Non-Hydrodynamic Modes from Linear Response in Effective Kinetic Theory

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CRC-TR 211 Strong-interaction matter under extreme conditions

HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

Outline

- Introduction
- What are non-hydrodynamic modes?
- What do we know about (non-)hydrodynamic modes?
- How can we find (non-)hydrodynamic modes?
- Conclusion and Outlook

Introduction

- In Heavy-Ion-Collisions a new state of matter named Quark Gluon Plasma (QGP) is formed
- The QGP can be modelled very well via hydrodynamics at later times when it equilibrates
- Important question: On what time and length scales does hydrodynamics apply?
- We use kinetic theory to study the non-equilibrium behavior

What are non-hydrodynamic modes?

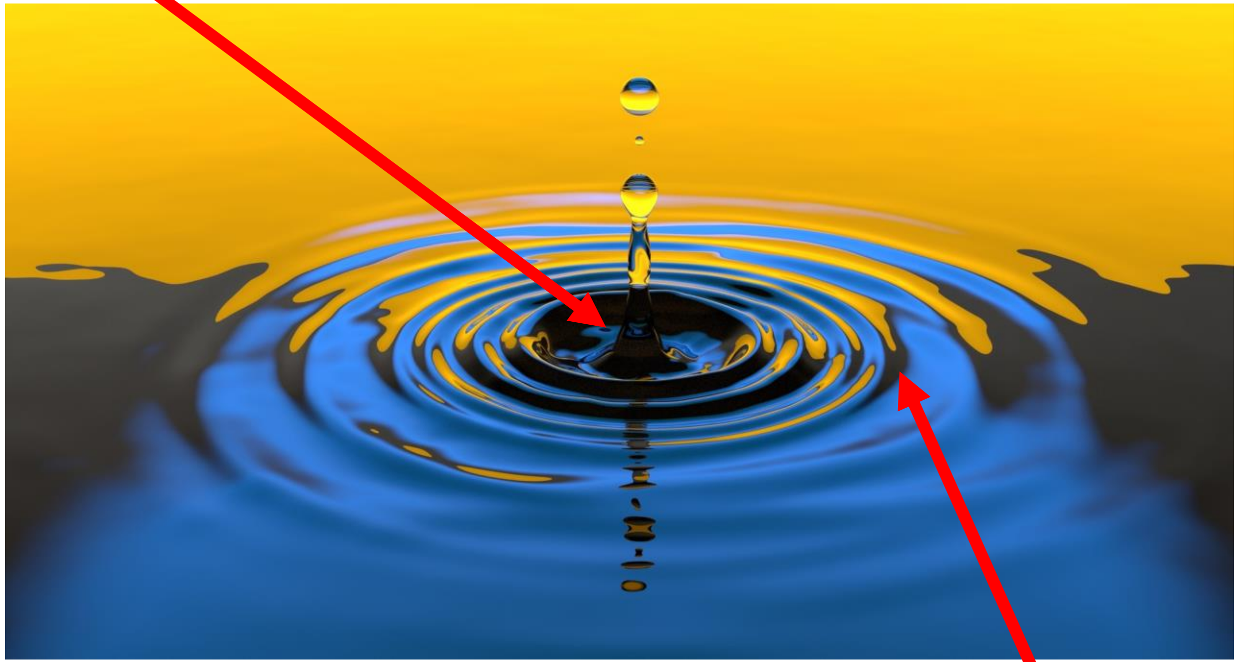
- Hydrodynamic modes define the long time long wavelength limit, i.e., defining the hydrodynamic regime ($\omega \to 0\ ,\ k \to 0$)
- Known mode e.g., sound mode $\omega = \pm k v_s - i\frac{\Gamma}{2}k^2$

- Non-hydrodynamic modes are everything that is not hydrodynamic
  - They even appear in hydrodynamic theories like Müller-Israel-Stewart
  - Example from RTA $\omega = -\frac{i}{\tau_R}$

- We study excitations of an equilibrium system to find hydrodynamic and non-hydrodynamic modes

What are non-hydrodynamic modes?

Non-Hydrodynamic



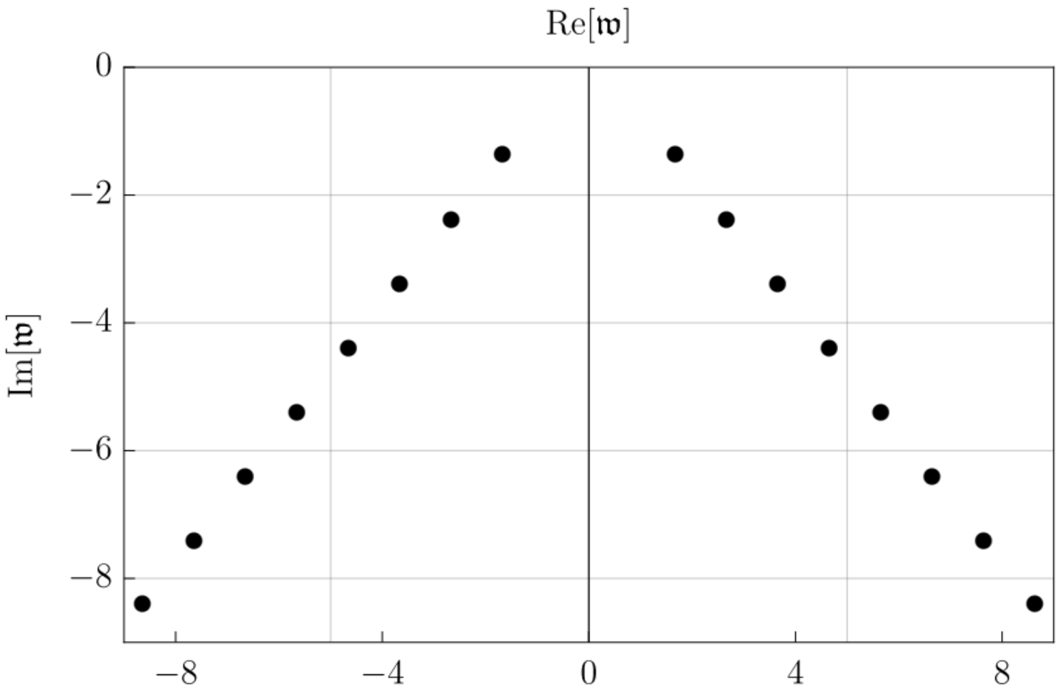
Source: *https://pxhere.com/en/photo/788515*

Hydrodynamic

What do we know about (non-)hydrodynamic modes?

