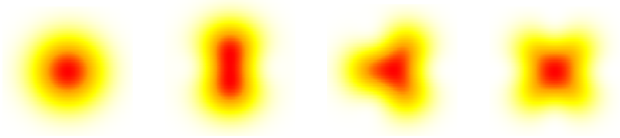
Model and Setup

- describing time evolution of boost-invariant phase space distribution of massless bosons using Boltzmann equation in conformal RTA

$$p^\mu \partial_\mu f = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\rm eq}) \ , \quad \tau_R = 5\frac{\eta}{s}T^{-1}$$

  - initialized with vanishing longitudinal pressure and no momentum anisotropies
  - time evolution of f depends only on opacity $\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1}\left(\frac{30}{\nu_{\rm eff}\pi^2}\frac{1}{\pi}\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}R\right)^{1/4}$

    Kurkela, Wiedemann, Wu EPJC 79 (2019) 965
  - energy weighted d.o.f.: dependence on IS only in energy density
- first study: simple initial energy density introducing only one eccentricity at a time



(no ecc.) $(n = 2)$ $(n = 3)$ $(n = 4)$

Analytical and numerical treatment

- typical values of $\hat{\gamma}$
  - min. bias pp: $\hat{\gamma} \approx 0.88 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.4\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{5\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{-1/4}$
  - central PbPb: $\hat{\gamma} \approx 9.2 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{6\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{4000\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{-1/4}$

$\Rightarrow$ treat problem both analytically (for small $\hat{\gamma}$) and numerically

linearized analytical treatment

- "opacity expansion" in number of scatterings

$$\text{0th order}: \; p^\mu \partial_\mu f^{(0)} = 0 \; ,$$

$$\text{1st order}: \; p^\mu \partial_\mu f^{(1)} = C[f^{(0)}]$$

Heiselberg, Levy PRC 59 (1999) 2716

Borghini, Gombeaud EPJC 71 (2011) 1612

- expansion parameter $C_{\mathrm{RTA}}[f] \sim \hat{\gamma}$
- linearize also in eccentricity

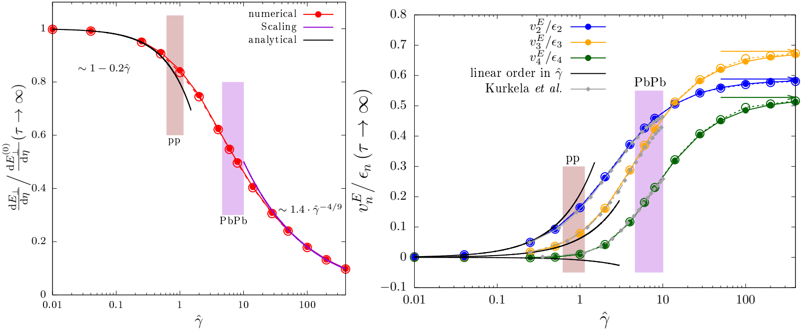
numerical treatment

- nonlinear in both opacity and eccentricity
- Relativistic Lattice Boltzmann solver for energy-weighted d.o.f.

Ambrus, Blaga PRC 98 (2018) 035201

Results and Comparisons

Opacity Dependence



Cooling:

- numerical curve smoothly connects the small-$\hat{\gamma}$ linearized and large-$\hat{\gamma}$ Bjorken scaling results

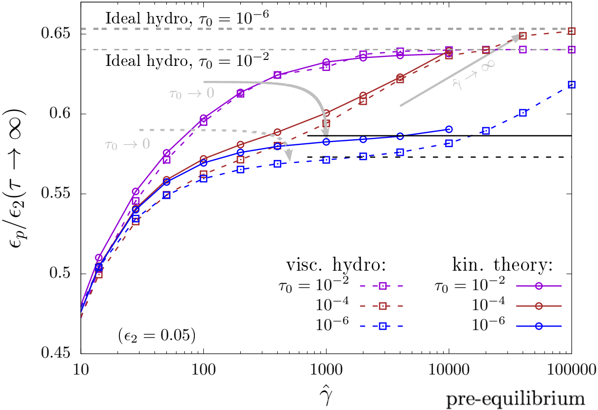
Anisotropic flow:

- linear order results tangential to numerical curve at small opacities
- agreement with previous results in identical setup

  Kurkela, Taghavi, Wiedemann, Wu PLB 811 (2020) 135901
- saturation at higher $\hat{\gamma}$

$\Rightarrow$ expectation: hydrodynamic behaviour at large opacities

Comparison to Hydrodynamics

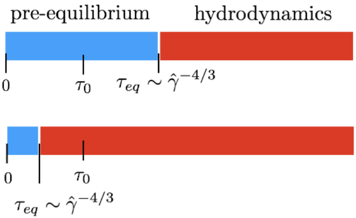


unphysical limit

physical limit

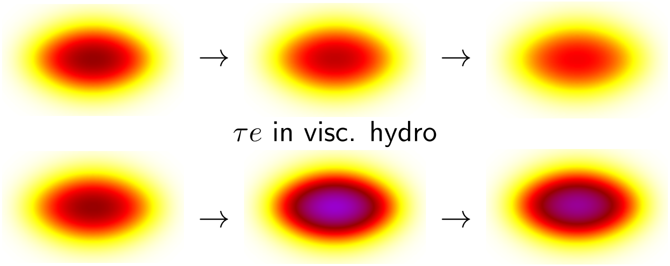
- agreement at large τ_0: no pre-equilibrium
- small τ_0: pre-equilibrium causes discrepancies
- convergence only in unphysical order of limits

$\Rightarrow$ Why is pre-equilibrium important for observables that develop at $\tau \sim R$?



Early time longitudinal cooling

- $\tau \ll R$: only longitudinal expansion $\Rightarrow$ local Bjorken flow cooling
  - follows universal attractor curve in scaling variable $\tilde{w} = \frac{T\tau}{4\pi\eta/s}$
    early times: $\tau^{4/3}e\,, \tau^{1/3}\frac{dE_\perp}{dy} \propto \hat{\gamma}^{-4/9}\tilde{w}^{\gamma}$ late times: $\tau^{4/3}e\,, \tau^{1/3}\frac{dE_\perp}{dy} \propto \hat{\gamma}^{-4/9}$



τe in kin. theory

τe in visc. hydro

$\tau = 3 \cdot 10^{-6}$fm $\tau = 8 \cdot 10^{-4}$fm $\tau = 3 \cdot 10^{-3}$fm

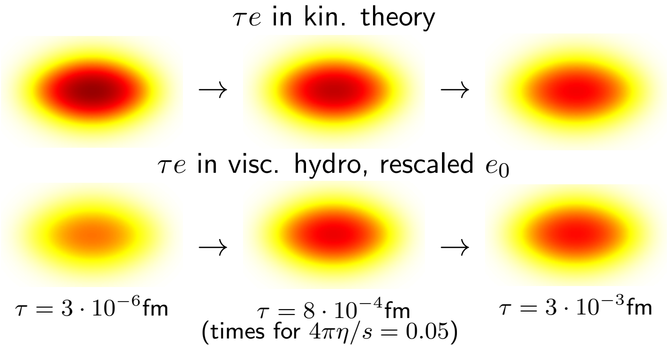
(times for $4\pi\eta/s = 0.05$)

- dynamics depend on local energy density $\Rightarrow$ inhomogeneous cooling
  - decrease of eccentricity before transverse flow develops

Improved Hydro Setups

How to "fix" Hydro?