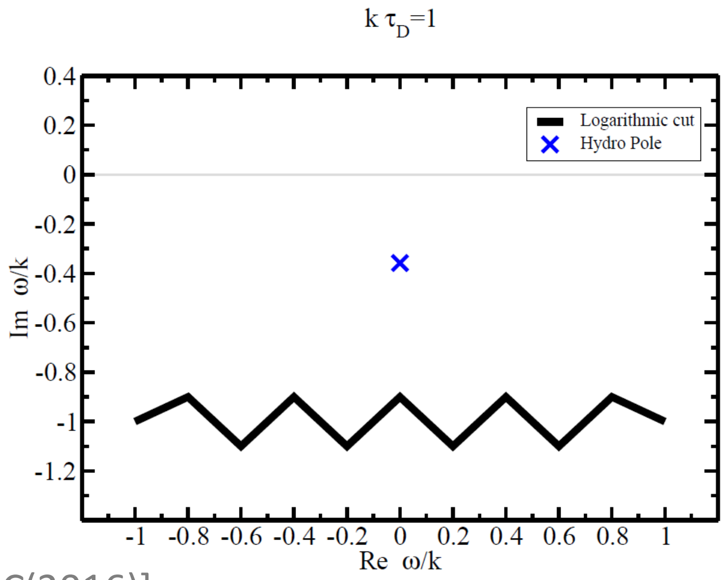
- The QGP is described by QCD
- It helps to study other theories to get some insight on non-hydrodynamic behavior
- AdS/CFT Holography:
  - Called Quasi-normal modes

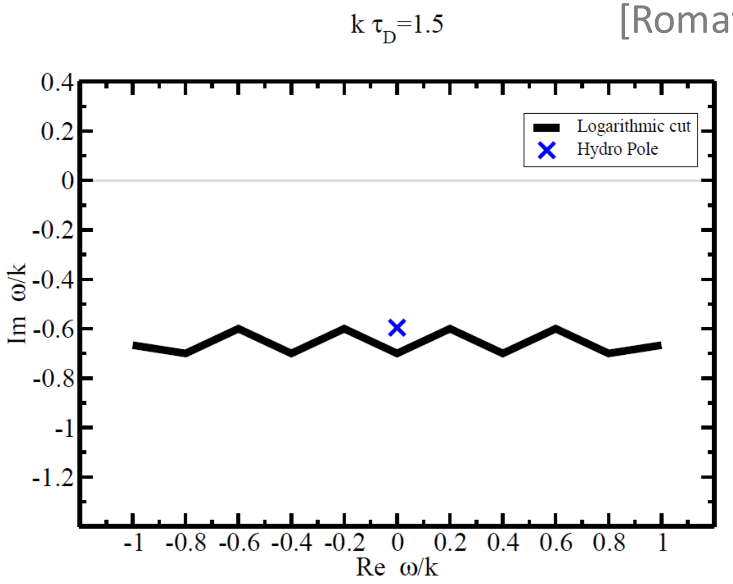
Anti-de Sitter/Conformal Field Theory



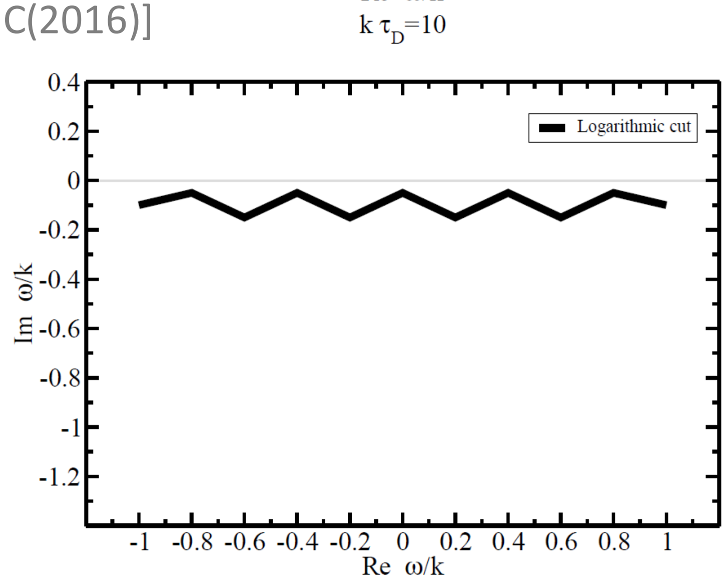
[Grozdanov et al., JHEP(2019)]

- Relaxation Time Approximation:
  - Branch cut $\ln\left(\frac{\omega - k + \frac{i}{\tau_R}}{\omega + k + \frac{i}{\tau_R}}\right)$
  - Analytical Calculations possible





[Romatschke, EPJ C(2016)]



How can we find (non-)hydrodynamic modes?

- Work in kinetic theory
  - Use Boltzmann equation with corresponding collision kernel

$$\partial_t f(t,p) - \frac{k^i p_i}{p^0} f(t,p) = C[f](p)$$

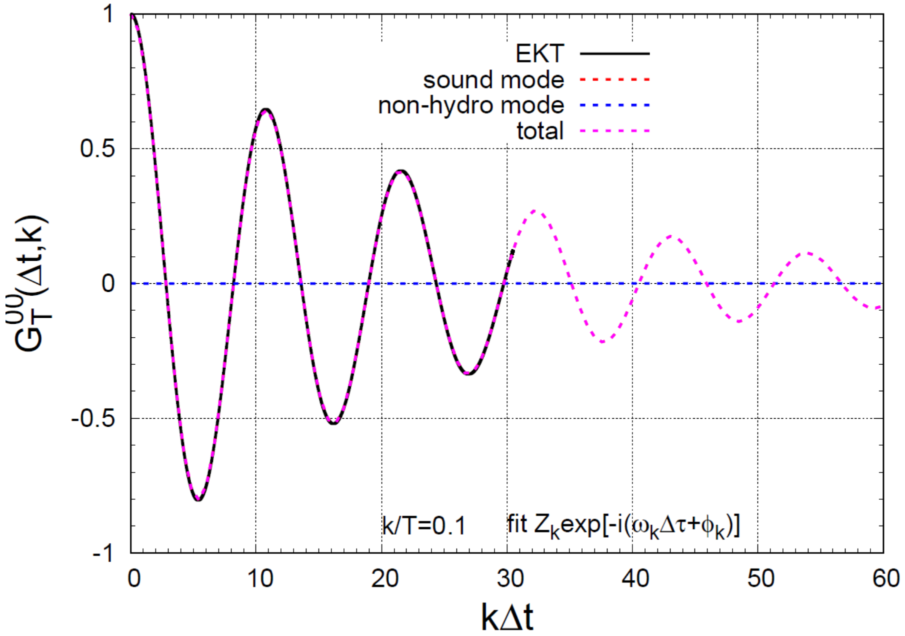
  - Introduce perturbations and linearize

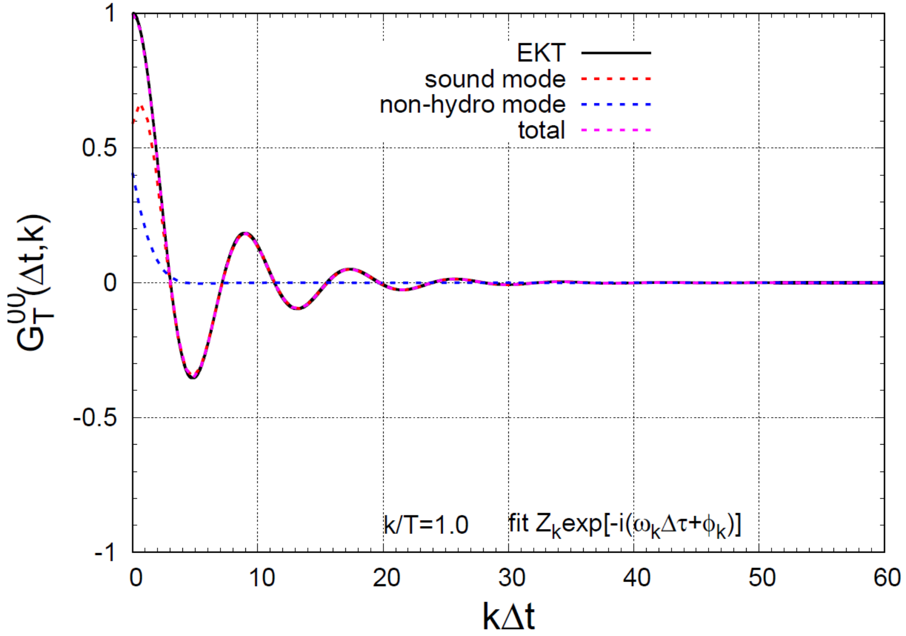
- One could numerically calculate real time Green's functions and Laplace transform (complex frequencies)
  - Numerical Laplace transform breaks down for multiple poles/non-analyticities

How can we find (non-)hydrodynamic modes?

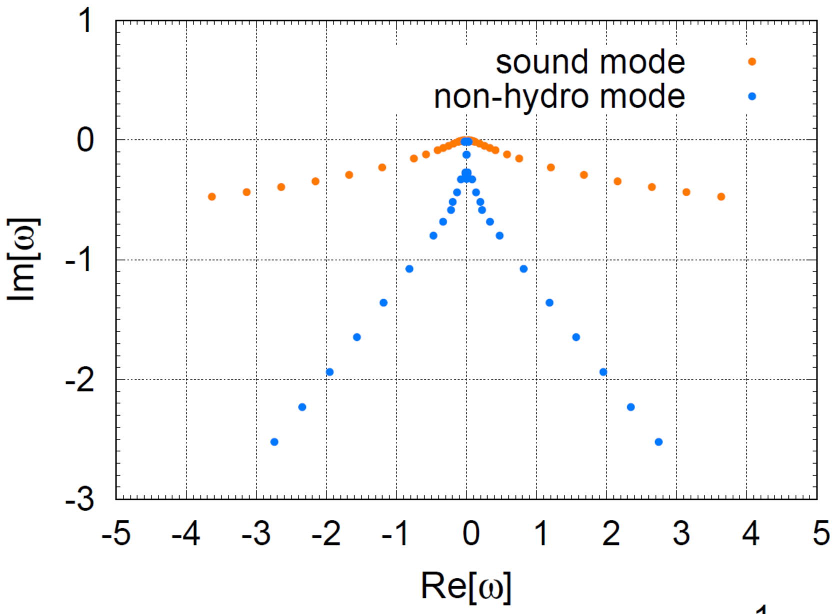
- Possibility: Calculate real time Green's functions and fit data to extract poles
  - Problems: Strong dependence on assumptions of fit function
  - Example: Yang-Mills

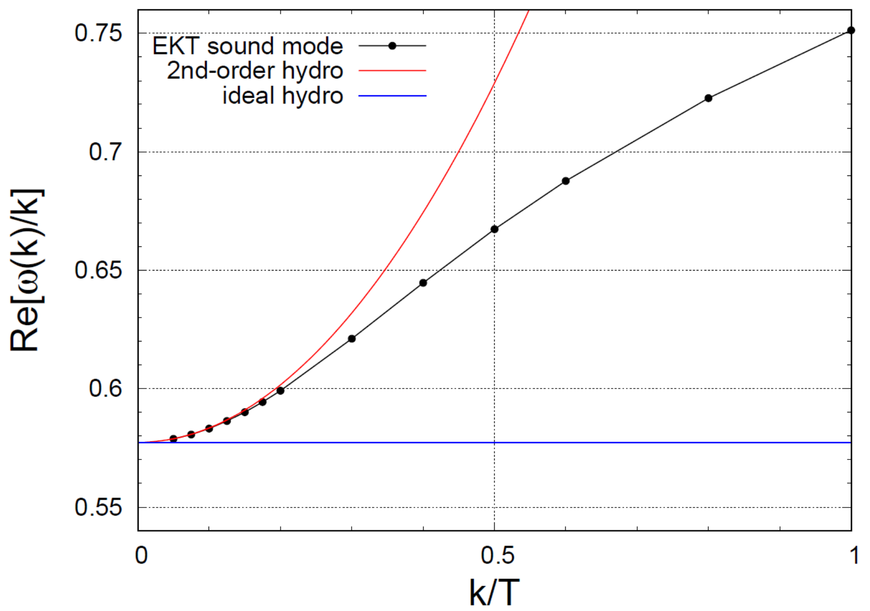
$$f(t) = \cos(\omega_1 t)e^{-w_2 t}$$
$$\Rightarrow \omega = \pm\omega_1 - i\omega_2$$





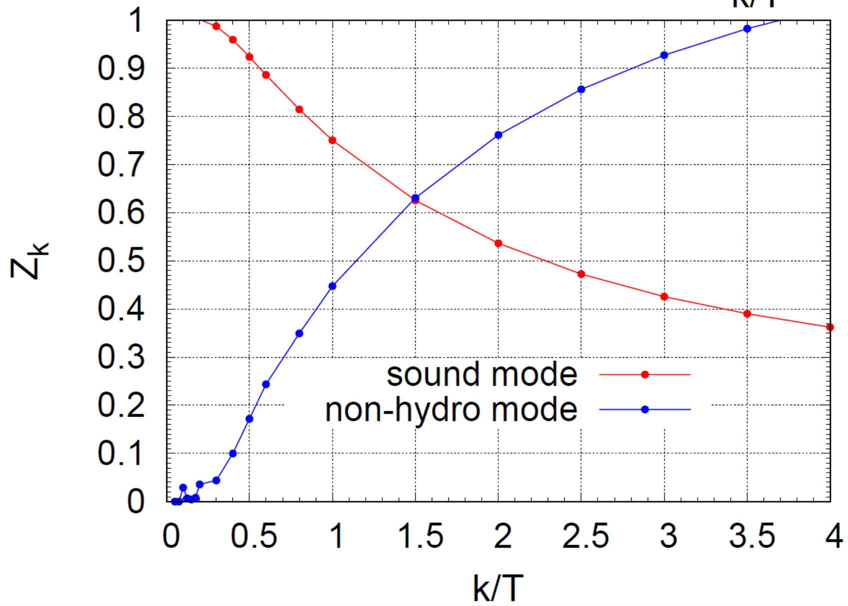
- Extract modes for multiple k and compare to hydro





- Non-Hydro mode dominates for larger k

- Behavior well described by two modes



How can we find (non-)hydrodynamic modes?

- Other approach: Expand Boltzmann equation in moments and calculate eigenvalues of operator

$$\partial_t f(t) = C f(t)$$
$$\Leftrightarrow f(t) = e^{Ct} f(0)$$

- Eigenvalues can be easily translated into frequencies

$$f(t) = \sum_i \langle \gamma_i | f(0) \rangle \gamma_i e^{c_i t}$$
$$\tilde{f}(\omega) = \sum_i \langle \gamma_i | f(0) \rangle \frac{-\gamma_i}{i\omega + c_i}$$

How can we find (non-)hydrodynamic modes?