- idea: counteract difference in pre-equilibrium by different hydro initialization



τe in kin. theory

τe in visc. hydro, rescaled e_0

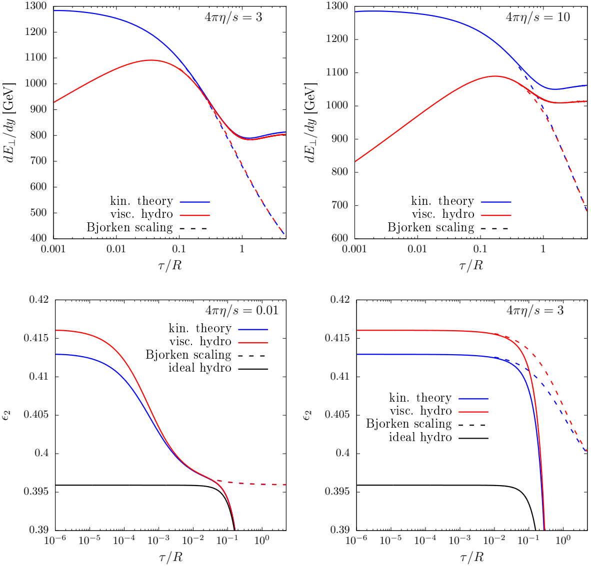
$\tau = 3 \cdot 10^{-6}$fm $\rightarrow$ $\tau = 8 \cdot 10^{-4}$fm $\rightarrow$ $\tau = 3 \cdot 10^{-3}$fm

(times for $4\pi\eta/s = 0.05$)

- more realistic initial condition: average profile (Pb+Pb 30-40%)
  - fixed profile: vary $\hat{\gamma}$ via η/s: $\hat{\gamma} \approx 11 \cdot (4\pi\eta/s)^{-1}$

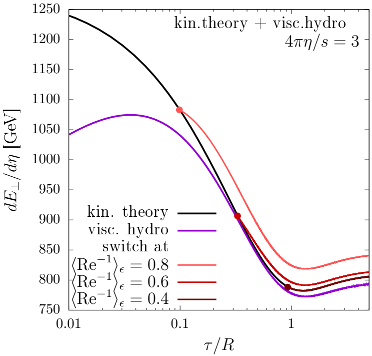
Initializing on the attractor

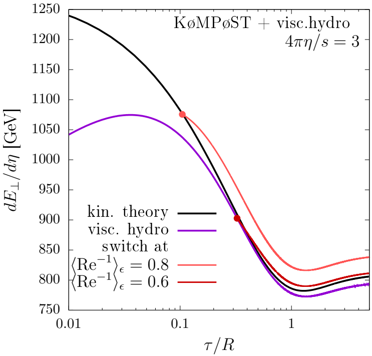
- accuracy depends on timescale separation of pre-equilibrium and transv. expansion



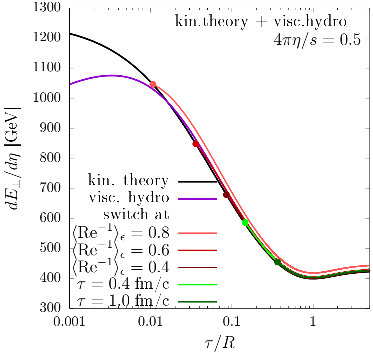
Hybrid schemes

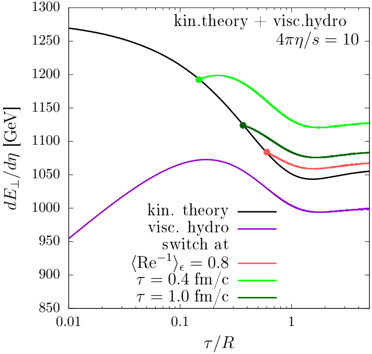
- idea: evolve system in kinetic theory until $\langle \mathrm{Re}^{-1} \rangle_\epsilon$ drops to specific value, then match $T^{\mu\nu}$ to hydro code
- system immediately starts following similar evolution to a pure hydro run
  - switching too early causes errors in pre-equilibrium
  - results from late switching times more accurate than rescaled hydro
- works just as well with KøMPøST, but with limited range of applicability