- Moments defined by a discrete number of momenta p and angles θ

$$N_k(t) = \int \frac{d^3p}{(2\pi)^3} w_i(p) w_j(cos\theta) \delta f(p,t)$$

$$C_{ij} = \frac{\delta C_i}{\delta N_j}$$

$$w_i(x) = \begin{cases} \frac{x - x_{i-1}}{x_i - x_{i-1}} \,, & x_{i-1} < x < x_i \\ \frac{x_{i+1} - x}{x_{i+1} - x_i} \,, & x_i \leq x < x_{i+1} \\ 0 \,, & x > x_{i+1} \text{ or } x < x_{i-1} \end{cases}$$

- Collision kernel and k contribution become matrix in moment space

$$\partial_t f - ik\cos(\theta) f = C[f] \qquad \partial_t \vec{N}(t) = C\vec{N}(t)$$

- We can solve ODE numerically and obtain energy by moments

$$\delta e(t) = \sum_k p_k N_k(t)$$

How can we find (non-)hydrodynamic modes?

- We can use eigenvectors of matrix to reconstruct Green's functions also

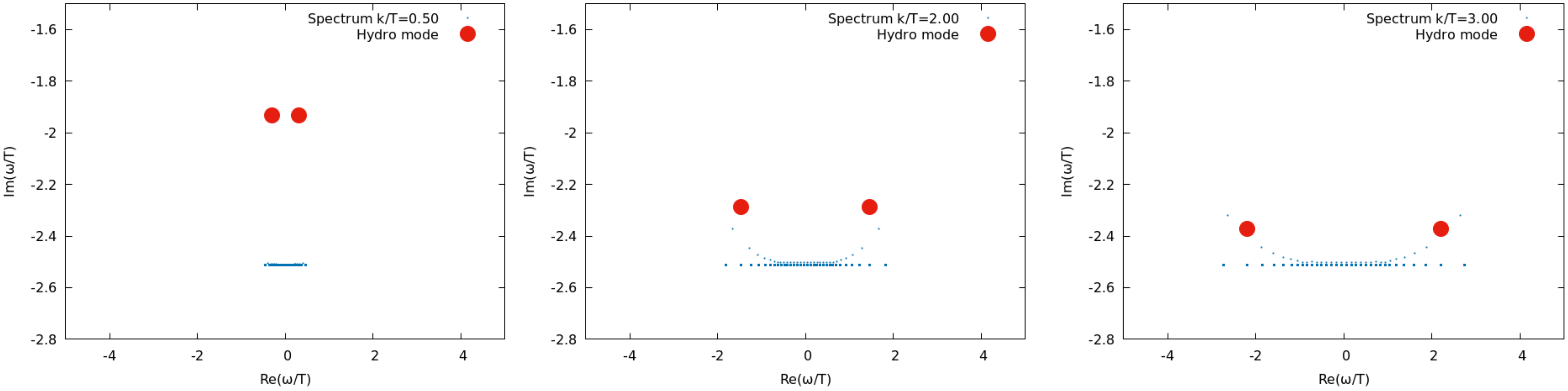
$$G(t) = \sum e^{\lambda_i t} \langle O|a_i\rangle \langle b_i|I\rangle \qquad G(\omega) = \sum -\frac{\langle O|a_i\rangle \langle b_i|I\rangle}{i\omega + \lambda_i}$$

- O is observable we want to get linear response of, e.g. energy

$$\langle O| = \begin{pmatrix} p_0 \\ p_1 \\ \vdots \\ p_{Np-1} \end{pmatrix}$$

How can we find (non-)hydrodynamic modes?

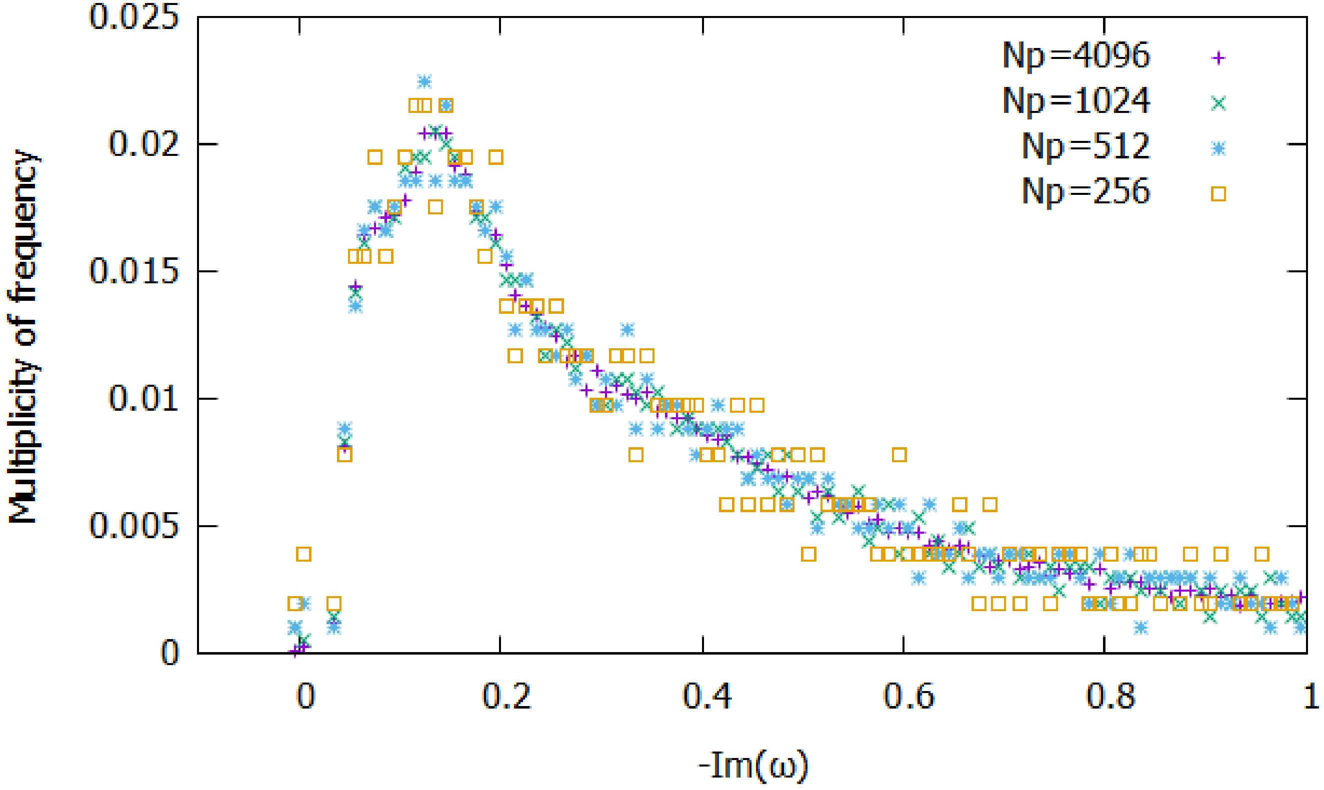
- Testing the method with RTA



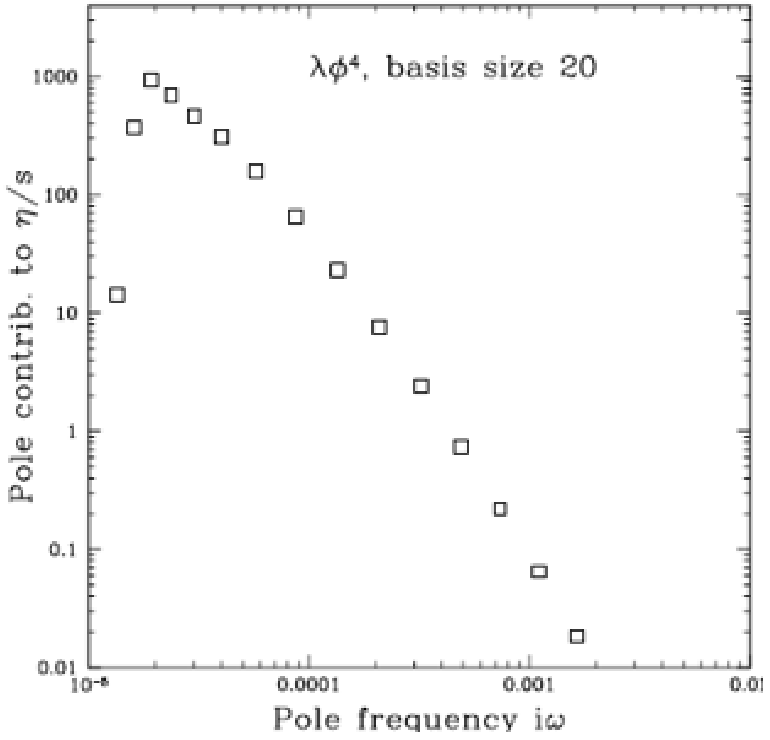
- Cut structure is visible, also two hydro poles wandering towards cut

How can we find (non-)hydrodynamic modes?

- Start analyzing scalar ϕ^4-theory



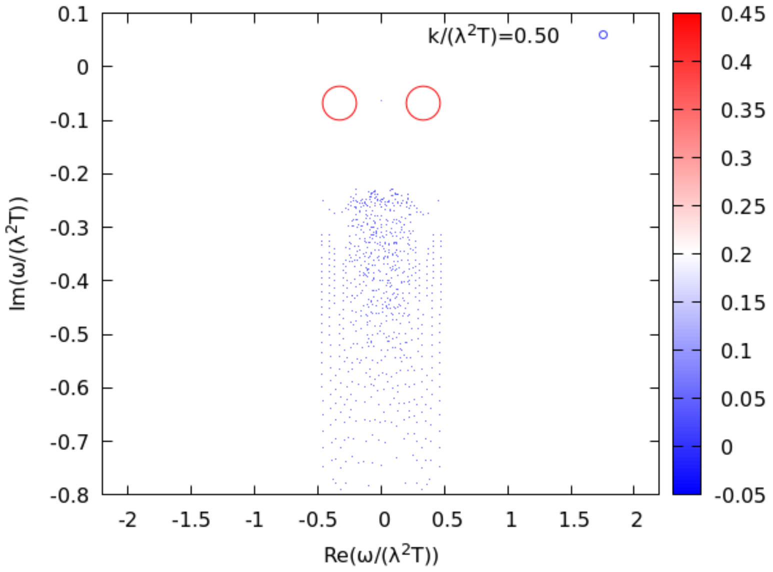
[Moore, JHEP(2018)]

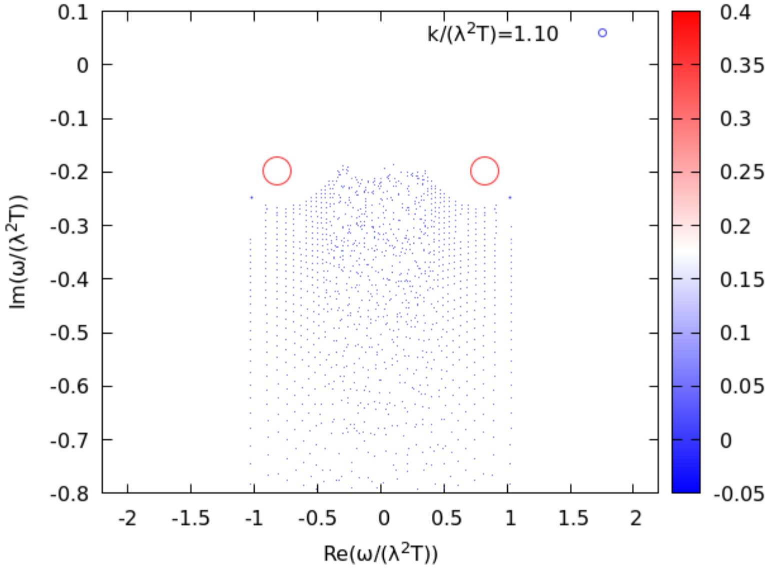


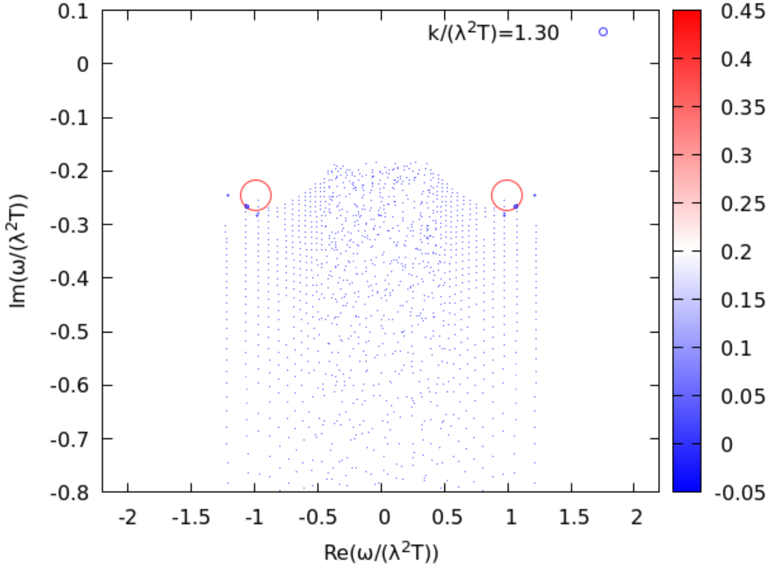
- k=0, increasing Np shows signs of continuous spectrum

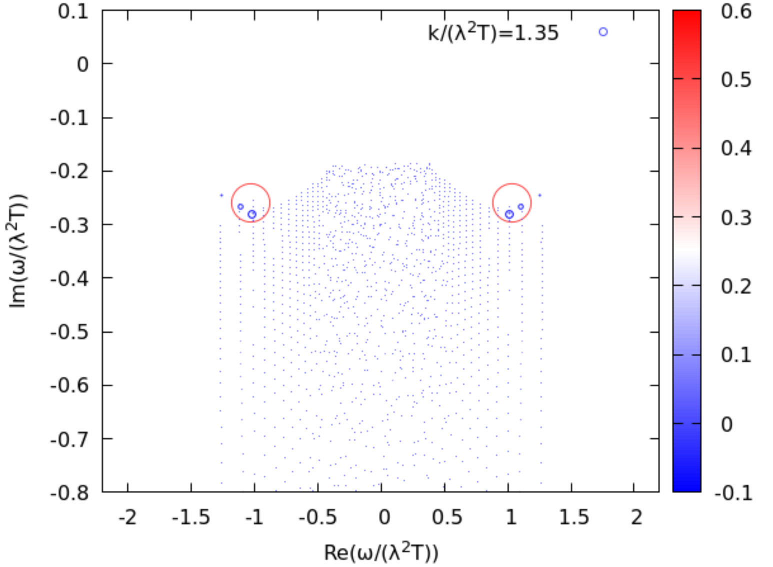
How can we find (non-)hydrodynamic modes?