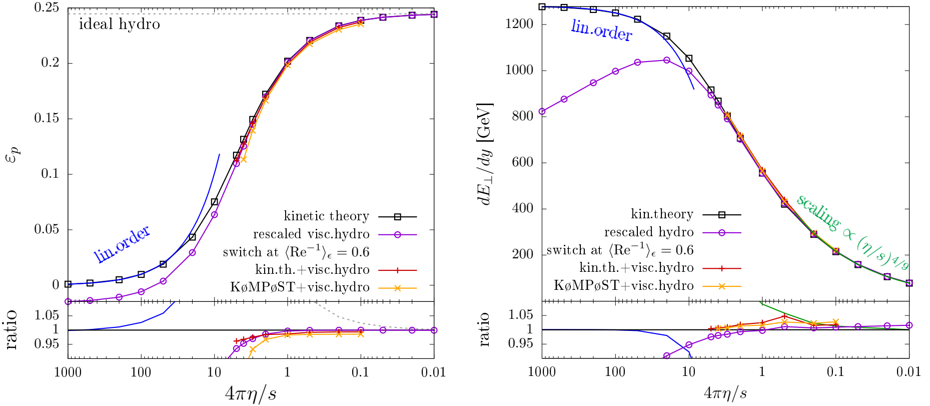
Switching at fixed time





- large opacities: "everything works"
- small opacities: "nothing works"
- benefit of switching at fixed $\langle \mathrm{Re}^{-1} \rangle$: accuracy independent of η/s

Comparison of improved hydro schemes with kin. theory



- perfect agreement at large $\hat{\gamma}$
- rescaled hydro accurate if $4\pi\eta/s \lesssim 3$ (for Pb+Pb 30-40%)
- still good agreement with linear order results at small opacities
  - computed (mostly) numerically, since initial condition no longer analytical
- Hybrid schemes work well, can improve on rescaled hydro at intermediate opacities, but KøMPøST slightly underestimates ϵ_p

Summary

- kinetic theory description covers full range in opacity from small to large systems

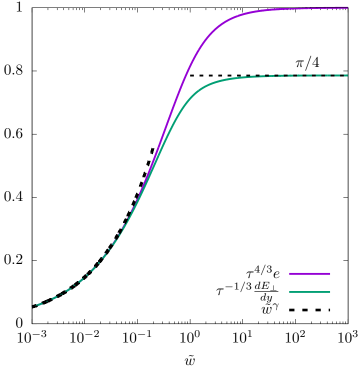
- naive comparison to hydrodynamics: disagreement even at large opacities!
  - difference during pre-equilibrium
  - eccentricity decreases before onset of transverse expansion

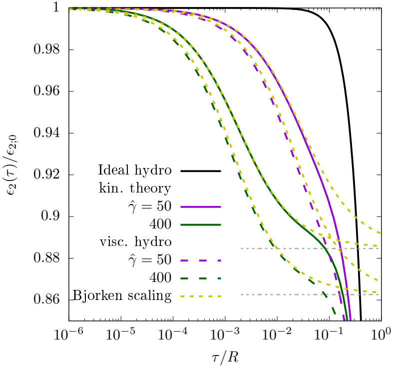
- different setup of hydrodynamic simulations can bring agreement at large opacities
  - initializing hydrodynamics on its early-time attractor
  - hybrid models with kinetic theory for pre-equilibrium

Backup

Early time Bjorken scaling

- $\tau \ll R$: no transverse expansion, system locally behaves like 0+1D Bjorken flow
  - universal attractor curve scaling in the variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau,\mathbf{x}_\perp)\tau}{4\pi\eta/s}$
    Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301
  - $\tilde{w} \gg 1$: $\tau^{4/3}e = \text{const.}$, $\tau^{1/3}\frac{dE_\perp}{dy} = \text{const.}$
  - $\tilde{w} \ll 1$: model dependent power law $\tau^{4/3}e \sim \tilde{w}^\gamma$





- inhomogeneous cooling changes energy density profile