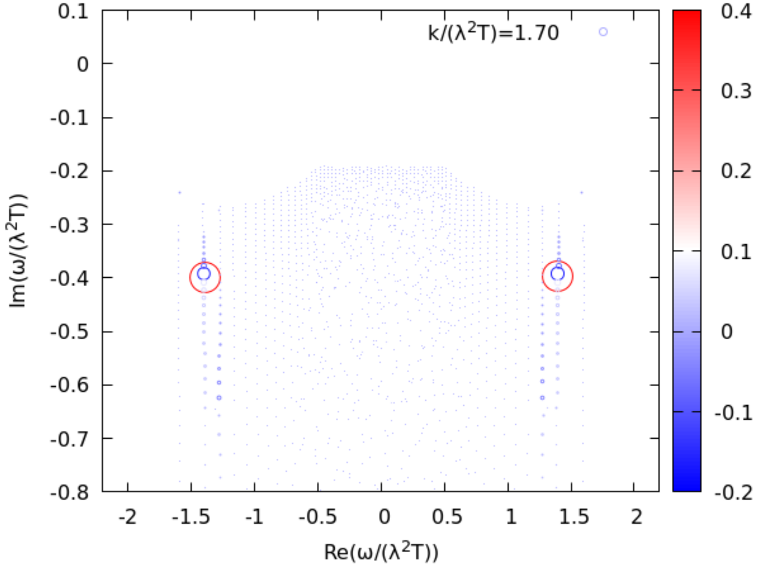
- Eigenvalues for k $\neq$ 0
- First evidence for more complicated analytic structure



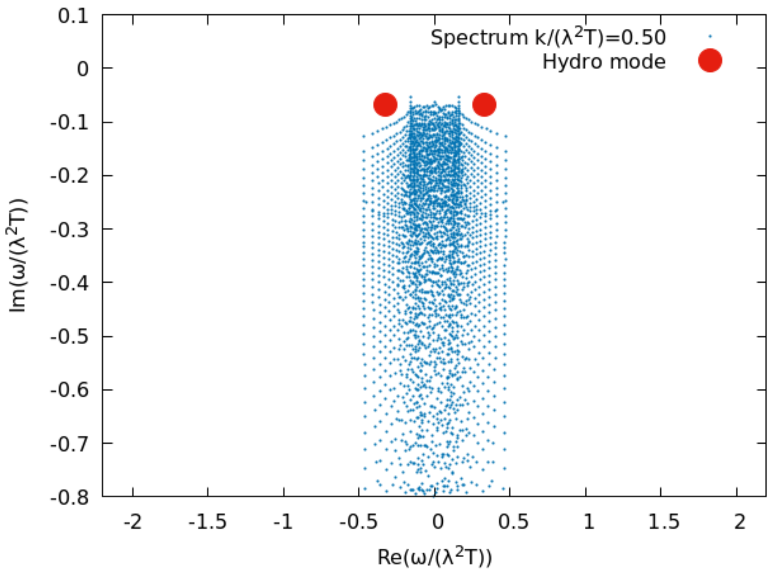


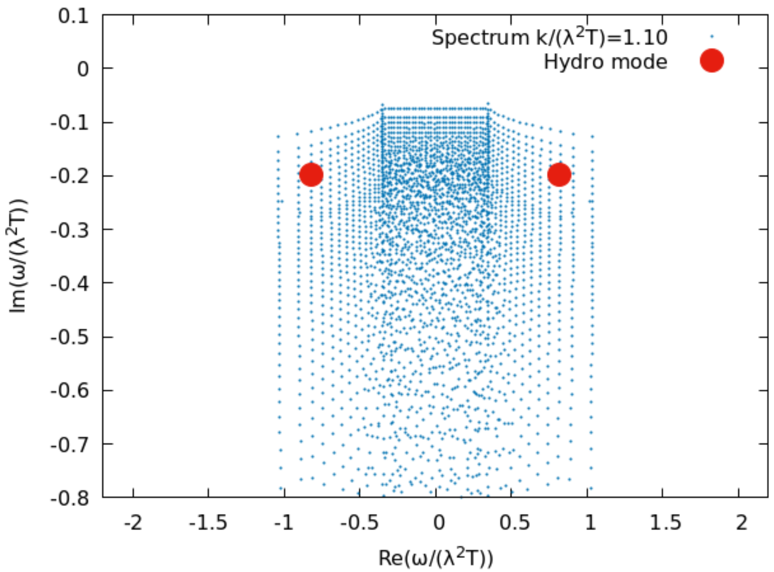


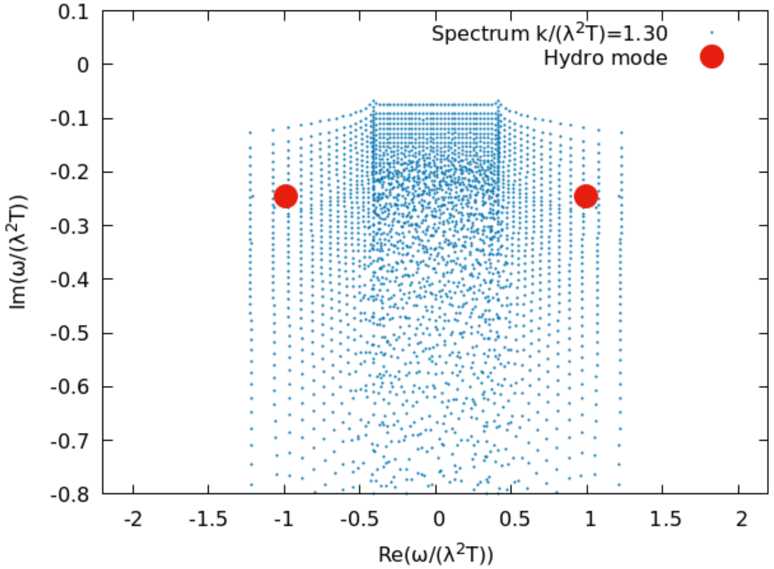


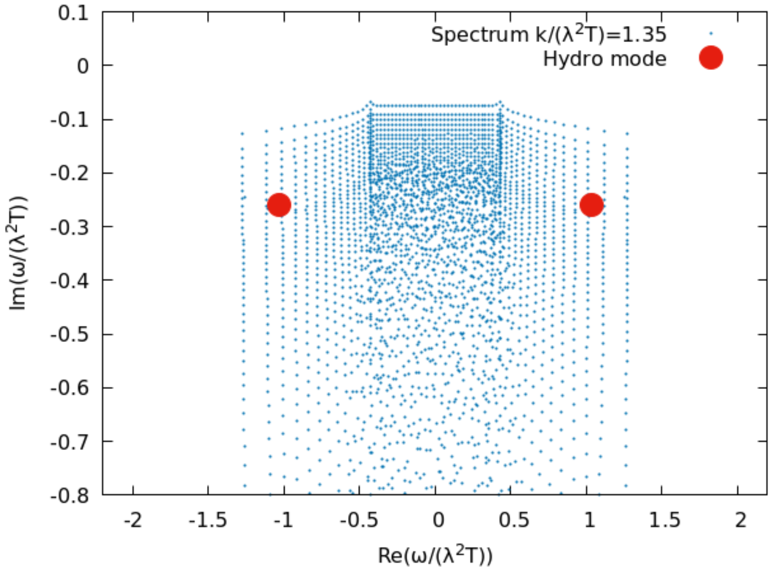


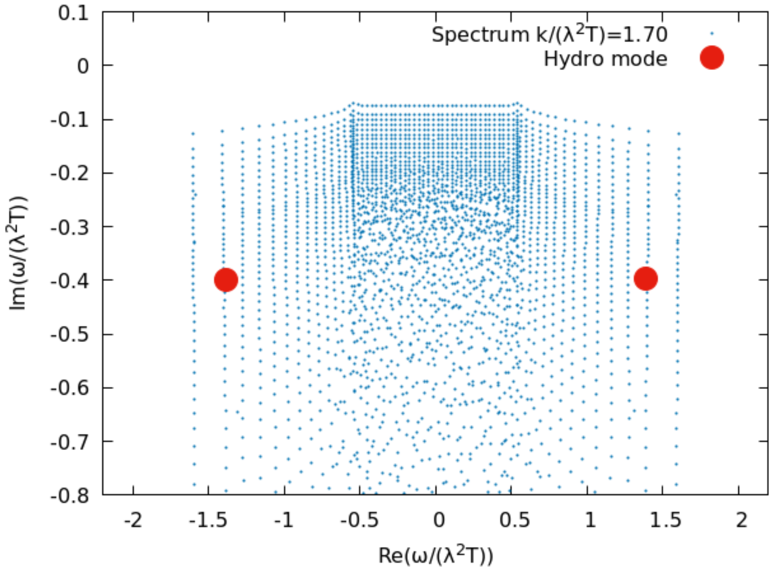
- Spectrum without residue scaling



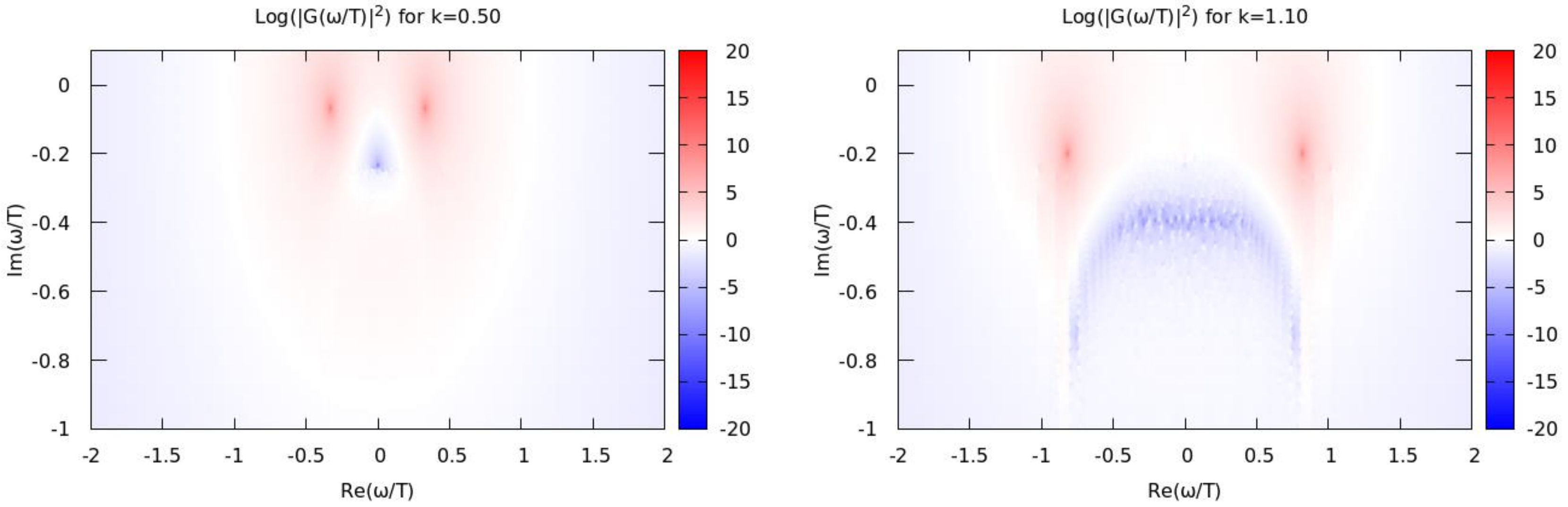


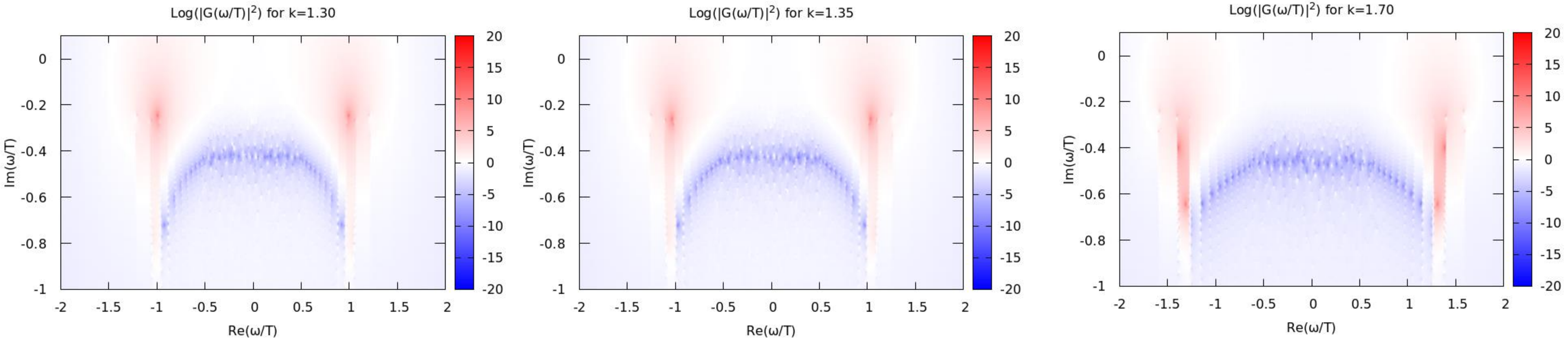






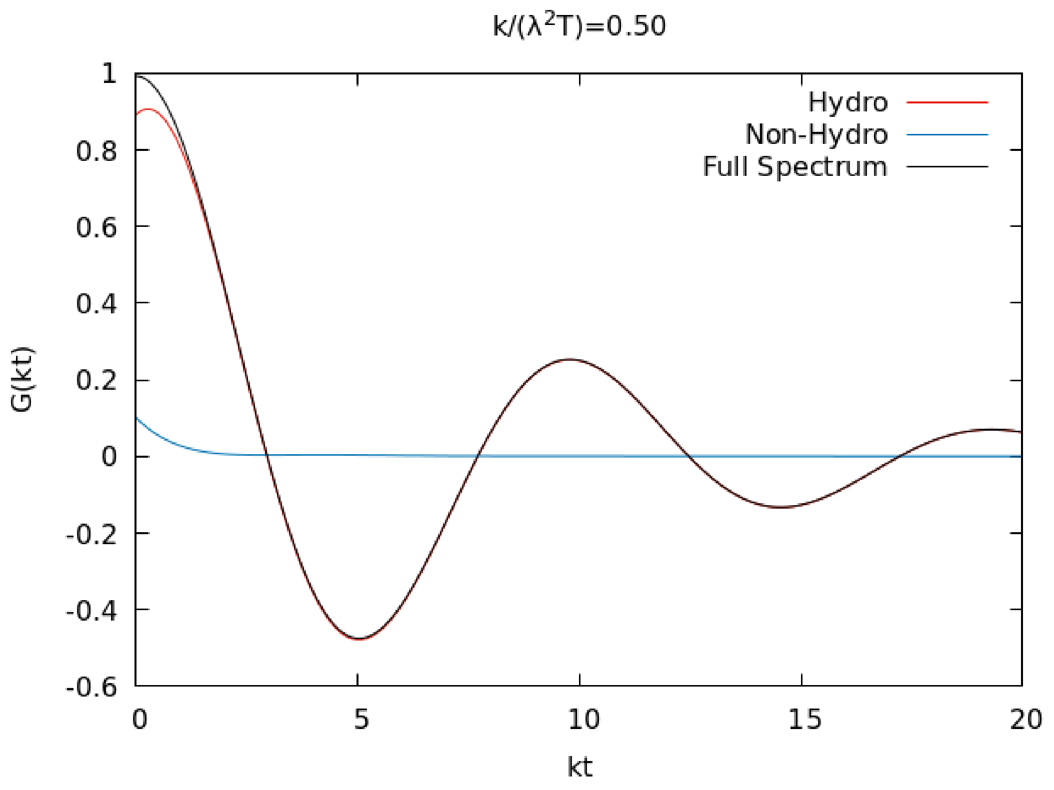
- Spectrum as 2D map

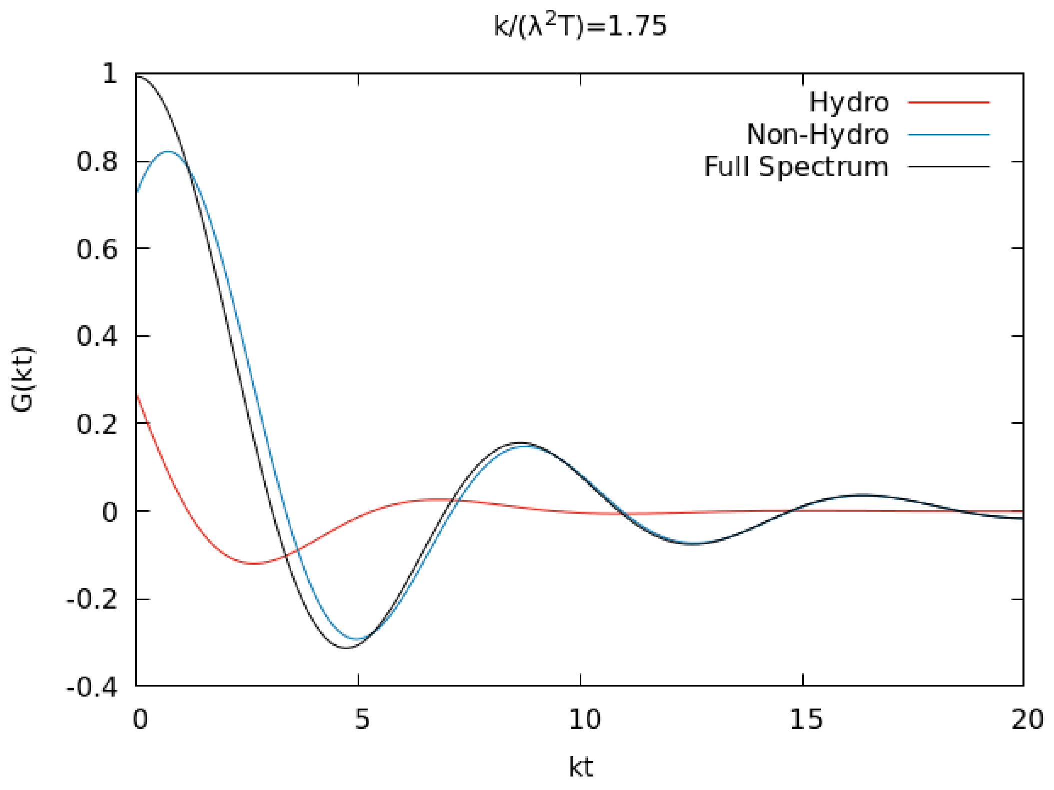




How can we find (non-)hydrodynamic modes?

- We can split Green's functions into hydro and non-hydro parts

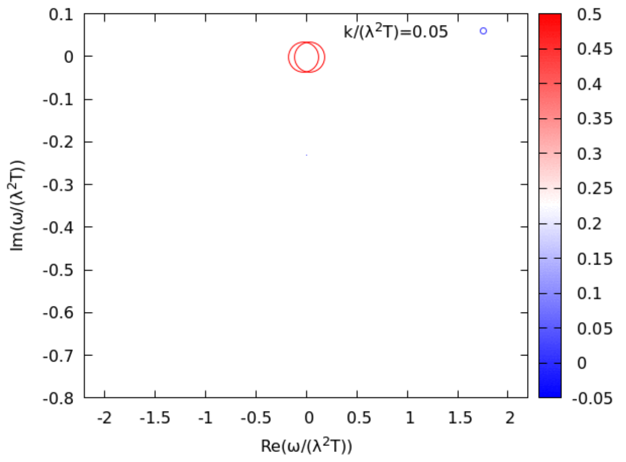




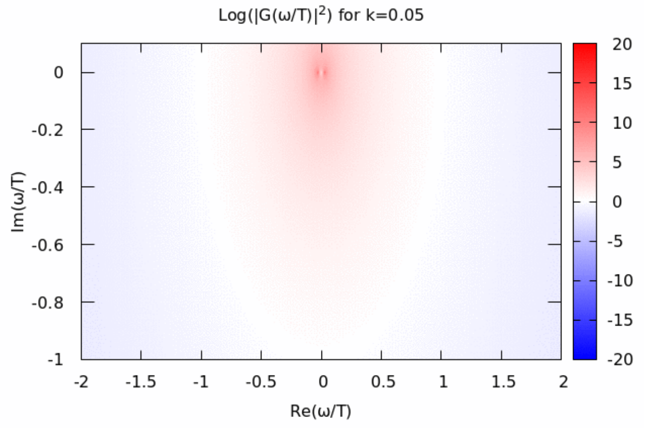
- Small k well described by hydro, large k well described by non-hydro

Conclusion and Outlook

- Non-Hydro modes help understand the hydro regime
- Analytical structure of QCD Green's function expected to have more complicated structure than poles and cuts



- Scalar theory shows sign of continuous spectrum that is more than poles and cuts



- Extract dominant modes and observe approach to hydro
- Extend formalism to QCD kinetic theory